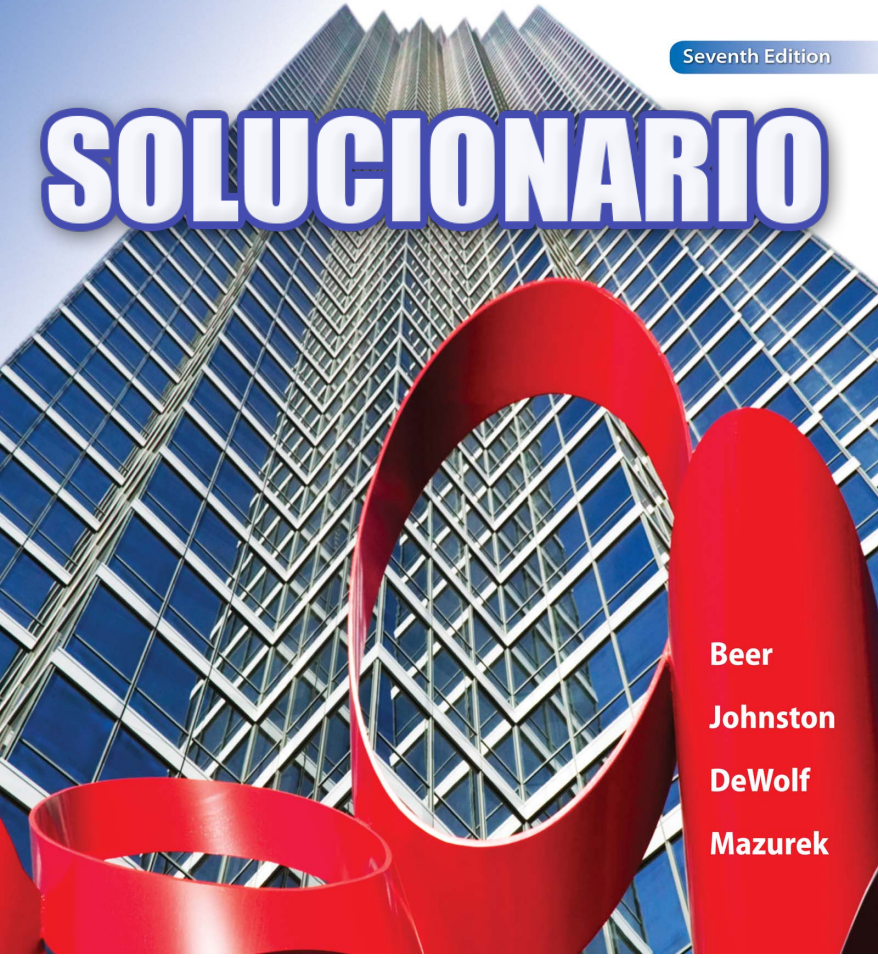
MECHANICS of MATERIALS

Seventh Edition

SOLUCIONARIO



Beer
Johnston
DeWolf
Mazurek

Instructors and Solutions Manual

to accompany

Mechanics of Materials

Seventh Edition

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TO THE INSTRUCTOR

As indicated in its preface, *Mechanics of Materials* is designed for the first course in mechanics of materials—or strength of materials—offered to engineering students in the sophomore or junior year. However, because of the large number of optional sections that have been included and the maturity of approach that has been achieved, this text can also be used to teach a more advanced course.

The text has been divided into units, each corresponding to a well-defined topic and consisting of one or several theory sections followed by sample problems and a large number of problems to be assigned. In order to accommodate courses of varying emphases, considerably more material has been ‘included than can be covered effectively in a single three-credit-hour course. To assist the instructors in making up a schedule of assignments that best fits their classes, the various topics presented in the text have been listed in Table I and both a minimum and a maximum number of periods to be spent on each topic have been suggested. Topics have been divided into three categories: core topics that will probably be covered in every course; additional topics that can be selected to complement this core to form courses of various emphases; and finally topics that can be used with more advanced students.

The problems have been grouped according to the portions of material they illustrate and have been arranged in order of increasing difficulty, with problems requiring special attention indicated by asterisks. The instructor’s attention is called to the fact that problems have been arranged in groups of six or more, all problems of the same group being closely related. This means that the instructor will easily find additional problems to amplify a particular point that has been brought up in the discussion of a problem assigned for homework. Since half of the problems are stated in SI units and half in U.S. customary units, it also means that the instructor has the choice of assigning problems using SI units and problems using U.S. customary units in whatever proportion is found to be most desirable for a given class. To assist in the preparation of homework assignments Table II provides a brief description of all groups of problems and a classification of the problems in each group according to the units used. It should also be noted that answers to all problems with a number set in roman type are given at the end of the text, while problems with a number set in italic are not.

In Table III six alternative lists of possible assignments have been suggested. Four of these lists consist of problems whose answers are given at the end of the text, and two of problems whose answers are not. Half of the problems in each list are stated in SI units and half in U.S. customary units. For those instructors who wish to emphasize the use of SI units, four additional lists of problems have been given in Table IV, in which 75% of the problems use SI units. Since the lists in Tables III and IV cover the entire text, instructors will want to select those groups of sections that are best suited to the course they wish to teach. For a typical one-semester course consisting of 42 class meetings and including four quizzes, as many as 38 of the 46 available groups can be selected.

Since the approach used in this text differs in a number of respects from the approach used in other books, the instructor is advised to read the preface to *Mechanics of Materials*, in which the authors have outlined their general philosophy. Attention is particularly called to the fact that

statically indeterminate problems are first discussed in Chapter 2 and are considered throughout the text for the various loading conditions encountered. Thus, students are presented at an early stage with a method of solution that combines the analysis of deformations with the conventional analysis of forces used in statics, and will have become thoroughly familiar with it by the end of the course. The concept of plastic deformation is also introduced in Chap. 2, where it is applied to the analysis of members under axial loading, while problems involving the plastic deformation of circular shafts and of prismatic beams are considered in optional sections of Chaps. 3 and 4, respectively. On the other hand, while the concept of stress at a point is introduced in Chap. 1, the discussion of the transformation of stresses is delayed until Chap. 7, after students have discovered for themselves the need for special techniques, such as Mohr's circle. In this edition, shear and bending-moment diagrams are introduced at the beginning of Chap. 5 and applied immediately to the design of beams in that chapter and in the next.

A brief description, chapter by chapter, of the topics included in the text will be found in the following pages. It is hoped that this material will help instructors organize their courses to best fit the needs of their students.

The authors of *Mechanics of Materials, 7/e*, wish to thank Professor Dean P. Updike of the Department of Mechanical Engineering and Mechanics at Lehigh University and Amy Mazurek for having written the problem solutions contained in this Manual.

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David F. Mazurek

DESCRIPTION OF THE MATERIAL CONTAINED IN
***MECHANICS OF MATERIALS,* Seventh Edition**

Chapter 1
Introduction–Concept of Stress

The main purpose of this chapter is to introduce the concept of stress. After a short review of Statics in Section 1.1 emphasizing the use of free-body diagrams, Sections 1.2 through 1.2 discuss *normal stresses* under an axial loading, *shearing stresses*—with applications to pins and bolts in single and double shear—and *bearing stresses*. This section also introduces the student to the concepts of analysis and design. Section 1.2A emphasizes the fact that stresses are inherently statically indeterminate and that, at this point, normal stresses under an axial loading can only be assumed to be uniformly distributed. Moreover, such an assumption requires that the axial loading be centric.

Section 1.2D is devoted to the application of these concepts to the analysis of a *simple structure*. Section 1.2E describes how students should approach the solution of a problem in mechanics of materials using the SMART methodology: Strategy, Modeling, Analysis and Reflect & Think. Section 1.2E also discusses the *numerical accuracy* to be expected in such a solution. Problems included in the first lesson also serve as a review of the methods of analysis of trusses, frames, and mechanisms learned in statics.

Section 1.3 discusses the determination of normal and shearing stresses *on oblique planes* under an axial loading, while Section 1.4 introduces the components of stress under general loading conditions. This section emphasizes the fact that the components of the shearing stresses exerted on perpendicular planes, such as τ_{xy} and τ_{yx}, must be equal. It also introduces the students to the concept of transformation of stress. However, the study of the computational techniques associated with the transformation of stress at a point is delayed until Chapter 7, after students have discovered for themselves the need for such techniques.

Section 1.5 is devoted to design considerations. It introduces the concepts of *ultimate load*, *ultimate stress*, and *factor of safety*. It also discusses the reasons for the use of factors of safety in engineering practice. The section ends with an optional presentation of an alternative method of design, *Load and Resistance Factor Design.*

Chapter 2
Stress and Strain–Axial Loading

This chapter is devoted to the analysis and design of members under a centric axial loading. Section 2.1A introduces the concept of *normal strain*, while Section 2.1B describes the general properties of the stress-strain diagrams of ductile and brittle materials and defines the *yield strength*, *ultimate strength*, and *breaking strength* of a material. Section 21C, which is optional, defines *true stress* and *true strain*. Section 2.1D introduces *Hooke's law*, the *modulus of elasticity*, and the *proportional limit* of a material. It defines as *isotropic* those materials whose mechanical properties are independent of the direction considered and as *anisotropic* those

whose mechanical properties depend upon that direction. Among the latter are *fiber-reinforced composite materials*, that are described in this section.

Section 2.1E discusses the elastic and the plastic behavior of a material and defines its *elastic limit*, while Section 2.1F is devoted to fatigue and the behavior of materials under repeated loadings. The first lesson of Chapter 2 ends with Section 2.1G, which shows how Hooke's law can be used to determine the deformation of a rod of uniform or variable cross section under one or several loads, and introduces the concept of *relative displacement*. Section 2.2 discusses *statically indeterminate problems* involving members under an axial load. As indicated in the preface of the text and in the introduction to this manual, the authors believe it is important to introduce the students at an early stage to the concept of statical indeterminacy and to show them how the analysis of deformations can be used in the solution of problems that cannot be solved by the methods of statics alone. It will also help them realize that stresses, being statically indeterminate, can be computed only by considering the corresponding distribution of strains. Section 2.3 discusses the *thermal expansion* of rods and shows how to determine stresses in statically indeterminate members subjected to temperature changes.

Section 2.4 introduces the concept of *lateral strain* for an isotropic material and defines *Poisson's ratio*. Section 2.5 discusses the *multiaxial loading* of a structural element and derives the generalized Hooke's law for such a loading. Since this derivation is based on the application of the *principle of superposition*, this principle is also introduced in Section 2.5, and the conditions under which it can be used are clearly stated. Section 2.6 is optional. It discusses the change in volume of a material under a multiaxial loading and defines the *dilatation* and the *bulk modulus* or *modulus of compression* of a given material.

Section 2.7 introduces the concept of *shearing strain*. It should be noted that the authors define the shearing strain as the change in the angle formed by the faces of the element of material considered, and not as the angle through which one of these faces rotates. Hooke's law for shearing stress and strain and the *modulus of rigidity* are also introduced in this section, as well as the *generalized Hooke's law* for a homogeneous, isotropic material under the most general stress conditions. Section 2.8 points out that strains, just as stresses, depend upon the orientation of the planes considered. It also establishes the fact that the constants E, ν, and G are not independent from each other and derives Eq. (2.35), that expresses the relation among these three constants. Section 2.9, which is optional, extends the stress-strain relationships to *fiber-reinforced composite materials.* The relations obtained are expressed by Eqs. (2.37) and (2.39) and involve three different values of the modulus of elasticity and six different values of Poisson's ratio.

Section 2.10 discusses the *distribution of the normal stresses* under a centric axial loading and shows that this distribution depends upon the manner in which the loads are applied. However, except in the immediate vicinity of the points of application of the loads, the distribution of stresses can be assumed uniform. This result verifies Saint-Venant's principle. Section 2.11 discusses *stress concentrations* near circular holes and fillets in flat bars under axial loading.

Section 2.12 is devoted to the *plastic deformation* of members under centric axial loads and introduces the concept of an *elastoplastic material*. As stated in the preface of the text, the authors believe that students should be exposed to the concept of plastic deformation in the first

course in mechanics of materials, if only to let them realize the limitations of the assumption of a linear stress-strain relation in engineering applications. By introducing this concept early in the course in connection with axial loading, rather than later with torsion or bending, one makes it easier for the students to understand and accept it. For the same reason, *residual stresses* are discussed in Section 2.13 in connection with axial loading. However, since some instructors may not want to include the concept of residual stresses in an elementary course, this section is optional and can be omitted without any prejudice to the understanding of the rest of the text.

Chapter 3
Torsion

The Introduction introduces this type of loading, while Section 3.1 establishes the relation that must be satisfied, on the basis of statics, by the shearing stresses in a given section of a shaft subjected to a torque. This condition, however, does not suffice to determine the stresses, and one must analyze the deformations that occur in the shaft. This is done in Section 3.1A, where it is proved that the distribution of shearing strains in a circular shaft is linear. It should be noted that the discussion presented in See. 3.1B is based solely on the assumption of rigid end plates, rather than on arbitrary and gratuitous assumptions regarding the deformations of a shaft. The results obtained in this and the following sections clearly depend upon the validity of this assumption, but can be extended to other loading conditions through the application of Saint-Venant's principle.

Section 3.1C is devoted to the analysis of the shearing stresses in the elastic range and presents the derivation of the *elastic torsion formulas* for circular shafts. The section ends with remarks on the transformation of stresses in torsion and the comparison between the failures of ductile and brittle materials in torsion.

The formula for the *angle of twist* of a shaft in the elastic range is derived in Section 3.2. This section also contains various applications involving the twisting of single and gear-connected shafts. Section 3.3 deals with the solution of problems involving *statically indeterminate shafts.*

Section 3.4 is devoted to the *design of transmission shafts* and begins with the determination of the torque required to transmit a given power at a given speed, both in SI and U.S. customary units. Note that the effect of bending on the design of transmission shafts will be discussed in Section 8.2, which is optional. Section 3.5 discusses *stress concentrations* at fillets in circular shafts.

Sections 3.6 through 3.7 deal with the *plastic deformations* and *residual stresses* in circular shafts and are optional. Since a similar presentation of the plastic deformations and residual stresses of members in pure bending is given in Chapter 4, the instructor may decide to include only one of these presentations in the course. Section 3.6 describes the general method for the determination of the torque corresponding to a given maximum shearing stress in a shaft made of a material with a *nonlinear stress-strain diagram*, while Sections 3.7 and 3.8 deal, respectively, with the deformations and the residual stresses in shafts made of an *elastoplastic material*. Sections 3.9 and 3.10 are also optional. They are devoted, respectively, to the torsion of solid members and thin-walled hollow shafts of *noncircular section.*

Chapter 4
Pure Bending

The Introduction defines this type of loading and shows how the results obtained in the following sections can be applied to the analysis of other types of loadings as well, namely, eccentric axial loadings and transverse loadings. Sections 4.1 and 4.1A establish the relation that must be satisfied, on the basis of statics, by the *normal stresses* in a given section of a member subjected to pure bending. This condition, however, does not suffice to determine the stresses, and one must analyze the *deformations* that occur in the member. This is done in Section 4.1B, where it is proved that the distribution of normal stresses in a symmetric member in pure bending is linear. It should be noted that no assumption is made in this discussion regarding the deformations of the member, except that the couples should be applied in such a way that the ends of the member remain plane. Whether this can actually be accomplished is discussed at the end of Section 4.3.

Section 4.2 is devoted to the analysis of the normal stresses in the elastic range and presents the derivation of the *elastic flexure formulas*. It also defines the elastic section modulus and ends with the derivation of the formula for the *curvature* of an elastic beam. Section 4.3 discusses the *anticlastic curvature* of members in pure bending and also states the loading conditions required for the ends of the member to remain plane.

Section 4.4 discusses the determination of stresses in members made of several materials and defines the *transformed section* of such members. It also shows how the transformed section can be used to determine the radius of curvature of the member. The section ends with a discussion of the stresses in *reinforced-concrete beams*. Section 4.5 deals with the stress concentrations at fillets and grooves in flat bars under pure bending.

Section 4.6 is optional. This section discusses the plastic deformations and residual stresses in members subjected to pure bending in much the same way that these were discussed in Sections 3.6 through 3.8 in the case of members in torsion. Section 4.6 describes the general method for the determination of the bending moment corresponding to a given maximum normal stress in a member possessing two planes of symmetry and made of a material with a nonlinear stress-strain diagram. Section 4.6A deals with members made of an *elastoplastic material* and derives formulas relating the thickness of the elastic core and the radius of curvature with the applied bending moment in the case of members with a rectangular cross section. It also defines the *shape factor* and the *plastic section modulus* of members with a nonrectangular section. Section 4.6B deals with the determination of the plastic moment of members made of an elastoplastic material and possessing a single plane of symmetry, while Section 4.6C discusses *residual stresses*.

Section 4.7 shows how the stresses due to a *two-dimensional eccentric axial loading* can be obtained by replacing the given eccentric load by a centric load and a couple, and superposing the corresponding stresses. Attention is called to the fact that the neutral axis *does not pass* through the centroid of the section.

Section 4.8 deals with the *unsymmetric bending* of elastic members. It is first shown that the neutral axis of a cross section will coincide with the axis of the bending couple if, and only if, the

axis of the couple is directed along one of the principal centroidal axes of the cross section. It is then shown that stresses due to unsymmetric bending can always be determined by resolving the given bending couple into two component couples directed along the principal axes of the section and superposing the corresponding stresses. Sample Problem 4.10 has been included in this section to provide an opportunity for students to use material on Mohr's Circle to determine stresses in non-symmetrical sections. The material is based on what students will have learned in a study of Statics where the transformation of moments of inertia is covered for problems where it is necessary to look at the rotation of coordinate axes. Problems 4.141 through 4.143 are also asterisked and are best solved with the use of Mohr's Circle.

This method of analysis is extended in Section 4.9 to the determination of the stresses due to an *eccentric axial loading in three-dimensional space*. The eccentric load is replaced by an equivalent system consisting of a centric load and two bending couples, and the corresponding stresses are superposed.

Section 4.10 is optional; it deals with the bending of *curved members*.

Chapter 5
Analysis and Design of Beams
for Bending

In the Introduction beams are defined as slender prismatic members subjected to transverse loads and are classified according to the way in which they are supported. It is shown that the internal forces in any given cross section are equivalent to a shear force **V** and a bending couple **M.** The bending couple **M** creates *normal stresses* in the section, while the shear force **V** creates *shearing stresses.* The former is determined in this chapter, using the flexure formula (5.1), while the latter will be discussed in Chapter 6.

Since the dominant criterion in the design of beams for strength is usually the bending stresses in the beam, the determination of the maximum value of the bending moment in the beam is the most important factor to be considered. To facilitate the determination of the bending moment in any given section of the beam, the concept of *shear and bending-moment diagrams* will be introduced in Section 5.1, using free-body diagrams of various portions of the beam.

An alternative method for the determination of shear and bending-moment diagrams, based on relations among load, shear, and bending moment, is presented in Section 5.2. To maintain the interest of the students, most of the problems to be assigned are focused on the engineering applications of these methods and call for the determination, not only of the shear and bending moment, but also of the *normal stresses in the beam.*

Section 5.3 is devoted to the *design of prismatic beams* based on the allowable normal stress for the material used. Sample Problems and problems to be assigned include wooden beams of rectangular cross section, as well as rolled-steel W and S beams. An optional paragraph on page 372 describes the application of *Load and Resistance Factor Design* to beams under transverse loading.

Section 5.4 introduces the concept of *singularity functions* and shows how these functions can provide an alternative and effective method for the determination of the shear and bending

moment at any point of a beam under the most general loading condition. While this section is optional, it should be included in the lesson schedule if singularity functions are to be used later for the determination of the slope and deflection of a beam (Section 9.3). It is pointed out on page 387 that singularity functions are particularly well suited to the *use of computers*, and several optional problems requiring the use of a computer (Probs. 5.118 through 5.125) have been included in this assignment.

Section 5.5, which is optional, is devoted to *nonprismatic beams*, such as forged or cast beams designed to be of constant strength, and rolled-steel beams reinforced with cover plates.

Chapter 6
Shearing Stresses in Beams and Thin-Walled Members

It is shown in the Introduction that a transverse load creates *shearing stresses* as well as normal stresses in a beam. Considering first the horizontal face of a beam element, it is shown in Sections 61A that the horizontal shear per unit length *q*, or *shear flow*, is equal to *VQ/I*. This result is applied in Concept Application 6.1 to the determination of the shear force in the nails connecting three planks forming a wooden beam, as well as in Probs. 6.1 through 6.4. Probs. 6.5 through 6.8 apply the same concepts to steel beams made of sections bolted together

In Section 6.1B the *average shearing stress* τ_{ave} exerted on the horizontal face of the beam element is obtained by dividing the shear flow *q* by the width *t* of the beam:

$$\tau_{\text{ave}} = \frac{VQ}{It} \tag{6.6}$$

Note that since the shearing stresses τ_{xy} and τ_{yx} exerted at a given point are equal, the expression obtained also represents the average shearing stress exerted at a given height on a vertical section of the beam. This formula is used to determine shearing stresses in a beam made of glued planks in Sample Prob. 6.1 and to design a timber beam in Sample Prob. 6.2. Problems 6.9 through 6.12 and 6.21 and 6.22 call for the determination of shearing stresses in various types of beams.

Section 6.1C explores shearing stresses in common beam types In Concept Applications 6.2 and 6.3 the designs obtained on the basis of normal stresses, respectively, for a timber beam in Sample Prob. 5.7 and for a rolled-steel beam in Sample Prob. 5.8 are checked and found to be acceptable from the point of view of shearing stresses.

Section 6.2 is optional and discusses the distribution of stresses in a narrow rectangular beam.

In Section 6.3 the expression $q = VQ/I$ obtained on Section 6.1A for the shear flow on the horizontal face of a beam element is shown to remain valid for the curved surface of a beam element of arbitrary shape. It is then applied in Concept Application 6.4 and in Probs. 6.29 through 6.33 to the determination of the shearing forces and shearing stresses in nailed and glued vertical surfaces.

Section 6.4 deals with the determination of shearing stresses in *thin-walled members* and shows that Eq. (6.6) can be applied to the determination of the average shearing stress in a section of arbitrary orientation.

Section 6.5, which is optional, describes the formation of *plastic zones* in beams subjected to transverse loads.

Section 6.6, which is also optional, deals with the *unsymmetric loading* of thin-walled members, the determination of the *shear center*, and the computation of the shearing stresses caused by a shearing force exerted at the shear center.

Chapter 7
Transformations of Stress and Strain

After a short introduction, formulas for the *transformation of plane stress* under a rotation of axes are derived in Section 7.1A, while the principal planes of stress, principal stresses, and maximum shearing stress are determined in Section 7.1B.

Section 7.2 is devoted to the use of *Mohr's circle*. It should be noted that the convention used in the text provides for a rotation on Mohr's circle in the *same sense* as the corresponding rotation of the element; in other words, this convention is the same as that used in statics for the transformation of moments and products of inertia. Attention is called to the statement at the bottom of page 493 of the text and the accompanying Fig. 7.15.

Section 7.3 discusses the general (*three-dimensional*) state of stress and establishes the fact that three principal axes of stress and three principal stresses exist. Section 7.4 shows how three different Mohr's circles can be used to represent the transformations of stress associated with rotations of the element about the principal axes. The results obtained are used to show that in a state of plane stress, the maximum shearing stress does not necessarily occur in the plane of stress.

Section 7.5 is optional. Section 7.5A presents the two criteria most commonly used to predict *whether a ductile material will yield* under a given state of plane stress, while Section 7.5B discusses the two criteria used to predict the *fracture of brittle materials.*

Section 7.6 deals with stresses in *thin-walled pressure vessels*; it is limited to the analysis of cylindrical and spherical pressure vessels.

The second part of the chapter (Sections 7.7 through 7.9) deals with *transformations of strain* and is optional. Section 7.7A presents the derivation of the formulas for the transformation of strain under a rotation of axes. It should be noted that this derivation is based on the consideration of an oblique triangle (Fig. 7.51) and the use of the law of cosines, and that the determination of the shearing strain is facilitated by the use of Eq. (7.43), which relates it to the normal strain along the coordinate axes and their bisector.

Section 7.7B introduces *Mohr's circle for plane strain*, and Section 7.8 discusses the three-dimensional analysis of strain and its application to the determination of the maximum shearing strain in states of plane strain and of plane stress. Section 7.9 deals with the use of strain rosettes for the determination of states of plane strain.

Chapter 8
Principal Stresses under a Given Loading

This chapter is devoted to the determination of the *principal stresses* and *maximum shearing stress* in beams, transmission shafts subjected to transverse loads as well as to torques, and bodies of arbitrary shape under combined loadings.

In the Introduction it is shown that, while only normal stresses occur on a square element with horizontal and vertical faces located at the surface of a beam, shearing stresses will occur if the element is rotated through 45^o (Fig. 8.1). The reverse situation is observed for an element with horizontal and vertical faces subjected only to shearing stresses (Fig. 8.2). The analysis of beams, therefore, should include the determination of the principal stresses and maximum shearing stress at various points.

This is done in Section 8.1 for cantilever beams of various rectangular sections subjected to a single concentrated load at their free end. It is found that the principal stress $\sigma_{\max}$ does not exceed the maximum normal stress σ_m determined by the method of Chapter 5 except very close to the load. While this result holds for most beams of nonrectangular section, it may not be valid for rolled-steel W and S beams, and the analysis and design of such beams should include the determination of the principal stress $\sigma_{\max}$ *at the junction of the web with the flanges of the beam*. (See Sample Probs. 8.1 and 8.2, and Probs. 8.1 through 8.14).

Section 8.2 is devoted to the analysis and design of *transmission shafts* using gears or sprocket wheels to transmit power to and from the shaft. These shafts are subjected to transverse loads as well as to torques. The design of such shafts is the subject of Sample Prob. 8.3 and Probs. 8.15 through 8.30.

The determination of the stresses at a given point *K* of a body due to a *combined loading* is the subject of Section 8.3. First, the loading is reduced to an equivalent system of forces and couples in a section of the body containing *K*. Next, the normal and shearing stresses are determined at *K*. Finally, using one of the methods of transformation of stresses presented in Chapter 7, the principal planes, principal stresses, and maximum shearing stress may be determined at *K*. This procedure is illustrated in Concept Application 8.1 and Sample Probs. 8.4 and 8.5.

Chapter 9
Deflection of Beams

The relation derived in Chapter 4 between the curvature of a beam and the bending moment is recalled in Section 9.1 and used to predict the variation of the curvature along the beam. In Section 9.1A, the equation of the *elastic curve* for a beam is obtained through two successive integrations, after the bending moment has been expressed as a function of the coordinate *x*. Concept Applications 9.1 and 9.2 show how the boundary conditions can be used to determine the two constants of integration in the cases of a cantilever beam and of a simply supported beam. Concept Application 9.3 indicates how to proceed when the bending moment must be represented by two different functions of *x*.

Section 9.1B is optional; it shows in the case of a beam supporting a distributed load, how the equation of the elastic curve can be obtained directly from the function representing the load distribution through the use of four successive integrations.

Section 9.2 is devoted to the analysis of *statically indeterminate beams* and to the determination of the reactions at their supports. It is suggested that a minimum of two lessons be spent on Sections 9.1 through 9.2 if neither the use of singularity functions (Section 9.3) nor the moment-area method (Sections 9.5 through 9.6) are to be covered in the course.

Section 9.3 is devoted to the use of *singularity functions* for the determination of beam deflections and slopes. It is optional and assumes that Section 5.4 has been covered previously. It is recommended that both Sections 5.4 and 9.3 be included in the course, since singularity functions provide the students with an effective and versatile method for the determination of deflections and slopes under the most diverse loading conditions. In addition, and as indicated earlier, singularity functions are well suited to the use of computers.

Section 9.4A discusses the *method of superposition* for the determination of beam deflections and slopes. It shows how the expressions given in Appendix D for various simple loadings can be used to obtain the deflection and slope of a beam supporting a more complex loading. In Section 9.4B, the method of superposition is applied to the determination of the reactions at the supports of *statically indeterminate beams.*

Sections 9.5 through 9.6 are optional. They deal with the application of the moment-area methods to the determination of the deflection of beams and may be omitted in courses that place a greater emphasis on analytical methods and make use of singularity functions. It should be noted, however, that these methods provide a very practical means for the determination of the deflection and slope of beams of *variable cross section.*

The two moment-area theorems are derived in Section 9.5A and are immediately applied in Section 9.5B to the computation of the slope and deflection of *cantilever beams* and *beams with symmetric loadings* (simply supported or overhanging beams). Section 9.5C shows how to draw a *bending-moment diagram by parts*. This approach greatly facilitates the determination of moment areas in all but the simplest loading situations.

Section 9.6 deals with simply supported and overhanging beams with *unsymmetric loadings*. The analysis of such beams hinges on the use of a reference tangent drawn through one of the supports after the tangential deviation of the second support has been computed from the bending-moment diagram. Section 9.6B describes how to locate the point of *maximum deflection* and how to compute that deflection.

Section 9.6C deals with the analysis of *statically indeterminate beams* and the determination of the reactions at their supports.

Chapter 10
Columns

Section 10.1 introduces the concept of *stability of a structure*. The example considered in this section consists of a block supported by two spring-connected rigid rods. It is shown that the position of equilibrium in which both rods are aligned is stable if this position is the only possible position of equilibrium of the system. The same criterion is applied to an elastic pin-ended column in Section 10.1A in order to derive *Euler's formula*. Section 10.1B shows how Euler's formula for pin-ended columns can be used to determine the critical load of columns with other end conditions.

Section 10.2 is optional; it deals with the eccentric loading of a column and gives the derivation of the *secant formula.*

Section 10.3 discusses the *design of columns under a centric load*. Empirical formulas developed by various engineering associations for the design of steel columns, aluminum columns, and wood columns are presented in Section 10.3A. Section 10.3B is devoted to an optional discussion of the application of *Load and Resistance Factor Design* to steel columns. As noted at the end of this section, the design formulas presented in this section are intended to provide introductory examples of different design approaches. These formulas do not provide all the requirements that are needed for more comprehensive designs often encountered in engineering practice.

Section 10.4 discusses the *design of columns under an eccentric load* and presents two of the most frequently used methods: the *allowable-stress method* and the *interaction method.*

Chapter 11
Energy Methods

Section 11.1A introduces the concept of *strain energy* by considering the work required to stretch a rod of uniform cross section. This work, which is equal to the area under the load-deformation curve, represents the strain energy of the rod. The *strain-energy density* is defined in Section. 11.1B, as well as the *modulus of toughness* and the *modulus of resilience* of a given material. The formula for the elastic strain energy associated with normal stresses is derived in Section 11.2A, as well as the expressions for the strain energy corresponding to an axial loading and to pure bending. The formula for the strain energy associated with shearing stresses is derived in Section 11.2B, as well as the expressions corresponding to torsion and transverse loading.

Section 11.3, which is optional, covers the strain energy for a general state of stress and derives an expression for the *distortion energy per unit volume*, both in the general case of three-dimensional stress and in the particular case of plane stress.

Section 11.4A discusses *impact loadings* and Section 11.4B the design of a structure for an impact load. To facilitate the solution of impact-loading problems, it is shown in Section 11.5A that the strain energy of a structure subjected to a single concentrated load **P** can be obtained by equating the strain energy to the work of **P.** (Appendix D is used to express the deflection in terms of *P*). As shown in Section 11.5B, the reverse procedure can be used to determine the deflection of a structure at the point of application of a single load **P** or a single couple **M**; the strain energy of the structure is computed from one of the formulas derived in Section 11.2, and the work of **P** or **M** is equated to the expression obtained for the strain energy.

Sections 11.6 through 11.9 are optional. In Section 11.6 an expression for the strain energy of a *structure subjected to several loads* is obtained by computing the work of the loads as they are successively applied. Reversing the order in which the loads are applied, one proves *Maxwell's reciprocal theorem*. The expression obtained for the strain energy of the structure is used in Section 11.7 to prove *Castigliano's theorem*. Section 11.8 is devoted to the application of Castigliano's theorem to the determination of the deflection and slope of a beam and to the deflection of a point in a truss. Finally, Section 11.9 deals with the application of Castigliano's theorem to the determination of the reactions at the supports of statically indeterminate structures such as beams and trusses.

TABLE I: LIST OF TOPICS COVERED IN *MECHANICS OF MATERIALS,* Seventh Edition

Sections	Topics	Suggested Number of Periods: Core Topics	Additional Topics	Advanced Topics
Chapter 1: Introduction – Concept of Stress				
1.1-2	Stress Under Axial Loading	1-2		
1.3-5	Components of Stress; Factor of Safety	1		
Chapter 2: Stress and Strain – Axial Loading				
2.1	Stress-Strain Diagrams; Deformations Under Axial Loading	1-2		
2.2-3	Statically Indeterminate Problems	1		
2.4-5	Poisson's Ratio; Generalized Hooke's Law	1		
*2.6	Dilatation; Bulk Modulus			0.25-0.5
2.7-8	Shearing Strain	0.5		
*2.9	Stress-Strain Relationships for Fiber-Reinforced Composite Materials			0.5-1
2.10-12	Stress Concentrations; Plastic Deformations	0.5-1		
*2.13	Residual Stresses		0.5	
Chapter 3: Torsion				
3.1	Stresses in Elastic Range	1		
3.2-3	Angle of Twist; Statically Indeterminate Shafts	1-2		
3.4-5	Design of Transmission Shafts; Stress Concentrations	1		
*3.6-8	Plastic Deformations; Residual Stresses		1-2	
*3.9-10	Noncircular Members; Thin-Walled Hollow Shafts			1-2
Chapter 4: Pure Bending				
4.1-3	Stresses in Elastic Range	1-2		
4.4-5	Members Made of Several Materials; Stress Concentrations	1-2		
*4.6	Plastic Deformations; Residual Stresses		1-2	
4.7	Eccentric Axial Loading	1-2		
4.8-9	Unsymmetric Bending; General Eccentric Axial Loading	1-2		
*4.10	Bending of Curved Members			1-2
Chapter 5: Analysis and Design of Beams for Bending				
5.1	Shear and Bending-Moment Diagrams	1-1.5		
5.2	Using Relations Between *w, V,* and *M*	1-1.5		
5.3	Design of Prismatic Beams in Bending	1-2		
*5.4	Use of Singularity Functions to Determine *V* and *M*	1-2		
*5.5	Nonprismatic Beams		1-2	
Chapter 6: Shearing Stresses in Beams and Thin-Walled Members				
6.1	Shearing Stresses in Beams	1-2		
*6.2	Shearing Stresses in Narrow Rectangular Beam		0.25	
6.3-4	Shearing Stresses in Thin-Walled Members	1-2		
*6.5	Plastic Deformations		0.25	
*6.6	Unsymmetric Loading; Shear Center		1-2	

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TABLE I: LIST OF TOPICS COVERED IN *MECHANICS OF MATERIALS*, Seventh Edition (CONTINUED)

Sections	Topics	Suggested Number of Periods: Core Topics	Additional Topics	Advanced Topics
Chapter 7: Transformation of Stress and Strain				
7.1	Transformation of Plane Stress	1-2		
7.2	Mohr's Circle for Plane Stress	1-2		
7.3-4	Three-Dimensional Analysis of Stress	0.5-1		
*7.5	Yield and Fracture Criteria		0.5-1	
7.6	Thin-Walled Pressure Vessels	0.5-1		
*7.7-8	Analysis of Strain; Mohr's Circle		1-1.5	
*7.9	Strain Rosette		0.5	
Chapter 8: Principal Stresses under a Given Loading				
8.1	Principal Stresses in a Beam		0.5-1	
8.2	Design of Transmission Shafts		0.5-1	
8.3	Stresses under Combined Loadings		1-3	
Chapter 9: Deflection of Beams				
9.1-1A	Equation of Elastic Curve	0.5-1		
*9.1B	Direct Determination of Elastic Curve from Load Distribution		0.5	
9.2	Statically Indeterminate Beams	0.5 - 1		
*9.3	Use of Singularity Functions		1-2	
9.4	Method of Superposition	1-2		
	Application of Moment-Area Theorems to:			
*9.5	Cantilever Beams and Beams with Symmetric Loadings		1-2	
*9.6A-6B	Beams with Unsymmetric Loadings; Maximum Deflection		1-1.5	
*9.6C	Statically Indeterminate Beams		0.5	
Chapter 10: Columns				
10.1	Euler's Column Formula	1-2		
*10.2	Eccentric Loading; Secant Formula		1	
10.3	Design of Columns under a Centric Load	1-2		
10.4	Design of Columns under an Eccentric Load	1-2		
Chapter 11: Energy Methods				
11.1-2	Strain Energy	1-2		
11.3	Strain Energy for General State of Stress		0.5	
11.4	Impact Loading	0.5-1		
11.5	Deflections by Work-Energy Method	0.5-1		
*11.6-8	Castigliano's Theorem		1-2	
*11.9	Statically Indeterminate Structures		1-2	
	Total Number of Periods	24-41½	21-38½	3-6

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number* SI Units	U.S. Units	Problem description
		CHAPTER 1: INTRODUCTION–CONCEPT OF STRESS
		Normal stress under axial loading:
1.1,2	1.3,4	in bars
1.*5*,*6*		
1.7,9	1.8,10	in pin-connected structures
1.13,14	1.*11*,*12*	in trusses and mechanisms
1.15,18	1.16,17	Shearing stress
1.20,21	1.*19*,22	Bearing stress between flat surfaces
1.24,25	1.23,28	Shearing and bearing stresses at pin-connected joints
1.*26*,*27*		
1.29,30	1.31,32	Stresses on an oblique plane
1.35,36	1.*33*,*34*	
		Factor of safety:
1.*38*,*39*	1.37,42	in tension
1.40,41		
1.*43*,*44*	1.45,46	in shear
1.47,48	1.*49*,50	
1.53,54	1.*51*,52	in structures involving links and pins
1.55,56		
*1.57	*1.58	Load and Resistance Factor Design
1.59,62	1.60,*61*	Review problems
1.*63*,*66*	1.64,65	
1.67,68	1.*69*,70	
1.C2,C4,C6	1.C1,C3, C5	Computer problems
		CHAPTER 2: STRESS AND STRAIN - AXIAL LOADING
		Stresses and deformations in statically determinate structures:
2.1,3	2.2,*5*	simple rods and wires
2.4,6	2.*7*,*8*	
2.9,*10*	2.11,13	multiple-criteria problems
2.*12*,14		
2.*16*,*18*	2.15,17	composite rods and members
2.19,20		
2.21,24	2.*22*,23	members of trusses and simple frames
2.25,26	2.27,*28*	
2.29	2.30	computation of deformations by integration
2.31	2.32	true strain

* Problems that do not involve any specific system of units have been indicated by underlining their number. Answers are not given to problems with a number set in italic type.

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
Mechanics of Materials Seventh Edition (CONTINUED)

Problem Number*		
SI Units	**U.S. Units**	**Problem description**
		Statically indeterminate structures (constant temperature):
2.33,34	2.35,36	with members undergoing equal deformations
2.*37*,*38*		
2.41,42	2.39,*40*	composite rods with both ends restrained
2.43,45	2.44,*46*	with members undergoing unequal deformations
		Statically indeterminate structures (with temperature changes):
2.47,48	2.*49*,50	with members undergoing equal deformations
2.51,54	2.52,*53*	composite rods with both ends restrained
2.55,56		with unequal deformations
2.*57*		
2.*60*	2.58,59	rods with gaps
		Poisson's ratio and generalized Hooke's law:
2.*62*,63	2.61,66	uniaxial loading
2.64,*65*		
2.67,68	2.69,70	biaxial loading
2.73,74	2.*71*,*72*	derivation of formulas
2.77,78	2.*75*,*76*	Hooke's law for shearing stress and strain
2.81,82	2.79,80	
*2.84,**86*	*2.83,*85	dilatation
*2.*87*,*88		problems involving cylindrical coordinates
*2.89	*2.90	Theory problems
*2.91,*92		Problems involving composites
2.93,94	2.*95*,*96*	Stress concentrations in flat bars
2.97,98	2.99,100	
		Plastic deformations under axial loading:
2.*103*,*104*	2.101,102	simple problems
2.105,106		
2.107,108	2.111,112	more complex problems
2.*109*,*110*		
2.113,114		
*2.115		
2.117,*118	2.116	problems involving temperature change
*2.122,*123	*2.*119*,**120*	problems involving residual stresses
	*2.121	
2.124,127	2.125,*126*	Review problems
2.*129*,131	2.128,130	
2.133,*134*	2.*132*,135	
2.C1,C3,C6	2.C2,C4,C5	Computer problems

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
		CHAPTER 3: TORSION
		Shearing stresses:
3.1,2	3.4,*5*	in simple shafts
3.3,6	3.7,*8*	
3.11,*12*	3.9,10	in shafts subjected to several torques
3.13,*14*		
3.*17*,18	3.15,16	in composite shafts
3.*19*,20		
3.21,22	3.23,*24*	in gear-connected shafts
3.27,28	3.25,*26*	
3.29	3.*30*	special problems
		Angle of twist:
3.31,*34*	3.32,33	in simple shafts
3.35,*36*		in shafts subjected to several torques
3.37,38	3.*39*,*40*	in composite shafts
3.42,*43*	3.41,44	in gear-connected shafts
3.45,48	3.46,47	Design of shafts based on allowable stress and allowable angle of twist
3.49,50		
		Statically indeterminate shafts:
3.*55*,56	3.*51*,*52*	with inner core and outer shell of different materials
3.57,*58*	3.53,54	
3.59		
3.60,62	3.61,63	Special problems
		Design of shafts:
3.65,67	3.64,66	easy problems with solid shafts
3.68,69	3.*70*,*72*	hollow shafts
3.71,73		
3.*74*,*75*	3.76,77	gear-connected shafts
3.80,81	3.78,79	multiple-criteria problems
3.*82*,*83*		
3.84,87	3.*85*,86	Stress concentrations in stepped shafts
3.88,90	3.89,*91*	

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
		Plastic deformations of shafts:
		Shafts made of elastoplastic material:
3.92,95	3.93,94	stresses in solid shafts
3.*96*,*97*		
3.98,100	3.99,101	angle of twist for solid shafts
3.102,104	3.*103*,*105*	
3.106,107	3.*108*,*109*	hollow and tapered shafts
3.112,113	3.110,111	Shafts made of a material with a nonlinear stress-strain diagram
3.*116*,*117*	3.114,115	Residual stresses and permanent angle of twist in shafts made of an
3.118,119	3.120	elastoplastic material
		Bars with rectangular cross section:
3.123,124	3.121,122	easy problems
3.127,128	3.*125*,*126*	
3.129,*130*	3.131,132	comparing circular and rectangular shafts
3.*134*	3.133	
3.135,137	3.136,138	application to structural shapes
		Thin-walled hollow shafts:
3.*140*,*141*	3.139,*145*	determine shearing stresses
3.142,143	3.146	
3.144		
3.147,149	3.*148*,150	Special problems
3.*152*,153	3.151,*154*	Review problems
3.156,157	3.155,*159*	
3.158,161	3.160,162	
3.C1,C2,C3	3.C4,C5,C6	Computer problems

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
		CHAPTER 4: PURE BENDING
		Normal stresses:
4.1,3	4.2,5	in beams with horizontal plane of symmetry
4.4,6	4.*7*,*8*	
4.11	4.9,10	in unsymmetrical beams (first locate centroid)
4.12,*13*		Resultant force on portion of cross section
4.*14*		
4.16,*17*	4.15,18	Beams with different allowable stresses in tension and compression
4.19,*20*		
4.21,22	4.23	Maximum stress and radius of curvature
4.24		
4.25,29	4.26,27	Maximization of beam strength
	4.*28*	
4.31	4.30	Anticlastic curvature
4.32		Special problem on theory
		Stresses in composite beams:
4.33,34	4.37,38	symmetric beams of two materials
4.*35*,*36*		
4.39,40	4.41,*42*	unsymmetric beams of two materials
4.43,44	4.45,*46*	Radius of curvature in composite beams
4.49,50	4.47,48	Stresses in reinforced concrete beams
4.53,54	4.*51*,*52*	
	4.55,*56*	Beams of three materials
4.*58*	4.57	Composite beams with circular cross section
4.59	*4.60	Beams with different moduli of elasticity in tension and compression
4.61,*62*	4.63,64	Stress concentrations in flat bars in pure bending
4.65,*66*		
		Plastic deformation in pure bending (elastoplastic material):
4.67,68	4.69,*70*	plastic zone in rectangular beams
4.71,72		
4.*73*,*74*	4.75,*76*	plastic zone in symmetric beams
4.77,78	4.79,80	plastic moment and shape factor of symmetric beams
4.81,82	4.84,86	plastic moment of unsymmetric beams
4.*83*,*85*		
		Residual stresses in symmetric beams:
4.87,88	4.*89*,*90*	after M_p has been applied and removed
4.91	4.92	after a given plastic zone has been developed
		Special problems:
4.94,*95*	4.93	residual radius of curvature
4.96	4.*97*,98	Plastic deformation of beams with a nonlinear stress-strain diagram

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
		Eccentric loading in plane of symmetry of member:
4.99,*102*	4.100,101	find stress in symmetric section
4.103,*104*		
4.106,107	4.105,109	design of symmetric section
4.108,110	4.*111*,*112*	
4.115,116	4.113,114	find stress in unsymmetric section
4.117,120	4.*118*,*119*	
4.121,122	4.124,125	computation of loads from strain measurements
4.*123*,*126*		
		Unsymmetric bending with one or two planes of symmetry:
4.127,129	4.128,130	bending moment at an angle with horizontal
4.133,134	4.*131*,*132*	
4.*136*,137	4.135,138	section at an angle with horizontal
4.139,*140*		
*4.141	*4.*142*,*143	Bending of unsymmetric section (principal axes must be determined)
		General eccentric bending:
4.144,145	4.146,147	symmetric beam; find stresses
4.*148*,*149*		
4.152,153	4.150,151	Bending of unsymmetric beams; determine largest bending moment
4.155	4.*154*,*156*	
4.157,158	4.159,160	Special problems and problems on theory
		Curved beams:
4.161,162	4.163,164	with rectangular cross section
4.*165*,*166*		
4.169,170	4.167,*168*	under eccentric loading
4.173,174	4.171,*172*	with unsymmetric cross section
4.177,178	4.175,*176*	with circular cross section
4.179		
4.180	4.181,*182*	with triangular cross section
4.185,*186*	4.183,184	with trapezoidal cross section
*4.187,188	4.189,*191	special problems and derivations of formulas
4.190		

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
4.192,*195*	4.*193*,194	Review problems
4.196,*197*	4.198,*200*	
4.199,201	4.202,203	
4.C1,C3	4.C2,C4	Computer problems
4.C5,C7	4.C6	
	CHAPTER 5: ANALYSIS AND DESIGN OF BEAMS FOR BENDING	
		Using the free-body diagram of a portion of a beam:
5.1,2	5.3,4	draw *V* and *M* diagrams (easy problems)
5.5,6		
5.7,9	5.8,*10*	draw *V* and *M* diagrams and determine maximum values of $\|V\|$ and $\|M\|$
5.11,13	5.12,*14*	
5.15,*17*	5.16,20	find maximum normal stress in given beam section
5.18,*19*		
5.*22*,23	5.21,25	draw *V* and *M* diagrams and find maximum normal stress in beam
5.*24*,26		
5.27,29	5.*28*,31	determine given parameter to minimize normal stress in beam
5.30,33	5.32	
		Using relations among *w*, *V* and *M* whenever appropriate:
5.34,35	5.36,37	draw *V* and *M* diagrams (easy problems)
5.38,39		
5.41,42	5.40,43	draw *V* and *M* diagrams and determine maximum values of $\|V\|$ and $\|M\|$
5.*44*,*45*		
5.46,48	5.47,49	find maximum normal stress in a given beam section
5.52,53	5.*50*,*51*	write equations for *V* and *M* and find maximum value of $\|M\|$
5.55,56	5.54,*57*	draw *V* and *M* diagrams and find maximum normal stress in beam
5.59,60	5.*58*,61	
*5.*62*,*64	*5.63	Special problems

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
Mechanics of Materials Seventh Edition (CONTINUED)

Problem Number*		
SI Units	**U.S. Units**	**Problem description**
5.65,*66* 5.69,70	5.*67*,68	Design of timber beams
5.73,74	5.71,72	Design of steel beams, W shapes
5.77,*78*	5.*75*,76	Design of steel beams, S shapes
5.79,80	5.81,82	Design of steel beams, miscellaneous shapes
5.83	5.84	Design of beams resting on ground
5.85,86 5.89,*90*	5.*87,88*	Find allowable load for beam of unsymmetric cross section with allowable stresses in tension and compression
5.*93*	5.91,92	Design of steel beams, W shapes
*5.94,*95	*5.96,*97	Design of beams using LRFD
		Using singularity functions write equations for $V(x)$ and $M(x)$ and
5.98,100	5.*99*,*101*	find M at given point in beam
5.102,105	5.103,104	
5.106,108	5.107,109	find $\|M\|_{max}$ in beam
5.110,*111*		find σ_{max} in beam
5.*112*,*113*		find $\|M\|_{max}$ and σ_{max} in beam
5.*116*,117	5.114,115	design beam, knowing allowable stress
		Using a computer and step functions,
5.118,119	5.120,*121*	calculate V and M along the beam
5.122,123	5.124,*125*	calculate V and M along the beam , and determine $\boldsymbol{\sigma}_{max}$ in the beam
		Nonuniform beams
		Beams of constant strength:
5.128,*129* 5.130,*131*	5.126,127	beams of uniform width and variable depth
5.134,135	5.132,*133*	built-up timber beams
5.136	5.*137*	beams of circular cross section
5.138	5.139	beams of uniform depth and variable width
5.140,143	5.141,*142*	rolled-steel beams with cover plates
5.144,145	5.*146*,147	
5.*148*,149	5.150,151	tapered beams
5.152,*153*	5.154,*155*	Review problems
5.156,157	5.158,159	
5.161,*162*	5.*160*,163	
5.C1,C2 5.C6	5.C3,C4 5.C5	Computer problems

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
	CHAPTER 6: SHEARING STRESSES IN BEAMS AND THIN-WALLED MEMBERS	
6.1,2 6.4,5 6.9,*10* 6.13,*14* 6.*16*,18 6.19,*20* 6.22,24 6.26,28	6.3,*6* 6.7,*8* 6.11,12 6.15,17 6.21,23 6.*25*,*27*	Shearing forces in nails and bolts, using horizontal cuts Shearing stresses in beams Designs of beams for normal and shearing stresses Beams with singly-symmetric sections Beams with various geometric sections
6.30,32 6.*33*,*34* 6.35,36 6.37,42 6.44,46 6.48,49 6.*53*,*54* 6.55,56 6.57	6.29,31 6.38,*39* 6.40,*41* 6.43,45 6.*47*,*50* 6.51,52 6.58,59 6.60	Shearing forces and shearing stresses on arbitrary cuts due to vertical shear Shearing stresses in extruded beams Shearing stresses in bolts Shearing stresses and shear flow in thin-walled members Special problems Shearing stresses in composite beams Plastic behavior
6.61,62 6.65,68 6.70,72 6.*73*,*74* 6.76,77 6.*79*,*80* *6.81,*82 *6.83,*84	6.63,64 6.*66*,*67* 6.69,71 6.75,78 *6.*85*,**86* *6.87,*88	Shear center in thin-walled beams with horizontal and vertical portions Shear center and shearing stresses in extruded beams Shear center in thin-walled beams with oblique portions Shear center in thin-walled beams with circular portions Problems involving location of shear center Special problems Shearing stresses in semicircular shapes, angle shapes and Z shapes
6.89,*90* 6.94,*95* 6.97,99	6.91,*92* 6.93,96 6.*98*,100	Review problems
6.C1 6.C3,C4	6.C2 6.C5,C6	Computer problems

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
	CHAPTER 7: TRANSFORMATION OF STRESS AND STRAIN	
7.2,4	7.1,3	Find stresses on oblique plane from equilibrium of wedge
7.5,7	7.*6,8*	Find principal planes and stresses
7.9,11	7.10,12	Find planes of maximum shearing stress and corresponding stresses
7.*14,16*	7.13,15	Find stresses on a given plane
7.18,19	7.17,21	Stresses on oblique planes - simple applications
7.*20,22*		
7.23,26	7.24,25	Find principal stresses and/or maximum shearing stress in loaded shaft
7.27,29	7.*28,30*	Special problems involving determination of a stress to satisfy a given requirement
		Using Mohr's circle, determine:
7.31,33	7.32,34	principal planes and stresses, and maximum shearing stress
7.*36,38*	7.35,37	stresses on oblique plane
7.40,41	7.39,43	stresses on oblique plane - simple applications
7.*42,44*		
7.45,48	7.46,47	principal stresses and/or maximum shearing stress in loaded shaft
7.49,51	7.*50,52*	
7.53		
7.*55*,56	7.*54*,57	find principal planes and stresses resulting from superposition of two states of stress
7.58,59	7.*60*,63	find range of values of a parameter for which a certain stress will not be exceeded
7.61,62		
7.64	7.65	Derivation of a formula involving Mohr's circle
7.68,69	7.66,*67*	In-plane and out-of-plane maximum shearing stress
7.70,71	7.*72*,73	Maximum shearing stress in a three dimensional state of stress
7.75,*76*	7.74,77	Maximum shearing stress (more advanced problems)
7.*78*,79		
*7.80		
7.81,82	7.83,84	Determine if material will yield under given state of stress. If not, find
7.*85,86*	7.87,88	the factor of safety
7.89,90	7.91,92	Will rupture occur under a given state of stress?
7.94,95	7.*93,97*	Find stress or load for which rupture will occur
7.96		

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
7.98,*99*	7.100,102	Spherical pressure vessels (easy problems)
7.*101*,103		
7.104,105	7.*107*,109	Cylindrical pressure vessels (easy problems)
7.106,108	7.*110*,111	
7.112,113	7.*116*,117	Stresses in welds in cylindrical pressure vessels
7.114,115	7.118,*119*	
7.122,*123*	7.120,121	Pressure vessels subjected to external loadings
7.124,*125*		
	7.126,127	Shrunk fit rings
		Find state of strain associated with given rotation:
7.129,131	7.128,*130*	using formulas of Sec. 7.10
7.133,135	7.132,*134*	using Mohr's circle
		Find principal strains and maximum shearing strain (in plane and out of plane):
7.*138*,139	7.136,137	for a state of plane stress
7.*142*,143	7.140,141	for a state of plane strain
7.*144*,*145*	7.146,149	Problems involving strain rosettes
7.147,148		
7.154,155	7.150,151	Problems involving use of Mohr's circle and Hooke's law
7.156,157	7.*152*,*153*	
7.159,*160*	7.158,*162*	Review problems
7.161,163	7.164,167	
7.165,*166*	7.*168*,169	
7.C1,C2	7.C3,C4	Computer problems
7.C5,C6	7.C7,C8	

CHAPTER 8: PRINCIPAL STRESSES UNDER GIVEN LOADING CONDITIONS

		Principal stresses in rolled-steel beams:
8.3,4	8.1,2	find σ_m in beam and σ_{max} at junction of flanges and web
8.5,6	8.*7*,*8*	design beam, taking σ_m, τ_m, and σ_{max} into account
8.9,*10*	8.11,12	check earlier design for σ_{max} at junction of flanges and web
8.13,*14*		
		Design of transmission shafts:
8.15,*16*	8.19,20	loading represented by forces and couples
8.17,*18*	8.21,22	
8.23,24	8.*25*,*26*	loading represented by input and output power
8.27,28	8.29,30	

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
		For beam of rectangular cross section under axial, bending and transverse loading (easy problems), find:
8.*33*,*34*	8.31,32	normal and shearing stresses
8.35,36		For beams of circular cross section under axial, bending, and transverse loading, find:
8.38,40	8.37,39	normal and shearing stresses
8.42,43	8.*41*,*44*	principal stresses and maximum shearing stress
		For beams of rectangular cross section under axial, bending and transverse loading, find:
8.47,48	8.*45*,46	normal and shearing stresses
8.51,52	8.49,*50*	principal stresses and maximum shearing stress
		For steel beams and structural tubes under axial, bending and transverse loading, find:
8.53,*54*		normal and shearing stresses
8.55,*56*	8.57,58	principal stresses, principal planes, and maximum shearing stress
8.60	8.*59*	Special problems
*8.61	*8.62,**63*	Problems involving torsion of rectangular sections or structural shapes
	*8.64	
8.66,*67*	8.65,69	Review problems
8.68,*70*	8.*71*,*72*	
8.74,76	8.*73*,75	
8.C1,C2	8.C4,C5	Computer problems
8.C3,C6	8.C7	

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
		CHAPTER 9: DEFLECTION OF BEAMS
		Using the integration method, determine the equation of the elastic curve and the deflection and/or slope at specific points for:
9.3,4	9.1,2	cantilever beams
9.5	9.*6*	
9.7	9.*8*	overhanging beams
		simply supported beams
9.10	9.9	symmetrical loading
9.12	9.11	unsymmetrical loading
9.*14,15*	9.13	beams and loadings requiring the use of 2 equations and 4 constants of integration
9.16		
9.17	9.18	direct determination of the elastic curve from an analytic function of $w(x)$
		For a statically indeterminate beam (first degree), determine:
9.19,*21*	9.20,23	reaction at the roller support
9.*22*,24		
9.25,28	9.26,27	reaction at the roller support and draw the M diagram (use of 2 equations and 4 constants of integration required)
9.30,32	9.*29,31*	reaction at the roller support and the deflection at a given point (use of 2 equations and 4 constants of integration required)
9.33	9.34	For a statically indeterminate beam (second degree), determine the reaction at one end and draw the M diagram
		Using singularity functions, determine the equation of the elastic curve and the deflection and/or slope at specified points:
9.35,36	9.37,38	easy problems
9.*39,40*		problems involving overhanging beams
9.41,44	9.*42*,43	problems with distributed loads
9.45,48	9.46,*47*	problems with numerical data
9.49,51	9.50,*52*	For a statically indeterminate beam (first degree), determine the reaction at
9.53,54	9.*55*,56	the roller support and the deflection at a specified point.
9.57	9.58	For a statically indeterminate beam (second degree), determine the reaction at the roller support and the deflection at a specified point.
9.59,62	9.60,61	determine the maximum deflection
9.*63,64*		Problems involving the reduction of a load to a force-couple system.

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
		Using method of superposition, determine the deflection and slope at specified points in:
9.65,67	9.66,68	cantilever beams
9.71,72	9.*69,70*	simply supported beams
9.73,*74*	9.75,76	cantilever beams (with numerical data)
9.77,*78*		simply supported beams (with numerical data)
9.79,80	9.82	statically indeterminate beams (first degree)
9.*81*		
9.*83*	9.84	statically indeterminate beams (second degree)
9.85,88	9.86,87	combined beams, determinate (with numerical data)
9.90,91	9.*89,92*	statically indeterminate beams (with numerical data)
9.94	9.93	Combined bending and torsion of rods
		Using the moment-area method, determine the slope and/or deflection at specified points in
		cantilever beams
9.95,96	9.97,98	with simple loadings
9.*99,100*		
9.102,104	9.101,103	with numerical data
9.105,107	9.*106,108*	with variable *EI*
		simply supported beams
9.109,110	9.111,112	with symmetric loadings
9.*114*	9.113	
9.*116*	9.115	with variable *EI*
9.118,119	9.*117,121*	with numerical data
9.120,122		
*9.123	*9.124	special problems
		Simply supported and overhanging beams with unsymmetric loadings
9.125,126	9.*127*,128	simply supported beams (easy problems)
9.129,130	9.*131*,132	simply supported beams with numerical data
9.133,*134*		overhanging beams
9.136,*138*	9.135,137	overhanging beams (with numerical data)
9.139	9.140	simply supported beams with variable *EI*
		Find maximum deflection for:
9.*141*,142		simply supported beams
9.*143*	9.144	with numerical data
9.145	9.146	overhanging beams
		Statically indeterminate beams (first degree)
9.147,150	9.148,*149*	single span
9.152	9.*151*	two span beams, find all reactions
9.153	9.154	single span beams (with numerical data)
9.155	9.156	simply supported beams with additional elastic support at midspan

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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
9.158,161	9.157,*159*	Review problems
9.*164*,165	9.160,*162*	
9.*167*,168	9.163,166	
9.C1,C3	9.C2,C5	Computer problems
9.C4,C8	9.C6,C7	
	CHAPTER 10: COLUMNS	
		Stability of rigid-rod-and-spring systems:
10.1,2	10.4,*5*	single spring
10.3		
10.7	10.6,*8*	systems with two or more springs
		Application of Euler's formula to the critical loading or pin-ended columns:
10.9	10.10	short struts
10.*12*,13	10.11	comparison of critical loads for various cross sections
10.*14*		
		Allowable loading for pin-ended columns:
10.16,*18*	10.15,17	rolled-steel shapes
10.19,*20*		multiple-member structures
10.24,25	10.21,22	columns with various end conditions
10.27,28	10.*23*,*26*	
		Application of the secant formula to the eccentric loading of columns:
10.29,30	10.32,34	find σ_{max} and either deflection or e for a given load
10.*31*,*33*		
10.36,37	10.35,*38*	find σ_{max} and load for a given deflection an eccentricity
10.39,40		
	10.41,*42*	temperature induced loading
10.43,44	10.45,*46*	find P_{all} for given e, σ_{max} and F.S. (using Fig. 10.24)
		Design problems
10.47,50	10.*48*,49	find column length
10.51,*52*	10.53,54	find cross section of column
10.*55*,56		find factor of safety of column

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
		Analysis of columns under centric load:
		columns with simple cross section:
10.57,59	10.58	steel columns
10.60	10.*61*	wood columns
10.62	10.*63*,64	aluminum columns
		columns with built-up cross sections:
10.66,*67*	10.65,68	steel columns
10.69		wood columns
10.*70*		aluminum column
		Design of columns under a centric load:
10.71,*73*	10.72	wood columns
10.75,*76*	10.74	aluminum columns
10.77,80	10.78,79	steel columns
10.83,84	10.*81*,*82*	
		Application of LRFD formulas:
*10.86	*10.85	analysis of columns under a centric loading
*10.88	*10.87	design of columns under a centric loading
		Analysis of columns under an eccentric load:
10.89,*90*		steel columns, find allowable load
	10.91,92	wood columns, find allowable load
10.93,*94*		aluminum columns, find allowable load
		Design of columns under an eccentric load:
		find maximum allowable length or allowable eccentricity:
	10.95,*96*	steel columns
	10.97,*98*	
10.99,100		wood columns
10.101,102		aluminum columns
		design cross section:
10.103,104		steel columns, rectangular cross section
10.105,106		steel columns, tube
10.107,*108*		wood columns, rectangular or circular cross sections
	10.109,*110*	aluminum columns
	10.111,*112*	
10.*115*,116	10.113,114	steel column, wide-flange shape
10.118,*119*	10.117,120	Review problems
10.121,*124*	10.*122*,123	
10.125,126	10.*127*,128	
10.C1,C3,C6	10.C2,C4,C5	Computer problems

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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
		CHAPTER 11: ENERGY METHODS
11.2,3	11.1,*4*	Modulus of resilience
		Modulus of resilience and modulus of toughness:
11.6	11.5	from stress-strain diagram
11.7	11.*8*	from load-deflection diagram of a tensile test
		Strain energy under axial loads:
11.10	11.9,11	strain energy under a given load
11.12,*13*		maximum allowable strain energy
11.*14*,15		factor of safety
11.16	11.17	strain energy by integration of approximate methods
11.*19*,20	11.18	strain energy of trusses
11.21		
11.*22*	11.23	with numerical data
11.24,27	11.25,*26*	Strain energy in bending:
11.30,31	11.28,*29*	with numerical data
11.32		derivation of formulas
11.33	11.*34*	Strain energy in torsion:
11.35		by integration
11.*36*,37	11.*38*,39	Maximum-distortion-energy criterion for 3-dimensional state of stress
11.40	11.*41	Special problems
		Impact loading:
11.42	11.43,44	of rods (horizontal impact)
11.45,*46*		of rods (vertical impact)
11.*47*		
11.50,51	11.48,*49*	of beams (horizontal impact)
11.52,53	11.54,*55*	of beams (vertical impact)
11.56,57		problems on theory
		Use of work-energy method to determine deflection or slope of:
11.58,61	11.59,*60*	prismatic beams
11.62,65	11.63,*64*	nonprismatic beams
11.66,*69*	11.67, 68	angle of twist of shafts
11.70		angle of twist of a thin-walled hollow shaft
11.*71*,72	11.73,75	deflection of a joint of a truss
11.*74*,76		

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TABLE II: CLASSIFICATION AND DESCRIPTION OF PROBLEMS
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Problem Number*		
SI Units	**U.S. Units**	**Problem description**
11.77,78 11.*79*,*81*	11.80,82	Work of several loads applied to a beam
		Determinate structures. Use Castigliano's theorem to determine:
11.83,86	11.*84*,85	deflection and/or slope of beams
11.88,89	11.*87*,90	
11.93,94	11.91,*92*	deflection and/of slope of beams (with numerical data)
11.95,96	11.97,*98*	
11.99,*100*	11.101,102	deflection of a given joint in a truss
11.103,*104*		
11.105,106 11.*108*,*110*	11.107,109	deflection and/or slope at a given point in a bent or curved rod
		Indeterminate structures (first degree). Use Castigliano's theorem to determine:
11.113,114	11.111,112	reaction at roller support and draw bending-moment diagram
	11.*115*,*116*	
	11.117,118	force in member(s) of a truss
11.119,121	11.*120*,*122*	
11.123,126	11.124,*125*	Review problems
11.*127*,129	11.128,*131*	
11.130,*133*	11.132,134	
11.C3,C4,C5	11.C1,C2,C6	Computer problems

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TABLE III: SAMPLE ASSIGNMENT SCHEDULE FOR A COURSE IN MECHANICS OF MATERIALS *Mechanics of Materials 7th Edition*

The six lists of suggested assignments contained in this table cover the entire text. The instructor can select those assignments that are covered in the course.

50% OF THE PROBLEMS IN EACH OF THESE LISTS USE SI UNITS AND 50% U.S. CUSTOMARY UNITS

| < -----For these lists answers to all of the problems are given in the back of the book----------->| |<---- For Lists 5 and 6 no answers are given->|

Group	Sections	Topics	List 1	List 2	List 3	List 4	List 5	List 6
1	1.1	Introduction						
2.	1.2	Stresses in the Members of a Structure	1.3,7,17,25	1.4,13,16,24	1.1,10,15,23	1.2,8,18,28	1.5,12,19,27	1.6,11,22,26
3	1.3-5	Components of Stress; Factor of Safety	1.29,37,48,52	1.30,42,47,50	1.31,39,46,56	1.32,41,45,55	1.34,38,44,51	1.33,39,43,49
4	2.1	Stress-Strain Diagram	2.1,13,19,27	2.4,11,20,23	2.2,14,15,24	2.7,9,17,21	2.5,12,16,28	2.8,10,18,22
5	2.2-3	Statically Indeterminate Problems	2.35,42,52,55	2.36,41,50,56	2.33,44,48,58	2.34,39,47,59	2.38,40,53,57	2.37,46,49,60
6	2.4-9	Generalized Hooke's Law; Poisson's Ratio	2.66,68,79,84	2.61,67,80,88	2.63,70,77,85	2.64,69,78,83	2.62,72,76,87	2.65,71,75,86
7	2.10-13	Stress Concentration; Plastic Behavior	2.93,102,108,116	2.94,101,107,121	2.99,105,112,117	2.100,106,111,118	2.96,104,110,119	2.95,103,109,120
8	3.1	Stresses in Torsion	3.1,9,18,25	3.2,10,20,23	3.4,11,15,22	3.7,13,16,21	3.5,12,17,26	3.8,14,19,24
9	3.2-3	Angle of Twist, Indeterminate Shafts	3.32,38,46,57	3.33,41,47,56	3.31,40,45,53	3.35,44,48,54	3.34,39,51,58	3.36,43,52,55
10	3.4-5	Transmission Shafts: Stress Concentrations	3.67,77,81,89	3.65,76,80,86	3.66,73,79,90	3.64,71,78,84	3.72,75,83,91	3.70,74,82,85
11	3.6-8	Plastic Deformations; Residual Stresses	3.94,100,111,119	3.93,98,110,118	3.95,101,113,115	3.92,99,112,114	3.97,105,109,117	3.96,103,108,116
12	3.9-10	Noncircular and Thin-Walled members	3.124,133,135,139	3.123,129,137,146	3.122,132,136,142	3.121,131,138,143	3.126,134,141,145	3.125,130,140,148
13	4.1-3	Stresses and Deformations in the Elastic Range	4.3,10,19,26	4.1,9,16,23	4.4,12,18,24	4.2,11,15,21	4.8,14,20,28	4.7,13,17,27
14	4.4-5	Members Made of Composite Materials	4.37,40,47,61	4.38,44,48,65	4.33,41,49,63	4.34,45,50,64	4.35,42,51,62	4.36,46,52,66
15	4.6	Plastic Deformations; Residual Stresses	4.68,79,82,92	4.67,80,81,92	4.75,77,86,91	4.69,78,84,91	4.73,76,83,90	4.70,74,85,89
16	4.7	Eccentric Axial Loading in a Plane of Symmetry	4.101,108,114,121	4.100,106,113,122	4.103,109,115,124	4.99,105,116,125	4.104,112,118,123	4.102,111,119,126
17	4.8-9	Unsymmetric Bending	4.127,135,144,151	4.129,138,145,150	4.128,137,146,152	4.130,139,147,153	4.131,136,148,154	4.132,140,149,156
18	4.10	Bending of Curved Members	4.162,169,175,185	4.161,173,181,185	4.164,167,177,183	4.163,171,179,184	4.166,168,176,186	4.165,172,182,186
19	5.1	Shear and Bending-Moment Diagrams	5.7,16,26,31	5.9,20,23,32	5.8,15,25,29	5.12,18,21,33	5.14,19,24,28	5.10,17,22,28
20	5.2	Relations Between *w*, *V*, and *M*	5.41,46,61,64	5.43,48,54,64	5.40,47,55,63	5.42,49,56,63	5.44,50,58,62	5.45,51,57,62
21	5.3	Design of Prismatic Beams in Bending	5.69,71,79,92	5.65,76,80,91	5.68,73,81,89	5.70,72,82,89	5.67,78,88,93	5.66,75,87,90
22	5.4	Use of Singularity Functions to Determine *V* and *M*	5.103,108,115,118	5.104,106,114,119	5.100,109,117,120	5.98,107,110,124	5.99,111,113,125	5.101,112,116,121
23	5.5	Nonprismatic Beams	5.128,132,143,151	5.130,139,140,150	5.126,134,147,149	5.127,135,141,149	5.131,137,142,148	5.129,133,146,148
24	6.1-2	Shearing Stresses in a Beam	6.2,12,19,23	6.1,11,18,21	6.7,13,17,24	6.3,9,15,22	6.8,14,20,27	6.6,10,16,25
25	6.3-5	Shearing Stresses in Thin-Walled Members	6.31,37,43,56	6.29,35,45,57	6.32,40,44,51	6.30,38,46,59	6.34,41,50,53	6.33,39,47,54
26	6.6	Shear Center	6.61,69,76,87	6.62,71,77,88	6.63,70,75,81	6.64,72,78,82	6.66,73,79,85	6.67,74,80,86
27	7.1	Transformation of Plane Stress	7.2,12,18,24	7.4,10,19,25	7.1,9,21,26	7.3,11,17,23	7.6,14,20,28	7.8,16,22,30
28	7.2	Mohr's Circle for Plane Stress	7.33,39,41,61	7.34,40,43,55	7.32,35,47,49	7.31,37,46,51	7.38,44,50,60	7.36,42,52,54
29	7.3-5	Three-Dimensional Stress Analysis; Fracture Criteria	7.69,77,82,91	7.68,74,81,92	7.73,75,84,89	7.66,79,83,90	7.72,76,86,93	7.67,78,85,97
30	7.6	Stresses in Thin-Walled Pressure Vessels	7.100,108,117,124	7.102,106,118,122	7.98,111,112,120	7.103,109,113,121	7.99,110,116,123	7.101,107,119,125
31	7.7-9	Analysis and Measurement of Strain	7.131,140,147,150	7.133,136,148,151	7.128,143,146,157	7.132,139,149,156	7.130,138,145,152	7.134,142,144,153
32	8.1-2	Principal Stresses in a Beam; Design of Shafts	8.4,12,17,20	8.3,11,15,19	8.2,13,21,24	8.1,9,22,23	8.7,10,16,25	8.8,14,18,26
33	8.3	Stresses under Combined Loadings	8.31,40,49,53	8.32,38,46,55	8.35,39,48,58	8.36,37,47,57	8.34,44,50,54	8.33,41,45,56
34	9.1-2	Deflection of Beams by Integration	9.3,11,24,27	9.4,10,19,26	9.1,12,23,25	9.2,9,20,28	9.8,14,22,29	9.6,15,21,31
35	9.3	Use of Singularity Functions	9.37,45,50,62	9.38,41,56,59	9.35,46,49,61	9.36,43,53,60	9.39,47,55,63	9.40,42,52,64
36	9.4	Method of Superposition; Statically Indet. Beams	9.65,76,79,87	9.67,75,80,86	9.66,77,82,88	9.68,73,84,85	9.70,74,83,92	9.69,78,81,89
37	9.5	Moment-Area Method	9.97,104,111,119	9.98,102,112,118	9.96,103,109,113	9.95,101,110,115	9.100,106,114,117	9.99,108,116,121
38	9.6	Moment-Area Method; Max Deflection; Indet. Beams	9.129,137,146,148	9.125,135,142,154	9.132,133,144,153	9.128,136,145,147	9.131,134,143,149	9.127,138,141,151
39	10.1	Columns: Euler's Formula	10.2,11,19,22	10.1,10,16,21	10.6,13,17,27	10.4,9,15,24	10.8,14,20,26	10.5,12,18,23
40	10.2	Eccentric Loading and Secant Formula	10.34,40,49,56	10.32,39,45,51	10.30,41,47,54	10.29,35,43,53	10.33,42,48,55	10.31,38,46,52
41	10.3	Design of Columns under a Centric Load	10.62,65,71,79	10.60,68,75,78	10.64,66,72,80	10.58,69,74,77	10.63,67,73,81	10.61,70,76,82
42	10.4	Design of Columns under an Eccentric Load	10.92,101,107,114	10.91,102,105,113	10.93,97,109,116	10.89,95,103,111	10.94,98,112,115	10.90,96,108,110
43	11.1-3	Elastic Strain Energy	11.6,11,20,28	11.2,9,21,25	11.5,12,18,30	11.1,10,23,24	11.8,14,19,29	11.4,13,22,26
44	11.4-5	Design for Impact Loading; Deflections	11.44,52,63,76	11.43,50,59,66	11.45,54,62,75	11.42,48,58,68	11.47,55,64,74	11.46,49,60,69
45	11.6-8	Castigliano's Theorem	11.80,83,91,99	11.82,88,97,103	11.77,85,93,101	11.78,90,95,102	11.79,84,92,100	11.81,87,98,104
46	11.9	Statically Indeterminate Structures	11.109,113,117	11.107,114,118	11.106,111,117	11.105,112,119	11.110,116,120	11.108,115,122

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TABLE IV: SAMPLE ASSIGNMENT SCHEDULE FOR A COURSE IN MECHANICS OF MATERIALS *Mechanics of Materials 7th Edition*

The four lists of suggested assignments contained in this table cover the entire text. The instructor can select those assignments that are covered in the course.

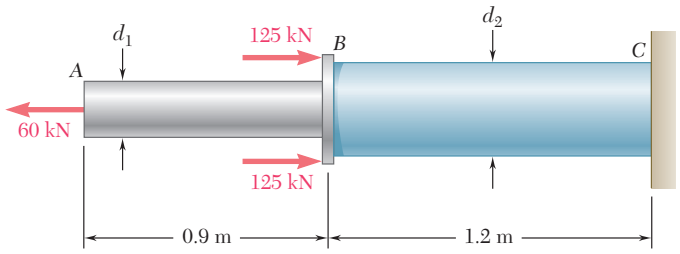
75% OF THE PROBLEMS IN EACH OF THESE LISTS USE SI UNITS AND 25% U.S. CUSTOMARY UNITS

ANSWERS TO ALL OF THESE PROBLEMS ARE GIVEN IN THE BACK OF THE BOOK

Group	Sections	Topics	List 1a	List 2a	List 3a	List 4a
1	1.1	Introduction				
2	1.2	Stresses in the Members of a Structure	1.3,7,20,25	1.4,13,21,24	1.1,9,15,23	1.2,14,18,28
3	1.3-5	Components of Stress; Factor of Safety	1.29,37,48,54	1.30,42,47,53	1.36,40,46,56	1.35,41,45,55
4	2.1	Stress-Strain Diagram	2.1,13,19,26	2.4,11,20,25	2.6,14,15,24	2.3,9,17,21
5	2.2-3	Statically Indeterminate Problems	2.35,42,54,55	2.36,41,51,56	2.33,45,48,58	2.34,43,47,59
6	2.4-9	Generalized Hooke's Law; Poisson's Ratio	2.66,68,81,84	2.61,67,82,88	2.63,74,77,85	2.64,73,78,83
7	2.10-13	Stress Concentration; Plastic Behavior	2.93,102,108,122	2.94,101,107,123	2.98,105,112,117	2.97,106,111,118
8	3.1	Stresses in Torsion	3.1,9,18,28	3.2,10,20,27	3.3,11,15,22	3.6,13,16,21
9	3.2-3	Angle of Twist, Indeterminate Shafts	3.32,38,49,57	3.33,42,50,56	3.31,37,45,53	3.35,42,48,54
10	3.4-5	Transmission Shafts: Stress Concentrations	3.67,77,81,88	3.65,76,80,87	3.69,73,79,90	3.68,71,78,84
11	3.6-8	Plastic Deformations; Residual Stresses	3.94,100,107,119	3.93,98,106,118	3.95,104,113,115	3.92,102,112,114
12	3.9-10	Noncircular and Thin-Walled members	3.124,133,135,144	3.123,129,137,147	3.128,132,136,142	3.127,131,138,143
13	4.1-3	Stresses and Deformations in the Elastic Range	4.3,10,19,25	4.1,9,16,22	4.6,12,18,24	4.4,11,15,21
14	4.4-5	Members Made of Composite Materials	4.37,40,53,61	4.38,44,54,65	4.33,39,49,63	4.34,43,50,64
15	4.6	Plastic Deformations; Residual Stresses	4.68,79,82,88	4.67,80,81,87	4.72,77,86,91	4.71,78,84,91
16	4.7	Eccentric Axial Loading in a Plane of Symmetry	4.101,108,117,121	4.100,106,120,122	4.103,110,115,124	4.99,107,116,125
17	4.8-9	Unsymmetric Bending	4.127,135,144,153	4.129,138,145,155	4.133,137,146,152	4.134,139,147,153
18	4.10	Bending of Curved Members	4.162,169,178,185	4.161,173,180,185	4.164,170,177,183	4.163,174,179,184
19	5.1	Shear and Bending-Moment Diagrams	5.7,16,26,30	5.9,20,23,27	5.13,15,25,29	5.11,18,21,33
20	5.2	Relations Between *w, V,* and *M*	5.41,46,60,64	5.43,48,59,64	5.40,53,55,63	5.42,52,56,63
21	5.3	Design of Prismatic Beams in Bending	5.69,71,79,85	5.65,76,80,86	5.73,77,81,89	5.70,74,82,89
22	5.4	Use of Singularity Functions to Determine *V* and *M*	5.103,108,118,123	5.104,106,119,122	5.100,105,117,120	5.98,102,110,124
23	5.5	Nonprismatic Beams	5.128,132,143,145	5.130,139,140,144	5.134,136,147,149	5.135,138,141,149
24	6.1-2	Shearing Stresses in a Beam	6.2,12,19,28	6.1,11,18,26	6.5,13,17,24	6.4,9,15,22
25	6.3-5	Shearing Stresses in Thin-Walled Members	6.31,37,49,56	6.29,35,48,57	6.32,42,44,51	6.30,36,46,59
26	6.6	Shear center	6.61,69,76,83	6.62,71,77,84	6.65,70,75,81	6.68,72,78,82
27	7.1	Transformation of Plane Stress	7.2,12,18,27	7.4,10,19,29	7.7,9,21,26	7.5,11,17,23
28	7.2	Mohr's Circle for Plane Stress	7.33,41,58,61	7.34,40,48,55	7.32,47,49,59	7.31,45,46,51
29	7.3-5	Three-Dimensional Stress Analysis; Fracture Criteria	7.69,77,82,94	7.68,74,81,95	7.70,75,84,89	7.71,79,83,90
30	7.6	Stresses in Thin-Walled Pressure Vessels	7.100,108,114,124	7.102,106,115,122	7.98,105,112,120	7.103,104,113,121
31	7.7-9	Analysis and Measurement of Strain	7.131,140,147,155	7.133,136,148,154	7.129,143,146,157	7.135,139,149,156
32	8.1-2	Principal Stresses in a Beam; Design of Shafts	8.4,12,17,27	8.3,11,15,28	8.5,13,21,24	8.6,9,22,23
33	8.3	Stresses under Combined Loadings	8.31,40,51,53	8.32,38,52,55	8.35,43,48,58	8.36,42,47,57
34	9.1-2	Deflection of Beams by Integration	9.3,11,24,30	9.4,10,19,32	9.7,12,23,25	9.5,9,20,28
35	9.3	Use of Singularity Functions	9.37,45,54,62	9.38,41,51,59	9.35,48,49,61	9.36,44,53,60
36	9.4	Method of Superposition; Statically Indet. Beams	9.65,76,79,91	9.67,75,80,90	9.72,77,82,88	9.71,73,84,85
37	9.5	Moment-Area Method	9.97,104,119,122	9.98,102,118,120	9.96,105,109,113	9.95,107,110,115
38	9.6	Moment-Area Method; Max Deflection; Indet. Beams	9.129,137,146,152	9.125,135,142,150	9.130,133,144,153	9.126,136,145,147
39	10.1	Columns: Euler's Formula	10.2,11,19,28	10.1,10,16,25	10.3,13,17,27	10.7,9,15,24
40	10.2	Eccentric Loading; Secant Formula	10.34,40,50,56	10.32,39,44,51	10.30,37,47,54	10.29,36,43,53
41	10.3	Design of Columns under a Centric Load	10.62,65,71,83	10.60,68,75,84	10.59,66,72,80	10.57,69,74,77
42	10.4	Design of Columns under an Eccentric Load	10.92,101,106,107	10.91,102,104,105	10.93,99,109,116	10.89,100,103,111
43	11.1-3	Elastic Strain Energy	11.6,11,20,31	11.2,9,21,27	11.7,12,18,30	11.3,10,23,24
44	11.4-5	Design for Impact Loading; Deflections	11.44,52,65,76	11.43,50,61,66	11.45,53,62,75	11.42,51,58,68
45	11.6-8	Castigliano's Theorem	11.80,83,94,99	11.82,88,96,103	11.77,86,93,101	11.78,89,95,102
46	11.9	Statically Indeterminate Structures	11.109,113,121	11.107,114,121	11.106,113,117	11.105,112,119

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CHAPTER 1



PROBLEM 1.1

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $d_1 = 30$ mm and $d_2 = 50$ mm, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

(*a*) Rod *AB*:

Force: $P = 60 \times 10^3$ N tension

Area: $A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30 \times 10^{-3})^2 = 706.86 \times 10^{-6}\ \text{m}^2$

Normal stress: $\sigma_{AB} = \frac{P}{A} = \frac{60 \times 10^3}{706.86 \times 10^{-6}} = 84.882 \times 10^6\ \text{Pa}$

$\sigma_{AB} = 84.9$ MPa ◀

(*b*) Rod *BC*:

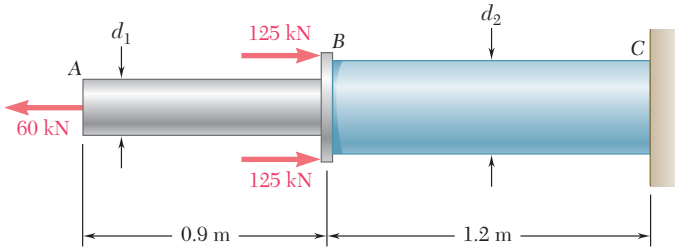
Force: $P = 60 \times 10^3 - (2)(125 \times 10^3) = -190 \times 10^3$ N

Area: $A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(50 \times 10^{-3})^2 = 1.96350 \times 10^{-3}\ \text{m}^2$

Normal stress: $\sigma_{BC} = \frac{P}{A} = \frac{-190 \times 10^3}{1.96350 \times 10^{-3}} = -96.766 \times 10^6\ \text{Pa}$

$\sigma_{BC} = -96.8$ MPa ◀

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PROBLEM 1.2

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that the average normal stress must not exceed 150 MPa in either rod, determine the smallest allowable values of the diameters d_1 and d_2.

SOLUTION

(*a*) Rod *AB*:

Force: $P = 60 \times 10^3 \text{ N}$

Stress: $\sigma_{AB} = 150 \times 10^6 \text{ Pa}$

Area: $A = \dfrac{\pi}{4} d_1^2$

$$\sigma_{AB} = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma_{AB}}$$

$$\frac{\pi}{4} d_1^2 = \frac{P}{\sigma_{AB}}$$

$$d_1^2 = \frac{4P}{\pi \sigma_{AB}} = \frac{(4)(60 \times 10^3)}{\pi (150 \times 10^6)} = 509.30 \times 10^{-6} \text{ m}^2$$

$$d_1 = 22.568 \times 10^{-3} \text{ m} \qquad d_1 = 22.6 \text{ mm} \blacktriangleleft$$

(*b*) Rod *BC*:

Force: $P = 60 \times 10^3 - (2)(125 \times 10^3) = -190 \times 10^3 \text{ N}$

Stress: $\sigma_{BC} = -150 \times 10^6 \text{ Pa}$

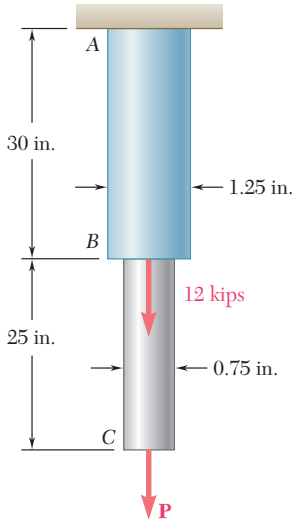
Area: $A = \dfrac{\pi}{4} d_2^2$

$$\sigma_{BC} = \frac{P}{A} = \frac{4P}{\pi d_2^2}$$

$$d_2^2 = \frac{4P}{\pi \sigma_{BC}} = \frac{(4)(-190 \times 10^3)}{\pi (-150 \times 10^6)} = 1.61277 \times 10^{-3} \text{ m}^2$$

$$d_2 = 40.159 \times 10^{-3} \text{ m} \qquad d_2 = 40.2 \text{ mm} \blacktriangleleft$$

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PROBLEM 1.3

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $P = 10$ kips, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

(*a*) Rod *AB*:

$$P = 12 + 10 = 22 \text{ kips}$$

$$A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(1.25)^2 = 1.22718 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{22}{1.22718} = 17.927 \text{ ksi} \qquad \sigma_{AB} = 17.93 \text{ ksi} \blacktriangleleft$$

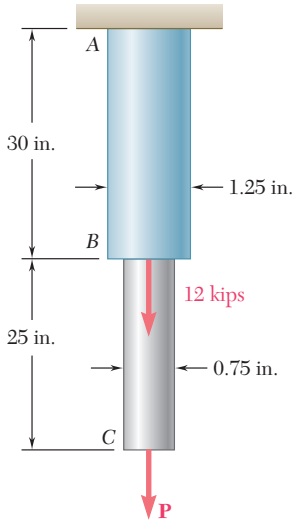
(*b*) Rod *BC*:

$$P = 10 \text{ kips}$$

$$A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{10}{0.44179} = 22.635 \text{ ksi} \qquad \sigma_{AB} = 22.6 \text{ ksi} \blacktriangleleft$$

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PROBLEM 1.4

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force **P** for which the tensile stresses in rods *AB* and *BC* are equal.

SOLUTION

(*a*) Rod *AB*:

$$P = P + 12 \text{ kips}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4}(1.25 \text{ in.})^2$$

$$A = 1.22718 \text{ in}^2$$

$$\sigma_{AB} = \frac{P + 12 \text{ kips}}{1.22718 \text{ in}^2}$$

(*b*) Rod *BC*:

$$P = P$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.75 \text{ in.})^2$$

$$A = 0.44179 \text{ in}^2$$

$$\sigma_{BC} = \frac{P}{0.44179 \text{ in}^2}$$

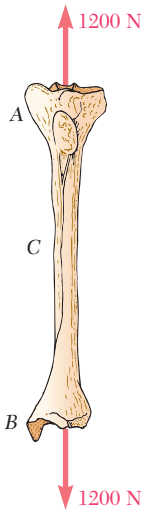
$$\sigma_{AB} = \sigma_{BC}$$

$$\frac{P + 12 \text{ kips}}{1.22718 \text{ in}^2} = \frac{P}{0.44179 \text{ in}^2}$$

$$5.3015 = 0.78539P$$

$$P = 6.75 \text{ kips} \blacktriangleleft$$

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PROBLEM 1.5

A strain gage located at *C* on the surface of bone *AB* indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at *C* to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at *C*.

SOLUTION

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

Geometry: $A = \frac{\pi}{4}(d_1^2 - d_2^2)$

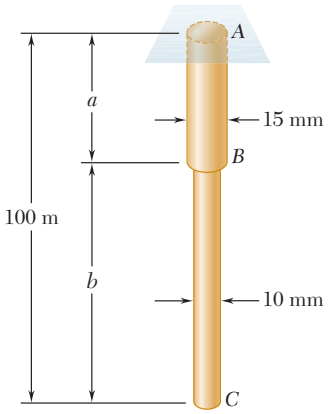
$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

$$d_2^2 = (25 \times 10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80 \times 10^6)}$$

$$= 222.92 \times 10^{-6}\ \text{m}^2$$

$$d_2 = 14.93 \times 10^{-3}\ \text{m} \qquad d_2 = 14.93\ \text{mm} \blacktriangleleft$$

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PROBLEM 1.6

Two brass rods *AB* and *BC*, each of uniform diameter, will be brazed together at *B* to form a nonuniform rod of total length 100 m, which will be suspended from a support at *A* as shown. Knowing that the density of brass is 8470 kg/m^3, determine (*a*) the length of rod *AB* for which the maximum normal stress in *ABC* is minimum, (*b*) the corresponding value of the maximum normal stress.

SOLUTION

Areas: $$A_{AB} = \frac{\pi}{4}(15\text{ mm})^2 = 176.715\text{ mm}^2 = 176.715\times10^{-6}\text{m}^2$$

$$A_{BC} = \frac{\pi}{4}(10\text{ mm})^2 = 78.54\text{ mm}^2 = 78.54\times10^{-6}\text{m}^2$$

From geometry, $$b = 100 - a$$

Weights: $$W_{AB} = \rho g\, A_{AB}\ell_{AB} = (8470)(9.81)(176.715\times10^{-6})\,a = 14.683a$$

$$W_{BC} = \rho g\, A_{BC}\ell_{BC} = (8470)(9.81)(78.54\times10^{-6})(100 - a) = 652.59 - 6.526a$$

Normal stresses:

At *A*, $$P_A = W_{AB} + W_{BC} = 652.59 + 8.157a \qquad (1)$$

$$\sigma_A = \frac{P_A}{A_{AB}} = 3.6930\times10^6 + 46.160\times10^3 a$$

At *B*, $$P_B = W_{BC} = 652.59 - 6.526a \qquad (2)$$

$$\sigma_B = \frac{P_B}{A_{BC}} = 8.3090\times10^6 - 83.090\times10^3 a$$

(*a*) Length of rod *AB*. The maximum stress in *ABC* is minimum when $\sigma_A = \sigma_B$ or

$$4.6160\times10^6 - 129.25\times10^3 a = 0$$

$$a = 35.71\text{ m} \qquad \ell_{AB} = a = 35.7\text{ m} \blacktriangleleft$$

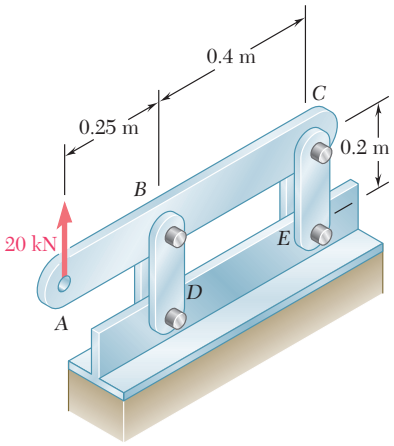
(*b*) Maximum normal stress.

$$\sigma_A = 3.6930\times10^6 + (46.160\times10^3)(35.71)$$

$$\sigma_B = 8.3090\times10^6 - (83.090\times10^3)(35.71)$$

$$\sigma_A = \sigma_B = 5.34\times10^6\text{ Pa} \qquad \sigma = 5.34\text{ MPa} \blacktriangleleft$$

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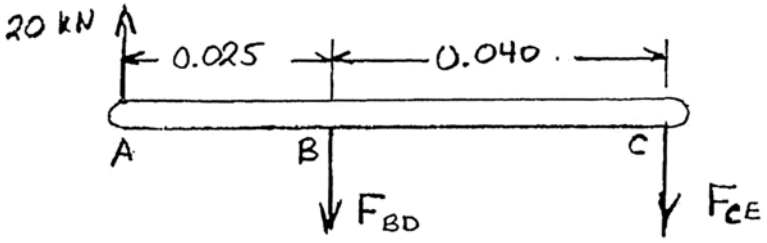


PROBLEM 1.7

Each of the four vertical links has an 8×36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

SOLUTION

Use bar *ABC* as a free body.



$$\Sigma M_C = 0: \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N} \qquad \text{Link } BD \text{ is in tension.}$$

$$\Sigma M_B = 0: \quad -(0.040)F_{CE} - (0.025)(20 \times 10^3) = 0$$

$$F_{CE} = -12.5 \times 10^3 \text{ N} \qquad \text{Link } CE \text{ is in compression.}$$

Net area of one link for tension $= (0.008)(0.036 - 0.016) = 160 \times 10^{-6} \text{ m}^2$

For two parallel links, $A_{\text{net}} = 320 \times 10^{-6} \text{ m}^2$

(*a*) $$\sigma_{BD} = \frac{F_{BD}}{A_{\text{net}}} = \frac{32.5 \times 10^3}{320 \times 10^{-6}} = 101.563 \times 10^6$$

$\sigma_{BD} = 101.6$ MPa ◀

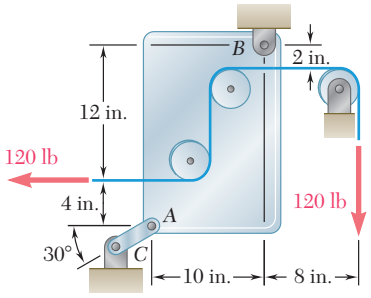
Area for one link in compression $= (0.008)(0.036) = 288 \times 10^{-6} \text{ m}^2$

For two parallel links, $A = 576 \times 10^{-6} \text{ m}^2$

(*b*) $$\sigma_{CE} = \frac{F_{CE}}{A} = \frac{-12.5 \times 10^3}{576 \times 10^{-6}} = -21.701 \times 10^{-6}$$

$\sigma_{CE} = -21.7$ MPa ◀

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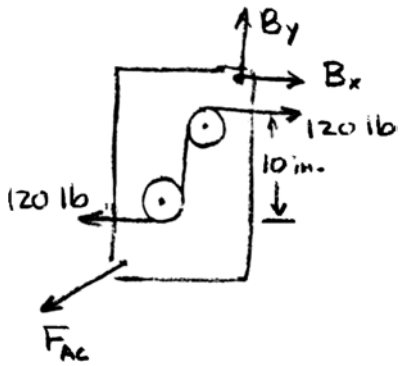


PROBLEM 1.8

Link AC has a uniform rectangular cross section $\frac{1}{8}$ in. thick and 1 in. wide. Determine the normal stress in the central portion of the link.

SOLUTION

Use the plate together with two pulleys as a free body. Note that the cable tension causes at 1200 lb-in. clockwise couple to act on the body.



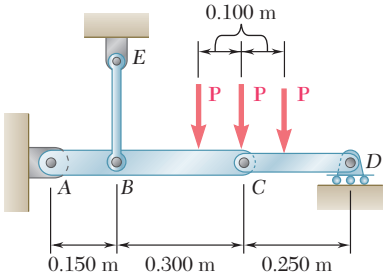
$$+\circlearrowleft \Sigma M_B = 0: \quad -(12+4)(F_{AC}\cos 30°) + (10)(F_{AC}\sin 30°) - 1200 \text{ lb} = 0$$

$$F_{AC} = -\frac{1200 \text{ lb}}{16\cos 30° - 10\sin 30°} = -135.500 \text{ lb}$$

Area of link *AC*: $A = 1 \text{ in.} \times \frac{1}{8} \text{ in.} = 0.125 \text{ in}^2$

Stress in link *AC*: $\sigma_{AC} = \frac{F_{AC}}{A} = -\frac{135.50}{0.125} = 1084 \text{ psi} = 1.084 \text{ ksi}$ ◀

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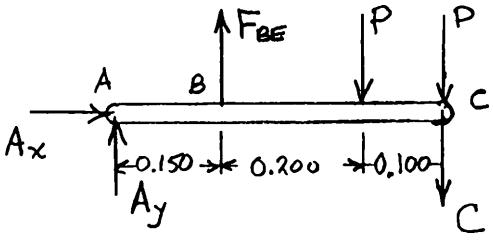


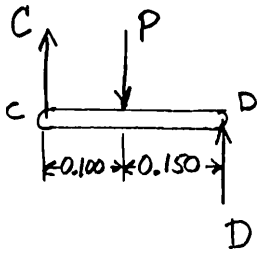
PROBLEM 1.9

Three forces, each of magnitude $P = 4$ kN, are applied to the mechanism shown. Determine the cross-sectional area of the uniform portion of rod *BE* for which the normal stress in that portion is +100 MPa.

SOLUTION

Draw free body diagrams of *AC* and *CD*.





Free Body *CD*:

$$+\circlearrowleft \Sigma M_D = 0: \quad 0.150P - 0.250C = 0$$

$$C = 0.6P$$

Free Body *AC*:

$$+\circlearrowleft M_A = 0: \quad 0.150F_{BE} - 0.350P - 0.450P - 0.450C = 0$$

$$F_{BE} = \frac{1.07}{0.150}P = 7.1333\,P = (7.133)(4\text{ kN}) = 28.533\text{ kN}$$

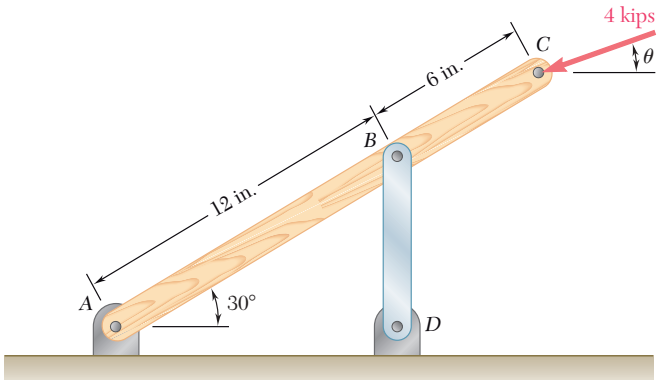
Required area of *BE*:

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}}$$

$$A_{BE} = \frac{F_{BE}}{\sigma_{BE}} = \frac{28.533 \times 10^3}{100 \times 10^6} = 285.33 \times 10^{-6}\text{m}^2$$

$$A_{BE} = 285\text{ mm}^2 \blacktriangleleft$$

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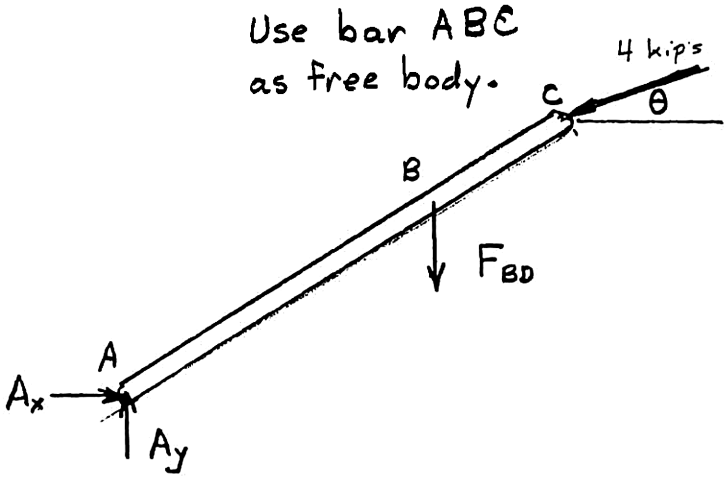


PROBLEM 1.10

Link BD consists of a single bar 1 in. wide and $\frac{1}{2}$ in. thick. Knowing that each pin has a $\frac{3}{8}$-in. diameter, determine the maximum value of the average normal stress in link BD if (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

SOLUTION

Use bar *ABC* as a free body.



(*a*) $\theta = 0$.

$+\circlearrowleft \Sigma M_A = 0$: $(18 \sin 30°)(4) - (12 \cos 30°)F_{BD} = 0$

$F_{BD} = 3.4641$ kips (tension)

Area for tension loading: $A = (b - d)t = \left(1 - \frac{3}{8}\right)\left(\frac{1}{2}\right) = 0.31250 \text{ in}^2$

Stress: $\sigma = \frac{F_{BD}}{A} = \frac{3.4641 \text{ kips}}{0.31250 \text{ in}^2}$ $\sigma = 11.09$ ksi ◀

(*b*) $\theta = 90°$.

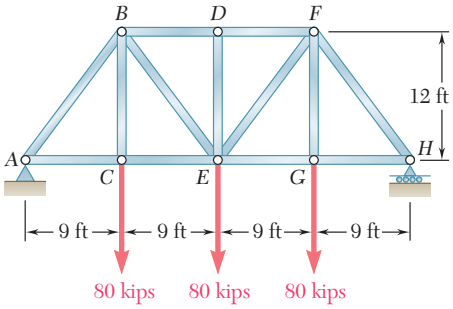
$+\circlearrowleft \Sigma M_A = 0$: $-(18 \cos 30°)(4) - (12 \cos 30°)F_{BD} = 0$

$F_{BD} = -6$ kips i.e. compression.

Area for compression loading: $A = bt = (1)\left(\frac{1}{2}\right) = 0.5 \text{ in}^2$

Stress: $\sigma = \frac{F_{BD}}{A} = \frac{-6 \text{ kips}}{0.5 \text{ in}^2}$ $\sigma = 12.00$ ksi ◀

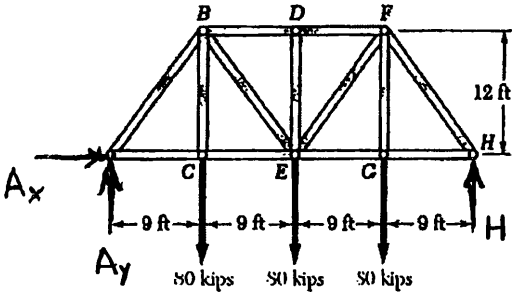
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PROBLEM 1.11

For the Pratt bridge truss and loading shown, determine the average normal stress in member *BE*, knowing that the cross-sectional area of that member is 5.87 in^2.

SOLUTION

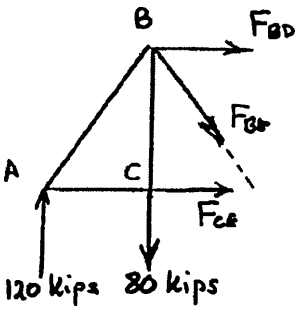


Use entire truss as free body.

$$+\circlearrowleft \Sigma M_H = 0: \quad (9)(80) + (18)(80) + (27)(80) - 36A_y = 0$$

$$A_y = 120 \text{ kips}$$

Use portion of truss to the left of a section cutting members *BD*, *BE*, and *CE*.

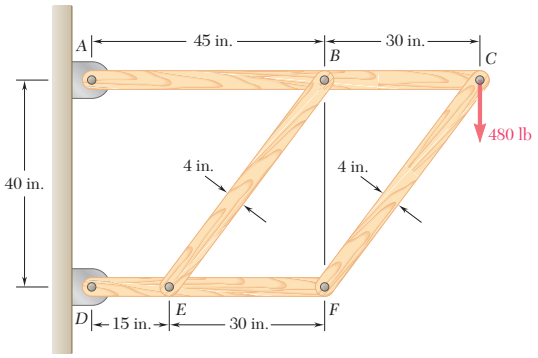


$$+\uparrow \Sigma F_y = 0: \quad 120 - 80 - \frac{12}{15}F_{BE} = 0 \qquad \therefore F_{BE} = 50 \text{ kips}$$

$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{50 \text{ kips}}{5.87 \text{ in}^2}$$

$$\sigma_{BE} = 8.52 \text{ ksi} \blacktriangleleft$$

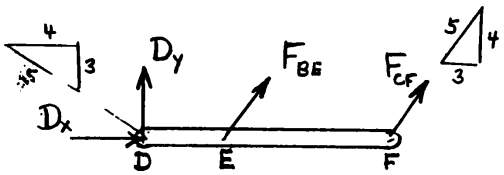
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PROBLEM 1.12

The frame shown consists of *four* wooden members, *ABC*, *DEF*, *BE*, and *CF*. Knowing that each member has a 2 × 4-in. rectangular cross section and that each pin has a $\frac{1}{2}$-in. diameter, determine the maximum value of the average normal stress (*a*) in member *BE*, (*b*) in member *CF*.

SOLUTION



Add support reactions to figure as shown.

Using entire frame as free body,

$$\Sigma M_A = 0: \quad 40D_x - (45 + 30)(480) = 0$$

$$D_x = 900 \text{ lb}$$

Use member DEF as free body.

Reaction at D must be parallel to F_{BE} and F_{CF}.

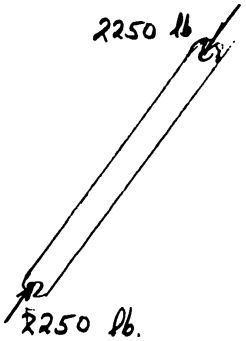
$$D_y = \frac{4}{3}D_x = 1200 \text{ lb}$$

$$\Sigma M_F = 0: \quad -(30)\left(\frac{4}{5}F_{BE}\right) - (30 + 15)D_Y = 0$$

$$F_{BE} = -2250 \text{ lb}$$

$$\Sigma M_E = 0: \quad (30)\left(\frac{4}{5}F_{CE}\right) - (15)D_Y = 0$$

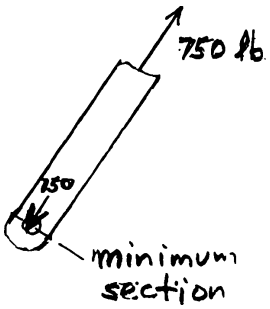
$$F_{CE} = 750 \text{ lb}$$



Stress in compression member *BE*:

Area: $A = 2 \text{ in.} \times 4 \text{ in.} = 8 \text{ in}^2$

(*a*) $$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{-2250}{8} \qquad \sigma_{BE} = -281 \text{ psi} \blacktriangleleft$$



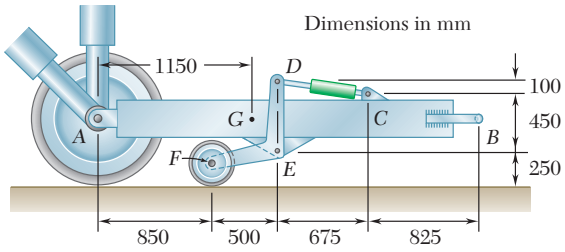
Minimum section area occurs at pin.

$$A_{\min} = (2)(4.0 - 0.5) = 7.0 \text{ in}^2$$

Stress in tension member *CF*:

(*b*) $$\sigma_{CF} = \frac{F_{CF}}{A_{\min}} = \frac{750}{7.0} \qquad \sigma_{CF} = 107.1 \text{ psi} \blacktriangleleft$$

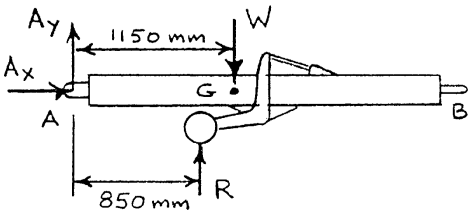
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PROBLEM 1.13

An aircraft tow bar is positioned by means of a single hydraulic cylinder connected by a 25-mm-diameter steel rod to two identical arm-and-wheel units *DEF*. The mass of the entire tow bar is 200 kg, and its center of gravity is located at *G*. For the position shown, determine the normal stress in the rod.

SOLUTION

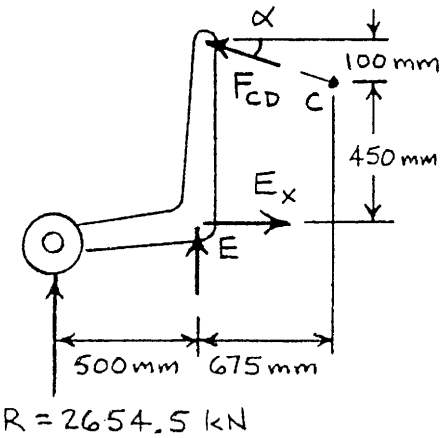


FREE BODY – ENTIRE TOW BAR:

$$W = (200\text{ kg})(9.81\text{ m/s}^2) = 1962.00\text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 850R - 1150(1962.00\text{ N}) = 0$$

$$R = 2654.5\text{ N}$$



FREE BODY – BOTH ARM & WHEEL UNITS:

$$\tan\alpha = \frac{100}{675} \qquad \alpha = 8.4270°$$

$$+\circlearrowleft \Sigma M_E = 0: \quad (F_{CD}\cos\alpha)(550) - R(500) = 0$$

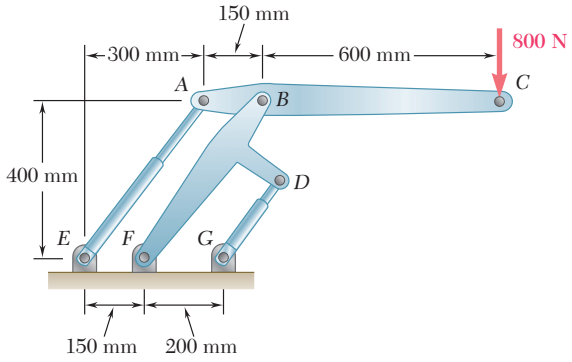
$$F_{CD} = \frac{500}{550\cos 8.4270°}(2654.5\text{ N})$$

$$= 2439.5\text{ N} \quad \text{(comp.)}$$

$$\sigma_{CD} = -\frac{F_{CD}}{A_{CD}} = -\frac{2439.5\text{ N}}{\pi(0.0125\text{ m})^2}$$

$$= -4.9697 \times 10^6\text{ Pa} \qquad \sigma_{CD} = -4.97\text{ MPa} \blacktriangleleft$$

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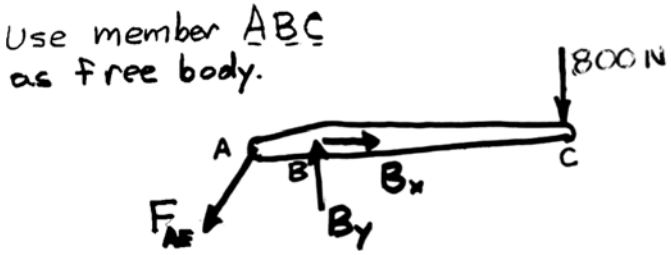


PROBLEM 1.14

Two hydraulic cylinders are used to control the position of the robotic arm *ABC*. Knowing that the control rods attached at *A* and *D* each have a 20-mm diameter and happen to be parallel in the position shown, determine the average normal stress in (*a*) member *AE*, (*b*) member *DG*.

SOLUTION

Use member *ABC* as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad (0.150)\frac{4}{5}F_{AE} - (0.600)(800) = 0$$

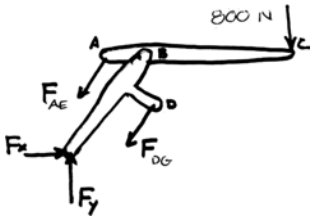
$$F_{AE} = 4 \times 10^3 \text{ N}$$

Area of rod in member *AE* is $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20 \times 10^{-3})^2 = 314.16 \times 10^{-6}\text{m}^2$

Stress in rod *AE*: $\sigma_{AE} = \frac{F_{AE}}{A} = \frac{4 \times 10^3}{314.16 \times 10^{-6}} = 12.7324 \times 10^6 \text{ Pa}$

(*a*) $\sigma_{AE} = 12.73 \text{ MPa}$ ◀

Use combined members *ABC* and *BFD* as free body.



$$+\circlearrowleft \Sigma M_F = 0: \quad (0.150)\left(\frac{4}{5}F_{AE}\right) - (0.200)\left(\frac{4}{5}F_{DG}\right) - (1.050 - 0.350)(800) = 0$$

$$F_{DG} = -1500 \text{ N}$$

Area of rod *DG*: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20 \times 10^{-3})^2 = 314.16 \times 10^{-6}\text{ m}^2$

Stress in rod *DG*: $\sigma_{DG} = \frac{F_{DG}}{A} = \frac{-1500}{3.1416 \times 10^{-6}} = -4.7746 \times 10^6 \text{ Pa}$

(*b*) $\sigma_{DG} = -4.77 \text{ MPa}$ ◀

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PROBLEM 1.15

Determine the diameter of the largest circular hole that can be punched into a sheet of polystyrene 6 mm thick, knowing that the force exerted by the punch is 45 kN and that a 55-MPa average shearing stress is required to cause the material to fail.

SOLUTION

For cylindrical failure surface: $A = \pi dt$

Shearing stress: $\tau = \dfrac{P}{A}$ or $A = \dfrac{P}{\tau}$

Therefore, $\dfrac{P}{\tau} = \pi dt$

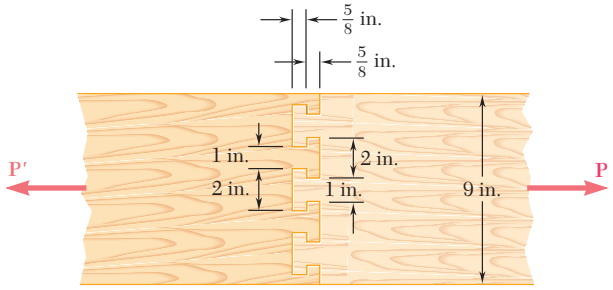
Finally, $d = \dfrac{P}{\pi t \tau}$

$$= \frac{45 \times 10^3 \text{ N}}{\pi(0.006 \text{ m})(55 \times 10^6 \text{ Pa})}$$

$$= 43.406 \times 10^{-3} \text{ m}$$

$d = 43.4$ mm ◀

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PROBLEM 1.16

Two wooden planks, each $\frac{1}{2}$ in. thick and 9 in. wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 1.20 ksi, determine the magnitude P of the axial load that will cause the joint to fail.

SOLUTION

Six areas must be sheared off when the joint fails. Each of these areas has dimensions $\frac{5}{8}$ in. $\times \frac{1}{2}$ in., its area being

$$A = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16} \text{ in}^2 = 0.3125 \text{ in}^2$$

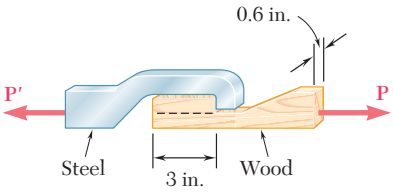
At failure, the force carried by each area is

$$F = \tau A = (1.20 \text{ ksi})(0.3125 \text{ in}^2) = 0.375 \text{ kips}$$

Since there are six failure areas,

$$P = 6F = (6)(0.375) \qquad P = 2.25 \text{ kips} \blacktriangleleft$$

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PROBLEM 1.17

When the force **P** reached 1600 lb, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.

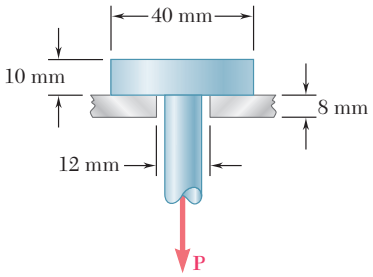
SOLUTION

Area being sheared: $A = 3 \text{ in.} \times 0.6 \text{ in.} = 1.8 \text{ in}^2$

Force: $P = 1600 \text{ lb}$

Shearing stress: $\tau = \dfrac{P}{A} = \dfrac{1600 \text{ lb}}{1.8 \text{ in}^2} = 8.8889 \times 10^2 \text{ psi}$ $\tau = 889 \text{ psi}$ ◀

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PROBLEM 1.18

A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 12-mm-diameter hole has been drilled. Knowing that the shearing stress must not exceed 180 MPa in the steel rod and 70 MPa in the aluminum plate, determine the largest load **P** that can be applied to the rod.

SOLUTION

For steel:

$$A_1 = \pi dt = \pi(0.012\text{ m})(0.010\text{ m})$$
$$= 376.99 \times 10^{-6}\text{ m}^2$$

$$\tau_1 = \frac{P}{A} \therefore P = A_1\tau_1 = (376.99 \times 10^{-6}\text{ m}^2)(180 \times 10^{6}\text{ Pa})$$
$$= 67.858 \times 10^{3}\text{ N}$$

For aluminum:

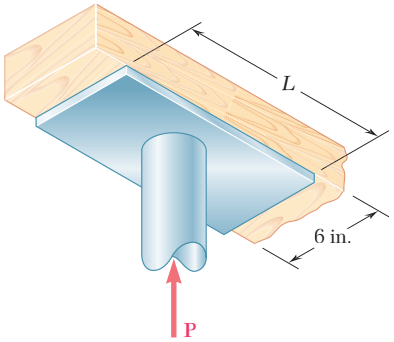
$$A_2 = \pi dt = \pi(0.040\text{ m})(0.008\text{ m}) = 1.00531 \times 10^{-3}\text{ m}^2$$

$$\tau_2 = \frac{P}{A_2} \therefore P = A_2\tau_2 = (1.00531 \times 10^{-3}\text{ m}^2)(70 \times 10^{6}\text{ Pa}) = 70.372 \times 10^{3}\text{ N}$$

Limiting value of P is the smaller value, so

$$P = 67.9\text{ kN} \blacktriangleleft$$

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PROBLEM 1.19

The axial force in the column supporting the timber beam shown is $P = 20$ kips. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 400 psi.

SOLUTION

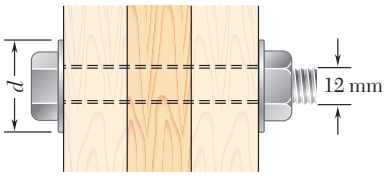
Bearing area: $A_b = Lw$

$$\sigma_b = \frac{P}{A_b} = \frac{P}{Lw}$$

$$L = \frac{P}{\sigma_b w} = \frac{20 \times 10^3 \text{ lb}}{(400 \text{ psi})(6 \text{ in.})} = 8.33 \text{ in.}$$

$L = 8.33$ in. ◀

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PROBLEM 1.20

Three wooden planks are fastened together by a series of bolts to form a column. The diameter of each bolt is 12 mm and the inner diameter of each washer is 16 mm, which is slightly larger than the diameter of the holes in the planks. Determine the smallest allowable outer diameter d of the washers, knowing that the average normal stress in the bolts is 36 MPa and that the bearing stress between the washers and the planks must not exceed 8.5 MPa.

SOLUTION

Bolt: $$A_{\text{Bolt}} = \frac{\pi d^2}{4} = \frac{\pi(0.012\text{ m})^2}{4} = 1.13097 \times 10^{-4}\text{ m}^2$$

Tensile force in bolt: $$\sigma = \frac{P}{A} \quad \Rightarrow \quad P = \sigma A$$

$$= (36 \times 10^6\text{ Pa})(1.13097 \times 10^{-4}\text{ m}^2)$$

$$= 4.0715 \times 10^3\text{ N}$$

Bearing area for washer: $$A_w = \frac{\pi}{4}\left(d_o^2 - d_i^2\right)$$

and $$A_w = \frac{P}{\sigma_{BRG}}$$

Therefore, equating the two expressions for A_w gives

$$\frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{P}{\sigma_{BRG}}$$

$$d_o^2 = \frac{4P}{\pi\sigma_{BRG}} + d_i^2$$

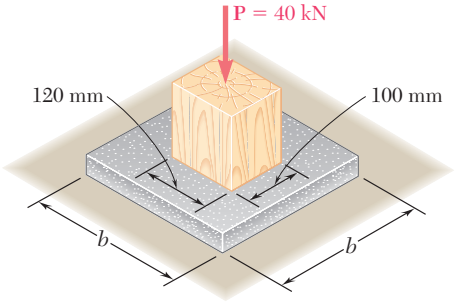
$$d_o^2 = \frac{4}{\pi}\frac{(4.0715 \times 10^3\text{ N})}{(8.5 \times 10^6\text{ Pa})} + (0.016\text{ m})^2$$

$$d_o^2 = 8.6588 \times 10^{-4}\text{ m}^2$$

$$d_o = 29.426 \times 10^{-3}\text{ m}$$

$$d_o = 29.4\text{ mm} \blacktriangleleft$$

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PROBLEM 1.21

A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

SOLUTION

(*a*) Bearing stress on concrete footing.

$$P = 40 \text{ kN} = 40 \times 10^3 \text{ N}$$

$$A = (100)(120) = 12 \times 10^3 \text{ mm}^2 = 12 \times 10^{-3} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{12 \times 10^{-3}} = 3.3333 \times 10^6 \text{ Pa} \qquad 3.33 \text{ MPa} \blacktriangleleft$$

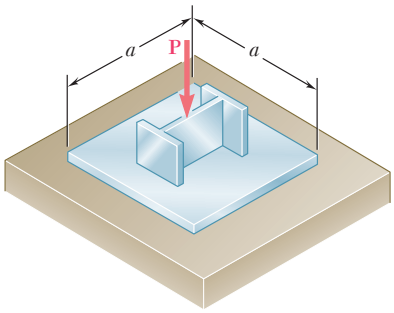
(*b*) Footing area. $P = 40 \times 10^3$ N $\quad \sigma = 145$ kPa $= 45 \times 10^3$ Pa

$$\sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586 \text{ m}^2$$

Since the area is square, $A = b^2$

$$b = \sqrt{A} = \sqrt{0.27586} = 0.525 \text{ m} \qquad b = 525 \text{ mm} \blacktriangleleft$$

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PROBLEM 1.22

An axial load **P** is supported by a short $W8 \times 40$ column of cross-sectional area $A = 11.7\ \text{in}^2$ and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 30 ksi and that the bearing stress on the concrete foundation must not exceed 3.0 ksi, determine the side a of the plate that will provide the most economical and safe design.

SOLUTION

For the column, $\sigma = \dfrac{P}{A}$ or

$$P = \sigma A = (30)(11.7) = 351 \text{ kips}$$

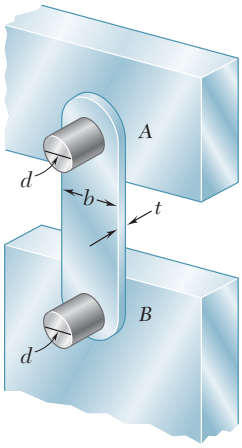
For the $a \times a$ plate, $\sigma = 3.0$ ksi

$$A = \frac{P}{\sigma} = \frac{351}{3.0} = 117\ \text{in}^2$$

Since the plate is square, $A = a^2$

$$a = \sqrt{A} = \sqrt{117} \qquad a = 10.82 \text{ in.} \blacktriangleleft$$

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PROBLEM 1.23

Link AB, of width $b = 2$ in. and thickness $t = \frac{1}{4}$ in., is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is −20 ksi and that the average shearing stress in each of the two pins is 12 ksi, determine (*a*) the diameter *d* of the pins, (*b*) the average bearing stress in the link.

SOLUTION

Rod *AB* is in compression.

$$A = bt \quad \text{where} \quad b = 2 \text{ in.} \quad \text{and} \quad t = \frac{1}{4} \text{ in.}$$

$$P = -\sigma A = -(-20)(2)\left(\frac{1}{4}\right) = 10 \text{ kips}$$

Pin: $$\tau_P = \frac{P}{A_P}$$

and $$A_P = \frac{\pi}{4} d^2$$

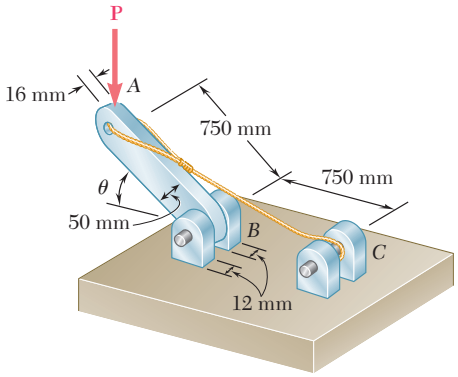
(*a*) $$d = \sqrt{\frac{4A_P}{\pi}} = \sqrt{\frac{4P}{\pi\tau_P}} = \sqrt{\frac{(4)(10)}{\pi(12)}} = 1.03006 \text{ in.}$$

$$d = 1.030 \text{ in.} \blacktriangleleft$$

(*b*) $$\sigma_b = \frac{P}{dt} = \frac{10}{(1.03006)(0.25)} = 38.833 \text{ ksi}$$

$$\sigma_b = 38.8 \text{ ksi} \blacktriangleleft$$

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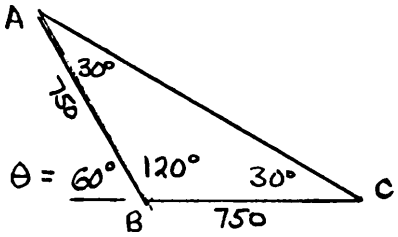


PROBLEM 1.24

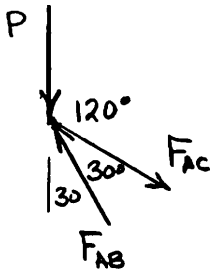
Determine the largest load **P** which may be applied at *A* when $\theta = 60°$, knowing that the average shearing stress in the 10-mm-diameter pin at *B* must not exceed 120 MPa and that the average bearing stress in member *AB* and in the bracket at *B* must not exceed 90 MPa.

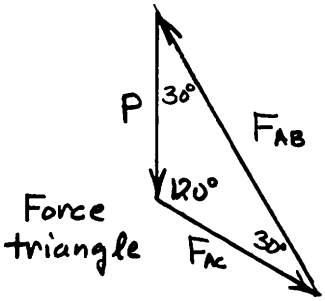
SOLUTION

Geometry: Triangle *ABC* is an isoseles triangle with angles shown here.



Use joint *A* as a free body.





Law of sines applied to force triangle:

$$\frac{P}{\sin 30°} = \frac{F_{AB}}{\sin 120°} = \frac{F_{AC}}{\sin 30°}$$

$$P = \frac{F_{AB} \sin 30°}{\sin 120°} = 0.57735 F_{AB}$$

$$P = \frac{F_{AC} \sin 30°}{\sin 30°} = F_{AC}$$

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PROBLEM 1.24 *(Continued)*

If shearing stress in pin at *B* is critical,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\,\text{m}^2$$

$$F_{AB} = 2A\tau = (2)(78.54 \times 10^{-6})(120 \times 10^{6}) = 18.850 \times 10^{3}\,\text{N}$$

If bearing stress in member *AB* at bracket at *A* is critical,

$$A_b = td = (0.016)(0.010) = 160 \times 10^{-6}\,\text{m}^2$$

$$F_{AB} = A_b\sigma_b = (160 \times 10^{-6})(90 \times 10^{6}) = 14.40 \times 10^{3}\,\text{N}$$

If bearing stress in the bracket at *B* is critical,

$$A_b = 2td = (2)(0.012)(0.010) = 240 \times 10^{-6}\,\text{m}^2$$

$$F_{AB} = A_b\sigma_b = (240 \times 10^{-6})(90 \times 10^{6}) = 21.6 \times 10^{3}\,\text{N}$$

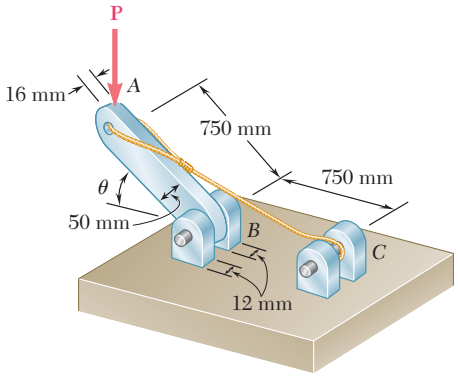
Allowable F_{AB} is the smallest, i.e., 14.40×10^{3} N

Then from statics, $P_{\text{allow}} = (0.57735)(14.40 \times 10^{3})$

$= 8.31 \times 10^{3}$ N

8.31 kN ◀

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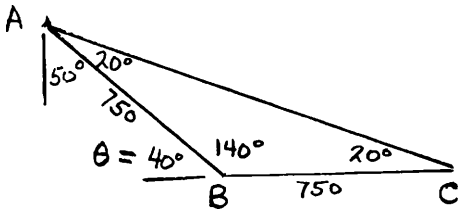


PROBLEM 1.25

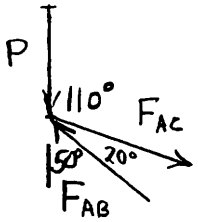
Knowing that $\theta = 40°$ and $P = 9$ kN, determine (*a*) the smallest allowable diameter of the pin at *B* if the average shearing stress in the pin is not to exceed 120 MPa, (*b*) the corresponding average bearing stress in member *AB* at *B*, (*c*) the corresponding average bearing stress in each of the support brackets at *B*.

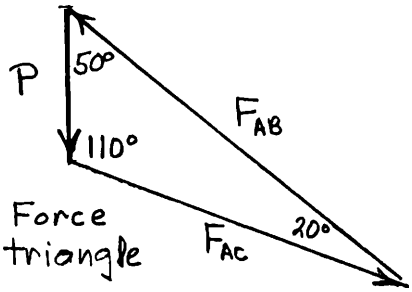
SOLUTION

Geometry: Triangle *ABC* is an isoseles triangle with angles shown here.



Use joint A as a free body.





Law of sines applied to force triangle:

$$\frac{P}{\sin 20°} = \frac{F_{AB}}{\sin 110°} = \frac{F_{AC}}{\sin 50°}$$

$$F_{AB} = \frac{P \sin 110°}{\sin 20°}$$

$$= \frac{(9)\sin 110°}{\sin 20°} = 24.727 \text{ kN}$$

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PROBLEM 1.25 *(Continued)*

(*a*) Allowable pin diameter.

$$\tau = \frac{F_{AB}}{2A_P} = \frac{F_{AB}}{2\frac{\pi}{4}d^2} = \frac{2F_{AB}}{\pi d^2} \text{ where } F_{AB} = 24.727 \times 10^3 \text{ N}$$

$$d^2 = \frac{2F_{AB}}{\pi\tau} = \frac{(2)(24.727 \times 10^3)}{\pi(120 \times 10^6)} = 131.181 \times 10^{-6} \text{ m}^2$$

$$d = 11.4534 \times 10^{-3} \text{ m} \qquad 11.45 \text{ mm} \blacktriangleleft$$

(*b*) Bearing stress in *AB* at *A*.

$$A_b = td = (0.016)(11.4534 \times 10^{-3}) = 183.254 \times 10^{-6} \text{ m}^2$$

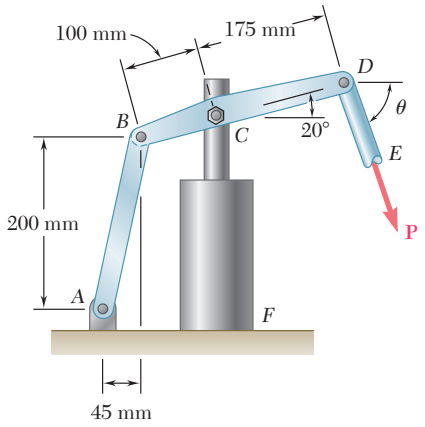
$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{24.727 \times 10^3}{183.254 \times 10^{-6}} = 134.933 \times 10^6 \text{ Pa} \qquad 134.9 \text{ MPa} \blacktriangleleft$$

(*c*) Bearing stress in support brackets at *B*.

$$A = td = (0.012)(11.4534 \times 10^{-3}) = 137.441 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{\frac{1}{2}F_{AB}}{A} = \frac{(0.5)(24.727 \times 10^3)}{137.441 \times 10^{-6}} = 89.955 \times 10^6 \text{ Pa} \qquad 90.0 \text{ MPa} \blacktriangleleft$$

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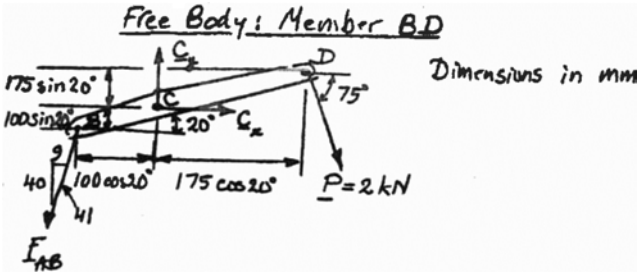


PROBLEM 1.26

The hydraulic cylinder *CF*, which partially controls the position of rod *DE*, has been locked in the position shown. Member *BD* is 15 mm thick and is connected at C to the vertical rod by a 9-mm-diameter bolt. Knowing that $P = 2$ kN and $\theta = 75°$, determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at *C* in member *BD*.

SOLUTION

Free Body: Member *BD*.



$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{40}{41}F_{AB}(100\cos 20°) - \frac{9}{4}F_{AB}(100\sin 20°)$$
$$-(2\text{ kN})\cos 75°(175\sin 20°) - (2\text{ kN})\sin 75°(175\cos 20°) = 0$$
$$\frac{100}{41}F_{AB}(40\cos 20° - 9\sin 20°) = (2\text{ kN})(175)\sin(75° + 20°)$$
$$F_{AB} = 4.1424\text{ kN}$$

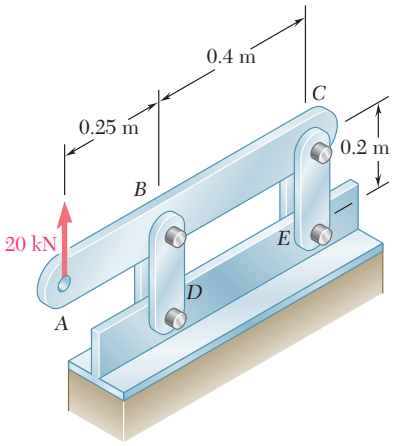
$$\overset{+}{\rightarrow}\Sigma F_x = 0: \quad C_x - \frac{9}{41}(4.1424\text{ kN}) + (2\text{ kN})\cos 75° = 0$$
$$C_x = 0.39167\text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - \frac{40}{41}(4.1424\text{ kN}) - (2\text{ kN})\sin 75° = 0$$
$$C_y = 5.9732\text{ kN}$$
$$\mathbf{C} = 5.9860\text{ kN} \measuredangle 86.2°$$

(*a*) $$\tau_{\text{ave}} = \frac{C}{A} = \frac{5.9860 \times 10^3\text{ N}}{\pi(0.0045\text{ m})^2} = 94.1 \times 10^6\text{ Pa} = 94.1\text{ MPa}$$ ◀

(*b*) $$\tau_b = \frac{C}{td} = \frac{5.9860 \times 10^3\text{ N}}{(0.015\text{ m})(0.009\text{ m})} = 44.3 \times 10^6\text{ Pa} = 44.3\text{ MPa}$$ ◀

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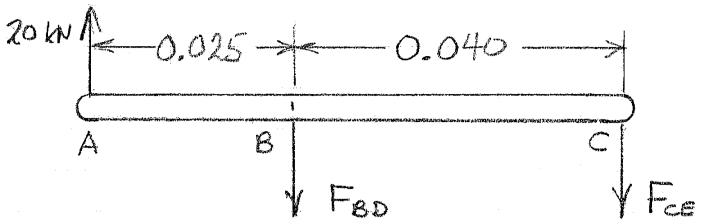
PROBLEM 1.27

For the assembly and loading of Prob. 1.7, determine (*a*) the average shearing stress in the pin at *B*, (*b*) the average bearing stress at *B* in member *BD*, (*c*) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10 × 50-mm uniform rectangular cross section.

PROBLEM 1.7 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

SOLUTION

Use bar *ABC* as a free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N}$$

(*a*) Shear pin at *B*. $\tau = \dfrac{F_{BD}}{2A}$ for double shear

where $A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{m}^2$

$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.822 \times 10^6 \text{ Pa} \qquad \tau = 80.8 \text{ MPa} \blacktriangleleft$$

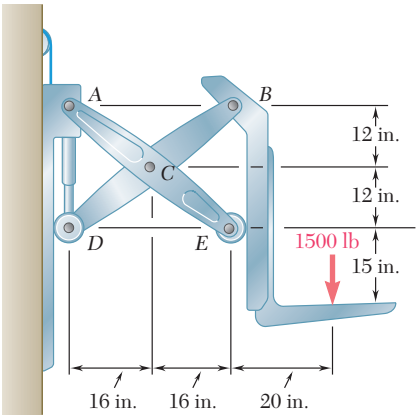
(*b*) Bearing: link *BD*. $A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{m}^2$

$$\sigma_b = \frac{\frac{1}{2}F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \text{ Pa} \qquad \sigma_b = 127.0 \text{ MPa} \blacktriangleleft$$

(*c*) Bearing in *ABC* at *B*. $A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{m}^2$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203.12 \times 10^6 \text{ Pa} \qquad \sigma_b = 203 \text{ MPa} \blacktriangleleft$$

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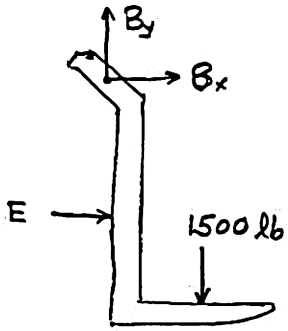


PROBLEM 1.28

Two identical linkage-and-hydraulic-cylinder systems control the position of the forks of a fork-lift truck. The load supported by the one system shown is 1500 lb. Knowing that the thickness of member *BD* is $\frac{5}{8}$ in., determine (*a*) the average shearing stress in the $\frac{1}{2}$-in.-diameter pin at *B*, (*b*) the bearing stress at *B* in member *BD*.

SOLUTION

Use one fork as a free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 24E - (20)(1500) = 0$$

$$E = 1250 \text{ lb} \rightarrow$$

$$\pm \Sigma F_x = 0: \quad E + B_x = 0$$

$$B_x = -E$$

$$B_x = 1250 \text{ lb} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 1500 = 0 \qquad B_y = 1500 \text{ lb}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1250^2 + 1500^2} = 1952.56 \text{ lb}$$

(*a*) Shearing stress in pin at *B*.

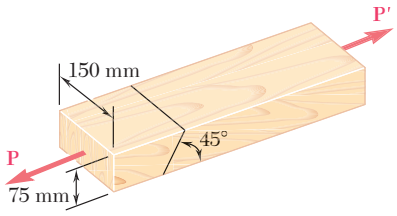
$$A_{\text{pin}} = \frac{\pi}{4} d_{\text{pin}}^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.196350 \text{ in}^2$$

$$\tau = \frac{B}{A_{\text{pin}}} = \frac{1952.56}{0.196350} = 9.94 \times 10^3 \text{ psi} \qquad \tau = 9.94 \text{ ksi} \blacktriangleleft$$

(*b*) Bearing stress at *B*.

$$\sigma = \frac{B}{dt} = \frac{1952.56}{\left(\frac{1}{2}\right)\left(\frac{5}{8}\right)} = 6.25 \times 10^3 \text{ psi} \qquad \sigma = 6.25 \text{ ksi} \blacktriangleleft$$

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PROBLEM 1.29

Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11$ kN, determine the normal and shearing stresses in the glued splice.

SOLUTION

$$\theta = 90° - 45° = 45°$$

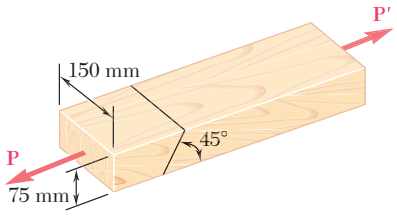
$$P = 11 \text{ kN} = 11 \times 10^3 \text{ N}$$

$$A_0 = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(11 \times 10^3)\cos^2 45°}{11.25 \times 10^{-3}} = 489 \times 10^3 \text{ Pa} \qquad \sigma = 489 \text{ kPa} \blacktriangleleft$$

$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(11 \times 10^3)(\sin 90°)}{(2)(11.25 \times 10^{-3})} = 489 \times 10^3 \text{ Pa} \qquad \tau = 489 \text{ kPa} \blacktriangleleft$$

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PROBLEM 1.30

Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 620 kPa, determine (*a*) the largest load **P** that can be safely applied, (*b*) the corresponding tensile stress in the splice.

SOLUTION

$$\theta = 90° - 45° = 45°$$

$$A_0 = (150)(75) = 11.25 \times 10^3 \text{mm}^2 = 11.25 \times 10^{-3} \text{m}^2$$

$$\tau = 620 \text{ kPa} = 620 \times 10^3 \text{Pa}$$

$$\tau = \frac{P \sin 2\theta}{2A_0}$$

(*a*) $$P = \frac{2A_0\tau}{\sin 2\theta} = \frac{(2)(11.25 \times 10^{-3})(620 \times 10^3)}{\sin 90°}$$

$$= 13.95 \times 10^3 \text{ N}$$

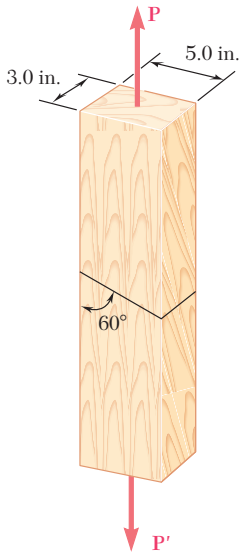
$$P = 13.95 \text{ kN} \blacktriangleleft$$

(*b*) $$\sigma = \frac{P \cos^2 \theta}{A_0} = \frac{(13.95 \times 10^3)(\cos 45°)^2}{11.25 \times 10^{-3}}$$

$$= 620 \times 10^3 \text{ Pa}$$

$$\sigma = 620 \text{ kPa} \blacktriangleleft$$

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PROBLEM 1.31

The 1.4-kip load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

SOLUTION

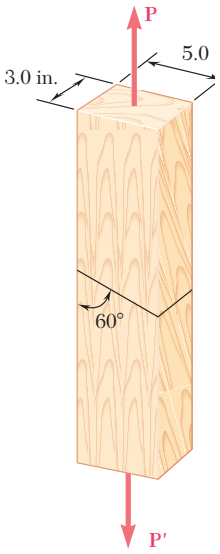
$$P = 1400 \text{ lb} \qquad \theta = 90° - 60° = 30°$$

$$A_0 = (5.0)(3.0) = 15 \text{ in}^2$$

$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(1400)(\cos 30°)^2}{15} \qquad \sigma = 70.0 \text{ psi} \blacktriangleleft$$

$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(1400)\sin 60°}{(2)(15)} \qquad \tau = 40.4 \text{ psi} \blacktriangleleft$$

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PROBLEM 1.32

Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 75 psi, determine (*a*) the largest load **P** that can be safely supported, (*b*) the corresponding shearing stress in the splice.

SOLUTION

$$A_0 = (5.0)(3.0) = 15\ \text{in}^2$$

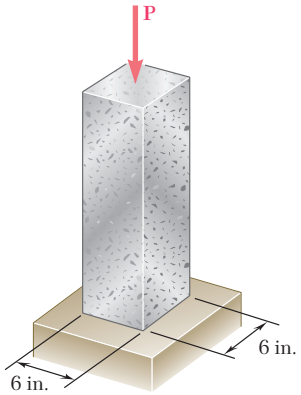
$$\theta = 90° - 60° = 30°$$

$$\sigma = \frac{P\cos^2\theta}{A_0}$$

(*a*)
$$P = \frac{\sigma A_0}{\cos^2\theta} = \frac{(75)(15)}{\cos^2 30°} = 1500\ \text{lb}$$
$P = 1.500$ kips ◀

(*b*)
$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(1500)\sin 60°}{(2)(15)}$$
$\tau = 43.3$ psi ◀

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PROBLEM 1.33

A centric load **P** is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 2.5 ksi, determine (*a*) the magnitude of **P**, (*b*) the orientation of the surface on which the maximum shearing stress occurs, (*c*) the normal stress exerted on that surface, (*d*) the maximum value of the normal stress in the block.

SOLUTION

$$A_0 = (6)(6) = 36 \text{ in}^2$$

$$\tau_{\max} = 2.5 \text{ ksi}$$

$$\theta = 45° \text{ for plane of } \tau_{\max}$$

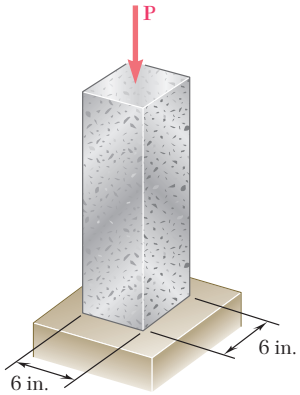
(*a*) $\tau_{\max} = \dfrac{|P|}{2A_0} \quad \therefore \quad |P| = 2A_0\tau_{\max} = (2)(36)(2.5)$ $\qquad P = 180.0 \text{ kips} \blacktriangleleft$

(*b*) $\sin 2\theta = 1 \quad 2\theta = 90°$ $\qquad \theta = 45.0° \blacktriangleleft$

(*c*) $\sigma_{45} = \dfrac{P}{A_0}\cos^2 45° = \dfrac{P}{2A_0} = -\dfrac{180}{(2)(36)}$ $\qquad \sigma_{45} = -2.50 \text{ ksi} \blacktriangleleft$

(*d*) $\sigma_{\max} = \dfrac{P}{A_0} = \dfrac{-180}{36}$ $\qquad \sigma_{\max} = -5.00 \text{ ksi} \blacktriangleleft$

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PROBLEM 1.34

A 240-kip load **P** is applied to the granite block shown. Determine the resulting maximum value of (*a*) the normal stress, (*b*) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.

SOLUTION

$$A_0 = (6)(6) = 36 \text{ in}^2$$

$$\sigma = \frac{P}{A_0}\cos^2\theta = \frac{-240}{36}\cos^2\theta = -6.67\cos^2\theta$$

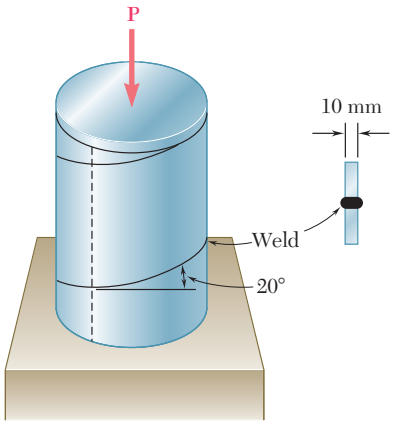
(*a*) max tensile stress = 0 at $\theta = 90.0°$

max. compressive stress = 6.67 ksi at $\theta = 0°$ ◀

(*b*) $\tau_{\max} = \dfrac{P}{2A_0} = \dfrac{240}{(2)(36)}$

$\tau_{\max} = 3.33$ ksi ◀
at $\theta = 45°$

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PROBLEM 1.35

A steel pipe of 400-mm outer diameter is fabricated from 10-mm thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that a 300-kN axial force **P** is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

SOLUTION

$$d_o = 0.400 \text{ m}$$

$$r_o = \frac{1}{2} d_o = 0.200 \text{ m}$$

$$r_i = r_o - t = 0.200 - 0.010 = 0.190 \text{ m}$$

$$A_o = \pi(r_o^2 - r_i^2) = \pi(0.200^2 - 0.190^2)$$

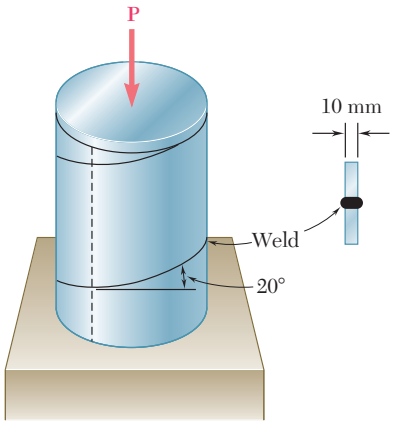
$$= 12.2522 \times 10^{-3} \text{ m}^2$$

$$\theta = 20°$$

$$\sigma = \frac{P}{A_o} = \cos^2\theta = \frac{-300 \times 10^3 \cos^2 20°}{12.2522 \times 10^{-3}} = 21.621 \times 10^6 \text{ Pa} \qquad \sigma = -21.6 \text{ MPa} \blacktriangleleft$$

$$\tau = \frac{P}{2A_0} = \sin 2\theta = \frac{-300 \times 10^3 \sin 40°}{(2)(12.2522 \times 10^{-3})} = 7.8695 \times 10^6 \text{ Pa} \qquad \tau = 7.87 \text{ MPa} \blacktriangleleft$$

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PROBLEM 1.36

A steel pipe of 400-mm outer diameter is fabricated from 10-mm thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in the directions respectively normal and tangential to the weld are $\sigma = 60$ MPa and $\tau = 36$ MPa, determine the magnitude P of the largest axial force that can be applied to the pipe.

SOLUTION

$$d_o = 0.400 \text{ m}$$

$$r_o = \frac{1}{2} d_o = 0.200 \text{ m}$$

$$r_i = r_o - t = 0.200 - 0.010 = 0.190 \text{ m}$$

$$A_o = \pi(r_o^2 - r_i^2) = \pi(0.200^2 - 0.190^2)$$

$$= 12.2522 \times 10^{-3} \text{ m}^2$$

$$\theta = 20°$$

Based on $|\sigma| = 60$ MPa: $\sigma = \dfrac{P}{A_0} \cos^2 \theta$

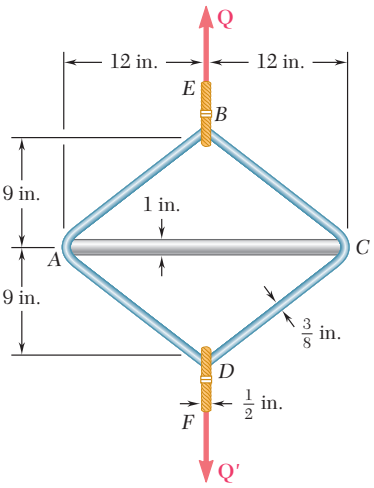
$$P = \frac{A_o \sigma}{\cos^2 \theta} = \frac{(12.2522 \times 10^{-3})(60 \times 10^6)}{\cos^2 20°} = 832.52 \times 10^3 \text{ N}$$

Based on $|\tau| = 30$ MPa: $\tau = \dfrac{P}{2A_o} \sin 2\theta$

$$P = \frac{2A_o \tau}{\sin 2\theta} = \frac{(2)(12.2522 \times 10^{-3})(36 \times 10^6)}{\sin 40°} = 1372.39 \times 10^3 \text{ N}$$

Smaller value is the allowable value of P. $P = 833$ kN ◀

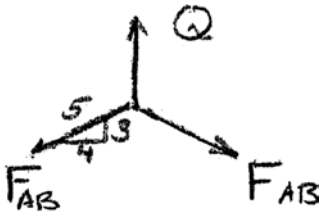
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PROBLEM 1.37

A steel loop *ABCD* of length 5 ft and of $\frac{3}{8}$-in. diameter is placed as shown around a 1-in.-diameter aluminum rod *AC*. Cables *BE* and *DF*, each of $\frac{1}{2}$-in. diameter, are used to apply the load **Q**. Knowing that the ultimate strength of the steel used for the loop and the cables is 70 ksi, and that the ultimate strength of the aluminum used for the rod is 38 ksi, determine the largest load **Q** that can be applied if an overall factor of safety of 3 is desired.

SOLUTION



Using joint *B* as a free body and considering symmetry,

$$2 \cdot \frac{3}{5} F_{AB} - Q = 0 \quad Q = \frac{6}{5} F_{AB}$$

Using joint *A* as a free body and considering symmetry,

$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{5}{6} Q - F_{AC} = 0 \quad \therefore \quad Q = \frac{3}{4} F_{AC}$$



Based on strength of cable *BE*,

$$Q_U = \sigma_U A = \sigma_U \frac{\pi}{4} d^2 = (70) \frac{\pi}{4} \left(\frac{1}{2} \right)^2 = 13.7445 \text{ kips}$$

Based on strength of steel loop,

$$Q_U = \frac{6}{5} F_{AB,U} = \frac{6}{5} \sigma_U A = \frac{6}{5} \sigma_U \frac{\pi}{4} d^2$$

$$= \frac{6}{5} (70) \frac{\pi}{4} \left(\frac{3}{8} \right)^2 = 9.2775 \text{ kips}$$

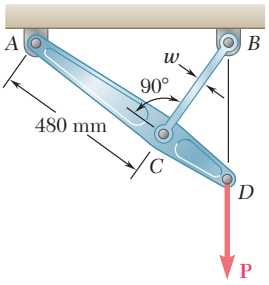
Based on strength of rod *AC*,

$$Q_U = \frac{3}{4} F_{AC,U} = \frac{3}{4} \sigma_U A = \frac{3}{4} \sigma_U \frac{\pi}{4} d^2 = \frac{3}{4} (38) \frac{\pi}{4} (1.0)^2 = 22.384 \text{ kips}$$

Actual ultimate load Q_U is the smallest, $\therefore \quad Q_U = 9.2775$ kips

Allowable load: $$Q = \frac{Q_U}{F.S.} = \frac{9.2775}{3} = 3.0925 \text{ kips}$$ $Q = 3.09$ kips ◀

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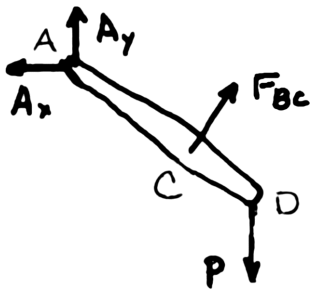


PROBLEM 1.38

Link *BC* is 6 mm thick, has a width $w = 25$ mm, and is made of a steel with a 480-MPa ultimate strength in tension. What was the safety factor used if the structure shown was designed to support a 16-kN load **P**?

SOLUTION

Use bar *ACD* as a free body and note that member *BC* is a two-force member.



$$\Sigma M_A = 0:$$

$$(480)F_{BC} - (600)P = 0$$

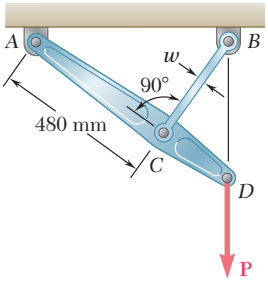
$$F_{BC} = \frac{600}{480}P = \frac{(600)(16\times10^3)}{480} = 20\times10^3\ \text{N}$$

Ultimate load for member *BC*: $F_U = \sigma_U A$

$$F_U = (480\times10^6)(0.006)(0.025) = 72\times10^3\ \text{N}$$

Factor of safety: $\text{F.S.} = \dfrac{F_U}{F_{BC}} = \dfrac{72\times10^3}{20\times10^3}$ F.S. = 3.60 ◀

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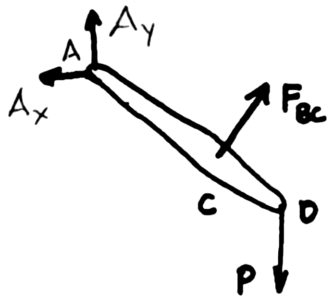


PROBLEM 1.39

Link *BC* is 6 mm thick and is made of a steel with a 450-MPa ultimate strength in tension. What should be its width *w* if the structure shown is being designed to support a 20-kN load **P** with a factor of safety of 3?

SOLUTION

Use bar *ACD* as a free body and note that member *BC* is a two-force member.



$$\Sigma M_A = 0:$$

$$(480)F_{BC} - 600P = 0$$

$$F_{BC} = \frac{600P}{480} = \frac{(600)(20\times10^3)}{480} = 25\times10^3\ \text{N}$$

For a factor of safety F.S. = 3, the ultimate load of member *BC* is

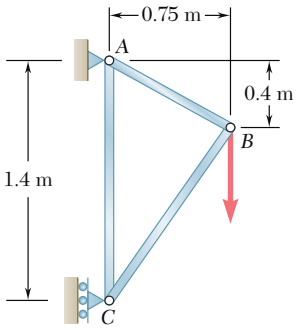
$$F_U = (\text{F.S.})(F_{BC}) = (3)(25\times10^3) = 75\times10^3\ \text{N}$$

But $F_U = \sigma_U A \quad \therefore \quad A = \dfrac{F_U}{\sigma_U} = \dfrac{75\times10^3}{450\times10^6} = 166.667\times10^{-6}\ \text{m}^2$

For a rectangular section, $A = wt$ or $w = \dfrac{A}{t} = \dfrac{166.667\times10^{-6}}{0.006} = 27.778\times10^{-3}\ \text{m}$

$$w = 27.8\ \text{mm} \blacktriangleleft$$

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PROBLEM 1.40

Members *AB* and *BC* of the truss shown are made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If a factor of safety of 3.2 is to be achieved for both bars, determine the required cross-sectional area of (*a*) bar *AB*, (*b*) bar *AC*.

SOLUTION

Length of member *AB*:

$$\ell_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

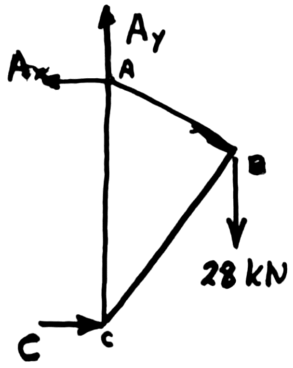
Use entire truss as a free body.

$$+\circlearrowleft \Sigma M_c = 0: \quad 1.4A_x - (0.75)(28) = 0$$

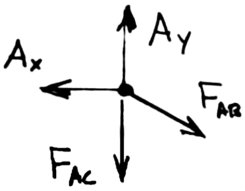
$$A_x = 15 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 28 = 0$$

$$A_y = 28 \text{ kN}$$



Use Joint *A* as free body.



$$\pm\rightarrow \Sigma F_x = 0: \quad \frac{0.75}{0.85}F_{AB} - A_x = 0$$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - F_{AC} - \frac{0.4}{0.85}F_{AB} = 0$$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar, $A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \quad P_U = 120 \times 10^3 \text{ N}$

For the material, $\sigma_U = \frac{P_U}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

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PROBLEM 1.40 ***(Continued)***

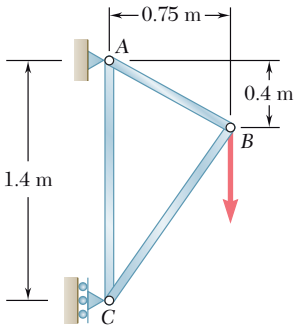
(*a*) For member *AB*: $\text{F.S.} = \dfrac{P_U}{F_{AB}} = \dfrac{\sigma_U A_{AB}}{F_{AB}}$

$$A_{AB} = \frac{(\text{F.S.})F_{AB}}{\sigma_U} = \frac{(3.2)(17 \times 10^3)}{300 \times 10^6} = 181.333 \times 10^{-6}\,\text{m}^2 \qquad A_{AB} = 181.3\,\text{mm}^2 \blacktriangleleft$$

(*b*) For member *AC*: $\text{F.S.} = \dfrac{P_U}{F_{AC}} = \dfrac{\sigma_U A_{AC}}{F_{AC}}$

$$A_{AC} = \frac{(\text{F.S.})F_{AC}}{\sigma_U} = \frac{(3.2)(20 \times 10^3)}{300 \times 10^6} = 213.33 \times 10^{-6}\,\text{m}^2 \qquad A_{AC} = 213\,\text{mm}^2 \blacktriangleleft$$

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PROBLEM 1.41

Members *AB* and *BC* of the truss shown are made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If bar *AB* has a cross-sectional area of 225 mm^2, determine (*a*) the factor of safety for bar *AB* and (*b*) the cross-sectional area of bar *AC* if it is to have the same factor of safety as bar *AB*.

SOLUTION

Length of member *AB*:

$$\ell_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

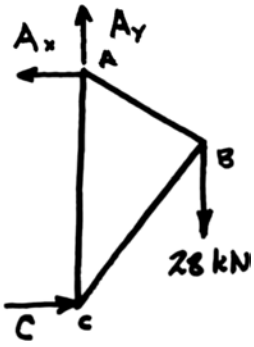
Use entire truss as a free body.

$$+\circlearrowleft \Sigma M_c = 0: \quad 1.4A_x - (0.75)(28) = 0$$

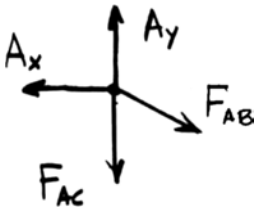
$$A_x = 15 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 28 = 0$$

$$A_y = 28 \text{ kN}$$



Use Joint *A* as free body.



$$\pm \rightarrow \Sigma F_x = 0: \quad \frac{0.75}{0.85} F_{AB} - A_x = 0$$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar, $A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \quad P_U = 120 \times 10^3 \text{ N}$

For the material, $\sigma_U = \frac{P_U}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

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PROBLEM 1.41 ***(Continued)***

(*a*) For bar *AB*:

$$\text{F.S.} = \frac{F_U}{F_{AB}} = \frac{\sigma_U A_{AB}}{F_{AB}} = \frac{(300 \times 10^6)(225 \times 10^{-6})}{17 \times 10^3}$$

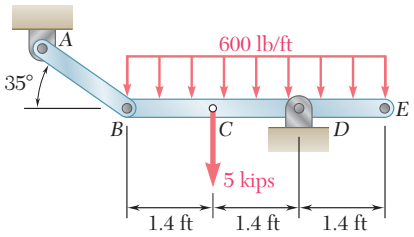
$$\text{F.S.} = 3.97 \blacktriangleleft$$

(*b*) For bar *AC*:

$$\text{F.S.} = \frac{F_U}{F_{AC}} = \frac{\sigma_U A_{AC}}{F_{AC}}$$

$$A_{AC} = \frac{(\text{F.S.})F_{AC}}{\sigma_U} = \frac{(3.97)(20 \times 10^3)}{300 \times 10^6} = 264.67 \times 10^{-6}\,\text{m}^2 \qquad A_{AC} = 265\,\text{mm}^2 \blacktriangleleft$$

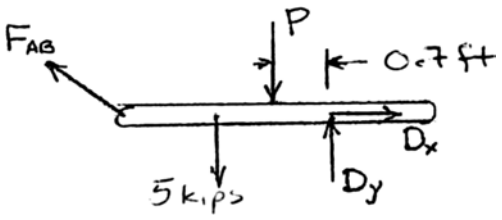
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PROBLEM 1.42

Link *AB* is to be made of a steel for which the ultimate normal stress is 65 ksi. Determine the cross-sectional area of *AB* for which the factor of safety will be 3.20. Assume that the link will be adequately reinforced around the pins at *A* and *B*.

SOLUTION



$$P = (4.2)(0.6) = 2.52 \text{ kips}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(2.8)(F_{AB}\sin 35°) + (0.7)(2.52) + (1.4)(5) = 0$$

$$F_{AB} = 5.4570 \text{ kips}$$

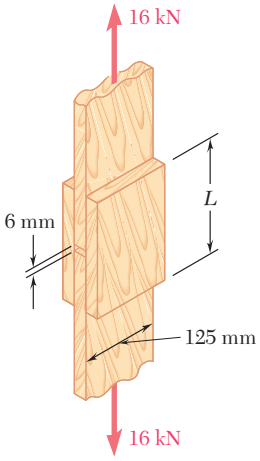
$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{\sigma_{\text{ult}}}{F.S.}$$

$$A_{AB} = \frac{(F.S.)F_{AB}}{\sigma_{\text{ult}}} = \frac{(3.20)(5.4570 \text{ kips})}{65 \text{ ksi}}$$

$$= 0.26854 \text{ in}^2$$

$$A_{AB} = 0.268 \text{ in}^2 \blacktriangleleft$$

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PROBLEM 1.43

Two wooden members are joined by plywood splice plates that are fully glued on the contact surfaces. Knowing that the clearance between the ends of the members is 6 mm and that the ultimate shearing stress in the glued joint is 2.5 MPa, determine the length L for which the factor of safety is 2.75 for the loading shown.

SOLUTION

$$\tau_{\text{all}} = \frac{2.5 \text{ MPa}}{2.75} = 0.90909 \text{ MPa}$$

On one face of the upper contact surface,

$$A = \frac{L - 0.006 \text{ m}}{2}(0.125 \text{ m})$$

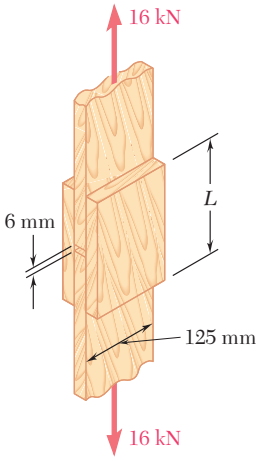
Since there are 2 contact surfaces,

$$\tau_{\text{all}} = \frac{P}{2A}$$

$$0.90909 \times 10^{6} = \frac{16 \times 10^{3}}{(L - 0.006)(0.125)}$$

$$L = 0.14680 \text{ m} \qquad 146.8 \text{ mm} \blacktriangleleft$$

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PROBLEM 1.44

For the joint and loading of Prob. 1.43, determine the factor of safety when $L = 180$ mm.

PROBLEM 1.43 Two wooden members are joined by plywood splice plates that are fully glued on the contact surfaces. Knowing that the clearance between the ends of the members is 6 mm and that the ultimate shearing stress in the glued joint is 2.5 MPa, determine the length L for which the factor of safety is 2.75 for the loading shown.

SOLUTION

Area of one face of upper contact surface:

$$A = \frac{0.180\text{ m} - 0.006\text{ m}}{2}(0.125\text{ m})$$

$$A = 10.8750 \times 10^{-3}\text{ m}^2$$

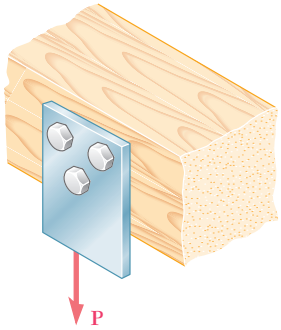
Since there are two surfaces,

$$\tau_{\text{all}} = \frac{P}{2A} = \frac{16 \times 10^3\text{ N}}{2(10.8750 \times 10^{-3}\text{ m}^2)}$$

$$\tau_{\text{all}} = 0.73563\text{ MPa}$$

$$\text{F.S.} = \frac{\tau_u}{\tau_{\text{all}}} = \frac{2.5\text{ MPa}}{0.73563\text{ MPa}} = 3.40 \quad \blacktriangleleft$$

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PROBLEM 1.45

Three $\frac{3}{4}$-in.-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a load $P = 24$ kips and that the ultimate shearing stress for the steel used is 52 ksi, determine the factor of safety for this design.

SOLUTION

For each bolt,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.44179 \text{ in}^2$$

$$P_U = A\tau_U = (0.44179)(52)$$

$$= 22.973 \text{ kips}$$

For the three bolts,

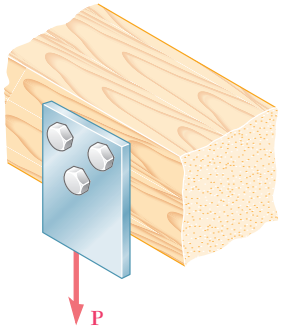
$$P_U = (3)(22.973) = 68.919 \text{ kips}$$

Factor of safety:

$$F.S. = \frac{P_U}{P} = \frac{68.919}{24}$$

$$F.S. = 2.87 \blacktriangleleft$$

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PROBLEM 1.46

Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a load $P = 28$ kips, that the ultimate shearing stress for the steel used is 52 ksi, and that a factor of safety of 3.25 is desired, determine the required diameter of the bolts.

SOLUTION

For each bolt, $P = \frac{24}{3} = 8$ kips

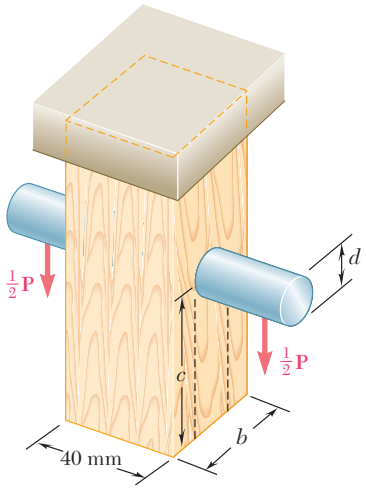
Required: $P_U = (F.S.)P = (3.25)(8.0) = 26.0$ kips

$$\tau_U = \frac{P_U}{A} = \frac{P_U}{\frac{\pi}{4}d^2} = \frac{4P_U}{\pi d^2}$$

$$d = \sqrt{\frac{4P_U}{\pi\tau_U}} = \sqrt{\frac{(4)(26.0)}{\pi(52)}} = 0.79789 \text{ in.}$$

$d = 0.798$ in. ◀

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PROBLEM 1.47

A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION

Based on double shear in pin,

$$P_U = 2A\tau_U = 2\frac{\pi}{4}d^2\tau_U$$

$$= \frac{\pi}{4}(2)(0.012)^2(145\times10^6) = 32.80\times10^3\ \text{N}$$

Based on tension in wood,

$$P_U = A\sigma_U = w(b-d)\sigma_U$$

$$= (0.040)(0.040-0.012)(60\times10^6)$$

$$= 67.2\times10^3\ \text{N}$$

Based on double shear in the wood,

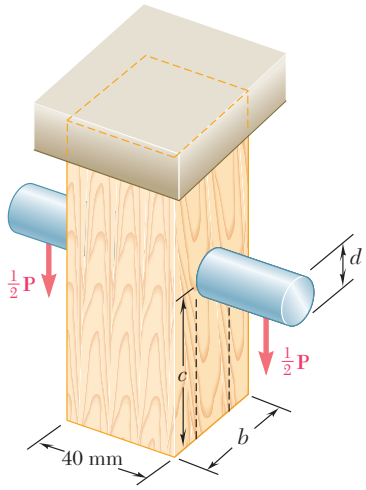
$$P_U = 2A\tau_U = 2wc\tau_U = (2)(0.040)(0.055)(7.5\times10^6)$$

$$= 33.0\times10^3\ \text{N}$$

Use smallest $P_U = 32.8\times10^3\ \text{N}$

Allowable: $P = \frac{P_U}{F.S.} = \frac{32.8\times10^3}{3.2} = 10.25\times10^3\ \text{N}$ 10.25 kN ◀

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PROBLEM 1.48

For the support of Prob. 1.47, knowing that the diameter of the pin is $d = 16$ mm and that the magnitude of the load is $P = 20$ kN, determine (*a*) the factor of safety for the pin, (*b*) the required values of *b* and *c* if the factor of safety for the wooden members is the same as that found in part *a* for the pin.

PROBLEM 1.47 A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION

$$P = 20\text{ kN} = 20 \times 10^3\text{ N}$$

(*a*) Pin: $$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.016)^2 = 2.01.06 \times 10^{-6}\text{ m}^2$$

Double shear: $$\tau = \frac{P}{2A} \quad \tau_U = \frac{P_U}{2A}$$

$$P_U = 2A\tau_U = (2)(201.16 \times 10^{-6})(145 \times 10^6) = 58.336 \times 10^3\text{ N}$$

$$F.S. = \frac{P_U}{P} = \frac{58.336 \times 10^3}{20 \times 10^3} \qquad F.S. = 2.92 \blacktriangleleft$$

(*b*) Tension in wood: $P_U = 58.336 \times 10^3$ N for same F.S.

$$\sigma_U = \frac{P_U}{A} = \frac{P_U}{w(b-d)} \quad \text{where} \quad w = 40\text{ mm} = 0.040\text{ m}$$

$$b = d + \frac{P_U}{w\sigma_U} = 0.016 + \frac{58.336 \times 10^3}{(0.040)(60 \times 10^6)} = 40.3 \times 10^{-3}\text{ m} \qquad b = 40.3\text{ mm} \blacktriangleleft$$

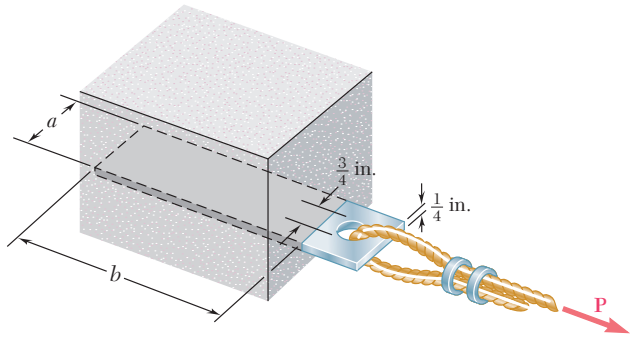
Shear in wood: $P_U = 58.336 \times 10^3$ N for same F.S.

Double shear: each area is $A = wc$ $\qquad \tau_U = \frac{P_U}{2A} = \frac{P_U}{2wc}$

$$c = \frac{P_U}{2w\tau_U} = \frac{58.336 \times 10^3}{(2)(0.040)(7.5 \times 10^6)} = 97.2 \times 10^{-3}\text{ m} \qquad c = 97.2\text{ mm} \blacktriangleleft$$

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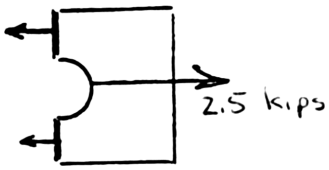
PROBLEM 1.49



A steel plate $\frac{1}{4}$ in. thick is embedded in a concrete wall to anchor a high-strength cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (*a*) the required width *a* of the plate, (*b*) the minimum depth *b* to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the end of the plate.)

SOLUTION

Based on tension in plate,



$$A = (a - d)t$$

$$P_U = \sigma_U A$$

$$F.S. = \frac{P_U}{P} = \frac{\sigma_U (a - d)t}{P}$$

Solving for *a*,

$$a = d + \frac{(F.S.)P}{\sigma_U t} = \frac{3}{4} + \frac{(3.60)(2.5)}{(36)\left(\frac{1}{4}\right)}$$

(*a*) $a = 1.750$ in. ◀

Based on shear between plate and concrete slab,

$$A = \text{perimeter} \times \text{depth} = 2(a + t)b \qquad \tau_U = 0.300 \text{ ksi}$$

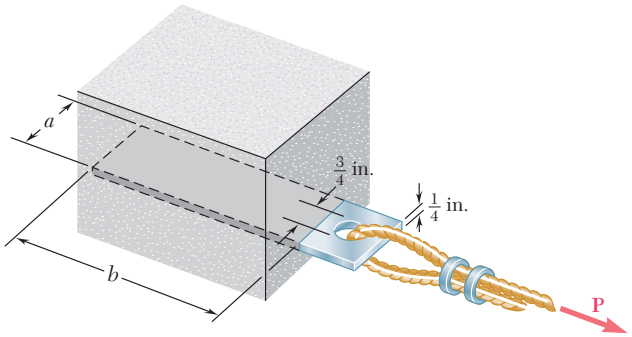
$$P_U = \tau_U A = 2\tau_U (a + t)b \qquad F.S. = \frac{P_U}{P}$$

Solving for *b*,

$$b = \frac{(F.S.)P}{2(a + t)\tau_U} = \frac{(3.6)(2.5)}{(2)\left(1.75 + \frac{1}{4}\right)(0.300)}$$

(*b*) $b = 7.50$ in. ◀

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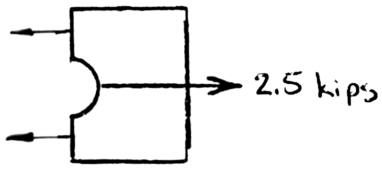
PROBLEM 1.50

Determine the factor of safety for the cable anchor in Prob. 1.49 when $P = 2.5$ kips, knowing that $a = 2$ in. and $b = 6$ in.

PROBLEM 1.49 A steel plate $\frac{1}{4}$ in. thick is embedded in a concrete wall to anchor a high-strength cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (*a*) the required width *a* of the plate, (*b*) the minimum depth *b* to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the end of the plate.)

SOLUTION

Based on tension in plate,



$$A = (a - d)t$$

$$= \left(2 - \frac{3}{4}\right)\left(\frac{1}{4}\right) = 0.31250 \text{ in}^2$$

$$P_U = \sigma_U A$$

$$= (36)(0.31250) = 11.2500 \text{ kips}$$

$$F.S. = \frac{P_U}{P} = \frac{11.2500}{3.5} = 4.50$$

Based on shear between plate and concrete slab,

$$A = \text{perimeter} \times \text{depth} = 2(a + t)b = 2\left(2 + \frac{1}{4}\right)(6.0)$$

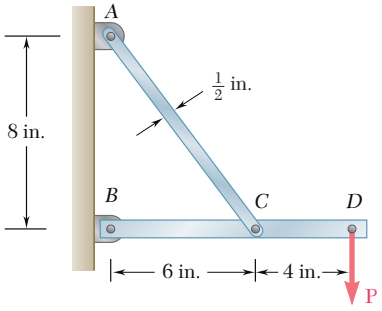
$$A = 27.0 \text{ in}^2 \qquad \tau_U = 0.300 \text{ ksi}$$

$$P_U = \tau_U A = (0.300)(27.0) = 8.10 \text{ kips}$$

$$F.S. = \frac{P_U}{P} = \frac{8.10}{2.5} = 3.240$$

Actual factor of safety is the smaller value. $F.S. = 3.24$ ◀

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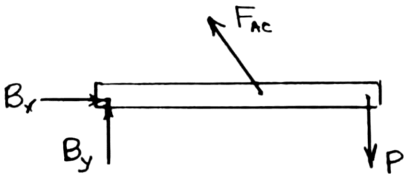


PROBLEM 1.51

Link *AC* is made of a steel with a 65-ksi ultimate normal stress and has a $\frac{1}{4} \times \frac{1}{2}$-in. uniform rectangular cross section. It is connected to a support at *A* and to member *BCD* at *C* by $\frac{3}{4}$-in.-diameter pins, while member *BCD* is connected to its support at *B* by a $\frac{5}{16}$-in.-diameter pin. All of the pins are made of a steel with a 25-ksi ultimate shearing stress and are in single shear. Knowing that a factor of safety of 3.25 is desired, determine the largest load **P** that can be applied at *D*. Note that link *AC* is not reinforced around the pin holes.

SOLUTION

Use free body *BCD*.



$$+\circlearrowleft M_B = 0: \quad (6)\left(\frac{8}{10}F_{AC}\right) - 10P = 0$$

$$P = 0.48F_{AC} \tag{1}$$

$$\pm\Sigma F_x = 0: \quad B_x - \frac{6}{10}F_{AC} = 0$$

$$B_x = \frac{6}{10}F_{AC} = 1.25P \rightarrow$$

$$+\circlearrowleft M_C = 0: \quad -6B_y - 4P = 0$$

$$B_y = -\frac{2}{3}P \qquad \text{i.e.} \quad B_y = \frac{2}{3}P \downarrow$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1.25^2 + \left(\frac{2}{3}\right)^2}P = 1.41667P \qquad P = 0.70588B \tag{2}$$

Shear in pins at *A* and *C*.

$$F_{AC} = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{25}{3.25}\right)\left(\frac{\pi}{4}\right)\left(\frac{3}{8}\right)^2 = 0.84959 \text{ kips}$$

Tension on net section of *A* and *C*.

$$F_{AC} = \sigma A_{\text{net}} = \frac{\sigma_U}{F.S.}A_{\text{net}} = \left(\frac{65}{3.25}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2} - \frac{3}{8}\right) = 0.625 \text{ kips}$$

Smaller value of F_{AC} is 0.625 kips.

From (1), $P = (0.48)(0.625) = 0.300$ kips

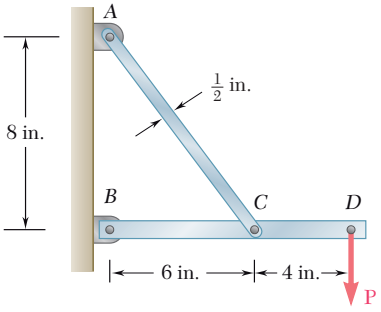
Shear in pin at *B*. $B = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{25}{3.25}\right)\left(\frac{\pi}{4}\right)\left(\frac{5}{16}\right)^2 = 0.58999$ kips

From (2), $P = (0.70588)(0.58999) = 0.416$ kips

Allowable value of *P* is the smaller value. $P = 0.300$ kips or $P = 300$ lb ◀

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PROBLEM 1.52

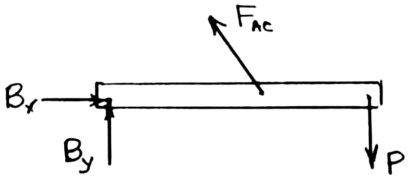


Solve Prob. 1.51, assuming that the structure has been redesigned to use $\frac{5}{16}$-in-diameter pins at *A* and *C* as well as at *B* and that no other changes have been made.

PROBLEM 1.51 Link *AC* is made of a steel with a 65-ksi ultimate normal stress and has a $\frac{1}{4} \times \frac{1}{2}$-in. uniform rectangular cross section. It is connected to a support at *A* and to member *BCD* at *C* by $\frac{3}{4}$-in.-diameter pins, while member *BCD* is connected to its support at *B* by a $\frac{5}{16}$-in.-diameter pin. All of the pins are made of a steel with a 25-ksi ultimate shearing stress and are in single shear. Knowing that a factor of safety of 3.25 is desired, determine the largest load **P** that can be applied at *D*. Note that link *AC* is not reinforced around the pin holes.

SOLUTION

Use free body *BCD*.



$$+\circlearrowleft M_B = 0: \quad (6)\left(\frac{8}{10}F_{AC}\right) - 10P = 0$$

$$P = 0.48F_{AC} \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad B_x - \frac{6}{10}F_{AC} = 0$$

$$B_x = \frac{6}{10}F_{AC} = 1.25P \rightarrow$$

$$+\circlearrowleft M_C = 0: \quad -6B_y - 4P = 0$$

$$B_y = -\frac{2}{3}P \qquad \text{i.e.} \quad B_y = \frac{2}{3}P \downarrow$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1.25^2 + \left(\frac{2}{3}\right)^2}\,P = 1.41667P \qquad P = 0.70583B \tag{2}$$

Shear in pins at *A* and *C*.

$$F_{AC} = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{25}{3.25}\right)\left(\frac{\pi}{4}\right)\left(\frac{5}{16}\right)^2 = 0.58999 \text{ kips}$$

Tension on net section of *A* and *C*.

$$F_{AC} = \sigma A_{\text{net}} = \frac{\sigma_U}{F.S.}A_{\text{net}} = \left(\frac{65}{3.25}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2} - \frac{5}{16}\right) = 0.9375 \text{ kips}$$

Smaller value of F_{AC} is 0.58999 kips.

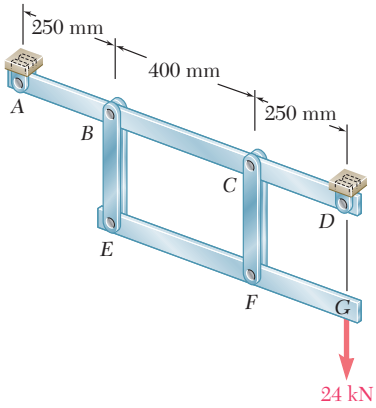
From (1), $P = (0.48)(0.58999) = 0.283$ kips

Shear in pin at *B*. $B = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{25}{3.25}\right)\left(\frac{\pi}{4}\right)\left(\frac{5}{16}\right)^2 = 0.58999$ kips

From (2), $P = (0.70588)(0.58999) = 0.416$ kips

Allowable value of *P* is the smaller value. $P = 0.283$ kips or $P = 283$ lb ◀

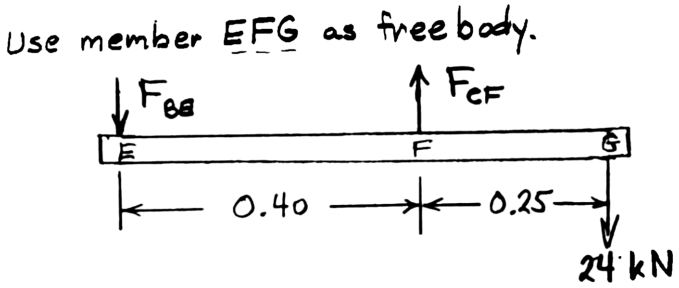
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PROBLEM 1.53

Each of the two vertical links *CF* connecting the two horizontal members *AD* and *EG* has a 10 × 40-mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at *C* and *F* has a 20-mm diameter and are made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links *CF* and the pins connecting them to the horizontal members.

SOLUTION



$$+\circlearrowleft \Sigma M_E = 0: \quad 0.40F_{CF} - (0.65)(24 \times 10^3) = 0$$

$$F_{CF} = 39 \times 10^3 \text{ N}$$

Based on tension in links *CF*,

$$A = (b - d)t = (0.040 - 0.02)(0.010) = 200 \times 10^{-6}\,\text{m}^2 \quad \text{(one link)}$$

$$F_U = 2\sigma_U A = (2)(400 \times 10^6)(200 \times 10^{-6}) = 160.0 \times 10^3 \text{ N}$$

Based on double shear in pins,

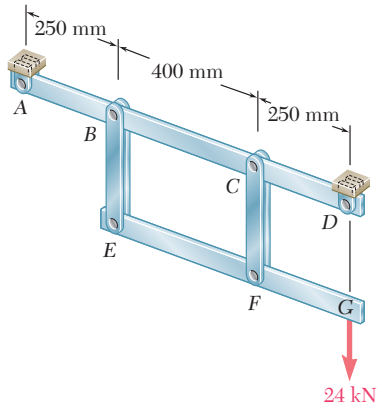
$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\,\text{m}^2$$

$$F_U = 2\tau_U A = (2)(150 \times 10^6)(314.16 \times 10^{-6}) = 94.248 \times 10^3 \text{ N}$$

Actual F_U is smaller value, i.e. $F_U = 94.248 \times 10^3$ N

Factor of safety: $$F.S. = \frac{F_U}{F_{CF}} = \frac{94.248 \times 10^3}{39 \times 10^3}$$ $F.S. = 2.42$ ◀

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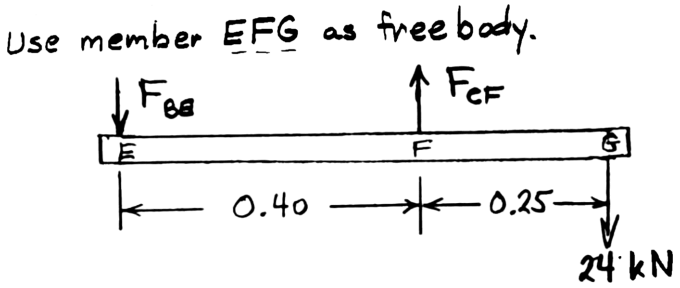
PROBLEM 1.54

Solve Prob. 1.53, assuming that the pins at *C* and *F* have been replaced by pins with a 30-mm diameter.

PROBLEM 1.53 Each of the two vertical links *CF* connecting the two horizontal members *AD* and *EG* has a 10 × 40-mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at *C* and *F* has a 20-mm diameter and are made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links *CF* and the pins connecting them to the horizontal members.

SOLUTION

Use member *EFG* as free body.



$$+\circlearrowleft \Sigma M_E = 0: \quad 0.40F_{CF} - (0.65)(24\times10^3) = 0$$

$$F_{CF} = 39\times10^3\ \text{N}$$

Based on tension in links *CF*,

$$A = (b-d)t = (0.040-0.030)(0.010) = 100\times10^{-6}\,\text{m}^2 \quad \text{(one link)}$$

$$F_U = 2\sigma_U A = (2)(400\times10^6)(100\times10^{-6}) = 80.0\times10^3\ \text{N}$$

Based on double shear in pins,

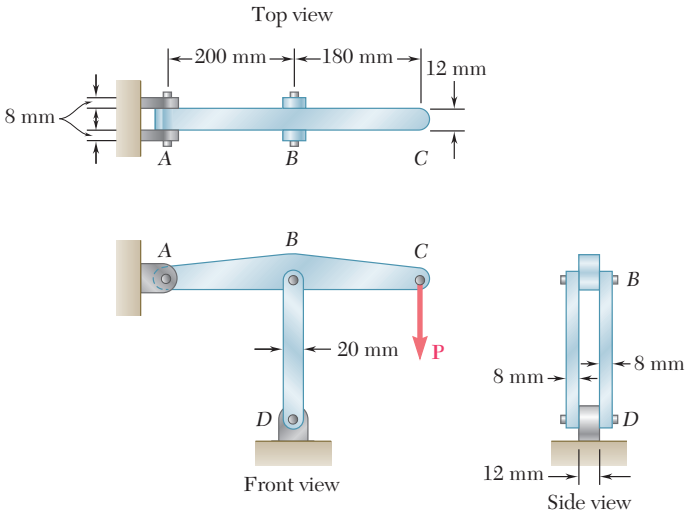
$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.030)^2 = 706.86\times10^{-6}\,\text{m}^2$$

$$F_U = 2\tau_U A = (2)(150\times10^6)(706.86\times10^{-6}) = 212.06\times10^3\ \text{N}$$

Actual F_U is smaller value, i.e. $F_U = 80.0\times10^3$ N

Factor of safety: $$F.S. = \frac{F_U}{F_{CF}} = \frac{80.0\times10^3}{39\times10^3}$$ $F.S. = 2.05$ ◀

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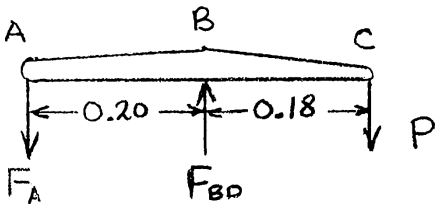


PROBLEM 1.55

In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

SOLUTION

Statics: Use *ABC* as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 0.20F_A - 0.18P = 0 \qquad P = \frac{10}{9}F_A$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.20F_{BD} - 0.38P = 0 \qquad P = \frac{10}{19}F_{BD}$$

Based on double shear in pin *A*, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.008)^2 = 50.266 \times 10^{-6}\ \text{m}^2$

$$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(50.266 \times 10^{-6})}{3.0} = 3.351 \times 10^3\ \text{N}$$

$$P = \frac{10}{9}F_A = 3.72 \times 10^3\ \text{N}$$

Based on double shear in pins at *B* and *D*, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\ \text{m}^2$

$$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3\ \text{N}$$

$$P = \frac{10}{19}F_{BD} = 3.97 \times 10^3\ \text{N}$$

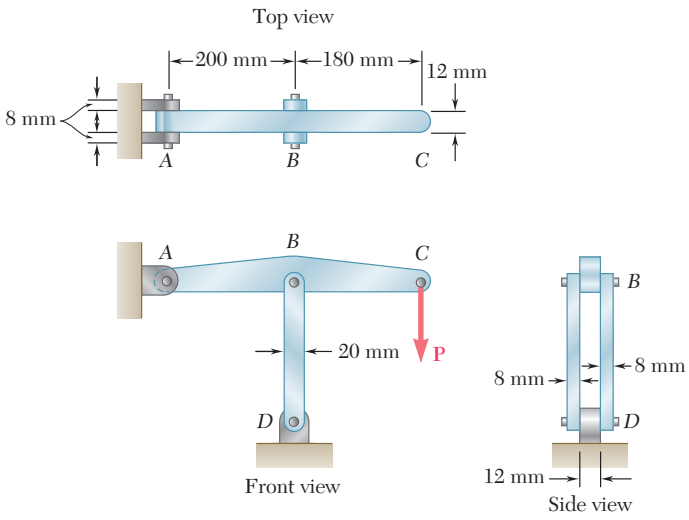
Based on compression in links *BD*, for one link, $A = (0.020)(0.008) = 160 \times 10^{-6}\ \text{m}^2$

$$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3\ \text{N}$$

$$P = \frac{10}{19}F_{BD} = 14.04 \times 10^3\ \text{N}$$

Allowable value of *P* is smallest, $\therefore \quad P = 3.72 \times 10^3\ \text{N}$ $\qquad P = 3.72\ \text{kN} \blacktriangleleft$

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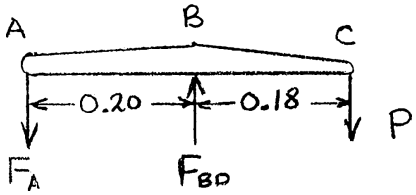
PROBLEM 1.56

In an alternative design for the structure of Prob. 1.55, a pin of 10-mm-diameter is to be used at *A*. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

PROBLEM 1.55 In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

SOLUTION

Statics: Use *ABC* as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 0.20F_A - 0.18P = 0 \qquad P = \frac{10}{9}F_A$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.20F_{BD} - 0.38P = 0 \qquad P = \frac{10}{19}F_{BD}$$

Based on double shear in pin *A*, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\ \text{m}^2$

$$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(78.54 \times 10^{-6})}{3.0} = 5.236 \times 10^3\ \text{N}$$

$$P = \frac{10}{9}F_A = 5.82 \times 10^3\ \text{N}$$

Based on double shear in pins at *B* and *D*, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\ \text{m}^2$

$$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3\ \text{N}$$

$$P = \frac{10}{19}F_{BD} = 3.97 \times 10^3\ \text{N}$$

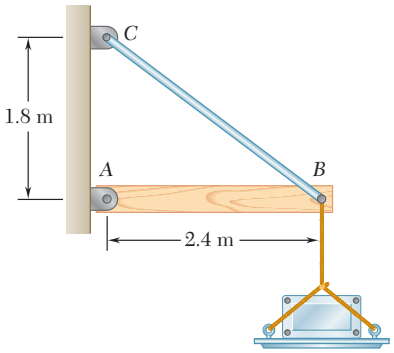
Based on compression in links *BD*, for one link, $A = (0.020)(0.008) = 160 \times 10^{-6}\ \text{m}^2$

$$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3\ \text{N}$$

$$P = \frac{10}{19}F_{BD} = 14.04 \times 10^3\ \text{N}$$

Allowable value of *P* is smallest, $\therefore\ P = 3.97 \times 10^3\ \text{N}$ $\qquad P = 3.97\ \text{kN}$ ◀

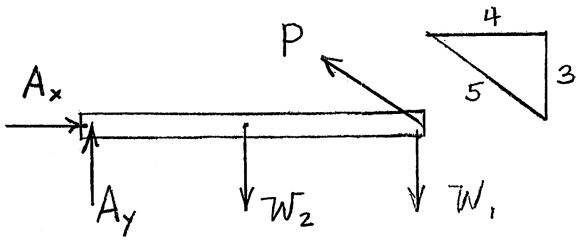
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PROBLEM 1.57

A 40-kg platform is attached to the end B of a 50-kg wooden beam AB, which is supported as shown by a pin at A and by a slender steel rod BC with a 12-kN ultimate load. (*a*) Using the Load and Resistance Factor Design method with a resistance factor $\phi = 0.90$ and load factors $\gamma_D = 1.25$ and $\gamma_L = 1.6$, determine the largest load that can be safely placed on the platform. (*b*) What is the corresponding conventional factor of safety for rod BC?

SOLUTION



$$+\circlearrowleft \Sigma M_A = 0: \quad (2.4)\frac{3}{5}P - 2.4W_1 - 1.2W_2$$

$$\therefore \quad P = \frac{5}{3}W_1 + \frac{5}{6}W_2$$

For dead loading, $W_1 = (40)(9.81) = 392.4$ N, $W_2 = (50)(9.81) = 490.5$ N

$$P_D = \left(\frac{5}{3}\right)(392.4) + \left(\frac{5}{6}\right)(490.5) = 1.0628 \times 10^3 \text{ N}$$

For live loading, $W_1 = mg \quad W_2 = 0 \quad P_L = \frac{5}{3}mg$

From which $m = \frac{3}{5}\frac{P_L}{g}$

Design criterion: $\gamma_D P_D + \gamma_L P_L = \phi P_U$

$$P_L = \frac{\phi P_U - \gamma_D P_D}{\gamma_L} = \frac{(0.90)(12 \times 10^3) - (1.25)(1.0628 \times 10^{-3})}{1.6}$$

$$= 5.920 \times 10^3 \text{ N}$$

(*a*) Allowable load. $m = \frac{3}{5}\frac{5.92 \times 10^3}{9.81}$ $m = 362$ kg ◀

Conventional factor of safety:

$$P = P_D + P_L = 1.0628 \times 10^3 + 5.920 \times 10^3 = 6.983 \times 10^3 \text{ N}$$

(*b*) $F.S. = \frac{P_U}{P} = \frac{12 \times 10^3}{6.983 \times 10^3}$ $F.S. = 1.718$ ◀

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PROBLEM 1.58

The Load and Resistance Factor Design method is to be used to select the two cables that will raise and lower a platform supporting two window washers. The platform weighs 160 lb and each of the window washers is assumed to weigh 195 lb with equipment. Since these workers are free to move on the platform, 75% of their total weight and the weight of their equipment will be used as the design live load of each cable. (*a*) Assuming a resistance factor $\phi = 0.85$ and load factors $\gamma_D = 1.2$ and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (*b*) What is the corresponding conventional factor of safety for the selected cables?

SOLUTION

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

(*a*) $$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\phi}$$

$$= \frac{(1.2)\left(\frac{1}{2} \times 160\right) + (1.5)\left(\frac{3}{4} \times 2 \times 195\right)}{0.85}$$

$$P_U = 629 \text{ lb} \blacktriangleleft$$

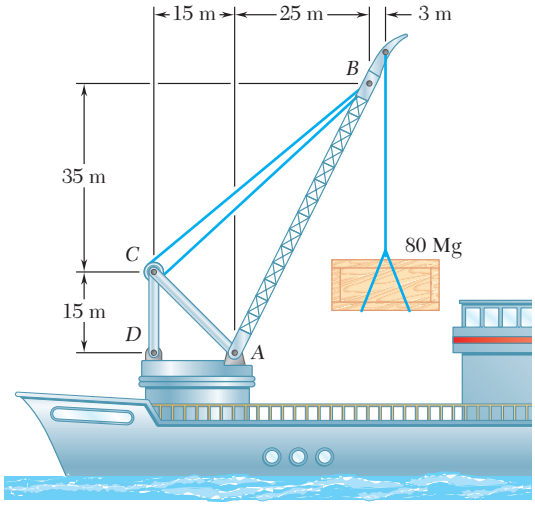
Conventional factor of safety:

$$P = P_D + P_L = \frac{1}{2} \times 160 + 0.75 \times 2 \times 195 = 372.5 \text{ lb}$$

(*b*) $$F.S. = \frac{P_U}{P} = \frac{629}{372.5}$$

$$F.S. = 1.689 \blacktriangleleft$$

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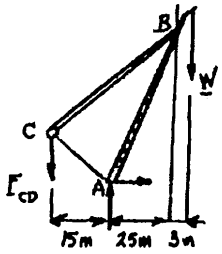
PROBLEM 1.59

In the marine crane shown, link *CD* is known to have a uniform cross section of 50 × 150 mm. For the loading shown, determine`` the normal stress in the central portion of that link.

SOLUTION

Weight of loading: $W = (80\text{ Mg})(9.81\text{ m/s}^2) = 784.8\text{ kN}$

Free Body: Portion *ABC*.



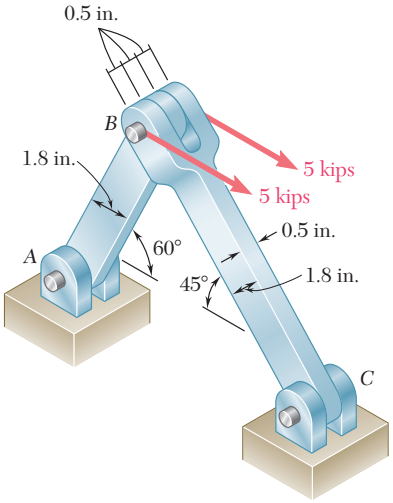
$$+\circlearrowleft \sum M_A = 0: \quad F_{CD}(15\text{ m}) - W(28\text{ m}) = 0$$

$$F_{CD} = \frac{28}{15}W = \frac{28}{15}(784.8\text{ kN})$$

$$F_{CD} = +1465\text{ kN}$$

$$\sigma_{CD} = \frac{F_{CD}}{A} = \frac{+1465 \times 10^3\text{ N}}{(0.050\text{ m})(0.150\text{ m})} = +195.3 \times 10^6\text{ Pa} \qquad \sigma_{CD} = +195.3\text{ MPa} \blacktriangleleft$$

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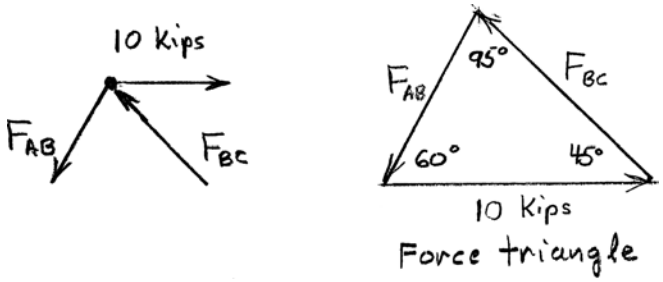


PROBLEM 1.60

Two horizontal 5-kip forces are applied to pin *B* of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.

SOLUTION

Use joint *B* as free body.



Law of Sines:

$$\frac{F_{AB}}{\sin 45°} = \frac{F_{BC}}{\sin 60°} = \frac{10}{\sin 95°}$$

$$F_{AB} = 7.3205 \text{ kips}$$

$$F_{BC} = 8.9658 \text{ kips}$$

Link *AB* is a tension member.

Minimum section at pin: $A_{\text{net}} = (1.8 - 0.8)(0.5) = 0.5 \text{ in}^2$

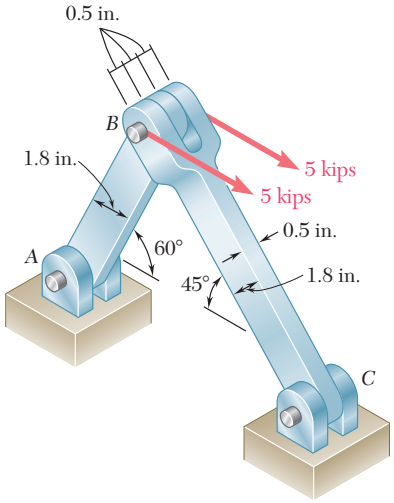
(*a*) Stress in *AB* : $\sigma_{AB} = \dfrac{F_{AB}}{A_{\text{net}}} = \dfrac{7.3205}{0.5}$ $\sigma_{AB} = 14.64 \text{ ksi}$ ◀

Link *BC* is a compression member.

Cross sectional area is $A = (1.8)(0.5) = 0.9 \text{ in}^2$

(*b*) Stress in *BC:* $\sigma_{BC} = \dfrac{-F_{BC}}{A} = \dfrac{-8.9658}{0.9}$ $\sigma_{BC} = -9.96 \text{ ksi}$ ◀

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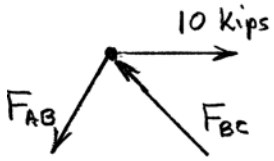
PROBLEM 1.61

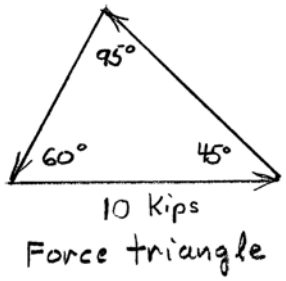
For the assembly and loading of Prob. 1.60, determine (*a*) the average shearing stress in the pin at *C*, (*b*) the average bearing stress at *C* in member *BC*, (*c*) the average bearing stress at *B* in member *BC*.

PROBLEM 1.60 Two horizontal 5-kip forces are applied to pin *B* of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.

SOLUTION

Use joint *B* as free body.





Law of Sines:

$$\frac{F_{AB}}{\sin 45°} = \frac{F_{BC}}{\sin 60°} = \frac{10}{\sin 95°} \qquad F_{BC} = 8.9658 \text{ kips}$$

(*a*) Shearing stress in pin at *C*. $\tau = \dfrac{F_{BC}}{2A_P}$

$$A_P = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.8)^2 = 0.5026 \text{ in}^2$$

$$\tau = \frac{8.9658}{(2)(0.5026)} = 8.92 \qquad \tau = 8.92 \text{ ksi} \blacktriangleleft$$

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PROBLEM 1.61 ***(Continued)***

(*b*) Bearing stress at *C* in member *BC*. $\sigma_b = \dfrac{F_{BC}}{A}$

$$A = td = (0.5)(0.8) = 0.4 \text{ in}^2$$

$$\sigma_b = \frac{8.9658}{0.4} = 22.4 \qquad \sigma_b = 22.4 \text{ ksi} \blacktriangleleft$$

(*c*) Bearing stress at *B* in member *BC*. $\sigma_b = \dfrac{F_{BC}}{A}$

$$A = 2td = 2(0.5)(0.8) = 0.8 \text{ in}^2$$

$$\sigma_b = \frac{8.9658}{0.8} = 11.21 \qquad \sigma_b = 11.21 \text{ ksi} \blacktriangleleft$$

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PROBLEM 1.62

Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.

SOLUTION

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time, the spacer pushes that plate upward with a compressive force P_s in order to maintain equilibrium.

$$P_b = P_s$$

For the bolt, $\sigma_b = \dfrac{F_b}{A_b} = \dfrac{4P_b}{\pi d_b^2}$ or $P_b = \dfrac{\pi}{4}\sigma_b d_b^2$

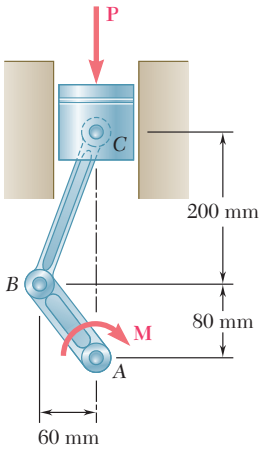
For the spacer, $\sigma_s = \dfrac{P_s}{A_s} = \dfrac{4P_s}{\pi(d_s^2 - d_b^2)}$ or $P_s = \dfrac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$

Equating P_b and P_s,

$$\frac{\pi}{4}\sigma_b d_b^2 = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$$

$$d_s = \sqrt{\left(1 + \frac{\sigma_b}{\sigma_s}\right)} d_b = \sqrt{\left(1 + \frac{200}{130}\right)}(16) \qquad d_s = 25.2 \text{ mm} \blacktriangleleft$$

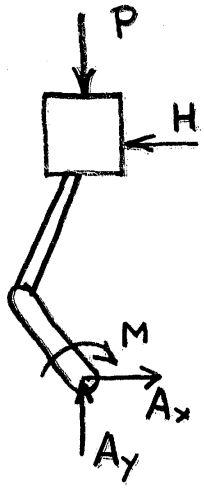
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PROBLEM 1.63

A couple **M** of magnitude 1500 N · m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm^2 uniform cross section.

SOLUTION



Use piston, rod, and crank together as free body. Add wall reaction H and bearing reactions A_x and A_y.

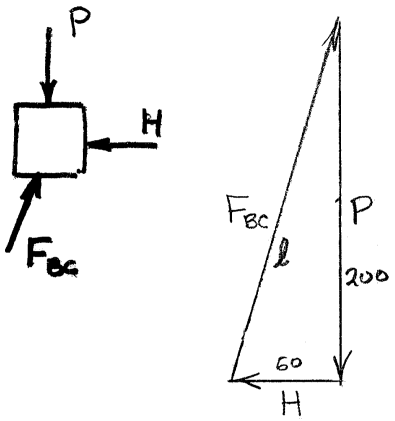
$$+\circlearrowleft \Sigma M_A = 0: \quad (0.280\ \text{m})H - 1500\ \text{N}\cdot\text{m} = 0$$

$$H = 5.3571\times 10^3\ \text{N}$$

Use piston alone as free body. Note that rod is a two-force member; hence the direction of force F_{BC} is known. Draw the force triangle and solve for P and F_{BE} by proportions.

$$l = \sqrt{200^2 + 60^2} = 208.81\ \text{mm}$$

$$\frac{P}{H} = \frac{200}{60} \quad \therefore \quad P = 17.86\times 10^3\ \text{N}$$



(*a*) $P = 17.86\ \text{kN}$ ◀

$$\frac{F_{BC}}{H} = \frac{208.81}{60} \quad \therefore \quad F_{BC} = 18.6436\times 10^3\ \text{N}$$

Rod *BC* is a compression member. Its area is

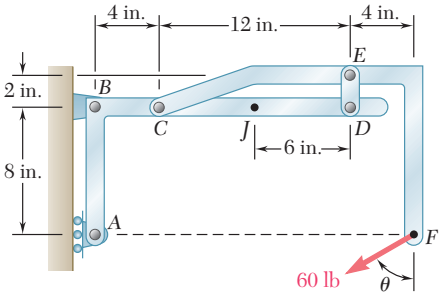
$$450\ \text{mm}^2 = 450\times 10^{-6}\ \text{m}^2$$

Stress:

$$\sigma_{BC} = \frac{-F_{BC}}{A} = \frac{-18.6436\times 10^3}{450\times 10^{-6}} = -41.430\times 10^6\ \text{Pa}$$

(*b*) $\sigma_{BC} = -41.4\ \text{MPa}$ ◀

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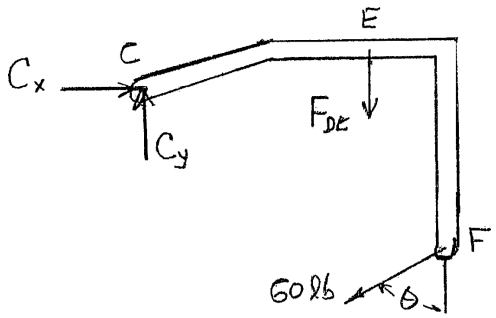


PROBLEM 1.64

Knowing that the link *DE* is $\frac{1}{8}$ in. thick and 1 in. wide, determine the normal stress in the central portion of that link when (*a*) $\theta = 0°$, (*b*) $\theta = 90°$.

SOLUTION

Use member *CEF* as a free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad -12\,F_{DE} - (8)(60\sin\theta) - (16)(60\cos\theta) = 0$$

$$F_{DE} = -40\sin\theta - 80\cos\theta \text{ lb}$$

$$A_{DE} = (1)\left(\frac{1}{8}\right) = 0.125 \text{ in}^2$$

$$\sigma_{DE} = \frac{F_{DE}}{A_{DE}}$$

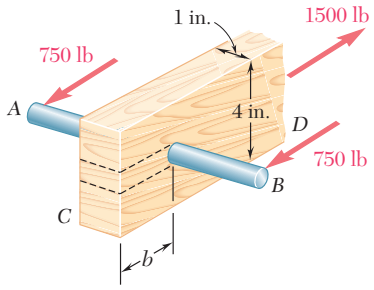
(*a*) $\underline{\theta = 0}$: $F_{DE} = -80$ lb

$$\sigma_{DE} = \frac{-80}{0.125} \qquad \sigma_{DE} = -640 \text{ psi} \blacktriangleleft$$

(*b*) $\underline{\theta = 90°}$: $F_{DE} = -40$ lb

$$\sigma_{DE} = \frac{-40}{0.125} \qquad \sigma_{DE} = -320 \text{ psi} \blacktriangleleft$$

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PROBLEM 1.65

A $\frac{5}{8}$-in.-diameter steel rod *AB* is fitted to a round hole near end *C* of the wooden member *CD*. For the loading shown, determine (*a*) the maximum average normal stress in the wood, (*b*) the distance *b* for which the average shearing stress is 100 psi on the surfaces indicated by the dashed lines, (*c*) the average bearing stress on the wood.

SOLUTION

(*a*) Maximum normal stress in the wood.

$$A_{\text{net}} = (1)\left(4 - \frac{5}{8}\right) = 3.375 \text{ in}^2$$

$$\sigma = \frac{P}{A_{\text{net}}} = \frac{1500}{3.375} = 444 \text{ psi} \qquad \sigma = 444 \text{ psi} \blacktriangleleft$$

(*b*) Distance *b* for $\tau = 100$ psi.

For sheared area see dotted lines.

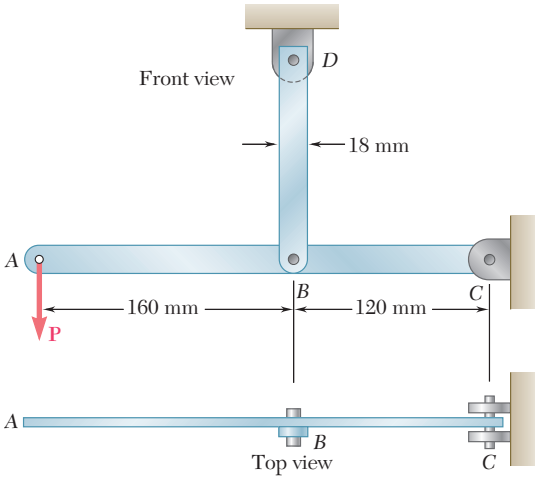
$$\tau = \frac{P}{A} = \frac{P}{2bt}$$

$$b = \frac{P}{2t\tau} = \frac{1500}{(2)(1)(100)} = 7.50 \text{ in.} \qquad b = 7.50 \text{ in.} \blacktriangleleft$$

(*c*) Average bearing stress on the wood.

$$\sigma_b = \frac{P}{A_b} = \frac{P}{dt} = \frac{1500}{\left(\frac{5}{8}\right)(1)} = 2400 \text{ psi} \qquad \sigma_b = 2400 \text{ psi} \blacktriangleleft$$

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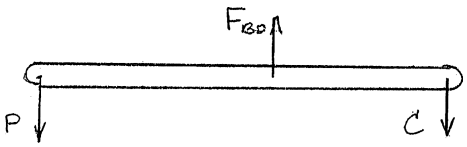


PROBLEM 1.66

In the steel structure shown, a 6-mm-diameter pin is used at *C* and 10-mm-diameter pins are used at *B* and *D*. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link *BD*. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at *A*. Note that link *BD* is not reinforced around the pin holes.

SOLUTION

Use free body *ABC*.



$$+\circlearrowleft \Sigma M_C = 0: \quad 0.280P - 0.120F_{BD} = 0$$

$$P = \frac{3}{7}F_{BD} \tag{1}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.160P - 0.120C = 0$$

$$P = \frac{3}{4}C \tag{2}$$

Tension on net section of link *BD*:

$$F_{BD} = \sigma A_{\text{net}} = \frac{\sigma_U}{F.S.}A_{\text{net}} = \left(\frac{400\times10^6}{3}\right)(6\times10^{-3})(18-10)(10^{-3}) = 6.40\times10^3\ \text{N}$$

Shear in pins at *B* and *D*:

$$F_{BD} = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(10\times10^{-3})^2 = 3.9270\times10^3\ \text{N}$$

Smaller value of F_{BD} is 3.9270×10^3 N.

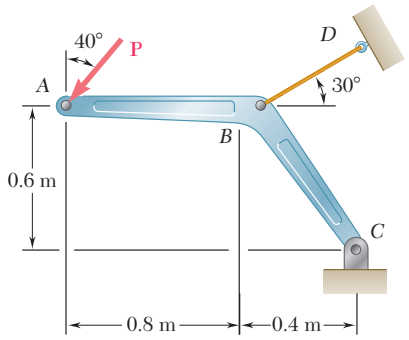
From (1), $$P = \left(\frac{3}{7}\right)(3.9270\times10^3) = 1.683\times10^3\ \text{N}$$

Shear in pin at *C*: $$C = 2\tau A_{\text{pin}} = 2\frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = (2)\left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(6\times10^{-3})^2 = 2.8274\times10^3\ \text{N}$$

From (2), $$P = \left(\frac{3}{4}\right)(2.8274\times10^3) = 2.12\times10^3\ \text{N}$$

Smaller value of *P* is allowable value. $P = 1.683\times10^3$ N $P = 1.683$ kN ◀

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PROBLEM 1.67

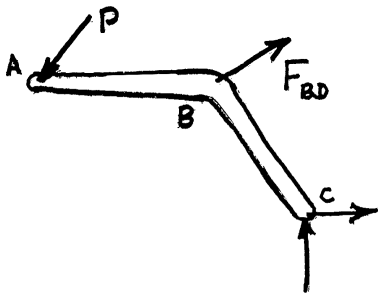
Member *ABC*, which is supported by a pin and bracket at *C* and a cable *BD*, was designed to support the 16-kN load **P** as shown. Knowing that the ultimate load for cable *BD* is 100 kN, determine the factor of safety with respect to cable failure.

SOLUTION

Use member *ABC* as a free body, and note that member *BD* is a two-force member.

$$+\circlearrowleft \Sigma M_c = 0: \quad (P\cos 40°)(1.2) + (P\sin 40°)(0.6) - (F_{BD}\cos 30°)(0.6) - (F_{BD}\sin 30°)(0.4) = 0$$

$$1.30493P - 0.71962F_{BD} = 0$$



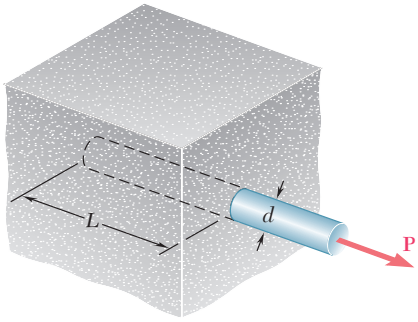
$$F_{BD} = 1.81335P = (1.81335)(16\times 10^3) = 29.014\times 10^3 \text{ N}$$

$$F_U = 100\times 10^3 \text{ N}$$

$$F.S. = \frac{F_U}{F_{BD}} = \frac{100\times 10^3}{29.014\times 10^3}$$

$F.S. = 3.45$ ◀

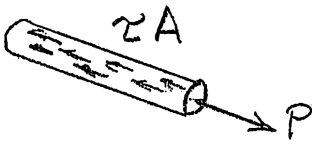
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PROBLEM 1.68

A force **P** is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. Determine the smallest length L for which the full allowable normal stress in the bar can be developed. Express the result in terms of the diameter d of the bar, the allowable normal stress σ_{all} in the steel, and the average allowable bond stress τ_{all} between the concrete and the cylindrical surface of the bar. (Neglect the normal stresses between the concrete and the end of the bar.)

SOLUTION



For shear, $A = \pi dL$

$$P = \tau_{all} A = \tau_{all} \pi dL$$

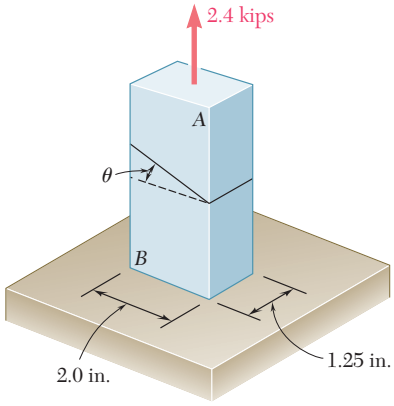
For tension, $A = \frac{\pi}{4} d^2$

$$P = \sigma_{all} A = \sigma_{all} \left(\frac{\pi}{4} d^2 \right)$$

Equating, $$\tau_{all} \pi dL = \sigma_{all} \frac{\pi}{4} d^2$$

Solving for L, $$L_{min} = \sigma_{all} d / 4\tau_{all} \blacktriangleleft$$

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PROBLEM 1.69

The two portions of member *AB* are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine (*a*) the value of θ for which the factor of safety of the member is maximum, (*b*) the corresponding value of the factor of safety. (*Hint:* Equate the expressions obtained for the factors of safety with respect to the normal and shearing stresses.)

SOLUTION

$$A_0 = (2.0)(1.25) = 2.50 \text{ in}^2$$

At the optimum angle, $(F.S.)_\sigma = (F.S.)_\tau$

Normal stress: $\sigma = \dfrac{P}{A_0}\cos^2\theta \quad \therefore \quad P_{U,\sigma} = \dfrac{\sigma_U A_0}{\cos^2\theta}$

$$(F.S.)_\sigma = \frac{P_{U,\sigma}}{P} = \frac{\sigma_U A_0}{P\cos^2\theta}$$

Shearing stress: $\tau = \dfrac{P}{A_0}\sin\theta\cos\theta \quad \therefore \quad P_{U,\tau} = \dfrac{\tau_U A_0}{\sin\theta\cos\theta}$

$$(F.S.)_\tau = \frac{P_{U,\tau}}{P} = \frac{\tau_U A_0}{P\sin\theta\cos\theta}$$

Equating, $\dfrac{\sigma_U A_0}{P\cos^2\theta} = \dfrac{\tau_U A_0}{P\sin\theta\cos\theta}$

Solving, $\dfrac{\sin\theta}{\cos\theta} = \tan\theta = \dfrac{\tau_U}{\sigma_U} = \dfrac{1.3}{2.5} = 0.520$

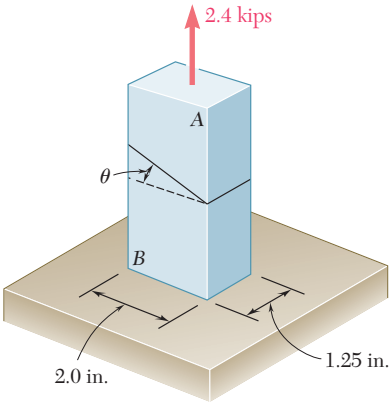
(*a*) $\theta_{\text{opt}} = 27.5°$ ◀

(*b*) $P_U = \dfrac{\sigma_U A_0}{\cos^2\theta} = \dfrac{(12.5)(2.50)}{\cos^2 27.5°} = 7.94 \text{ kips}$

$$F.S. = \frac{P_U}{P} = \frac{7.94}{2.4}$$

$F.S. = 3.31$ ◀

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PROBLEM 1.70

The two portions of member *AB* are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine the range of values of θ for which the factor of safety of the members is at least 3.0.

SOLUTION

$$A_0 = (2.0)(1.25) = 2.50 \text{ in.}^2$$

$$P = 2.4 \text{ kips}$$

$$P_U = (F.S.)P = 7.2 \text{ kips}$$

Based on tensile stress,

$$\sigma_U = \frac{P_U}{A_0}\cos^2\theta$$

$$\cos^2\theta = \frac{\sigma_U A_0}{P_U} = \frac{(2.5)(2.50)}{7.2} = 0.86806$$

$$\cos\theta = 0.93169 \qquad \theta = 21.3° \qquad \theta > 21.3°$$

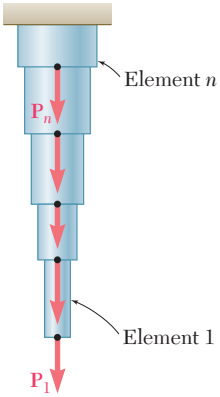
Based on shearing stress, $$\tau_U = \frac{P_U}{A_0}\sin\theta\cos\theta = \frac{P_U}{2A_0}\sin 2\theta$$

$$\sin 2\theta = \frac{2A_0\tau_U}{P_U} = \frac{(2)(2.50)(1.3)}{7.2} = 0.90278$$

$$2\theta = 64.52° \qquad \theta = 32.3° \qquad \theta < 32.3°$$

Hence, $21.3° < \theta < 32.3°$ ◀

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PROBLEM 1.C1

A solid steel rod consisting of n cylindrical elements welded together is subjected to the loading shown. The diameter of element i is denoted by d_i and the load applied to its lower end by $\mathbf{P}_i$ with the magnitude P_i of this load being assumed positive if $\mathbf{P}_i$ is directed downward as shown and negative otherwise. (*a*) Write a computer program that can be used with either SI or U.S. customary units to determine the average stress in each element of the rod. (*b*) Use this program to solve Problems 1.1 and 1.3.

SOLUTION

Force in element i:

It is the sum of the forces applied to that element and all lower ones:

$$F_i = \sum_{k=1}^{i} P_k$$

Average stress in element i:

$$\text{Area} = A_i = \frac{1}{4}\pi d_i^2$$

$$\text{Ave. stress} = \frac{F_i}{A_i}$$

Program outputs:

Problem 1.1

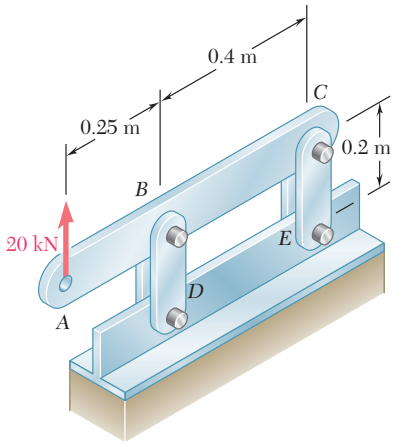
Element	Stress (MPa)
1	84.883
2	–96.766

Problem 1.3

Element	Stress (ksi)
1	22.635
2	17.927

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PROBLEM 1.C2

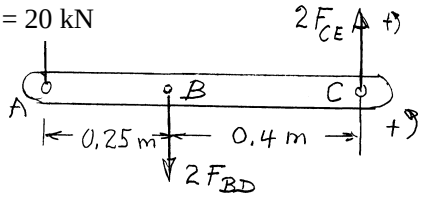


A 20-kN load is applied as shown to the horizontal member *ABC*. Member *ABC* has a 10×50-mm uniform rectangular cross section and is supported by four vertical links, each of 8×36-mm uniform rectangular cross section. Each of the four pins at *A*, *B*, *C*, and *D* has the same diameter *d* and is in double shear. (*a*) Write a computer program to calculate for values of *d* from 10 to 30 mm, using 1-mm increments, (i) the maximum value of the average normal stress in the links connecting pins *B* and *D*, (ii) the average normal stress in the links connecting pins *C* and *E*, (iii) the average shearing stress in pin *B*, (iv) the average shearing stress in pin *C*, (v) the average bearing stress at *B* in member *ABC*, and (vi) the average bearing stress at *C* in member *ABC*. (*b*) Check your program by comparing the values obtained for $d = 16$ mm with the answers given for Probs. 1.7 and 1.27. (*c*) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 150 MPa, 90 MPa, and 230 MPa. (*d*) Solve Part *c*, assuming that the thickness of member *ABC* has been reduced from 10 to 8 mm.

SOLUTION

Forces in links.

F.B. diagram of *ABC*:



$$+\circlearrowleft \Sigma M_C = 0: \quad 2F_{BD}(BC) - P(AC) = 0$$

$$F_{BD} = P(AC)/2(BC) \quad \text{(tension)}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 2F_{CE}(BC) - P(AB) = 0$$

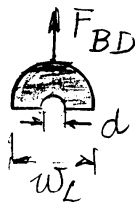
$$F_{CE} = P(AB)/2(BC) \quad \text{(comp.)}$$

(i) Link *BD*.

Thickness $= t_L$

$A_{BD} = t_L(w_L - d)$

$\sigma_{BD} = +F_{BD}/A_{BD}$



(iii) Pin *B*.

$\tau_B = F_{BD}/(\pi d^2/4)$

(v) Bearing stress at *B*.

Thickness of member $AC = t_{AC}$

Sig Bear $B = F_{BD}/(dt_{AC})$

(vi) Bearing stress at *C*.

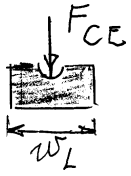
Sig Bear $C = F_{CE}/(dt_{AC})$

(ii) Link *CE*.

Thickness $= t_L$

$A_{CE} = t_L w_L$

$\sigma_{CE} = -F_{CE}/A_{CE}$



(iv) Pin *C*.

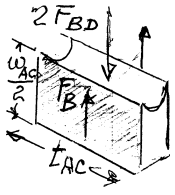
$\tau_C = F_{CE}/(\pi d^2/4)$

Shearing stress in *ABC* under Pin *B*.

$F_B = \tau_{AC} t_{AC}(w_{AC}/2)$

$\Sigma F_y = 0: \quad 2F_B = 2F_{BD}$

$$\tau_{AC} = \frac{2F_{BD}}{\tau_{AC} w_{AC}}$$



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PROBLEM 1.C2 *(Continued)*

Program Outputs

Input data for Parts (*a*), (*b*), (*c*):

$P = 20$ kN, $AB = 0.25$ m, $BC = 0.40$ m, $AC = 0.65$ m,
$TL = 8$ mm, $WL = 36$ mm, $TAC = 10$ mm, $WAC = 50$ mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	325.00	125.00
11.00	81.25	-21.70	170.99	65.77	295.45	113.64
12.00	84.64	-21.70	143.68	55.26	270.83	104.17
13.00	88.32	-21.70	122.43	47.09	250.00	96.15
14.00	92.33	-21.70	105.56	40.60	232.14	89.29
15.00	96.73	-21.70	91.96	35.37	216.67	83.33
16.00	101.56	-21.70	80.82	31.08	203.12	78.13 ← (b)
17.00	106.91	-21.70	71.59	27.54	191.18	73.53
18.00	112.85	-21.70	63.86	24.56	180.56	69.44
19.00	119.49	-21.70	57.31	22.04	171.05	65.79
20.00	126.95	-21.70	51.73	19.89	162.50	62.50
21.00	135.42	-21.70	46.92	18.04	154.76	59.52
22.00	145.09	-21.70	42.75	16.44	147.73	56.82
23.00	156.25	-21.70	39.11	15.04	141.30	54.35
24.00	169.27	-21.70	35.92	13.82	135.42	52.08
25.00	184.66	-21.70	33.10	12.73	130.00	50.00
26.00	203.13	-21.70	30.61	11.77	125.00	48.08
27.00	225.69	-21.70	28.38	10.92	120.37	46.30
28.00	253.91	-21.70	26.39	10.15	116.07	44.64
29.00	290.18	-21.70	24.60	9.46	112.07	43.10
30.00	338.54	-21.70	22.99	8.84	108.33	41.67

(c) Answer: 16 mm $\le d \le$ 22 mm ◀ (c)

Check: For $d = 22$ mm, Tau $AC = 65$ MPa < 90 MPa O.K.

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PROBLEM 1.C2 *(Continued)*

Input data for Part (*d*): $P = 20$ kN,

$AB = 0.25$ m, $BC = 0.40$ m,

$AC = 0.65$ m, $TL = 8$ mm, $WL = 36$ mm,

$TAC = 8$ mm, $WAC = 50$ mm

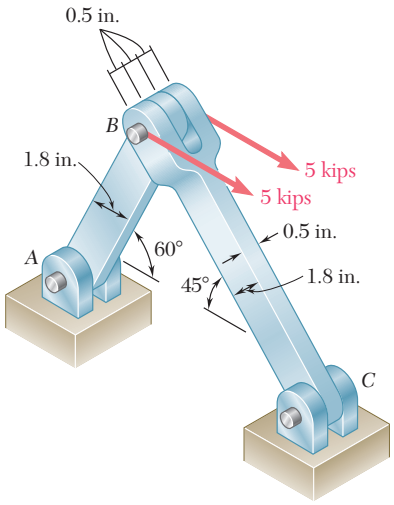
d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	406.25	156.25
11.00	81.25	-21.70	170.99	65.77	369.32	142.05
12.00	84.64	-21.70	143.68	55.26	338.54	130.21
13.00	88.32	-21.70	122.43	47.09	312.50	120.19
14.00	92.33	-21.70	105.56	40.60	290.18	111.61
15.00	96.73	-21.70	91.96	35.37	270.83	104.17
16.00	101.56	-21.70	80.82	31.08	253.91	97.66
17.00	106.91	-21.70	71.59	27.54	238.97	91.91
18.00	112.85	-21.70	63.86	24.56	225.69	86.81
19.00	119.49	-21.70	57.31	22.04	213.82	82.24
20.00	126.95	-21.70	51.73	19.89	203.12	78.13
21.00	135.42	-21.70	46.92	18.04	193.45	74.40
22.00	145.09	-21.70	42.75	16.44	184.66	71.02
23.00	156.25	-21.70	39.11	15.04	176.63	67.93
24.00	169.27	-21.70	35.92	13.82	169.27	65.10
25.00	184.66	-21.70	33.10	12.73	162.50	62.50
26.00	203.13	-21.70	30.61	11.77	156.25	60.10
27.00	225.69	-21.70	28.38	10.92	150.46	57.87
28.00	253.91	-21.70	26.39	10.15	145.09	55.80
29.00	290.18	-21.70	24.60	9.46	140.09	53.88
30.00	338.54	-21.70	22.99	8.84	135.42	52.08

(d) Answer: $18 \text{ mm} \le d \le 22 \text{ mm}$ ◀ (d)

Check: For $d = 22$ mm, Tau $AC = 81.25$ MPa < 90 MPa O.K.

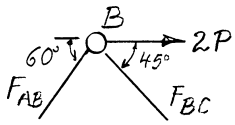
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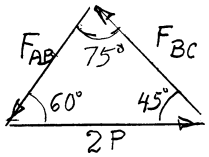
PROBLEM 1.C3



Two horizontal 5-kip forces are applied to Pin *B* of the assembly shown. Each of the three pins at *A*, *B*, and *C* has the same diameter *d* and is double shear. (*a*) Write a computer program to calculate for values of *d* from 0.50 to 1.50 in., using 0.05-in. increments, (i) the maximum value of the average normal stress in member *AB*, (ii) the average normal stress in member *BC*, (iii) the average shearing stress in pin *A*, (iv) the average shearing stress in pin *C*, (v) the average bearing stress at *A* in member *AB*, (vi) the average bearing stress at *C* in member *BC*, and (vii) the average bearing stress at *B* in member *BC*. (*b*) Check your program by comparing the values obtained for $d = 0.8$ in. with the answers given for Problems 1.60 and 1.61. (*c*) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 22 ksi, 13 ksi, and 36 ksi. (*d*) Solve Part *c*, assuming that a new design is being investigated in which the thickness and width of the two members are changed, respectively, from 0.5 to 0.3 in. and from 1.8 to 2.4 in.

SOLUTION





Forces in members *AB* and *BC*.

Free body: Pin *B*.

From force triangle:

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{2P}{\sin 75^\circ}$$

$$F_{AB} = 2P(\sin 45^\circ / \sin 75^\circ)$$

$$F_{BC} = 2P(\sin 60^\circ / \sin 75^\circ)$$

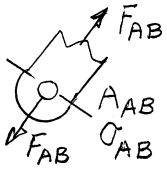
(i) Max. ave. stress in *AB*.

Width = w

Thickness = t

$A_{AB} = (w - d)t$

$\sigma_{AB} = F_{AB}/A_{AB}$



(ii) Ave. stress in *BC*.

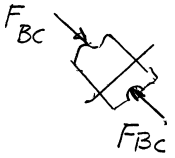
$A_{BC} = wt$

$\sigma_{BC} = F_{BC}/A_{BC}$

(iii) Pin *A*.

$\tau_A = (F_{AB}/2)/(\pi d^2/4)$

(iv) Pin *C*.



$\tau_C = (F_{BC}/2)/(\pi d^2/4)$

(v) Bearing stress at *A*.

Sig Bear $A = F_{AB}/dt$

(vi) Bearing stress at *C*.

Sig Bear $C = F_{BC}/dt$

(vii) Bearing stress at *B* in member *BC*.

Sig Bear $B = F_{BC}/2dt$

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PROBLEM 1.C3 *(Continued)*

Program Outputs

Input data for Parts (*a*), (*b*), (*c*):

$P = 5$ kips, $w = 1.8$ in., $t = 0.5$ in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	11.262	-9.962	18.642	22.831	29.282	35.863	17.932
0.550	11.713	-9.962	15.406	18.869	26.620	32.603	16.301
0.600	12.201	-9.962	12.945	15.855	24.402	29.886	14.943
0.650	12.731	-9.962	11.030	13.510	22.525	27.587	13.793
0.700	13.310	-9.962	9.511	11.649	20.916	25.616	12.808
0.750	13.944	-9.962	8.285	10.147	19.521	23.909	11.954
0.800	14.641	-9.962	7.282	8.918	18.301	22.414	11.207
0.850	15.412	-9.962	6.450	7.900	17.225	21.096	10.548
0.900	16.268	-9.962	5.754	7.047	16.268	19.924	9.962
0.950	17.225	-9.962	5.164	6.324	15.412	18.875	9.438
1.000	18.301	-9.962	4.660	5.708	14.641	17.932	8.966
1.050	19.521	-9.962	4.227	5.177	13.944	17.078	8.539
1.100	20.916	-9.962	3.852	4.717	13.310	16.301	8.151
1.150	22.525	-9.962	3.524	4.316	12.731	15.593	7.796
1.200	24.402	-9.962	3.236	3.964	12.201	14.943	7.471
1.250	26.620	-9.962	2.983	3.653	11.713	14.345	7.173
1.300	29.282	-9.962	2.758	3.377	11.262	13.793	6.897
1.350	32.536	-9.962	2.557	3.132	10.845	13.283	6.641
1.400	36.603	-9.962	2.378	2.912	10.458	12.808	6.404
1.450	41.831	-9.962	2.217	2.715	10.097	12.367	6.183
1.500	48.803	-9.962	2.071	2.537	9.761	11.954	5.977

(b)

(c) Answer: $0.70 \text{ in.} \le d \le 1.10 \text{ in.}$ ◀ (c)

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PROBLEM 1.C3 *(Continued)*

Input data for Part (*d*),

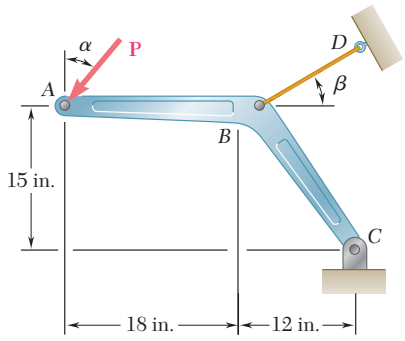
$P = 5$ kips, $w = 2.4$ in., $t = 0.3$ in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	12.843	-12.452	18.642	22.831	46.803	59.772	29.886
0.550	13.190	-12.452	15.406	18.869	44.367	54.338	27.169
0.600	13.556	-12.452	12.945	15.855	40.669	49.810	24.905
0.650	13.944	-12.452	11.030	13.510	37.541	45.978	22.989
0.700	14.354	-12.452	9.511	11.649	34.860	42.694	21.347
0.750	14.789	-12.452	8.285	10.147	32.536	39.848	19.924
0.800	15.251	-12.452	7.282	8.918	30.502	37.357	18.679
0.850	15.743	-12.452	6.450	7.900	28.708	35.160	17.580
0.900	16.268	-12.452	5.754	7.047	27.113	33.206	16.603
0.950	16.829	-12.452	5.164	6.324	25.686	31.459	15.729
1.000	17.430	-12.452	4.660	5.708	24.402	29.886	14.943
1.050	18.075	-12.452	4.227	5.177	23.240	28.463	14.231
1.100	18.771	-12.452	3.852	4.717	22.183	27.169	13.584
1.150	19.521	-12.452	3.524	4.316	21.219	25.988	12.994
1.200	20.335	-12.452	3.236	3.964	20.335	24.905	12.452
1.250	21.219	-12.452	2.983	3.653	19.521	23.909	11.954
1.300	22.183	-12.452	2.758	3.377	18.771	22.989	11.495
1.350	23.240	-12.452	2.557	3.132	18.075	22.138	11.069
1.400	24.402	-12.452	2.378	2.912	17.430	21.347	10.674
1.450	25.686	-12.452	2.217	2.715	16.829	20.611	10.305
1.500	27.113	-12.452	2.071	2.537	16.268	19.924	9.962

(d) Answer: 0.85 in. $\le d \le$ 1.25 in. ◀ (d)

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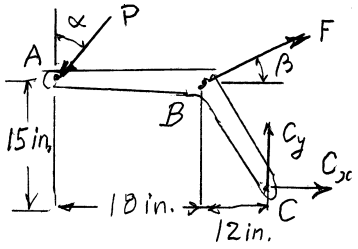
PROBLEM 1.C4



A 4-kip force **P** forming an angle α with the vertical is applied as shown to member *ABC*, which is supported by a pin and bracket at *C* and by a cable *BD* forming an angle β with the horizontal. (*a*) Knowing that the ultimate load of the cable is 25 kips, write a computer program to construct a table of the values of the factor of safety of the cable for values of α and β from 0 to 45°, using increments in α and β corresponding to 0.1 increments in tan α and tan β. (*b*) Check that for any given value of α, the maximum value of the factor of safety is obtained for $\beta = 38.66°$ and explain why. (*c*) Determine the smallest possible value of the factor of safety for $\beta = 38.66°$, as well as the corresponding value of α, and explain the result obtained.

SOLUTION

(a) Draw *F.B.* diagram of *ABC*:



$$+\circlearrowleft \Sigma M_C = 0: \quad (P\sin\alpha)(1.5\text{ in.}) + (P\cos\alpha)(30\text{ in.}) - (F\cos\beta)(15\text{ in.}) - (F\sin\beta)(12\text{ in.}) = 0$$

$$F = P\,\frac{15\sin\alpha + 30\cos\alpha}{15\cos\beta + 12\sin\beta}$$

$$F.S. = F_{ult}/F$$

Output for $P = 4$ kips and $F_{ult} = 20$ kips:

	VALUES OF FS									
	BETA									
	0	5.71	11.31	16.70	21.80	26.56	30.96	34.99	38.66	41.99	45.00
ALPHA											
0.000	3.125	3.358	3.555	3.712	3.830	3.913	3.966	3.994	4.002	3.995	3.977
5.711	2.991	3.214	3.402	3.552	3.666	3.745	3.796	3.823	3.830	3.824	3.807
11.310	2.897	3.113	3.295	3.441	3.551	3.628	3.677	3.703	3.710	3.704	3.687
16.699	2.837	3.049	3.227	3.370	3.477	3.553	3.600	3.626	3.633	3.627	3.611
21.801	2.805	3.014	3.190	3.331	3.438	3.512	3.560	3.585	3.592	3.586	3.570
26.565	2.795	3.004	3.179	3.320	3.426	3.500	3.547	3.572	3.579	3.573	3.558
30.964	2.803	3.013	3.189	3.330	3.436	3.510	3.558	3.583	3.590	3.584	3.568
34.992	2.826	3.036	3.214	3.356	3.463	3.538	3.586	3.611	3.619	3.612	3.596
38.660	2.859	3.072	3.252	3.395	3.503	3.579	3.628	3.653	3.661	3.655	3.638
41.987	2.899	3.116	3.298	3.444	3.554	3.631	3.680	3.706	3.713	3.707	3.690
45.000	2.946	3.166	3.351	3.499	3.611	3.689	3.739	3.765	3.773	3.767	3.750

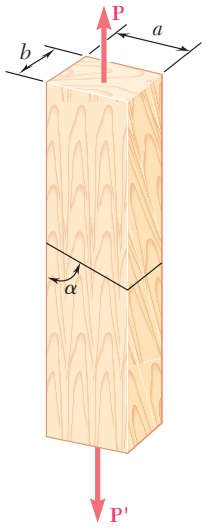
↑(b)

(b) When $\beta = 38.66°$, $\tan\beta = 0.8$ and cable *BD* is perpendicular to the lever arm *BC*.

(c) $F.S. = 3.579$ for $\alpha = 26.6°$; *P* is perpendicular to the lever arm *AC*.

Note: The value $F.S. = 3.579$ is the smallest of the values of *F.S.* corresponding to $\beta = 38.66°$ and the largest of those corresponding to $\alpha = 26.6°$. The point $\alpha = 26.6°$, $\beta = 38.66°$ is a "saddle point," or "minimax" of the function $F.S.(\alpha, \beta)$.

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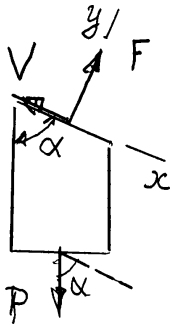


PROBLEM 1.C5

A load **P** is supported as shown by two wooden members of uniform rectangular cross section that are joined by a simple glued scarf splice. (*a*) Denoting by σ_U and τ_U, respectively, the ultimate strength of the joint in tension and in shear, write a computer program which, for given values of *a*, *b*, *P*, σ_U and τ_U, expressed in either SI or U.S. customary units, and for values of α from 5 to 85° at 5° intervals, can be used to calculate (i) the normal stress in the joint, (ii) the shearing stress in the joint, (iii) the factor of safety relative to failure in tension, (iv) the factor of safety relative to failure in shear, and (v) the overall factor of safety for the glued joint. (*b*) Apply this program, using the dimensions and loading of the members of Probs. 1.29 and 1.31, knowing that $\sigma_U = 150$ psi and $\tau_U = 214$ psi for the glue used in Prob. 1.29, and that $\sigma_U = 1.26$ MPa and $\tau_U = 1.50$ MPa for the glue used in Prob. 1.31. (*c*) Verify in each of these two cases that the shearing stress is maximum for $a = 45°$.

SOLUTION

(i) and (ii) Draw the *F.B.* diagram of lower member:



$$+\searrow \Sigma F_x = 0: \quad -V + P\cos\alpha = 0 \qquad V = P\cos\alpha$$

$$+\nearrow \Sigma F_y = 0: \quad F - P\sin\alpha = 0 \qquad F = P\sin\alpha$$

$\text{Area} = ab/\sin\alpha$

Normal stress: $$\sigma = \frac{F}{\text{Area}} = (P/ab)\sin^2\alpha$$

Shearing stress: $$\tau = \frac{V}{\text{Area}} = (P/ab)\sin\alpha\cos\alpha$$

(iii) *F.S.* for tension (normal stresses):

$$FSN = \sigma_U/\sigma$$

(iv) *F.S.* for shear:

$$FSS = \tau_U/\tau$$

(v) Overall *F.S.*:

F.S. = The smaller of *FSN* and *FSS*.

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PROBLEM 1.C5 *(Continued)*

Program Outputs

Problem 1.29

$a = 150$ mm
$b = 75$ mm
$P = 11$ kN
$\sigma_U = 1.26$ MPa
$\tau_U = 1.50$ MPa

ALPHA	SIG (MPa)	TAU (MPa)	FSN	FSS	FS	
5	0.007	0.085	169.644	17.669	17.669	
10	0.029	0.167	42.736	8.971	8.971	
15	0.065	0.244	19.237	6.136	6.136	
20	0.114	0.314	11.016	4.773	4.773	
25	0.175	0.375	7.215	4.005	4.005	
30	0.244	0.423	5.155	3.543	3.543	
35	0.322	0.459	3.917	3.265	3.265	
40	0.404	0.481	3.119	3.116	3.116	
45	0.489	0.489	2.577	3.068	2.577	◀ (b), (c)
50	0.574	0.481	2.196	3.116	2.196	
55	0.656	0.459	1.920	3.265	1.920	
60	0.733	0.423	1.718	3.543	1.718	
65	0.803	0.375	1.569	4.005	1.569	
70	0.863	0.314	1.459	4.773	1.459	
75	0.912	0.244	1.381	6.136	1.381	
80	0.948	0.167	1.329	8.971	1.329	
85	0.970	0.085	1.298	17.669	1.298	

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PROBLEM 1.C5 *(Continued)*

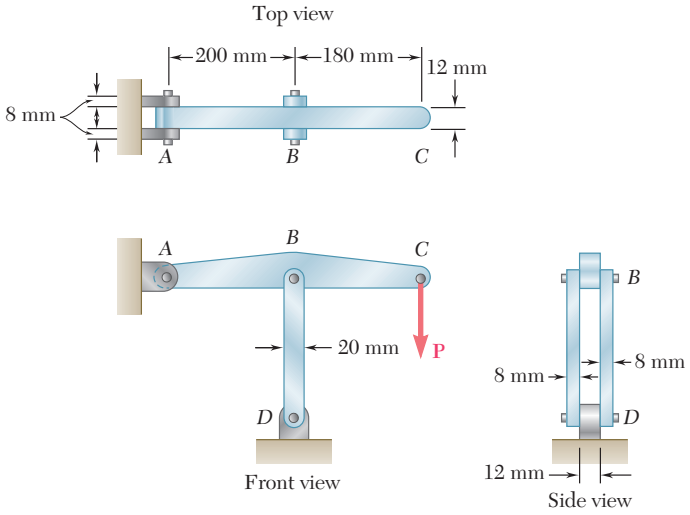
Problem 1.31

$a = 5$ in.
$b = 3$ in.
$P = 1400$ lb
$\sigma_U = 150$ psi
$\tau_U = 214$ psi

ALPHA	SIG (psi)	TAU (psi)	FSN	FSS	FS	
5	0.709	8.104	211.574	26.408	26.408	
10	2.814	15.961	53.298	13.408	13.408	
15	6.252	23.333	23.992	9.171	9.171	
20	10.918	29.997	13.739	7.134	7.134	
25	16.670	35.749	8.998	5.986	5.986	
30	23.333	40.415	6.429	5.295	5.295	
35	30.706	43.852	4.885	4.880	4.880	
40	38.563	45.958	3.890	4.656	3.890	
45	46.667	46.667	3.214	4.586	3.214	◀ (c)
50	54.770	45.958	2.739	4.656	2.739	
55	62.628	43.852	2.395	4.880	2.395	
60	70.000	40.415	2.143	5.295	2.143	◀ (b)
65	76.663	35.749	1.957	5.986	1.957	
70	82.415	29.997	1.820	7.134	1.820	
75	87.081	23.333	1.723	9.171	1.723	
80	90.519	15.961	1.657	13.408	1.657	
85	92.624	8.104	1.619	26.408	1.619	

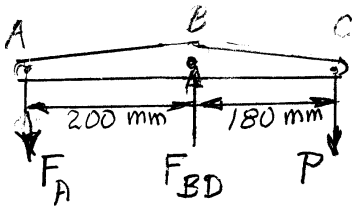
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PROBLEM 1.C6



Member *ABC* is supported by a pin and bracket at *A* and by two links, which are pin-connected to the member at *B* and to a fixed support at *D*. (*a*) Write a computer program to calculate the allowable load P_{all} for any given values of (i) the diameter d_1 of the pin at *A*, (ii) the common diameter d_2 of the pins at *B* and *D*, (iii) the ultimate normal stress σ_U in each of the two links, (iv) the ultimate shearing stress τ_U in each of the three pins, and (v) the desired overall factor of safety *F.S.* (*b*) Your program should also indicate which of the following three stresses is critical: the normal stress in the links, the shearing stress in the pin at *A*, or the shearing stress in the pins at *B* and *D*. (*c*) Check your program by using the data of Probs. 1.55 and 1.56, respectively, and comparing the answers obtained for P_{all} with those given in the text. (*d*) Use your program to determine the allowable load P_{all}, as well as which of the stresses is critical, when $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa for aluminum links, $\tau_U = 100$ MPa for steel pins, and *F.S.* = 3.2.

SOLUTION



(a) F.B. diagram of *ABC*:

$$\Sigma M_A = 0: \quad P = \frac{200}{380} F_{BD}$$

$$\Sigma M_B = 0: \quad P = \frac{200}{180} F_A$$

(i) For given d_1 of Pin *A*: $F_A = 2(\tau_U/FS)(\pi d_1^2/4)$, $P_1 = \dfrac{200}{180} F_A$

(ii) For given d_2 of Pins *B* and *D*: $F_{BD} = 2(\tau_U/FS)(\pi d_2^2/4)$, $P_2 = \dfrac{200}{380} F_{BD}$

(iii) For ultimate stress in links *BD*: $F_{BD} = 2(\sigma_U/FS)(0.02)(0.008)$, $P_3 = \dfrac{200}{380} F_{BD}$

(iv) For ultimate shearing stress in pins: P_4 is the smaller of P_1 and P_2.

(v) For desired overall *F.S.*: P_5 is the smaller of P_3 and P_4.

If $P_3 < P_4$, stress is critical in links.

If $P_4 < P_3$ and $P_1 < P_2$, stress is critical in Pin *A*.

If $P_4 < P_3$ and $P_2 < P_1$, stress is critical in Pins *B* and *D*.

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PROBLEM 1.C6 *(Continued)*

Program Outputs

(b) Problem 1.55. Data: $d_1 = 8$ mm, $d_2 = 12$ mm, $\sigma_U, = 250$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.0$

$P_{\text{all}} = 3.72$ kN. Stress in Pin *A* is critical. ◀

(c) Problem 1.56. Data: $d_1 = 10$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.0$

$P_{\text{all}} = 3.97$ kN. Stress in Pins *B* and *D* is critical. ◀

(d) Data: $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.2$

$P_{\text{all}} = 5.79$ kN. Stress in links is critical. ◀

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CHAPTER 2

PROBLEM 2.1

A nylon thread is subjected to a 8.5-N tension force. Knowing that $E = 3.3\,\text{GPa}$ and that the length of the thread increases by 1.1%, determine (*a*) the diameter of the thread, (*b*) the stress in the thread.

SOLUTION

(*a*) Strain: $\varepsilon = \dfrac{\delta}{L} = \dfrac{1.1}{100} = 0.011$

Stress: $\sigma = E\varepsilon = (3.3 \times 10^9)(0.011) = 36.3 \times 10^6\,\text{Pa}$

$$\sigma = \frac{P}{A}$$

Area: $A = \dfrac{P}{\sigma} = \dfrac{8.5}{36.3 \times 10^6} = 234.16 \times 10^{-9}\,\text{m}^2$

Diameter: $d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(234.16 \times 10^{-9})}{\pi}} = 546 \times 10^{-6}\,\text{m}$ $d = 0.546\,\text{mm}$ ◀

(*b*) Stress: $\sigma = 36.3\,\text{MPa}$ ◀

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PROBLEM 2.2

A 4.8-ft-long steel wire of $\frac{1}{4}$ -in.-diameter is subjected to a 750-lb tensile load. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the elongation of the wire, (*b*) the corresponding normal stress.

SOLUTION

(*a*) Deformation: $\delta = \dfrac{PL}{AE}; \qquad A = \dfrac{\pi d^2}{4}$

Area: $A = \dfrac{\pi(0.25 \text{ in.})^2}{4} = 4.9087 \times 10^{-2} \text{ in}^2$

$$\delta = \frac{(750 \text{ lb})(4.8 \text{ ft} \times 12 \text{ in./ft})}{(4.9087 \times 10^{-2} \text{ in}^2)(29 \times 10^6 \text{ psi})}$$

$$\delta = 3.0347 \times 10^{-2} \text{ in.} \qquad \delta = 0.0303 \text{ in.} \blacktriangleleft$$

(*b*) Stress: $\sigma = \dfrac{P}{A}$

Area: $\sigma = \dfrac{(750 \text{ lb})}{(4.9087 \times 10^{-2} \text{ in}^2)}$

$$\sigma = 1.52790 \times 10^4 \text{ psi} \qquad \sigma = 15.28 \text{ ksi} \blacktriangleleft$$

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PROBLEM 2.3

An 18-m-long steel wire of 5-mm diameter is to be used in the manufacture of a prestressed concrete beam. It is observed that the wire stretches 45 mm when a tensile force **P** is applied. Knowing that $E = 200$ GPa, determine (*a*) the magnitude of the force **P**, (*b*) the corresponding normal stress in the wire.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}$, or $P = \dfrac{\delta AE}{L}$

with $A = \dfrac{1}{4}\pi d^2 = \dfrac{1}{4}\pi(0.005)^2 = 19.6350 \times 10^{-6}\ \text{m}^2$

$$P = \frac{(0.045\ \text{m})(19.6350 \times 10^{-6}\ \text{m}^2)(200 \times 10^{9}\ \text{N/m}^2)}{18\ \text{m}} = 9817.5\ \text{N}$$

$P = 9.82$ kN ◀

(*b*) $\sigma = \dfrac{P}{A} = \dfrac{9817.5\ \text{N}}{19.6350 \times 10^{-6}\ \text{m}^2} = 500 \times 10^{6}\ \text{Pa}$

$\sigma = 500$ MPa ◀

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PROBLEM 2.4

Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod with E = 73 GPa and an ultimate strength of 140 MPa. Knowing that the distance between the gage marks is 250.28 mm after a load is applied, determine (*a*) the stress in the rod, (*b*) the factor of safety.

SOLUTION

(*a*) $\delta = L - L_0$

$= 250.28 \text{ mm} - 250 \text{ mm}$

$= 0.28 \text{ mm}$

$\varepsilon = \dfrac{\delta}{L_0}$

$= \dfrac{0.28 \text{ mm}}{250 \text{ mm}}$

$= 1.11643 \times 10^{-4}$

$\sigma = E\varepsilon$

$= (73 \times 10^9 \text{ Pa})(1.11643 \times 10^{-4})$

$= 8.1760 \times 10^7 \text{ Pa}$

$\sigma = 81.8 \text{ MPa}$ ◀

(*b*) $\text{F.S.} = \dfrac{\sigma_u}{\sigma}$

$= \dfrac{140 \text{ MPa}}{81.760 \text{ MPa}}$

$= 1.71233$ $\text{F.S.} = 1.712$ ◀

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PROBLEM 2.5

An aluminum pipe must not stretch more than 0.05 in. when it is subjected to a tensile load. Knowing that $E = 10.1 \times 10^6$ psi and that the maximum allowable normal stress is 14 ksi, determine (*a*) the maximum allowable length of the pipe, (*b*) the required area of the pipe if the tensile load is 127.5 kips.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}$

Thus, $L = \dfrac{EA\delta}{P} = \dfrac{E\delta}{\sigma} = \dfrac{(10.1 \times 10^6)(0.05)}{14 \times 10^3}$

$L = 36.1$ in. ◀

(*b*) $\sigma = \dfrac{P}{A}$

Thus, $A = \dfrac{P}{\sigma} = \dfrac{127.5 \times 10^3}{14 \times 10^3}$ $A = 9.11 \text{ in}^2$ ◀

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PROBLEM 2.6

A control rod made of yellow brass must not stretch more than 3 mm when the tension in the wire is 4 kN. Knowing that E = 105 GPa and that the maximum allowable normal stress is 180 MPa, determine (*a*) the smallest diameter rod that should be used, (*b*) the corresponding maximum length of the rod.

SOLUTION

(*a*) $\sigma = \dfrac{P}{A}; \qquad A = \dfrac{\pi d^2}{4}$

Substituting, we have

$$\sigma = \frac{P}{\left(\dfrac{\pi d^2}{4}\right)} \quad \Rightarrow \quad d = \sqrt{\frac{4P}{\sigma\pi}}$$

$$d = \sqrt{\frac{4(4\times10^{3}\ \text{N})}{(180\times10^{6}\ \text{Pa})\,\pi}}$$

$$d = 5.3192\times10^{-3}\ \text{m}$$

$$d = 5.32\ \text{mm} \blacktriangleleft$$

(*b*) $\sigma = E\varepsilon; \qquad \varepsilon = \dfrac{\delta}{L}$

Substituting, we have

$$\sigma = E\frac{\delta}{L} \quad \Rightarrow \quad L = \frac{E\partial}{\sigma}$$

$$L = \frac{(105\times10^{9}\ \text{Pa})(3\times10^{-3}\ \text{m})}{(180\times10^{6}\ \text{Pa})}$$

$$L = 1.750\ \text{m} \blacktriangleleft$$

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PROBLEM 2.7

A steel control rod is 5.5 ft long and must not stretch more than 0.04 in. when a 2-kip tensile load is applied to it. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the smallest diameter rod that should be used, (*b*) the corresponding normal stress caused by the load.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}$: $0.04 \text{ in.} = \dfrac{(2000 \text{ lb})(5.5 \times 12 \text{ in.})}{A(29 \times 10^6 \text{ psi})}$

$$A = \frac{1}{4}\pi d^2 = 0.113793 \text{ in}^2$$

$$d = 0.38063 \text{ in.} \qquad d = 0.381 \text{ in.} \blacktriangleleft$$

(*b*) $\sigma = \dfrac{P}{A} = \dfrac{2000 \text{ lb}}{0.113793 \text{ in}^2} = 17575.8 \text{ psi}$ $\qquad \sigma = 17.58 \text{ ksi} \blacktriangleleft$

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PROBLEM 2.8

A cast-iron tube is used to support a compressive load. Knowing that $E = 10 \times 10^6$ psi and that the maximum allowable change in length is 0.025%, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 1600 lb if the outside diameter of the tube is 2.0 in.

SOLUTION

(*a*) $\dfrac{\delta}{L} = \dfrac{\delta}{100} = 0.00025$

$\sigma = E\varepsilon; \qquad \varepsilon = \dfrac{\delta}{L}$

$\therefore \qquad \sigma = E\dfrac{\delta}{L}$

$\sigma = (10 \times 10^6 \text{ psi})(0.00025)$

$\sigma = 2.50 \times 10^3 \text{ psi}$

$\sigma = 2.50 \text{ ksi}$ ◀

(*b*) $\sigma = \dfrac{P}{A}; \qquad \therefore \qquad A = \dfrac{P}{\sigma} = \dfrac{1600 \text{ lb}}{2.50 \times 10^3 \text{ psi}} = 0.64 \text{ in}^2$

$A = \dfrac{\pi}{4}\left(d_o^2 - d_i^2\right)$

$d_i^2 = d_o^2 - \dfrac{4A}{\pi}$

$d_i^2 = (2.0 \text{ in.})^2 - \dfrac{4(0.64 \text{ in}^2)}{\pi} = 3.1851 \text{ in}^2$

$\therefore \qquad d_i = 1.78469 \text{ in.}$

$t = \dfrac{1}{2}(d_o - d_i) = \dfrac{1}{2}(2.0 \text{ in.} - 1.78469 \text{ in.})$

$t = 0.107655 \text{ in.}$

$t = 0.1077$ ◀

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PROBLEM 2.9

A 4-m-long steel rod must not stretch more than 3 mm and the normal stress must not exceed 150 MPa when the rod is subjected to a 10-kN axial load. Knowing that $E = 200$ GPa, determine the required diameter of the rod.

SOLUTION

$L = 4$ m

$\delta = 3 \times 10^{-3}$ m, $\sigma = 150 \times 10^{6}$ Pa

$E = 200 \times 10^{9}$ Pa, $P = 10 \times 10^{3}$ N

Stress: $\sigma = \dfrac{P}{A}$

$$A = \frac{P}{\sigma} = \frac{10 \times 10^{3}\ \text{N}}{150 \times 10^{6}\ \text{Pa}} = 66.667 \times 10^{-6}\text{m}^2 = 66.667\ \text{mm}^2$$

Deformation: $\delta = \dfrac{PL}{AE}$

$$A = \frac{PL}{E\delta} = \frac{(10 \times 10^{3})(4)}{(200 \times 10^{9})(3 \times 10^{-3})} = 66.667 \times 10^{-6}\text{m}^2 = 66.667\ \text{mm}^2$$

The larger value of A governs: $A = 66.667\ \text{mm}^2$

$$A = \frac{\pi}{4} d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(66.667)}{\pi}}$$

$d = 9.21$ mm ◀

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PROBLEM 2.10

A nylon thread is to be subjected to a 10-N tension. Knowing that $E = 3.2$ GPa, that the maximum allowable normal stress is 40 MPa, and that the length of the thread must not increase by more than 1%, determine the required diameter of the thread.

SOLUTION

Stress criterion:

$$\sigma = 40 \text{ MPa} = 40 \times 10^6 \text{ Pa} \quad P = 10 \text{ N}$$

$$\sigma = \frac{P}{A}: \quad A = \frac{P}{\sigma} = \frac{10 \text{ N}}{40 \times 10^6 \text{ Pa}} = 250 \times 10^{-9} \text{m}^2$$

$$A = \frac{\pi}{4} d^2: \quad d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{250 \times 10^{-9}}{\pi}} = 564.19 \times 10^{-6} \text{m}$$

$$d = 0.564 \text{ mm}$$

Elongation criterion:

$$\frac{\delta}{L} = 1\% = 0.01$$

$$\delta = \frac{PL}{AE}:$$

$$A = \frac{P/E}{\delta/L} = \frac{10 \text{ N}/3.2 \times 10^9 \text{ Pa}}{0.01} = 312.5 \times 10^{-9} \text{ m}^2$$

$$d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{312.5 \times 10^{-9}}{\pi}} = 630.78 \times 10^{-6} \text{m}^2$$

$$d = 0.631 \text{ mm}$$

The required diameter is the larger value: $d = 0.631$ mm ◀

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PROBLEM 2.11

A block of 10-in. length and 1.8 × 1.6-in. cross section is to support a centric compressive load **P**. The material to be used is a bronze for which $E = 14 \times 10^6$ psi. Determine the largest load that can be applied, knowing that the normal stress must not exceed 18 ksi and that the decrease in length of the block should be at most 0.12% of its original length.

SOLUTION

Considering allowable stress, $\sigma = 18$ ksi or 18×10^3 psi

Cross-sectional area: $A = (1.8 \text{ in.})(1.6 \text{ in.}) = 2.880 \text{ in}^2$

$$\sigma = \frac{P}{A} \quad \Rightarrow \quad P = \sigma A$$

$$= (18 \times 10^3 \text{ psi})(2.880 \text{ in}^2)$$

$$= 5.1840 \times 10^4 \text{ lb}$$

$$\text{or } 51.840 \text{ kips}$$

Considering allowable deformation, $\frac{\delta}{L} = 0.12\%$ or 0.0012 in.

$$\delta = \frac{PL}{AE} \quad \Rightarrow \quad P = AE\left(\frac{\delta}{L}\right)$$

$$P = (2.880 \text{ in}^2)(14 \times 10^6 \text{ psi})(0.0012 \text{ in.})$$

$$= 4.8384 \times 10^4 \text{ lb}$$

$$\text{or } 48.384 \text{ kips}$$

The smaller value for **P** resulting from the required deformation criteria governs.

48.4 kips ◀

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PROBLEM 2.12

A square yellow-brass bar must not stretch more than 2.5 mm when it is subjected to a tensile load. Knowing that $E = 105$ GPa and that the allowable tensile strength is 180 MPa, determine (*a*) the maximum allowable length of the bar, (*b*) the required dimensions of the cross section if the tensile load is 40 kN.

SOLUTION

$\sigma = 180 \times 10^6$ Pa $\quad P = 40 \times 10^3$ N

$E = 105 \times 10^9$ Pa $\quad \delta = 2.5 \times 10^{-3}$ m

(*a*) $\quad \delta = \dfrac{PL}{AE} = \dfrac{\sigma L}{E}$

$$L = \frac{E\delta}{\sigma} = \frac{(105 \times 10^9)(2.5 \times 10^{-3})}{180 \times 10^6} = 1.45833 \text{ m}$$

$L = 1.458$ m ◀

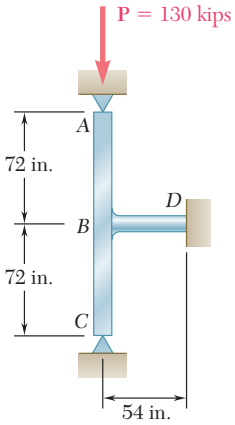
(*b*) $\quad \sigma = \dfrac{P}{A}$

$$A = \frac{P}{\sigma} = \frac{40 \times 10^3}{180 \times 10^6} = 222.22 \times 10^{-6} \text{ m}^2 = 222.22 \text{ mm}^2$$

$A = a^2 \quad a = \sqrt{A} = \sqrt{222.22}$

$a = 14.91$ mm ◀

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PROBLEM 2.13

Rod *BD* is made of steel ($E = 29\times10^6$ psi) and is used to brace the axially compressed member *ABC*. The maximum force that can be developed in member *BD* is 0.02*P*. If the stress must not exceed 18 ksi and the maximum change in length of *BD* must not exceed 0.001 times the length of *ABC*, determine the smallest-diameter rod that can be used for member *BD*.

SOLUTION

$$F_{BD} = 0.02P = (0.02)(130) = 2.6 \text{ kips} = 2.6\times10^3 \text{ lb}$$

Considering stress, $\sigma = 18 \text{ ksi} = 18\times10^3 \text{ psi}$

$$\sigma = \frac{F_{BD}}{A} \qquad \therefore \quad A = \frac{F_{BD}}{\sigma} = \frac{2.6}{18} = 0.14444 \text{ in}^2$$

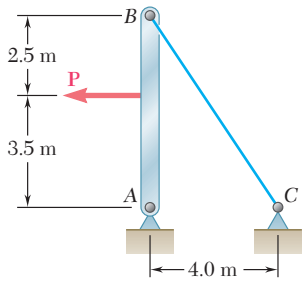
Considering deformation, $\delta = (0.001)(144) = 0.144$ in.

$$\delta = \frac{F_{BD}L_{BD}}{AE} \qquad \therefore \quad A = \frac{F_{BD}L_{BD}}{E\delta} = \frac{(2.6\times10^3)(54)}{(29\times10^6)(0.144)} = 0.03362 \text{ in}^2$$

Larger area governs. $A = 0.14444 \text{ in}^2$

$$A = \frac{\pi}{4}d^2 \qquad \therefore \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.14444)}{\pi}} \qquad d = 0.429 \text{ in.} \blacktriangleleft$$

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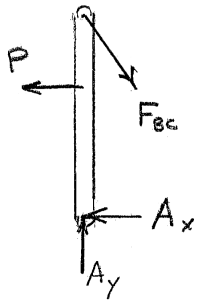
PROBLEM 2.14

The 4-mm-diameter cable *BC* is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.

SOLUTION

$$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$$

Use bar *AB* as a free body.



$$+\circlearrowleft \Sigma M_A = 0: \quad 3.5P - (6)\left(\frac{4}{7.2111}F_{BC}\right) = 0$$

$$P = 0.9509F_{BC}$$

Considering allowable stress, $\sigma = 190 \times 10^6$ Pa

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F_{BC}}{A} \quad \therefore \quad F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{ N}$$

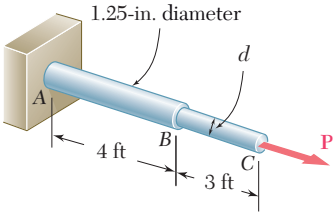
Considering allowable elongation, $\delta = 6 \times 10^{-3}$ m

$$\delta = \frac{F_{BC}L_{BC}}{AE} \quad \therefore \quad F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{ N}$$

Smaller value governs. $F_{BC} = 2.091 \times 10^3$ N

$$P = 0.9509F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3 \text{ N} \qquad P = 1.988 \text{ kN} \blacktriangleleft$$

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PROBLEM 2.15

A single axial load of magnitude $P = 15$ kips is applied at end C of the steel rod ABC. Knowing that $E = 30 \times 10^6$ psi, determine the diameter d of portion BC for which the deflection of point C will be 0.05 in.

SOLUTION

$$\delta_C = \sum \frac{PL_i}{A_i E_i} = \left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC}$$

$$L_{AB} = 4\text{ ft} = 48\text{ in.}; \qquad L_{BC} = 3\text{ ft} = 36\text{ in.}$$

$$A_{AB} = \frac{\pi d^2}{4} = \frac{\pi(1.25\text{ in.})^2}{4} = 1.22718\text{ in}^2$$

Substituting, we have

$$0.05\text{ in.} = \left(\frac{15 \times 10^3\text{ lb}}{30 \times 10^6\text{ psi}}\right)\left(\frac{48\text{ in.}}{1.22718\text{ in}^2} + \frac{36\text{ in.}}{A_{BC}}\right)$$

$$A_{BC} = 0.59127\text{ in}^2$$

$$A_{BC} = \frac{\pi d^2}{4}$$

or

$$d = \sqrt{\frac{4A_{BC}}{\pi}}$$

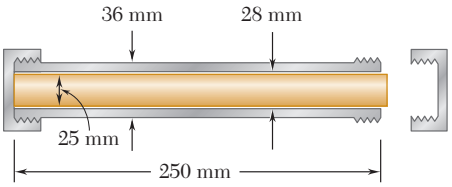
$$d = \sqrt{\frac{4(0.59127\text{ in}^2)}{\pi}}$$

$$d = 0.86766\text{ in.}$$

$d = 0.868$ in. ◀

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PROBLEM 2.16



A 250-mm-long aluminum tube ($E = 70$ GPa) of 36-mm outer diameter and 28-mm inner diameter can be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105$ GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

SOLUTION

$$A_{\text{tube}} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{\text{rod}} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$\delta_{\text{tube}} = \frac{PL}{E_{\text{tube}}A_{\text{tube}}} = \frac{P(0.250)}{(70 \times 10^9)(402.12 \times 10^{-6})} = 8.8815 \times 10^{-9} P$$

$$\delta_{\text{rod}} = -\frac{PL}{E_{\text{rod}}A_{\text{rod}}} = \frac{P(0.250)}{(105 \times 10^6)(490.87 \times 10^{-6})} = -4.8505 \times 10^{-9} P$$

$$\delta^* = \left(\frac{1}{4} \text{ turn}\right) \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375 \times 10^{-6} \text{ m}$$

$$\delta_{\text{tube}} = \delta^* + \delta_{\text{rod}} \quad \text{or} \quad \delta_{\text{tube}} - \delta_{\text{rod}} = \delta^*$$

$$8.8815 \times 10^{-9} P + 4.8505 \times 10^{-9} P = 375 \times 10^{-6}$$

$$P = \frac{0.375 \times 10^{-3}}{(8.8815 + 4.8505)(10^{-9})} = 27.308 \times 10^3 \text{ N}$$

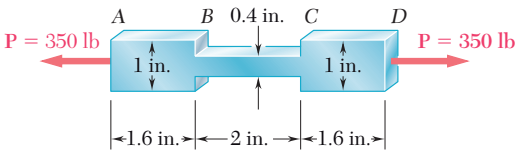
(*a*) $$\sigma_{\text{tube}} = \frac{P}{A_{\text{tube}}} = \frac{27.308 \times 10^3}{402.12 \times 10^{-6}} = 67.9 \times 10^6 \text{ Pa}$$ $\sigma_{\text{tube}} = 67.9$ MPa ◀

$$\sigma_{\text{rod}} = -\frac{P}{A_{\text{rod}}} = -\frac{27.308 \times 10^3}{490.87 \times 10^{-6}} = -55.6 \times 10^6 \text{ Pa}$$ $\sigma_{\text{rod}} = -55.6$ MPa ◀

(*b*) $$\delta_{\text{tube}} = (8.8815 \times 10^{-9})(27.308 \times 10^3) = 242.5 \times 10^{-6} \text{ m}$$ $\delta_{\text{tube}} = 0.243$ mm ◀

$$\delta_{\text{rod}} = -(4.8505 \times 10^{-9})(27.308 \times 10^3) = -132.5 \times 10^{-6} \text{ m}$$ $\delta_{\text{rod}} = -0.1325$ mm ◀

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PROBLEM 2.17

The specimen shown has been cut from a $\frac{1}{4}$-in.-thick sheet of vinyl ($E = 0.45 \times 10^6$ psi) and is subjected to a 350-lb tensile load. Determine (*a*) the total deformation of the specimen, (*b*) the deformation of its central portion *BC*.

SOLUTION

$$\delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(350\text{ lb})(1.6\text{ in.})}{(0.45\times10^6\text{ psi})(1\text{ in.})(0.25\text{ in.})} = 4.9778\times10^{-3}\text{ in.}$$

$$\delta_{BC} = \frac{PL_{BC}}{EA_{BC}} = \frac{(350\text{ lb})(2\text{ in.})}{(0.45\times10^6\text{ psi})(0.4\text{ in.})(0.25\text{ in.})} = 15.5556\times10^{-3}\text{ in.}$$

$$\delta_{CD} = \delta_{AB} = 4.9778\times10^{-3}\text{ in.}$$

(*a*) Total deformation:

$$\delta = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

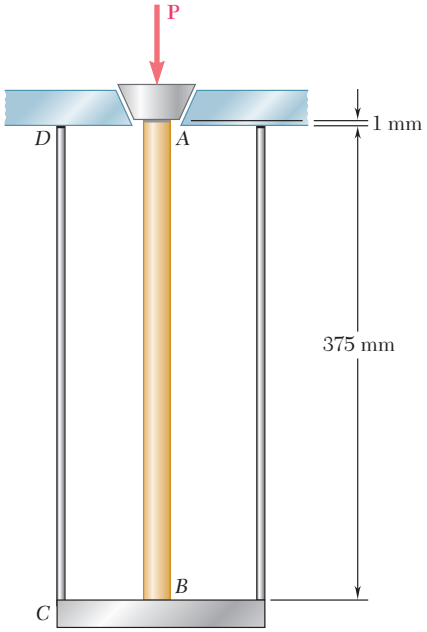
$$\delta = 25.511\times10^{-3}\text{ in.}$$

$$\delta = 25.5\times10^{-3}\text{ in.} \blacktriangleleft$$

(*b*) Deformation of portion *BC* :

$$\delta_{BC} = 15.56\times10^{-3}\text{ in.} \blacktriangleleft$$

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PROBLEM 2.18

The brass tube AB ($E = 105$ GPa) has a cross-sectional area of 140 mm^2 and is fitted with a plug at A. The tube is attached at B to a rigid plate that is itself attached at C to the bottom of an aluminum cylinder ($E = 72$ GPa) with a cross-sectional area of 250 mm^2. The cylinder is then hung from a support at D. In order to close the cylinder, the plug must move down through 1 mm. Determine the force **P** that must be applied to the cylinder.

SOLUTION

Shortening of brass tube AB:

$$L_{AB} = 375 + 1 = 376 \text{ mm} = 0.376 \text{ m} \quad A_{AB} = 140 \text{ mm}^2 = 140 \times 10^{-6} \text{ m}^2$$

$$E_{AB} = 105 \times 10^9 \text{ Pa}$$

$$\delta_{AB} = \frac{PL_{AB}}{E_{AB}A_{AB}} = \frac{P(0.376)}{(105 \times 10^9)(140 \times 10^{-6})} = 25.578 \times 10^{-9} P$$

Lengthening of aluminum cylinder CD:

$$L_{CD} = 0.375 \text{ m} \quad A_{CD} = 250 \text{ mm}^2 = 250 \times 10^{-6} \text{ m}^2 \quad E_{CD} = 72 \times 10^9 \text{ Pa}$$

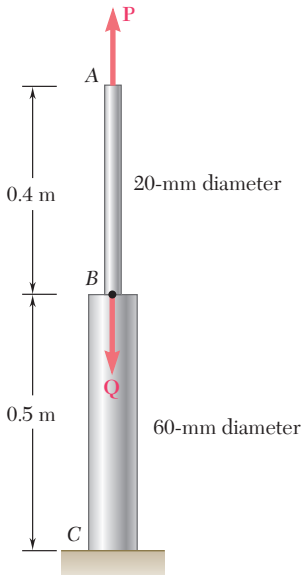
$$\delta_{CD} = \frac{PL_{CD}}{E_{CD}A_{CD}} = \frac{P(0.375)}{(72 \times 10^9)(250 \times 10^{-6})} = 20.833 \times 10^{-9} P$$

Total deflection: $\delta_A = \delta_{AB} + \delta_{CD}$ where $\delta_A = 0.001$ m

$$0.001 = (25.578 \times 10^{-9} + 20.833 \times 10^{-9})P$$

$$P = 21.547 \times 10^3 \text{ N} \qquad P = 21.5 \text{ kN} \blacktriangleleft$$

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PROBLEM 2.19

Both portions of the rod *ABC* are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at *A* is zero, (*b*) the corresponding deflection of *B*.

SOLUTION

(*a*) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\ \text{m}^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\ \text{m}^2$

Force in member *AB* is *P* tension.

Elongation:

$$\delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(4 \times 10^3)(0.4)}{(70 \times 10^9)(314.16 \times 10^{-6})} = 72.756 \times 10^{-6}\ \text{m}$$

Force in member *BC* is $Q - P$ compression.

Shortening:

$$\delta_{BC} = \frac{(Q-P)L_{BC}}{EA_{BC}} = \frac{(Q-P)(0.5)}{(70 \times 10^9)(2.8274 \times 10^{-3})} = 2.5263 \times 10^{-9}(Q-P)$$

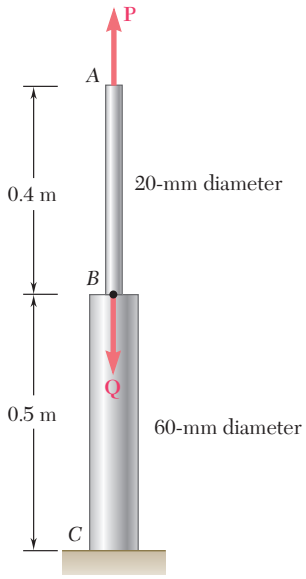
For zero deflection at *A*, $\delta_{BC} = \delta_{AB}$

$$2.5263 \times 10^{-9}(Q-P) = 72.756 \times 10^{-6} \quad \therefore \quad Q - P = 28.8 \times 10^3\ \text{N}$$

$$Q = 28.3 \times 10^3 + 4 \times 10^3 = 32.8 \times 10^3\ \text{N} \qquad Q = 32.8\ \text{kN} \blacktriangleleft$$

(*b*) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756 \times 10^{-6}\ \text{m}$ $\qquad \delta_{AB} = 0.0728\ \text{mm} \downarrow \blacktriangleleft$

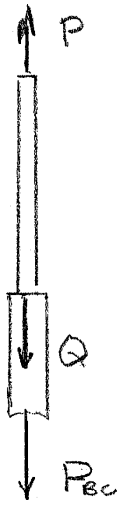
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PROBLEM 2.20

The rod *ABC* is made of an aluminum for which $E = 70$ GPa. Knowing that $\mathbf{P} = 6$ kN and $\mathbf{Q} = 42$ kN, determine the deflection of (*a*) point *A*, (*b*) point *B*.

SOLUTION



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\,\text{m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\,\text{m}^2$$

$$P_{AB} = P = 6 \times 10^3\,\text{N}$$

$$P_{BC} = P - Q = 6 \times 10^3 - 42 \times 10^3 = -36 \times 10^3\,\text{N}$$

$$L_{AB} = 0.4\,\text{m} \quad L_{BC} = 0.5\,\text{m}$$

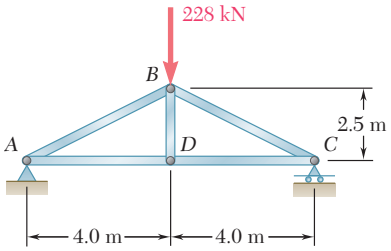
$$\delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E_A} = \frac{(6 \times 10^3)(0.4)}{(314.16 \times 10^{-6})(70 \times 10^9)} = 109.135 \times 10^{-6}\,\text{m}$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(-36 \times 10^3)(0.5)}{(2.8274 \times 10^{-3})(70 \times 10^9)} = -90.947 \times 10^{-6}\,\text{m}$$

(*a*) $\delta_A = \delta_{AB} + \delta_{BC} = 109.135 \times 10^{-6} - 90.947 \times 10^{-6}\,\text{m} = 18.19 \times 10^{-6}\,\text{m}$ $\qquad \delta_A = 0.01819\ \text{mm} \uparrow$ ◀

(*b*) $\delta_B = \delta_{BC} = -90.9 \times 10^{-6}\,\text{m} = -0.0909\ \text{mm}$ $\qquad$ or $\qquad \delta_B = 0.0909\ \text{mm} \downarrow$ ◀

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PROBLEM 2.21

For the steel truss ($E = 200$ GPa) and loading shown, determine the deformations of the members *AB* and *AD*, knowing that their cross-sectional areas are 2400 mm^2 and 1800 mm^2, respectively.

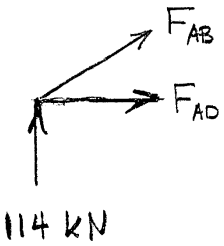
SOLUTION

Statics: Reactions are 114 kN upward at *A* and *C*.

Member *BD* is a zero force member.

$$L_{AB} = \sqrt{4.0^2 + 2.5^2} = 4.717 \text{ m}$$

Use joint *A* as a free body.



$$+\uparrow \Sigma F_y = 0: \; 114 + \frac{2.5}{4.717} F_{AB} = 0$$

$$F_{AB} = -215.10 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0: \; F_{AD} + \frac{4}{4.717} F_{AB} = 0$$

$$F_{AD} = -\frac{(4)(-215.10)}{4.717} = 182.4 \text{ kN}$$

Member *AB*:

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(-215.10 \times 10^3)(4.717)}{(200 \times 10^9)(2400 \times 10^{-6})}$$

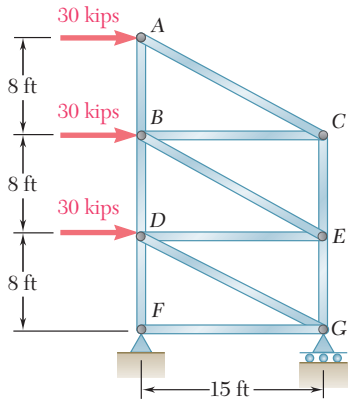
$$= -2.11 \times 10^{-3} \text{ m} \qquad \delta_{AB} = -2.11 \text{ mm} \blacktriangleleft$$

Member *AD*:

$$\delta_{AD} = \frac{F_{AD} L_{AD}}{E A_{AD}} = \frac{(182.4 \times 10^3)(4.0)}{(200 \times 10^9)(1800 \times 10^{-6})}$$

$$= 2.03 \times 10^{-3} \text{ m} \qquad \delta_{AD} = 2.03 \text{ mm} \blacktriangleleft$$

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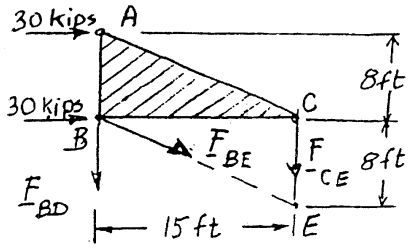


PROBLEM 2.22

For the steel truss $(E = 29\times10^6\,\text{psi})$ and loading shown, determine the deformations of the members *BD* and *DE*, knowing that their cross-sectional areas are 2 in^2 and 3 in^2, respectively.

SOLUTION

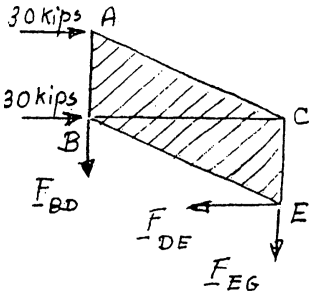
Free body: Portion *ABC* of truss



$$+\circlearrowleft \Sigma M_E = 0:\ F_{BD}(15\text{ ft}) - (30\text{ kips})(8\text{ ft}) - (30\text{ kips})(16\text{ ft}) = 0$$

$$F_{BD} = +48.0\text{ kips}$$

Free body: Portion *ABEC* of truss



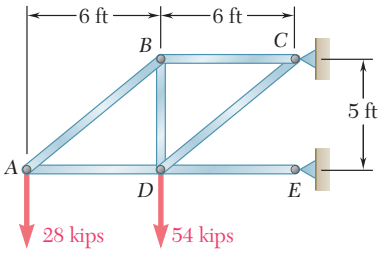
$$\rightarrow \Sigma F_x = 0:\ 30\text{ kips} + 30\text{ kips} - F_{DE} = 0$$

$$F_{DE} = +60.0\text{ kips}$$

$$\delta_{BD} = \frac{PL}{AE} = \frac{(+48.0\times10^3\text{ lb})(8\times12\text{ in.})}{(2\text{ in}^2)(29\times10^6\text{ psi})} \qquad \delta_{BD} = +79.4\times10^{-3}\text{ in.} \blacktriangleleft$$

$$\delta_{DE} = \frac{PL}{AE} = \frac{(+60.0\times10^3\text{ lb})(15\times12\text{ in.})}{(3\text{ in}^2)(29\times10^6\text{ psi})} \qquad \delta_{DE} = +124.1\times10^{-3}\text{ in.} \blacktriangleleft$$

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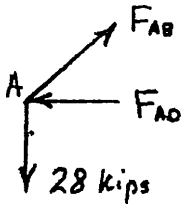
PROBLEM 2.23

Members *AB* and *BC* are made of steel ($E = 29 \times 10^6$ psi) with cross-sectional areas of 0.80 in^2 and 0.64 in^2, respectively. For the loading shown, determine the elongation of (*a*) member *AB*, (*b*) member *BC*.

SOLUTION

(*a*) $L_{AB} = \sqrt{6^2 + 5^2} = 7.810 \text{ ft} = 93.72 \text{ in.}$

Use joint *A* as a free body.

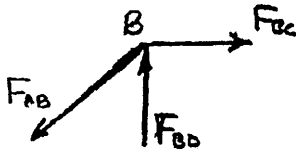


$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{7.810} F_{AB} - 28 = 0$$

$$F_{AB} = 43.74 \text{ kip} = 43.74 \times 10^3 \text{ lb}$$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(43.74 \times 10^3)(93.72)}{(29 \times 10^6)(0.80)} \qquad \delta_{AB} = 0.1767 \text{ in.} \blacktriangleleft$$

(*b*) Use joint *B* as a free body.

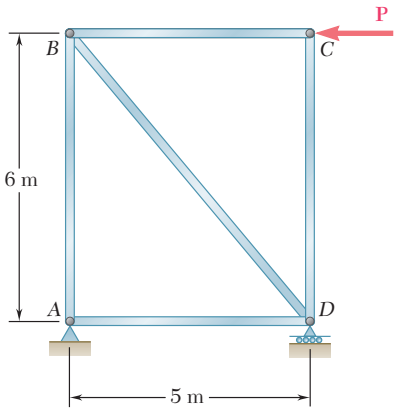


$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC} - \frac{6}{7.810} F_{AB} = 0$$

$$F_{BC} = \frac{(6)(43.74)}{7.810} = 33.60 \text{ kip} = 33.60 \times 10^3 \text{ lb}$$

$$\delta_{BC} = \frac{F_{BC} L_{BC}}{E A_{BC}} = \frac{(33.60 \times 10^3)(72)}{(29 \times 10^6)(0.64)} \qquad \delta_{BC} = 0.1304 \text{ in.} \blacktriangleleft$$

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PROBLEM 2.24

The steel frame ($E = 200$ GPa) shown has a diagonal brace *BD* with an area of 1920 mm^2. Determine the largest allowable load **P** if the change in length of member *BD* is not to exceed 1.6 mm.

SOLUTION

$$\delta_{BD} = 1.6 \times 10^{-3}\ \text{m}, \quad A_{BD} = 1920\ \text{mm}^2 = 1920 \times 10^{-6}\ \text{m}^2$$

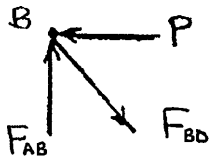
$$L_{BD} = \sqrt{5^2 + 6^2} = 7.810\ \text{m}, \quad E_{BD} = 200 \times 10^9\ \text{Pa}$$

$$\delta_{BD} = \frac{F_{BD} L_{BD}}{E_{BD} A_{BD}}$$

$$F_{BD} = \frac{E_{BD} A_{BD} \delta_{BD}}{L_{BD}} = \frac{(200 \times 10^9)(1920 \times 10^{-6})(1.6 \times 10^{-3})}{7.81}$$

$$= 78.67 \times 10^3\ \text{N}$$

Use joint *B* as a free body. $\overset{+}{\rightarrow} \Sigma F_x = 0$:



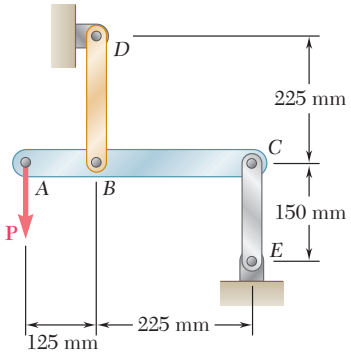
$$\frac{5}{7.810} F_{BD} - P = 0$$

$$P = \frac{5}{7.810} F_{BD} = \frac{(5)(78.67 \times 10^3)}{7.810}$$

$$= 50.4 \times 10^3\ \text{N}$$

$$P = 50.4\ \text{kN} \blacktriangleleft$$

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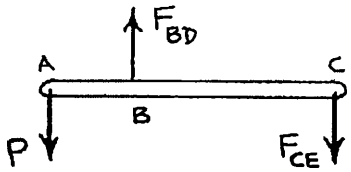


PROBLEM 2.25

Link *BD* is made of brass ($E = 105$ GPa) and has a cross-sectional area of 240 mm^2. Link *CE* is made of aluminum ($E = 72$ GPa) and has a cross-sectional area of 300 mm^2. Knowing that they support rigid member *ABC*, determine the maximum force **P** that can be applied vertically at point *A* if the deflection of *A* is not to exceed 0.35 mm.

SOLUTION

Free body member *AC*:



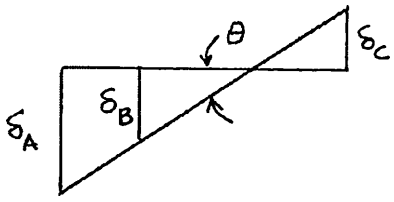
$$+\circlearrowleft \Sigma M_C = 0: \quad 0.350P - 0.225F_{BD} = 0$$
$$F_{BD} = 1.55556P$$
$$+\circlearrowleft \Sigma M_B = 0: \quad 0.125P - 0.225F_{CE} = 0$$
$$F_{CE} = 0.55556P$$

$$\delta_B = \delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}} = \frac{(1.55556P)(0.225)}{(105\times10^9)(240\times10^{-6})} = 13.8889\times10^{-9}P$$

$$\delta_C = \delta_{CE} = \frac{F_{CE}L_{CE}}{E_{CE}A_{CE}} = \frac{(0.55556P)(0.150)}{(72\times10^9)(300\times10^{-6})} = 3.8581\times10^{-9}P$$

Deformation Diagram:

From the deformation diagram,



Slope: $$\theta = \frac{\delta_B + \delta_C}{L_{BC}} = \frac{17.7470\times10^{-9}P}{0.225} = 78.876\times10^{-9}P$$

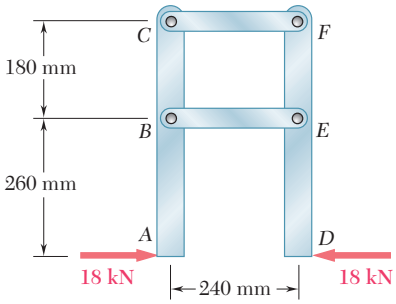
$$\delta_A = \delta_B + L_{AB}\theta$$
$$= 13.8889\times10^{-9}P + (0.125)(78.876\times10^{-9}P)$$
$$= 23.748\times10^{-9}P$$

Apply displacement limit. $\delta_A = 0.35\times10^{-3}\text{ m} = 23.748\times10^{-9}P$

$$P = 14.7381\times10^3\text{ N}$$

$$P = 14.74\text{ kN} \blacktriangleleft$$

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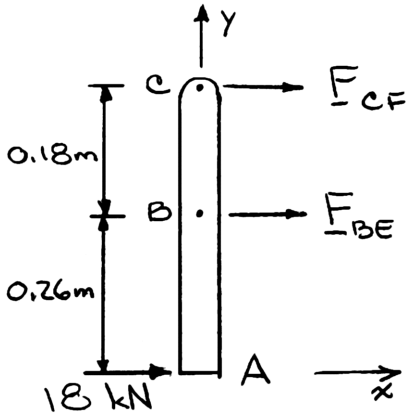


PROBLEM 2.26

Members *ABC* and *DEF* are joined with steel links (E = 200 GPa). Each of the links is made of a pair of 25 × 35-mm plates. Determine the change in length of (*a*) member *BE*, (*b*) member *CF*.

SOLUTION

Free body diagram of Member *ABC*:



$+\circlearrowleft \Sigma M_B = 0$:

$$(0.26\text{ m})(18\text{ kN}) - (0.18\text{ m})F_{CF} = 0$$

$$F_{CF} = 26.0\text{ kN}$$

$+\rightarrow \Sigma F_x = 0$:

$$18\text{ kN} + F_{BE} + 26.0\text{ kN} = 0$$

$$F_{BE} = -44.0\text{ kN}$$

Area for link made of two plates:

$$A = 2(0.025\text{ m})(0.035\text{ m}) = 1.750\times10^{-3}\text{ m}^2$$

(*a*) $$\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{(-44.0\times10^{3}\text{ N})(0.240\text{ m})}{(200\times10^{9}\text{ Pa})(1.75\times10^{-3}\text{ m}^2)}$$

$$= -30.171\times10^{-6}\text{ m}$$

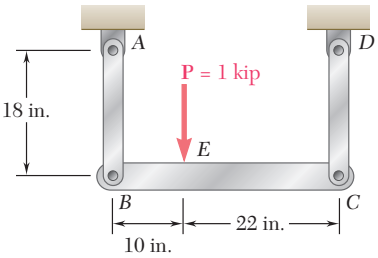
$$\delta_{BE} = -0.0302\text{ mm} \blacktriangleleft$$

(*b*) $$\delta_{CF} = \frac{F_{BF}L}{EA} = \frac{(26.0\times10^{3}\text{ N})(0.240\text{ m})}{(200\times10^{9}\text{ Pa})(1.75\times10^{-3}\text{ m}^2)}$$

$$= 17.8286\times10^{-6}\text{ m}$$

$$\delta_{CF} = 0.01783\text{ mm} \blacktriangleleft$$

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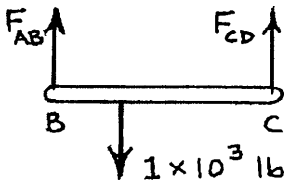


PROBLEM 2.27

Each of the links *AB* and *CD* is made of aluminum ($E = 10.9 \times 10^6$ psi) and has a cross-sectional area of 0.2 in^2. Knowing that they support the rigid member *BC*, determine the deflection of point *E*.

SOLUTION

Free body *BC*:



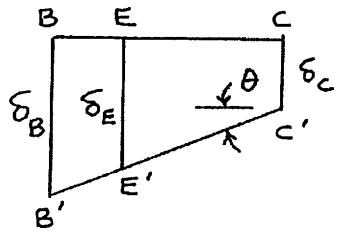
$$+\circlearrowleft \Sigma M_C = 0: \; -(32)F_{AB} + (22)(1\times10^3) = 0$$
$$F_{AB} = 687.5 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \; 687.5 - 1\times10^3 + F_{CD} = 0$$
$$F_{CD} = 312.5 \text{ lb}$$

$$\delta_{AB} = \frac{F_{AB}\,L_{AB}}{EA} = \frac{(687.5)(18)}{(10.9\times10^6)(0.2)} = 5.6766\times10^{-3} \text{ in.} = \delta_B$$

$$\delta_{CD} = \frac{F_{CD}\,L_{CD}}{EA} = \frac{(312.5)(18)}{(10.9\times10^6)(0.2)} = 2.5803\times10^{-3} \text{ in.} = \delta_C$$

Deformation diagram:

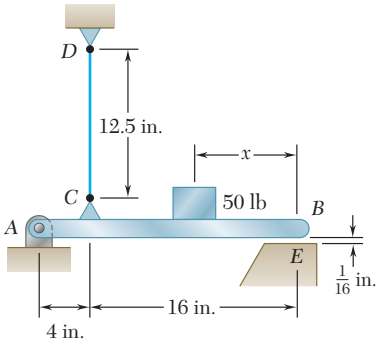


$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{L_{BC}} = \frac{3.0963\times10^{-3}}{32}$$
$$= 96.759\times10^{-6} \text{ rad}$$

$$\delta_E = \delta_C + L_{EC}\theta$$
$$= 2.5803\times10^{-3} + (22)(96.759\times10^{-6})$$
$$= 4.7090\times10^{-3} \text{ in.}$$

$$\delta_E = 4.71\times10^{-3} \text{ in.} \downarrow \blacktriangleleft$$

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PROBLEM 2.28

The length of the $\frac{3}{32}$-in.-diameter steel wire *CD* has been adjusted so that with no load applied, a gap of $\frac{1}{16}$ in. exists between the end *B* of the rigid beam *ACB* and a contact point *E*. Knowing that $E = 29 \times 10^6$ psi, determine where a 50-lb block should be placed on the beam in order to cause contact between *B* and *E*.

SOLUTION

Rigid beam *ACB* rotates through angle θ to close gap.

$$\theta = \frac{1/16}{20} = 3.125 \times 10^{-3} \text{ rad}$$

Point *C* moves downward.

$$\delta_C = 4\theta = 4(3.125 \times 10^{-3}) = 12.5 \times 10^{-3} \text{ in.}$$

$$\delta_{CD} = \delta_C = 12.5 \times 10^{-3} \text{ in.}$$

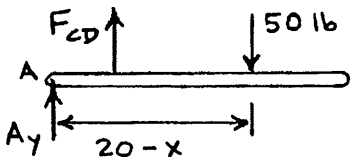
$$A_{CD} = \frac{\pi}{d} d^2 = \frac{\pi}{4}\left(\frac{3}{32}\right)^2 = 6.9029 \times 10^{-3} \text{ in}^2$$

$$\delta_{CD} = \frac{F_{CD} L_{CD}}{E A_{CD}}$$

$$F_{CD} = \frac{E A_{CD} \delta_{CD}}{L_{CD}} = \frac{(29 \times 10^6)(6.9029 \times 10^{-3})(12.5 \times 10^{-3})}{12.5}$$

$$= 200.18 \text{ lb}$$

Free body *ACB*:



$$+\circlearrowleft \Sigma M_A = 0: \quad 4F_{CD} - (50)(20 - x) = 0$$

$$20 - x = \frac{(4)(200.18)}{50} = 16.0144$$

$$x = 3.9856 \text{ in.}$$

For contact, $x < 3.99$ in. ◀

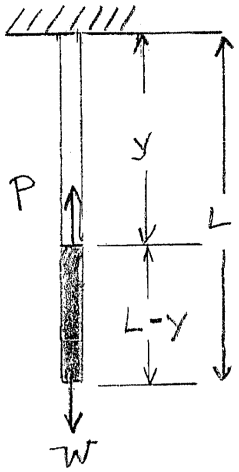
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PROBLEM 2.29

A homogeneous cable of length L and uniform cross section is suspended from one end. (*a*) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (*b*) Show that the same elongation would be obtained if the cable were horizontal and if a force equal to half of its weight were applied at each end.

SOLUTION

(*a*) For element at point identified by coordinate y,



P = weight of portion below the point

$$= \rho g A(L - y)$$

$$d\delta = \frac{P\,dy}{EA} = \frac{\rho g A(L-y)\,dy}{EA} = \frac{\rho g(L-y)}{E}\,dy$$

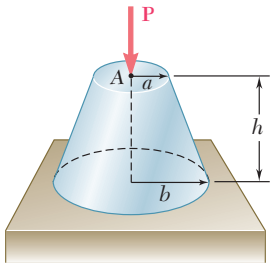
$$\delta = \int_0^L \frac{\rho g(L-y)}{E}\,dy = \frac{\rho g}{E}\left(Ly - \frac{1}{2}y^2\right)\Big|_0^L$$

$$= \frac{\rho g}{E}\left(L^2 - \frac{L^2}{2}\right) \qquad \delta = \frac{1}{2}\frac{\rho g L^2}{E} \blacktriangleleft$$

(*b*) Total weight: $W = \rho g A L$

$$F = \frac{EA\delta}{L} = \frac{EA}{L}\cdot\frac{1}{2}\frac{\rho g L^2}{E} = \frac{1}{2}\rho g A L \qquad F = \frac{1}{2}W \blacktriangleleft$$

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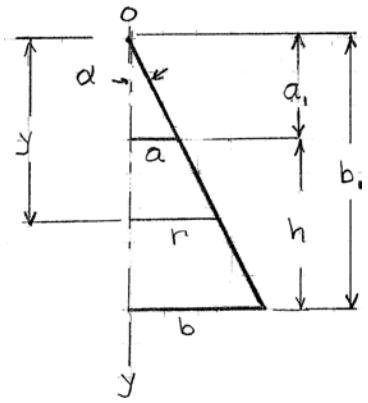


PROBLEM 2.30

A vertical load **P** is applied at the center *A* of the upper section of a homogeneous frustum of a circular cone of height *h*, minimum radius *a*, and maximum radius *b*. Denoting by *E* the modulus of elasticity of the material and neglecting the effect of its weight, determine the deflection of point *A*.

SOLUTION

Extend the slant sides of the cone to meet at a point *O* and place the origin of the coordinate system there.



From geometry, $\tan\alpha = \dfrac{b-a}{h}$

$$a_1 = \frac{a}{\tan\alpha}, \quad b_1 = \frac{b}{\tan\alpha}, \quad r = y\tan\alpha$$

At coordinate point *y*, $A = \pi r^2$

Deformation of element of height *dy*: $d\delta = \dfrac{P\,dy}{AE}$

$$d\delta = \frac{P}{E\pi}\frac{dy}{r^2} = \frac{P}{\pi E\tan^2\alpha}\frac{dy}{y^2}$$

Total deformation:

$$\delta_A = \frac{P}{\pi E\tan^2\alpha}\int_{a_1}^{b_1}\frac{dy}{y^2} = \frac{P}{\pi E\tan^2\alpha}\left(-\frac{1}{y}\right)\Bigg|_{a_1}^{b_1} = \frac{P}{\pi E\tan^2\alpha}\left(\frac{1}{a_1}-\frac{1}{b_1}\right)$$

$$= \frac{P}{\pi E\tan^2\alpha}\frac{b_1-a_1}{a_1b_1} = \frac{P(b_1-a_1)}{\pi Eab}$$

$$\delta_A = \frac{Ph}{\pi Eab}\downarrow \blacktriangleleft$$

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PROBLEM 2.31

Denoting by ε the "engineering strain" in a tensile specimen, show that the true strain is $\varepsilon_t = \ln(1+\varepsilon)$.

SOLUTION

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\frac{L_0+\delta}{L_0} = \ln\left(1+\frac{\delta}{L_0}\right) = \ln(1+\varepsilon)$$

Thus, $\varepsilon_t = \ln(1+\varepsilon)$ ◀

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PROBLEM 2.32

The volume of a tensile specimen is essentially constant while plastic deformation occurs. If the initial diameter of the specimen is d_1, show that when the diameter is d, the true strain is $\varepsilon_t = 2\ln(d_1/d)$.

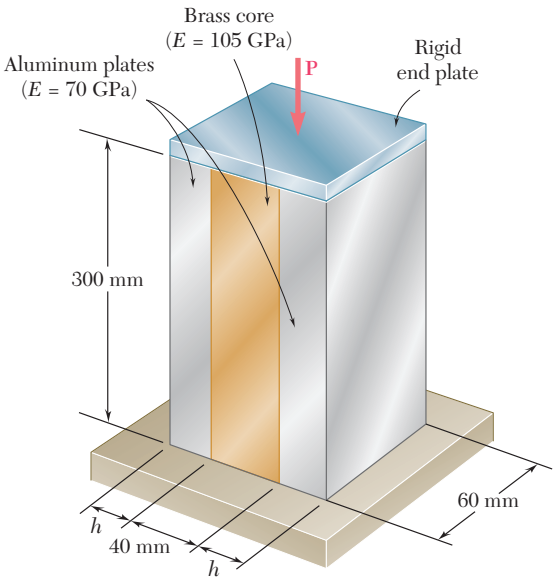
SOLUTION

If the volume is constant, $\frac{\pi}{4}d^2L = \frac{\pi}{4}d_1^2L_0$

$$\frac{L}{L_0} = \frac{d_1^2}{d^2} = \left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\left(\frac{d_1}{d}\right)^2 \qquad \varepsilon_t = 2\ln\frac{d_1}{d} \blacktriangleleft$$

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PROBLEM 2.33

An axial centric force of magnitude $P = 450$ kN is applied to the composite block shown by means of a rigid end plate. Knowing that $h = 10$ mm, determine the normal stress in (*a*) the brass core, (*b*) the aluminum plates.

SOLUTION

$$\delta_A = \delta_B = \delta; \qquad P = P_A + P_B$$

$$\delta = \frac{P_A L}{E_A A_A} \quad \text{and} \quad \delta = \frac{P_B L}{E_B A_B}$$

Therefore,
$$P_A = (E_A A_A)\left(\frac{\delta}{L}\right); \qquad P_B = (E_B A_B)\left(\frac{\delta}{L}\right)$$

Substituting,
$$P_A = \left(E_A A_A + E_B A_B\right)\left(\frac{\delta}{L}\right)$$

$$\in = \frac{\delta}{L} = \frac{P}{\left(E_A A_A + E_B A_B\right)}$$

$$\in = \frac{(450 \times 10^3 \text{ N})}{(70 \times 10^9 \text{ Pa})(2)(0.06 \text{ m})(0.01 \text{ m}) + (105 \times 10^9 \text{ Pa})(0.06 \text{ m})(0.04 \text{ m})}$$

$$\in = 1.33929 \times 10^{-3}$$

Now,
$$\sigma = E\in$$

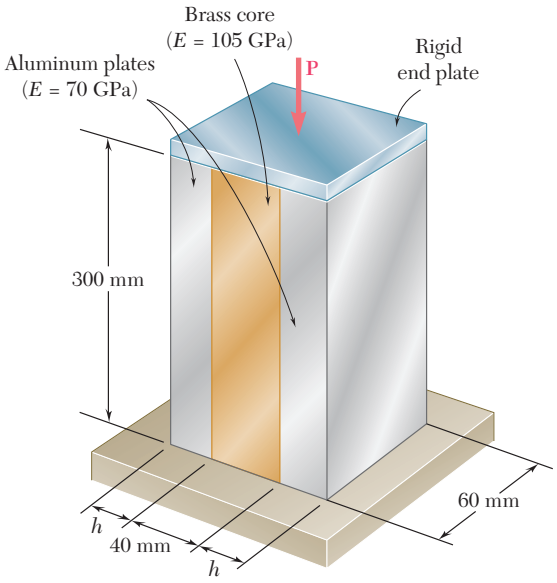
(*a*) Brass-core:
$$\sigma_B = (105 \times 10^9 \text{ Pa})(1.33929 \times 10^{-3})$$
$$= 1.40625 \times 10^8 \text{ Pa}$$

$$\sigma_B = 140.6 \text{ MPa} \blacktriangleleft$$

(*b*) Aluminum:
$$\sigma_A = (70 \times 10^9 \text{ Pa})(1.33929 \times 10^{-3})$$
$$= 9.3750 \times 10^7 \text{ Pa}$$

$$\sigma_A = 93.8 \text{ MPa} \blacktriangleleft$$

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PROBLEM 2.34

For the composite block shown in Prob. 2.33, determine (*a*) the value of *h* if the portion of the load carried by the aluminum plates is half the portion of the load carried by the brass core, (*b*) the total load if the stress in the brass is 80 MPa.

PROBLEM 2.33. An axial centric force of magnitude $P = 450$ kN is applied to the composite block shown by means of a rigid end plate. Knowing that $h = 10$ mm, determine the normal stress in (*a*) the brass core, (*b*) the aluminum plates.

SOLUTION

$$\delta = \delta_a = \delta_b; \qquad P = P_a + P_b$$

$$\delta = \frac{P_a L}{E_a A_a} \quad \text{and} \quad \delta = \frac{P_b L}{E_b A_b}$$

Therefore,

$$P_a = (E_a A_a)\frac{\delta}{L}; \qquad P_b = (E_b A_b)\left(\frac{\delta}{L}\right)$$

(*a*) $$P_a = \frac{1}{2}P_b$$

$$(E_a A_a)\left(\frac{\delta}{L}\right) = \frac{1}{2}(E_b A_b)\left(\frac{\delta}{L}\right)$$

$$A_a = \frac{1}{2}\left(\frac{E_b}{E_a}\right)A_b$$

$$A_a = \frac{1}{2}\left(\frac{105\text{ GPa}}{70\text{ GPa}}\right)(40\text{ mm})(60\text{ mm})$$

$$A_a = 1800\text{ mm}^2$$

$$1800\text{ mm}^2 = 2(60\text{ mm})(h)$$

$$h = 15.00\text{ mm} \blacktriangleleft$$

(*b*) $$\sigma_b = \frac{P_b}{A_b} \Rightarrow P_b = \sigma_b A_b \text{ and } P_a = \frac{1}{2}P_b$$

$$P = P_a + P_b$$

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PROBLEM 2.34 ***(Continued)***

$$P = \frac{1}{2}(\sigma_b A_b) + \sigma_b A_b$$

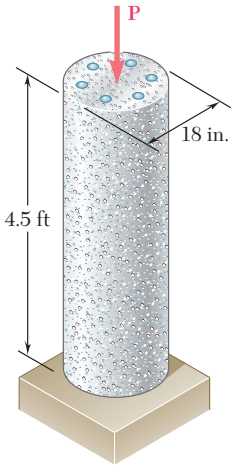
$$P = (\sigma_b A_b)1.5$$

$$P = (80 \times 10^6 \text{ Pa})(0.04 \text{ m})(0.06 \text{ m})(1.5)$$

$$P = 2.880 \times 10^5 \text{ N}$$

$$P = 288 \text{ kN} \blacktriangleleft$$

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PROBLEM 2.35

The 4.5-ft concrete post is reinforced with six steel bars, each with a $1\frac{1}{8}$-in. diameter. Knowing that $E_s = 29 \times 10^6$ psi and $E_c = 4.2 \times 10^6$ psi, determine the normal stresses in the steel and in the concrete when a 350-kip axial centric force **P** is applied to the post.

SOLUTION

Let P_c = portion of axial force carried by concrete.

P_s = portion carried by the six steel rods.

$$\delta = \frac{P_c L}{E_c A_c} \qquad P_c = \frac{E_c A_c \delta}{L}$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{-P}{E_c A_c + E_s A_s}$$

$$A_s = 6\frac{\pi}{4}d_s^2 = \frac{6\pi}{4}(1.125\text{ in.})^2 = 5.9641\text{ in}^2$$

$$A_c = \frac{\pi}{4}d_c^2 - A_s = \frac{\pi}{4}(18\text{ in.})^2 - 5.9641\text{ in}^2$$

$$= 248.51\text{ in}^2$$

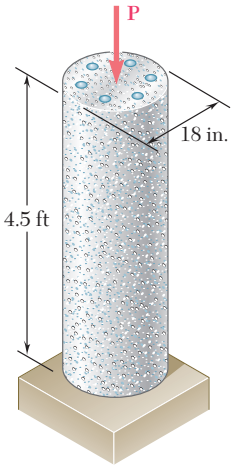
$$L = 4.5\text{ ft} = 54\text{ in.}$$

$$\varepsilon = \frac{-350\times10^3\text{ lb}}{(4.2\times10^6\text{ psi})(248.51\text{ in}^2) + (29\times10^6\text{ psi})(5.9641\text{ in}^2)} = -2.8767\times10^{-4}$$

$$\sigma_s = E_s\varepsilon = (29\times10^6\text{ psi})(-2.8767\times10^{-4}) = -8.3424\times10^3\text{ psi} \qquad \sigma_s = -8.34\text{ ksi} \blacktriangleleft$$

$$\sigma_c = E_c\varepsilon = (4.2\times10^6\text{ psi})(-2.8767\times10^{-4}) = 1.20821\times10^3\text{ psi} \qquad \sigma_c = -1.208\text{ ksi} \blacktriangleleft$$

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PROBLEM 2.36

For the post of Prob. 2.35, determine the maximum centric force that can be applied if the allowable normal stress is 20 ksi in the steel and 2.4 ksi in the concrete.

PROBLEM 2.35 The 4.5-ft concrete post is reinforced with six steel bars, each with a $1\frac{1}{8}$-in. diameter. Knowing that $E_s = 29 \times 10^6$ psi and $E_c = 4.2 \times 10^6$ psi, determine the normal stresses in the steel and in the concrete when a 350-kip axial centric force **P** is applied to the post.

SOLUTION

Allowable strain in each material:

Steel: $$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{20\times10^3 \text{ psi}}{29\times10^6 \text{ psi}} = 6.8966\times10^{-4}$$

Concrete: $$\varepsilon_c = \frac{\sigma_c}{E_c} = \frac{2.4\times10^3 \text{ psi}}{4.2\times10^6 \text{ psi}} = 5.7143\times10^{-4}$$

Smaller value governs. $$\varepsilon = \frac{\delta}{L} = 5.7143\times10^{-4}$$

Let P_c = Portion of load carried by concrete.

P_s = Portion of load carried by 6 steel rods.

$$\delta = \frac{P_c L}{E_c A_c} \qquad \therefore \qquad P_c = E_c A_c \left(\frac{\delta}{L}\right) = E_c A_c \in$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad \therefore \qquad P_s = E_s A_s \left(\frac{\delta}{L}\right) = E_s A_s \in$$

$$A_s = 6\left(\frac{\pi}{4}\right)d_s^2 = \frac{6\pi}{4}(1.125 \text{ in.})^2 = 5.9641 \text{ in}^2$$

$$A_c = \left(\frac{\pi}{4}\right)d_c^2 - A_s = \frac{\pi}{4}(18 \text{ in.})^2 - 5.9641 \text{ in}^2 = 2.4851\times10^2 \text{ in}^2$$

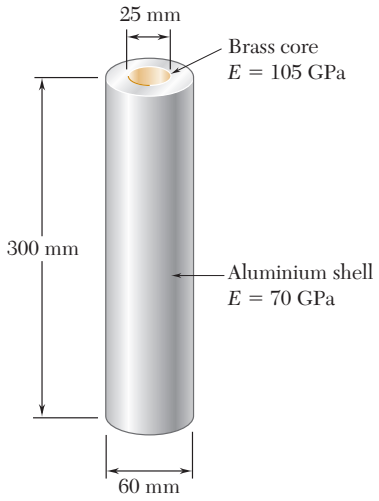
$$P = P_c + P_s = E_c A_c \in + E_s A_s \in$$

$$P = [(4.2\times10^6 \text{ psi})(2.4851\times10^2 \text{ in}^2) + (29\times10^6 \text{ psi})(5.9641 \text{ in}^2)](5.7143\times10^{-4})$$

$$P = 6.9526\times10^5 \text{ lb}$$

$$P = 695 \text{ kips} \blacktriangleleft$$

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PROBLEM 2.37

An axial force of 200 kN is applied to the assembly shown by means of rigid end plates. Determine (*a*) the normal stress in the aluminum shell, (*b*) the corresponding deformation of the assembly.

SOLUTION

Let P_a = Portion of axial force carried by shell.

P_b = Portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a}, \quad \text{or} \quad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_b L}{E_b A_b}, \quad \text{or} \quad P_b = \frac{E_b A_b}{L}\delta$$

Thus,
$$P = P_a + P_b = (E_a A_a + E_b A_b)\frac{\delta}{L}$$

with
$$A_a = \frac{\pi}{4}[(0.060)^2 - (0.025)^2] = 2.3366 \times 10^{-3}\,\text{m}^2$$

$$A_b = \frac{\pi}{4}(0.025)^2 = 0.49087 \times 10^{-3}\,\text{m}^2$$

$$P = [(70 \times 10^9)(2.3366 \times 10^{-3}) + (105 \times 10^9)(0.49087 \times 10^{-3})]\frac{\delta}{L}$$

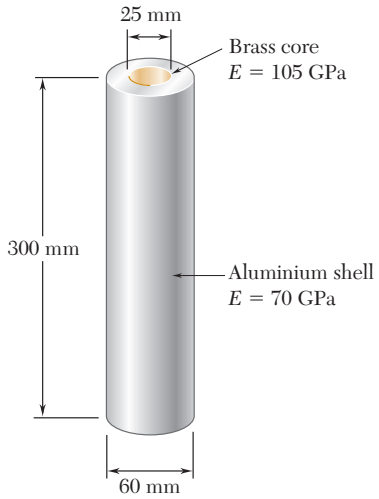
$$P = 215.10 \times 10^6 \frac{\delta}{L}$$

Strain:
$$\varepsilon = \frac{\delta}{L} = \frac{P}{215.10 \times 10^6} = \frac{200 \times 10^3}{215.10 \times 10^6} = 0.92980 \times 10^{-3}$$

(*a*) $\sigma_a = E_a \varepsilon = (70 \times 10^9)(0.92980 \times 10^{-3}) = 65.1 \times 10^6\,\text{Pa}$ $\sigma_a = 65.1\,\text{MPa}$ ◀

(*b*) $\delta = \varepsilon L = (0.92980 \times 10^{-3})(300\ \text{mm})$ $\delta = 0.279\ \text{mm}$ ◀

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PROBLEM 2.38

The length of the assembly shown decreases by 0.40 mm when an axial force is applied by means of rigid end plates. Determine (*a*) the magnitude of the applied force, (*b*) the corresponding stress in the brass core.

SOLUTION

Let P_a = Portion of axial force carried by shell and P_b = Portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a}, \quad \text{or} \quad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_b L}{E_b A_b}, \quad \text{or} \quad P_b = \frac{E_b A_b}{L}\delta$$

Thus, $$P = P_a + P_b = (E_a A_a + E_b A_b)\frac{\delta}{L}$$

with $$A_a = \frac{\pi}{4}[(0.060)^2 - (0.025)^2] = 2.3366 \times 10^{-3}\,\text{m}^2$$

$$A_b = \frac{\pi}{4}(0.025)^2 = 0.49087 \times 10^{-3}\,\text{m}^2$$

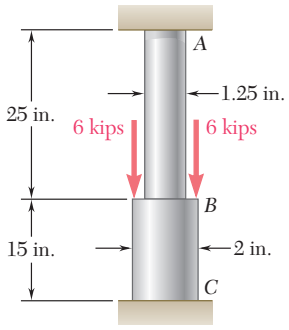
$$P = [(70 \times 10^9)(2.3366 \times 10^{-3}) + (105 \times 10^9)(0.49087 \times 10^{-3})]\frac{\delta}{L} = 215.10 \times 10^6 \frac{\delta}{L}$$

with $\delta = 0.40$ mm, $L = 300$ mm

(*a*) $$P = (215.10 \times 10^6)\frac{0.40}{300} = 286.8 \times 10^3\ \text{N}$$ $P = 287$ kN ◀

(*b*) $$\sigma_b = \frac{P_b}{A_b} = \frac{E_b \delta}{L} = \frac{(105 \times 10^9)(0.40 \times 10^{-3})}{300 \times 10^{-3}} = 140 \times 10^6\ \text{Pa}$$ $\sigma_b = 140.0$ MPa ◀

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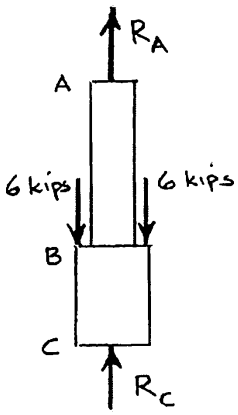


PROBLEM 2.39

A polystyrene rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends and supports two 6-kip loads as shown. Knowing that $E = 0.45\times10^{6}$ psi, determine (*a*) the reactions at *A* and *C*, (*b*) the normal stress in each portion of the rod.

SOLUTION

(*a*) We express that the elongation of the rod is zero.



$$\delta = \frac{P_{AB}L_{AB}}{\frac{\pi}{4}d_{AB}^2E} + \frac{P_{BC}L_{BC}}{\frac{\pi}{4}d_{BC}^2E} = 0$$

But $P_{AB} = +R_A \qquad P_{BC} = -R_C$

Substituting and simplifying,

$$\frac{R_AL_{AB}}{d_{AB}^2} - \frac{R_CL_{BC}}{d_{BC}^2} = 0$$

$$R_C = \frac{L_{AB}}{L_{BC}}\left(\frac{d_{BC}}{d_{AB}}\right)^2 R_A = \frac{25}{15}\left(\frac{2}{1.25}\right)^2 R_A$$

$$R_C = 4.2667R_A \qquad (1)$$

From the free body diagram, $R_A + R_C = 12$ kips (2)

Substituting (1) into (2), $5.2667R_A = 12$

$$R_A = 2.2785 \text{ kips} \qquad R_A = 2.28 \text{ kips} \uparrow \blacktriangleleft$$

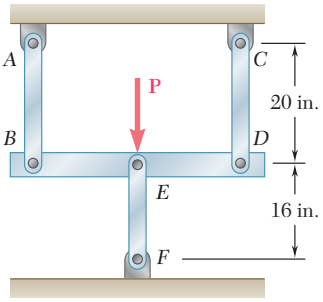
From (1), $R_C = 4.2667\,(2.2785) = 9.7217$ kips

$$R_C = 9.72 \text{ kips} \uparrow \blacktriangleleft$$

(*b*) $$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{+R_A}{A_{AB}} = \frac{2.2785}{\frac{\pi}{4}(1.25)^2} \qquad \sigma_{AB} = +1.857 \text{ ksi} \blacktriangleleft$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{-R_C}{A_{BC}} = \frac{-9.7217}{\frac{\pi}{4}(2)^2} \qquad \sigma_{BC} = -3.09 \text{ ksi} \blacktriangleleft$$

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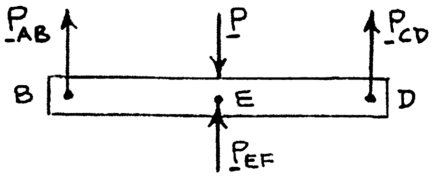


PROBLEM 2.40

Three steel rods ($E = 29 \times 10^6$ psi) support an 8.5-kip load **P**. Each of the rods *AB* and *CD* has a 0.32-in^2 cross-sectional area and rod *EF* has a 1-in^2 cross-sectional area. Neglecting the deformation of bar *BED*, determine (*a*) the change in length of rod *EF*, (*b*) the stress in each rod.

SOLUTION

Use member *BED* as a free body.



By symmetry, or by $\sum M_E = 0$:

$$P_{CD} = P_{AB}$$

$$+\uparrow \sum F_y = 0: \; P_{AB} + P_{CD} + P_{EF} - P = 0$$

$$P = 2P_{AB} + P_{EF}$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{EA_{AB}} \qquad \delta_{CD} = \frac{P_{CD}L_{CD}}{EA_{CD}} \qquad \delta_{EF} = \frac{P_{EF}L_{EF}}{EA_{EF}}$$

Since $L_{AB} = L_{CD}$ and $A_{AB} = A_{CD}$, $\delta_{AB} = \delta_{CD}$

Since points *A*, *C*, and *F* are fixed, $\delta_B = \delta_{AB}, \delta_D = \delta_{CD}, \; \delta_E = \delta_{EF}$

Since member *BED* is rigid, $\delta_E = \delta_B = \delta_C$

$$\frac{P_{AB}L_{AB}}{EA_{AB}} = \frac{P_{EF}L_{EF}}{EA_{EF}} \qquad \therefore \qquad P_{AB} = \frac{A_{AB}}{A_{EF}} \cdot \frac{L_{EF}}{L_{AB}} P_{EF} = \frac{0.32}{1} \cdot \frac{16}{20} P_{EF} = 0.256 P_{EF}$$

$$P = 2P_{AB} + P_{EF} = 2(0.256P_{EF}) + P_{EF} = 1.512P_{EF}$$

$$P_{EF} = \frac{P}{1.512} = \frac{8.5}{1.512} = 5.6217 \text{ kips}$$

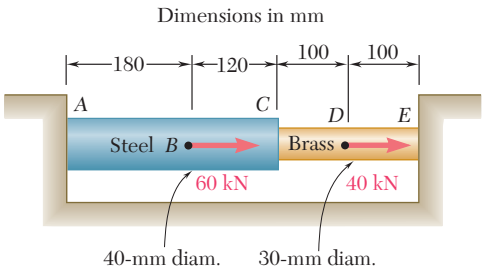
$$P_{AB} = P_{CD} = 0.256(5.6217) = 1.43916 \text{ kips}$$

(*a*) $$\delta_{EF} = \frac{P_{EF}L_{EF}}{EA_{EF}} = \frac{(5.6217)(16)}{(29 \times 10^3)(1)} = 0.0031016 \text{ in.}$$ $\delta_{EF} = 0.00310$ in. ◀

(*b*) $$\sigma_{AB} = \sigma_{CD} = \frac{P_{AB}}{A_{AB}} = \frac{1.43916}{0.32} = 4.4974 \text{ ksi}$$ $\sigma_{AB} = \sigma_{CD} = 4.50$ ksi ◀

$$\sigma_{EF} = \frac{P_{EF}}{A_{EF}} = \frac{5.6217}{1} = 5.6217 \text{ ksi}$$ $\sigma_{EF} = 5.62$ ksi ◀

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PROBLEM 2.41

Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at A and E, (*b*) the deflection of point C.

SOLUTION

$\underline{A}$ to $\underline{C}$: $E = 200 \times 10^9$ Pa

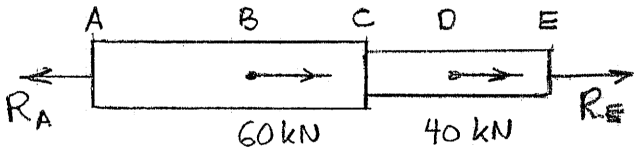
$$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$$

$$EA = 251.327 \times 10^6 \text{ N}$$

$\underline{C}$ to $\underline{E}$: $E = 105 \times 10^9$ Pa

$$A = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$EA = 74.220 \times 10^6 \text{ N}$$



$\underline{A}$ to $\underline{B}$: $P = R_A$

$$L = 180 \text{ mm} = 0.180 \text{ m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327 \times 10^6}$$

$$= 716.20 \times 10^{-12} R_A$$

$\underline{B}$ to $\underline{C}$: $P = R_A - 60 \times 10^3$

$$L = 120 \text{ mm} = 0.120 \text{ m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{251.327 \times 10^6}$$

$$= 447.47 \times 10^{-12} R_A - 26.848 \times 10^{-6}$$

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PROBLEM 2.41 *(Continued)*

<u>*C* to *D*</u>: $P = R_A - 60\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9}R_A - 80.841\times10^{-6}$$

<u>*D* to *E*</u>: $P = R_A - 100\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9}R_A - 134.735\times10^{-6}$$

<u>*A* to *E*</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.85837\times10^{-9}R_A - 242.424\times10^{-6}$$

Since point *E* cannot move relative to *A*, $\delta_{AE} = 0$

(*a*) $3.85837\times10^{-9}R_A - 242.424\times10^{-6} = 0 \quad R_A = 62.831\times10^3 \text{ N}$ $\quad R_A = 62.8 \text{ kN} \leftarrow$ ◀

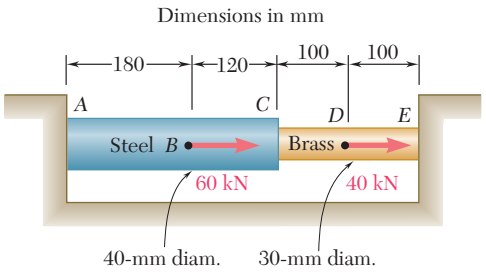
$R_E = R_A - 100\times10^3 = 62.8\times10^3 - 100\times10^3 = -37.2\times10^3 \text{ N}$ $\quad R_E = 37.2 \text{ kN} \leftarrow$ ◀

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367\times10^{-9}R_A - 26.848\times10^{-6}$

$$= (1.16369\times10^{-9})(62.831\times10^3) - 26.848\times10^{-6}$$

$$= 46.3\times10^{-6} \text{ m}$$ $\quad \delta_C = 46.3\ \mu\text{m} \rightarrow$ ◀

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PROBLEM 2.42

Solve Prob. 2.41, assuming that rod *AC* is made of brass and rod *CE* is made of steel.

PROBLEM 2.41 Two cylindrical rods, one of steel and the other of brass, are joined at *C* and restrained by rigid supports at *A* and *E*. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at *A* and *E*, (*b*) the deflection of point *C*.

SOLUTION

<u>*A* to *C*</u>: $E = 105\times10^{9}\ \text{Pa}$

$$A = \frac{\pi}{4}(40)^2 = 1.25664\times10^{3}\ \text{mm}^2 = 1.25664\times10^{-3}\ \text{m}^2$$

$$EA = 131.947\times10^{6}\ \text{N}$$

<u>*C* to *E*</u>: $E = 200\times10^{9}\ \text{Pa}$

$$A = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times10^{-6}\ \text{m}^2$$

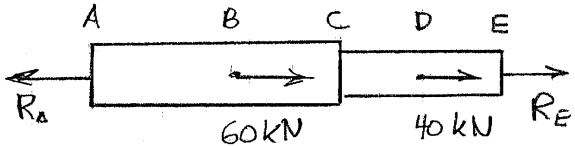
$$EA = 141.372\times10^{6}\ \text{N}$$

<u>*A* to *B*</u>: $P = R_A$

$$L = 180\ \text{mm} = 0.180\ \text{m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{131.947\times10^{6}}$$

$$= 1.36418\times10^{-9}R_A$$



<u>*B* to *C*</u>: $P = R_A - 60\times10^{3}$

$$L = 120\ \text{mm} = 0.120\ \text{m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^{3})(0.120)}{131.947\times10^{6}}$$

$$= 909.456\times10^{-12}R_A - 54.567\times10^{-6}$$

<u>*C* to *D*</u>: $P = R_A - 60\times10^{3}$

$$L = 100\ \text{mm} = 0.100\ \text{m}$$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 60\times10^{3})(0.100)}{141.372\times10^{6}}$$

$$= 707.354\times10^{-12}R_A - 42.441\times10^{-6}$$

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PROBLEM 2.42 *(Continued)*

<u>*D* to *E*</u>: $P = R_A - 100\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times10^3)(0.100)}{141.372\times10^6}$$

$$= 707.354\times10^{-12}R_A - 70.735\times10^{-6}$$

<u>*A* to *E*</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.68834\times10^{-9}R_A - 167.743\times10^{-6}$$

Since point *E* cannot move relative to *A*, $\delta_{AE} = 0$

(*a*) $3.68834\times10^{-9}R_A - 167.743\times10^{-6} = 0 \quad R_A = 45.479\times10^3 \text{ N}$ $R_A = 45.5 \text{ kN} \leftarrow$ ◀

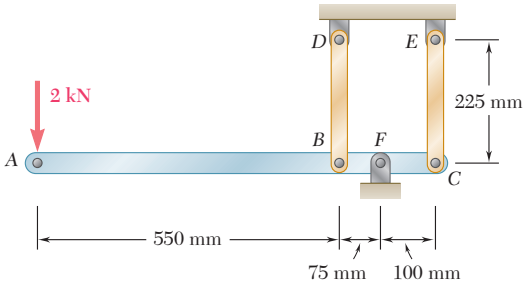
$R_E = R_A - 100\times10^3 = 45.479\times10^3 - 100\times10^3 = -54.521\times10^3$ $R_E = 54.5 \text{ kN} \leftarrow$ ◀

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 2.27364\times10^{-9}R_A - 54.567\times10^{-6}$

$= (2.27364\times10^{-9})(45.479\times10^3) - 54.567\times10^{-6}$

$= 48.8\times10^{-6} \text{ m}$ $\delta_C = 48.8\,\mu\text{m} \rightarrow$ ◀

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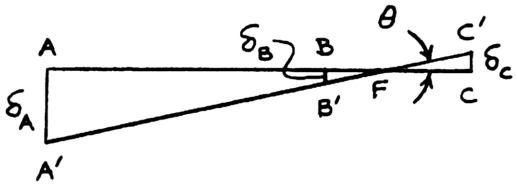


PROBLEM 2.43

Each of the rods *BD* and *CE* is made of brass ($E = 105$ GPa) and has a cross-sectional area of 200 mm^2. Determine the deflection of end *A* of the rigid member *ABC* caused by the 2-kN load.

SOLUTION

Let θ be the rotation of member *ABC* as shown.



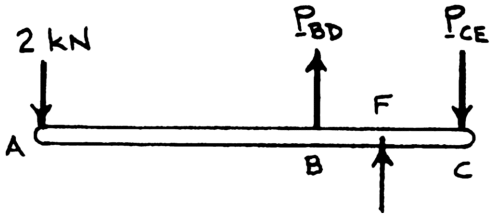
Then $\delta_A = 0.625\theta_1 \qquad \delta_B = 0.075\theta_1 \qquad \delta_C = 0.1\theta$

But $\delta_B = \dfrac{P_{BD}L_{BD}}{AE}$

$$P_{BD} = \frac{EA\delta_B}{L_{BD}} = \frac{(105\times10^9)(200\times10^{-6})(0.075\theta)}{0.225}$$

$$= 7\times10^6\ \theta$$

Free body *ABC*:



$$\delta_C = \frac{P_{CE}L_{CE}}{AE}$$

$$P_{CE} = \frac{EA\delta_C}{L_{CE}} = \frac{(105\times10^9)(200\times10^{-6})(0.1\,\theta)}{0.225}$$

$$= 9.3333\times10^6\ \theta$$

From free body of member *ABC*:

$$+\circlearrowleft \Sigma \mathbf{M}_F = 0: \quad (0.625)(2000) - 0.075P_{BD} - 0.1P_{CE} = 0$$

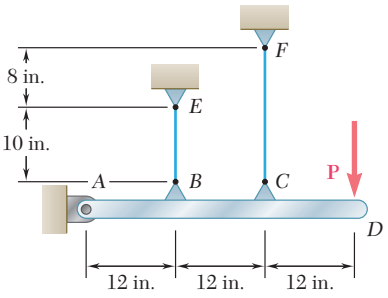
or $(0.625)(2000) - 0.075(7\times10^6\ \theta) - 0.1(9.3333\times10^6\ \theta) = 0$

$$\theta = 0.85714\times10^{-3}\ \text{rad}$$

and $\delta_A = 0.625\theta = 0.625(0.85714\times10^{-3}) = 0.53571\times10^{-3}\ \text{m}$

$$\delta_A = 0.536\ \text{mm} \downarrow \blacktriangleleft$$

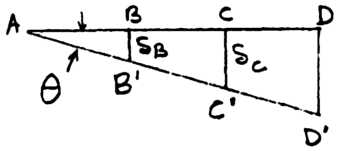
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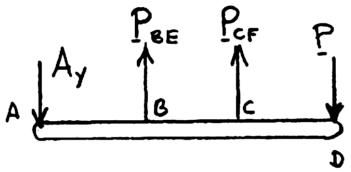


PROBLEM 2.44

The rigid bar *AD* is supported by two steel wires of $\frac{1}{16}$-in. diameter ($E = 29 \times 10^6$ psi) and a pin and bracket at *A*. Knowing that the wires were initially taut, determine (*a*) the additional tension in each wire when a 220-lb load **P** is applied at *D*, (*b*) the corresponding deflection of point *D*.

SOLUTION





Let θ be the notation of bar *ABCD*.

Then $\delta_B = 12\,\theta$

$$\delta_C = 24\,\theta$$

$$\delta_B = \frac{P_{BE} L_{BE}}{AE}$$

$$P_{BE} = \frac{EA\delta_{BE}}{L_{BE}} = \frac{(29\times10^6)\frac{\pi}{4}\left(\frac{1}{6}\right)^2(12\,\theta)}{10}$$

$$= 106.765\times10^3\,\theta$$

$$\delta_C = \frac{P_{CF} L_{CF}}{EA}$$

$$P_{CF} = \frac{EA\delta_{CE}}{L_{CF}} = \frac{(29\times10^6)\frac{\pi}{4}\left(\frac{1}{16}\right)^2(24\,\theta)}{18}$$

$$= 118.628\times10^3\,\theta$$

Using free body *ABCD*,

$+\circlearrowleft \sum \mathbf{M}_A = 0$: $\quad 12P_{BE} + 24P_{CF} - 36P = 0$

$$(12)(106.765\times10^3\,\theta) + (24)(118.628\times10^6\,\theta) - (36)(220) = 0$$

$$4.1283\times10^6\,\theta = (36)(220)$$

$$\theta = 1.91847\times10^{-3}\ \text{rad}\ \circlearrowright$$

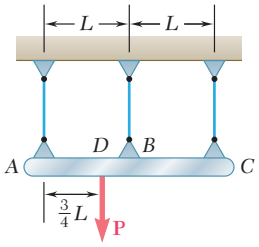
(*a*) $P_{BE} = (106.765\times10^3)(1.91847\times10^{-3}) = 204.83\ \text{lb}$ $\qquad P_{BE} = 205\ \text{lb}$ ◀

$P_{CF} = (118.628\times10^3)(1.91847\times10^{-3}) = 227.58\ \text{lb}$ $\qquad P_{CF} = 228\ \text{lb}$ ◀

(*b*) $\delta_D = 36\ \theta = (36)(1.91847\times10^{-3}) = 69.1\times10^{-3}\ \text{in.}$

$0.0691\ \text{in.}\downarrow$ ◀

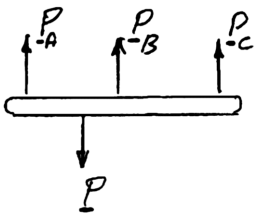
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PROBLEM 2.45

The rigid bar *ABC* is suspended from three wires of the same material. The cross-sectional area of the wire at *B* is equal to half of the cross-sectional area of the wires at *A* and *C*. Determine the tension in each wire caused by the load **P** shown.

SOLUTION



$$+\circlearrowleft \Sigma \mathbf{M}_A = 0: \quad 2LP_C + LP_B - \frac{3}{4}LP = 0$$

$$P_C = \frac{3}{8}P - \frac{1}{2}P_B$$

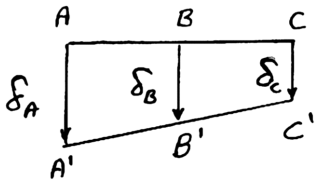
$$+\circlearrowleft \Sigma \mathbf{M}_C = 0: \quad -2LP_A - LP_B + \frac{5}{4}LP = 0$$

$$P_A = \frac{5}{8}P - \frac{1}{2}P_B$$

Let l be the length of the wires.

$$\delta_A = \frac{P_A l}{EA} = \frac{l}{EA}\left(\frac{5}{8}P - \frac{1}{2}P_B\right)$$

$$\delta_B = \frac{P_B l}{E(A/2)} = \frac{2l}{EA}P_B$$



$$\delta_C = \frac{P_C l}{EA} = \frac{l}{EA}\left(\frac{3}{8}P - \frac{1}{2}P_B\right)$$

From the deformation diagram,

$$\delta_A - \delta_B = \delta_B - \delta_C$$

or $$\delta_B = \frac{1}{2}(\delta_A + \delta_c)$$

$$\frac{l}{E(A/2)}P_B = \frac{1}{2}\frac{l}{EA}\left(\frac{5}{8}P - \frac{1}{2}P_B + \frac{3}{8}P - \frac{1}{2}P_B\right)$$

$$\frac{5}{2}P_B = \frac{1}{2}P; \qquad P_B = \frac{1}{5}P \qquad P_B = 0.200P \blacktriangleleft$$

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PROBLEM 2.45 ***(Continued)***

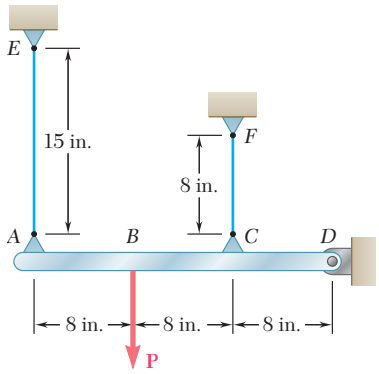
$$P_A = \frac{5}{8}P - \frac{1}{2}\left(\frac{P}{5}\right) = \frac{21}{40}P \qquad P_A = 0.525P \blacktriangleleft$$

$$P_C = \frac{3}{8}P - \frac{1}{2}\left(\frac{P}{5}\right) = \frac{11}{40}P \qquad P_C = 0.275P \blacktriangleleft$$

Check:

$$P_A + P_B + P_C = 1.000P \qquad \text{Ok}$$

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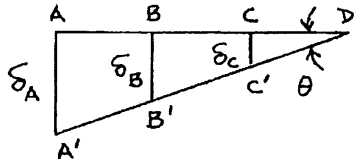
PROBLEM 2.46

The rigid bar *AD* is supported by two steel wires of $\frac{1}{16}$-in. diameter ($E = 29 \times 10^6$ psi) and a pin and bracket at *D*. Knowing that the wires were initially taut, determine (*a*) the additional tension in each wire when a 120-lb load **P** is applied at *B*, (*b*) the corresponding deflection of point *B*.

SOLUTION

Let θ be the rotation of bar *ABCD*.

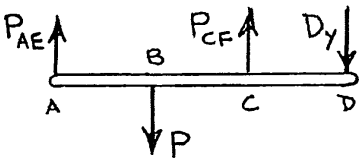
Then $\delta_A = 24\theta \qquad \delta_C = 8\theta$



$$\delta_A = \frac{P_{AE} L_{AE}}{AE}$$

$$P_{AE} = \frac{EA\delta_A}{L_{AE}} = \frac{(29\times10^6)\frac{\pi}{4}(\frac{1}{16})^2(24\theta)}{15}$$

$$= 142.353\times10^3\theta$$



$$\delta_C = \frac{P_{CF} L_{CF}}{AE}$$

$$P_{CF} = \frac{EA\delta_C}{L_{CF}} = \frac{(29\times10^6)\frac{\pi}{4}\left(\frac{1}{16}\right)^2(8\theta)}{8}$$

$$= 88.971\times10^3\theta$$

Using free body *ABCD*,

$$+\circlearrowleft \Sigma M_D = 0: \qquad -24P_{AE} + 16P - 8P_{CF} = 0$$

$$-24(142.353\times10^3\theta) + 16(120) - 8(88.971\times10^3\theta) = 0$$

$$\theta = 0.46510\times10^{-3} \text{ rad}\circlearrowright$$

(*a*) $P_{AE} = (142.353\times10^3)(0.46510\times10^{-3})$ $\qquad P_{AE} = 66.2 \text{ lb} \blacktriangleleft$

$P_{CF} = (88.971\times10^3)(0.46510\times10^{-3})$ $\qquad P_{CF} = 41.4 \text{ lb} \blacktriangleleft$

(*b*) $\delta_B = 16\theta = 16(0.46510\times10^{-3})$ $\qquad \delta_B = 7.44\times10^{-3} \text{ in.}\downarrow \blacktriangleleft$

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PROBLEM 2.47

The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15°C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195°C.

SOLUTION

Brass core:

$$E = 105 \text{ GPa}$$
$$\alpha = 20.9 \times 10^{-6}/^{\circ}\text{C}$$

Aluminum shell:

$$E = 70 \text{ GPa}$$
$$\alpha = 23.6 \times 10^{-6}/^{\circ}\text{C}$$

Let L be the length of the assembly.

Free thermal expansion:

$$\Delta T = 195 - 15 = 180\,^{\circ}\text{C}$$

Brass core: $(\delta_T)_b = L\alpha_b(\Delta T)$

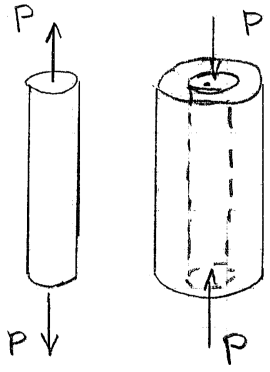
Aluminum shell: $(\delta_T)_a = L\alpha_a(\Delta T)$

Net expansion of shell with respect to the core: $\delta = L(\alpha_a - \alpha_b)(\Delta T)$

Let P be the tensile force in the core and the compressive force in the shell.

Brass core:

$$E_b = 105 \times 10^9 \text{ Pa}$$
$$A_b = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2$$
$$= 490.87 \times 10^{-6} \text{ m}^2$$
$$(\delta_P)_b = \frac{PL}{E_b A_b}$$



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PROBLEM 2.47 *(Continued)*

Aluminum shell:

$$(\delta_p)_a = \frac{PL}{E_a A_a}$$

$$E_a = 70 \times 10^9 \text{ Pa}$$

$$A_a = \frac{\pi}{4}(60^2 - 25^2)$$

$$= 2.3366 \times 10^3 \text{ mm}^2$$

$$= 2.3366 \times 10^{-3} \text{ m}^2$$

$$\delta = (\delta_P)_b + (\delta_P)_a$$

$$L(\alpha_b - \alpha_a)(\Delta T) = \frac{PL}{E_b A_b} + \frac{PL}{E_a A_a} = KPL$$

where

$$K = \frac{1}{E_b A_b} + \frac{1}{E_a A_a}$$

$$= \frac{1}{(105 \times 10^9)(490.87 \times 10^{-6})} + \frac{1}{(70 \times 10^9)(2.3366 \times 10^{-3})}$$

$$= 25.516 \times 10^{-9} \text{ N}^{-1}$$

Then

$$P = \frac{(\alpha_b - \alpha_a)(\Delta T)}{K}$$

$$= \frac{(23.6 \times 10^{-6} - 20.9 \times 10^{-6})(180)}{25.516 \times 10^{-9}}$$

$$= 19.047 \times 10^3 \text{ N}$$

Stress in aluminum:

$$\sigma_a = -\frac{P}{A_a} = -\frac{19.047 \times 10^3}{2.3366 \times 10^{-3}} = -8.15 \times 10^6 \text{ Pa} \qquad \sigma_a = -8.15 \text{ MPa} \blacktriangleleft$$

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PROBLEM 2.48

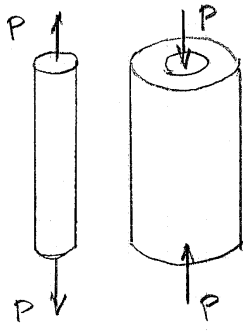
Solve Prob. 2.47, assuming that the core is made of steel (E_s = 200 GPa, $\alpha_s = 11.7\times10^{-6}/°C$) instead of brass.

PROBLEM 2.47 The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15°C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195°C.

SOLUTION

Aluminum shell: $E = 70$ GPa $\alpha = 23.6\times10^{-6}/°C$

Let L be the length of the assembly.



Free thermal expansion: $\Delta T = 195 - 15 = 180\,°C$

Steel core: $(\delta_T)_s = L\alpha_s(\Delta T)$

Aluminum shell: $(\delta_T)_a = L\alpha_a(\Delta T)$

Net expansion of shell with respect to the core: $\delta = L(\alpha_a - \alpha_s)(\Delta T)$

Let P be the tensile force in the core and the compressive force in the shell.

Steel core: $E_s = 200\times10^9\,\text{Pa}, \quad A_s = \dfrac{\pi}{4}(25)^2 = 490.87\ \text{mm}^2 = 490.87\times10^{-6}\,\text{m}^2$

$$(\delta_P)_s = \frac{PL}{E_s A_s}$$

Aluminum shell: $E_a = 70\times10^9\,\text{Pa}$

$$(\delta_P)_a = \frac{PL}{E_a A_a}$$

$$A_a = \frac{\pi}{4}(60^2 - 25)^2 = 2.3366\times10^3\,\text{mm}^2 = 2.3366\times10^{-3}\,\text{m}^2$$

$$\delta = (\delta_P)_s + (\delta_P)_a$$

$$L(\alpha_a - \alpha_s)(\Delta T) = \frac{PL}{E_s A_s} + \frac{PL}{E_a A_a} = KPL$$

where

$$K = \frac{1}{E_s A_s} + \frac{1}{E_a A_a}$$

$$= \frac{1}{(200\times10^9)(490.87\times10^{-6})} + \frac{1}{(70\times10^9)(2.3366\times10^{-3})}$$

$$= 16.2999\times10^{-9}\,\text{N}^{-1}$$

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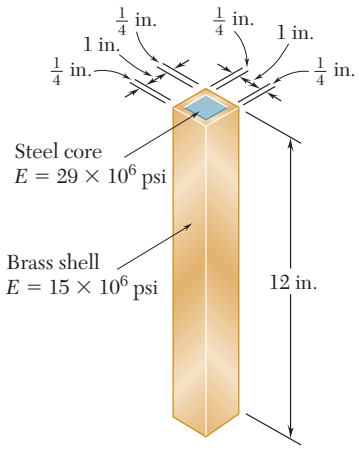
PROBLEM 2.48 ***(Continued)***

Then

$$P = \frac{(\alpha_a - \alpha_s)(\Delta T)}{K} = \frac{(23.6 \times 10^{-6} - 11.7 \times 10^{-6})(180)}{16.2999 \times 10^{-9}} = 131.412 \times 10^{3}\,\text{N}$$

Stress in aluminum: $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{131.412 \times 10^{3}}{2.3366 \times 10^{-3}} = -56.241 \times 10^{6}\,\text{Pa}$ $\sigma_a = -56.2\ \text{MPa}$ ◀

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PROBLEM 2.49

The brass shell ($\alpha_b = 11.6 \times 10^{-6}/°\text{F}$) is fully bonded to the steel core ($\alpha_s = 6.5 \times 10^{-6}/°\text{F}$). Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 8 ksi.

SOLUTION

Let P_s = axial force developed in the steel core.

For equilibrium with zero total force, the compressive force in the brass shell is P_s.

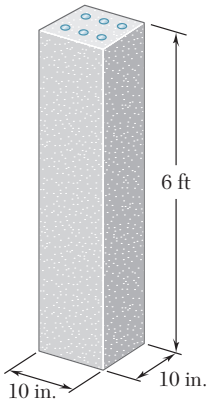
Strains:
$$\varepsilon_s = \frac{P_s}{E_s A_s} + \alpha_s(\Delta T)$$
$$\varepsilon_b = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

Matching:
$$\varepsilon_s = \varepsilon_b$$
$$\frac{P_s}{E_s A_s} + \alpha_s(\Delta T) = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$
$$\left(\frac{1}{E_s A_s} + \frac{1}{E_b A_b}\right) P_s = (\alpha_b - \alpha_s)(\Delta T) \qquad (1)$$

$$A_b = (1.5)(1.5) - (1.0)(1.0) = 1.25 \text{ in}^2$$
$$A_s = (1.0)(1.0) = 1.0 \text{ in}^2$$
$$\alpha_b - \alpha_s = 5.1 \times 10^{-6}/°\text{F}$$
$$P_s = \sigma_s A_s = (8 \times 10^3)(1.0) = 8 \times 10^3 \text{ lb}$$
$$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(29 \times 10^6)(1.0)} + \frac{1}{(15 \times 10^6)(1.25)} = 87.816 \times 10^{-9} \text{ lb}^{-1}$$

From (1),
$$(87.816 \times 10^{-9})(8 \times 10^3) = (5.1 \times 10^{-6})(\Delta T)$$
$$\Delta T = 137.8°\text{F} \blacktriangleleft$$

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PROBLEM 2.50

The concrete post ($E_c = 3.6\times10^6$ psi and $\alpha_c = 5.5\times10^{-6}$/°F) is reinforced with six steel bars, each of $\frac{7}{8}$-in. diameter ($E_s = 29\times10^6$ psi and $\alpha_s = 6.5\times10^{-6}$/°F). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 65°F.

SOLUTION

$$A_s = 6\frac{\pi}{4}d^2 = 6\frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 3.6079 \text{ in}^2$$

$$A_c = 10^2 - A_s = 10^2 - 3.6079 = 96.392 \text{ in}^2$$

Let $P_c =$ tensile force developed in the concrete.

For equilibrium with zero total force, the compressive force in the six steel rods equals P_c.

Strains: $\varepsilon_s = -\dfrac{P_c}{E_s A_s} + \alpha_s(\Delta T)$ $\qquad$ $\varepsilon_c = \dfrac{P_c}{E_c A_c} + \alpha_c(\Delta T)$

Matching: $\varepsilon_c = \varepsilon_s$ $\qquad$ $\dfrac{P_c}{E_c A_c} + \alpha_c(\Delta T) = -\dfrac{P_c}{E_s A_s} + \alpha_s(\Delta T)$

$$\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s}\right)P_c = (\alpha_s - \alpha_c)(\Delta T)$$

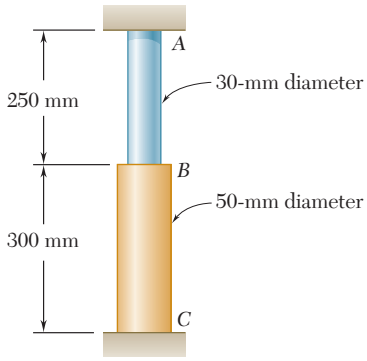
$$\left[\frac{1}{(3.6\times10^6)(96.392)} + \frac{1}{(29\times10^6)(3.6079)}\right]P_c = (1.0\times10^{-6})(65)$$

$$P_c = 5.2254\times10^3 \text{ lb}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{5.2254\times10^3}{96.392} = 54.210 \text{ psi} \qquad \sigma_c = 54.2 \text{ psi}$$

$$\sigma_s = -\frac{P_c}{A_s} = -\frac{5.2254\times10^3}{3.6079} = -1448.32 \text{ psi} \qquad \sigma_s = -1.448 \text{ ksi}$$

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PROBLEM 2.51

A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7\times10^{-6}/°C$) and portion *BC* is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9\times10^{-6}/°C$). Knowing that the rod is initially unstressed, determine the compressive force induced in *ABC* when there is a temperature rise of $50°C$.

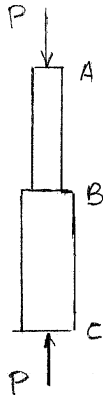
SOLUTION

$$A_{AB} = \frac{\pi}{4}d_{AB}^2 = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times10^{-6}\ \text{m}^2$$

$$A_{BC} = \frac{\pi}{4}d_{BC}^2 = \frac{\pi}{4}(50)^2 = 1.9635\times10^{3}\ \text{mm}^2 = 1.9635\times10^{-3}\ \text{m}^2$$

Free thermal expansion:

$$\begin{aligned}\delta_T &= L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)\\ &= (0.250)(11.7\times10^{-6})(50) + (0.300)(20.9\times10^{-6})(50)\\ &= 459.75\times10^{-6}\ \text{m}\end{aligned}$$



Shortening due to induced compressive force *P*:

$$\begin{aligned}\delta_P &= \frac{PL}{E_s A_{AB}} + \frac{PL}{E_b A_{BC}}\\ &= \frac{0.250P}{(200\times10^{9})(706.86\times10^{-6})} + \frac{0.300P}{(105\times10^{9})(1.9635\times10^{-3})}\\ &= 3.2235\times10^{-9}P\end{aligned}$$

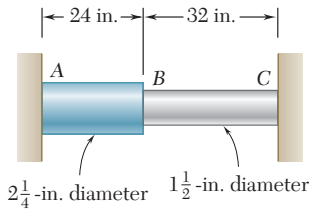
For zero net deflection, $\delta_P = \delta_T$

$$3.2235\times10^{-9}P = 459.75\times10^{-6}$$

$$P = 142.624\times10^{3}\ \text{N}$$

$$P = 142.6\ \text{kN} \blacktriangleleft$$

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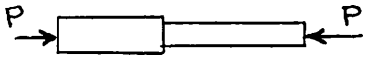
PROBLEM 2.52

A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}$/°F) and portion *BC* is made of aluminum ($E_a = 10.4 \times 10^6$ psi, $\alpha_a = 13.3 \times 10^{-6}$/°F). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions *AB* and *BC* by a temperature rise of 70°F, (*b*) the corresponding deflection of point *B*.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2 \quad A_{BC} = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$

Free thermal expansion. $\Delta T = 70°\text{F}$



$$(\delta_T)_{AB} = L_{AB}\alpha_s(\Delta T) = (24)(6.5 \times 10^{-6})(70) = 10.92 \times 10^{-3} \text{ in.}$$

$$(\delta_T)_{BC} = L_{BC}\alpha_a(\Delta T) = (32)(13.3 \times 10^{-6})(70) = 29.792 \times 10^{-3} \text{ in.}$$

Total: $\delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 40.712 \times 10^{-3}$ in.

Shortening due to induced compressive force *P*.

$$(\delta_P)_{AB} = \frac{PL_{AB}}{E_s A_{AB}} = \frac{24P}{(29 \times 10^6)(3.9761)} = 208.14 \times 10^{-9} P$$

$$(\delta_P)_{BC} = \frac{PL_{BC}}{E_a A_{BC}} = \frac{32P}{(10.4 \times 10^6)(1.76715)} = 1741.18 \times 10^{-9} P$$

Total: $\delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 1949.32 \times 10^{-9} P$

For zero net deflection, $\delta_P = \delta_T$ $\quad 1949.32 \times 10^{-9} P = 40.712 \times 10^{-3}$ $\quad P = 20.885 \times 10^3$ lb

(*a*) $\sigma_{AB} = -\dfrac{P}{A_{AB}} = -\dfrac{20.885 \times 10^3}{3.9761} = -5.25 \times 10^3$ psi $\quad \sigma_{AB} = -5.25$ ksi ◀

$\sigma_{BC} = -\dfrac{P}{A_{BC}} = -\dfrac{20.885 \times 10^3}{1.76715} = -11.82 \times 10^3$ psi $\quad \sigma_{BC} = -11.82$ ksi ◀

(*b*) $(\delta_P)_{AB} = (208.14 \times 10^{-9})(20.885 \times 10^3) = 4.3470 \times 10^{-3}$ in.

$\delta_B = (\delta_T)_{AB} \rightarrow + (\delta_P)_{AB} \leftarrow = 10.92 \times 10^{-3} \rightarrow + 4.3470 \times 10^{-3} \leftarrow$ $\quad \delta_B = 6.57 \times 10^{-3}$ in. $\rightarrow$ ◀

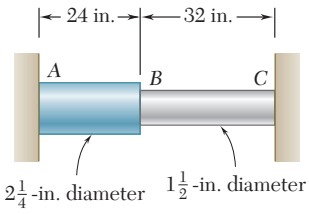
or

$(\delta_P)_{BC} = (1741.18 \times 10^{-9})(20.885 \times 10^3) = 36.365 \times 10^{-3}$ in.

$\delta_B = (\delta_T)_{BC} \leftarrow + (\delta_P)_{BC} \rightarrow = 29.792 \times 10^{-3} \leftarrow + 36.365 \times 10^{-3} \rightarrow = 6.57 \times 10^{-3}$ in. $\rightarrow$ (checks)

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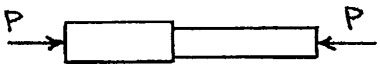
PROBLEM 2.53



Solve Prob. 2.52, assuming that portion *AB* of the composite rod is made of aluminum and portion *BC* is made of steel.

PROBLEM 2.52 A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}$/°F) and portion *BC* is made of aluminum ($E_a = 10.4 \times 10^6$ psi, $\alpha_a = 13.3 \times 10^{-6}$/°F). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions *AB* and *BC* by a temperature rise of 70°F, (*b*) the corresponding deflection of point *B*.

SOLUTION



$$A_{AB} = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2 \qquad A_{BC} = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$

Free thermal expansion. $\Delta T = 70°\text{F}$

$$(\delta_T)_{AB} = L_{AB}\alpha_a(\Delta T) = (24)(13.3\times10^{-6})(70) = 22.344\times10^{-3}\text{in.}$$
$$(\delta_T)_{BC} = L_{BC}\alpha_s(\Delta T) = (32)(6.5\times10^{-6})(70) = 14.56\times10^{-3}\text{in.}$$

Total: $\delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 36.904\times10^{-3}\text{in.}$

Shortening due to induced compressive force *P*.

$$(\delta_P)_{AB} = \frac{PL_{AB}}{E_a A_{AB}} = \frac{24P}{(10.4\times10^6)(3.9761)} = 580.39\times10^{-9}P$$
$$(\delta_P)_{BC} = \frac{PL_{BC}}{E_s A_{BC}} = \frac{32P}{(29\times10^6)(1.76715)} = 624.42\times10^{-9}P$$

Total: $\delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 1204.81\times10^{-9}P$

For zero net deflection, $\delta_P = \delta_T$ $\qquad 1204.81\times10^{-9}P = 36.904\times10^{-3}$ $\qquad P = 30.631\times10^3$ lb

(*a*) $$\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{30.631\times10^3}{3.9761} = -7.70\times10^3 \text{ psi}$$ $\qquad \sigma_{AB} = -7.70$ ksi ◀

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{30.631\times10^3}{1.76715} = -17.33\times10^3 \text{ psi}$$ $\qquad \sigma_{BC} = -17.33$ ksi ◀

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PROBLEM 2.53 ***(Continued)***

(*b*) $(\delta_P)_{AB} = (580.39\times10^{-9})(30.631\times10^{3}) = 17.7779\times10^{-3}$ in.

$\delta_B = (\delta_T)_{AB} \rightarrow + (\delta_P)_{AB} \leftarrow = 22.344\times10^{-3} \rightarrow + 17.7779\times10^{-3} \leftarrow \qquad \delta_B = 4.57\times10^{-3}$ in. $\rightarrow$ ◀

or $(\delta_P)_{BC} = (624.42\times10^{-9})(30.631\times10^{3}) = 19.1266\times10^{-3}$ in.

$\delta_B = (\delta_T)_{BC} \leftarrow + (\delta_P)_{BC} \rightarrow = 14.56\times10^{-3} \leftarrow + 19.1266\times10^{-3} \rightarrow = 4.57\times10^{-3}$ in. $\rightarrow$ (checks)

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PROBLEM 2.54

The steel rails of a railroad track (E_s = 200 GPa, α_s = 11.7 × 102^{-6}/°C) were laid at a temperature of 6°C. Determine the normal stress in the rails when the temperature reaches 48°C, assuming that the rails (*a*) are welded to form a continuous track, (*b*) are 10 m long with 3-mm gaps between them.

SOLUTION

(*a*) $\delta_T = \alpha(\Delta T)L = (11.7\times10^{-6})(48-6)(10) = 4.914\times10^{-3}$ m

$$\delta_P = \frac{PL}{AE} = \frac{L\sigma}{E} = \frac{(10)\sigma}{200\times10^{9}} = 50\times10^{-12}\sigma$$

$$\delta = \delta_T + \delta_P = 4.914\times10^{-3} + 50\times10^{-12}\sigma = 0$$

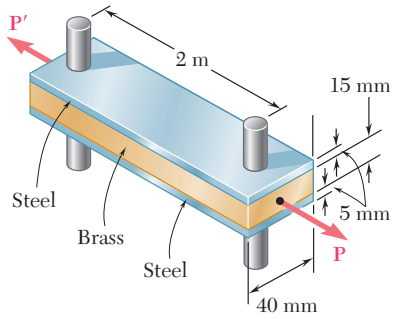
$$\sigma = -98.3\times10^{6} \text{ Pa} \qquad \sigma = -98.3 \text{ MPa}$$

(*b*) $\delta = \delta_T + \delta_P = 4.914\times10^{-3} + 50\times10^{-12}\sigma = 3\times10^{-3}$

$$\sigma = \frac{3\times10^{-3} - 4.914\times10^{-3}}{50\times10^{-12}}$$

$$= -38.3\times10^{6} \text{ Pa} \qquad \sigma = -38.3 \text{ MPa}$$

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PROBLEM 2.55

Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7\times10^{-6}/°C$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9\times10^{-6}/°C$) that is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes that were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

SOLUTION

(*a*) Required temperature change for fabrication:

$$\delta_T = 0.5 \text{ mm} = 0.5\times10^{-3}\text{m}$$

Temperature change required to expand steel bar by this amount:

$$\delta_T = L\alpha_s\Delta T, \quad 0.5\times10^{-3} = (2.00)(11.7\times10^{-6})(\Delta T),$$
$$\Delta T = 0.5\times10^{-3} = (2)(11.7\times10^{-6})(\Delta T)$$
$$\Delta T = 21.368\,°C \qquad 21.4\,°C \blacktriangleleft$$

(*b*) Once assembled, a tensile force P^* develops in the steel, and a compressive force P^* develops in the brass, in order to elongate the steel and contract the brass.

Elongation of steel: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400\times10^{-6}\text{m}^2$

$$(\delta_P)_s = \frac{F^*L}{A_sE_s} = \frac{P^*(2.00)}{(400\times10^{-6})(200\times10^9)} = 25\times10^{-9}P^*$$

Contraction of brass: $A_b = (40)(15) = 600 \text{ mm}^2 = 600\times10^{-6}\text{m}^2$

$$(\delta_P)_b = \frac{P^*L}{A_bE_b} = \frac{P^*(2.00)}{(600\times10^{-6})(105\times10^9)} = 31.746\times10^{-9}P^*$$

But $(\delta_P)_s + (\delta_P)_b$ is equal to the initial amount of misfit:

$$(\delta_P)_s + (\delta_P)_b = 0.5\times10^{-3}, \quad 56.746\times10^{-9}P^* = 0.5\times10^{-3}$$
$$P^* = 8.8112\times10^3 \text{ N}$$

Stresses due to fabrication:

Steel: $$\sigma_s^* = \frac{P^*}{A_s} = \frac{8.8112\times10^3}{400\times10^{-6}} = 22.028\times10^6\text{Pa} = 22.028 \text{ MPa}$$

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PROBLEM 2.55 *(Continued)*

Brass: $$\sigma_b^* = -\frac{P^*}{A_b} = -\frac{8.8112\times10^3}{600\times10^{-6}} = -14.6853\times10^6 \text{ Pa} = -14.685 \text{ MPa}$$

To these stresses must be added the stresses due to the 25-kN load.

For the added load, the additional deformation is the same for both the steel and the brass. Let δ' be the additional displacement. Also, let P_s and P_b be the additional forces developed in the steel and brass, respectively.

$$\delta' = \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$$

$$P_s = \frac{A_s E_s}{L}\delta' = \frac{(400\times10^{-6})(200\times10^9)}{2.00}\delta' = 40\times10^6\,\delta'$$

$$P_b = \frac{A_b E_b}{L}\delta' = \frac{(600\times10^{-6})(105\times10^9)}{2.00}\delta' = 31.5\times10^6\,\delta'$$

Total: $$P = P_s + P_b = 25\times10^3 \text{ N}$$

$$40\times10^6\,\delta' + 31.5\times10^6\,\delta' = 25\times10^3 \qquad \delta' = 349.65\times10^{-6} \text{ m}$$

$$P_s = (40\times10^6)(349.65\times10^{-6}) = 13.9860\times10^3 \text{ N}$$

$$P_b = (31.5\times10^6)(349.65\times10^{-6}) = 11.0140\times10^3 \text{ N}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{13.9860\times10^3}{400\times10^{-6}} = 34.965\times10^6 \text{ Pa}$$

$$\sigma_b = \frac{P_b}{A_b} = \frac{11.0140\times10^3}{600\times10^{-6}} = 18.3566\times10^6 \text{ Pa}$$

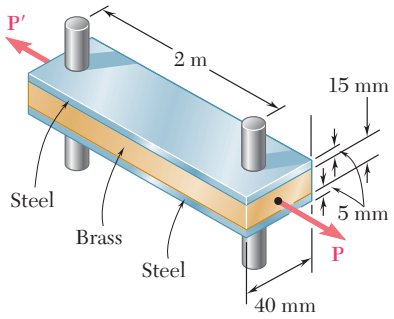
Add stress due to fabrication.

Total stresses:

$$\sigma_s = 34.965\times10^6 + 22.028\times10^6 = 56.991\times10^6 \text{ Pa} \qquad \sigma_s = 57.0 \text{ MPa}$$

$$\sigma_b = 18.3566\times10^6 - 14.6853\times10^6 = 3.6713\times10^6 \text{ Pa} \qquad \sigma_b = 3.67 \text{ MPa} \blacktriangleleft$$

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PROBLEM 2.56

Determine the maximum load P that may be applied to the brass bar of Prob. 2.55 if the allowable stress in the steel bars is 30 MPa and the allowable stress in the brass bar is 25 MPa.

PROBLEM 2.55 Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) that is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes that were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

SOLUTION

See solution to Problem 2.55 to obtain the fabrication stresses.

$$\sigma_s^* = 22.028 \text{ MPa}$$

$$\sigma_b^* = 14.6853 \text{ MPa}$$

Allowable stresses: $\sigma_{s,\text{all}} = 30$ MPa, $\sigma_{b,\text{all}} = 25$ MPa

Available stress increase from load.

$$\sigma_s = 30 - 22.028 = 7.9720 \text{ MPa}$$

$$\sigma_b = 25 + 14.6853 = 39.685 \text{ MPa}$$

Corresponding available strains.

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{7.9720 \times 10^6}{200 \times 10^9} = 39.860 \times 10^{-6}$$

$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{39.685 \times 10^6}{105 \times 10^9} = 377.95 \times 10^{-6}$$

Smaller value governs $\therefore \quad \varepsilon = 39.860 \times 10^{-6}$

Areas: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$

$$A_b = (15)(40) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$$

Forces $P_s = E_s A_s \varepsilon = (200 \times 10^9)(400 \times 10^{-6})(39.860 \times 10^{-6}) = 3.1888 \times 10^3 \text{ N}$

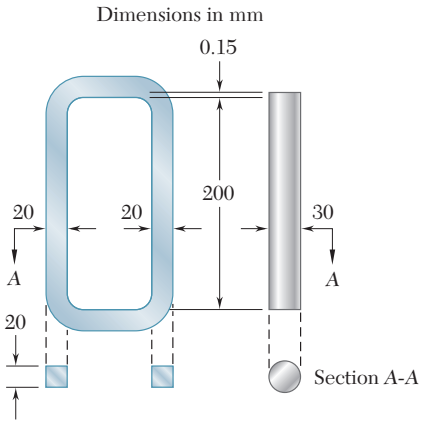
$$P_b = E_b A_b \varepsilon = (105 \times 10^9)(600 \times 10^{-6})(39.860 \times 10^{-6}) = 2.5112 \times 10^{-3} \text{ N}$$

Total allowable additional force:

$$P = P_s + P_b = 3.1888 \times 10^3 + 2.5112 \times 10^3 = 5.70 \times 10^3 \text{ N}$$

$P = 5.70$ kN ◀

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PROBLEM 2.57

An aluminum rod (E_a = 70 GPa, $\alpha_a = 23.6 \times 10^{-6}$/°C) and a steel link ($E_s$ × 200 GPa, $\alpha_a = 11.7 \times 10^{-6}$/°C) have the dimensions shown at a temperature of 20°C. The steel link is heated until the aluminum rod can be fitted freely into the link. The temperature of the whole assembly is then raised to 150°C. Determine the final normal stress (*a*) in the rod, (*b*) in the link.

SOLUTION

$$\Delta T = T_f - T_i = 150°\text{C} - 20°\text{C} = 130°\text{C}$$

Unrestrained thermal expansion of each part:

Aluminum rod: $(\delta_T)_a = L\alpha_a(\Delta T)$

$$(\delta_T)_a = (0.200\text{ m})(23.6\times10^{-6}/°\text{C})(130°\text{C})$$

$$= 6.1360\times10^{-4}\text{ m}$$

Steel link: $(\delta_T)_s = L\alpha_s(\Delta T)$

$$(\delta_T)_s = (0.200\text{ m})(11.7\times10^{-6}/°\text{C})(130°\text{C})$$

$$= 3.0420\times10^{-4}\text{ m}$$

Let *P* be the compressive force developed in the aluminum rod. It is also the tensile force in the steel link.

Aluminum rod: $(\delta_P)_a = \dfrac{PL}{E_a A_a}$

$$= \frac{P(0.200\text{ m})}{(70\times10^9\text{ Pa})(\pi/4)(0.03\text{ m})^2}$$

$$= 4.0420\times10^{-9}P$$

Steel link: $(\delta_P)_s = \dfrac{PL}{E_s A_s}$

$$= \frac{P(0.200)}{(200\times10^9\text{ Pa})(2)(0.02\text{ m})^2}$$

$$= 1.250\times10^{-9}P$$

Setting the total deformed lengths in the link and rod equal gives

$$(0.200) + (\delta_T)_s + (\delta_P)_s = (0.200) + (0.15\times10^{-3}) + (\delta_T)_a - (\delta_P)_a$$

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PROBLEM 2.57 *(Continued)*

$$(\delta_P)_s + (\delta_P)_a = 0.15 \times 10^{-3} + (\delta_T)_a - (\delta_T)_s$$

$$1.25 \times 10^{-9} P + 4.0420 \times 10^{-9} P = 0.15 \times 10^{-3} + 6.1360 \times 10^{-4} - 3.0420 \times 10^{-4}$$

$$P = 8.6810 \times 10^{4} \text{ N}$$

(*a*) Stress in rod: $\sigma = \dfrac{P}{A}$

$$\sigma_R = \frac{-8.6810 \times 10^{4} \text{ N}}{(\pi/4)(0.030 \text{ m})^2} = -1.22811 \times 10^{8} \text{ Pa}$$

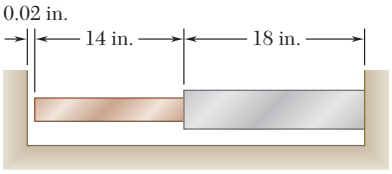
$$\sigma_R = -122.8 \text{ MPa} \blacktriangleleft$$

(b) Stress in link:

$$\sigma_L = \frac{8.6810 \times 10^{4} \text{ N}}{(2)(0.020 \text{ m})^2} = 1.08513 \times 10^{8} \text{ Pa}$$

$$\sigma_L = 108.5 \text{ MPa} \blacktriangleleft$$

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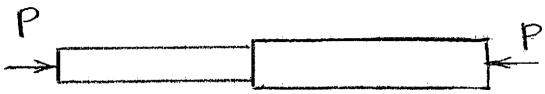
PROBLEM 2.58

Knowing that a 0.02-in. gap exists when the temperature is 75°F, determine (*a*) the temperature at which the normal stress in the aluminum bar will be equal to –11 ksi, (*b*) the corresponding exact length of the aluminum bar.

SOLUTION

$$\sigma_a = -11\ \text{ksi} = -11\times10^3\ \text{psi}$$
$$P = -\sigma_a A_a = (11\times10^3)(2.8) = 30.8\times10^3\ \text{lb}$$

Shortening due to *P*:



$$\delta_P = \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a}$$
$$= \frac{(30.8\times10^3)(14)}{(15\times10^6)(2.4)} + \frac{(30.8\times10^3)(18)}{(10.6\times10^6)(2.8)}$$
$$= 30.657\times10^{-3}\ \text{in.}$$

Available elongation for thermal expansion:

$$\delta_T = 0.02 + 30.657\times10^{-3} = 50.657\times10^{-3}\ \text{in.}$$

But $\delta_T = L_b\alpha_b(\Delta T) + L_a\alpha_a(\Delta T)$

$$= (14)(12\times10^{-6})(\Delta T) + (18)(12.9\times10^{-6})(\Delta T) = (400.2\times10^{-6})\Delta T$$

Equating, $(400.2\times10^{-6})\Delta T = 50.657\times10^{-3} \quad \Delta T = 126.6°\text{F}$

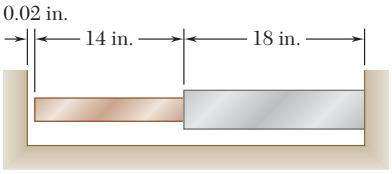
(*a*) $T_{\text{hot}} = T_{\text{cold}} + \Delta T = 75 + 126.6 = 201.6°\text{F}$ $\quad T_{\text{hot}} = 201.6°\text{F}$ ◀

(*b*) $\delta_a = L_a\alpha_a(\Delta T) - \frac{PL_a}{E_a A_a}$

$$= (18)(12.9\times10^{-6})(26.6) - \frac{(30.8\times10^3)(18)}{(10.6\times10^6)(2.8)} = 10.712\times10^{-3}\ \text{in.}$$

$$L_{\text{exact}} = 18 + 10.712\times10^{-3} = 18.0107\ \text{in.} \quad L = 18.0107\ \text{in.}$$ ◀

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Bronze
$A = 2.4\ \text{in}^2$
$E = 15 \times 10^6\ \text{psi}$
$\alpha = 12 \times 10^{-6}/°\text{F}$

Aluminum
$A = 2.8\ \text{in}^2$
$E = 10.6 \times 10^6\ \text{psi}$
$\alpha = 12.9 \times 10^{-6}/°\text{F}$

PROBLEM 2.59

Determine (*a*) the compressive force in the bars shown after a temperature rise of 180°F, (*b*) the corresponding change in length of the bronze bar.

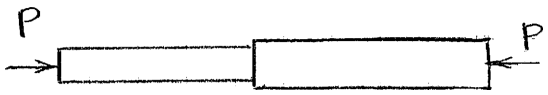
SOLUTION

Thermal expansion if free of constraint:

$$\begin{aligned}\delta_T &= L_b\alpha_b(\Delta T) + L_a\alpha_a(\Delta T)\\ &= (14)(12\times10^{-6})(180) + (18)(12.9\times10^{-6})(180)\\ &= 72.036\times10^{-3}\ \text{in.}\end{aligned}$$

Constrained expansion: $\delta = 0.02$ in.

Shortening due to induced compressive force *P*:



$$\delta_P = 72.036\times10^{-3} - 0.02 = 52.036\times10^{-3}\ \text{in.}$$

But
$$\delta_P = \frac{PL_b}{E_bA_b} + \frac{PL_a}{E_aA_a} = \left(\frac{L_b}{E_bA_b} + \frac{L_a}{E_aA_a}\right)P$$

$$= \left(\frac{14}{(15\times10^6)(2.4)} + \frac{18}{(10.6\times10^6)(2.8)}\right)P = 995.36\times10^{-9}P$$

Equating,
$$995.36\times10^{-9}P = 52.036\times10^{-3}$$
$$P = 52.279\times10^3\ \text{lb}$$

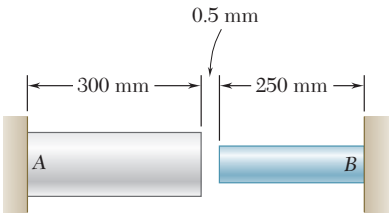
(*a*) $P = 52.3$ kips ◀

(*b*) $\delta_b = L_b\alpha_b(\Delta T) - \dfrac{PL_b}{E_bA_b}$

$$= (14)(12\times10^{-6})(180) - \frac{(52.279\times10^3)(14)}{(15\times10^6)(2.4)} = 9.91\times10^{-3}\ \text{in.}$$

$\delta_b = 9.91\times10^{-3}$ in. ◀

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Aluminum
$A = 2000\ \text{mm}^2$
$E = 75\ \text{GPa}$
$\alpha = 23 \times 10^{-6}/°\text{C}$

Stainless steel
$A = 800\ \text{mm}^2$
$E = 190\ \text{GPa}$
$\alpha = 17.3 \times 10^{-6}/°\text{C}$

PROBLEM 2.60

At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 140°C, determine (*a*) the normal stress in the aluminum rod, (*b*) the change in length of the aluminum rod.

SOLUTION

$$\Delta T = 140 - 20 = 120°\text{C}$$

Free thermal expansion:

$$\begin{aligned}\delta_T &= L_a\alpha_a(\Delta T) + L_s\alpha_s(\Delta T)\\ &= (0.300)(23\times10^{-6})(120) + (0.250)(17.3\times10^{-6})(120)\\ &= 1.347\times10^{-3}\,\text{m}\end{aligned}$$

Shortening due to P to meet constraint:

$$\delta_P = 1.347\times10^{-3} - 0.5\times10^{-3} = 0.847\times10^{-3}\,\text{m}$$

$$\begin{aligned}\delta_P &= \frac{PL_a}{E_aA_a} + \frac{PL_s}{E_sA_s} = \left(\frac{L_a}{E_aA_a} + \frac{L_s}{E_sA_s}\right)P\\ &= \left(\frac{0.300}{(75\times10^9)(2000\times10^{-6})} + \frac{0.250}{(190\times10^9)(800\times10^{-6})}\right)P\\ &= 3.6447\times10^{-9}P\end{aligned}$$

Equating, $3.6447\times10^{-9}P = 0.847\times10^{-3}$

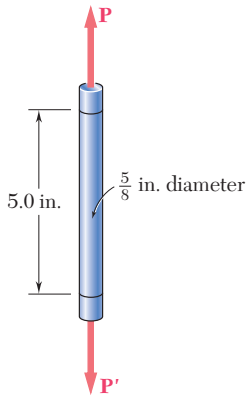
$$P = 232.39\times10^3\ \text{N}$$

(*a*) $$\sigma_a = -\frac{P}{A_a} = -\frac{232.39\times10^3}{2000\times10^{-6}} = -116.2\times10^6\,\text{Pa}$$ $\sigma_a = -116.2\ \text{MPa}$ ◀

(*b*) $$\delta_a = L_a\alpha_a(\Delta T) - \frac{PL_a}{E_aA_a}$$

$$= (0.300)(23\times10^{-6})(120) - \frac{(232.39\times10^3)(0.300)}{(75\times10^9)(2000\times10^{-6})} = 363\times10^{-6}\,\text{m}$$ $\delta_a = 0.363\ \text{mm}$ ◀

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PROBLEM 2.61

A standard tension test is used to determine the properties of an experimental plastic. The test specimen is a $\frac{5}{8}$-in.-diameter rod and it is subjected to an 800-lb tensile force. Knowing that an elongation of 0.45 in. and a decrease in diameter of 0.025 in. are observed in a 5-in. gage length, determine the modulus of elasticity, the modulus of rigidity, and Poisson's ratio for the material.

SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.306796 \text{ in}^2$$

$$P = 800 \text{ lb}$$

$$\sigma_y = \frac{P}{A} = \frac{800}{0.306796} = 2.6076 \times 10^3 \text{ psi}$$

$$\varepsilon_y = \frac{\delta_y}{L} = \frac{0.45}{5.0} = 0.090$$

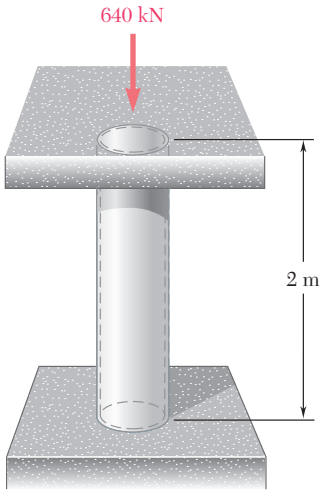
$$\varepsilon_x = \frac{\delta_x}{d} = \frac{-0.025}{0.625} = -0.040$$

$$E = \frac{\delta_y}{\varepsilon_y} = \frac{2.6076 \times 10^3}{0.090} = 28.973 \times 10^3 \text{ psi} \qquad E = 29.0 \times 10^3 \text{ psi} \blacktriangleleft$$

$$\nu = -\frac{\varepsilon_x}{\varepsilon_y} = \frac{-0.040}{0.090} = 0.44444 \qquad \nu = 0.444 \blacktriangleleft$$

$$\sigma = \frac{E}{2(1+\nu)} = \frac{28.973 \times 10^3}{(2)(1+0.44444)} = 10.0291 \times 10^3 \text{ psi} \qquad \sigma = 10.03 \times 10^3 \text{ psi} \blacktriangleleft$$

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PROBLEM 2.62

A 2-m length of an aluminum pipe of 240-mm outer diameter and 10-mm wall thickness is used as a short column to carry a 640-kN centric axial load. Knowing that $E = 73$ GPa and $\nu = 0.33$, determine (*a*) the change in length of the pipe, (*b*) the change in its outer diameter, (*c*) the change in its wall thickness.

SOLUTION

$$d_o = 0.240 \qquad t = 0.010 \qquad L = 2.0$$

$$d_i = d_o - 2t = 0.240 - 2(0.010) = 0.220 \text{ m} \qquad P = 640 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4}(0.240 - 0.220) = 7.2257 \times 10^{-3} \text{ m}^2$$

(*a*) $$\delta = -\frac{PL}{EA} = -\frac{(640 \times 10^3)(2.0)}{(73 \times 10^9)(7.2257 \times 10^{-3})}$$

$$= -2.4267 \times 10^{-3} \text{ m} \qquad \delta = -2.43 \text{ mm} \blacktriangleleft$$

$$\varepsilon = \frac{\delta}{L} = \frac{-2.4267}{2.0} = -1.21335 \times 10^{-3}$$

$$\varepsilon_{LAT} = -\nu\varepsilon = -(0.33)(-1.21335 \times 10^{-3})$$

$$= 4.0041 \times 10^{-4}$$

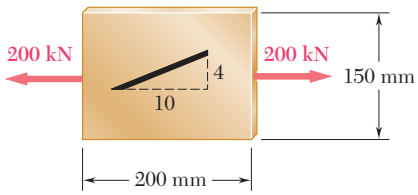
(b) $$\Delta d_o = d_o \varepsilon_{LAT} = (240 \text{ mm})(4.0041 \times 10^{-4}) = 9.6098 \times 10^{-2} \text{ mm}$$

$$\Delta d_o = 0.0961 \text{ mm} \blacktriangleleft$$

$$\Delta t = t\varepsilon_{LAT} = (10 \text{ mm})(4.0041 \times 10^{-4}) = 4.0041 \times 10^{-3} \text{ mm}$$

$$\Delta t = 0.00400 \text{ mm} \blacktriangleleft$$

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PROBLEM 2.63

A line of slope 4:10 has been scribed on a cold-rolled yellow-brass plate, 150 mm wide and 6 mm thick. Knowing that $E = 105$ GPa and $\nu = 0.34$, determine the slope of the line when the plate is subjected to a 200-kN centric axial load as shown.

SOLUTION

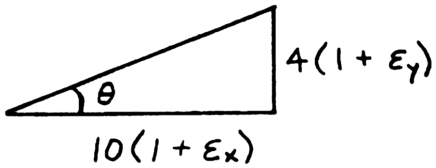
$$A = (0.150)(0.006) = 0.9 \times 10^{-3} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{200 \times 10^3}{0.9 \times 10^{-3}} = 222.22 \times 10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{222.22 \times 10^6}{105 \times 10^9} = 2.1164 \times 10^{-3}$$

$$\varepsilon_y = -\nu\varepsilon_x = -(0.34)(2.1164 \times 10^{-3})$$

$$= -0.71958 \times 10^{-3}$$



$$\tan\theta = \frac{4(1+\varepsilon_y)}{10(1+\varepsilon_x)}$$

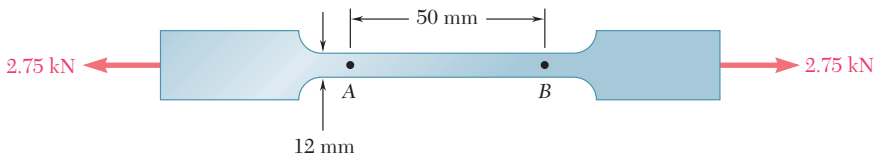
$$= \frac{4(1 - 0.71958 \times 10^{-3})}{10(1 + 2.1164 \times 10^{-3})}$$

$$= 0.39887$$

$$\tan\theta = 0.399 \blacktriangleleft$$

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PROBLEM 2.64



A 2.75-kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate (E = 200 GPa, ν = 0.30). Determine the resulting change (*a*) in the 50-mm gage length, (*b*) in the width of portion *AB* of the test coupon, (*c*) in the thickness of portion *AB*, (*d*) in the cross-sectional area of portion *AB*.

SOLUTION

$$A = (1.6)(12) = 19.20 \text{ mm}^2$$

$$= 19.20 \times 10^{-6} \text{ m}^2$$

$$P = 2.75 \times 10^{3} \text{ N}$$

$$\sigma_x = \frac{P}{A} = \frac{2.75 \times 10^{3}}{19.20 \times 10^{-6}}$$

$$= 143.229 \times 10^{6} \text{ Pa}$$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{143.229 \times 10^{6}}{200 \times 10^{9}} = 716.15 \times 10^{-6}$$

$$\varepsilon_y = \varepsilon_z = -\upsilon\varepsilon_x = -(0.30)(716.15 \times 10^{-6}) = -214.84 \times 10^{-6}$$

(*a*) $L = 0.050$ m $\quad \delta_x = L\varepsilon_x = (0.50)(716.15 \times 10^{-6}) = 35.808 \times 10^{-6}$ m

0.0358 mm ◀

(*b*) $w = 0.012$ m $\quad \delta_y = w\varepsilon_y = (0.012)(-214.84 \times 10^{-6}) = -2.5781 \times 10^{-6}$ m

–0.00258 mm ◀

(*c*) $t = 0.0016$ m $\quad \delta_z = t\varepsilon_z = (0.0016)(-214.84 \times 10^{-6}) = -343.74 \times 10^{-9}$ m

–0.000344 mm ◀

(*d*) $A = w_0(1+\varepsilon_y)t_0(1+\varepsilon_z) = w_0t_0(1+\varepsilon_y+\varepsilon_z+\varepsilon_y\varepsilon_z) \qquad A_0 = w_0t_0$

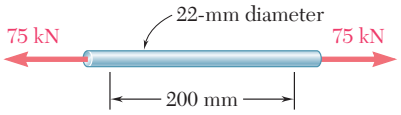
$$\Delta A = A - A_0 = w_0t_0(\varepsilon_y + \varepsilon_z + \text{negligible term}) = 2w_0t_0\varepsilon_y$$

$$= (2)(0.012)(0.0016)(-214.84 \times 10^{-6}) = -8.25 \times 10^{-9} \text{ m}^2$$

–0.00825 mm^2 ◀

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PROBLEM 2.65



In a standard tensile test, a steel rod of 22-mm diameter is subjected to a tension force of 75 kN. Knowing that $\nu = 0.3$ and $E = 200$ GPa, determine (*a*) the elongation of the rod in a 200-mm gage length, (*b*) the change in diameter of the rod.

SOLUTION

$P = 75 \text{ kN} = 75 \times 10^3 \text{ N}$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.022)^2 = 380.13 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{75 \times 10^3}{380.13 \times 10^{-6}} = 197.301 \times 10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{\sigma}{E} = \frac{197.301 \times 10^6}{200 \times 10^9} = 986.51 \times 10^{-6}$$

$$\delta_x = L\varepsilon_x = (200 \text{ mm})(986.51 \times 10^{-6})$$

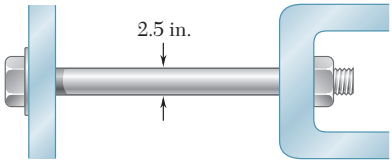
(*a*) $\delta_x = 0.1973 \text{ mm}$ ◀

$$\varepsilon_y = -\nu\varepsilon_x = -(0.3)(986.51 \times 10^{-6}) = -295.95 \times 10^{-6}$$

$$\delta_y = d\varepsilon_y = (22 \text{ mm})(-295.95 \times 10^{-6})$$

(*b*) $\delta_y = -0.00651 \text{ mm}$ ◀

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PROBLEM 2.66

The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the internal force in the bolt if the diameter is observed to decrease by 0.5×10^{-3} in.

SOLUTION

$$\delta_y = -0.5 \times 10^{-3} \text{ in.} \qquad d = 2.5 \text{ in.}$$

$$\varepsilon_y = \frac{\varepsilon_y}{d} = -\frac{0.5 \times 10^{-3}}{2.5} = -0.2 \times 10^{-3}$$

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}: \qquad \varepsilon_x = \frac{-\varepsilon_y}{\nu} = \frac{0.2 \times 10^{-3}}{0.3} = 0.66667 \times 10^{-3}$$

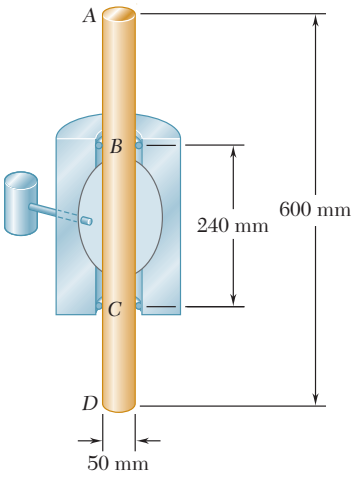
$$\sigma_x = E\varepsilon_x = (29 \times 10^6)(0.66667 \times 10^{-3}) = 19.3334 \times 10^3 \text{ psi}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(2.5)^2 = 4.9087 \text{ in}^2$$

$$F = \sigma_x A = (19.3334 \times 10^3)(4.9087) = 94.902 \times 10^3 \text{ lb}$$

$$F = 94.9 \text{ kips} \blacktriangleleft$$

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PROBLEM 2.67

The brass rod *AD* is fitted with a jacket that is used to apply a hydrostatic pressure of 48 MPa to the 240-mm portion *BC* of the rod. Knowing that $E = 105$ GPa and $\nu = 0.33$, determine (*a*) the change in the total length *AD*, (*b*) the change in diameter at the middle of the rod.

SOLUTION

$$\sigma_x = \sigma_z = -p = -48\times10^6 \text{ Pa}, \qquad \sigma_y = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{105\times10^9}\left[-48\times10^6 - (0.33)(0) - (0.33)(-48\times10^6)\right]$$

$$= 306.29\times10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x - \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{105\times10^9}\left[-(0.33)(-48\times10^6) + 0 - (0.33)(-48\times10^6)\right]$$

$$= 301.71\times10^{-6}$$

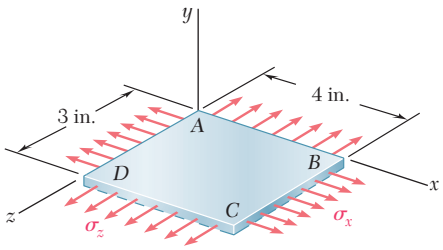
(*a*) Change in length: only portion *BC* is strained. $L = 240$ mm

$$\delta_y = L\varepsilon_y = (240)(-301.71\times10^{-6}) = -0.0724 \text{ mm}$$ ◀

(*b*) Change in diameter: $d = 50$ mm

$$\delta_x = \delta_z = d\varepsilon_x = (50)(-306.29\times10^{-6}) = -0.01531 \text{ mm}$$ ◀

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PROBLEM 2.68

A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 18$ ksi and $\sigma_z = 24$ ksi. Knowing that the properties of the fabric can be approximated as $E = 12.6 \times 10^6$ psi and $\nu = 0.34$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

$$\sigma_x = 18 \text{ ksi} \qquad \sigma_y = 0 \qquad \sigma_z = 24 \text{ ksi}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{12.6\times10^6}\left[18{,}000 - (0.34)(24{,}000)\right] = 780.95\times10^{-6}$$

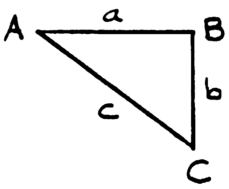
$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{12.6\times10^6}\left[-(0.34)(18{,}000) + 24{,}000\right] = 1.41905\times10^{-3}$$

(*a*) $\delta_{AB} = (AB)\varepsilon_x = (4 \text{ in.})(780.95\times10^{-6}) = 0.0031238 \text{ in.}$

0.00312 in. ◀

(*b*) $\delta_{BC} = (BC)\varepsilon_z = (3 \text{ in.})(1.41905\times10^{-3}) = 0.0042572 \text{ in.}$

0.00426 in. ◀



Label sides of right triangle *ABC* as *a*, *b*, *c*.

Then $c^2 = a^2 + b^2$

Obtain differentials by calculus.

$$2cdc = 2ada + 2bdb$$

$$dc = \frac{a}{c}da + \frac{b}{c}db$$

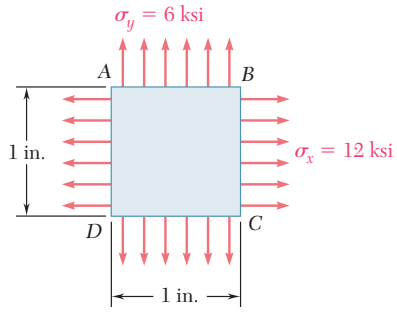
But $a = 4$ in. $\quad b = 3$ in. $\quad c = \sqrt{4^2 + 3^2} = 5$ in.

$da = \delta_{AB} = 0.0031238$ in. $\quad db = \delta_{BC} = 0.0042572$ in.

(*c*) $\delta_{AC} = dc = \frac{4}{5}(0.0031238) + \frac{3}{5}(0.0042572)$

0.00505 in. ◀

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PROBLEM 2.69

A 1-in. square was scribed on the side of a large steel pressure vessel. After pressurization the biaxial stress condition at the square is as shown. Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \upsilon\delta_y) = \frac{1}{29\times10^6}\left[12\times10^3 - (0.30)(6\times10^3)\right]$$
$$= 351.72\times10^{-6}$$
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \upsilon\delta_x) = \frac{1}{29\times10^6}\left[6\times10^3 - (0.30)(12\times10^3)\right]$$
$$= 82.759\times10^{-6}$$

(*a*) $\delta_{AB} = (AB)_0\,\varepsilon_x = (1.00)(351.72\times10^{-6}) = 352\times10^{-6}$ in. ◀

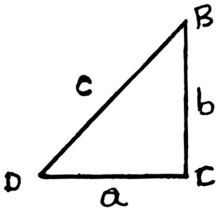
(*b*) $\delta_{BC} = (BC)_0\,\varepsilon_y = (1.00)(82.759\times10^{-6}) = 82.8\times10^{-6}$ in. ◀

(*c*) $(AC) = \sqrt{(AB)^2 + (BC)^2} = \sqrt{(AB_0 + \delta_x)^2 + (BC_0 + \delta_y)^2}$

$$= \sqrt{(1 + 351.72\times10^{-6})^2 + (1 + 82.759\times10^{-6})^2}$$
$$= 1.41452$$

$(AC)_0 = \sqrt{2}$ $\qquad AC - (AC)_0 = 307\times10^{-6}$ ◀

or use calculus as follows:



Label sides using *a*, *b*, and *c* as shown.

$c^2 = a^2 + b^2$

Obtain differentials. $\qquad 2c\,dc = 2a\,da + 2b\,dc$

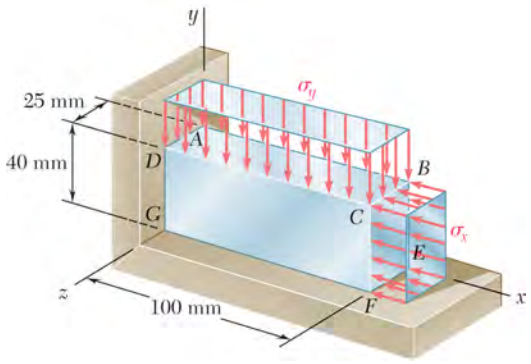
from which $\qquad dc = \frac{a}{c}da + \frac{b}{c}dc$

But $a = 100$ in., $b = 1.00$ in., $c = \sqrt{2}$ in.

$da = \delta_{AB} = 351.72\times10^{-6}$ in., $\quad db = \delta_{BC} = 82.8\times10^{-6}$ in.

$$\delta_{AC} = dc = \frac{1.00}{\sqrt{2}}(351.7\times10^{-6}) + \frac{1.00}{\sqrt{2}}(82.8\times10^{-6})$$
$$= 307\times10^{-6}\text{ in.}$$

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PROBLEM 2.70

The block shown is made of a magnesium alloy, for which $E = 45$ GPa and $\nu = 0.35$. Knowing that $\sigma_x = -180$ MPa, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.

SOLUTION

(*a*) $\delta_y = 0 \qquad \varepsilon_y = 0 \qquad \sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$\sigma_y = \nu\sigma_x = (0.35)(-180\times10^6)$$

$$= -63\times10^6 \text{ Pa} \qquad \sigma_y = -63.0 \text{ MPa} \blacktriangleleft$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x - \nu\sigma_y) = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{(0.35)(-243\times10^6)}{45\times10^9} = +1.890\times10^{-3}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_Z) = \frac{\sigma_x - \nu\sigma_y}{E} = -\frac{157.95\times10^6}{45\times10^9} = -3.510\times10^{-3}$$

(*b*) $A_0 = L_x L_z$

$$A = L_x(1+\varepsilon_x)L_z(1+\varepsilon_z) = L_x L_z(1+\varepsilon_x+\varepsilon_z+\varepsilon_x\varepsilon_z)$$

$$\Delta A = A - A_0 = L_x L_z(\varepsilon_x+\varepsilon_z+\varepsilon_x\varepsilon_z) \approx L_x L_z(\varepsilon_x+\varepsilon_z)$$

$$\Delta A = (100 \text{ mm})(25 \text{ mm})(-3.510\times10^{-3} + 1.890\times10^{-3}) \qquad \Delta A = -4.05 \text{ mm}^2 \blacktriangleleft$$

(*c*) $V_0 = L_x L_y L_z$

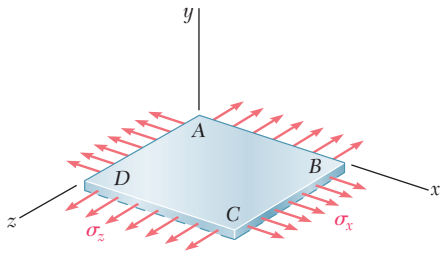
$$V = L_x(1+\varepsilon_x)L_y(1+\varepsilon_y)L_z(1+\varepsilon_z)$$

$$= L_x L_y L_z(1+\varepsilon_x+\varepsilon_y+\varepsilon_z+\varepsilon_x\varepsilon_y+\varepsilon_y\varepsilon_z+\varepsilon_z\varepsilon_x+\varepsilon_x\varepsilon_y\varepsilon_z)$$

$$\Delta V = V - V_0 = L_x L_y L_z(\varepsilon_x+\varepsilon_y+\varepsilon_z+\text{small terms})$$

$$\Delta V = (100)(40)(25)(-3.510\times10^{-3} + 0 + 1.890\times10^{-3}) \qquad \Delta V = -162.0 \text{ mm}^3 \blacktriangleleft$$

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PROBLEM 2.71

The homogeneous plate *ABCD* is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the *x* direction must be zero, that is, $\varepsilon_x = 0$. Denoting by *E* the modulus of elasticity and by *v* Poisson's ratio, determine (*a*) the required magnitude of σ_x, (*b*) the ratio σ_0 / ε_z.

SOLUTION

$$\sigma_z = \sigma_0, \quad \sigma_y = 0, \quad \varepsilon_x = 0$$

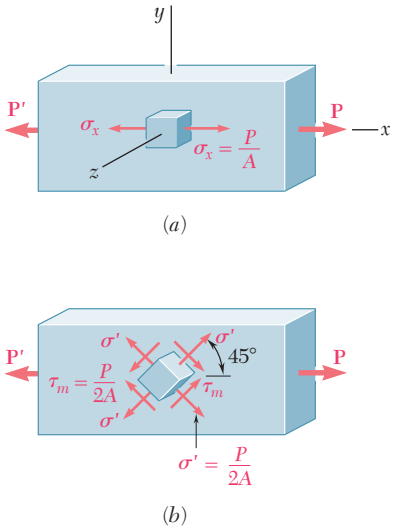
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}(\sigma_x - \nu\sigma_0)$$

(*a*) $\sigma_x = \nu\sigma_0$ ◀

(*b*) $\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{E}(-\nu^2\sigma_0 - 0 + \sigma_0) = \frac{1-\nu^2}{E}\sigma_0$

$\frac{\sigma_0}{\varepsilon_z} = \frac{E}{1-\nu^2}$ ◀

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PROBLEM 2.72

For a member under axial loading, express the normal strain ε' in a direction forming an angle of 45° with the axis of the load in terms of the axial strain ε_x by (*a*) comparing the hypotenuses of the triangles shown in Fig. 2.43, which represent, respectively, an element before and after deformation, (*b*) using the values of the corresponding stresses of σ' and σ_x shown in Fig. 1.38, and the generalized Hooke's law.

SOLUTION

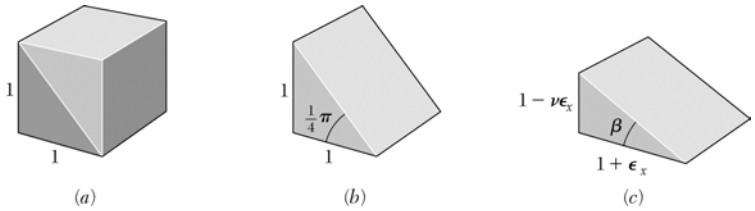


Figure 2.49

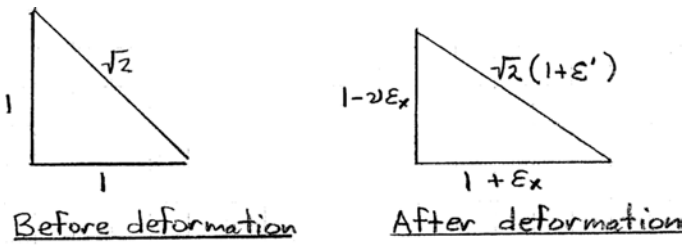
(*a*) $$[\sqrt{2}(1+\varepsilon')]^2 = (1+\varepsilon_x)^2 + (1-\nu\varepsilon_x)^2$$

$$2(1+2\varepsilon'+\varepsilon'^2) = 1+2\varepsilon_x+\varepsilon_x^2+1-2\nu\varepsilon_x+\nu^2\varepsilon_x^2$$

$$4\varepsilon'+2\varepsilon'^2 = 2\varepsilon_x+\varepsilon_x^2-2\nu\varepsilon_x+\nu^2\varepsilon_x^2$$

Neglect squares as small. $$4\varepsilon' = 2\varepsilon_x - 2\nu\varepsilon_x$$

$$\varepsilon' = \frac{1-\nu}{2}\varepsilon_x \blacktriangleleft$$

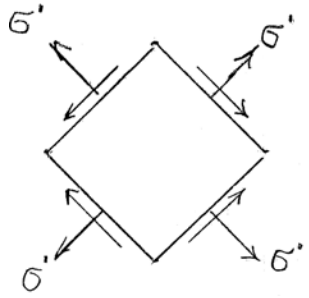


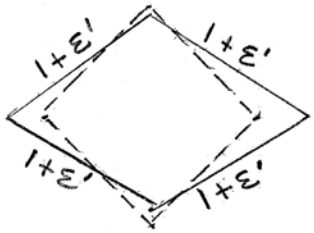
(A) **(B)**

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PROBLEM 2.72 ***(Continued)***

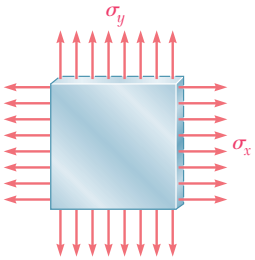
(*b*)
$$\varepsilon' = \frac{\sigma'}{E} - \frac{\nu\sigma'}{E}$$
$$= \frac{1-\nu}{E} \cdot \frac{P}{2A}$$
$$= \frac{1-\nu}{2E}\sigma_x$$
$$= \frac{1-\nu}{2}\varepsilon_x$$





◀

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PROBLEM 2.73

In many situations, it is known that the normal stress in a given direction is zero. For example, $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress*, show that if the strains ε_x and ε_y have been determined experimentally, we can express σ_x, σ_y, and ε_z as follows:

$$\sigma_x = E\frac{\varepsilon_x + \nu\varepsilon_y}{1-\nu^2} \qquad \sigma_y = E\frac{\varepsilon_y + \nu\varepsilon_x}{1-\nu^2} \qquad \varepsilon_z = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

SOLUTION

$$\sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \tag{1}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y) \tag{2}$$

Multiplying (2) by ν and adding to (1),

$$\varepsilon_x + \nu\varepsilon_y = \frac{1-\nu^2}{E}\sigma_x \qquad \text{or} \qquad \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \quad \blacktriangleleft$$

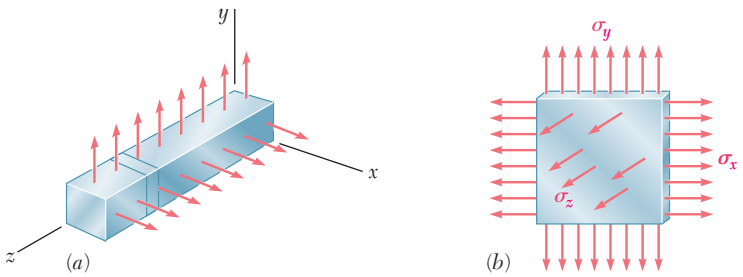
Multiplying (1) by ν and adding to (2),

$$\varepsilon_y + \nu\varepsilon_x = \frac{1-\nu^2}{E}\sigma_y \qquad \text{or} \qquad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) \quad \blacktriangleleft$$

$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) = -\frac{\nu}{\cancel{E}}\cdot\frac{\cancel{E}}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y + \varepsilon_y + \nu\varepsilon_x)$$

$$= -\frac{\nu(1+\nu)}{1-\nu^2}(\varepsilon_x + \varepsilon_y) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) \quad \blacktriangleleft$$

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PROBLEM 2.74

In many situations, physical constraints prevent strain from occurring in a given direction. For example, $\varepsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for this situation, which is known as *plane strain*, we can express σ_z, ε_x, and ε_y as follows:

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\varepsilon_x = \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y]$$

$$\varepsilon_y = \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x]$$

SOLUTION

$$\varepsilon_z = 0 = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) \quad \text{or} \quad \sigma_z = \nu(\sigma_x + \sigma_y) \qquad \blacktriangleleft$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{E}[\sigma_x - \nu\sigma_y - \nu^2(\sigma_x + \sigma_y)]$$

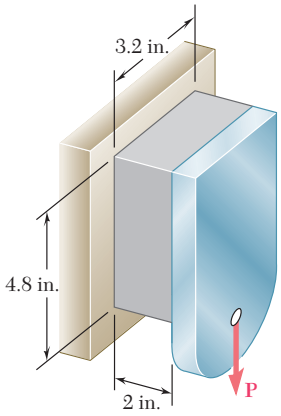
$$= \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y] \qquad \blacktriangleleft$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{E}[-\nu\sigma_x + \sigma_y - \nu^2(\sigma_x + \sigma_y)]$$

$$= \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x] \qquad \blacktriangleleft$$

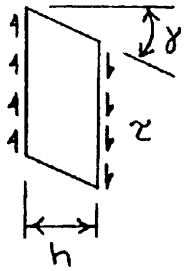
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PROBLEM 2.75

The plastic block shown is bonded to a rigid support and to a vertical plate to which a 55-kip load **P** is applied. Knowing that for the plastic used $G = 150$ ksi, determine the deflection of the plate.

SOLUTION



$$A = (3.2)(4.8) = 15.36 \text{ in}^2$$

$$P = 55 \times 10^3 \text{ lb}$$

$$\tau = \frac{P}{A} = \frac{55 \times 10^3}{15.36} = 3580.7 \text{ psi}$$

$$G = 150 \times 10^3 \text{ psi}$$

$$\gamma = \frac{\tau}{G} = \frac{3580.7}{150 \times 10^3} = 23.871 \times 10^{-3}$$

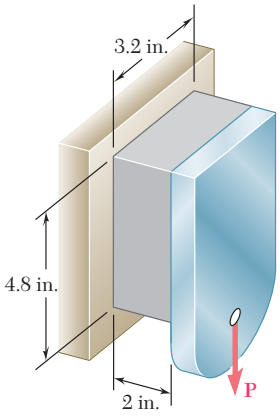
$$h = 2 \text{ in.}$$

$$\delta = h\gamma = (2)(23.871 \times 10^{-3})$$

$$= 47.7 \times 10^{-3} \text{ in.}$$

$$\delta = 0.0477 \text{ in.} \downarrow \blacktriangleleft$$

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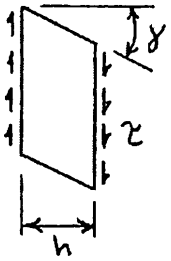


PROBLEM 2.76

What load **P** should be applied to the plate of Prob. 2.75 to produce a $\frac{1}{16}$-in. deflection?

PROBLEM 2.75 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 55-kip load **P** is applied. Knowing that for the plastic used $G = 150$ ksi, determine the deflection of the plate.

SOLUTION



$$\delta = \frac{1}{16}\text{ in.} = 0.0625\text{ in.}$$

$$h = 2\text{ in.}$$

$$\gamma = \frac{\delta}{h} = \frac{0.0625}{2} = 0.03125$$

$$G = 150 \times 10^3\text{ psi}$$

$$\tau = G\gamma = (150 \times 10^3)(0.03125)$$

$$= 4687.5\text{ psi}$$

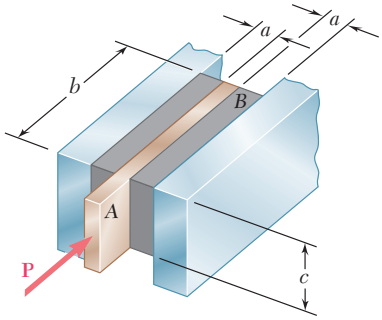
$$A = (3.2)(4.8) = 15.36\text{ in}^2$$

$$P = \tau A = (4687.5)(15.36)$$

$$= 72.0 \times 10^3\text{ lb}$$

72.0 kips ◀

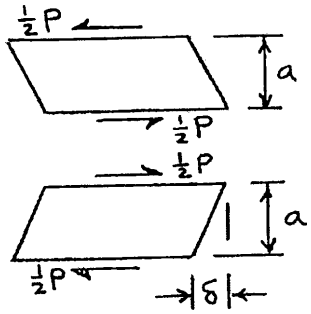
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PROBLEM 2.77

Two blocks of rubber with a modulus of rigidity $G = 12$ MPa are bonded to rigid supports and to a plate AB. Knowing that $c = 100$ mm and $P = 45$ kN, determine the smallest allowable dimensions a and b of the blocks if the shearing stress in the rubber is not to exceed 1.4 MPa and the deflection of the plate is to be at least 5 mm.

SOLUTION



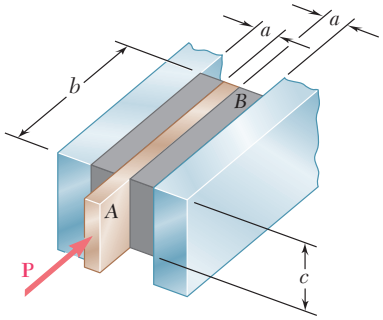
Shearing strain: $\gamma = \dfrac{\delta}{a} = \dfrac{\tau}{G}$

$$a = \frac{G\delta}{\tau} = \frac{(12\times10^{6}\ \text{Pa})(0.005\ \text{m})}{1.4\times10^{6}\ \text{Pa}} = 0.0429\ \text{m} \qquad a = 42.9\ \text{mm} \blacktriangleleft$$

Shearing stress: $\tau = \dfrac{\frac{1}{2}P}{A} = \dfrac{P}{2bc}$

$$b = \frac{P}{2c\tau} = \frac{45\times10^{3}\ \text{N}}{2(0.1\ \text{m})(1.4\times10^{6}\ \text{Pa})} = 0.1607\ \text{m} \qquad b = 160.7\ \text{mm} \blacktriangleleft$$

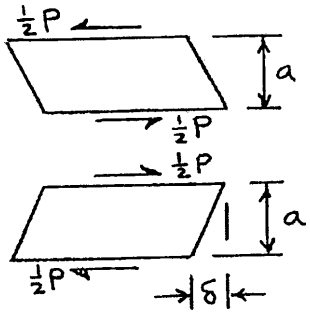
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PROBLEM 2.78

Two blocks of rubber with a modulus of rigidity $G = 10$ MPa are bonded to rigid supports and to a plate AB. Knowing that $b = 200$ mm and $c = 125$ mm, determine the largest allowable load P and the smallest allowable thickness a of the blocks if the shearing stress in the rubber is not to exceed 1.5 MPa and the deflection of the plate is to be at least 6 mm.

SOLUTION



Shearing stress: $\tau = \dfrac{\frac{1}{2}P}{A} = \dfrac{P}{2bc}$

$$P = 2bc\tau = 2(0.2\text{ m})(0.125\text{ m})(1.5\times10^{3}\text{ kPa})$$

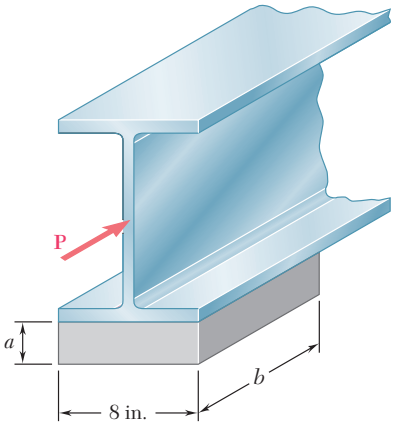
$P = 75.0$ kN ◀

Shearing strain: $\gamma = \dfrac{\delta}{a} = \dfrac{\tau}{G}$

$$a = \frac{G\delta}{\tau} = \frac{(10\times10^{6}\text{ Pa})(0.006\text{ m})}{1.5\times10^{6}\text{ Pa}} = 0.04\text{ m}$$

$a = 40.0$ mm ◀

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PROBLEM 2.79

An elastomeric bearing ($G = 130$ psi) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than $\frac{3}{8}$ in. when a 5-kip lateral load is applied as shown. Knowing that the maximum allowable shearing stress is 60 psi, determine (*a*) the smallest allowable dimension *b*, (*b*) the smallest required thickness *a*.

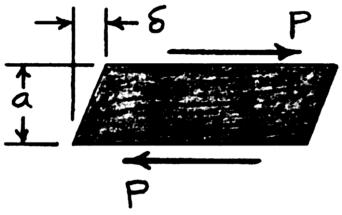
SOLUTION

Shearing force: $P = 5 \text{ kips} = 5000 \text{ lb}$

Shearing stress: $\tau = 60 \text{ psi}$

$$\tau = \frac{P}{A}, \quad \text{or} \quad A = \frac{P}{\tau} = \frac{5000}{60} = 83.333 \text{ in}^2$$

and $A = (8 \text{ in.})(b)$

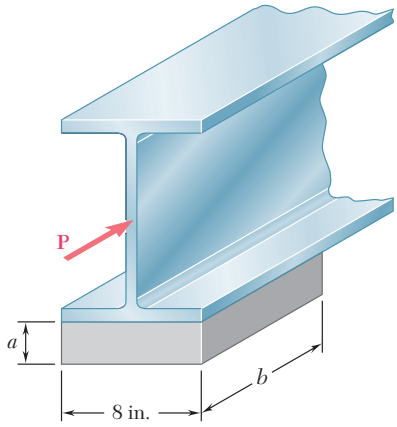


(*a*) $$b = \frac{A}{8} = \frac{83.333}{8} = 10.4166 \text{ in.}$$ $b = 10.42$ in. ◀

$$\gamma = \frac{\tau}{\sigma} = \frac{60}{130} = 461.54 \times 10^{-3} \text{ rad}$$

(*b*) But $\gamma = \frac{\delta}{a}$, or $$a = \frac{\delta}{\gamma} = \frac{0.375 \text{ in.}}{461.54 \times 10^{-3}}$$ $a = 0.813$ in. ◀

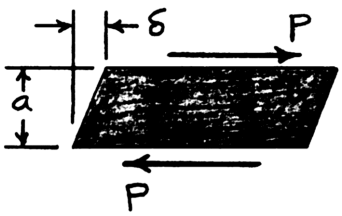
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PROBLEM 2.80

For the elastomeric bearing in Prob. 2.79 with $b = 10$ in. and $a = 1$ in., determine the shearing modulus G and the shear stress τ for a maximum lateral load $P = 5$ kips and a maximum displacement $\delta = 0.4$ in.

SOLUTION



Shearing force: $P = 5 \text{ kips} = 5000 \text{ lb}$

Area: $A = (8 \text{ in.})(10 \text{ in.}) = 80 \text{ in}^2$

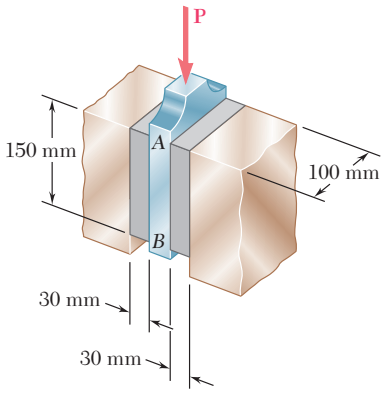
Shearing stress:

$$\tau = \frac{P}{A} = \frac{5000}{80} \qquad \tau = 62.5 \text{ psi} \blacktriangleleft$$

Shearing strain: $$\gamma = \frac{\delta}{a} = \frac{0.4 \text{ in.}}{1 \text{ in.}} = 0.400 \text{ rad}$$

Shearing modulus: $$G = \frac{\tau}{\gamma} = \frac{62.5}{0.400} \qquad G = 156.3 \text{ psi} \blacktriangleleft$$

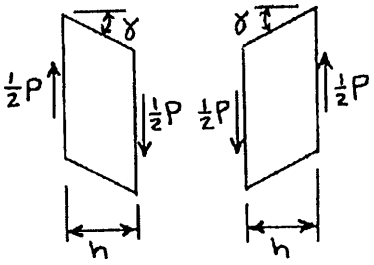
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PROBLEM 2.81

A vibration isolation unit consists of two blocks of hard rubber bonded to a plate AB and to rigid supports as shown. Knowing that a force of magnitude $P = 25\text{ kN}$ causes a deflection $\delta = 1.5\text{ mm}$ of plate AB, determine the modulus of rigidity of the rubber used.

SOLUTION



$$F = \frac{1}{2}P = \frac{1}{2}(25\times10^{3}\text{ N}) = 12.5\times10^{3}\text{ N}$$

$$\tau = \frac{F}{A} = \frac{(12.5\times10^{3}\text{ N})}{(0.15\text{ m})(0.1\text{ m})} = 833.33\times10^{3}\text{ Pa}$$

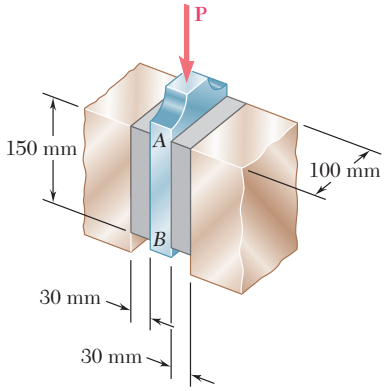
$$\delta = 1.5\times10^{-3}\text{ m} \qquad h = 0.03\text{ m}$$

$$\gamma = \frac{\delta}{h} = \frac{1.5\times10^{-3}}{0.03} = 0.05$$

$$G = \frac{\tau}{\gamma} = \frac{833.33\times10^{3}}{0.05} = 16.67\times10^{6}\text{ Pa}$$

$$G = 16.67\text{ MPa} \blacktriangleleft$$

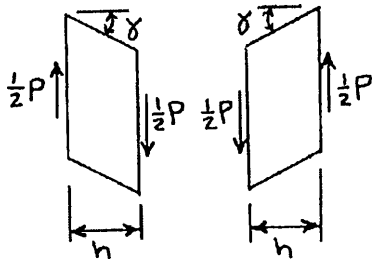
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PROBLEM 2.82

A vibration isolation unit consists of two blocks of hard rubber with a modulus of rigidity $G = 19$ MPa bonded to a plate AB and to rigid supports as shown. Denoting by P the magnitude of the force applied to the plate and by δ the corresponding deflection, determine the effective spring constant, $k = P/\delta$, of the system.

SOLUTION



Shearing strain: $\gamma = \dfrac{\delta}{h}$

Shearing stress: $\tau = G\gamma = \dfrac{G\delta}{h}$

Force: $\dfrac{1}{2}P = A\tau = \dfrac{GA\delta}{h}$ or $P = \dfrac{2GA\delta}{h}$

Effective spring constant: $k = \dfrac{P}{\delta} = \dfrac{2GA}{h}$

with $A = (0.15)(0.1) = 0.015\ \text{m}^2 \quad h = 0.03\ \text{m}$

$$k = \frac{2(19 \times 10^6\ \text{Pa})(0.015\ \text{m}^2)}{0.03\ \text{m}} = 19.00 \times 10^6\ \text{N/m}$$

$$k = 19.00 \times 10^3\ \text{kN/m} \blacktriangleleft$$

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PROBLEM 2.83*

A 6-in.-diameter solid steel sphere is lowered into the ocean to a point where the pressure is 7.1 ksi (about 3 miles below the surface). Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine (*a*) the decrease in diameter of the sphere, (*b*) the decrease in volume of the sphere, (*c*) the percent increase in the density of the sphere.

SOLUTION

For a solid sphere, $V_0 = \frac{\pi}{6} d_0^3$

$$= \frac{\pi}{6}(6.00)^3$$

$$= 113.097 \text{ in}^3$$

$$\sigma_x = \sigma_y = \sigma_z = -p$$

$$= -7.1 \times 10^3 \text{ psi}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= -\frac{(1-2\nu)p}{E} = -\frac{(0.4)(7.1\times10^3)}{29\times10^6}$$

$$= -97.93 \times 10^{-6}$$

Likewise, $\varepsilon_y = \varepsilon_z = -97.93 \times 10^{-6}$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = -293.79 \times 10^{-6}$$

(*a*) $-\Delta d = -d_0 \varepsilon_x = -(6.00)(-97.93 \times 10^{-6}) = 588 \times 10^{-6}$ in. $\quad -\Delta d = 588 \times 10^{-6}$ in. ◀

(*b*) $-\Delta V = -V_0 e = -(113.097)(-293.79 \times 10^{-6}) = 33.2 \times 10^{-3}$ in^3 $\quad -\Delta V = 33.2 \times 10^{-3}$ in^3 ◀

(*c*) Let m = mass of sphere. m = constant.

$$m = \rho_0 V_0 = \rho V = \rho V_0 (1+e)$$

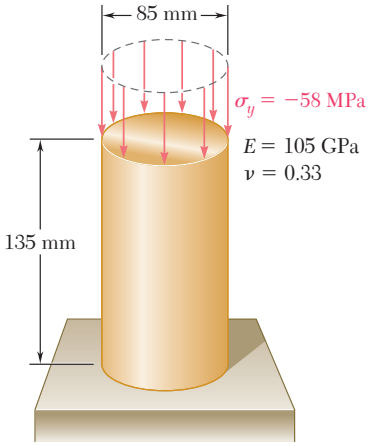
$$\frac{\rho - \rho_0}{\rho_0} = \frac{\rho}{\rho_0} - 1 = \frac{m}{V_0(1+e)} \times \frac{V_0}{m} - 1 = \frac{1}{1+e} - 1$$

$$= (1 - e + e^2 - e^3 + \cdots) - 1 = -e + e^2 - e^3 + \cdots$$

$$\approx -e = 293.79 \times 10^{-6}$$

$$\frac{\rho - \rho_0}{\rho_0} \times 100\% = (293.79 \times 10^{-6})(100\%) \quad 0.0294\% \text{ ◀}$$

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PROBLEM 2.84*

(*a*) For the axial loading shown, determine the change in height and the change in volume of the brass cylinder shown. (*b*) Solve part *a*, assuming that the loading is hydrostatic with $\sigma_x = \sigma_y = \sigma_z = -70$ MPa.

SOLUTION

$$h_0 = 135 \text{ mm} = 0.135 \text{ m}$$

$$A_0 = \frac{\pi}{4} d_0^2 = \frac{\pi}{4}(85)^2 = 5.6745 \times 10^3 \text{ mm}^2 = 5.6745 \times 10^{-3} \text{ m}^2$$

$$V_0 = A_0 h_0 = 766.06 \times 10^3 \text{ mm}^3 = 766.06 \times 10^{-6} \text{ m}^3$$

(*a*) $\sigma_x = 0, \quad \sigma_y = -58 \times 10^6 \text{ Pa}, \quad \sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{\sigma_y}{E} = -\frac{58 \times 10^6}{105 \times 10^9} = -552.38 \times 10^{-6}$$

$$\Delta h = h_0 \varepsilon_y = (135 \text{ mm})(-552.38 \times 10^{-6}) \qquad \Delta h = -0.0746 \text{ mm} \blacktriangleleft$$

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(1-2\nu)\sigma_y}{E} = \frac{(0.34)(-58 \times 10^6)}{105 \times 10^9} = -187.81 \times 10^{-6}$$

$$\Delta V = V_0 e = (766.06 \times 10^3 \text{ mm}^3)(-187.81 \times 10^{-6}) \qquad \Delta V = -143.9 \text{ mm}^3 \blacktriangleleft$$

(*b*) $\sigma_x = \sigma_y = \sigma_z = -70 \times 10^6 \text{ Pa} \qquad \sigma_x + \sigma_y + \sigma_z = -210 \times 10^6 \text{ Pa}$

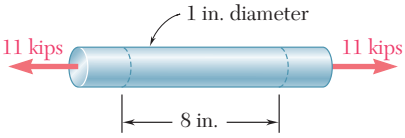
$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1-2\nu}{E}\sigma_y = \frac{(0.34)(-70 \times 10^6)}{105 \times 10^9} = -226.67 \times 10^{-6}$$

$$\Delta h = h_0 \varepsilon_y = (135 \text{ mm})(-226.67 \times 10^{-6}) \qquad \Delta h = -0.0306 \text{ mm} \blacktriangleleft$$

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(0.34)(-210 \times 10^6)}{105 \times 10^9} = -680 \times 10^{-6}$$

$$\Delta V = V_0 e = (766.06 \times 10^3 \text{ mm}^3)(-680 \times 10^{-6}) \qquad \Delta V = -521 \text{ mm}^3 \blacktriangleleft$$

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PROBLEM 2.85*

Determine the dilatation e and the change in volume of the 8-in. length of the rod shown if (a) the rod is made of steel with $E = 29 \times 10^6$ psi and $\nu = 0.30$, (b) the rod is made of aluminum with $E = 10.6 \times 10^6$ psi and $\nu = 0.35$.

SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1)^2 = 0.78540 \text{ in}^2$$

$$P = 11 \times 10^3 \text{ lb}$$

Stresses : $$\sigma_x = \frac{P}{A} = \frac{11 \times 10^3}{0.78540} = 14.0056 \times 10^3 \text{ psi}$$

$$\sigma_y = \sigma_z = 0$$

(a) Steel. $E = 29 \times 10^6$ psi $\nu = 0.30$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x}{E} = \frac{14.0056 \times 10^3}{29 \times 10^6} = 482.95 \times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = -\frac{\nu\sigma_x}{E} = -\nu\varepsilon_x = -(0.30)(482.95 \times 10^{-6})$$

$$= -144.885 \times 10^{-6}$$

$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = -\frac{\nu\sigma_x}{E} = \varepsilon_y = -144.885 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 193.2 \times 10^{-6} \quad \blacktriangleleft$$

$$\Delta v = ve = \Delta L e = (0.78540)(8)(193.2 \times 10^{-6}) = 1.214 \times 10^{-3} \text{ in}^3 \quad \blacktriangleleft$$

(b) Aluminum. $E = 10.6 \times 10^6$ psi $\nu = 0.35$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{14.0056 \times 10^3}{10.6 \times 10^6} = 1.32128 \times 10^{-3}$$

$$\varepsilon_y = -\nu\varepsilon_x = -(0.35)(1.32128 \times 10^{-3}) = -462.45 \times 10^{-6}$$

$$\varepsilon_z = \varepsilon_y = -462.45 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 396 \times 10^{-6} \quad \blacktriangleleft$$

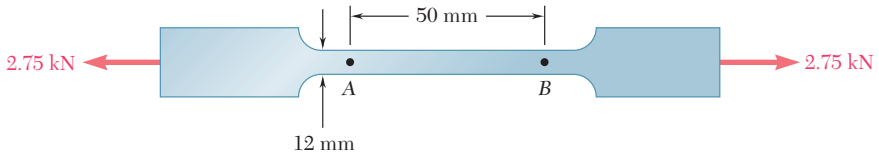
$$\Delta v = ve = \Delta L e = (0.78540)(8)(396 \times 10^{-6}) = 2.49 \times 10^{-3} \text{ in}^3 \quad \blacktriangleleft$$

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PROBLEM 2.86

Determine the change in volume of the 50-mm gage length segment *AB* in Prob. 2.64 (*a*) by computing the dilatation of the material, (*b*) by subtracting the original volume of portion *AB* from its final volume.

PROBLEM 2.64 A 2.75-kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate (E = 200 GPa, ν = 0.30). Determine the resulting change (*a*) in the 50-mm gage length, (*b*) in the width of portion *AB* of the test coupon, (*c*) in the thickness of portion *AB*, (*d*) in the cross-sectional area of portion *AB*.



SOLUTION

(*a*) $A_0 = (12)(1.6) = 19.2 \text{ mm}^2 = 19.2 \times 10^{-6} \text{ m}^2$

Volume $V_0 = L_0 A_0 = (50)(19.2) = 960 \text{ mm}^3$

$$\sigma_x = \frac{P}{A_0} = \frac{2.75 \times 10^3}{19.2 \times 10^{-6}} = 143.229 \times 10^6 \text{ Pa} \qquad \sigma_y = \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \upsilon\sigma_y - \upsilon\sigma_z) = \frac{\sigma_x}{E} = \frac{143.229 \times 10^6}{200 \times 10^9} = 716.15 \times 10^{-6}$$

$$\varepsilon_y = \varepsilon_z = -\upsilon\varepsilon_x = -(0.30)(716.15 \times 10^{-3}) = -214.84 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 286.46 \times 10^{-6}$$

$$\Delta v = v_0 e = (960)(286.46 \times 10^{-6}) = 0.275 \text{ mm}^3 \quad \blacktriangleleft$$

(*b*) From the solution to problem 2.64,

$$\delta_x = 0.035808 \text{ mm} \qquad \delta_y = -0.0025781 \qquad \delta_z = -0.00034374 \text{ mm}$$

The dimensions when under the 2.75-kN load are

Length: $L = L_0 + \delta_x = 50 + 0.035808 = 50.035808 \text{ mm}$

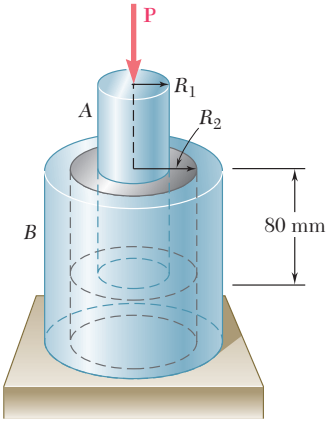
Width: $w = w_0 + \delta_y = 12 - 0.0025781 = 11.997422 \text{ mm}$

Thickness: $t = t_0 + \delta_z = 1.6 - 0.00034374 = 1.599656 \text{ mm}$

Volume: $V = Lwt = (50.03581)(11.997422)(1.599656) = 960.275 \text{ mm}^3$

$$\Delta V = V - V_0 = 960.275 - 960 = 0.275 \text{ mm}^3 \quad \blacktriangleleft$$

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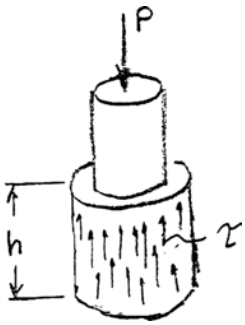


PROBLEM 2.87*

A vibration isolation support consists of a rod *A* of radius $R_1 = 10$ mm and a tube *B* of inner radius $R_2 = 25$ mm bonded to an 80-mm-long hollow rubber cylinder with a modulus of rigidity *G* = 12 MPa. Determine the largest allowable force **P** that can be applied to rod *A* if its deflection is not to exceed 2.50 mm.

SOLUTION

Let *r* be a radial coordinate. Over the hollow rubber cylinder, $R_1 \le r \le R_2$.



Shearing stress τ acting on a cylindrical surface of radius *r* is

$$\tau = \frac{P}{A} = \frac{P}{2\pi rh}$$

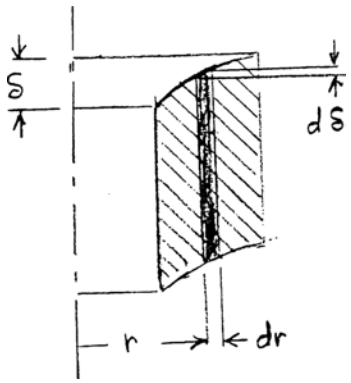
The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi Ghr}$$

Shearing deformation over radial length *dr*:

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi Gh}\frac{dr}{r}$$



Total deformation.

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi Gh}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi Gh}\ln r\Big|_{R_1}^{R_2} = \frac{P}{2\pi Gh}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi Gh}\ln\frac{R_2}{R_1} \quad \text{or} \quad P = \frac{2\pi Gh\delta}{\ln(R_2/R_1)}$$

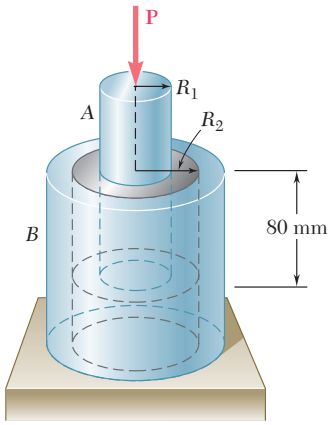
Data: $R_1 = 10$ mm $= 0.010$ m, $R_2 = 25$ mm $= 0.025$ m, $h = 80$ mm $= 0.080$ m

$$G = 12\times10^6 \text{ Pa} \qquad \delta = 2.50\times10^{-3} \text{ m}$$

$$P = \frac{(2\pi)(12\times10^6)(0.080)(2.50\times10^{-3})}{\ln(0.025/0.010)} = 16.46\times10^3 \text{ N}$$

16.46 kN ◀

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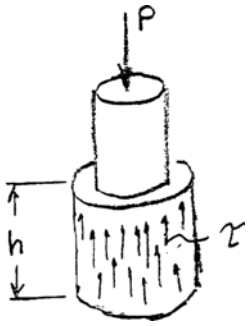


PROBLEM 2.88

A vibration isolation support consists of a rod *A* of radius R_1 and a tube *B* of inner radius R_2 bonded to a 80-mm-long hollow rubber cylinder with a modulus of rigidity $G = 10.93$ MPa. Determine the required value of the ratio R_2/R_1 if a 10-kN force **P** is to cause a 2-mm deflection of rod *A*.

SOLUTION

Let *r* be a radial coordinate. Over the hollow rubber cylinder, $R_1 \le r \le R_2$.



Shearing stress τ acting on a cylindrical surface of radius *r* is

$$\tau = \frac{P}{A} = \frac{P}{2\pi rh}$$

The shearing strain is

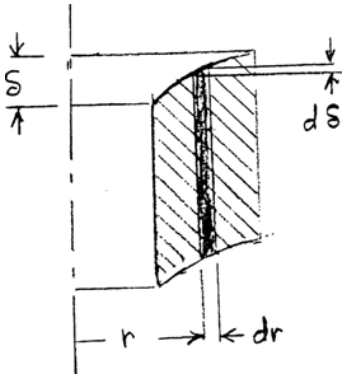
$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi Ghr}$$

Shearing deformation over radial length *dr*:

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr$$

$$dr\delta = \frac{P}{2\pi Gh}\frac{dr}{r}$$



Total deformation.

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi Gh}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi Gh}\ln r\,\Big|_{R_1}^{R_2} = \frac{P}{2\pi Gh}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi Gh}\ln\frac{R_2}{R_1}$$

$$\ln\frac{R_2}{R_1} = \frac{2\pi Gh\delta}{P} = \frac{(2\pi)(10.93\times10^6)(0.080)(0.002)}{10.10^3} = 1.0988$$

$$\frac{R_2}{R_1} = \exp(1.0988) = 3.00$$

$R_2/R_1 = 3.00$ ◀

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PROBLEM 2.89*

The material constants E, G, k, and v are related by Eqs. (2.33) and (2.43). Show that any one of these constants may be expressed in terms of any other two constants. For example, show that (*a*) $k = GE/(9G - 3E)$ and (*b*) $v = (3k - 2G)/(6k + 2G)$.

SOLUTION

$$k = \frac{E}{3(1-2v)} \quad \text{and} \quad G = \frac{E}{2(1+v)}$$

(*a*) $$1 + v = \frac{E}{2G} \quad \text{or} \quad v = \frac{E}{2G} - 1$$

$$k = \frac{E}{3\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]} = \frac{2EG}{3[2G - 2E + 4G]} = \frac{2EG}{18G - 6E}$$

$$k = \frac{EG}{9G - 6E} \blacktriangleleft$$

(*b*) $$\frac{k}{G} = \frac{2(1+v)}{3(1-2v)}$$

$$3k - 6kv = 2G + 2Gv$$

$$3k - 2G = 2G + 6k$$

$$v = \frac{3k - 2G}{6k + 2G} \blacktriangleleft$$

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PROBLEM 2.90*

Show that for any given material, the ratio G/E of the modulus of rigidity over the modulus of elasticity is always less than $\frac{1}{2}$ but more than $\frac{1}{3}$. [*Hint:* Refer to Eq. (2.43) and to Sec. 2.13.]

SOLUTION

$$G = \frac{E}{2(1+\nu)} \qquad \text{or} \qquad \frac{E}{G} = 2(1+\nu)$$

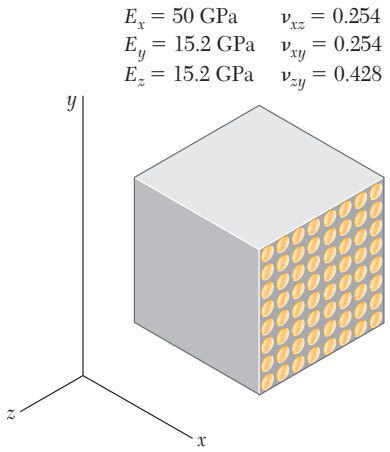
Assume $\nu > 0$ for almost all materials, and $\nu < \frac{1}{2}$ for a positive bulk modulus.

Applying the bounds, $$2 \le \frac{E}{G} < 2\left(1+\frac{1}{2}\right) = 3$$

Taking the reciprocals, $$\frac{1}{2} > \frac{G}{E} > \frac{1}{3}$$

or $$\frac{1}{3} < \frac{G}{E} < \frac{1}{2} \blacktriangleleft$$

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PROBLEM 2.91*

A composite cube with 40-mm sides and the properties shown is made with glass polymer fibers aligned in the *x* direction. The cube is constrained against deformations in the *y* and *z* directions and is subjected to a tensile load of 65 kN in the *x* direction. Determine (*a*) the change in the length of the cube in the *x* direction, (*b*) the stresses σ_x, σ_y, and σ_z.

$$E_x = 50 \text{ GPa} \quad \nu_{xz} = 0.254$$
$$E_y = 15.2 \text{ GPa} \quad \nu_{xy} = 0.254$$
$$E_z = 15.2 \text{ GPa} \quad \nu_{zy} = 0.428$$

SOLUTION

Stress-to-strain equations are

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \tag{1}$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \tag{2}$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \tag{3}$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \tag{4}$$

$$\frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \tag{5}$$

$$\frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \tag{6}$$

The constraint conditions are $\varepsilon_y = 0$ and $\varepsilon_z = 0$.

Using (2) and (3) with the constraint conditions gives

$$\frac{1}{E_y}\sigma_y - \frac{\nu_{zy}}{E_z}\sigma_z = \frac{\nu_{xy}}{E_x}\sigma_x \tag{7}$$

$$-\frac{\nu_{yz}}{E_y}\sigma_y + \frac{1}{E_z}\sigma_z = \frac{V_{xz}}{E_x}\sigma_x \tag{8}$$

$$\frac{1}{15.2}\sigma_y - \frac{0.428}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad \sigma_y - 0.428\sigma_z = 0.077216\sigma_x$$

$$-\frac{0.428}{15.2}\sigma_y + \frac{1}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad -0.428\sigma_y + \sigma_z = 0.077216\sigma_x$$

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PROBLEM 2.91* *(Continued)*

Solving simultaneously, $\sigma_y = \sigma_z = 0.134993\sigma_x$

Using (4) and (5) in (1), $\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{xy}}{E_x}\sigma_y - \frac{\nu_{xz}}{E}\sigma_z$

$$E_x = \frac{1}{E_x}[1-(0.254)(0.134993)-(0.254)(0.134993)]\sigma_x$$

$$= \frac{0.93142\sigma_x}{E_x}$$

$$A = (40)(40) = 1600 \text{ mm}^2 = 1600\times10^{-6} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{65\times10^3}{1600\times10^{-6}} = 40.625\times10^6 \text{ Pa}$$

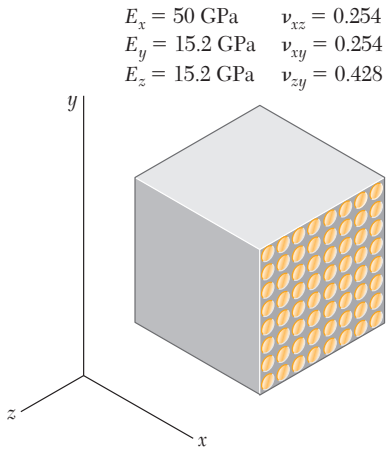
$$\varepsilon_x = \frac{(0.93142)(40.625\times10^3)}{50\times10^9} = 756.78\times10^{-6}$$

(*a*) $\delta_x = L_x\varepsilon_x = (40 \text{ mm})(756.78\times10^{-6})$ $\delta_x = 0.0303 \text{ mm}$ ◀

(*b*) $\sigma_x = 40.625\times10^6 \text{ Pa}$ $\sigma_x = 40.6 \text{ MPa}$ ◀

$\sigma_y = \sigma_z = (0.134993)(40.625\times10^6) = 5.48\times10^6 \text{ Pa}$ $\sigma_y = \sigma_z = 5.48 \text{ MPa}$ ◀

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PROBLEM 2.92*

The composite cube of Prob. 2.91 is constrained against deformation in the *z* direction and elongated in the *x* direction by 0.035 mm due to a tensile load in the *x* direction. Determine (*a*) the stresses σ_x, σ_y, and σ_z and (*b*) the change in the dimension in the *y* direction.

$$E_x = 50 \text{ GPa} \quad \nu_{xz} = 0.254$$
$$E_y = 15.2 \text{ GPa} \quad \nu_{xy} = 0.254$$
$$E_z = 15.2 \text{ GPa} \quad \nu_{zy} = 0.428$$

SOLUTION

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \tag{1}$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \tag{2}$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \tag{3}$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \tag{4}$$

$$\frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \tag{5}$$

$$\frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \tag{6}$$

Constraint condition: $\varepsilon_z = 0$

Load condition: $\sigma_y = 0$

From Equation (3),
$$0 = -\frac{\nu_{xz}}{E_x}\sigma_x + \frac{1}{E_z}\sigma_z$$

$$\sigma_z = \frac{\nu_{xz}E_z}{E_x}\sigma_x = \frac{(0.254)(15.2)}{50} = 0.077216\sigma_x$$

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PROBLEM 2.92* *(Continued)*

From Equation (1) with $\sigma_y = 0$,

$$\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{zx}}{E_z}\sigma_z = \frac{1}{E_x}\sigma_x - \frac{\nu_{xz}}{E_x}\sigma_z$$

$$= \frac{1}{E_x}[\sigma_x - 0.254\sigma_z] = \frac{1}{E_x}[1-(0.254)(0.077216)]\sigma_x$$

$$= \frac{0.98039}{E_x}\sigma_x$$

$$\sigma_x = \frac{E_x\varepsilon_x}{0.98039}$$

But $\varepsilon_x = \dfrac{\delta_x}{L_x} = \dfrac{0.035 \text{ mm}}{40 \text{ mm}} = 875\times10^{-6}$

(*a*) $\sigma_x = \dfrac{(50\times10^9)(875\times10^{-6})}{0.98039} = 44.625\times10^3 \text{ Pa}$ $\sigma_x = 44.6 \text{ MPa}$ ◀

$\sigma_y = 0$ ◀

$\sigma_z = (0.077216)(44.625\times10^6) = 3.446\times10^6 \text{ Pa}$ $\sigma_z = 3.45 \text{ MPa}$ ◀

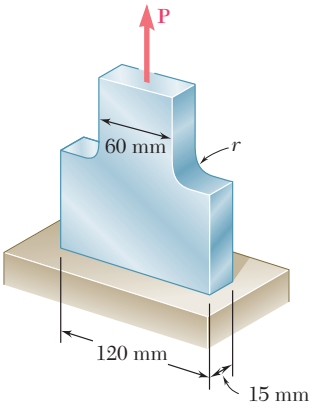
From (2), $\varepsilon_y = \dfrac{\nu_{xy}}{E_x}\sigma_x + \dfrac{1}{E_y}\sigma_y - \dfrac{\nu_{zy}}{E_z}\sigma_z$

$$= -\frac{(0.254)(44.625\times10^6)}{50\times10^9} + 0 - \frac{(0.428)(3.446\times10^6)}{15.2\times10^9}$$

$$= -323.73\times10^{-6}$$

(*b*) $\delta_y = L_y\varepsilon_y = (40 \text{ mm})(-323.73\times10^{-6})$ $\delta_y = -0.0129 \text{ mm}$ ◀

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PROBLEM 2.93

Knowing that, for the plate shown, the allowable stress is 125 MPa, determine the maximum allowable value of P when (a) $r = 12$ mm, (b) $r = 18$ mm.

SOLUTION

$$A = (60)(15) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\frac{D}{d} = \frac{120 \text{ mm}}{60 \text{ mm}} = 2.00$$

(a) $r = 12$ mm $\qquad \dfrac{r}{d} = \dfrac{12 \text{ mm}}{60 \text{ mm}} = 0.2$

From Fig. 2.60b, $\qquad K = 1.92 \qquad \sigma_{\max} = K\dfrac{P}{A}$

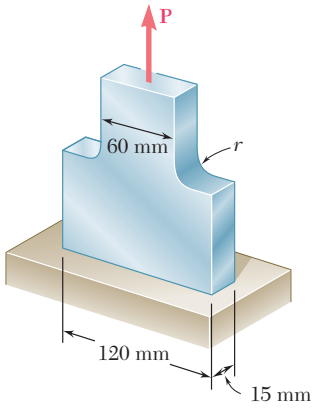
$$P = \frac{A\sigma_{\max}}{K} = \frac{(900 \times 10^{-6})(125 \times 10^{6})}{1.92} = 58.6 \times 10^{3} \text{ N}$$

$$= 58.3 \text{ kN} \blacktriangleleft$$

(b) $r = 18$ mm, $\qquad \dfrac{r}{d} = \dfrac{18 \text{ mm}}{60 \text{ mm}} = 0.30$ $\qquad$ From Fig 2.60b, $\qquad K = 1.75$

$$P = \frac{A\sigma_{\max}}{K} = \frac{(900 \times 10^{-6})(125 \times 10^{6})}{1.75} = 64.3 \times 10^{3} \text{ N} \qquad = 64.3 \text{ kN} \blacktriangleleft$$

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PROBLEM 2.94

Knowing that $P = 38$ kN, determine the maximum stress when (*a*) $r = 10$ mm, (*b*) $r = 16$ mm, (*c*) $r = 18$ mm.

SOLUTION

$$A = (60)(15) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\frac{D}{d} = \frac{10 \text{ mm}}{60 \text{ mm}} = 2.00$$

(*a*) $r = 10$ mm $\qquad \dfrac{r}{d} = \dfrac{10 \text{ mm}}{60 \text{ mm}} = 0.1667$

From Fig. 2.60b, $\qquad K = 2.06 \qquad \sigma_{\max} = \dfrac{KP}{A}$

$$\sigma_{\max} = \frac{(2.06)(38 \times 10^3)}{900 \times 10^{-6}} = 87.0 \times 10^6 \text{ Pa} \qquad = 87.0 \text{ MPa} \blacktriangleleft$$

(*b*) $r = 16$ mm $\qquad \dfrac{r}{d} = \dfrac{16 \text{ mm}}{60 \text{ mm}} = 0.2667$

From Fig. 2.60b, $\qquad K = 1.78$

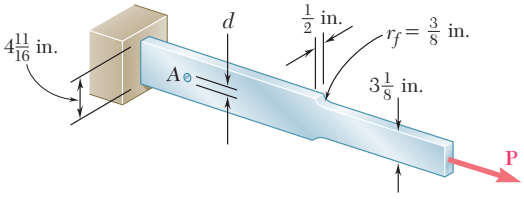
$$\sigma_{\max} = \frac{(1.78)(38 \times 10^3)}{900 \times 10^{-6}} = 75.2 \times 10^6 \text{ Pa} \qquad = 75.2 \text{ MPa} \blacktriangleleft$$

(*c*) $r = 18$ mm, $\qquad \dfrac{r}{d} = \dfrac{18 \text{ mm}}{60 \text{ mm}} = 0.30$

From Fig 2.60b, $\qquad K = 1.75$

$$\sigma_{\max} = \frac{(1.75)(38 \times 10^3)}{900 \times 10^{-6}} = 73.9 \times 10^6 \text{ Pa} \qquad = 73.9 \text{ MPa} \blacktriangleleft$$

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PROBLEM 2.95

A hole is to be drilled in the plate at *A*. The diameters of the bits available to drill the hole range from $\frac{1}{2}$ to $1\frac{1}{2}$ in. in $\frac{1}{4}$-in. increments. If the allowable stress in the plate is 21 ksi, determine (*a*) the diameter *d* of the largest bit that can be used if the allowable load **P** at the hole is to exceed that at the fillets, (*b*) the corresponding allowable load **P**.

SOLUTION

At the fillets: $\dfrac{D}{d} = \dfrac{4.6875}{3.125} = 1.5 \qquad \dfrac{r}{d} = \dfrac{0.375}{3.125} = 0.12$

From Fig. 2.60b, $K = 2.10$

$$A_{\min} = (3.125)(0.5) = 1.5625 \text{ in}^2$$

$$\sigma_{\max} = K\frac{P_{\text{all}}}{A_{\min}} = \sigma_{\text{all}}$$

$$P_{\text{all}} = \frac{A_{\min}\sigma_{\text{all}}}{K} = \frac{(1.5625)(21)}{2.10} = 15.625 \text{ kips}$$

At the hole: $A_{\text{net}} = (D - 2r)t$, K from Fig. 2.60a

$$\sigma_{\max} = K\frac{P}{A_{\text{net}}} = \sigma_{\text{all}} \qquad \therefore \qquad P_{\text{all}} = \frac{A_{\text{net}}\sigma_{\text{all}}}{K}$$

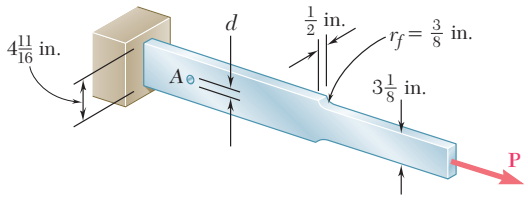
with $D = 4.6875$ in. $\quad t = 0.5$ in. $\quad \sigma_{\text{all}} = 21$ ksi

Hole diam.	r	$d = D - 2r$	$2r/D$	K	A_{net}	P_{all}
0.5 in.	0.25 in.	4.1875 in.	0.107	2.68	2.0938 in^2	16.41 kips
0.75 in.	0.375 in.	3.9375 in.	0.16	2.58	1.96875 in^2	16.02 kips
1 in.	0.5 in.	3.6875 in.	0.213	2.49	1.84375 in^2	15.55 kips
1.25 in.	0.625 in.	3.4375 in.	0.267	2.41	1.71875 in^2	14.98 kips
1.5 in.	0.75 in.	3.1875 in.	0.32	2.34	1.59375 in^2	14.30 kips

(*a*) Largest hole with $P_{\text{all}} > 15.625$ kips is the $\frac{3}{4}$-in.-diameter hole. ◀

(*b*) Allowable load $P_{\text{all}} = 15.63$ kips ◀

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PROBLEM 2.96

(*a*) For $P = 13$ kips and $d = \frac{1}{2}$ in., determine the maximum stress in the plate shown. (*b*) Solve part *a*, assuming that the hole at *A* is not drilled.

SOLUTION

Maximum stress at hole:

Use Fig. 2.60a for values of *K*.

$$\frac{2r}{D} = \frac{0.5}{4.6875} = 0.017, \qquad K = 2.68$$

$$A_{\text{net}} = (0.5)(4.6875 - 0.5) = 2.0938 \text{ in}^2$$

$$\sigma_{\max} = K\frac{P}{A_{\text{net}}} = \frac{(2.68)(13)}{2.0938} = 16.64 \text{ ksi}$$

Maximum stress at fillets:

Use Fig. 2.60b for values of *K*.

$$\frac{r}{d} = \frac{0.375}{3.125} = 0.12 \qquad \frac{D}{d} = \frac{4.6875}{3.125} = 1.5 \qquad K = 2.10$$

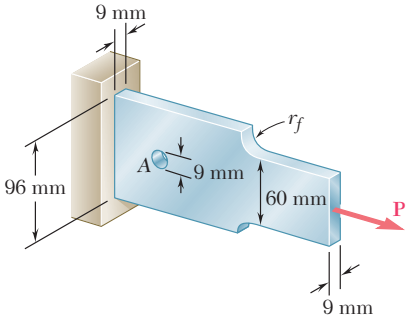
$$A_{\min} = (0.5)(3.125) = 1.5625 \text{ in}^2$$

$$\sigma_{\max} = K\frac{P}{A_{\min}} = \frac{(2.10)(13)}{1.5625} = 17.47 \text{ ksi}$$

(*a*) With hole and fillets: 17.47 ksi ◀

(*b*) Without hole: 17.47 ksi ◀

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PROBLEM 2.97

Knowing that the hole has a diameter of 9 mm, determine (*a*) the radius r_f of the fillets for which the same maximum stress occurs at the hole *A* and at the fillets, (*b*) the corresponding maximum allowable load **P** if the allowable stress is 100 MPa.

SOLUTION

For the circular hole, $r = \left(\frac{1}{2}\right)(9) = 4.5 \text{ mm}$

$$d = 96 - 9 = 87 \text{ mm} \qquad \frac{2r}{D} = \frac{2(4.5)}{96} = 0.09375$$

$$A_{\text{net}} = dt = (0.087 \text{ m})(0.009 \text{ m}) = 783 \times 10^{-6} \text{ m}^2$$

From Fig. 2.60a, $K_{\text{hole}} = 2.72$

$$\sigma_{\max} = \frac{K_{\text{hole}} P}{A_{\text{net}}}$$

$$P = \frac{A_{\text{net}} \sigma_{\max}}{K_{\text{hole}}} = \frac{(783 \times 10^{-6})(100 \times 10^{6})}{2.72} = 28.787 \times 10^{3} \text{ N}$$

(*a*) For fillet, $D = 96 \text{ mm}, d = 60 \text{ mm}$

$$\frac{D}{d} = \frac{96}{60} = 1.60$$

$$A_{\min} = dt = (0.060 \text{ m})(0.009 \text{ m}) = 540 \times 10^{-6} \text{ m}^2$$

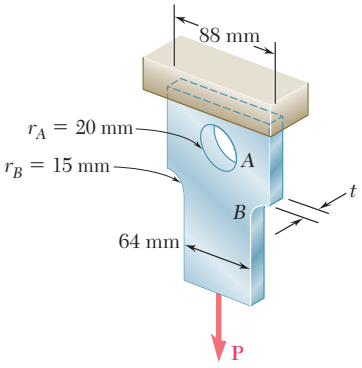
$$\sigma_{\max} = \frac{K_{\text{fillet}} P}{A_{\min}} \quad \therefore \quad K_{\text{fillet}} = \frac{A_{\min} \sigma_{\max}}{P} = \frac{(5.40 \times 10^{-6})(100 \times 10^{6})}{28.787 \times 10^{3}}$$

$$= 1.876$$

From Fig. 2.60b, $\frac{r_f}{d} \approx 0.19 \quad \therefore \quad r_f \approx 0.19d = 0.19(60)$ $\qquad r_f = 11.4 \text{ mm}$ ◀

(*b*) $P = 28.8 \text{ kN}$ ◀

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PROBLEM 2.98

For $P = 100$ kN, determine the minimum plate thickness t required if the allowable stress is 125 MPa.

SOLUTION

At the hole: $r_A = 20$ mm $\quad d_A = 88 - 40 = 48$ mm

$$\frac{2r_A}{D_A} = \frac{2(20)}{88} = 0.455$$

From Fig. 2.60a, $K = 2.20$

$$\sigma_{\max} = \frac{KP}{A_{\text{net}}} = \frac{KP}{d_A t} \quad \therefore \quad t = \frac{KP}{d_A \sigma_{\max}}$$

$$t = \frac{(2.20)(100 \times 10^3 \text{ N})}{(0.048 \text{ m})(125 \times 10^6 \text{ Pa})} = 36.7 \times 10^{-3} \text{ m} = 36.7 \text{ mm}$$

At the fillet: $D = 88$ mm, $\quad d_B = 64$ mm $\quad \dfrac{D}{d_B} = \dfrac{88}{64} = 1.375$

$$r_B = 15 \text{ mm} \qquad \frac{r_B}{d_B} = \frac{15}{64} = 0.2344$$

From Fig. 2.60b, $K = 1.70$

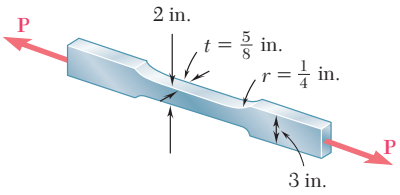
$$\sigma_{\max} = \frac{KP}{A_{\min}} = \frac{KP}{d_B t}$$

$$t = \frac{KP}{d_B \sigma_{\max}} = \frac{(1.70)(100 \times 10^3 \text{ N})}{(0.064 \text{ m})(125 \times 10^6 \text{ Pa})} = 21.25 \times 10^{-3} \text{ m} = 21.25 \text{ mm}$$

The larger value is the required minimum plate thickness.

$t = 36.7$ mm ◀

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PROBLEM 2.99

(*a*) Knowing that the allowable stress is 20 ksi, determine the maximum allowable magnitude of the centric load **P**. (*b*) Determine the percent change in the maximum allowable magnitude of **P** if the raised portions are removed at the ends of the specimen.

SOLUTION

$$\frac{D}{d} = \frac{3}{2} = 1.50 \qquad \frac{r}{d} = \frac{0.250}{2} = 0.125$$

From Fig. 2.60b, $K = 2.08$

$$A_{\min} = td = (0.625)(2) = 1.25 \text{ in}^2$$

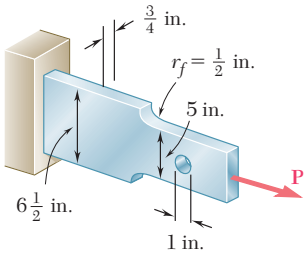
(*a*) $\sigma_{\max} = \dfrac{KP}{A_{\min}} \quad \therefore \quad P = \dfrac{A_{\min}\sigma_{\max}}{K} = \dfrac{(1.25)(20)}{2.08} = 12.0192 \text{ kips}$ $\qquad P = 12.02 \text{ kips}$ ◀

(*b*) Without raised section, $K = 1.00$

$$P = A_{\min}\sigma_{\max} = (1.25)(20) = 25 \text{ kips}$$

$$\% \text{ change} = \left(\frac{25 - 12.02}{12.02}\right) \times 100\% \qquad = 108.0\%$$ ◀

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PROBLEM 2.100

A centric axial force is applied to the steel bar shown. Knowing that $\sigma_{all} = 20$ ksi, determine the maximum allowable load **P**.

SOLUTION

At the hole: $r = 0.5$ in. $\quad d = 5 - 1 = 4$ in.

$$\frac{2r}{d} = \frac{2(0.5)}{5} = 0.2 \qquad \text{From Fig. 2.60a,} \qquad K = 2.51$$

$$A_{net} = td = (0.75)(4) = 3 \text{ in}^2$$

$$\sigma_{max} = \frac{KP}{A_{net}}$$

$$P = \frac{A_{net}\sigma_{max}}{K} = \frac{(3)(20)}{2.51} = 23.9 \text{ kips}$$

At the fillet : $D = 6.5$ in., $\quad d = 5$ in., $\quad \frac{D}{d} = \frac{6.5}{5} = 1.3$

$r = 0.5$ in. $\qquad \frac{r}{d} = \frac{0.5}{5} = 0.1$

From Fig. 2.60b, $\quad K = 2.04$

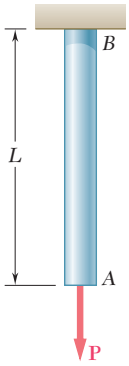
$$A_{min} = td = (0.75)(5) = 3.75 \text{ in}^2$$

$$\sigma_{max} = \frac{KP}{A_{min}}$$

$$P = \frac{A_{min}\sigma_{max}}{K} = \frac{(3.75)(20)}{2.04} = 36.8 \text{ kips}$$

Smaller value for P controls. $\qquad P = 23.9$ kips ◀

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PROBLEM 2.101

The cylindrical rod *AB* has a length $L = 5$ ft and a 0.75-in. diameter; it is made of a mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 36$ ksi . A force **P** is applied to the bar and then removed to give it a permanent set δ_P. Determine the maximum value of the force **P** and the maximum amount δ_m by which the bar should be stretched if the desired value of δ_P is (*a*) 0.1 in., (*b*) 0.2 in.

SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2 \qquad L = 5 \text{ ft} = 60 \text{ in.}$$

$$\delta_y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(60)(36\times10^3)}{29\times10^3} = 0.074483 \text{ in.}$$

When δ_m exceeds δ_Y, thus causing permanent stretch δ_p, the maximum force is

$$P_m = A\sigma_Y = (0.44179)(36\times10^3) = 15.9043\times10^3 \text{ lb}$$

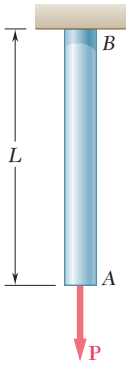
$$P = 15.90 \text{ kips} \blacktriangleleft$$

$\delta_p = \delta_m - \delta' = \delta_m - \delta_Y$ so that $\delta_m = \delta_p + \delta_Y$

(*a*) $\delta_p = 0.1$ in. $\delta_m = 0.1 + 0.074483 = 0.1745$ in. ◀

(*b*) $\delta_p = 0.2$ in. $\delta_m = 0.2 + 0.074483 = 0.274$ in. ◀

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PROBLEM 2.102

The cylindrical rod *AB* has a length $L = 6$ ft and a 1.25-in. diameter; it is made of a mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 36$ ksi. A force **P** is applied to the bar until end *A* has moved down by an amount δ_m. Determine the maximum value of the force **P** and the permanent set of the bar after the force has been removed, knowing (*a*) $\delta_m = 0.125$ in., (*b*) $\delta_m = 0.250$ in.

SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1.25)^2 = 1.22718 \text{ in}^2 \qquad L = 6 \text{ ft} = 72 \text{ in.}$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(72)(36\times10^3)}{29\times10^3} = 0.089379 \text{ in.}$$

If $\delta_m \geq \delta_Y$, $\quad P_m = A\sigma_Y = (1.22718)(36\times10^3)$

$$= 44.179\times10^3 \text{ lb} = 44.2 \text{ kips}$$

(*a*) $\delta_m = 0.125 \text{ in.} > \delta_Y$ so that $P_m = 44.2$ kips ◀

$$\delta' = \frac{P_m L}{AE} = \frac{\sigma_Y L}{E} = \delta_Y = 0.089379$$

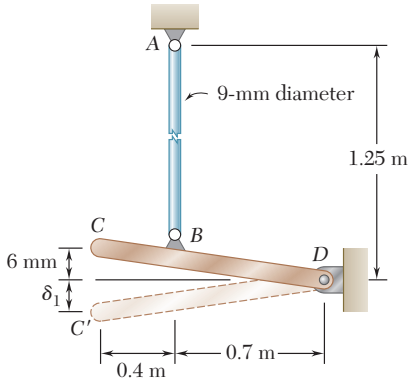
$$\delta_p = \delta_m - \delta' = 0.125 - 0.089379 = 0.356 \text{ in.}$$ ◀

(*b*) $\delta_m = 0.250 \text{ in.} > \delta_Y$ so that $P_m = 44.2$ kips ◀

$$\delta' = \delta_Y$$

$$\delta_p = \delta_m - \delta' = 0.250 - 0.089379 = 0.1606 \text{ in.}$$ ◀

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PROBLEM 2.103

Rod *AB* is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. After the rod has been attached to the rigid lever *CD*, it is found that end *C* is 6 mm too high. A vertical force **Q** is then applied at *C* until this point has moved to position C'. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

Since rod *AB* is to be stretched permanently,

$$(F_{AB})_{\max} = A_{AB}\sigma_Y = (63.617 \times 10^{-6})(345 \times 10^6)$$

$$= 21.948 \times 10^3 \text{ N}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad 1.1Q - 0.7F_{AB} = 0$$

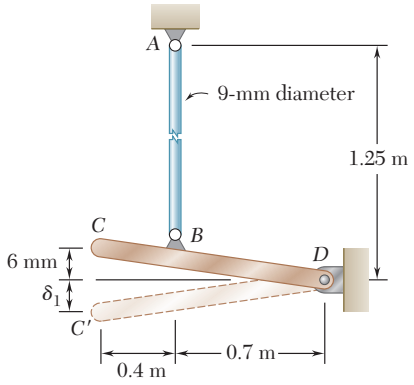
$$Q_{\max} = \frac{0.7}{1.1}(21.948 \times 10^3) = 13.9669 \times 10^{-3} \text{ N} \qquad 13.97 \text{ kN} \blacktriangleleft$$

$$\delta'_{AB} = \frac{(F_{AB})_{\max} L_{AB}}{EA_{AB}} = \frac{(21.948 \times 10^3)(1.25)}{(200 \times 10^9)(63.617 \times 10^{-6})} = 2.15625 \times 10^{-3} \text{ m}$$

$$\theta' = \frac{\delta_{AB'}}{0.7} = 3.0804 \times 10^{-3} \text{ rad}$$

$$\delta_1 = 1.1\theta' = 3.39 \times 10^{-3} \text{ m} \qquad 3.39 \text{ mm} \blacktriangleleft$$

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PROBLEM 2.104

Solve Prob. 2.103, assuming that the yield point of the mild steel is 250 MPa.

PROBLEM 2.103 Rod *AB* is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. After the rod has been attached to the rigid lever *CD*, it is found that end *C* is 6 mm too high. A vertical force **Q** is then applied at *C* until this point has moved to position C'. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

Since rod *AB* is to be stretched permanently,

$$(F_{AB})_{\max} = A_{AB}\sigma_Y = (63.617 \times 10^{-6})(250 \times 10^{6})$$

$$= 15.9043 \times 10^{3} \text{ N}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad 1.1Q - 0.7F_{AB} = 0$$

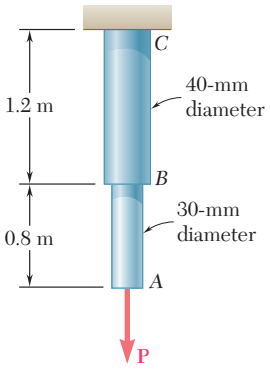
$$Q_{\max} = \frac{0.7}{1.1}(15.9043 \times 10^{3}) = 10.12 \times 10^{3} \text{ N} \qquad 10.12 \text{ kN} \blacktriangleleft$$

$$\delta'_{AB} = \frac{(F_{AB})_{\max} L_{AB}}{EA_{AE}} = \frac{(15.9043 \times 10^{3})(1.25)}{(200 \times 10^{9})(63.617 \times 10^{-6})} = 1.5625 \times 10^{-3} \text{ m}$$

$$\theta' = \frac{\delta'_{AB}}{0.7} = 2.2321 \times 10^{-3} \text{ rad}$$

$$\delta_1 = 1.1\theta' = 2.46 \times 10^{-3} \text{ m} \qquad 2.46 \text{ mm} \blacktriangleleft$$

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PROBLEM 2.105

Rod *ABC* consists of two cylindrical portions *AB* and *BC*; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod and then removed to give it a permanent set $\delta_p = 2$ mm. Determine the maximum value of the force **P** and the maximum amount δ_m by which the rod should be stretched to give it the desired permanent set.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^{3} \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$$

$$P_{\max} = A_{\min}\sigma_Y = (706.86 \times 10^{-6})(250 \times 10^{6}) = 176.715 \times 10^{3} \text{ N}$$

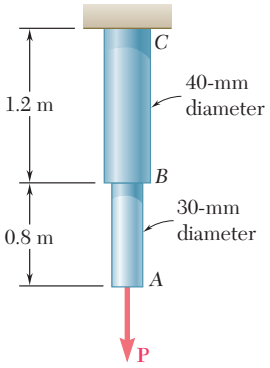
$$P_{\max} = 176.7 \text{ kN} \blacktriangleleft$$

$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715 \times 10^{3})(0.8)}{(200 \times 10^{9})(706.86 \times 10^{-6})} + \frac{(176.715 \times 10^{3})(1.2)}{(200 \times 10^{9})(1.25664 \times 10^{-3})}$$

$$= 1.84375 \times 10^{-3} \text{ m} = 1.84375 \text{ mm}$$

$$\delta_p = \delta_m - \delta' \text{ or } \delta_m = \delta_p + \delta' = 2 + 1.84375 \qquad \delta_m = 3.84 \text{ mm} \blacktriangleleft$$

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PROBLEM 2.106

Rod *ABC* consists of two cylindrical portions *AB* and *BC*; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod until its end *A* has moved down by an amount $\delta_m = 5$ mm. Determine the maximum value of the force **P** and the permanent set of the rod after the force has been removed.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^{3} \text{ mm}^2 = 1.25644 \times 10^{-3} \text{ m}^2$$

$$P_{\max} = A_{\min}\sigma_Y = (706.86 \times 10^{-6})(250 \times 10^{6}) = 176.715 \times 10^{3} \text{ N}$$

$$P_{\max} = 176.7 \text{ kN} \blacktriangleleft$$

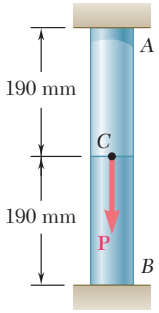
$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715 \times 10^{3})(0.8)}{(200 \times 10^{9})(706.68 \times 10^{-6})} + \frac{(176.715 \times 10^{3})(1.2)}{(200 \times 10^{9})(1.25664 \times 10^{-3})}$$

$$= 1.84375 \times 10^{-3} \text{ m} = 1.84375 \text{ mm}$$

$$\delta_p = \delta_m - \delta' = 5 - 1.84375 = 3.16 \text{ mm}$$

$$\delta_p = 3.16 \text{ mm} \blacktriangleleft$$

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PROBLEM 2.107

Rod *AB* consists of two cylindrical portions *AC* and *BC*, each with a cross-sectional area of 1750 mm^2. Portion *AC* is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion *CB* is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load **P** is applied at *C* as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of *C* if *P* is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of *C*.

SOLUTION

Displacement at *C* to cause yielding of *AC*.

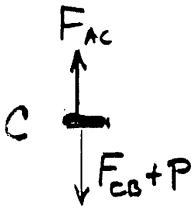
$$\delta_{C,Y} = L_{AC}\varepsilon_{Y,AC} = \frac{L_{AC}\sigma_{Y,AC}}{E} = \frac{(0.190)(250\times10^6)}{200\times10^9} = 0.2375\times10^{-3}\text{ m}$$

Corresponding force. $F_{AC} = A\sigma_{Y,AC} = (1750\times10^{-6})(250\times10^6) = 437.5\times10^3\text{ N}$

$$F_{CB} = -\frac{EA\delta_C}{L_{CB}} = -\frac{(200\times10^9)(1750\times10^{-6})(0.2375\times10^{-3})}{0.190} = -437.5\times10^3\text{ N}$$

For equilibrium of element at *C*,

$$F_{AC} - (F_{CB} + P_Y) = 0 \qquad P_Y = F_{AC} - F_{CB} = 875\times10^3\text{ N}$$



Since applied load $P = 975\times10^3\text{ N} > 875\times10^3\text{ N}$, portion *AC* yields.

$$F_{CB} = F_{AC} - P = 437.5\times10^3 - 975\times10^3\text{ N} = -537.5\times10^3\text{ N}$$

(*a*) $$\delta_C = -\frac{F_{CB}L_{CD}}{EA} = \frac{(537.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.29179\times10^{-3}\text{ m}$$

0.292 mm ◀

(*b*) Maximum stresses: $\sigma_{AC} = \sigma_{Y,AC} = 250$ MPa 250 MPa ◀

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{537.5\times10^3}{1750\times10^{-6}} = -307.14\times10^6\text{ Pa} = -307\text{ MPa}$$ −307 MPa ◀

(*c*) Deflection and forces for unloading.

$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \qquad \therefore \quad P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{AB}} = -P'_{AC}$$

$$P' = 975\times10^3 = P'_{AC} - P'_{CB} = 2P'_{AC} \quad P'_{AC} = 487.5\times10^{-3}\text{ N}$$

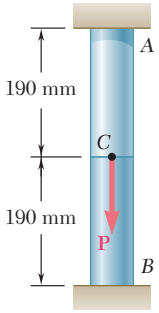
$$\delta' = \frac{(487.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.26464\times10^3\text{ m}$$

$$\delta_p = \delta_m - \delta' = 0.29179\times10^{-3} - 0.26464\times10^{-3}$$

$$= 0.02715\times10^{-3}\text{ m}$$ 0.0272 mm ◀

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PROBLEM 2.108



For the composite rod of Prob. 2.107, if P is gradually increased from zero until the deflection of point C reaches a maximum value of $\delta_m = 0.3$ mm and then decreased back to zero, determine (*a*) the maximum value of P, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C after the load is removed.

PROBLEM 2.107 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 1750 mm^2. Portion AC is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion CB is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C.

SOLUTION

Displacement at C is $\delta_m = 0.30$ mm. The corresponding strains are

$$\varepsilon_{AC} = \frac{\delta_m}{L_{AC}} = \frac{0.30 \text{ mm}}{190 \text{ mm}} = 1.5789 \times 10^{-3}$$

$$\varepsilon_{CB} = -\frac{\delta_m}{L_{CB}} = -\frac{0.30 \text{ mm}}{190 \text{ mm}} = -1.5789 \times 10^{-3}$$

Strains at initial yielding:

$$\varepsilon_{Y,\,AC} = \frac{\sigma_{Y,\,AC}}{E} = \frac{250 \times 10^6}{200 \times 10^9} = 1.25 \times 10^{-3} \qquad \text{(yielding)}$$

$$\varepsilon_{Y,\,CB} = \frac{\sigma_{Y,\,BC}}{E} = -\frac{345 \times 10^6}{200 \times 10^9} = -1.725 \times 10^{-3} \qquad \text{(elastic)}$$

(*a*) Forces: $F_{AC} = A\sigma_Y = (1750 \times 10^{-6})(250 \times 10^6) = 437.5 \times 10^{-3}$ N

$$F_{CB} = EA\varepsilon_{CB} = (200 \times 10^9)(1750 \times 10^{-6})(-1.5789 \times 10^{-3}) = -552.6 \times 10^{-3} \text{ N}$$

For equilibrium of element at C, $F_{AC} - F_{CB} - P = 0$

$$P = F_{AC} - F_{CD} = 437.5 \times 10^3 + 552.6 \times 10^3 = 990.1 \times 10^3 \text{ N} \qquad 990 \text{ kN} \blacktriangleleft$$

(*b*) Stresses: AC: $\sigma_{AC} = \sigma_{Y,\,AC}$ 250 MPa ◀

$$CB: \quad \sigma_{CB} = \frac{F_{CB}}{A} = -\frac{552.6 \times 10^3}{1750 \times 10^{-6}} = -316 \times 10^6 \text{ Pa} \qquad -316 \text{ MPa} \blacktriangleleft$$

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PROBLEM 2.108 *(Continued)*

(*c*) Deflection and forces for unloading.

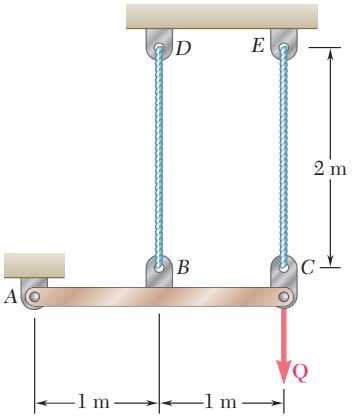
$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore \quad P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{AB}} = -P_{AC}$$

$$P' = P'_{AC} - P'_{CB} = 2P'_{AC} = 990.1\times10^{3}\,\text{N} \quad \therefore \quad P'_{AC} = 495.05\times10^{3}\,\text{N}$$

$$\delta' = \frac{(495.05\times10^{3})(0.190)}{(200\times10^{9})(1750\times10^{-6})} = 0.26874\times10^{-3}\,\text{m} = 0.26874\ \text{mm}$$

$$\delta_p = \delta_m - \delta' = 0.30\ \text{mm} - 0.26874\ \text{mm} \qquad 0.0313\,\text{mm} \blacktriangleleft$$

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PROBLEM 2.109

Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at *C* to the rigid bar *ABC* and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable *BD*, (*b*) the maximum deflection of point *C*, (*c*) the final displacement of point *C*. (*Hint:* In part *c*, cable *CE* is not taut.)

SOLUTION

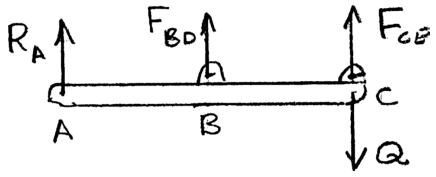
Elongation constraints for taut cables.

Let $\theta =$ rotation angle of rigid bar *ABC*.

$$\theta = \frac{\delta_{BD}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\delta_{CE} = \frac{1}{2}\delta_{CE} \tag{1}$$

Equilibrium of bar *ABC*.



$$+\circlearrowleft M_A = 0: \; L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}\,Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}}F_{BD} = F_{CE} + \frac{1}{2}F_{BD} \tag{2}$$

Assume cable *CE* is yielded. $F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^{3}\,\text{N}$

From (2), $F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^{3} - 34.5\times10^{3}) = 31.0\times10^{3}\,\text{N}$

Since $F_{BD} < A\sigma_Y = 34.5\times10^{3}\,\text{N}$, cable *BD* is elastic when $Q = 50$ kN.

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PROBLEM 2.109 *(Continued)*

(*a*) Maximum stresses. $\sigma_{CE} = \sigma_Y = 345 \text{ MPa}$

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0 \times 10^3}{100 \times 10^{-6}} = 310 \times 10^6 \text{ Pa} \qquad \sigma_{BD} = 310 \text{ MPa} \blacktriangleleft$$

(*b*) Maximum of deflection of point *C*.

$$\delta_{BD} = \frac{F_{BD} L_{BD}}{EA} = \frac{(31.0 \times 10^3)(2)}{(200 \times 10^9)(100 \times 10^{-6})} = 3.1 \times 10^{-3} \text{ m}$$

From (1), $\delta_C = \delta_{CE} = 2\delta_{BD} = 6.2 \times 10^{-3} \text{ m}$ 6.20 mm ↓ ◀

Permanent elongation of cable *CE*: $(\delta_{CE})_p = (\delta_{CE}) - \dfrac{\sigma_Y L_{CE}}{E}$

$$(\delta_{CE})_P = (\delta_{CE})_{\max} - \frac{\sigma_Y L_{CE}}{E}$$

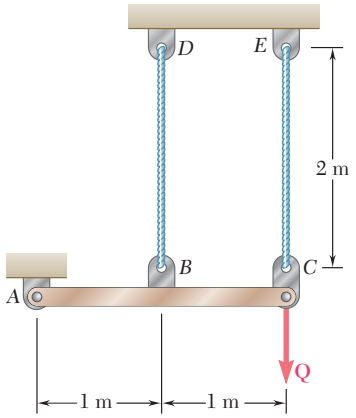
$$= 6.20 \times 10^{-3} - \frac{(345 \times 10^6)(2)}{200 \times 10^9} = 2.75 \times 10^{-3} \text{ m}$$

(*c*) Unloading. Cable *CE* is slack $(F_{CE} = 0)$ at $Q = 0$.

From (2), $F_{BD} = 2(Q - F_{CE}) = 2(0 - 0) = 0$

Since cable *BD* remained elastic, $\delta_{BD} = \dfrac{F_{BD} L_{BD}}{EA} = 0.$ 0 ◀

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PROBLEM 2.110

Solve Prob. 2.109, assuming that the cables are replaced by rods of the same cross-sectional area and material. Further assume that the rods are braced so that they can carry compressive forces.

PROBLEM 2.109 Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at *C* to the rigid bar *ABC* and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable *BD*, (*b*) the maximum deflection of point *C*, (*c*) the final displacement of point *C*. (*Hint:* In part *c*, cable *CE* is not taut.)

SOLUTION

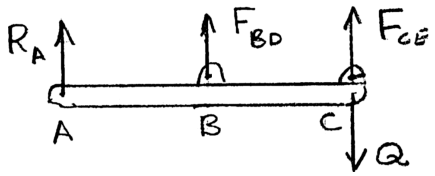
Elongation constraints.

Let $\theta =$ rotation angle of rigid bar *ABC*.

$$\theta = \frac{\delta_{BC}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\delta_{CE} = \frac{1}{2}\delta_{CE} \tag{1}$$

Equilibrium of bar *ABC*.



$$+\circlearrowleft M_A = 0: \quad L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}}F_{BD} = F_{CE} + \frac{1}{2}F_{BD} \tag{2}$$

Assume cable *CE* is yielded. $F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^{3}$ N

From (2), $F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^{3} - 34.5\times10^{3}) = 31.0\times10^{3}$ N

Since $F_{BD} < A\sigma_Y = 34.5\times10^{3}$ N, cable *BD* is elastic when $Q = 50$ kN.

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PROBLEM 2.110 *(Continued)*

(*a*) Maximum stresses. $\sigma_{CE} = \sigma_Y = 345$ MPa

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0 \times 10^3}{100 \times 10^{-6}} = 310 \times 10^6 \text{ Pa} \qquad \sigma_{BD} = 310 \text{ MPa} \blacktriangleleft$$

(*b*) Maximum of deflection of point *C*.

$$\delta_{BD} = \frac{F_{BD} L_{BD}}{EA} = \frac{(31.0 \times 10^3)(2)}{(200 \times 10^9)(100 \times 10^{-6})} = 3.1 \times 10^{-3} \text{ m}$$

From (1), $\delta_C = \delta_{CE} = 2\delta_{BD} = 6.2 \times 10^{-3}$ m 6.20 mm ↓ ◀

Unloading. $Q' = 50 \times 10^3$ N, $\delta'_{CE} = \delta'_C$

From (1), $\delta'_{BD} = \frac{1}{2}\delta'_C$

Elastic $F''_{BD} = \dfrac{EA\delta'_{BD}}{L_{BD}} = \dfrac{(200 \times 10^9)(100 \times 10^{-6})(\frac{1}{2}\delta'_C)}{2} = 5 \times 10^6 \delta'_C$

$$F'_{CE} = \frac{EA\delta'_{CE}}{L_{CE}} = \frac{(200 \times 10^9)(100 \times 10^{-6})(\delta'_C)}{2} = 10 \times 10^6 \delta'_C$$

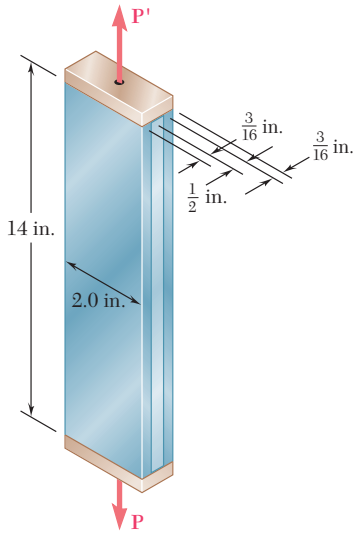
From (2), $Q' = F'_{CE} + \frac{1}{2}F'_{BD} = 12.5 \times 10^6 \delta'_C$

Equating expressions for Q', $12.5 \times 10^6 \delta'_C = 50 \times 10^3$

$$\delta'_C = 4 \times 10^{-3} \text{ m}$$

(*c*) Final displacement. $\delta_C = (\delta_C)_m - \delta'_C = 6.2 \times 10^{-3} - 4 \times 10^{-3} = 2.2 \times 10^{-3}$ m 2.20 mm ↓ ◀

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PROBLEM 2.111

Two tempered-steel bars, each $\frac{3}{16}$ in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

For the mild steel, $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$ $\quad \delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(14)(50\times10^3)}{29\times10^6} = 0.024138$ in.

For the tempered steel, $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$ $\quad \delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(14)(100\times10^3)}{29\times10^3} = 0.048276$ in.

Total area: $A = A_1 + A_2 = 1.75 \text{ in}^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$. The mild steel yields. Tempered steel is elastic.

(*a*) Forces: $P_1 = A_1\sigma_{Y1} = (1.00)(50\times10^3) = 50\times10^3$ lb

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times10^3)(0.75)(0.04)}{14} = 62.14\times10^3 \text{ lb}$$

$$P = P_1 + P_2 = 112.14\times10^3 \text{ lb} = 112.1 \text{ kips}$$ $\quad P = 112.1$ kips ◀

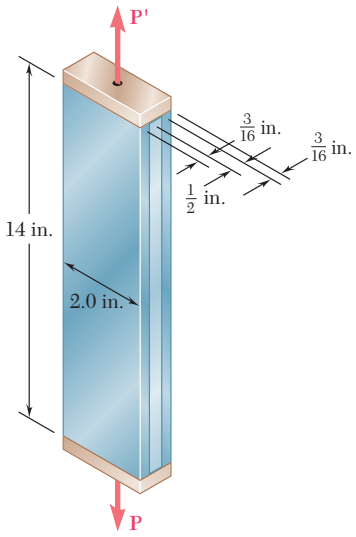
(*b*) Stresses: $\sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 50\times10^3$ psi $= 50$ ksi

$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14\times10^3}{0.75} = 82.86\times10^3 \text{ psi} = 82.86 \text{ ksi}$$ $\quad 82.86$ ksi ◀

Unloading: $\delta' = \frac{PL}{EA} = \frac{(112.14\times10^3)(14)}{(29\times10^6)(1.75)} = 0.03094$ in.

(*c*) Permanent set: $\delta_p = \delta_m - \delta' = 0.04 - 0.03094 = 0.00906$ in. $\quad 0.00906$ in. ◀

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PROBLEM 2.112

For the composite bar of Prob. 2.111, if P is gradually increased from zero to 98 kips and then decreased back to zero, determine (*a*) the maximum deformation of the bar, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$ in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

Areas: Mild steel: $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$

Tempered steel: $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$

Total: $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel:

$$\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50\times10^3) = 87.50\times10^3 \text{ lb}$$

$P > P_Y$, therefore, mild steel yields.

Let P_1 = force carried by mild steel.

P_2 = force carried by tempered steel.

$$P_1 = A_1\sigma_1 = (1.00)(50\times10^3) = 50\times10^3 \text{ lb}$$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98\times10^3 - 50\times10^3 = 48\times10^3 \text{ lb}$$

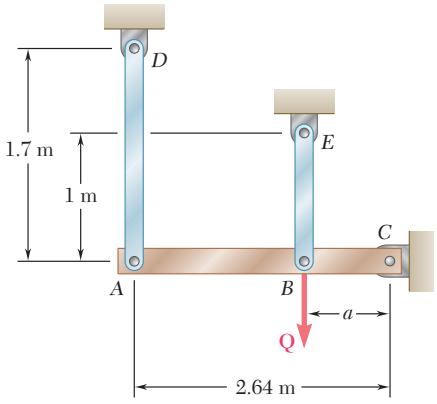
(*a*) $\delta_m = \frac{P_2 L}{EA_2} = \frac{(48\times10^3)(14)}{(29\times10^6)(0.75)}$ 0.0309 in. ◀

(*b*) $\sigma_2 = \frac{P_2}{A_2} = \frac{48\times10^3}{0.75} = 64\times10^3 \text{ psi}$ 64.0 ksi ◀

Unloading: $\delta' = \frac{PL}{EA} = \frac{(98\times10^3)(14)}{(29\times10^6)(1.75)} = 0.02703$ in.

(*c*) $\delta_P = \delta_m - \delta' = 0.03090 - 0.02703 = 0.003870$ in. 0.00387 in. ◀

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PROBLEM 2.113

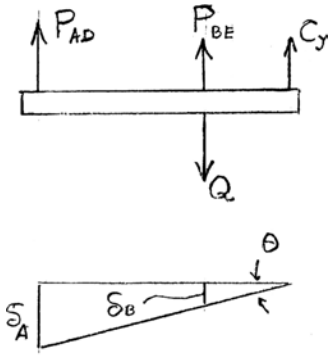
The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

SOLUTION

Statics: $\Sigma M_C = 0: \; 0.640(Q - P_{BE}) - 2.64P_{AD} = 0$

Deformation: $\delta_A = 2.64\theta, \;\; \delta_B = a\theta = 0.640\theta$

Elastic analysis:



$$A = (37.5)(6) = 225 \text{ mm}^2 = 225 \times 10^{-6} \text{ m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.7}\delta_A = 26.47 \times 10^6 \delta_A$$

$$= (26.47 \times 10^6)(2.64\theta) = 69.88 \times 10^6 \theta$$

$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6 \times 10^9 \theta$$

$$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.0}\delta_B = 45 \times 10^6 \delta_B$$

$$= (45 \times 10^6)(0.640\theta) = 28.80 \times 10^6 \theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 128 \times 10^9 \theta$$

From statics, $Q = P_{BE} + \frac{2.64}{0.640}P_{AD} = P_{BE} + 4.125P_{AD}$

$$= [28.80 \times 10^6 + (4.125)(69.88 \times 10^6)]\theta = 317.06 \times 10^6 \theta$$

θ_Y at yielding of link *AD*: $\sigma_{AD} = \sigma_Y = 250 \times 10^6 = 310.6 \times 10^9 \theta$

$$\theta_Y = 804.89 \times 10^{-6}$$

$$Q_Y = (317.06 \times 10^6)(804.89 \times 10^{-6}) = 255.2 \times 10^3 \text{ N}$$

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PROBLEM 2.113 ***(Continued)***

(*a*) Since $Q = 260\times10^3 > Q_Y$, link *AD* yields. $\sigma_{AD} = 250$ MPa ◀

$$P_{AD} = A\sigma_Y = (225\times10^{-6})(250\times10^{-6}) = 56.25\times10^3 \text{ N}$$

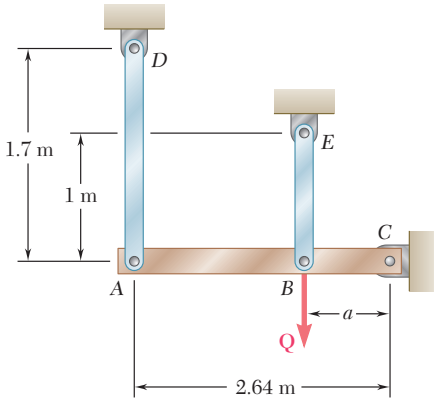
From statics, $P_{BE} = Q - 4.125P_{AD} = 260\times10^3 - (4.125)(56.25\times10^3)$

$$P_{BE} = 27.97\times10^3 \text{ N}$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = \frac{27.97\times10^3}{225\times10^{-6}} = 124.3\times10^6 \text{ Pa}$$ $\sigma_{BE} = 124.3$ MPa ◀

(*b*) $$\delta_B = \frac{P_{BE}L_{BE}}{EA} = \frac{(27.97\times10^3)(1.0)}{(200\times10^9)(225\times10^{-6})} = 621.53\times10^{-6} \text{ m}$$ $\delta_B = 0.622$ mm ↓ ◀

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PROBLEM 2.114

Solve Prob. 2.113, knowing that $a = 1.76$ m and that the magnitude of the force **Q** applied at *B* is gradually increased from zero to 135 kN.

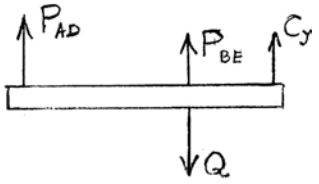
PROBLEM 2.113 The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

SOLUTION

Statics: $\Sigma M_C = 0: \; 1.76(Q - P_{BE}) - 2.64P_{AD} = 0$

Deformation: $\delta_A = 2.64\theta, \quad \delta_B = 1.76\theta$

Elastic Analysis:

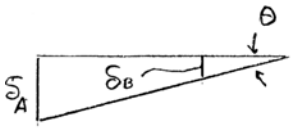


$$A = (37.5)(6) = 225 \text{ mm}^2 = 225 \times 10^{-6} \text{ m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.7}\delta_A = 26.47 \times 10^6 \delta_A$$

$$= (26.47 \times 10^6)(2.64\theta) = 69.88 \times 10^6 \theta$$

$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6 \times 10^9 \theta$$



$$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.0}\delta_B = 45 \times 10^6 \delta_B$$

$$= (45 \times 10^6)(1.76\theta) = 79.2 \times 10^6 \theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 352 \times 10^9 \theta$$

From statics, $Q = P_{BE} + \frac{2.64}{1.76}P_{AD} = P_{BE} + 1.500P_{AD}$

$$= [73.8 \times 10^6 + (1.500)(69.88 \times 10^6)]\theta = 178.62 \times 10^6 \theta$$

θ_Y at yielding of link *BE*: $\sigma_{BE} = \sigma_Y = 250 \times 10^6 = 352 \times 10^9 \theta_Y$

$$\theta_Y = 710.23 \times 10^{-6}$$

$$Q_Y = (178.62 \times 10^6)(710.23 \times 10^{-6}) = 126.86 \times 10^3 \text{ N}$$

(*a*) Since $Q = 135 \times 10^3 \text{ N} > Q_Y$, link *BE* yields. $\sigma_{BE} = \sigma_Y = 250$ MPa ◀

$$P_{BE} = A\sigma_Y = (225 \times 10^{-6})(250 \times 10^6) = 56.25 \times 10^3 \text{ N}$$

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PROBLEM 2.114 *(Continued)*

From statics, $P_{AD} = \frac{1}{1.500}(Q - P_{BE}) = 52.5 \times 10^3$ N

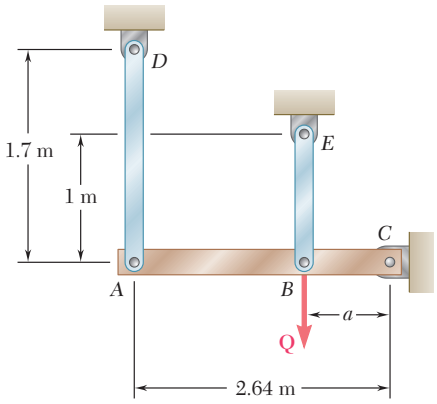
$$\sigma_{AD} = \frac{P_{AD}}{A} = \frac{52.5 \times 10^3}{225 \times 10^{-6}} = 233.3 \times 10^6 \qquad \sigma_{AD} = 233 \text{ MPa} \blacktriangleleft$$

From elastic analysis of *AD*, $\theta = \frac{P_{AD}}{69.88 \times 10^6} = 751.29 \times 10^{-3}$ rad

(*b*) $\delta_B = 1.76\theta = 1.322 \times 10^{-3}$ m $\qquad \delta_B = 1.322 \text{ mm} \downarrow \blacktriangleleft$

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PROBLEM 2.115*



Solve Prob. 2.113, assuming that the magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that $a = 0.640$ m, determine (*a*) the residual stress in each link, (*b*) the final deflection of point *B*. Assume that the links are braced so that they can carry compressive forces without buckling.

PROBLEM 2.113 The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

SOLUTION

See solution to Problem 2.113 for the normal stresses in each link and the deflection of Point *B* after loading.

$$\sigma_{AD} = 250 \times 10^6 \text{ Pa}$$
$$\sigma_{BE} = 124.3 \times 10^6 \text{ Pa}$$
$$\delta_B = 621.53 \times 10^{-6} \text{ m}$$

The elastic analysis given in the solution to Problem 2.113 applies to the unloading.

$$Q' = 317.06 \times 10^6 \theta'$$
$$Q' = \frac{Q}{317.06 \times 10^6} = \frac{260 \times 10^3}{317.06 \times 10^6} = 820.03 \times 10^{-6}$$
$$\sigma'_{AD} = 310.6 \times 10^9 \theta = (310.6 \times 10^9)(820.03 \times 10^{-6}) = 254.70 \times 10^6 \text{ Pa}$$
$$\sigma'_{BE} = 128 \times 10^9 \theta = (128 \times 10^9)(820.03 \times 10^{-6}) = 104.96 \times 10^6 \text{ Pa}$$
$$\delta'_B = 0.640\theta' = 524.82 \times 10^{-6} \text{ m}$$

(*a*) Residual stresses.

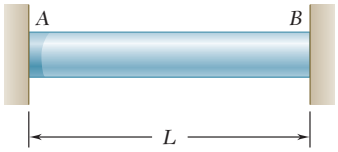
$$\sigma_{AD,\text{res}} = \sigma_{AD} - \sigma'_{AD} = 250 \times 10^6 - 254.70 \times 10^{-6} = -4.70 \times 10^6 \text{ Pa} \qquad = -4.70 \text{ MPa} \blacktriangleleft$$

$$\sigma_{BE,\text{res}} = \sigma_{BE} - \sigma'_{BE} = 124.3 \times 10^6 - 104.96 \times 10^6 = 19.34 \times 10^6 \text{ Pa} \qquad = 19.34 \text{ MPa} \blacktriangleleft$$

(*b*) $\delta_{B,P} = \delta_B - \delta'_B = 621.53 \times 10^{-6} - 524.82 \times 10^{-6} = 96.71 \times 10^{-6} \text{ m} \qquad = 0.0967 \text{ mm} \downarrow \blacktriangleleft$

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PROBLEM 2.116



A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of 45°F. The steel is assumed to be elastoplastic with $\sigma_Y = 36$ ksi and $E = 29\times10^6$ psi. Knowing that $\alpha = 6.5\times10^{-6}/°\text{F}$, determine the stress in the bar (*a*) when the temperature is raised to 320°F, (*b*) after the temperature has returned to 45°F.

SOLUTION

Let P be the compressive force in the rod.

Determine temperature change to cause yielding.

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_Y L}{E} + L\alpha(\Delta T)_Y = 0$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E\alpha} = \frac{36\times10^3}{(29\times10^6)(6.5\times10^{-6})} = 190.98°\text{F}$$

But $\Delta T = 320 - 45 = 275°\text{F} > (\Delta T_Y)$

(*a*) Yielding occurs. $\sigma = -\sigma_Y = -36.0$ ksi ◀

Cooling: $(\Delta\text{T})' = 275°\text{F}$

$$\delta' = \delta'_P = \delta'_T = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

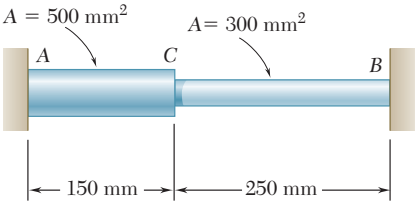
$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)'$$

$$= -(29\times10^6)(6.5\times10^{-6})(275) = -51.8375\times10^3 \text{ psi}$$

(*b*) Residual stress:

$$\sigma_{\text{res}} = -\sigma_Y - \sigma' = -36\times10^3 + 51.8375\times10^3 = 15.84\times10 \text{ psi}$$ 15.84 ksi ◀

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PROBLEM 2.117

The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 25°C. The steel is assumed elastoplastic, with E = 200 GPa and $\sigma_y = 250$ MPa. The temperature of both portions of the rod is then raised to 150°C. Knowing that $\alpha = 11.7\times10^{-6}/°C$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point *C*.

SOLUTION

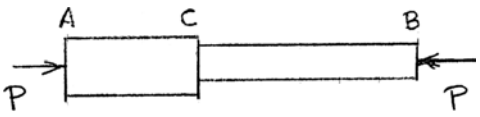
$$A_{AC} = 500\times10^{-6}\,\text{m}^2 \qquad L_{AC} = 0.150\ \text{m}$$
$$A_{CB} = 300\times10^{-6}\,\text{m}^2 \qquad L_{CB} = 0.250\ \text{m}$$

Constraint: $\delta_P + \delta_T = 0$

Determine ΔT to cause yielding in portion *CB*.

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\alpha(\Delta T)$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$



At yielding, $P = P_Y = A_{CB}\sigma_Y = (300\times10^{-6})(2.50\times10^{6}) = 75\times10^{3}\,\text{N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$
$$= \frac{75\times10^{3}}{(0.400)(200\times10^{9})(11.7\times10^{-6})}\left(\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}\right) = 90.812°C$$

Actual ΔT: $150°C - 25°C = 125°C > (\Delta T)_Y$

Yielding occurs. For $\Delta T > (\Delta T)_Y$, $P = P_Y = 75\times10^{3}\,\text{N}$

(*a*) $\sigma_{AC} = -\frac{P_Y}{A_{AC}} = -\frac{75\times10^{3}}{500\times10^{-6}} = -150\times10^{6}\,\text{Pa}$ $\qquad \sigma_{AC} = -150.0\ \text{MPa}$ ◀

$\sigma_{CB} = -\frac{P_Y}{A_{CB}} = -\sigma_Y$ $\qquad \sigma_{CB} = -250\ \text{MPa}$ ◀

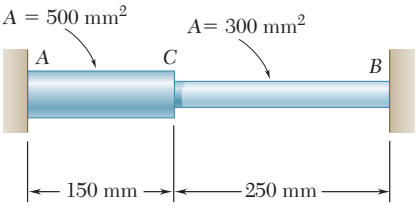
(*b*) For $\Delta T > (\Delta T)_Y$, portion *AC* remains elastic.

$$\delta_{C/A} = -\frac{P_Y L_{AC}}{EA_{AC}} + L_{AC}\alpha(\Delta T)$$
$$= -\frac{(75\times10^{3})(0.150)}{(200\times10^{9})(500\times10^{-6})} + (0.150)(11.7\times10^{-6})(125) = 106.9\times10^{-6}\,\text{m}$$

Since Point *A* is stationary, $\delta_C = \delta_{C/A} = 106.9\times10^{-6}\,\text{m}$ $\qquad \delta_C = 0.1069\ \text{mm}\rightarrow$ ◀

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PROBLEM 2.118*



Solve Prob. 2.117, assuming that the temperature of the rod is raised to 150°C and then returned to 25°C.

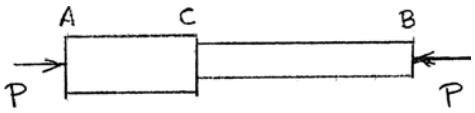
PROBLEM 2.117 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 25°C. The steel is assumed elastoplastic, with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The temperature of both portions of the rod is then raised to 150°C. Knowing that $\alpha = 11.7\times10^{-6}/°C$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point *C*.

SOLUTION

$$A_{AC} = 500\times10^{-6}\,\text{m}^2 \qquad L_{AC} = 0.150\ \text{m} \qquad A_{CB} = 300\times10^{-6}\,\text{m}^2 \qquad L_{CB} = 0.250\ \text{m}$$

Constraint: $\delta_P + \delta_T = 0$

Determine ΔT to cause yielding in portion *CB*.



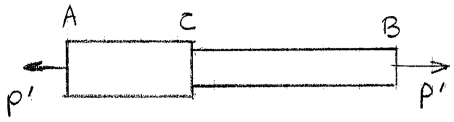
$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\alpha(\Delta T)$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

At yielding, $P = P_Y = A_{CB}\sigma_Y = (300\times10^{-6})(250\times10^{6}) = 75\times10^{3}\,\text{N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right) = \frac{75\times10^{3}}{(0.400)(200\times10^{9})(11.7\times10^{-6})}\left(\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}\right)$$
$$= 90.812\ °\text{C}$$

Actual ΔT: $150°\text{C} - 25°\text{C} = 125°\text{C} > (\Delta T)_Y$



Yielding occurs. For $\Delta T > (\Delta T)_Y$, $P = P_Y = 75\times10^{3}$ N

Cooling:

$$(\Delta T)' = 125°\text{C} \qquad P' = \frac{EL_{AB}\alpha(\Delta T)'}{\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)} = \frac{(200\times10^{9})(0.400)(11.7\times10^{-6})(125)}{\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}} = 103.235\times10^{3}\,\text{N}$$

Residual force: $P_{\text{res}} = P' - P_Y = 103.235\times10^{3} - 75\times10^{3} = 28.235\times10^{3}\,\text{N}$ (tension)

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PROBLEM 2.118* *(Continued)*

(*a*) Residual stresses.

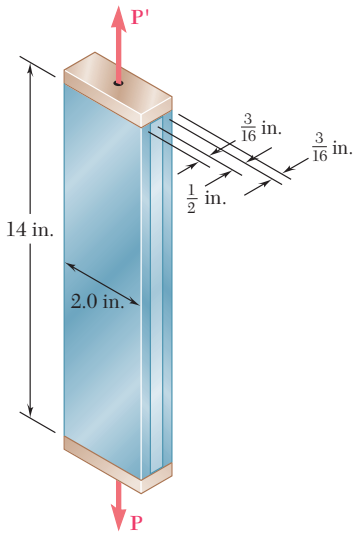
$$\sigma_{AC} = \frac{P_{\text{res}}}{A_{AC}} = \frac{28.235 \times 10^3}{500 \times 10^{-6}} \qquad \sigma_{AC} = 56.5 \text{ MPa} \blacktriangleleft$$

$$\sigma_{CB} = \frac{P_{\text{res}}}{A_{CB}} = \frac{28.235 \times 10^3}{300 \times 10^{-6}} \qquad \sigma_{CB} = 9.41 \text{ MPa} \blacktriangleleft$$

(*b*) Permanent deflection of point *C*.

$$\delta_C = \frac{P_{\text{res}} L_{AC}}{E A_{AC}} \qquad \delta_C = 0.0424 \text{ mm} \rightarrow \blacktriangleleft$$

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PROBLEM 2.119*

For the composite bar of Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero to 98 kips and then decreased back to zero.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$ in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

Areas. Mild steel: $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$

Tempered steel: $A_2 = (2)\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$

Total: $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel: $\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50\times10^3) = 87.50\times10^3 \text{ lb}$

$P > P_Y$; therefore, mild steel yields.

Let P_1 = force carried by mild steel

P_2 = force carried by tempered steel

$P_1 = A_1\sigma_{Y1} = (1.00)(50\times10^3) = 50\times10^3 \text{ lb}$

$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98\times10^3 - 50\times10^3 = 48\times10^3 \text{ lb}$

$\sigma_2 = \frac{P_2}{A_2} = \frac{48\times10^3}{0.75} = 64\times10^3 \text{ psi}$

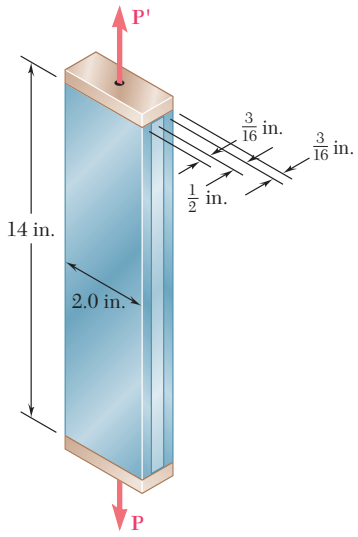
Unloading. $\sigma' = \frac{P}{A} = \frac{98\times10^3}{1.75} = 56\times10^3 \text{ psi}$

Residual stresses.

Mild steel: $\sigma_{1,\text{res}} = \sigma_1 - \sigma' = 50\times10^3 - 56\times10^3 = -6\times10^{-3} \text{ psi} = -6 \text{ ksi}$

Tempered steel: $\sigma_{2,\text{res}} = \sigma_2 - \sigma_1 = 64\times10^3 - 56\times10^3 = 8\times10^3 \text{ psi}$ 8.00 ksi ◀

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PROBLEM 2.120*

For the composite bar in Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and is then decreased back to zero.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$ in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

For the mild steel, $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$ $\quad \delta_{Y1} = \dfrac{L\delta_{Y1}}{E} = \dfrac{(14)(50\times10^3)}{29\times10^6} = 0.024138 \text{ in.}$

For the tempered steel, $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$ $\quad \delta_{Y2} = \dfrac{L\delta_{Y2}}{E} = \dfrac{(14)(100\times10^3)}{29\times10^6} = 0.048276 \text{ in.}$

Total area: $A = A_1 + A_2 = 1.75 \text{ in}^2$

$$\delta_{Y1} < \delta_m < \delta_{Y2}$$

The mild steel yields. Tempered steel is elastic.

Forces:

$$P_1 = A_1\delta_{Y1} = (1.00)(50\times10^3) = 50\times10^3 \text{ lb}$$

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times10^6)(0.75)(0.04)}{14} = 62.14\times10^3 \text{ lb}$$

Stresses: $\sigma_1 = \dfrac{P_1}{A_1} = \delta_{Y1} = 50\times10^3 \text{ psi}$ $\quad \sigma_2 = \dfrac{P_2}{A_2} = \dfrac{62.14\times10^3}{0.75} = 82.86\times10^3 \text{ psi}$

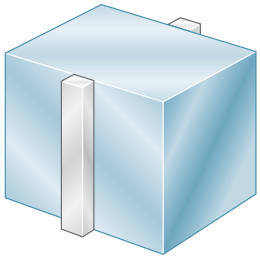
Unloading:

$$\sigma' = \frac{P}{A} = \frac{112.14}{1.75} = 64.08\times10^3 \text{ psi}$$

Residual stresses. $\sigma_{1,\text{res}} = \sigma_1 - \sigma' = 50\times10^3 - 64.08\times10^3 = -14.08\times10^3 \text{ psi} = -14.08 \text{ ksi}$

$$\sigma_{2,\text{res}} = \sigma_2 - \sigma' = 82.86\times10^3 - 64.08\times10^3 = 18.78\times10^3 \text{ psi} = 18.78 \text{ ksi}$$ ◀

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PROBLEM 2.121*

Narrow bars of aluminum are bonded to the two sides of a thick steel plate as shown. Initially, at $T_1 = 70°\text{F}$, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1, determine (*a*) the highest temperature T_2 that does *not* result in residual stresses, (*b*) the temperature T_2 that will result in a residual stress in the aluminum equal to 58 ksi. Assume $\alpha_a = 12.8\times10^{-6}/°\text{F}$ for the aluminum and $\alpha_s = 6.5\times10^{-6}/°\text{F}$ for the steel. Further assume that the aluminum is elastoplastic, with $E = 10.9\times10^{6}$ psi and $\sigma_Y = 58$ ksi. (*Hint:* Neglect the small stresses in the plate.)

SOLUTION

Determine temperature change to cause yielding.

$$\delta = \frac{PL}{EA} + L\alpha_a(\Delta T)_Y = L\alpha_s(\Delta T)_Y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_Y = -\sigma_Y$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{58\times10^3}{(10.9\times10^6)(12.8-6.5)(10^{-6})} = 844.62°\text{F}$$

(*a*) $T_{2Y} = T_1 + (\Delta T)_Y = 70 + 844.62 = 915°\text{F}$ 915°F ◀

After yielding,

$$\delta = \frac{\sigma_Y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

Cooling:

$$\delta' = \frac{P'L}{AE} + L\alpha_a(\Delta T)' = L\alpha_s(\Delta T)'$$

The residual stress is

$$\sigma_{\text{res}} = \sigma_Y - \frac{P'}{A} = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

Set $\sigma_{\text{res}} = -\sigma_Y$

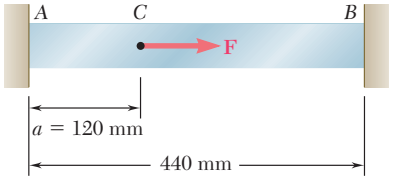
$$-\sigma_Y = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$\Delta T = \frac{2\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{(2)(58\times10^3)}{(10.9\times10^6)(12.8-6.5)(10^{-6})} = 1689°\text{F}$$

(*b*) $T_2 = T_1 + \Delta T = 70 + 1689 = 1759°\text{F}$ 1759°F ◀

If $T_2 > 1759°\text{F}$, the aluminum bar will most likely yield in compression.

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PROBLEM 2.122*

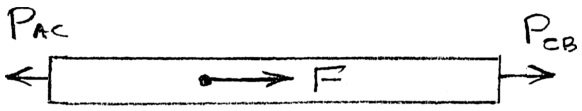
Bar *AB* has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point *C*, (*b*) the residual stress in the bar.

SOLUTION

$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

Force to yield portion *AC*:

$$P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^{6})$$
$$= 300 \times 10^{3} \text{ N}$$



For equilibrium, $F + P_{CB} - P_{AC} = 0$.

$$P_{CB} = P_{AC} - F = 300 \times 10^{3} - 520 \times 10^{3}$$
$$= -220 \times 10^{3} \text{ N}$$

$$\delta_C = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^{3})(0.440 - 0.120)}{(200 \times 10^{9})(1200 \times 10^{-6})}$$
$$= 0.29333 \times 10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220 \times 10^{3}}{1200 \times 10^{-6}}$$
$$= -183.333 \times 10^{6} \text{ Pa}$$

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PROBLEM 2.122* *(Continued)*

Unloading:

$$\delta'_C = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} = \frac{(F - P'_{AC})L_{CB}}{EA}$$

$$P'_{AC}\left(\frac{L_{AC}}{EA} + \frac{L_{BC}}{EA}\right) = \frac{FL_{CB}}{EA}$$

$$P'_{AC} = \frac{FL_{CB}}{L_{AC} + L_{CB}} = \frac{(520\times10^3)(0.440 - 0.120)}{0.440} = 378.18\times10^3\ \text{N}$$

$$P'_{CB} = P'_{AC} - F = 378.18\times10^3 - 520\times10^3 = -141.820\times10^3\ \text{N}$$

$$\sigma'_{AC} = \frac{P'_{AC}}{A} = \frac{378.18\times10^3}{1200\times10^{-6}} = 315.150\times10^6\ \text{Pa}$$

$$\sigma'_{BC} = \frac{P'_{BC}}{A} = -\frac{141.820\times10^3}{1200\times10^{-6}} = -118.183\times10^6\ \text{Pa}$$

$$\delta'_C = \frac{(378.18\times10^3)(0.120)}{(200\times10^9)(1200\times10^{-6})} = 0.189090\times10^{-3}\ \text{m}$$

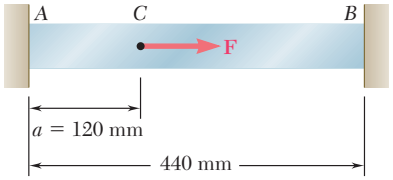
(*a*) $\delta_{C,p} = \delta_C - \delta'_C = 0.29333\times10^{-3} - 0.189090\times10^{-3} = 0.104240\times10^{-3}\ \text{m}$ $= 0.1042\ \text{mm}$ ◀

(*b*) $\sigma_{AC,\text{res}} = \sigma_Y - \sigma'_{AC} = 250\times10^6 - 315.150\times10^6 = -65.150\times10^6\ \text{Pa}$ $= -65.2\ \text{MPa}$ ◀

$\sigma_{CB,\text{res}} = \sigma_{CB} - \sigma'_{CB} = -183.333\times10^6 + 118.183\times10^6 = -65.150\times10^6\ \text{Pa}$ $= -65.2\ \text{MPa}$ ◀

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PROBLEM 2.123*



Solve Prob. 2.122, assuming that $a = 180$ mm.

PROBLEM 2.122 Bar *AB* has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point *C*, (*b*) the residual stress in the bar.

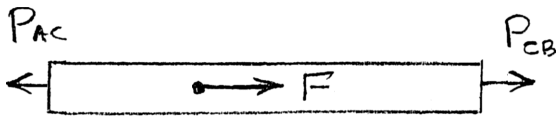
SOLUTION

$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

Force to yield portion *AC*: $P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^{6})$

$$= 300 \times 10^{3} \text{ N}$$

For equilibrium, $F + P_{CB} - P_{AC} = 0$.



$$P_{CB} = P_{AC} - F = 300 \times 10^{3} - 520 \times 10^{3}$$

$$= -220 \times 10^{3} \text{ N}$$

$$\delta_C = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^{3})(0.440 - 0.180)}{(200 \times 10^{9})(1200 \times 10^{-6})}$$

$$= 0.23833 \times 10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = -\frac{220 \times 10^{3}}{1200 \times 10^{-6}}$$

$$= -183.333 \times 10^{6} \text{ Pa}$$

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PROBLEM 2.123* *(Continued)*

Unloading:

$$\delta'_C = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} = \frac{(F - P'_{AC})L_{CB}}{EA}$$

$$= P'_{AC}\left(\frac{L_{AC}}{EA} + \frac{L_{BC}}{EA}\right) = \frac{FL_{CB}}{EA}$$

$$P'_{AC} = \frac{FL_{CB}}{L_{AC} + L_{CB}} = \frac{(520\times10^3)(0.440 - 0.180)}{0.440} = 307.27\times10^3 \text{ N}$$

$$P'_{CB} = P'_{AC} - F = 307.27\times10^3 - 520\times10^3 = -212.73\times10^3 \text{ N}$$

$$\delta'_C = \frac{(307.27\times10^3)(0.180)}{(200\times10^9)(1200\times10^{-6})} = 0.23045\times10^{-3} \text{ m}$$

$$\sigma'_{AC} = \frac{P'_{AC}}{A} = \frac{307.27\times10^3}{1200\times10^{-6}} = 256.058\times10^6 \text{ Pa}$$

$$\sigma'_{CB} = \frac{P'_{CB}}{A} = \frac{-212.73\times10^3}{1200\times10^{-6}} = -177.275\times10^6 \text{ Pa}$$

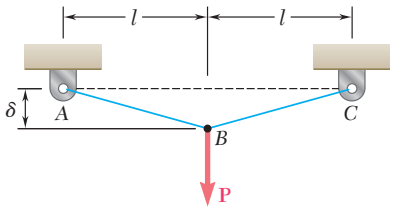
(*a*) $\delta_{C,p} = \delta_C - \delta'_C = 0.23833\times10^{-3} - 0.23045\times10^{-3} = 0.00788\times10^{-3}$ m $= 0.00788$ mm ◀

(*b*) $\sigma_{AC,\text{res}} = \sigma_{AC} - \sigma'_{AC} = 250\times10^6 - 256.058\times10^6 = -6.0580\times10^6$ Pa $= -6.06$ MPa ◀

$\sigma_{CB,\text{res}} = \sigma_{CB} - \sigma'_{CB} = -183.333\times10^6 + 177.275\times10^6 = -6.0580\times10^6$ Pa $= -6.06$ MPa ◀

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PROBLEM 2.124



The uniform wire *ABC*, of unstretched length 2*l*, is attached to the supports shown and a vertical load **P** is applied at the midpoint *B*. Denoting by *A* the cross-sectional area of the wire and by *E* the modulus of elasticity, show that, for $\delta \ll l$, the deflection at the midpoint *B* is

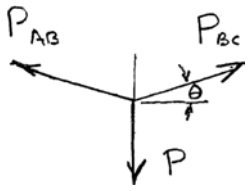
$$\delta = l\sqrt[3]{\frac{P}{AE}}$$

SOLUTION

Use approximation.

$$\sin\theta \approx \tan\theta \approx \frac{\delta}{l}$$

Statics: $+\uparrow \Sigma F_Y = 0: \quad 2P_{AB}\sin\theta - P = 0$

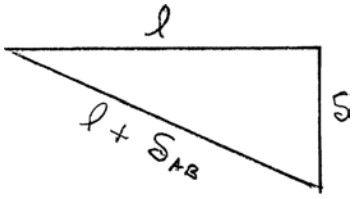


$$P_{AB} = \frac{P}{2\sin\theta} \approx \frac{Pl}{2\delta}$$

Elongation: $\delta_{AB} = \frac{P_{AB}l}{AE} = \frac{Pl^2}{2AE\delta}$

Deflection:

From the right triangle,



$$(l+\delta_{AB})^2 = l^2 + \delta^2$$

$$\delta^2 = l^2 + 2l\delta_{AB} + \delta_{AB}^2 - l^2$$

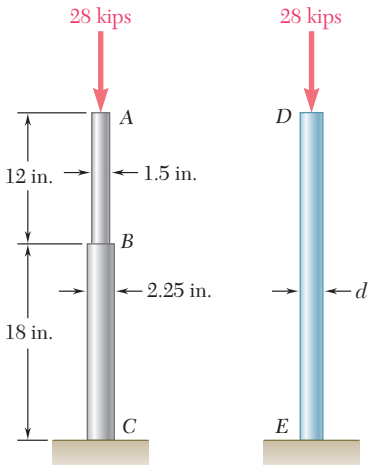
$$= 2l\delta_{AB}\left(1 + \frac{1}{2}\frac{\delta_{AB}}{l}\right) \approx 2l\delta_{AB}$$

$$\approx \frac{Pl^3}{AE\delta}$$

$$\delta^3 \approx \frac{Pl^3}{AE} \quad \therefore \quad \delta \approx l\sqrt[3]{\frac{P}{AE}}$$

◀

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PROBLEM 2.125

The aluminum rod ABC ($E = 10.1 \times 10^6$ psi), which consists of two cylindrical portions AB and BC, is to be replaced with a cylindrical steel rod DE ($E = 29 \times 10^6$ psi) of the same overall length. Determine the minimum required diameter d of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 24 ksi.

SOLUTION

Deformation of aluminum rod.

$$\delta_A = \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E}$$

$$= \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$= \frac{28 \times 10^3}{10.1 \times 10^6}\left(\frac{12}{\frac{\pi}{4}(1.5)^2} + \frac{18}{\frac{\pi}{4}(2.25)^2}\right)$$

$$= 0.031376 \text{ in.}$$

Steel rod. $\delta = 0.031376$ in.

$$\delta = \frac{PL}{EA} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(28 \times 10^3)(30)}{(29 \times 10^6)(0.031376)} = 0.92317 \text{ in}^2$$

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{28 \times 10^3}{24 \times 10^3} = 1.16667 \text{ in}^2$$

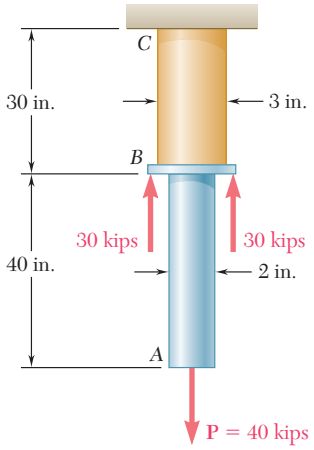
Required area is the larger value. $A = 1.16667$ in^2

Diameter:

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(1.16667)}{\pi}}$$

$d = 1.219$ in. ◀

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PROBLEM 2.126

Two solid cylindrical rods are joined at *B* and loaded as shown. Rod *AB* is made of steel ($E = 29\times10^6$ psi), and rod *BC* of brass ($E = 15\times10^6$ psi). Determine (*a*) the total deformation of the composite rod *ABC*, (*b*) the deflection of point *B*.

SOLUTION

Portion *AB*:

$$P_{AB} = 40\times10^3 \text{ lb}$$
$$L_{AB} = 40 \text{ in.}$$
$$d = 2 \text{ in.}$$
$$A_{AB} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(2)^2 = 3.1416 \text{ in}^2$$
$$E_{AB} = 29\times10^6 \text{ psi}$$
$$\delta_{AB} = \frac{P_{AB}L_{AB}}{E_{AB}A_{AB}} = \frac{(40\times10^3)(40)}{(29\times10^6)(3.1416)} = 17.5619\times10^{-3} \text{ in.}$$

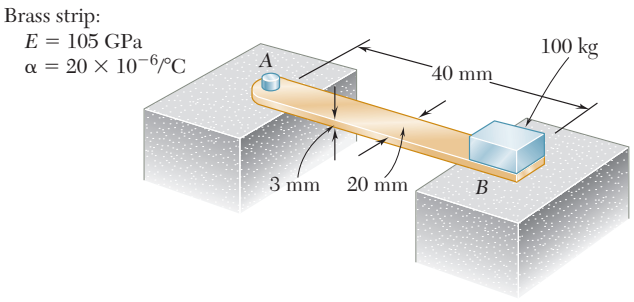
Portion *BC*:

$$P_{BC} = -20\times10^3 \text{ lb}$$
$$L_{BC} = 30 \text{ in.}$$
$$d = 3 \text{ in.}$$
$$A_{BC} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(3)^2 = 7.0686 \text{ in}^2$$
$$E_{BC} = 15\times10^6 \text{ psi}$$
$$\delta_{BC} = \frac{P_{BC}L_{BC}}{E_{BC}A_{BC}} = \frac{(-20\times10^3)(30)}{(15\times10^6)(7.0686)} = -5.6588\times10^{-3} \text{ in.}$$

(*a*) $\delta = \delta_{AB} + \delta_{BC} = 17.5619\times10^{-6} - 5.6588\times10^{-6}$ $\qquad \delta = 11.90\times10^{-3} \text{ in.}\downarrow$ ◀

(*b*) $\delta_B = -\delta_{BC}$ $\qquad \delta_B = 5.66\times10^{-3} \text{ in.}\uparrow$ ◀

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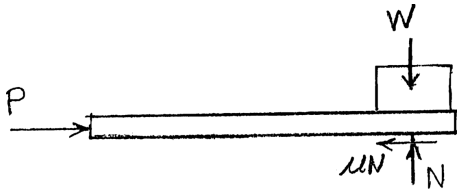


PROBLEM 2.127

The brass strip *AB* has been attached to a fixed support at *A* and rests on a rough support at *B*. Knowing that the coefficient of friction is 0.60 between the strip and the support at *B*, determine the decrease in temperature for which slipping will impend.

SOLUTION

Brass strip:



$$E = 105 \text{ GPa}$$
$$\alpha = 20 \times 10^{-6}/^{\circ}\text{C}$$

$$+\uparrow \Sigma F_y = 0: \quad N - W = 0 \qquad N = W$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - \mu N = 0 \qquad P = \mu W = \mu mg$$

$$\delta = -\frac{PL}{EA} + L\alpha(\Delta T) = 0 \qquad \Delta T = \frac{P}{EA\alpha} = \frac{\mu mg}{EA\alpha}$$

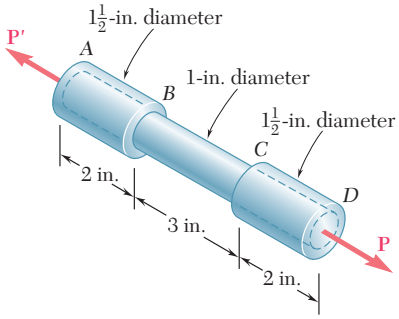
Data: $\mu = 0.60$

$$A = (20)(3) = 60 \text{ mm}^2 = 60 \times 10^{-6} \text{ m}^2$$
$$m = 100 \text{ kg}$$
$$g = 9.81 \text{ m/s}^2$$
$$E = 105 \times 10^9 \text{ Pa}$$

$$\Delta T = \frac{(0.60)(100)(9.81)}{(105 \times 10^9)(60 \times 10^{-6})(20 \times 10^6)}$$

$$\Delta T = 4.67^{\circ}\text{C} \blacktriangleleft$$

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PROBLEM 2.128

The specimen shown is made from a 1-in.-diameter cylindrical steel rod with two 1.5-in.-outer-diameter sleeves bonded to the rod as shown. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the load **P** so that the total deformation is 0.002 in., (*b*) the corresponding deformation of the central portion *BC*.

SOLUTION

(*a*) $$\delta = \Sigma \frac{P_i L_i}{A_i E_i} = \frac{P}{E} \Sigma \frac{L_i}{A_i}$$

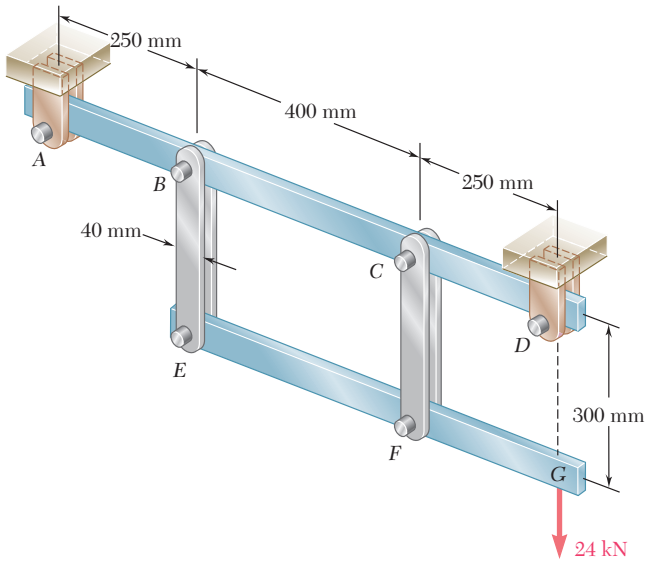
$$P = E\delta \left(\Sigma \frac{L_i}{A_i} \right)^{-1} \qquad A_i = \frac{\pi}{4} d_i^2$$

	L, in.	d, in.	A, in^2	L/A, in^{-1}
AB	2	1.5	1.7671	1.1318
BC	3	1.0	0.7854	3.8197
CD	2	1.5	1.7671	1.1318
				6.083 ← sum

$$P = (29 \times 10^6)(0.002)(6.083)^{-1} = 9.353 \times 10^3 \text{ lb} \qquad P = 9.53 \text{ kips} \blacktriangleleft$$

(*b*) $$\delta_{BC} = \frac{PL_{BC}}{A_{BC} E} = \frac{P}{E} \frac{L_{BC}}{A_{BC}} = \frac{9.535 \times 10^3}{29 \times 10^6} (3.8197) \qquad \delta = 1.254 \times 10^{-3} \text{ in.} \blacktriangleleft$$

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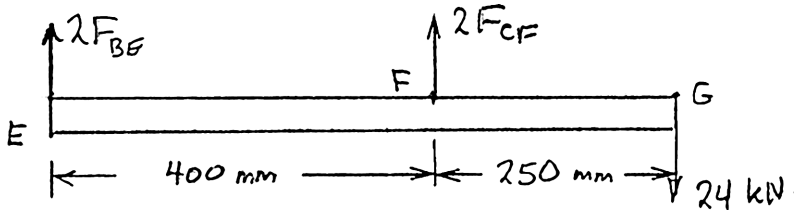


PROBLEM 2.129

Each of the four vertical links connecting the two rigid horizontal members is made of aluminum ($E = 70$ GPa) and has a uniform rectangular cross section of 10 × 40 mm. For the loading shown, determine the deflection of (*a*) point *E*, (*b*) point *F*, (c) point *G*.

SOLUTION

Statics. Free body *EFG*:



$$+\circlearrowleft \Sigma M_F = 0: \quad -(400)(2F_{BE}) - (250)(24) = 0$$

$$F_{BE} = -7.5 \text{ kN} = -7.5 \times 10^3 \text{ N}$$

$$+\circlearrowleft \Sigma M_E = 0: \quad (400)(2F_{CF}) - (650)(24) = 0$$

$$F_{CF} = 19.5 \text{ kN} = 19.5 \times 10^3 \text{ N}$$

Area of one link:

$$A = (10)(40) = 400 \text{ mm}^2$$

$$= 400 \times 10^{-6} \text{ m}^2$$

Length: $L = 300 \text{ mm} = 0.300 \text{ m}$

Deformations.

$$\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{(-7.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = -80.357 \times 10^{-6} \text{ m}$$

$$\delta_{CF} = \frac{F_{CF}L}{EA} = \frac{(19.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = 208.93 \times 10^{-6} \text{ m}$$

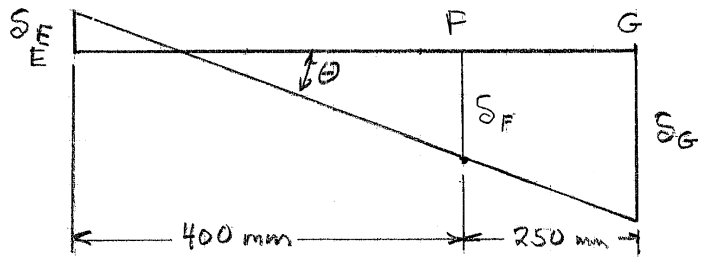
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PROBLEM 2.129 *(Continued)*

(*a*) Deflection of Point *E*. $\delta_E = |\delta_{BF}|$ $\delta_E = 80.4\ \mu\text{m} \uparrow$ ◀

(*b*) Deflection of Point *F*. $\delta_F = \delta_{CF}$ $\delta_F = 209\ \mu\text{m} \downarrow$ ◀

Geometry change.



Let θ be the small change in slope angle.

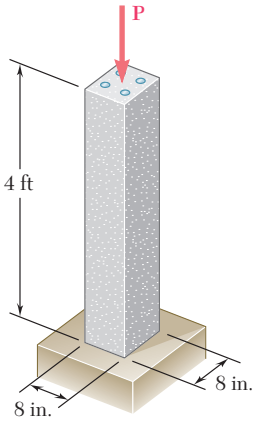
$$\theta = \frac{\delta_E + \delta_F}{L_{EF}} = \frac{80.357\times10^{-6} + 208.93\times10^{-6}}{0.400} = 723.22\times10^{-6}\ \text{radians}$$

(*c*) Deflection of Point *G*. $\delta_G = \delta_F + L_{FG}\,\theta$

$$\delta_G = \delta_F + L_{FG}\,\theta = 208.93\times10^{-6} + (0.250)(723.22\times10^{-6})$$
$$= 389.73\times10^{-6}\,\text{m}$$

$\delta_G = 390\ \mu\text{m} \downarrow$ ◀

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PROBLEM 2.130

A 4-ft concrete post is reinforced with four steel bars, each with a $\frac{3}{4}$-in. diameter. Knowing that $E_s = 29\times10^6$ psi and $E_c = 3.6\times10^6$ psi, determine the normal stresses in the steel and in the concrete when a 150-kip axial centric force **P** is applied to the post.

SOLUTION

$$A_s = 4\left[\frac{\pi}{4}\left(\frac{3}{4}\right)^2\right] = 1.76715 \text{ in}^2$$

$$A_c = 8^2 - A_s = 62.233 \text{ in}^2$$

$$\delta_s = \frac{P_s L}{A_s E_s} = \frac{P_s(48)}{(1.76715)(29\times10^6)} = 0.93663\times10^{-6} P_s$$

$$\delta_c = \frac{P_c L}{A_c E_c} = \frac{P_c(48)}{(62.233)(3.6\times10^6)} = 0.21425\times10^{-6} P_c$$

But $\delta_s = \delta_c$: $0.93663\times10^{-6} P_s = 0.21425\times10^{-6} P_c$

$$P_s = 0.22875 P_c \qquad (1)$$

Also, $$P_s + P_c = P = 150 \text{ kips} \qquad (2)$$

Substituting (1) into (2), $1.22875 P_c = 150$ kips

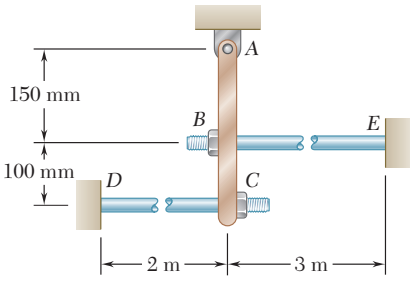
$$P_c = 122.075 \text{ kips}$$

From (1), $P_s = 0.22875(122.075) = 27.925$ kips

$$\sigma_s = -\frac{P_s}{A_s} = -\frac{27.925}{1.76715} \qquad \sigma_s = -15.80 \text{ ksi} \blacktriangleleft$$

$$\sigma_c = -\frac{P_c}{A_c} = -\frac{122.075}{62.233} \qquad \sigma_c = -1.962 \text{ ksi} \blacktriangleleft$$

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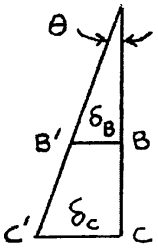


PROBLEM 2.131

The steel rods *BE* and *CD* each have a 16-mm diameter ($E = 200$ GPa); the ends of the rods are single-threaded with a pitch of 2.5 mm. Knowing that after being snugly fitted, the nut at *C* is tightened one full turn, determine (*a*) the tension in rod *CD*, (*b*) the deflection of point *C* of the rigid member *ABC*.

SOLUTION

Let θ be the rotation of bar *ABC* as shown.



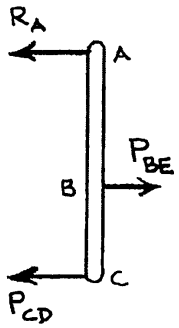
Then $\delta_B = 0.15\theta \qquad \delta_C = 0.25\theta$

But $\delta_C = \delta_{\text{turn}} - \dfrac{P_{CD}L_{CD}}{E_{CD}A_{CD}}$

$$P_{CD} = \frac{E_{CD}A_{CD}}{L_{CD}}(\delta_{\text{turn}} - \delta_C)$$

$$= \frac{(200\times10^9\ \text{Pa})\frac{\pi}{4}(0.016\ \text{m})^2}{2\,\text{m}}(0.0025\,\text{m} - 0.25\theta)$$

$$= 50.265\times10^3 - 5.0265\times10^6\theta$$



$$\delta_B = \frac{P_{BE}L_{BE}}{E_{BE}A_{BE}} \quad \text{or} \quad P_{BE} = \frac{E_{BE}A_{BE}}{L_{BE}}\delta_B$$

$$P_{BE} = \frac{(200\times10^9\ \text{Pa})\frac{\pi}{4}(0.016\ \text{m})^2}{3\,\text{m}}(0.15\theta)$$

$$= 2.0106\times10^6\theta$$

From free body of member *ABC*:

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.15P_{BE} - 0.25P_{CD} = 0$$

$$0.15(2.0106\times10^6\theta) - 0.25(50.265\times10^3 - 5.0265\times10^6\theta) = 0$$

$$\theta = 8.0645\times10^{-3}\ \text{rad}$$

(*a*) $P_{CD} = 50.265\times10^3 - 5.0265\times10^6(8.0645\times10^{-3})$

$$= 9.7288\times10^3\ \text{N}$$

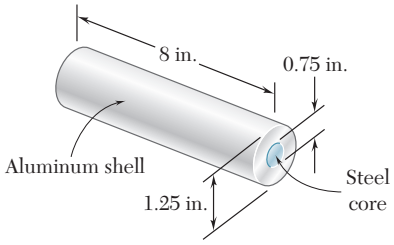
$P_{CD} = 9.73\ \text{kN}$ ◀

(*b*) $\delta_C = 0.25\theta = 0.25(8.0645\times10^{-3})$

$$= 2.0161\times10^{-3}\ \text{m}$$

$\delta_C = 2.02\ \text{mm} \leftarrow$ ◀

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PROBLEM 2.132

The assembly shown consists of an aluminum shell ($E_a = 10.6 \times 10^6$ psi, $\alpha_a = 12.9 \times 10^{-6}$/°F) fully bonded to a steel core ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}$/°F) and is unstressed. Determine (*a*) the largest allowable change in temperature if the stress in the aluminum shell is not to exceed 6 ksi, (*b*) the corresponding change in length of the assembly.

SOLUTION

Since $\alpha_a > \alpha_s$, the shell is in compression for a positive temperature rise.

Let $\sigma_a = -6 \text{ ksi} = -6 \times 10^3 \text{ psi}$

$$A_a = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4}(1.25^2 - 0.75^2) = 0.78540 \text{ in}^2$$

$$A_s = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2$$

$$P = -\sigma_a A_a = \sigma_s A_s$$

where P is the tensile force in the steel core.

$$\sigma_s = -\frac{\sigma_a A_a}{A_s} = \frac{(6 \times 10^3)(0.78540)}{0.44179} = 10.667 \times 10^3 \text{ psi}$$

$$\varepsilon = \frac{\sigma_s}{E_s} + \alpha_s(\Delta T) = \frac{\sigma_a}{E_a} + \alpha_a(\Delta T)$$

$$(\alpha_a - \alpha_s)(\Delta T) = \frac{\sigma_s}{E_s} - \frac{\sigma_a}{E_a}$$

$$(6.4 \times 10^{-6})(\Delta T) = \frac{10.667 \times 10^3}{29 \times 10^6} + \frac{6 \times 10^3}{10.6 \times 10^6} = 0.93385 \times 10^{-3}$$

(*a*) $\Delta T = 145.91°\text{F}$ $\Delta T = 145.9°\text{F}$ ◀

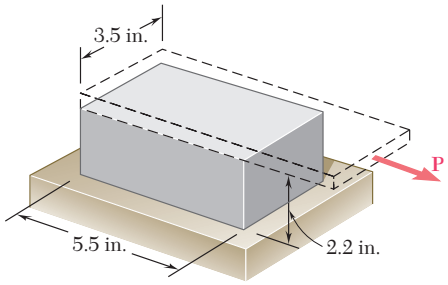
(*b*) $$\varepsilon = \frac{10.667 \times 10^3}{29 \times 10^6} + (6.5 \times 10^{-6})(145.91) = 1.3163 \times 10^{-3}$$

or

$$\varepsilon = \frac{-6 \times 10^3}{10.6 \times 10^6} + (12.9 \times 10^{-6})(145.91) = 1.3163 \times 10^{-3}$$

$\delta = L\varepsilon = (8.0)(1.3163 \times 10^{-3}) = 0.01053$ in. $\delta = 0.01053$ in. ◀

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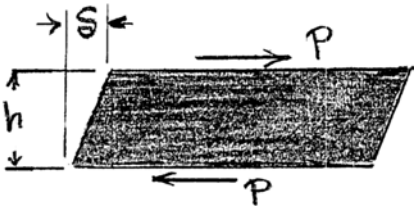
PROBLEM 2.133

The plastic block shown is bonded to a fixed base and to a horizontal rigid plate to which a force **P** is applied. Knowing that for the plastic used $G = 55$ ksi, determine the deflection of the plate when $P = 9$ kips.

SOLUTION

Consider the plastic block. The shearing force carried is $P = 9 \times 10^3$ lb

The area is $A = (3.5)(5.5) = 19.25 \text{ in}^2$



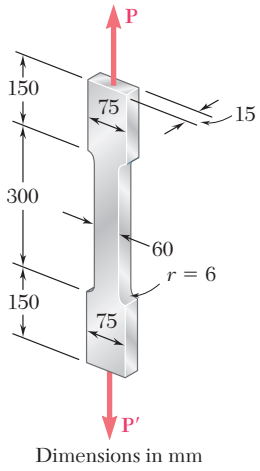
Shearing stress: $\tau = \dfrac{P}{A} = \dfrac{9 \times 10^3}{19.25} = 467.52 \text{ psi}$

Shearing strain: $\gamma = \dfrac{\tau}{G} = \dfrac{467.52}{55 \times 10^3} = 0.0085006$

But $\gamma = \dfrac{\delta}{h} \quad \therefore \quad \delta = h\gamma = (2.2)(0.0085006)$

$\delta = 0.01870 \text{ in.}$ ◀

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Dimensions in mm

PROBLEM 2.134

The aluminum test specimen shown is subjected to two equal and opposite centric axial forces of magnitude P. (*a*) Knowing that $E = 70$ GPa and $\sigma_{all} = 200$ MPa, determine the maximum allowable value of P and the corresponding total elongation of the specimen. (*b*) Solve part *a*, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60 × 15-mm rectangular cross section.

SOLUTION

$$\sigma_{all} = 200\times10^{6}\ \text{Pa} \quad E = 70\times10^{9}\ \text{Pa}$$

$$A_{min} = (60\ \text{mm})(15\ \text{mm}) = 900\ \text{mm}^2 = 900\times10^{-6}\ \text{m}^2$$

(*a*) Test specimen. $D = 75$ mm, $d = 60$ mm, $r = 6$ mm

$$\frac{D}{d} = \frac{75}{60} = 1.25 \quad \frac{r}{d} = \frac{6}{60} = 0.10$$

From Fig. 2.60b, $K = 1.95$ $\quad \sigma_{max} = K\dfrac{P}{A}$

$$P = \frac{A\sigma_{max}}{K} = \frac{(900\times10^{-6})(200\times10^{6})}{1.95} = 92.308\times10^{3}\ \text{N} \qquad P = 92.3\ \text{kN} \blacktriangleleft$$

Wide area $A^{*} = (75\ \text{mm})(15\ \text{mm}) = 1125\ \text{mm}^2 = 1.125\times10^{-3}\ \text{m}^2$

$$\delta = \Sigma\frac{P_iL_i}{A_iE_i} = \frac{P}{E}\Sigma\frac{L_i}{A_i} = \frac{92.308\times10^{3}}{70\times10^{9}}\left[\frac{0.150}{1.125\times10^{-3}} + \frac{0.300}{900\times10^{-6}} + \frac{0.150}{1.125\times10^{-3}}\right]$$

$$= 7.91\times10^{-6}\ \text{m} \qquad \delta = 0.791\ \text{mm} \blacktriangleleft$$

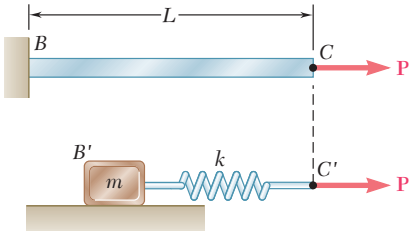
(*b*) Uniform bar.

$$P = A\sigma_{all} = (900\times10^{-6})(200\times10^{6}) = 180\times10^{3}\ \text{N} \qquad P = 180.0\ \text{kN} \blacktriangleleft$$

$$\delta = \frac{PL}{AE} = \frac{(180\times10^{3})(0.600)}{(900\times10^{-6})(70\times10^{9})} = 1.714\times10^{-3}\ \text{m} \qquad \delta = 1.714\ \text{mm} \blacktriangleleft$$

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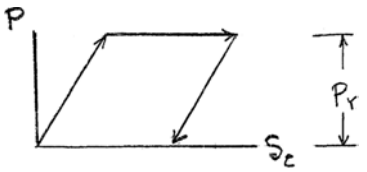
PROBLEM 2.135



The uniform rod BC has a cross-sectional area A and is made of a mild steel that can be assumed to be elastoplastic with a modulus of elasticity E and a yield strength σ_Y. Using the block-and-spring system shown, it is desired to simulate the deflection of end C of the rod as the axial force **P** is gradually applied and removed, that is, the deflection of points C and C' should be the same for all values of P. Denoting by μ the coefficient of friction between the block and the horizontal surface, derive an expression for (*a*) the required mass m of the block, (*b*) the required constant k of the spring.

SOLUTION

Force-deflection diagram for Point C or rod BC.

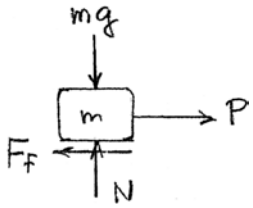


For $P < P_Y = A\sigma_Y$

$$\delta_C = \frac{PL}{EA} \qquad P = \frac{EA}{L}\delta_C$$

$$P_{\max} = P_Y = A\sigma_Y$$

Force-deflection diagram for Point C' of block-and-spring system.



$$+\uparrow \Sigma F_y = 0: \quad N - mg = 0 \qquad N = mg$$

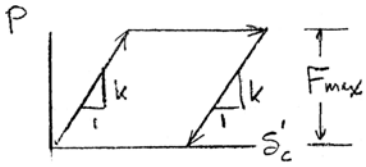
$$\xrightarrow{+} \Sigma F_x = 0: \quad P - F_f = 0 \qquad P = F_f$$

If block does not move, i.e., $F_f < \mu N = \mu mg$ or $P < \mu mg$,

then $$\delta_c' = \frac{P}{K} \quad \text{or} \quad P = k\delta_c'$$

If $P = \mu mg$, then slip at $P = F_m = \mu mg$ occurs.

If the force P is the removed, the spring returns to its initial length.

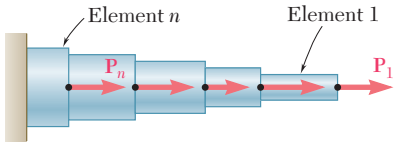


(*a*) Equating P_Y and $F_{\max}$, $A\sigma_Y = \mu mg \qquad m = \dfrac{A\sigma_Y}{\mu g}$ ◀

(*b*) Equating slopes, $k = \dfrac{EA}{L}$ ◀

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PROBLEM 2.C1



A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (*a*) Write a computer program that can be used to determine the average normal stress in each element, the deformation of each element, and the total deformation of the rod. (*b*) Use this program to solve Probs. 2.20 and 2.126.

SOLUTION

For each element, enter

$$L_i, \quad A_i, \quad E_i$$

Compute deformation

Update axial load $P = P + P_i$

Compute for each element

$$\sigma_i = P/A_i$$
$$\delta_i = PL_i/A_iE_i$$

Total deformation:

Update through n elements

$$\delta = \delta + \delta_i$$

Program Outputs

Problem 2.20

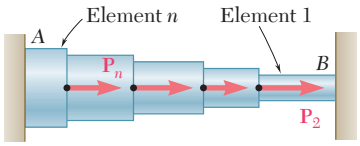
Element	Stress (MPa)	Deformation (mm)
1	19.0986	0.1091
2	–12.7324	–0.0909
Total Deformation =		0.0182 mm

Problem 2.126

Element	Stress (ksi)	Deformation (in.)
1	12.7324	0.0176
2	–2.8294	–0.0057
Total Deformation =		0.01190 in.

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PROBLEM 2.C2



Rod *AB* is horizontal with both ends fixed; it consists of *n* elements, each of which is homogeneous and of uniform cross section, and is subjected to the loading shown. The length of element *i* is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (Note that $P_1 = 0$.) (*a*) Write a computer program which can be used to determine the reactions at *A* and *B*, the average normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs. 2.41 and 2.42.

SOLUTION

We Consider the reaction at *B* redundant and release the rod at *B*

Compute δ_B with $R_B = 0$

For each element, enter

$$L_i, \quad A_i, \quad E_i$$

Update axial load

$$P = P + P_i$$

Compute for each element

$$\sigma_i = P/A_i$$
$$\delta_i = PL_i/A_iE_i$$

Update total deformation

$$\delta_B = \delta_B + \delta_i$$

Compute δ_B due to unit load at *B*

$$\text{Unit} \quad \sigma_i = 1/A_i$$
$$\text{Unit} \quad \delta_i = L_i/A_iE_i$$

Update total unit deformation

$$\text{Unit} \quad \delta_B = \text{Unit } \delta_B + \text{Unit } \delta_i$$

Superposition

For total displacement at $B = 0$

$$\delta_B + R_B \quad \text{Unit} \quad \delta_B = 0$$

Solving:

$$R_B = -\delta_B/\text{Unit } \delta_B$$

Then: $$R_A = \Sigma P_i + R_B$$

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PROBLEM 2.C2 *(Continued)*

For each element

$$\sigma = \sigma_i + R_B \quad \text{Unit } \sigma_i$$
$$\delta = \delta_i + R_B \quad \text{Unit } \delta_i$$

Program Outputs

Problem 2.41

RA = –62.809 kN
RB = –37.191 kN

Element	Stress (MPa)	Deformation (mm)
1	–52.615	–0.05011
2	3.974	0.00378
3	2.235	0.00134
4	49.982	0.04498

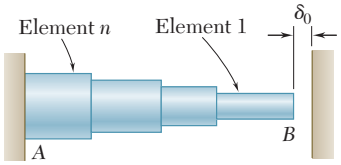
Problem 2.42

RA = –45.479 kN
RB = –54.521 kN

Element	Stress (MPa)	Deformation (mm)
1	–77.131	–0.03857
2	–20.542	–0.01027
3	–11.555	–0.01321
4	36.191	0.06204

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PROBLEM 2.C3



Rod *AB* consists of *n* elements, each of which is homogeneous and of uniform cross section. End *A* is fixed, while initially there is a gap δ_0 between end *B* and the fixed vertical surface on the right. The length of element *i* is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and its coefficient of thermal expansion by α_i. After the temperature of the rod has been increased by ΔT, the gap at *B* is closed and the vertical surfaces exert equal and opposite forces on the rod. (*a*) Write a computer program which can be used to determine the magnitude of the reactions at *A* and *B*, the normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs. 2.59 and 2.60.

SOLUTION

We compute the displacements at *B*.

Assuming there is no support at *B*,

enter $L_i, \quad A_i, \quad E_i, \quad \alpha_i$

Enter temperature change *T*. Compute for each element.

$$\delta_i = \alpha_i L_i T$$

Update total deformation.

$$\delta_B = \delta_B + \delta_i$$

Compute δ_B due to unit load at *B*.

$$\text{Unit } \delta_i = L_i / A_i E_i$$

Update total unit deformation.

$$\text{Unit } \delta_B = \text{Unit } \delta_B + \text{Unit } \delta_i$$

Compute reactions.

From superposition,

$$R_B = (\delta_B - \delta_0)/\text{Unit } \delta_B$$

Then

$$R_A = -R_B$$

For each element,

$$\sigma_i = -R_B / A_i$$
$$\delta_i = \alpha_i L_i T + R_B L_i / A_i E_i$$

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PROBLEM 2.C3 *(Continued)*

Program Outputs

Problem 2.59.

$R = 52.279$ kips

Element	Stress (ksi)	Deformation (10*–3 in.)
1	–21.783	9.909
2	–18.671	10.091

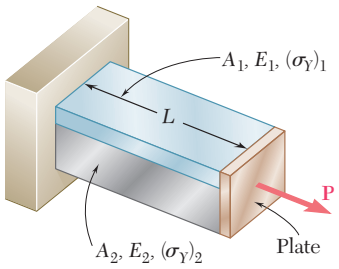
Problem 2.60.

$R = 232.390$ kN

Element	Stress (MPa)	Deformation (microm)
1	–116.195	363.220
2	–290.487	136.780

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PROBLEM 2.C4



Bar *AB* has a length L and is made of two different materials of given cross-sectional area, modulus of elasticity, and yield strength. The bar is subjected as shown to a load **P** that is gradually increased from zero until the deformation of the bar has reached a maximum value δ_m and then decreased back to zero. (*a*) Write a computer program that, for each of 25 values of δ_m equally spaced over a range extending from 0 to a value equal to 120% of the deformation causing both materials to yield, can be used to determine the maximum value P_m of the load, the maximum normal stress in each material, the permanent deformation δ_p of the bar, and the residual stress in each material. (*b*) Use this program to solve Probs. 2.111 and 2.112.

SOLUTION

Note: The following assumes $(\sigma_Y)_1 < (\sigma_Y)_2$

Displacement increment

$$\delta_m = 0.05(\sigma_Y)_2 L/E_2$$

Displacements at yielding

$$\delta_A = (\sigma_Y)_1 L/E_1 \quad \delta_B = (\sigma_Y)_2 L/E_2$$

For each displacement

If $\delta_m < \delta_A$:

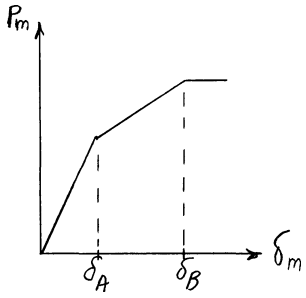
$$\sigma_1 = \delta_m E_1/L$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = (\delta_m/L)(A_1E_1 + A_2E_2)$$

If $\delta_A < \delta_m < \delta_B$:

$$\sigma_1 = (\sigma_Y)_1$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = A_1\sigma_1 + (\delta_m/L)A_2E_2$$

If $\delta_m > \delta_B$:

$$\sigma_1 = (\sigma_Y)_1$$
$$\sigma_2 = (\sigma_Y)_2$$
$$P_m = A_1\sigma_1 + A_2\sigma_2$$



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PROBLEM 2.C4 *(Continued)*

Permanent deformations, residual stresses

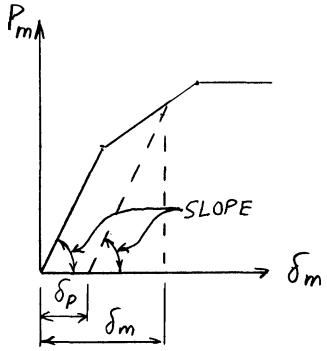
Slope of first (elastic) segment

$$\text{Slope} = (A_1E_1 + A_2E_2)/L$$

$$\delta_P = \delta_m - (P_m/\text{Slope})$$

$$(\sigma_1)_{\text{res}} = \sigma_1 - (E_1P_m/(L\ \text{Slope}))$$

$$(\sigma_2)_{\text{res}} = \sigma_2 - (E_2P_m/(L\ \text{Slope}))$$



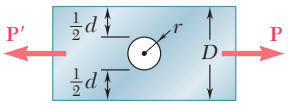
Program Outputs

Problems 2.111 and 2.112

DM 10** – 3 in.	PM kips	SIGM (1) ksi	SIGM (2) ksi	DP 10** – 3 in.	SIGR (1) ksi	SIG (2) ksi	
0.000	0.000	0.000	0.000	0.000	0.000	0.000	
2.414	8.750	5.000	5.000	0.000	0.000	0.000	
4.828	17.500	10.000	10.000	0.000	0.000	0.000	
7.241	26.250	15.000	15.000	0.000	0.000	0.000	
9.655	35.000	20.000	20.000	0.000	0.000	0.000	
12.069	43.750	25.000	25.000	0.000	0.000	0.000	
14.483	52.500	30.000	30.000	0.000	0.000	0.000	
16.897	61.250	35.000	35.000	0.000	0.000	0.000	
19.310	70.000	40.000	40.000	0.000	0.000	0.000	
21.724	78.750	45.000	45.000	0.000	0.000	0.000	
24.138	87.500	50.000	50.000	0.000	0.000	0.000	
26.552	91.250	50.000	55.000	1.379	–2.143	2.857	
28.966	95.000	50.000	60.000	2.759	–4.286	5.714	
31.379	98.750	50.000	65.000	4.138	–6.429	8.571	2.112 ◀
33.793	102.500	50.000	70.000	5.517	–8.571	11.429	
36.207	106.250	50.000	75.000	6.897	–10.714	14.286	
38.621	110.000	50.000	80.000	8.276	–12.857	17.143	
41.034	113.750	50.000	85.000	9.655	–15.000	20.000	2.111 ◀
43.448	117.500	50.000	90.000	11.034	–17.143	22.857	
45.862	121.250	50.000	95.000	12.414	–19.286	25.714	
48.276	125.000	50.000	100.000	13.793	–21.429	28.571	
50.690	125.000	50.000	100.000	16.207	–21.429	28.571	
53.103	125.000	50.000	100.000	18.621	–21.429	28.571	
55.517	125.000	50.000	100.000	21.034	–21.429	28.571	
57.931	125.000	50.000	100.000	23.448	–21.429	28.571	

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PROBLEM 2.C5



The plate has a hole centered across the width. The stress concentration factor for a flat bar under axial loading with a centric hole is

$$K = 3.00 - 3.13\left(\frac{2r}{D}\right) + 3.66\left(\frac{2r}{D}\right)^2 - 1.53\left(\frac{2r}{D}\right)^3$$

where r is the radius of the hole and D is the width of the bar. Write a computer program to determine the allowable load **P** for the given values of r, D, the thickness t of the bar, and the allowable stress σ_{all} of the material. Knowing that $t = \frac{1}{4}$ in., $D = 3.0$ in., and $\sigma_{all} = 16$ ksi, determine the allowable load **P** for values of r from 0.125 in. to 0.75 in., using 0.125 in. increments.

SOLUTION

Enter

$$r, \quad D, \quad t, \quad \sigma_{\text{all}}$$

Compute K

$$RD = 2.0r/D$$

$$K = 3.00 - 3.13RD + 3.66RD^2 - 1.53RD^3$$

Compute average stress

$$\sigma_{\text{ave}} = \sigma_{\text{all}}/K$$

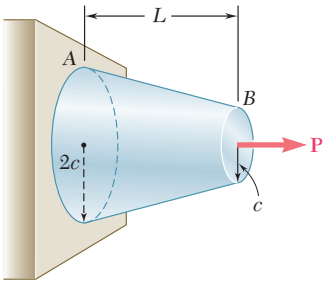
Allowable load

$$P_{\text{all}} = \sigma_{\text{ave}}(D - 2.0r)t$$

Program Output

Radius (in.)	Allowable Load (kips)
0.1250	3.9802
0.2500	3.8866
0.3750	3.7154
0.5000	3.4682
0.6250	3.1523
0.7500	2.7794

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PROBLEM 2.C6

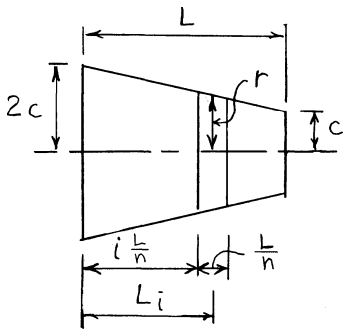
A solid truncated cone is subjected to an axial force **P** as shown. The exact elongation is $(PL)/(2\pi c^2 E)$. By replacing the cone by n circular cylinders of equal thickness, write a computer program that can be used to calculate the elongation of the truncated cone. What is the percentage error in the answer obtained from the program using (*a*) $n = 6$, (*b*) $n = 12$, (*c*) $n = 60$?

SOLUTION

For $i = 1$ to n:

$$L_i = (i + 0.5)(L/n)$$

$$r_i = 2c - c(L_i/L)$$



Area:

$$A = \pi r_i^2$$

Displacement:

$$\delta = \delta + P(L/n)/(AE)$$

Exact displacement:

$$\delta_{\text{exact}} = PL/(2.0\pi c^2 E)$$

Percentage error:

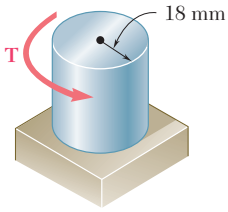
$$\text{Percent} = 100(\delta - \delta_{\text{exact}})/\delta_{\text{exact}}$$

Program Output

n	Approximate	Exact	Percent
6	0.15852	0.15915	–0.40083
12	0.15899	0.15915	–0.10100
60	0.15915	0.15915	–0.00405

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CHAPTER 3



PROBLEM 3.1

Determine the torque **T** that causes a maximum shearing stress of 70 MPa in the steel cylindrical shaft shown.

SOLUTION

$$\tau_{\max} = \frac{Tc}{J}; \quad J = \frac{\pi}{2}c^4$$

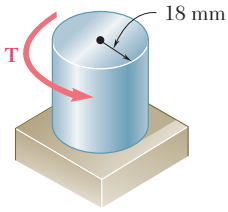
$$T = \frac{\pi}{2}c^3\tau_{\max}$$

$$= \frac{\pi}{2}(0.018\text{ m})^3(70 \times 10^6\text{ Pa})$$

$$= 641.26\text{ N} \cdot \text{m}$$

$$T = 641\text{ N} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 3.2

For the cylindrical shaft shown, determine the maximum shearing stress caused by a torque of magnitude $T = 800\ \text{N} \cdot \text{m}$.

SOLUTION

$$\tau_{\max} = \frac{Tc}{J}; \quad J = \frac{\pi}{2}c^4$$

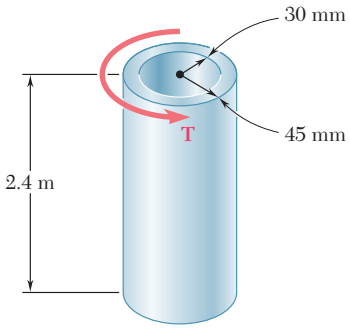
$$\tau_{\max} = \frac{2T}{\pi c^3}$$

$$= \frac{2(800\ \text{N} \cdot \text{m})}{\pi(0.018\ \text{m})^3}$$

$$= 87.328 \times 10^6\ \text{Pa}$$

$$\tau_{\max} = 87.3\ \text{MPa} \blacktriangleleft$$

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PROBLEM 3.3

(*a*) Determine the torque **T** that causes a maximum shearing stress of 45 MPa in the hollow cylindrical steel shaft shown. (*b*) Determine the maximum shearing stress caused by the same torque **T** in a solid cylindrical shaft of the same cross-sectional area.

SOLUTION

(*a*) Given shaft:

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)$$

$$J = \frac{\pi}{2}(45^4 - 30^4) = 5.1689 \times 10^6 \text{ mm}^4 = 5.1689 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c}$$

$$T = \frac{(5.1689 \times 10^{-6})(45 \times 10^6)}{45 \times 10^{-3}} = 5.1689 \times 10^3 \text{ N} \cdot \text{m}$$

$$T = 5.17 \text{ kN} \cdot \text{m} \blacktriangleleft$$

(*b*) Solid shaft of same area:

$$A = \pi\left(c_2^2 - c_1^2\right) = \pi(45^2 - 30^2) = 3.5343 \times 10^3 \text{ mm}^2$$

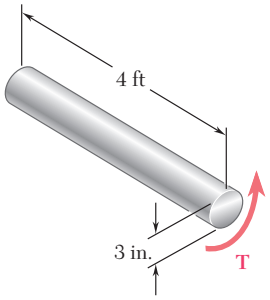
$$\pi c^2 = A \quad \text{or} \quad c = \sqrt{\frac{A}{\pi}} = 33.541 \text{ mm}$$

$$J = \frac{\pi}{2}c^4, \quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau = \frac{(2)(5.1689 \times 10^3)}{\pi(0.033541)^3} = 87.2 \times 10^6 \text{ Pa}$$

$$\tau = 87.2 \text{ MPa} \blacktriangleleft$$

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PROBLEM 3.4

(*a*) Determine the maximum shearing stress caused by a 40-kip · in. torque **T** in the 3-in.-diameter solid aluminum shaft shown. (*b*) Solve part *a*, assuming that the solid shaft has been replaced by a hollow shaft of the same outer diameter and of 1-in. inner diameter.

SOLUTION

(*a*)

$$\tau = \frac{Tc}{J}$$

$$= \frac{(40 \text{ kip} \cdot \text{in.})(1.5 \text{ in.})}{\left(\frac{\pi}{2}\right)(1.5 \text{ in.})^4}$$

$$= 7.5451 \text{ ksi}$$

$$\tau = 7.55 \text{ ksi} \blacktriangleleft$$

(*b*)

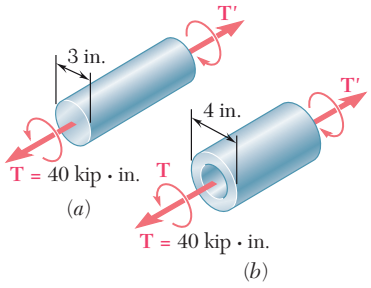
$$\tau = \frac{Tc}{J}$$

$$= \frac{(40 \text{ kip} \cdot \text{in.})(1.5 \text{ in.})}{\frac{\pi}{2}[(1.5 \text{ in.})^4 - (0.5 \text{ in.})^4]}$$

$$= 7.6394 \text{ ksi}$$

$$\tau = 7.64 \text{ ksi} \blacktriangleleft$$

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PROBLEM 3.5

(*a*) For the 3-in.-diameter solid cylinder and loading shown, determine the maximum shearing stress. (*b*) Determine the inner diameter of the 4-in.-diameter hollow cylinder shown, for which the maximum stress is the same as in part *a*.

SOLUTION

(*a*) Solid shaft:

$$c = \frac{1}{2}d = \frac{1}{2}(3.0 \text{ in.}) = 1.5 \text{ in.}$$

$$J = \frac{\pi}{2}c^4$$

$$\tau_{\max} = \frac{Tc}{J}$$

$$= \frac{2T}{\pi c^3}$$

$$= \frac{2(40 \text{ kip} \cdot \text{in.})}{\pi(1.5 \text{ in.})^3}$$

$$= 7.5451 \text{ ksi}$$

$$\tau_{\max} = 7.55 \text{ ksi} \blacktriangleleft$$

(*b*) Hollow shaft:

$$c_o = \frac{1}{2}d = \frac{1}{2}(4.0 \text{ in.}) = 2.0 \text{ in.}$$

$$\frac{J}{c_o} = \frac{\frac{\pi}{2}\left(c_o^4 - c_i^4\right)}{c_o} = \frac{T}{\tau_{\max}}$$

$$c_i^4 = c_o^4 - \frac{2Tc_o}{\pi\tau_{\max}}$$

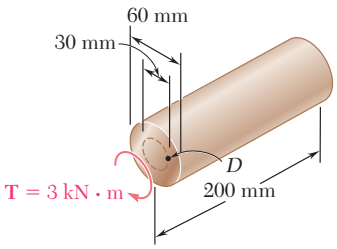
$$= (2.0 \text{ in.})^4 - \frac{2(40 \text{ kip} \cdot \text{in.})(2.0 \text{ in.})}{\pi(7.5451 \text{ ksi})}$$

$$= 9.2500 \text{ in}^4 \quad \therefore \quad c_i = 1.74395 \text{ in.}$$

and $d_i = 2c_i = 3.4879$ in.

$$d_i = 3.49 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.6

A torque $T = 3\ \text{kN} \cdot \text{m}$ is applied to the solid bronze cylinder shown. Determine (*a*) the maximum shearing stress, (*b*) the shearing stress at point *D* which lies on a 15-mm-radius circle drawn on the end of the cylinder, (*c*) the percent of the torque carried by the portion of the cylinder within the 15-mm radius.

SOLUTION

(*a*) $$c = \frac{1}{2}d = 30\ \text{mm} = 30 \times 10^{-3}\ \text{m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(30 \times 10^{-3})^4 = 1.27235 \times 10^{-6}\ \text{m}^4$$

$$T = 3\ \text{kN} = 3 \times 10^3\ \text{N}$$

$$\tau_m = \frac{Tc}{J} = \frac{(3 \times 10^3)(30 \times 10^{-3})}{1.27235 \times 10^{-6}} = 70.736 \times 10^6\ \text{Pa}$$

$\tau_m = 70.7\ \text{MPa}$ ◀

(*b*) $$\rho_D = 15\ \text{mm} = 15 \times 10^{-3}\ \text{m}$$

$$\tau_D = \frac{\rho_D}{c}\tau = \frac{(15 \times 10^{-3})(70.736 \times 10^{-6})}{(30 \times 10^{-3})}$$

$\tau_D = 35.4\ \text{MPa}$ ◀

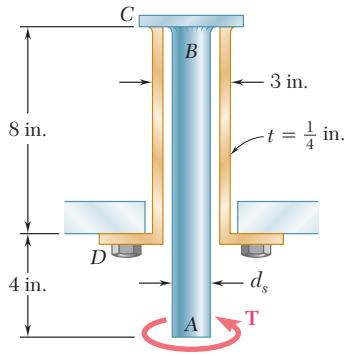
(*c*) $$\tau_D = \frac{T_D \rho_D}{J_D} \qquad T_D = \frac{J_D \tau_D}{\rho_D} = \frac{\pi}{2}\rho_D^3 \tau_D$$

$$T_D = \frac{\pi}{2}(15 \times 10^{-3})^3(35.368 \times 10^6) = 187.5\ \text{N} \cdot \text{m}$$

$$\frac{T_D}{T} \times 100\% = \frac{187.5}{3 \times 10^3}(100\%) = 6.25\%$$

6.25% ◀

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PROBLEM 3.7

The solid spindle *AB* is made of a steel with an allowable shearing stress of 12 ksi, and sleeve *CD* is made of a brass with an allowable shearing stress of 7 ksi. Determine (*a*) the largest torque **T** that can be applied at *A* if the allowable shearing stress is not to be exceeded in sleeve CD, (*b*) the corresponding required value of the diameter d_s of spindle *AB*.

SOLUTION

(*a*) Analysis of sleeve *CD*:

$$c_2 = \frac{1}{2}d_o = \frac{1}{2}(3) = 1.5 \text{ in.}$$

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

$$T = \frac{J\tau}{c_2} = \frac{(4.1172)(7 \times 10^3)}{1.5} = 19.21 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$T = 19.21 \text{ kip} \cdot \text{in.} \blacktriangleleft$$

(*b*) Analysis of solid spindle *AB*:

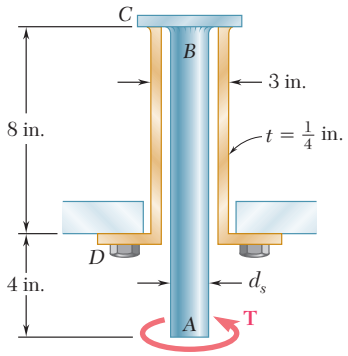
$$\tau = \frac{Tc}{J}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{T}{\tau} = \frac{19.21 \times 10^3}{12 \times 10^3} = 1.601 \text{ in}^3$$

$$c = \sqrt[3]{\frac{(2)(1.601)}{\pi}} = 1.006 \text{ in.} \quad d_s = 2c$$

$$d = 2.01 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.8

The solid spindle *AB* has a diameter $d_s = 1.5$ in. and is made of a steel with an allowable shearing stress of 12 ksi, while sleeve *CD* is made of a brass with an allowable shearing stress of 7 ksi. Determine the largest torque **T** that can be applied at *A*.

SOLUTION

Analysis of solid spindle *AB*:

$$c = \frac{1}{2} d_s = 0.75 \text{ in.}$$

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c} = \frac{\pi}{2} \tau c^3$$

$$T = \frac{\pi}{2} (12 \times 10^3)(0.75)^3 = 7.95 \times 10^3 \text{ lb} \cdot \text{in.}$$

Analysis of sleeve *CD*:

$$c_2 = \frac{1}{2} d_o = \frac{1}{2} (3) = 1.5 \text{ in.}$$

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2} \left(c_2^4 - c_1^4 \right) = \frac{\pi}{2} (1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

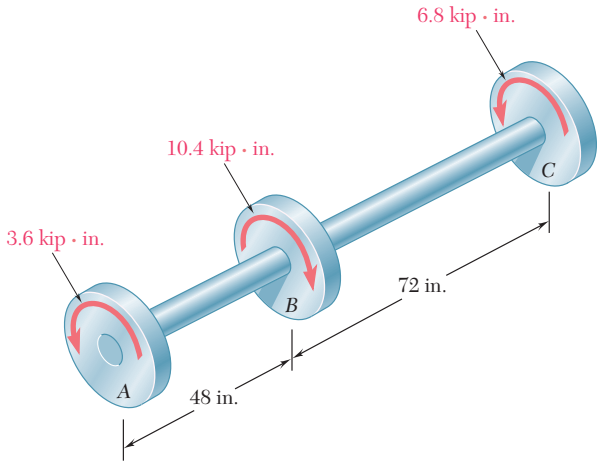
$$T = \frac{J\tau}{c_2} = \frac{(4.1172)(7 \times 10^3)}{1.5} = 19.21 \times 10^3 \text{ lb} \cdot \text{in.}$$

The smaller torque governs.

$$T = 7.95 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$T = 7.95 \text{ kip} \cdot \text{in.} \blacktriangleleft$$

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PROBLEM 3.9

The torques shown are exerted on pulleys *A*, *B*, and *C*. Knowing that both shafts are solid, determine the maximum shearing stress in (*a*) shaft *AB*, (*b*) shaft *BC*.

SOLUTION

(*a*) Shaft *AB*:

$$T_{AB} = 3.6 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$c = \frac{1}{2}d = \frac{1}{2}(1.3) = 0.65 \text{ in.}$$

$$J = \frac{\pi}{2}c^4$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau_{\max} = \frac{(2)(3.6 \times 10^3)}{\pi(0.65)^3} = 8.35 \times 10^3 \text{ psi}$$

$$\tau_{\max} = 8.35 \text{ ksi} \blacktriangleleft$$

(*b*) Shaft *BC*:

$$T_{BC} = 6.8 \times 10^3 \text{ lb} \cdot \text{in.}$$

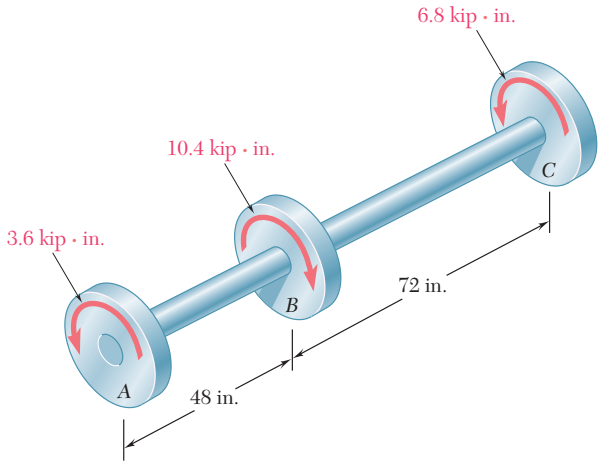
$$c = \frac{1}{2}d = \frac{1}{2}(1.8) = 0.9 \text{ in.}$$

$$J = \frac{\pi}{2}c^4$$

$$\tau_{\max} = \frac{2T_{BC}}{\pi c^3} = \frac{(2)(6.8 \times 10^3)}{\pi(0.9)^3} = 5.94 \times 10^3 \text{ psi}$$

$$\tau_{\max} = 5.94 \text{ ksi} \blacktriangleleft$$

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PROBLEM 3.10

The shafts of the pulley assembly shown are to be redesigned. Knowing that the allowable shearing stress in each shaft is 8.5 ksi, determine the smallest allowable diameter of (*a*) shaft *AB*, (*b*) shaft *BC*.

SOLUTION

(*a*) Shaft *AB*:

$$T_{AB} = 3.6 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\tau_{\max} = 8.5 \text{ ksi} = 8.5 \times 10^3 \text{ psi}$$

$$J = \frac{\pi}{2} c^4 \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T_{AB}}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(3.6 \times 10^3)}{\pi (8.5 \times 10^3)}} = 0.646 \text{ in.} \qquad d_{AB} = 2c = 1.292 \text{ in.} \blacktriangleleft$$

(*b*) Shaft *BC*:

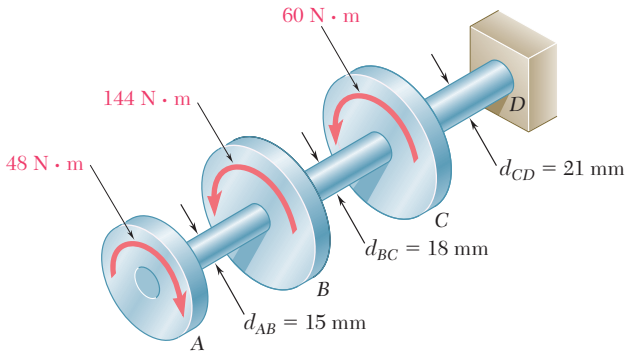
$$T_{BC} = 6.8 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\tau_{\max} = 8.5 \times 10^3 \text{ psi}$$

$$c = \sqrt[3]{\frac{2T_{BC}}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(6.8 \times 10^3)}{\pi (8.5 \times 10^3)}} = 0.7985 \text{ in.}$$

$$d_{BC} = 2c = 1.597 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.11

Knowing that each portion of the shafts *AB*, *BC*, and *CD* consist of a solid circular rod, determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.

SOLUTION

Shaft *AB*:

$$T = 48 \text{ N} \cdot \text{m}$$

$$c = \frac{1}{2}d = 7.5 \text{ mm} = 0.0075 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau_{\max} = \frac{(2)(48)}{\pi(0.0075)^3} = 72.433 \text{ MPa}$$

Shaft *BC*:

$$T = -48 + 144 = 96 \text{ N} \cdot \text{m}$$

$$c = \frac{1}{2}d = 9 \text{ mm} \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(96)}{\pi(0.009)^3} = 83.835 \text{ MPa}$$

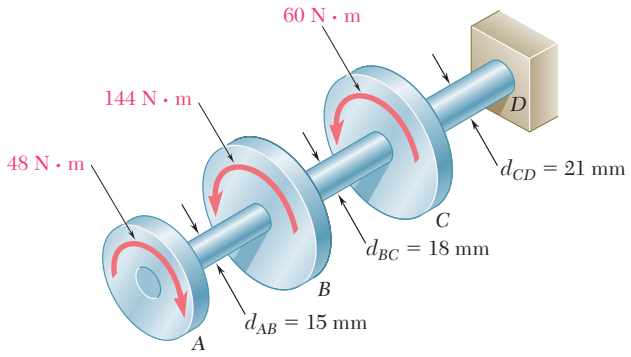
Shaft *CD*:

$$T = -48 + 144 + 60 = 156 \text{ N} \cdot \text{m}$$

$$c = \frac{1}{2}d = 10.5 \text{ mm} \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2 \times 156)}{\pi(0.0105)^3} = 85.79 \text{ MPa}$$

Answers: (*a*) Shaft *CD* (*b*) 85.8 MPa ◀

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PROBLEM 3.12

Knowing that an 8-mm-diameter hole has been drilled through each of the shafts *AB*, *BC*, and *CD*, determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.

SOLUTION

Hole: $c_1 = \frac{1}{2}d_1 = 4 \text{ mm}$

Shaft *AB*: $T = 48 \text{ N} \cdot \text{m}$

$$c_2 = \frac{1}{2}d_2 = 7.5 \text{ mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.0075^4 - 0.004^4) = 4.5679 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(48)(0.0075)}{4.5679 \times 10^{-9}} = 78.810 \text{ MPa}$$

Shaft *BC*: $T = -48 + 144 = 96 \text{ N} \cdot \text{m}$ $\quad c_2 = \frac{1}{2}d_2 = 9 \text{ mm}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.009^4 - 0.004^4) = 9.904 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(96)(0.009)}{9.904 \times 10^{-9}} = 87.239 \text{ MPa}$$

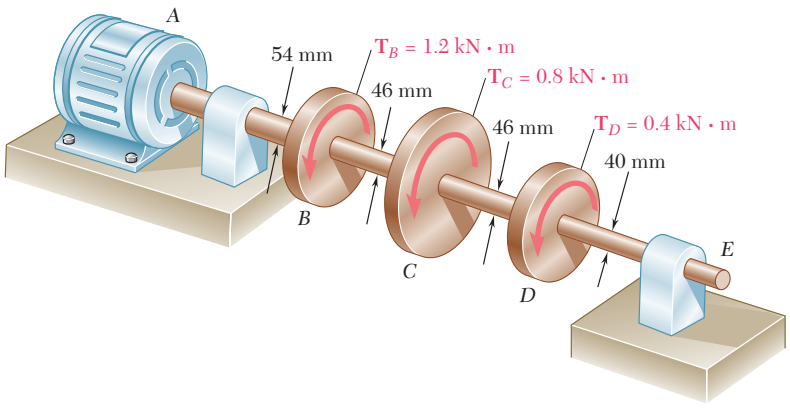
Shaft *CD*: $T = -48 + 144 + 60 = 156 \text{ N} \cdot \text{m}$ $\quad c_2 = \frac{1}{2}d_2 = 10.5 \text{ mm}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.0105^4 - 0.004^4) = 18.691 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(156)(0.0105)}{18.691 \times 10^{-9}} = 87.636 \text{ MPa}$$

Answers: (*a*) Shaft *CD* (*b*) 87.6 MPa ◀

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PROBLEM 3.13

Under normal operating conditions, the electric motor exerts a torque of 2.4 kN · m on shaft *AB*. Knowing that each shaft is solid, determine the maximum shearing stress in (*a*) shaft *AB*, (*b*) shaft *BC*, (*c*) shaft *CD*.

SOLUTION

(*a*) Shaft *AB*:

$$T_{AB} = 2.4 \times 10^3 \text{ N} \cdot \text{m}, \quad c = \frac{1}{2}d = 0.027 \text{ m}$$

$$\tau_{AB} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(2.4 \times 10^3)}{\pi (0.027)^3} = 77.625 \times 10^6 \text{ Pa}$$

77.6 MPa ◀

(*b*) Shaft *BC*:

$$T_{BC} = 2.4 \text{ kN} \cdot \text{m} - 1.2 \text{ kN} \cdot \text{m} = 1.2 \text{ kN} \cdot \text{m}, \quad c = \frac{1}{2}d = 0.023 \text{ m}$$

$$\tau_{BC} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1.2 \times 10^3)}{\pi (0.023)^3} = 62.788 \times 10^6 \text{ Pa}$$

62.8 MPa ◀

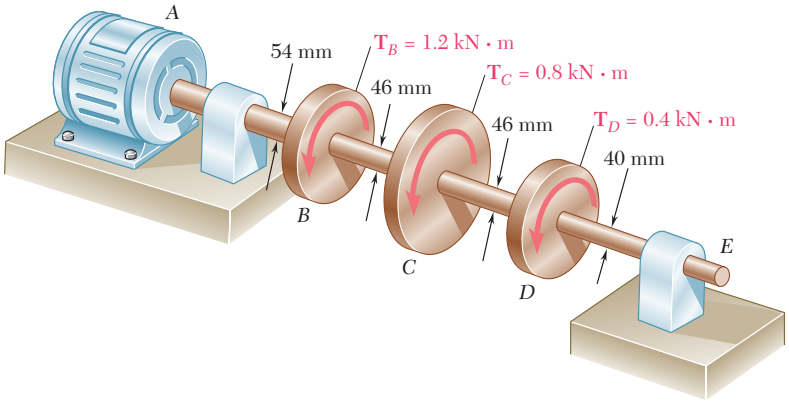
(*c*) Shaft *CD*:

$$T_{CD} = 0.4 \times 10^3 \text{ N} \cdot \text{m} \quad c = \frac{1}{2}d = 0.023 \text{ m}$$

$$\tau_{CD} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(0.4 \times 10^3)}{\pi (0.023)^3} = 20.929 \times 10^6 \text{ Pa}$$

20.9 MPa ◀

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PROBLEM 3.14

In order to reduce the total mass of the assembly of Prob. 3.13, a new design is being considered in which the diameter of shaft *BC* will be smaller. Determine the smallest diameter of shaft *BC* for which the maximum value of the shearing stress in the assembly will not be increased.

PROBLEM 3.13 Under normal operating conditions, the electric motor exerts a torque of 2.4 kN · m on shaft *AB*. Knowing that each shaft is solid, determine the maximum shearing stress in (*a*) shaft *AB*, (*b*) shaft *BC*, (*c*) shaft *CD*.

SOLUTION

See solution to Problem 3.13 for maximum shearing stresses in portions *AB*, *BC*, and *CD* of the shaft. The largest maximum shearing value is $\tau_{\max} = 77.625 \times 10^6$ Pa occurring in *AB*.

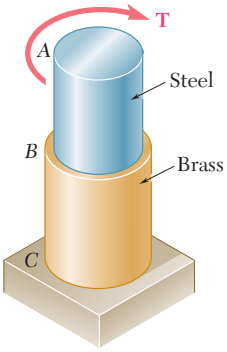
Adjust diameter of *BC* to obtain the same value of stress.

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi\tau} = \frac{(2)(1.2 \times 10^3)}{\pi(77.625 \times 10^6)} = 9.8415 \times 10^{-6}\ \text{m}^3$$

$$c = 21.43 \times 10^{-3}\ \text{m} \quad d = 2c = 42.8 \times 10^{-3}\ \text{m} \qquad 42.8\ \text{mm} \blacktriangleleft$$

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PROBLEM 3.15

The allowable shearing stress is 15 ksi in the 1.5-in.-diameter steel rod *AB* and 8 ksi in the 1.8-in.-diameter brass rod *BC*. Neglecting the effect of stress concentrations, determine the largest torque that can be applied at *A*.

SOLUTION

$$\tau_{\max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}c^4, \quad T = \frac{\pi}{2}c^3\tau_{\max}$$

Rod *AB*: $\tau_{\max} = 15 \text{ ksi} \qquad c = \frac{1}{2}d = 0.75 \text{ in.}$

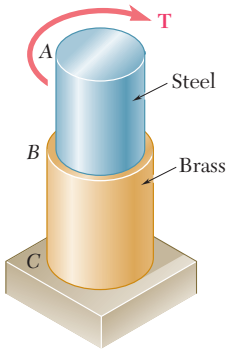
$$T = \frac{\pi}{2}(0.75)^3(15) = 9.94 \text{ kip} \cdot \text{in.}$$

Rod *BC*: $\tau_{\max} = 8 \text{ ksi} \qquad c = \frac{1}{2}d = 0.90 \text{ in.}$

$$T = \frac{\pi}{2}(0.90)^3(8) = 9.16 \text{ kip} \cdot \text{in.}$$

The allowable torque is the smaller value. $T = 9.16 \text{ kip} \cdot \text{in.}$ ◀

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PROBLEM 3.16

The allowable shearing stress is 15 ksi in the steel rod *AB* and 8 ksi in the brass rod *BC*. Knowing that a torque of magnitude $T = 10 \text{ kip} \cdot \text{in.}$ is applied at *A*, determine the required diameter of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

$$\tau_{\max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}, \quad c^3 = \frac{2T}{\pi\tau_{\max}}$$

(*a*) Rod *AB*: $T = 10 \text{ kip} \cdot \text{in.}$ $\tau_{\max} = 15 \text{ ksi}$

$$c^3 = \frac{(2)(10)}{\pi(15)} = 0.4244 \text{ in}^3$$

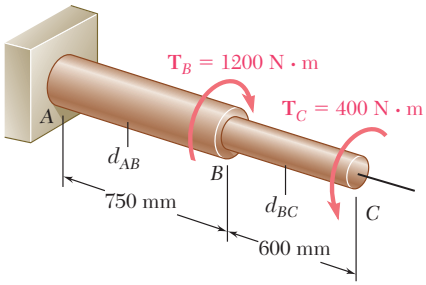
$$c = 0.7515 \text{ in.} \qquad d = 2c = 1.503 \text{ in.} \blacktriangleleft$$

(*b*) Rod *BC*: $T = 10 \text{ kip} \cdot \text{in.}$ $\tau_{\max} = 8 \text{ ksi}$

$$c^3 = \frac{(2)(10)}{\pi(8)} = 0.79577 \text{ in}^2$$

$$c = 0.9267 \text{ in.} \qquad d = 2c = 1.853 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.17

The solid shaft shown is formed of a brass for which the allowable shearing stress is 55 MPa. Neglecting the effect of stress concentrations, determine the smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded.

SOLUTION

$$\tau_{\max} = 55 \text{ MPa} = 55 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}}$$

Shaft *AB*:

$$T_{AB} = 1200 - 400 = 800 \text{ N} \cdot \text{m}$$

$$c = \sqrt[3]{\frac{(2)(800)}{\pi(55 \times 10^6)}} = 21.00 \times 10^{-3} \text{ m} = 21.0 \text{ m}$$

$$\text{minimum } d_{AB} = 2c = 42.0 \text{ mm} \blacktriangleleft$$

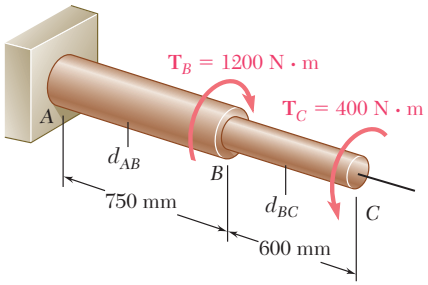
Shaft *BC*:

$$T_{BC} = 400 \text{ N} \cdot \text{m}$$

$$c = \sqrt[3]{\frac{(2)(400)}{\pi(55 \times 10^6)}} = 16.667 \times 10^{-3} \text{ m} = 16.67 \text{ mm}$$

$$\text{minimum } d_{BC} = 2c = 33.3 \text{ mm} \blacktriangleleft$$

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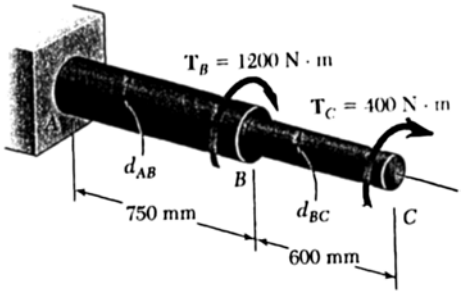
PROBLEM 3.18

Solve Prob. 3.17, assuming that the direction of $\mathbf{T}_C$ is reversed.

PROBLEM 3.17 The solid shaft shown is formed of a brass for which the allowable shearing stress is 55 MPa. Neglecting the effect of stress concentrations, determine the smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded.

SOLUTION

Note that the direction of T_C has been reversed in the figure.



$$\tau_{\max} = 55 \text{ MPa} = 55 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}}$$

Shaft *AB*:

$$T_{AB} = 1200 + 400 = 1600 \text{ N} \cdot \text{m}$$

$$c = \sqrt[3]{\frac{(2)(1600)}{\pi(55 \times 10^6)}} = 26.46 \times 10^{-3} \text{ m} = 26.46 \text{ mm}$$

$$\text{minimum } d_{AB} = 2c = 52.9 \text{ mm} \blacktriangleleft$$

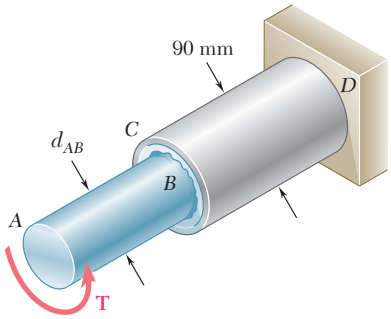
Shaft *BC*:

$$T_{BC} = 400 \text{ N} \cdot \text{m}$$

$$c = \sqrt[3]{\frac{(2)(400)}{\pi(55 \times 10^6)}} = 16.667 \times 10^{-3} \text{ m} = 16.67 \text{ mm}$$

$$\text{minimum } d_{BC} = 2c = 33.3 \text{ mm} \blacktriangleleft$$

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PROBLEM 3.19

The solid rod *AB* has a diameter $d_{AB} = 60$ mm and is made of a steel for which the allowable shearing stress is 85 MPa. The pipe *CD*, which has an outer diameter of 90 mm and a wall thickness of 6 mm, is made of an aluminum for which the allowable shearing stress is 54 MPa. Determine the largest torque **T** that can be applied at *A*.

SOLUTION

Rod *AB*:

$$\tau_{\text{all}} = 85 \times 10^6 \text{ Pa} \qquad c = \frac{1}{2}d = 0.030 \text{ m}$$

$$T_{\text{all}} = \frac{J\tau_{\text{all}}}{c} = \frac{\pi}{2}c^3\tau_{\text{all}}$$

$$= \frac{\pi}{2}(0.030)^3(85 \times 10^6) = 3.605 \times 10^3 \text{ N} \cdot \text{m}$$

Pipe *CD*:

$$\tau_{\text{all}} = 54 \times 10^6 \text{ Pa} \qquad c_2 = \frac{1}{2}d_2 = 0.045 \text{ m}$$

$$c_1 = c_2 - t = 0.045 - 0.006 = 0.039 \text{ m}$$

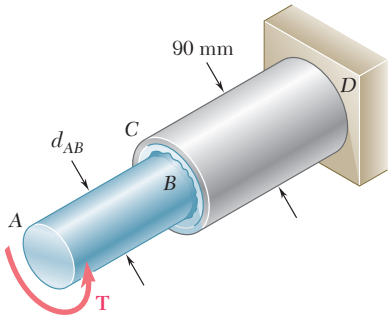
$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.045^4 - 0.039^4) = 2.8073 \times 10^{-6} \text{ m}^4$$

$$T_{\text{all}} = \frac{J\tau_{\text{all}}}{c_2} = \frac{(2.8073 \times 10^{-6})(54 \times 10^6)}{0.045} = 3.369 \times 10^3 \text{ N} \cdot \text{m}$$

Allowable torque is the smaller value. $T_{\text{all}} = 3.369 \times 10^3 \text{ N} \cdot \text{m}$

$3.37 \text{ kN} \cdot \text{m}$ ◀

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PROBLEM 3.20

The solid rod *AB* has a diameter $d_{AB} = 60$ mm. The pipe *CD* has an outer diameter of 90 mm and a wall thickness of 6 mm. Knowing that both the rod and the pipe are made of a steel for which the allowable shearing stress is 75 MPa, determine the largest torque **T** that can be applied at *A*.

SOLUTION

$$\tau_{\text{all}} = 75 \times 10^6 \text{ Pa} \quad T_{\text{all}} = \frac{J\tau_{\text{all}}}{c}$$

Rod *AB*:

$$c = \frac{1}{2}d = 0.030 \text{ m} \quad J = \frac{\pi}{2}c^4$$

$$T_{\text{all}} = \frac{\pi}{2}c^3\tau_{\text{all}} = \frac{\pi}{2}(0.030)^3(75 \times 10^6)$$

$$= 3.181 \times 10^3 \text{ N} \cdot \text{m}$$

Pipe *CD*:

$$c_2 = \frac{1}{2}d_2 = 0.045 \text{ m} \quad c_1 = c_2 - t = 0.045 - 0.006 = 0.039 \text{ m}$$

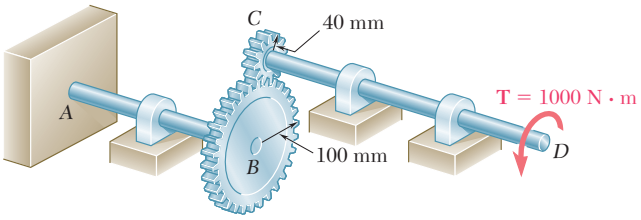
$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.045^4 - 0.039^4) = 2.8073 \times 10^{-6} \text{ m}^4$$

$$T_{\text{all}} = \frac{(2.8073 \times 10^{-6})(75 \times 10^6)}{0.045} = 4.679 \times 10^3 \text{ N} \cdot \text{m}$$

Allowable torque is the smaller value. $T_{\text{all}} = 3.18 \times 10^3 \text{ N} \cdot \text{m}$

$3.18 \text{ kN} \cdot \text{m}$ ◀

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PROBLEM 3.21

A torque of magnitude $T = 1000\ \text{N} \cdot \text{m}$ is applied at D as shown. Knowing that the allowable shearing stress is 60 MPa in each shaft, determine the required diameter of (*a*) shaft *AB*, (*b*) shaft *CD*.

SOLUTION

$$T_{CD} = 1000\ \text{N} \cdot \text{m}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500\ \text{N} \cdot \text{m}$$

(*a*) Shaft *AB*:

$$\tau_{\text{all}} = 60 \times 10^6\ \text{Pa}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(2500)}{\pi(60 \times 10^6)} = 26.526 \times 10^{-6}\ \text{m}^3$$

$$c = 29.82 \times 10^{-3} = 29.82\ \text{mm} \qquad d = 2c = 59.6\ \text{mm} \blacktriangleleft$$

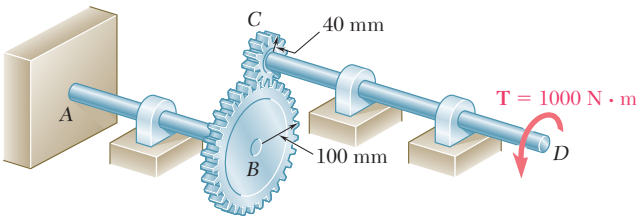
(*b*) Shaft *CD*:

$$\tau_{\text{all}} = 60 \times 10^6\ \text{Pa}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(1000)}{\pi(60 \times 10^6)} = 10.610 \times 10^{-6}\ \text{m}^3$$

$$c = 21.97 \times 10^{-3}\ \text{m} = 21.97\ \text{mm} \qquad d = 2c = 43.9\ \text{mm} \blacktriangleleft$$

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PROBLEM 3.22

A torque of magnitude $T = 1000\ \text{N} \cdot \text{m}$ is applied at D as shown. Knowing that the diameter of shaft AB is 56 mm and that the diameter of shaft CD is 42 mm, determine the maximum shearing stress in (*a*) shaft AB, (*b*) shaft CD.

SOLUTION

$$T_{CD} = 1000\ \text{N} \cdot \text{m}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500\ \text{N} \cdot \text{m}$$

(*a*) Shaft *AB*:

$$c = \frac{1}{2} d = 0.028\ \text{m}$$

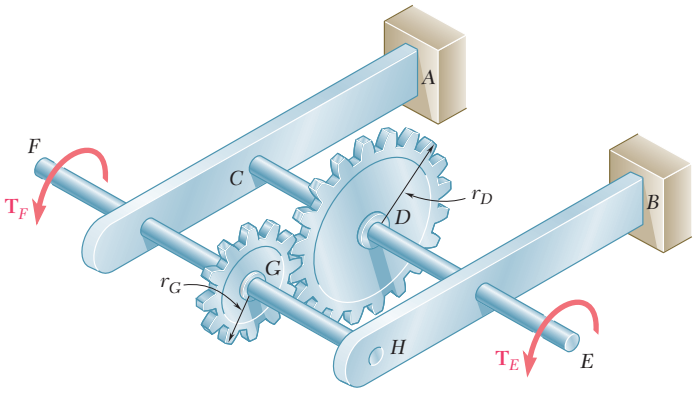
$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2500)}{\pi (0.028)^3} = 72.50 \times 10^6 \qquad 72.5\ \text{MPa} \blacktriangleleft$$

(*b*) Shaft *CD*:

$$c = \frac{1}{2} d = 0.020\ \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1000)}{\pi (0.020)^3} = 68.7 \times 10^6 \qquad 68.7\ \text{MPa} \blacktriangleleft$$

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PROBLEM 3.23

Under normal operating conditions, a motor exerts a torque of magnitude T_F at F. The shafts are made of a steel for which the allowable shearing stress is 12 ksi and have diameters $d_{CDE} = 0.900$ in. and $d_{FGH} = 0.800$ in. Knowing that $r_D = 6.5$ in. and $r_G = 4.5$ in., determine the largest allowable value of T_F.

SOLUTION

$$\tau_{\text{all}} = 12 \text{ ksi}$$

Shaft *FG*:

$$c = \frac{1}{2}d = 0.400 \text{ in.}$$

$$T_{F,\text{all}} = \frac{J\tau_{\text{all}}}{c} = \frac{\pi}{2}c^3\tau_{\text{all}}$$

$$= \frac{\pi}{2}(0.400)^3(12) = 1.206 \text{ kip} \cdot \text{in.}$$

Shaft *DE*:

$$c = \frac{1}{2}d = 0.450 \text{ in.}$$

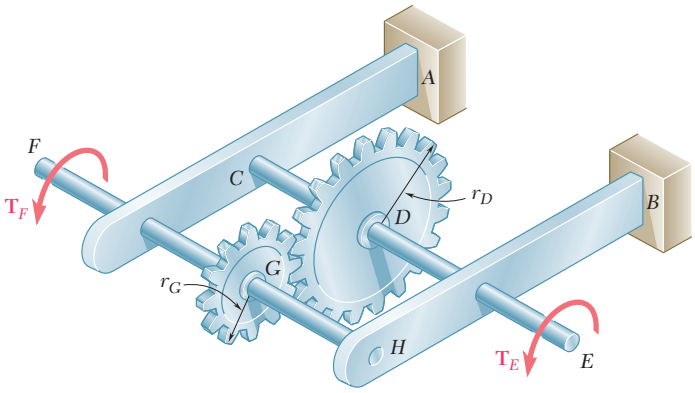
$$T_{E,\text{all}} = \frac{\pi}{2}c^3\tau_{all}$$

$$= \frac{\pi}{2}(0.450)^3(12) = 1.7177 \text{ kip} \cdot \text{in.}$$

$$T_F = \frac{r_G}{r_D}T_E \quad T_{F,\text{all}} = \frac{4.5}{6.5}(1.7177) = 1.189 \text{ kip} \cdot \text{in.}$$

Allowable value of T_F is the smaller. $T_F = 1.189 \text{ kip} \cdot \text{in.}$ ◀

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PROBLEM 3.24

Under normal operating conditions, a motor exerts a torque of magnitude $T_F = 1200$ lb · in. at F. Knowing that $r_D = 8$ in., $r_G = 3$ in., and the allowable shearing stress is 10.5 ksi in each shaft, determine the required diameter of (*a*) shaft *CDE*, (*b*) shaft *FGH*.

SOLUTION

$$T_F = 1200 \text{ lb} \cdot \text{in.}$$

$$T_E = \frac{r_D}{r_G} T_F = \frac{8}{3}(1200) = 3200 \text{ lb} \cdot \text{in.}$$

$$\tau_{\text{all}} = 10.5 \text{ ksi} = 10{,}500 \text{ psi}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \text{or} \quad c^3 = \frac{2T}{\pi\tau}$$

(*a*) Shaft *CDE*:

$$c^3 = \frac{(2)(3200)}{\pi(10{,}500)} = 0.194012 \text{ in}^3$$

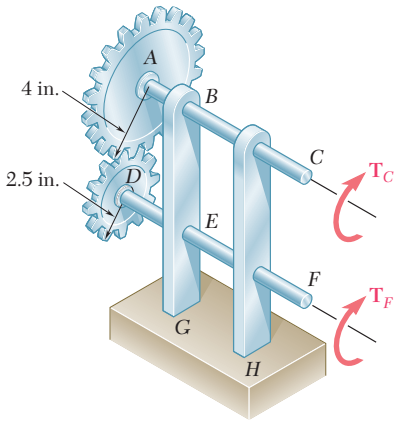
$$c = 0.5789 \text{ in.} \qquad d_{DE} = 2c \qquad d_{DE} = 1.158 \text{ in.} \blacktriangleleft$$

(*b*) Shaft *FGH*:

$$c^3 = \frac{(2)(1200)}{\pi(10{,}500)} = 0.012757 \text{ in}^3$$

$$c = 0.4174 \text{ in.} \qquad d_{FG} = 2c \qquad d_{FG} = 0.835 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.25

The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 7000 psi. Knowing the diameters of the two shafts are, respectively, $d_{BC} = 1.6$ in. and $d_{EF} = 1.25$ in., determine the largest torque $\mathbf{T}_C$ that can be applied at C.

SOLUTION

$$\tau_{\max} = 7000 \text{ psi} = 7.0 \text{ ksi}$$

Shaft *BC*: $d_{BC} = 1.6$ in.

$$c = \frac{1}{2}d = 0.8 \text{ in.}$$

$$T_C = \frac{J\tau_{\max}}{c} = \frac{\pi}{2}\tau_{\max}c^3$$

$$= \frac{\pi}{2}(7.0)(0.8)^3 = 5.63 \text{ kip} \cdot \text{in.}$$

Shaft *EF*: $d_{EF} = 1.25$ in.

$$c = \frac{1}{2}d = 0.625 \text{ in.}$$

$$T_F = \frac{J\tau_{\max}}{c} = \frac{\pi}{2}\tau_{\max}c^3$$

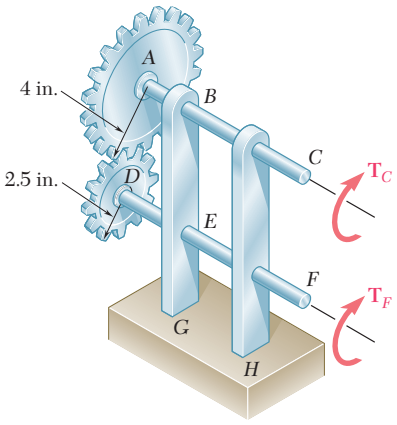
$$= \frac{\pi}{2}(7.0)(0.625)^3 = 2.684 \text{ kip} \cdot \text{in.}$$

By statics, $T_C = \frac{r_A}{r_D}T_F = \frac{4}{2.5}(2.684) = 4.30 \text{ kip} \cdot \text{in.}$

Allowable value of T_C is the smaller.

$$T_C = 4.30 \text{ kip} \cdot \text{in.} \blacktriangleleft$$

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PROBLEM 3.26

The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 8500 psi. Knowing that a torque of magnitude $T_C = 5$ kip · in. is applied at C and that the assembly is in equilibrium, determine the required diameter of (*a*) shaft *BC*, (*b*) shaft *EF*.

SOLUTION

$$\tau_{\max} = 8500 \text{ psi} = 8.5 \text{ ksi}$$

(*a*) Shaft *BC*:

$$T_C = 5 \text{ kip} \cdot \text{in.}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}}$$

$$c = \sqrt[3]{\frac{(2)(5)}{\pi(8.5)}} = 0.7208 \text{ in.}$$

$$d_{BC} = 2c = 1.442 \text{ in.} \blacktriangleleft$$

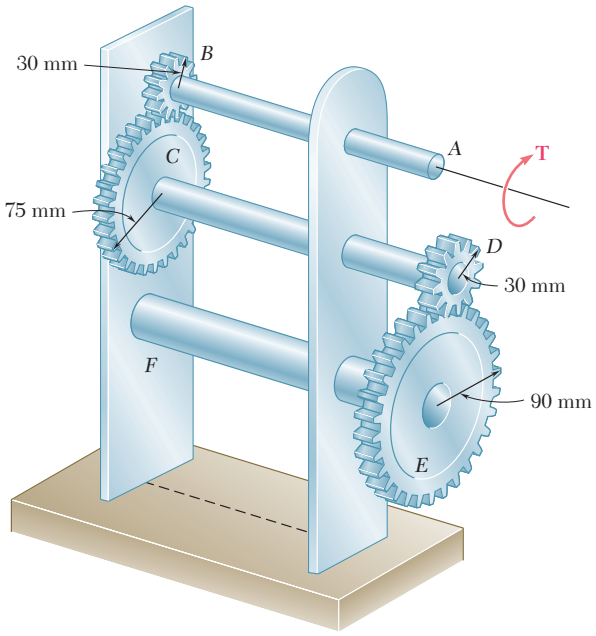
(*b*) Shaft *EF*:

$$T_F = \frac{r_D}{r_A} T_C = \frac{2.5}{4}(5) = 3.125 \text{ kip} \cdot \text{in.}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(3.125)}{\pi(8.5)}} = 0.6163 \text{ in.}$$

$$d_{EF} = 2c = 1.233 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.27

For the gear train shown, the diameters of the three solid shafts are:

$$d_{AB} = 20\,\text{mm} \quad d_{CD} = 25\,\text{mm} \quad d_{EF} = 40\,\text{mm}$$

Knowing that for each shaft the allowable shearing stress is 60 MPa, determine the largest torque **T** that can be applied.

SOLUTION

Statics: $T_{AB} = T$

$$\frac{T_{CD}}{r_C} = \frac{T_{AB}}{r_B} \qquad T_{CD} = \frac{r_C}{r_B}T_{AB} = \frac{75}{30}T = 2.5T$$

$$\frac{T_{EF}}{r_F} = \frac{T_{CD}}{r_D} \qquad T_{EF} = \frac{r_F}{r_D}T_{CD} = \frac{90}{30}(2.5T) = 7.5T$$

Determine the magnitude of T so that the stress is $60\,\text{MPa} = 60 \times 10^6\,\text{Pa}$.

$$\tau = \frac{Tc}{J} \qquad T_{\text{shaft}} = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3$$

Shaft *AB*: $c = \frac{1}{2}d_{AB} = 10\,\text{mm} = 0.010\,\text{m}$

$$T_{AB} = T = \frac{\pi}{2}(60 \times 10^6)(0.010)^3 \qquad T = 94.2\,\text{N} \cdot \text{m}$$

Shaft *CD*: $c = \frac{1}{2}d_{CD} = 12.5\,\text{mm} = 0.0125\,\text{m}$

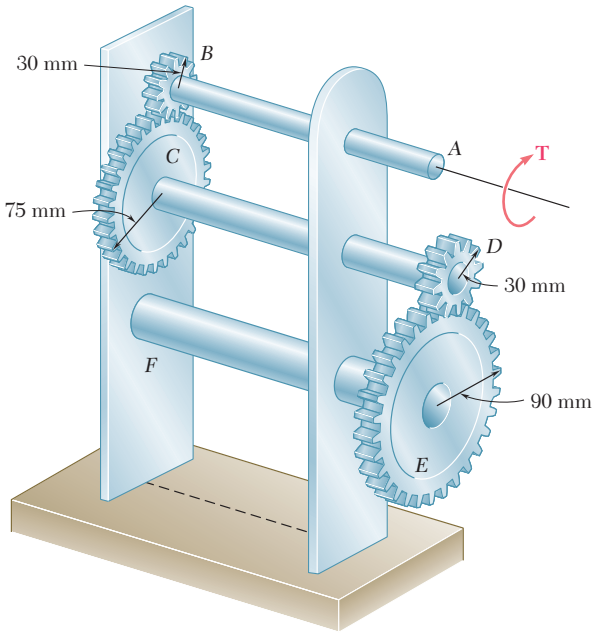
$$T_{CD} = 2.5T = \frac{\pi}{2}(60 \times 10^6)(0.0125)^3 \qquad T = 73.6\,\text{N} \cdot \text{m}$$

Shaft *EF*: $c = \frac{1}{2}d_{EF} = 20\,\text{mm} = 0.020\,\text{m}$

$$T_{EF} = 7.5T = \frac{\pi}{2}(60 \times 10^6)(0.020)^3 \qquad T = 100.5\,\text{N} \cdot \text{m}$$

The smallest value of **T** is the largest torque that can be applied. $T = 73.6\,\text{N} \cdot \text{m}$ ◀

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PROBLEM 3.28

A torque $T = 900$ N · m is applied to shaft *AB* of the gear train shown. Knowing that the allowable shearing stress is 80 MPa, determine the required diameter of (*a*) shaft *AB*, (*b*) shaft *CD*, (*c*) shaft *EF*.

SOLUTION

From statics,

$$T_{AB} = T$$

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{75 \text{ mm}}{30 \text{ mm}} T = 2.5T$$

$$T_{EF} = \frac{r_E}{r_D} T_{BC} = \frac{90 \text{ mm}}{30 \text{ mm}} (2.5T) = 7.5T$$

(*a*) Shaft *AB*:

$$T_{AB} = T = 900 \text{ N} \cdot \text{m}$$

$$\tau_{\text{all}} = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2} c^4}$$

$$c^3 = \frac{2}{\pi} \frac{T}{\tau_{\text{all}}}$$

$$c^3 = \frac{2}{\pi} \frac{T_{AB}}{\tau_{\text{all}}} = \frac{2}{\pi} \frac{900 \text{ N} \cdot \text{m}}{80 \text{ MPa}}$$

$$c = 19.2757 \text{ mm} \rightarrow d_{AB} = 38.6 \text{ mm} \blacktriangleleft$$

(*b*) Shaft *CD*:

$$T_{CD} = 2.5T = 2.5(900 \text{ N} \cdot \text{m})$$

$$c^3 = \frac{2}{\pi} \frac{T_{CD}}{\tau_{\text{all}}} = \frac{2}{\pi} \frac{2250 \text{ N} \cdot \text{m}}{80 \text{ MPa}}$$

$$c = 26.161 \text{ mm} \rightarrow d_{CD} = 52.3 \text{ mm} \blacktriangleleft$$

(*c*) Shaft *EF*:

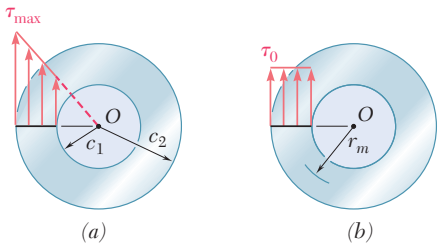
$$T_{EF} = 7.5T = 7.5(900 \text{ N} \cdot \text{m})$$

$$c^3 = \frac{2}{\pi} \frac{T_{EF}}{\tau_{\text{all}}} = \frac{2}{\pi} \frac{6750 \text{ N} \cdot \text{m}}{80 \text{ MPa}}$$

$$c = 37.731 \text{ mm} \rightarrow d_{EF} = 75.5 \text{ mm} \blacktriangleleft$$

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PROBLEM 3.29



While the exact distribution of the shearing stresses in a hollow-cylindrical shaft is as shown in Fig. *a*, an approximate value can be obtained for τ_{max} by assuming that the stresses are uniformly distributed over the area A of the cross section, as shown in Fig. *b*, and then further assuming that all of the elementary shearing forces act at a distance from O equal to the mean radius $\frac{1}{2}(c_1 + c_2)$ of the cross section. This approximate value $\tau_0 = T/Ar_m$, where T is the applied torque. Determine the ratio τ_{max}/τ_0 of the true value of the maximum shearing stress and its approximate value τ_0 for values of c_1/c_2, respectively, equal to 1.00, 0.95, 0.75, 0.50, and 0.

SOLUTION

For a hollow shaft,

$$\tau_{max} = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)} = \frac{2Tc_2}{\pi\left(c_2^2 - c_1^2\right)\left(c_2^2 + c_1^2\right)} = \frac{2Tc_2}{A\left(c_2^2 + c_1^2\right)}$$

By definition,

$$\tau_0 = \frac{T}{Ar_m} = \frac{2T}{A(c_2 + c_1)}$$

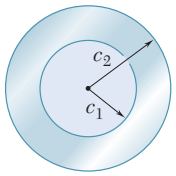
Dividing,

$$\frac{\tau_{max}}{\tau_0} = \frac{c_2(c_2 + c_1)}{c_2^2 + c_1^2} = \frac{1 + (c_1/c_2)}{1 + (c_1/c_2)^2}$$ ◀

c_1/c_2	1.0	0.95	0.75	0.5	0.0
τ_{max}/τ_0	1.0	1.025	1.120	1.200	1.0

◀

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PROBLEM 3.30

(*a*) For a given allowable shearing stress, determine the ratio T/w of the maximum allowable torque T and the weight per unit length w for the hollow shaft shown. (*b*) Denoting by $(T/w)_0$ the value of this ratio for a solid shaft of the same radius c_2, express the ratio T/w for the hollow shaft in terms of $(T/w)_0$ and c_1/c_2.

SOLUTION

w = weight per unit length

ρg = specific weight

W = total weight

L = length

$$w = \frac{W}{L} = \frac{\rho g L A}{L} = \rho g A = \rho g \pi \left(c_2^2 - c_1^2\right)$$

$$T_{\text{all}} = \frac{J\tau_{\text{all}}}{c_2} = \frac{\pi}{2}\frac{c_2^4 - c_1^4}{c_2}\tau_{\text{all}} = \frac{\pi}{2}\frac{\left(c_2^2 + c_1^2\right)\left(c_2^2 - c_1^2\right)}{c_2}\tau_{\text{all}}$$

(*a*) $\dfrac{T}{W} = \left(c_1^2 + c_2^2\right)\tau_{\text{all}}$ $\qquad \dfrac{T}{w} = \dfrac{\left(c_1^2 + c_2^2\right)\tau_{\text{all}}}{2\rho g c_2}$ (hollow shaft) ◀

$c_1 = 0$ for solid shaft $\qquad \left(\dfrac{T}{w}\right)_0 = \dfrac{c_2\tau_{\text{all}}}{2\rho g}$ (solid shaft)

(*b*) $\dfrac{(T/w)_h}{(T/w)_0} = 1 + \dfrac{c_1^2}{c_2^2}$ $\qquad \left(\dfrac{T}{w}\right) = \left(\dfrac{T}{w}\right)_0\left(1 + \dfrac{c_1^2}{c_2^2}\right)$ ◀

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PROBLEM 3.31

Determine the largest allowable diameter of a 3-m-long steel rod ($G = 77.2$ GPa) if the rod is to be twisted through 30° without exceeding a shearing stress of 80 MPa.

SOLUTION

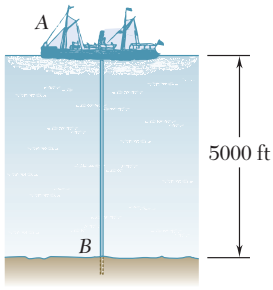
$$L = 3\,\text{m}, \qquad \varphi = \frac{30\pi}{180} = 523.6 \times 10^{-3}\,\text{rad}, \qquad \tau = 80 \times 10^{6}\,\text{Pa}$$

$$\varphi = \frac{TL}{GJ}, \qquad T = \frac{GJ\varphi}{L}, \qquad \tau = \frac{Tc}{J} = \frac{GJ\varphi c}{JL} = \frac{G\varphi c}{L}, \qquad c = \frac{\tau L}{G\varphi}$$

$$c = \frac{(80 \times 10^{6})(3.0)}{(77.2 \times 10^{9})(523.6 \times 10^{-3})} = 5.9374 \times 10^{-3}\,\text{m} = 5.9374\,\text{mm}$$

$$d = 2c = 11.87 \blacktriangleleft$$

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PROBLEM 3.32

The ship at *A* has just started to drill for oil on the ocean floor at a depth of 5000 ft. Knowing that the top of the 8-in.-diameter steel drill pipe ($G = 11.2 \times 10^6$ psi) rotates through two complete revolutions before the drill bit at *B* starts to operate, determine the maximum shearing stress caused in the pipe by torsion.

SOLUTION

$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L}$$

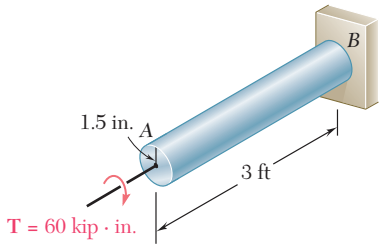
$$\tau = \frac{Tc}{J} = \frac{GJ\varphi c}{JL} = \frac{G\varphi c}{L}$$

$$\varphi = 2\,\text{rev} = (2)(2\pi) = 12.566\ \text{rad}, \qquad c = \frac{1}{2}d = 4.0\ \text{in.}$$

$$L = 5000\ \text{ft} = 60{,}000\ \text{in.}$$

$$\tau = \frac{(11.2 \times 10^6)(12.566)(4.0)}{60{,}000} = 9.3826 \times 10^3\ \text{psi} \qquad \tau = 9.38\ \text{ksi} \blacktriangleleft$$

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PROBLEM 3.33

(*a*) For the solid steel shaft shown, determine the angle of twist at *A*. Use $G = 11.5 \times 10^6$ psi. (*b*) Solve part *a*, assuming that the steel shaft is hollow with a 1.5-in. outer radius and a 0.75-in. inner radius.

SOLUTION

(*a*)

$$c = 1.5 \text{ in.} \qquad J = \frac{\pi}{2}(1.5 \text{ in.})^4 = 7.9522 \text{ in}^4$$

$$\phi = \frac{TL}{GJ} = \frac{(60 \text{ kip} \cdot \text{in.})(36 \text{ in.})}{(11{,}200 \text{ kips/in}^2)(7.9522 \text{ in}^4)}$$

$$\phi = 0.024252 \text{ radians}$$

$$\phi = 1.390° \blacktriangleleft$$

(*b*)

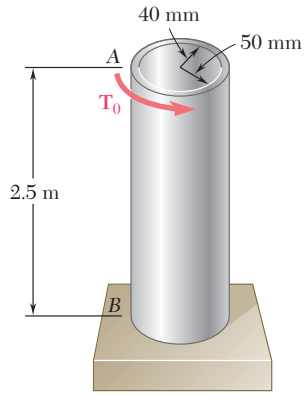
$$J = \frac{\pi}{2}[(1.5 \text{ in.})^4 - (0.75 \text{ in.})^4] = 7.4552 \text{ in}^4$$

$$\phi = \frac{(60 \text{ kip} \cdot \text{in.})(36 \text{ in.})}{(11{,}200 \text{ kips/in}^2)(7.4552 \text{ in}^4)}$$

$$\phi = 0.025869 \text{ radians}$$

$$\phi = 1.482° \blacktriangleleft$$

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PROBLEM 3.34

(*a*) For the aluminum pipe shown ($G = 27$ GPa), determine the torque $\mathbf{T}_0$ causing an angle of twist of 2°. (*b*) Determine the angle of twist if the same torque $\mathbf{T}_0$ is applied to a solid cylindrical shaft of the same length and cross-sectional area.

SOLUTION

(*a*)

$$c_o = 50 \text{ mm} = 0.050 \text{ m}, \quad c_i = 40 \text{ mm} = 0.040 \text{ m}$$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4) = \frac{\pi}{2}(0.050^4 - 0.040^4)$$

$$= 5.7962 \times 10^{-6} \text{ m}^4$$

$$\varphi = 2° = 34.907 \times 10^{-3} \text{ rad} \qquad L = 2.5 \text{ m}$$

$$G = 27 \times 10^9 \text{ Pa}$$

$$\varphi = \frac{TL}{GJ} \qquad T_0 = \frac{GJ\varphi}{L} = \frac{(27 \times 10^9)(5.7962 \times 10^{-6})(34.907 \times 10^{-3})}{2.5}$$

$$= 2.1851 \times 10^3 \text{ N} \cdot \text{m}$$

$$T_0 = 2.19 \text{ kN} \cdot \text{m} \blacktriangleleft$$

Area of pipe: $A = \pi(c_o^2 - c_i^2) = \pi(0.050^2 - 0.040^2) = 2.8274 \text{ m}^2$

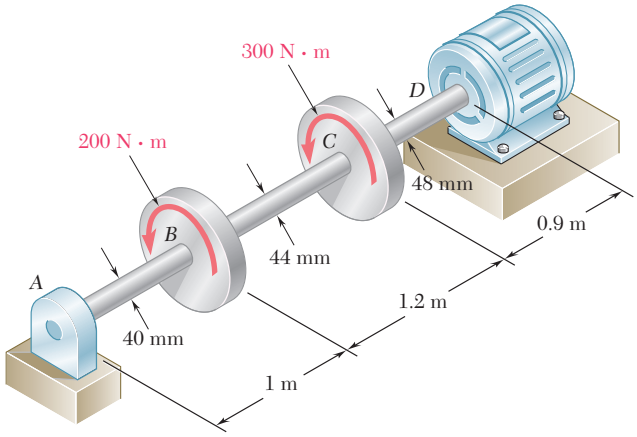
(*b*) Radius of solid of same area: $c_s = \sqrt{\frac{A}{\pi}} = 0.030 \text{ m}$

$$J_s = \frac{\pi}{2}c_s^4 = \frac{\pi}{2}(0.030)^4 = 1.27235 \times 10^{-6} \text{ m}^2$$

$$\varphi_s = \frac{T_0 L}{GJ} = \frac{(2.1851 \times 10^3)(2.5)}{(27 \times 10^9)(1.27235 \times 10^{-6})} = 0.15902 \text{ rad}$$

$$\varphi_s = 9.11° \blacktriangleleft$$

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PROBLEM 3.35

The electric motor exerts a 500-N · m torque on the aluminum shaft *ABCD* when it is rotating at a constant speed. Knowing that $G = 27$ GPa and that the torques exerted on pulleys *B* and *C* are as shown, determine the angle of twist between (*a*) *B* and *C*, (*b*) *B* and *D*.

SOLUTION

(*a*) Angle of twist between *B* and *C*.

$$T_{BC} = 200 \text{ N} \cdot \text{m}, \quad L_{BC} = 1.2 \text{ m}$$

$$c = \frac{1}{2}d = 0.022 \text{ m}, \quad G = 27 \times 10^9 \text{ Pa}$$

$$J_{BC} = \frac{\pi}{2}c^4 = 367.97 \times 10^{-9} \text{ m}$$

$$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(200)(1.2)}{(27 \times 10^9)(367.97 \times 10^9)} = 24.157 \times 10^{-3} \text{ rad} \qquad \varphi_{B/C} = 1.384° \blacktriangleleft$$

(*b*) Angle of twist between *B* and *D*.

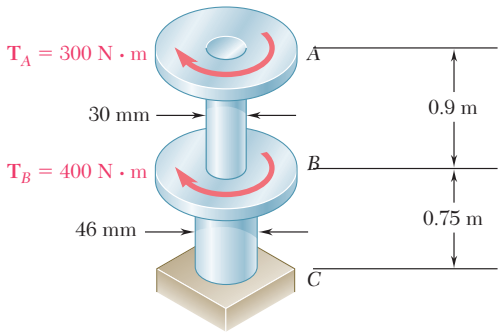
$$T_{CD} = 500 \text{ N} \cdot \text{m}, \quad L_{CD} = 0.9 \text{ m}, \quad c = \frac{1}{2}d = 0.024 \text{ m}, \quad G = 27 \times 10^9 \text{ Pa}$$

$$J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.024)^4 = 521.153 \times 10^{-9} \text{ m}^4$$

$$\varphi_{C/D} = \frac{(500)(0.9)}{(27 \times 10^9)(521.153 \times 10^9)} = 31.980 \times 10^{-3} \text{ rad}$$

$$\varphi_{B/D} = \varphi_{B/C} + \varphi_{C/D} = 24.157 \times 10^{-3} + 31.980 \times 10^{-3} = 56.137 \times 10^{-3} \text{ rad} \qquad \varphi_{B/D} = 3.22° \blacktriangleleft$$

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PROBLEM 3.36

The torques shown are exerted on pulleys *A* and *B*. Knowing that the shafts are solid and made of steel ($G = 77.2$ GPa), determine the angle of twist between (*a*) *A* and *B*, (*b*) *A* and *C*.

SOLUTION

(*a*) $T_{AB} = 300 \text{ N} \cdot \text{m}, \quad L_{AB} = 0.9 \text{ m}, \quad C_{AB} = \frac{1}{2}d = 0.015 \text{ m}$

$$J_{AB} = \frac{\pi}{2}(0.015)^4 = 79.522 \times 10^{-9} \text{ m}^4$$

$$\varphi_{AB} = \frac{T_{AB}L_{AB}}{GJ} = \frac{(300)(0.9)}{(77.2 \times 10^9)(79.522 \times 10^{-9})} = 43.980 \times 10^{-3} \text{ rad}$$

$$\varphi_{AB} = 2.52° \blacktriangleleft$$

(*b*) $T_{BC} = 300 + 400 = 700 \text{ N} \cdot \text{m}, \quad L_{BC} = 0.75 \text{ m}, \quad C_{BC} = \frac{1}{2}d = 0.023 \text{ m}$

$$J_{BC} = \frac{\pi}{2}(0.023)^4 = 439.57 \times 10^{-9} \text{ m}^4$$

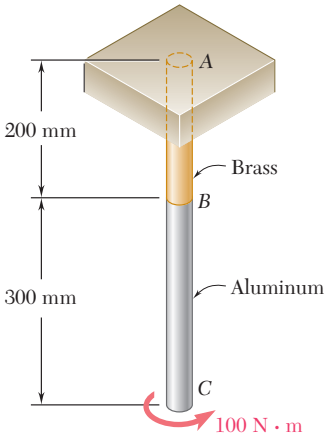
$$\varphi_{BC} = \frac{T_{BC}L_{BC}}{GJ_{BC}} = \frac{(700)(0.75)}{(77.2 \times 10^9)(439.57 \times 10^{-9})} = 15.4708 \times 10^{-3} \text{ rad}$$

$$\varphi_{BC} = 0.89°$$

$$\varphi_{AC} = \varphi_{AB} + \varphi_{BC} = 2.52° + 0.89°$$

$$\phi_{AC} = 3.41° \blacktriangleleft$$

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PROBLEM 3.37

The aluminum rod *BC* ($G = 26$ GPa) is bonded to the brass rod *AB* ($G = 39$ GPa). Knowing that each rod is solid and has a diameter of 12 mm, determine the angle of twist (*a*) at *B*, (*b*) at *C*.

SOLUTION

Both portions:

$$c = \frac{1}{2}d = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(6 \times 10^{-3})^4 = 2.03575 \times 10^{-9} \text{ m}^4$$

$$T = 100 \text{ N} \cdot \text{m}$$

Rod AB: $G_{AB} = 39 \times 10^9$ Pa, $L_{AB} = 0.200$ m

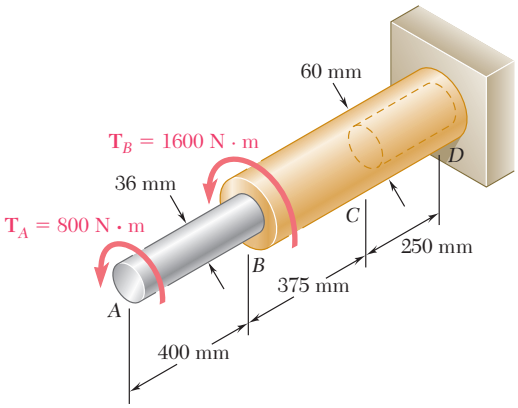
(*a*) $$\varphi_B = \varphi_{AB} = \frac{TL_{AB}}{G_{AB}J} = \frac{(100)(0.200)}{(39 \times 10^9)(2.03575 \times 10^{-9})} = 0.25191 \text{ rad}$$ $\varphi_B = 14.43°$ ◀

Rod BC: $G_{BC} = 26 \times 10^9$ Pa, $L_{BC} = 0.300$ m

$$\varphi_{BC} = \frac{TL_{BC}}{G_{BC}J} = \frac{(100)(0.300)}{(26 \times 10^9)(2.03575 \times 10^{-9})} = 0.56679 \text{ rad}$$

(*b*) $\varphi_C = \varphi_B + \varphi_{BC} = 0.25191 + 0.56679 = 0.81870$ rad $\varphi_C = 46.9°$ ◀

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PROBLEM 3.38

The aluminum rod *AB* ($G = 27$ GPa) is bonded to the brass rod *BD* ($G = 39$ GPa). Knowing that portion *CD* of the brass rod is hollow and has an inner diameter of 40 mm, determine the angle of twist at *A*.

SOLUTION

Rod *AB*:

$$G = 27 \times 10^9 \text{ Pa}, \quad L = 0.400 \text{ m}$$

$$T = 800 \text{ N} \cdot \text{m} \quad c = \frac{1}{2}d = 0.018 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896 \times 10^{-9} \text{ m}$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(800)(0.400)}{(27 \times 10^9)(164.896 \times 10^{-9})} = 71.875 \times 10^{-3} \text{ rad}$$

Part *BC*:

$$G = 39 \times 10^9 \text{ Pa} \quad L = 0.375 \text{ m}, \quad c = \frac{1}{2}d = 0.030 \text{ m}$$

$$T = 800 + 1600 = 2400 \text{ N} \cdot \text{m}, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(2400)(0.375)}{(39 \times 10^9)(1.27234 \times 10^{-6})} = 18.137 \times 10^{-3} \text{ rad}$$

Part *CD*:

$$c_1 = \frac{1}{2}d_1 = 0.020 \text{ m}$$

$$c_2 = \frac{1}{2}d_2 = 0.030 \text{ m}, \quad L = 0.250 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.030^4 - 0.020^4) = 1.02102 \times 10^{-6} \text{ m}^4$$

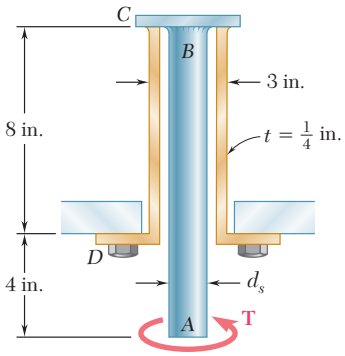
$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(2400)(0.250)}{(39 \times 10^9)(1.02102 \times 10^{-6})} = 15.068 \times 10^{-3} \text{ rad}$$

Angle of twist at *A*.

$$\varphi_A = \varphi_{A/B} + \varphi_{B/C} + \varphi_{C/D}$$
$$= 105.080 \times 10^{-3} \text{ rad}$$

$$\varphi_A = 6.02° \blacktriangleleft$$

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PROBLEM 3.39

The solid spindle *AB* has a diameter $d_s = 1.75$ in. and is made of a steel with $G = 11.2 \times 10^6$ psi and $\tau_{all} = 12$ ksi, while sleeve *CD* is made of a brass with $G = 5.6 \times 10^6$ psi and $\tau_{all} = 7$ ksi. Determine (*a*) the largest torque **T** that can be applied at *A* if the given allowable stresses are not to be exceeded and if the angle of twist of sleeve *CD* is not to exceed 0.375°, (*b*) the corresponding angle through which end *A* rotates.

SOLUTION

Spindle *AB*: $c = \frac{1}{2}(1.75 \text{ in.}) = 0.875 \text{ in.}$ $L = 12$ in., $\tau_{all} = 12$ ksi, $G = 11.2 \times 10^6$ psi

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.875)^4 = 0.92077 \text{ in}^4$$

Sleeve *CD*: $c_1 = 1.25$ in., $c_2 = 1.5$ in., $L = 8$ in., $\tau_{all} = 7$ ksi

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 4.1172 \text{ in}^4, \quad G = 5.6 \times 10^6 \text{ psi}$$

(*a*) Largest allowable torque *T*.

Ciriterion: Stress in spindle *AB*. $\tau = \frac{Tc}{J}$ $T = \frac{J\tau}{c}$

$$T = \frac{(0.92077)(12)}{0.875} = 12.63 \text{ kip} \cdot \text{in.}$$

Critrion: Stress in sleeve *CD*. $T = \frac{J\tau}{c_2} = \frac{4.1172 \text{ in}^4}{1.5 \text{ in.}}(7 \text{ ksi})$ $T = 19.21 \text{ kip} \cdot \text{in.}$

Criterion: Angle of twist of sleeve *CD*. $\phi = 0.375° = 6.545 \times 10^{-3}$ rad

$$\phi = \frac{TL}{JG} \quad T = \frac{JG}{L}\phi = \frac{(4.1172)(5.6 \times 10^6)}{8}(6.545 \times 10^{-3})$$

$$T = 18.86 \text{ kip} \cdot \text{in.}$$

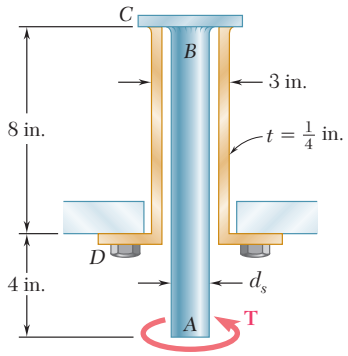
The largest allowable torque is $T = 12.63 \text{ kip} \cdot \text{in.}$ ◀

(*b*) Angle of rotation of end A. $\phi_A = \phi_{A/D} = \phi_{A/B} + \phi_{C/D} = \sum \frac{T_i L_i}{J_i G_i} = T \sum \frac{L_i}{J_i G_i}$

$$= (12.63 \times 10^3)\left[\frac{12}{(0.92077)(11.2 \times 10^6)} + \frac{8}{(4.1172)(5.6 \times 10^6)}\right]$$

$$= 0.01908 \text{ radians} \qquad \varphi_A = 1.093° \text{ ◀}$$

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PROBLEM 3.40

The solid spindle *AB* has a diameter $d_s = 1.5$ in. and is made of a steel with $G = 11.2 \times 10^6$ psi and $\tau_{all} = 12$ ksi, while sleeve *CD* is made of a brass with $G = 5.6 \times 10^6$ psi and $\tau_{all} = 7$ ksi. Determine the angle through which end *A* can be rotated.

SOLUTION

Stress analysis of solid spindle *AB*:

$$c = \frac{1}{2} d_s = 0.75 \text{ in.}$$

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3$$

$$T = \frac{\pi}{2}(12 \times 10^3)(0.75)^3 = 7.95 \times 10^3 \text{ lb} \cdot \text{in.}$$

Stress analysis of sleeve *CD*:

$$c_2 = \frac{1}{2} d_o = \frac{1}{2}(3) = 1.5 \text{ in.}$$

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

$$T = \frac{J\tau}{c_2} = \frac{(4.1172)(7 \times 10^{-3})}{1.5} = 19.21 \times 10^3 \text{ lb} \cdot \text{in.}$$

The smaller torque governs.

$$T = 7.95 \times 10^3 \text{ lb} \cdot \text{in.}$$

Deformation of spindle *AB*:

$$c = 0.75 \text{ in.}$$

$$J = \frac{\pi}{2} c^4 = 0.49701 \text{ in}^4, \quad L = 12 \text{ in.}, \quad G = 11.2 \times 10^6 \text{ psi}$$

$$\varphi_{AB} = \frac{TL}{GJ} = \frac{(7.95 \times 10^3)(12)}{(11.2 \times 10^6)(0.49701)} = 0.017138 \text{ radians}$$

Deformation of sleeve *CD*:

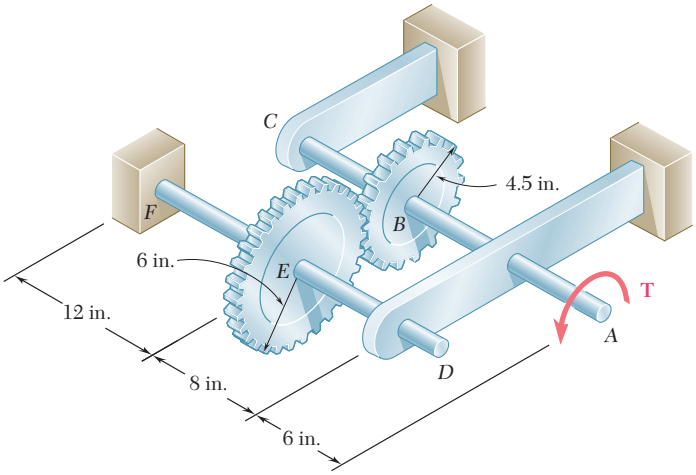
$$J = 4.1172 \text{ in}^4, \quad L = 8 \text{ in.}, \quad G = 5.6 \times 10^6 \text{ psi}$$

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(7.95 \times 10^3)(8)}{(5.6 \times 10^6)(4.1172)} = 0.002758 \text{ radians}$$

Total angle of twist:

$$\varphi_{AD} = \varphi_{AB} + \varphi_{CD} = 0.019896 \text{ radians} \qquad \varphi_{AD} = 1.140° \blacktriangleleft$$

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PROBLEM 3.41

Two shafts, each of $\frac{7}{8}$-in. diameter, are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi and that the shaft at F is fixed, determine the angle through which end A rotates when a 1.2-kip · in. torque is applied at A.

SOLUTION

Calculation of torques.

Circumferential contact force between gears B and E: $F = \dfrac{T_{AB}}{r_B} = \dfrac{T_{EF}}{r_E}$ $T_{EF} = \dfrac{r_E}{r_B} T_{AB}$

$$T_{AB} = 1.2 \text{ kip} \cdot \text{in.} = 1200 \text{ lb} \cdot \text{in.}$$

$$T_{EF} = \frac{6}{4.5}(1200) = 1600 \text{ lb} \cdot \text{in.}$$

Twist in shaft FE.

$$L = 12 \text{ in.}, \quad c = \frac{1}{2}d = \frac{7}{16} \text{ in.}, \quad G = 11.2 \times 10^6 \text{ psi}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{7}{16}\right)^4 = 57.548 \times 10^{-3} \text{ in}^4$$

$$\varphi_{E/F} = \frac{TL}{GJ} = \frac{(1600)(12)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 29.789 \times 10^{-3} \text{ rad}$$

Rotation at E. $\varphi_E = \varphi_{E/F} = 29.789 \times 10^{-3}$ rad

Tangential displacement at gear circle: $\delta = r_E \varphi_E = r_B \varphi_B$

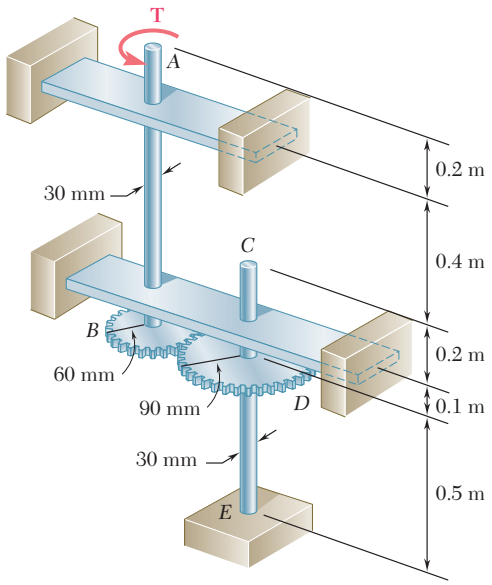
Rotation at B. $\varphi_B = \dfrac{r_E}{r_B}\varphi_E = \dfrac{6}{4.5}(29.789 \times 10^{-3}) = 39.718 \times 10^{-3}$ rad

Twist in shaft BA. $L = 8 + 6 = 14$ in. $J = 57.548 \times 10^{-3} \text{ in}^4$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(14)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 26.065 \times 10^{-3} \text{ rad}$$

Rotation at A. $\varphi_A = \varphi_B + \varphi_{A/B} = 65.783 \times 10^{-3}$ rad $\varphi_A = 3.77°$ ◀

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PROBLEM 3.42

Two solid steel shafts, each of 30-mm diameter, are connected by the gears shown. Knowing that $G = 77.2$ GPa, determine the angle through which end A rotates when a torque of magnitude $T = 200\ \text{N} \cdot \text{m}$ is applied at A.

SOLUTION

Calculation of torques.

Circumferential contact force between gears B and D: $F = \dfrac{T_{AB}}{r_B} = \dfrac{T_{DE}}{r_D}$

$$T_{DE} = \frac{r_D}{r_B} T_{AB} = \frac{90}{60}(200) = 300\ \text{N} \cdot \text{m}$$

Twist in shaft DE.

$$J_{DE} = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.015)^4 = 79.522 \times 10^{-9}\ \text{m}^4$$

$$L_{DE} = 0.5\ \text{m}$$

$$\varphi_{DE} = \frac{T_{DE} L_{DE}}{G J_{DE}} = \frac{(300)(0.5)}{(77.2 \times 10^9)(79.522 \times 10^{-9})} = 24.434 \times 10^{-3}\ \text{rad}$$

Rotation at D.

$$\varphi_D = \varphi_{DE} = 24.434 \times 10^{-3}\ \text{rad}$$

Circumferential displacement at gear circles: $\delta = r_D \varphi_D = r_B \varphi_B$

Rotation at B.

$$\varphi_B = \frac{r_D}{r_B} \varphi_D = \frac{90}{60}(24.434 \times 10^{-3}) = 36.651 \times 10^{-3}\ \text{rad}$$

Twist in shaft AB.

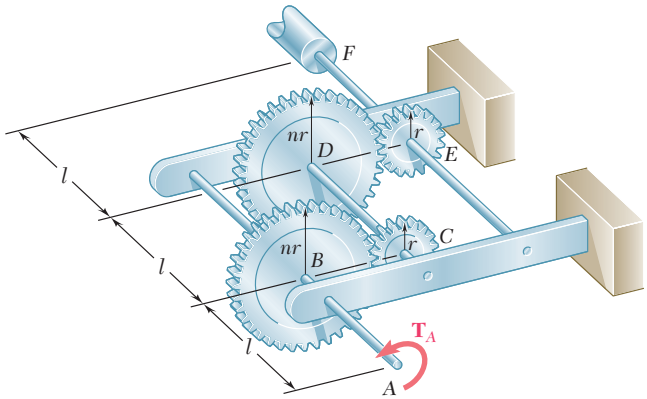
$$L_{AB} = 0.1 + 0.2 + 0.4 + 0.2 = 0.9\ \text{m}, \quad J_{AB} = 79.522 \times 10^{-9}\ \text{m}^4$$

$$\varphi_{AB} = \frac{T_{AB} L_{AB}}{G J_{AB}} = \frac{(200)(0.9)}{(77.2 \times 10^9)(79.522 \times 10^{-9})} = 29.320 \times 10^{-3}\ \text{rad}$$

Rotation at A.

$$\varphi_A = \varphi_B + \varphi_{AB}$$
$$= 36.651 \times 10^{-3} + 29.320 \times 10^{-3}\ \text{rad}$$
$$= 65.971 \times 10^{-3}\ \text{rad} \qquad 3.78° \blacktriangleleft$$

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PROBLEM 3.43

A coder *F*, used to record in digital form the rotation of shaft *A*, is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter *d*. Two of the gears have a radius *r* and the other two a radius *nr*. If the rotation of the coder *F* is prevented, determine in terms of *T*, *l*, *G*, *J*, and *n* the angle through which end *A* rotates.

SOLUTION

$$T_{AB} = T_A$$

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{T_{AB}}{n} = \frac{T_A}{n}$$

$$T_{EF} = \frac{r_E}{r_D} T_{CD} = \frac{T_{CD}}{n} = \frac{T_A}{n^2}$$

$$\varphi_E = \varphi_{EF} = \frac{T_{EF} l_{EF}}{GJ} = \frac{T_A l}{n^2 GJ}$$

$$\varphi_D = \frac{r_E}{r_D} \varphi_E = \frac{\varphi_E}{n} = \frac{T_A l}{n^3 GJ}$$

$$\varphi_{CD} = \frac{T_{CD} l_{CD}}{GJ} = \frac{T_A l}{nGJ}$$

$$\varphi_C = \varphi_D + \varphi_{CD} = \frac{T_A l}{n^3 GJ} + \frac{T_A l}{nGJ} = \frac{T_A l}{GJ}\left(\frac{1}{n^3} + \frac{1}{n}\right)$$

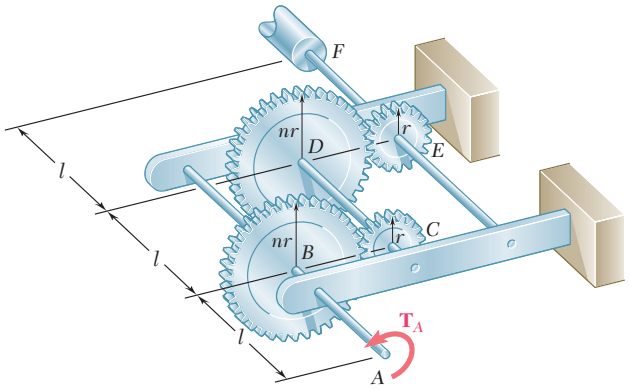
$$\varphi_B = \frac{r_C}{r_B} \varphi_C = \frac{\varphi_C}{n} = \frac{Tl}{GJ}\left(\frac{1}{n^4} + \frac{1}{n^2}\right)$$

$$\varphi_{AB} = \frac{T_{AB} l_{AB}}{GJ} = \frac{T_A l}{GJ}$$

$$\varphi_A = \varphi_B + \varphi_{AB} = \frac{T_A l}{GJ}\left(\frac{1}{n^4} + \frac{1}{n^2} + 1\right)$$

◀

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PROBLEM 3.44

For the gear train described in Prob. 3.43, determine the angle through which end A rotates when $T = 5\,\text{lb}\cdot\text{in.}$, $l = 2.4\,\text{in.}$, $d = 1/16\,\text{in.}$, $G = 11.2\times10^{6}\,\text{psi}$, and $n = 2$.

PROBLEM 3.43 A coder F, used to record in digital form the rotation of shaft A, is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d. Two of the gears have a radius r and the other two a radius nr. If the rotation of the coder F is prevented, determine in terms of T, l, G, J, and n the angle through which end A rotates.

SOLUTION

See solution to Prob. 3.43 for development of equation for φ_A.

$$\varphi_A = \frac{Tl}{GJ}\left(1+\frac{1}{n^2}+\frac{1}{n^4}\right)$$

Data: $T = 5\,\text{lb}\cdot\text{in.}$, $l = 2.4\,\text{in.}$, $c = \frac{1}{2}d = \frac{1}{32}\,\text{in.}$, $G = 11.2\times10^{6}\,\text{psi}$

$$n = 2,\quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{1}{32}\right)^4 = 1.49803\times10^{-6}\,\text{in}^4$$

$$\varphi_A = \frac{(5)(2.4)}{(11.2\times10^{6})(1.49803\times10^{-6})}\left(1+\frac{1}{4}+\frac{1}{16}\right) = 938.73\times10^{-3}\,\text{rad}$$

$$\varphi_A = 53.8^\circ \blacktriangleleft$$

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PROBLEM 3.45

The design specifications of a 1.2-m-long solid circular transmission shaft require that the angle of twist of the shaft not exceed 4° when a torque of 750 N · m is applied. Determine the required diameter of the shaft, knowing that the shaft is made of a steel with an allowable shearing stress of 90 MPa and a modulus of rigidity of 77.2 GPa.

SOLUTION

$$T = 750 \text{ N} \cdot \text{m}, \quad \varphi = 4^\circ = 69.813 \times 10^{-3} \text{ rad}, \quad L = 1.2 \text{ m}, \quad J = \frac{\pi}{2} c^4$$

$$\tau = 90 \text{ MPa} = 90 \times 10^6 \text{ Pa}$$

$$G = 77.2 \text{ GPa} = 77.2 \times 10^9 \text{ Pa}$$

Based on angle of twist.

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$c = \sqrt[4]{\frac{2TL}{\pi G \varphi}} = \sqrt[4]{\frac{(2)(750)(1.2)}{\pi (77.2 \times 10^9)(69.813 \times 10^{-3})}} = 18.0569 \times 10^{-3} \text{ m}$$

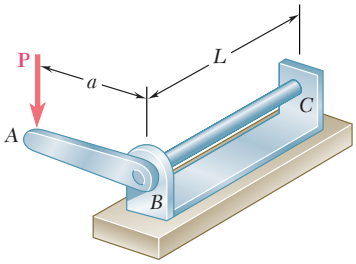
Based on shearing stress.

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(750)}{\pi (90 \times 10^6)}} = 17.441 \times 10^{-3} \text{ m}$$

Use larger value. $c = 18.0569 \times 10^{-3} \text{ m} = 18.0569 \text{ mm}$ $\quad d = 2c = 36.1 \text{ mm}$ ◀

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PROBLEM 3.46

The solid cylindrical rod *BC* of length $L = 24$ in. is attached to the rigid lever *AB* of length $a = 15$ in. and to the support at *C*. Design specifications require that the displacement of *A* not exceed 1 in. when a 100-lb force **P** is applied at *A*. For the material indicated, determine the required diameter of the rod.

Steel: $\tau_{all} = 15$ ksi, $G = 11.2 \times 10^6$ psi.

SOLUTION

At the allowable twist angle,

$$\sin\varphi = \frac{\Delta}{a} = \frac{1}{15} = 0.06667$$

$$\varphi = 3.8226^\circ = 0.066716 \text{ rad.}$$

$$T = Pa\cos\varphi = (100)(15)\cos 3.8226^\circ = 1496.7 \text{ lb} \cdot \text{in.}$$

Based on twist.

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4} \quad \therefore \quad c^4 = \frac{2TL}{\pi G \varphi}$$

$$c^4 = \frac{(2)(1496.7)(24)}{\pi(11.2 \times 10^6)(0.066716)}$$

$$= 30.603 \times 10^{-3} \text{ in}^4 \qquad c = 0.418 \text{ in.}$$

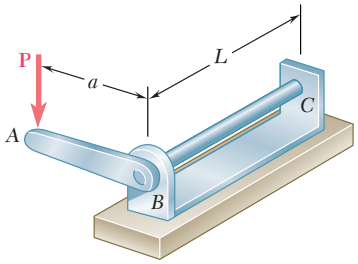
Based on stress.

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi\tau} \qquad (\tau = 15{,}000 \text{ psi})$$

$$c^3 = \frac{2(1496.7)}{\pi(15{,}000)} = 63.522 \times 10^{-3} \text{ in}^3 \qquad c = 0.399 \text{ in.}$$

Use larger value for design. $c = 0.399$ in. $\qquad d = 2c = 0.837$ in. ◀

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PROBLEM 3.47

The solid cylindrical rod *BC* of length $L = 24$ in. is attached to the rigid lever *AB* of length $a = 15$ in. and to the support at *C*. Design specifications require that the displacement of *A* not exceed 1 in. when a 100-lb force **P** is applied at *A*. For the material indicated, determine the required diameter of the rod.

Aluminum: $\tau_{all} = 10$ ksi, $G = 3.9 \times 10^6$ psi.

SOLUTION

At the allowable twist angle,

$$\sin\varphi = \frac{\Delta}{a} = \frac{1}{15} = 0.06667$$

$$\varphi = 3.8226^\circ = 0.066716 \text{ rad}$$

$$T = Pa\cos\varphi = (100)(15)\cos 3.8226^\circ = 1496.7 \text{ lb} \cdot \text{in.}$$

Based on twist.

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4} \quad \therefore \quad c^4 = \frac{2TL}{\pi G \varphi}$$

$$c^4 = \frac{(2)(1496.7)(24)}{\pi(3.9 \times 10^6)(0.066716)}$$

$$= 87.888 \times 10^{-3} \text{ in}^4 \quad c = 0.544 \text{ in.}$$

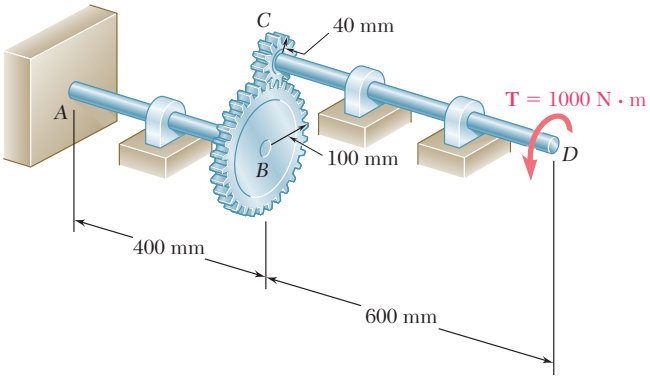
Based on stress.

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi\tau} \quad (\tau = 10{,}000 \text{ psi})$$

$$c^3 = \frac{2(1496.7)}{\pi(10{,}000)} = 95.283 \times 10^{-3} \text{ in}^3 \quad c = 0.457 \text{ in.}$$

Use larger value for design. $c = 0.544$ in. $d = 2c = 1.089$ in. ◀

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PROBLEM 3.48

The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both *AB* and *CD*. It is further required that $\tau_{\max} \le 60$ MPa, and that the angle ϕ_D through which end *D* of shaft *CD* rotates not exceed 1.5°. Knowing that $G = 77.2$ GPa, determine the required diameter of the shafts.

SOLUTION

$$T_{CD} = T_D = 1000 \text{ N} \cdot \text{m} \qquad T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500 \text{ N} \cdot \text{m}$$

For design based on stress, use larger torque. $T_{AB} = 2500 \text{ N} \cdot \text{m}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi\tau} = \frac{(2)(2500)}{\pi(60 \times 10^6)} = 26.526 \times 10^{-6} \text{ m}^3$$

$$c = 29.82 \times 10^{-3} \text{ m} = 29.82 \text{ mm}, \quad d = 2c = 59.6 \text{ mm}$$

Design based on rotation angle. $\varphi_D = 1.5° = 26.18 \times 10^{-3}$ rad

Shaft *AB*: $T_{AB} = 2500 \text{ N} \cdot \text{m}, \quad L = 0.4 \text{ m}$

$$\varphi_{AB} = \frac{TL}{GJ} = \frac{(2500)(0.4)}{GJ} = \frac{1000}{GJ}$$

$$\text{Gears} \begin{cases} \varphi_B = \varphi_{AB} = \dfrac{1000}{GJ} \\ \varphi_C = \dfrac{r_B}{r_C}\varphi_B = \left(\dfrac{100}{40}\right)\left(\dfrac{1000}{GJ}\right) = \dfrac{2500}{GJ} \end{cases}$$

Shaft *CD*: $T_{CD} = 1000 \text{ N} \cdot \text{m}, \quad L = 0.6 \text{ m}$

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(1000)(0.6)}{GJ} = \frac{600}{GJ}$$

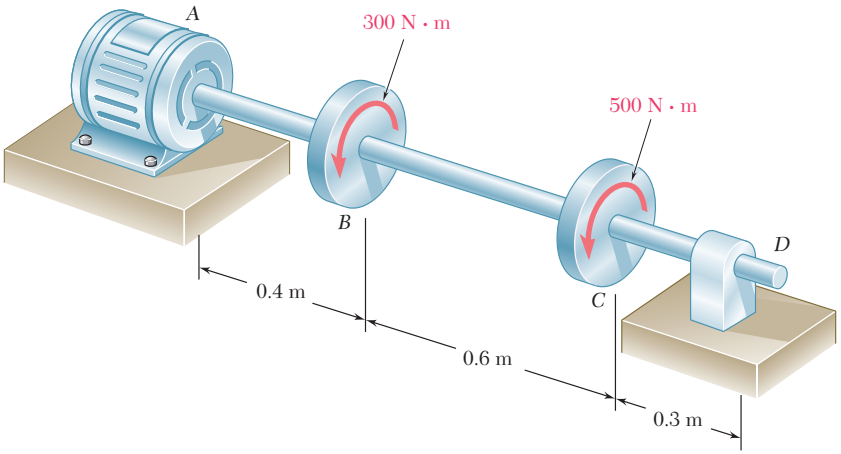
$$\varphi_D = \varphi_C + \varphi_{CD} = \frac{2500}{GJ} + \frac{600}{GJ} = \frac{3100}{GJ} = \frac{3100}{G\frac{\pi}{2}c^4}$$

$$c^4 = \frac{(2)(3100)}{\pi G \varphi_D} = \frac{(2)(3100)}{\pi(77.2 \times 10^9)(26.18 \times 10^{-3})} = 976.46 \times 10^{-9} \text{ m}^4$$

$$c = 31.435 \times 10^{-3} \text{ m} = 31.435 \text{ mm}, \quad d = 2c = 62.9 \text{ mm}$$

Design must use larger value for *d*. $d = 62.9$ mm ◀

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PROBLEM 3.49

The electric motor exerts a torque of 800 N · m on the steel shaft *ABCD* when it is rotating at constant speed. Design specifications require that the diameter of the shaft be uniform from *A* to *D* and that the angle of twist between *A* to *D* not exceed 1.5°. Knowing that $\tau_{max} \le 60$ MPa and $G = 77.2$ GPa, determine the minimum diameter shaft that can be used.

SOLUTION

Torques:

$$T_{AB} = 300 + 500 = 800 \text{ N} \cdot \text{m}$$
$$T_{BC} = 500 \text{ N} \cdot \text{m}, \qquad T_{CD} = 0$$

Design based on stress. $\tau = 60 \times 10^6$ Pa

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(800)}{\pi(60 \times 10^6)} = 8.488 \times 10^{-6} \text{ m}^3$$

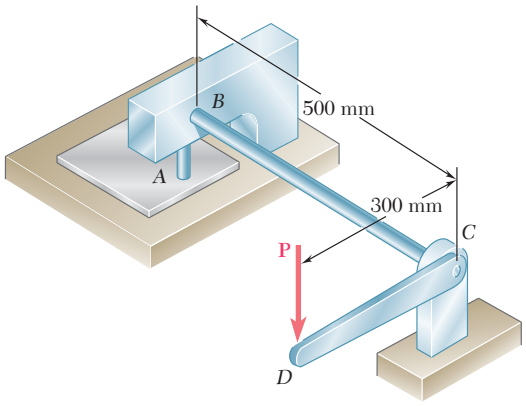
$$c = 20.40 \times 10^{-3} \text{ m} = 20.40 \text{ mm}, \qquad d = 2c = 40.8 \text{ mm}$$

Design based on deformation. $\varphi_{D/A} = 1.5° = 26.18 \times 10^{-3}$ rad

$$\varphi_{D/C} = 0$$
$$\varphi_{C/B} = \frac{T_{BC}L_{BC}}{GJ} = \frac{(500)(0.6)}{GJ} = \frac{300}{GJ}$$
$$\varphi_{B/A} = \frac{T_{AB}L_{AB}}{GJ} = \frac{(800)(0.4)}{GJ} = \frac{320}{GJ}$$
$$\varphi_{D/A} = \varphi_{D/C} + \varphi_{C/B} + \varphi_{B/A} = \frac{620}{GJ} = \frac{620}{G\frac{\pi}{2}c^4} = \frac{(2)(620)}{\pi G c^4}$$
$$c^4 = \frac{(2)(620)}{\pi G \varphi_{D/A}} = \frac{(2)(620)}{\pi(77.2 \times 10^9)(26.18 \times 10^{-3})} = 195.292 \times 10^{-9} \text{ m}^4$$
$$c = 21.022 \times 10^{-3} \text{ m} = 21.022 \text{ mm}, \qquad d = 2c = 42.0 \text{ mm}$$

Design must use larger value of *d*. $d = 42.0$ mm ◀

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PROBLEM 3.50

A hole is punched at *A* in a plastic sheet by applying a 600-N force **P** to end *D* of lever *CD*, which is rigidly attached to the solid cylindrical shaft *BC*. Design specifications require that the displacement of *D* should not exceed 15 mm from the time the punch first touches the plastic sheet to the time it actually penetrates it. Determine the required diameter of shaft *BC* if the shaft is made of a steel with $G = 77.2$ GPa and $\tau_{all} = 80$ MPa.

SOLUTION

Torque: $T = rP = (0.300\text{ m})(600\text{ N}) = 180\text{ N}\cdot\text{m}$

Shaft diameter based on displacement limit.

$$\varphi = \frac{\delta}{r} = \frac{15\text{ mm}}{300\text{ mm}} = 0.005\text{ rad}$$

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$c^4 = \frac{2TL}{\pi G\varphi} = \frac{(2)(180)(0.500)}{\pi(77.2\times10^9)(0.05)} = 14.843\times10^{-9}\text{ m}^4$$

$$c = 11.038\times10^{-3}\text{ m} = 11.038\text{ m} \qquad d = 2c = 22.1\text{ mm}$$

Shaft diameter based on stress.

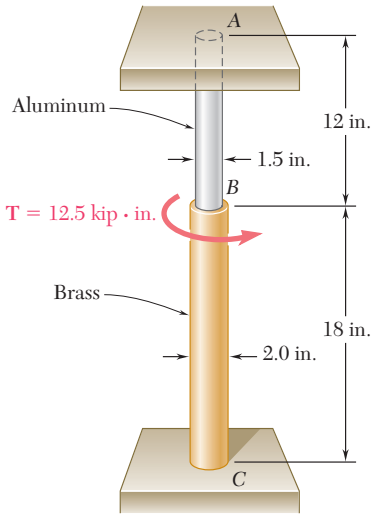
$$\tau = 80\times10^6\text{ Pa} \qquad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi\tau} = \frac{(2)(180)}{\pi(80\times10^6)} = 1.43239\times10^{-6}\text{ m}^3$$

$$c = 11.273\times10^{-3}\text{ m} = 11.273\text{ mm} \qquad d = 2c = 22.5\text{ mm}$$

Use the larger value to meet both limits. $d = 22.5\text{ mm}$ ◀

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PROBLEM 3.51

The solid cylinders *AB* and *BC* are bonded together at *B* and are attached to fixed supports at *A* and *C*. Knowing that the modulus of rigidity is 3.7×10^6 psi for aluminum and 5.6×10^6 psi for brass, determine the maximum shearing stress (*a*) in cylinder *AB*, (*b*) in cylinder *BC*.

SOLUTION

The torques in cylinders *AB* and *BC* are statically indeterminate. Match the rotation φ_B for each cylinder.

Cylinder *AB*: $c = \frac{1}{2}d = 0.75$ in. $L = 12$ in.

$$J = \frac{\pi}{2}c^4 = 0.49701 \text{ in}^4$$

$$\varphi_B = \frac{T_{AB}L}{GJ} = \frac{T_{AB}(12)}{(3.7 \times 10^6)(0.49701)} = 6.5255 \times 10^{-6} T_{AB}$$

Cylinder *BC*: $c = \frac{1}{2}d = 1.0$ in. $L = 18$ in.

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.0)^4 = 1.5708 \text{ in}^4$$

$$\varphi_B = \frac{T_{BC}L}{GJ} = \frac{T_{BC}(18)}{(5.6 \times 10^6)(1.5708)} = 2.0463 \times 10^{-6} T_{BC}$$

Matching expressions for φ_B: $6.5255 \times 10^{-6} T_{AB} = 2.0463 \times 10^{-6} T_{BC}$

$$T_{BC} = 3.1889\, T_{AB} \quad (1)$$

Equilibrium of connection at *B*: $T_{AB} + T_{BC} - T = 0$ $T = 12.5 \times 10^3$ lb · in.

$$T_{AB} + T_{BC} = 12.5 \times 10^3 \quad (2)$$

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PROBLEM 3.51 ***(Continued)***

Substituting (1) into (2), $4.1889\, T_{AB} = 12.5 \times 10^3$

$$T_{AB} = 2.9841 \times 10^3 \text{ lb} \cdot \text{in.} \quad T_{BC} = 9.5159 \times 10^3 \text{ lb} \cdot \text{in.}$$

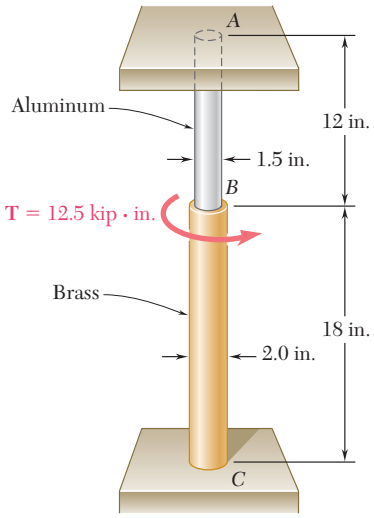
(*a*) Maximum stress in cylinder *AB*.

$$\tau_{AB} = \frac{T_{AB}c}{J} = \frac{(2.9841 \times 10^3)(0.75)}{0.49701} = 4.50 \times 10^3 \text{ psi} \qquad \tau_{AB} = 4.50 \text{ ksi} \blacktriangleleft$$

(*b*) Maximum stress in cylinder *BC*.

$$\tau_{BC} = \frac{T_{BC}c}{J} = \frac{(9.5159 \times 10^3)(1.0)}{1.5708} = 6.06 \times 10^3 \text{ psi} \qquad \tau_{BC} = 6.06 \text{ ksi} \blacktriangleleft$$

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PROBLEM 3.52

Solve Prob. 3.51, assuming that cylinder *AB* is made of steel, for which $G = 11.2 \times 10^6$ psi.

PROBLEM 3.51 The solid cylinders *AB* and *BC* are bonded together at *B* and are attached to fixed supports at *A* and *C*. Knowing that the modulus of rigidity is 3.7×10^6 psi for aluminum and 5.6×10^6 psi for brass, determine the maximum shearing stress (*a*) in cylinder *AB*, (*b*) in cylinder *BC*.

SOLUTION

The torques in cylinders *AB* and *BC* are statically indeterminate. Match the rotation φ_B for each cylinder.

Cylinder *AB*: $c = \frac{1}{2}d = 0.75$ in. $L = 12$ in. $J = \frac{\pi}{2}c^4 = 0.49701$ in^4

$$\varphi_B = \frac{T_{AB}L}{GJ} = \frac{T_{AB}(12)}{(11.2 \times 10^6)(0.49701)} = 2.1557 \times 10^{-6} T_{AB}$$

Cylinder *BC*: $c = \frac{1}{2}d = 1.0$ in. $L = 18$ in. $J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.0)^4 = 1.5708$ in^4

$$\varphi_B = \frac{T_{BC}L}{GJ} = \frac{T_{BC}(18)}{(5.6 \times 10^6)(1.5708)} = 2.0463 \times 10^{-6} T_{BC}$$

Matching expressions for φ_B: $2.1557 \times 10^{-6} T_{AB} = 2.0463 \times 10^{-6} T_{BC}$ $T_{BC} = 1.0535 T_{AB}$ (1)

Equilibrium of connection at *B*: $T_{AB} + T_{BC} - T = 0$ $T_{AB} + T_{BC} = 12.5 \times 10^3$ (2)

Substituting (1) into (2), $2.0535 T_{AB} = 12.5 \times 10^3$

$T_{AB} = 6.0872 \times 10^3$ lb · in. $T_{BC} = 6.4128 \times 10^3$ lb · in.

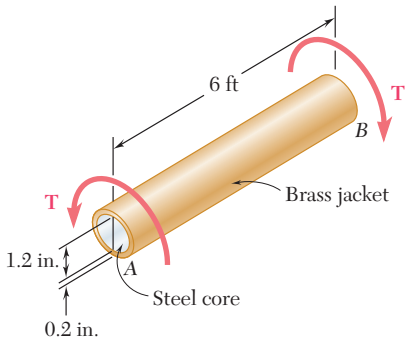
(*a*) Maximum stress in cylinder *AB*.

$$\tau_{AB} = \frac{T_{AB}c}{J} = \frac{(6.0872 \times 10^3)(0.75)}{0.49701} = 9.19 \times 10^3 \text{ psi} \qquad \tau_{AB} = 9.19 \text{ ksi} \blacktriangleleft$$

(*b*) Maximum stress in cylinder *BC*.

$$\tau_{BC} = \frac{T_{BC}c}{J} = \frac{(6.4128 \times 10^3)(1.0)}{1.5708} = 4.08 \times 10^3 \text{ psi} \qquad \tau_{BC} = 4.08 \text{ ksi} \blacktriangleleft$$

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PROBLEM 3.53

The composite shaft shown consists of a 0.2-in.-thick brass jacket ($G = 5.6 \times 10^6$ psi) bonded to a 1.2-in.-diameter steel core ($G_{\text{steel}} = 11.2 \times 10^6$ psi). Knowing that the shaft is subjected to 5-kip · in. torques, determine (*a*) the maximum shearing stress in the brass jacket, (*b*) the maximum shearing stress in the steel core, (*c*) the angle of twist of end *B* relative to end *A*.

SOLUTION

Steel core:
$$c_1 = \frac{1}{2}d = 0.6 \text{ in.}$$
$$J_1 = \frac{\pi}{2}c_1^4 = \frac{\pi}{2}(0.6)^4 = 0.20358 \text{ in}^4$$
$$G_1J_1 = (11.2 \times 10^6)(0.20358) = 2.2801 \times 10^6 \text{ lb} \cdot \text{in}^2$$

Torque carried by steel core. $T_1 = G_1J_1\frac{\varphi}{L}$

Brass jacket:
$$c_2 = c_1 + t = 0.6 + 0.2 = 0.8 \text{ in.}$$
$$J_2 = \frac{\pi}{2}\left(C_2^4 - C_1^4\right) = \frac{\pi}{2}(0.8^4 - 0.6^4) = 0.43982 \text{ in}^4$$
$$G_2J_2 = (5.6 \times 10^6)(0.43982) = 2.4630 \times 10^6 \text{ lb} \cdot \text{in}^2$$

Torque carried by brass jacket. $T_2 = G_2J_2\frac{\varphi}{L}$

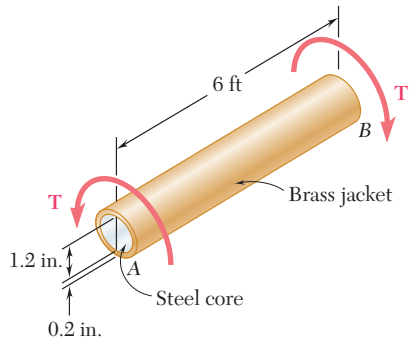
Total torque:
$$T = T_1 + T_2 = (G_1J_1 + G_2J_2)\frac{\varphi}{L}$$
$$\frac{\varphi}{L} = \frac{T}{G_1J_1 + G_2J_2} = \frac{5 \times 10^3}{2.2801 \times 10^6 + 2.4630 \times 10^6}$$
$$= 1.05416 \times 10^{-3} \text{ rad/in.}$$

(*a*) Maximum shearing stress in brass jacket.
$$\tau_{\max} = G_2\gamma_{\max} = G_2c_2\frac{\varphi}{L} = (5.6 \times 10^6)(0.8)(1.05416 \times 10^{-3})$$
$$= 4.72 \times 10^3 \text{ psi} \qquad 4.72 \text{ ksi} \blacktriangleleft$$

(*b*) Maximum shearing stress in steel core.
$$\tau_{\max} = G_1\gamma_{\max} = G_1c_1\frac{\varphi}{L} = (11.2 \times 10^6)(0.6)(1.05416 \times 10^{-3})$$
$$= 7.08 \times 10^3 \text{ psi} \qquad 7.08 \text{ ksi} \blacktriangleleft$$

(*c*) Angle of twist. ($L = 6$ ft $= 72$ in.)
$$\varphi = L\frac{\varphi}{L} = (72)(1.0542 \times 10^{-3}) = 75.9 \times 10^{-3} \text{ rad}$$
$$= 4.35° \blacktriangleleft$$

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PROBLEM 3.54

The composite shaft shown consists of a 0.2-in.-thick brass jacket ($G = 5.6 \times 10^6$ psi) bonded to a 1.2-in.-diameter steel core ($G_{\text{steel}} = 11.2 \times 10^6$ psi). Knowing that the shaft is being subjected to the torques shown, determine the largest angle through which it can be twisted if the following allowable stresses are not to be exceeded: $\tau_{\text{steel}} = 15$ ksi and $\tau_{\text{brass}} = 8$ ksi.

SOLUTION

$$\tau_{\max} = G\gamma_{\max} = Gc_{\max}\frac{\varphi}{L}$$

$$\frac{\varphi_{\text{all}}}{L} = \frac{\tau_{\text{all}}}{Gc_{\max}} \quad \text{for each material.}$$

Steel core:

$$\tau_{\text{all}} = 15 \text{ ksi} = 15{,}000 \text{ psi}, \quad c_{\max} = \frac{1}{2}d = 0.6 \text{ in.}$$

$$\frac{\varphi_{\text{all}}}{L} = \frac{15{,}000}{(11.2 \times 10^6)(0.6)} = 2.2321 \times 10^{-3} \text{ rad/in.}$$

Brass jacket:

$$\tau_{\text{all}} = 8 \text{ ksi} = 8000 \text{ psi}, \quad c_{\max} = 0.6 + 0.2 = 0.8 \text{ in.}$$

$$\frac{\varphi_{\text{all}}}{L} = \frac{8000}{(5.6 \times 10^6)(0.8)} = 1.78571 \times 10^{-3} \text{ rad/in.}$$

Smaller value governs.

$$\frac{\varphi_{\text{all}}}{L} = 1.78571 \times 10^{-3} \text{ rad/in.}$$

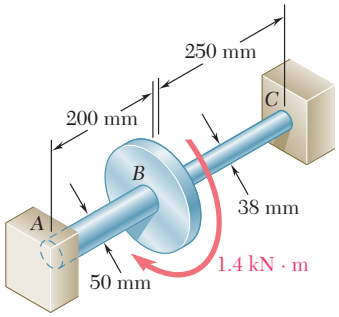
Allowable angle of twist:

$$L = 6 \text{ ft} = 72 \text{ in.}$$

$$\varphi_{\text{all}} = L\frac{\varphi_{\text{all}}}{L} = (72)(1.78571 \times 10^{-3}) = 128.571 \times 10^{-3} \text{ rad}$$

$$= 7.37° \quad \blacktriangleleft$$

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PROBLEM 3.55

Two solid steel shafts ($G = 77.2$ GPa) are connected to a coupling disk B and to fixed supports at A and C. For the loading shown, determine (*a*) the reaction at each support, (*b*) the maximum shearing stress in shaft AB, (*c*) the maximum shearing stress in shaft BC.

SOLUTION

Shaft AB: $T = T_{AB}, \quad L_{AB} = 0.200\text{ m}, \quad c = \frac{1}{2}d = 25\text{ mm} = 0.025\text{ m}$

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.025)^4 = 613.59\times10^{-9}\text{ m}^4 \qquad \varphi_B = \frac{T_{AB}L_{AB}}{GJ_{AB}}$$

$$T_{AB} = \frac{GJ_{AB}}{L_{AB}}\varphi_B = \frac{(77.2\times10^9)(613.59\times10^{-9})}{0.200}\varphi_B = 236.847\times10^3\,\varphi_B$$

Shaft BC: $T = T_{BC}, \quad L_{BC} = 0.250\text{ m}, \quad c = \frac{1}{2}d = 19\text{ mm} = 0.019\text{ m}$

$$J_{BC} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.019)^4 = 204.71\times10^{-9}\text{ m}^4 \qquad \varphi_B = \frac{T_{BC}L_{BC}}{GJ_{BC}}$$

$$T_{BC} = \frac{GJ_{BC}}{L_{BC}}\varphi_B = \frac{(77.2\times10^9)(204.71\times10^{-9})}{0.250} = 63.214\times10^3\varphi_B$$

Equilibrium of coupling disk. $T = T_{AB} + T_{BC}$

$$1.4\times10^3 = 236.847\times10^3\varphi_B + 63.214\times10^3\varphi_B \qquad \varphi_B = 4.6657\times10^{-3}\text{ rad}$$

$$T_{AB} = (236.847\times10^3)(4.6657\times10^{-3}) = 1.10506\times10^3\text{ N}\cdot\text{m}$$

$$T_{BC} = (63.214\times10^3)(4.6657\times10^{-3}) = 294.94\text{ N}\cdot\text{m}$$

(*a*) Reactions at supports. $T_A = T_{AB} = 1105\text{ N}\cdot\text{m}$ ◀

$T_C = T_{BC} = 295\text{ N}\cdot\text{m}$ ◀

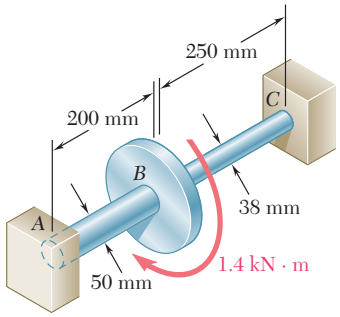
(*b*) Maximum shearing stress in AB.

$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(1.10506\times10^3)(0.025)}{613.59\times10^{-9}} = 45.0\times10^6\text{ Pa} \qquad \tau_{AB} = 45.0\text{ MPa}$$ ◀

(*c*) Maximum shearing stress in BC.

$$\tau_{BC} = \frac{T_{BC}c}{J_{BC}} = \frac{(294.94)(0.019)}{204.71\times10^{-9}} = 27.4\times10^6\text{ Pa} \qquad \tau_{BC} = 27.4\text{ MPa}$$ ◀

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PROBLEM 3.56

Solve Prob. 3.55, assuming that the shaft *AB* is replaced by a hollow shaft of the same outer diameter and 25-mm inner diameter.

PROBLEM 3.55 Two solid steel shafts ($G = 77.2$ GPa) are connected to a coupling disk *B* and to fixed supports at *A* and *C*. For the loading shown, determine (*a*) the reaction at each support, (*b*) the maximum shearing stress in shaft *AB*, (*c*) the maximum shearing stress in shaft *BC*.

SOLUTION

Shaft *AB*: $T = T_{AB}, \quad L_{AB} = 0.200 \text{ m}, \quad c_2 = 25 \text{ mm} = 0.025 \text{ m}$

$$c_1 = 12.5 \text{ mm} = 0.0125 \text{ m}$$

$$J_{AB} = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.025^4 - 0.0125^4) = 575.24 \times 10^{-9} \text{ m}^4$$

$$T_{AB} = \frac{GJ_{AB}}{L_{AB}}\varphi_B = \frac{(77.2 \times 10^9)(575.24 \times 10^{-9})}{0.200}\varphi_B = 222.04 \times 10^3 \varphi_B$$

Shaft *BC*: $T = T_{BC}, \quad L_{BC} = 0.250 \text{ m}, \quad c = \frac{1}{2}d = 19 \text{ mm} = 0.019 \text{ m}$

$$J_{BC} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.019)^4 = 204.71 \times 10^{-9} \text{ m}$$

$$T_{BC} = \frac{GJ_{BC}}{L_{BC}}\varphi_B = \frac{(77.2 \times 10^9)(204.71 \times 10^{-9})}{0.250}\varphi_B = 63.214 \times 10^3 \varphi_B$$

Equilibrium of coupling disk: $T = T_{AB} + T_{BC}$

$$1.4 \times 10^3 = 222.04 \times 10^3 \varphi_B + 63.214 \times 10^3 \varphi_B$$

$$\varphi_B = 4.9079 \times 10^{-3} \text{ rad}$$

$$T_{AB} = (222.04 \times 10^3)(4.9079 \times 10^{-3}) = 1.08975 \times 10^3 \text{ N} \cdot \text{m}$$

$$T_{BC} = (63.214 \times 10^3)(4.9079 \times 10^{-3}) = 310.25 \text{ N} \cdot \text{m}$$

(*a*) Reactions at supports:

$$T_A = T_{AB} = 1090 \text{ N} \cdot \text{m} \blacktriangleleft$$

$$T_C = T_{BC} = 310 \text{ N} \cdot \text{m} \blacktriangleleft$$

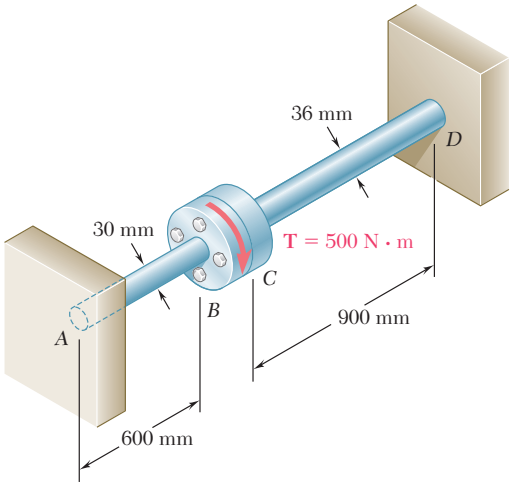
(*b*) Maximum shearing stress in *AB*:

$$\tau_{AB} = \frac{T_{AB}c_2}{J_{AB}} = \frac{(1.08975 \times 10^3)(0.025)}{575.243 \times 10^{-9}} = 47.4 \times 10^6 \text{ Pa} \qquad \tau_{AB} = 47.4 \text{ MPa} \blacktriangleleft$$

(*c*) Maximum shearing stress in *BC*:

$$\tau_{BC} = \frac{T_{BC}c}{J_{BC}} = \frac{(310.25)(0.019)}{204.71 \times 10^{-9}} = 28.8 \times 10^6 \text{ Pa} \qquad \tau_{BC} = 28.8 \text{ MPa} \blacktriangleleft$$

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PROBLEM 3.57

Two solid steel shafts are fitted with flanges that are then connected by bolts as shown. The bolts are slightly undersized and permit a 1.5° rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that $G = 77.2$ GPa, determine the maximum shearing stress in each shaft when a torque of **T** of magnitude 500 N · m is applied to the flange indicated.

The torque **T** is applied to flange *B*.

SOLUTION

Shaft *AB*: $T = T_{AB}, \quad L = 0.6\text{ m}, \quad c = \frac{1}{2}d = 0.015\text{ m}$

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4 = 79.522 \times 10^{-9}\text{ m}^4$$

$$\varphi_B = \frac{T_{AB}L_{AB}}{G_{AB}J_{AB}}$$

$$T_{AB} = \frac{G_{AB}J_{AB}}{L}\varphi_B = \frac{(77.2 \times 10^9)(79.522 \times 10^{-9})}{0.6}\varphi_B$$

$$= 10.2318 \times 10^3 \varphi_B$$

Shaft *CD*: $T = T_{CD}, \quad L_{CD} = 0.9\text{ m}, \quad c = \frac{1}{2}d = 0.018\text{ m}, \quad J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4$

$$J_{CD} = 164.896 \times 10^{-9}\text{ m}^4$$

$$T_{CD} = \frac{G_{CD}J_{CD}}{L_{CD}}\varphi_C = \frac{(77.2 \times 10^9)(164.896 \times 10^{-9})}{0.9}\varphi_C = 14.1444 \times 10^3 \varphi_C$$

Clearance rotation for flange *B*: $\varphi'_B = 1.5° = 26.178 \times 10^{-3}\text{ rad}$

Torque to remove clearance: $T'_{AB} = (10.2318 \times 10^{-3})(26.178 \times 10^{-3}) = 267.85\text{ N} \cdot \text{m}$

Torque T'' to cause additional rotation φ'' : $T'' = 500 - 267.85 = 232.15\text{ N} \cdot \text{m}$

$$T'' = T''_{AB} + T''_{CD}$$

$$232.15 = (10.2318 \times 10^3 + 14.1444 \times 10^3)\varphi'' \quad \therefore \quad \varphi'' = 9.5236 \times 10^{-3}\text{ rad}$$

$$T''_{AB} = (10.2318 \times 10^3)(9.5236 \times 10^{-3}) = 97.444\text{ N} \cdot \text{m}$$

$$T''_{CD} = (14.1444 \times 10^3)(9.5236 \times 10^{-3}) = 134.706\text{ N} \cdot \text{m}$$

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PROBLEM 3.57 (*Continued*)

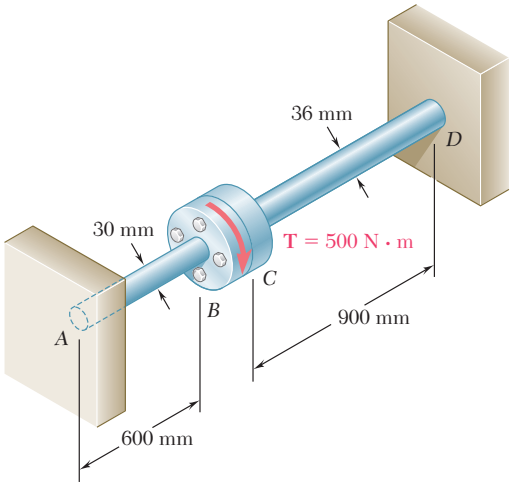
Maximum shearing stress in *AB*:

$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(267.85 + 97.444)(0.015)}{79.522 \times 10^{-9}} = 68.9 \times 10^{6}\ \text{Pa} \qquad 68.9\ \text{MPa} \blacktriangleleft$$

Maximum shearing stress in *CD*:

$$\tau_{CD} = \frac{T_{CD}c}{J_{CD}} = \frac{(134.706)(0.018)}{164.896 \times 10^{-9}} = 14.70 \times 10^{6}\ \text{Pa} \qquad 14.70\ \text{MPa} \blacktriangleleft$$

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PROBLEM 3.58

Two solid steel shafts are fitted with flanges that are then connected by bolts as shown. The bolts are slightly undersized and permit a 1.5° rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that $G = 77.2$ GPa, determine the maximum shearing stress in each shaft when a torque of **T** of magnitude 500 N · m is applied to the flange indicated.

The torque **T** is applied to flange *C*.

SOLUTION

Shaft *AB*: $T = T_{AB}, \quad L_{AB} = 0.6\text{ m}, \quad c = \frac{1}{2}d = 0.015\text{ m}$

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4 = 79.522 \times 10^{-9}\text{ m}^4$$

$$\varphi_B = \frac{T_{AB}L_{AB}}{G_{AB}J_{AB}}$$

$$T_{AB} = \frac{G_{AB}J_{AB}}{L_{AB}}\varphi_B = \frac{(77.2 \times 10^9)(79.522 \times 10^{-9})}{0.6}\varphi_B$$

$$= 10.2318 \times 10^3\,\varphi_B$$

Shaft *CD*: $T = T_{CD}, \quad L_{CD} = 0.9\text{ m}, \quad c = \frac{1}{2}d = 0.018\text{ m} \quad J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4$

$$J_{CD} = 164.896 \times 10^{-9}\text{ m}^4$$

$$T_{CD} = \frac{G_{CD}J_{CD}}{L_{CD}}\varphi_C = \frac{(77.2 \times 10^9)(164.896 \times 10^{-9})}{0.9}\varphi_C = 14.1444 \times 10^3\varphi_C$$

Clearance rotation for flange *C*: $\varphi'_C = 1.5° = 26.178 \times 10^{-3}\text{ rad}$

Torque to remove clearance: $T'_{CD} = (14.1444 \times 10^3)(26.178 \times 10^{-3}) = 370.27\text{ N} \cdot \text{m}$

Torque T'' to cause additional rotation φ'' : $T'' = 500 - 370.27 = 129.730\text{ N} \cdot \text{m}$

$$T'' = T''_{AB} + T''_{CD}$$

$$129.730 = (10.2318 \times 10^3 + 14.1444 \times 10^3)\varphi'' \qquad \varphi'' = 5.3220 \times 10^{-3}\text{ rad}$$

$$T''_{AB} = (10.2318 \times 10^3)(5.3220 \times 10^{-3}) = 54.454\text{ N} \cdot \text{m}$$

$$T''_{CD} = (14.1444 \times 10^3)(5.3220 \times 10^{-3}) = 75.277\text{ N} \cdot \text{m}$$

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PROBLEM 3.58 (*Continued*)

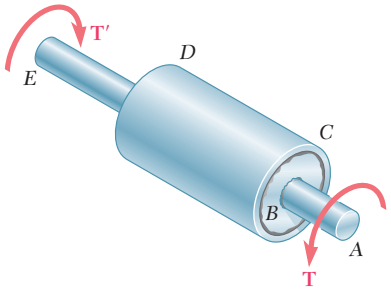
Maximum shearing stress in *AB*.

$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(54.454)(0.015)}{79.522 \times 10^{-9}} = 10.27 \times 10^{6}\ \text{Pa} \qquad 10.27\ \text{MPa} \blacktriangleleft$$

Maximum shearing stress in *CD*.

$$\tau_{CD} = \frac{T_{CD}c}{J_{CD}} = \frac{(370.27 + 75.277)(0.018)}{164.896 \times 10^{-9}} = 48.6 \times 10^{6}\ \text{Pa} = 48.6\ \text{MPa} \blacktriangleleft$$

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PROBLEM 3.59

The steel jacket *CD* has been attached to the 40-mm-diameter steel shaft *AE* by means of *rigid* flanges welded to the jacket and to the rod. The outer diameter of the jacket is 80 mm and its wall thickness is 4 mm. If 500-N · m torques are applied as shown, determine the maximum shearing stress in the jacket.

SOLUTION

Solid shaft:

$$c = \frac{1}{2}d = 0.020 \text{ m}$$

$$J_S = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

Jacket:

$$c_2 = \frac{1}{2}d = 0.040 \text{ m}$$

$$c_1 = c_2 - t = 0.040 - 0.004 = 0.036 \text{ m}$$

$$J_J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.040^4 - 0.036^4)$$

$$= 1.3829 \times 10^{-6} \text{ m}^4$$

Torque carried by shaft. $T_S = GJ_S\,\varphi/L$

Torque carried by jacket. $T_J = GJ_J\,\varphi/L$

Total torque. $T = T_S + T_J = (J_S + J_J)G\varphi/L \quad \therefore \quad \dfrac{G\varphi}{L} = \dfrac{T}{J_S + J_J}$

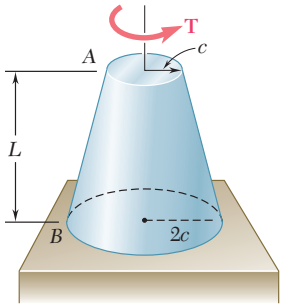
$$T_J = \frac{J_J}{J_S + J_J}T = \frac{(1.3829 \times 10^{-6})(500)}{1.3829 \times 10^{-6} + 251.33 \times 10^{-6}} = 423.1 \text{ N} \cdot \text{m}$$

Maximum shearing stress in jacket.

$$\tau = \frac{T_J c_2}{J_J} = \frac{(423.1)(0.040)}{1.3829 \times 10^{-6}} = 12.24 \times 10^6 \text{ Pa}$$

12.24 MPa ◀

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PROBLEM 3.60

A torque **T** is applied as shown to a solid tapered shaft *AB*. Show by integration that the angle of twist at *A* is

$$\phi = \frac{7TL}{12\pi Gc^4}$$

SOLUTION

Introduce coordinate *y* as shown.

$$r = \frac{cy}{L}$$

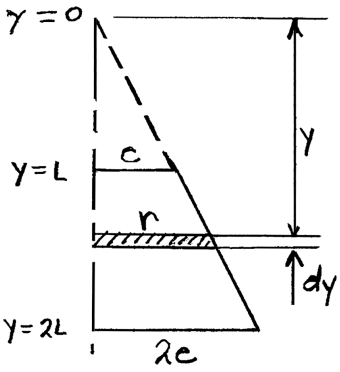
Twist in length *dy*:

$$d\varphi = \frac{T\,dy}{GJ} = \frac{T\,dy}{G\frac{\pi}{2}r^4} = \frac{2TL^4\,dy}{\pi Gc^4 y^4}$$

$$\varphi = \int_L^{2L} \frac{2TL^4}{\pi Gc^4}\frac{dy}{y^4} = \frac{2TL}{\pi Gc^4}\int_L^{2L}\frac{dy}{y^4}$$

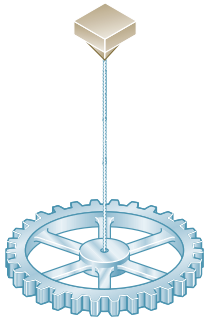
$$= \frac{2TL^4}{\pi Gc^4}\left\{-\frac{1}{3y^3}\right\}_L^{2L} = \frac{2TL^4}{\pi Gc^4}\left\{-\frac{1}{24L^3}+\frac{1}{3L^3}\right\}$$

$$= \frac{2TL^4}{\pi Gc^4}\left\{\frac{7}{24L^3}\right\} = \frac{7TL}{12\pi Gc^4}$$



◀

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PROBLEM 3.61

The mass moment of inertia of a gear is to be determined experimentally by using a torsional pendulum consisting of a 6-ft steel wire. Knowing that $G = 11.2 \times 10^6$ psi, determine the diameter of the wire for which the torsional spring constant will be 4.27 lb · ft/rad.

SOLUTION

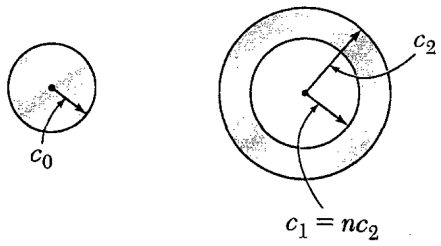
Torsion spring constant $K = 4.27$ lb · ft/rad $= 51.24$ lb · in./rad

$$K = \frac{T}{\varphi} = \frac{T}{TL/GJ} = \frac{GJ}{L} = \frac{\pi G c^4}{2L}$$

$$c^4 = \frac{2LK}{\pi G} = \frac{(2)(72)(51.24)}{\pi(11.2 \times 10^6)} = 209.7 \times 10^{-6}\ \text{in}^4$$

$$c = 0.1203 \text{ in.} \qquad d = 2c = 0.241 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.62

A solid shaft and a hollow shaft are made of the same material and are of the same weight and length. Denoting by n the ratio c_1/c_2, show that the ratio T_s/T_h of the torque T_s in the solid shaft to the torque T_h in the hollow shaft is (*a*) $\sqrt{(1-n^2)}/(1+n^2)$ if the maximum shearing stress is the same in each shaft, (*b*) $(1-n)/(1+n^2)$ if the angle of twist is the same for each shaft.

SOLUTION

For equal weight and length, the areas are equal.

$$\pi c_0^2 = \pi\left(c_2^2 - c_1^2\right) = \pi c_2^2(1-n^2) \qquad \therefore \qquad c_0 = c_2(1-n^2)^{1/2}$$

$$J_s = \frac{\pi}{2}c_0^4 = \frac{\pi}{2}c_2^4(1-n^2)^2 \qquad J_h = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}c_2^4(1-n^4)$$

(*a*) For equal stresses,

$$\tau = \frac{T_s c_0}{J_s} = \frac{T_h c_2}{J_h}$$

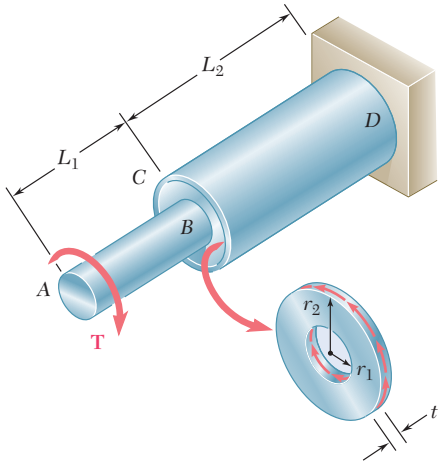
$$\frac{T_s}{T_h} = \frac{J_s c_2}{J_h c_0} = \frac{\frac{\pi}{2}c_2^4(1-n^2)^2 c_2}{\frac{\pi}{2}c_2^4(1-n^4)c_2(1-n^2)^{1/2}} = \frac{1-n^2}{(1+n^2)(1-n^2)^{1/2}} = \frac{(1-n^2)^{1/2}}{1+n^2} \quad \blacktriangleleft$$

(*b*) For equal angles of twist,

$$\varphi = \frac{T_s L}{G J_s} = \frac{T_h L}{G J_h}$$

$$\frac{T_s}{T_h} = \frac{J_s}{J_h} = \frac{\frac{\pi}{2}c_2^4(1-n^2)^2}{\frac{\pi}{2}c_2^4(1-n^4)} = \frac{(1-n^2)^2}{1-n^4} = \frac{1-n^2}{1+n^2} \quad \blacktriangleleft$$

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PROBLEM 3.63

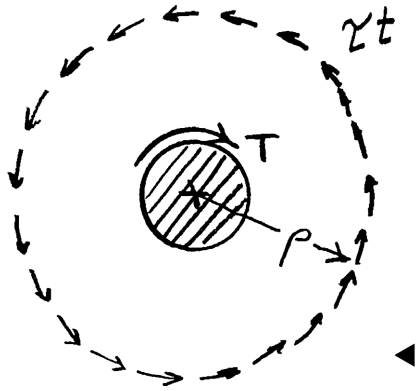
An annular plate of thickness t and modulus G is used to connect shaft AB of radius r_1 to tube CD of radius r_2. Knowing that a torque **T** is applied to end A of shaft AB and that end D of tube CD is fixed, (*a*) determine the magnitude and location of the maximum shearing stress in the annular plate, (*b*) show that the angle through which end B of the shaft rotates with respect to end C of the tube is

$$\varphi_{BC} = \frac{T}{4\pi Gt}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$

SOLUTION

Use a free body consisting of shaft AB and an inner portion of the plate BC, the outer radius of this portion being ρ.

The force per unit length of circumference is τt.



$$\Sigma M = 0$$

$$\tau t(2\pi\rho)\rho - T = 0$$

$$\tau = \frac{T}{2\pi t\rho^2}$$

(*a*) Maximum shearing stress occurs at $\rho = r_1$ $\qquad \tau_{\max} = \dfrac{T}{2\pi t r_1^2}$ ◀

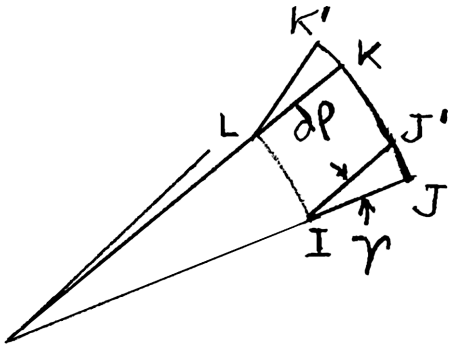
Shearing strain: $\qquad \gamma = \dfrac{\tau}{G} = \dfrac{T}{2\pi GT\rho^2}$

The relative circumferential displacement in radial length $d\rho$ is

$$d\delta = \gamma d\rho = \rho\, d\varphi$$

$$d\varphi = \gamma\frac{d\rho}{\rho}$$

$$d\varphi = \frac{T}{2\pi Gt\rho^2}\frac{d\rho}{\rho} = \frac{Td\rho}{2\pi GT\rho^3}$$



(*b*) $$\varphi_{B/C} = \int_{r_1}^{r_2}\frac{Td\rho}{2\pi Gt\rho^3} = \frac{T}{2\pi Gt}\int_{r_1}^{r_2}\frac{d\rho}{\rho^3} = \frac{T}{2\pi Gt}\left\{-\frac{1}{2\rho^2}\right\}\Bigg|_{r_1}^{r_2}$$

$$= \frac{T}{2\pi Gt}\left\{-\frac{1}{2r_2^2} + \frac{1}{2r_1^2}\right\} = \frac{T}{4\pi Gt}\left\{\frac{1}{r_1^2} - \frac{1}{r_2^2}\right\}$$ ◀

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PROBLEM 3.64

Determine the maximum shearing stress in a solid shaft of 1.5-in. diameter as it transmits 75 hp at a speed of (*a*) 750 rpm, (*b*) 1500 rpm.

SOLUTION

$$c = \frac{1}{2}d = 0.75 \text{ in.} \qquad P = 75 \text{ hp} = (75)(6600) = 495 \times 10^3 \text{ lb} \cdot \text{in./s}$$

(*a*) $f = \dfrac{750}{60} = 12.5 \text{ Hz}$

$$T = \frac{P}{2\pi f} = \frac{495 \times 10^3}{2\pi(12.5)} = 6.3025 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(6.3025 \times 10^3)}{\pi(0.75)^3} = 9.51 \times 10^3 \text{ psi} \qquad \tau = 9.51 \text{ ksi} \blacktriangleleft$$

(*b*) $f = \dfrac{1500}{60} = 25 \text{ Hz}$

$$T = \frac{495 \times 10^3}{2\pi(25)} = 3.1513 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\tau = \frac{(2)(3.1513 \times 10^3)}{\pi(0.75)^3} = 4.76 \times 10^3 \text{ psi} \qquad \tau = 4.76 \text{ ksi} \blacktriangleleft$$

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PROBLEM 3.65

Determine the maximum shearing stress in a solid shaft of 12-mm diameter as it transmits 2.5 kW at a frequency of (*a*) 25 Hz, (*b*) 50 Hz.

SOLUTION

$$c = \frac{1}{2}d = 6 \text{ mm} = 0.006 \text{ m} \qquad P = 2.5 \text{ kW} = 2500 \text{ W}$$

(*a*) $f = 25$ Hz. $\qquad T = \dfrac{P}{2\pi f} = \dfrac{2500}{2\pi(25)} = 15.9155 \text{ N} \cdot \text{m}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(15.9155)}{\pi(0.006)^3} = 46.9 \times 10^6 \text{ Pa} \qquad \tau = 46.9 \text{ MPa} \blacktriangleleft$$

(*b*) $f = 50$ Hz. $\qquad T = \dfrac{2500}{2\pi(50)} = 7.9577 \text{ N} \cdot \text{m}$

$$\tau = \frac{2(7.9577)}{\pi(0.006)^3} = 23.5 \times 10^6 \text{ Pa} \qquad \tau = 23.5 \text{ MPa} \blacktriangleleft$$

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PROBLEM 3.66

Using an allowable shearing stress of 4.5 ksi, design a solid steel shaft to transmit 12 hp at a speed of (*a*) 1200 rpm, (*b*) 2400 rpm.

SOLUTION

$$\tau_{\text{all}} = 4.5 \text{ ksi} = 4500 \text{ psi} \qquad P = 12 \text{ hp} = (12)(6600) = 79.2 \times 10^3 \text{ lb} \cdot \text{in./s}$$

(*a*) $$f = \frac{1200}{60} = 20 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{79.2 \times 10^3}{2\pi(20)} = 630.25 \text{ lb} \cdot \text{in.}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt{\frac{(2)(630.25)}{\pi(4500)}} = 0.44675 \text{ in.}$$

$$d = 2c = 0.893 \text{ in.} \blacktriangleleft$$

(*b*) $$f = \frac{2400}{60} = 40 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{79.2 \times 10^3}{2\pi(40)} = 315.13 \text{ lb} \cdot \text{in.}$$

$$c = \sqrt[3]{\frac{(2)(315.13)}{\pi(4500)}} = 0.35458 \text{ in.}$$

$$d = 2c = 0.709 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.67

Using an allowable shearing stress of 50 MPa, design a solid steel shaft to transmit 15 kW at a frequency of (*a*) 30 Hz, (*b*) 60 Hz.

SOLUTION

$$\tau_{\text{all}} = 50 \text{ MPa} = 50 \times 10^6 \text{ Pa} \qquad P = 15 \text{ kW} = 15 \times 10^3 \text{ W}$$

(*a*) $f = 30$ Hz:

$$T = \frac{P}{2\pi f} = \frac{15 \times 10^3}{2\pi(30)} = 79.577 \text{ N} \cdot \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(79.577)}{\pi(50 \times 10^6)}} = 10.0438 \times 10^{-3} \text{ m} = 10.0438 \text{ mm}$$

$$d = 2c = 20.1 \text{ mm} \blacktriangleleft$$

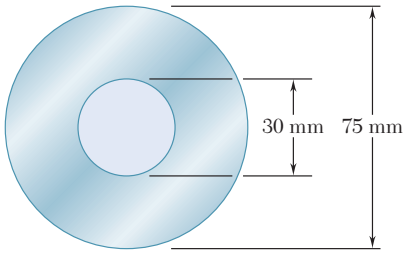
(*b*) $f = 60$ Hz:

$$T = \frac{P}{2\pi f} = \frac{15 \times 10^3}{2\pi(60)} = 39.789 \text{ N} \cdot \text{m}$$

$$c = \sqrt[3]{\frac{(2)(39.789)}{\pi(50 \times 10^6)}} = 7.9718 \times 10^{-3} \text{ m} = 7.9718 \text{ mm}$$

$$d = 2c = 15.94 \text{ mm} \blacktriangleleft$$

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PROBLEM 3.68

While a steel shaft of the cross section shown rotates at 120 rpm, a stroboscopic measurement indicates that the angle of twist is 2° in a 4-m length. Using $G = 77.2$ GPa, determine the power being transmitted.

SOLUTION

Twist angle: $\varphi = 2^\circ = 34.907 \times 10^{-3}$ rad

$$c_1 = \frac{1}{2} d_1 = 0.015 \text{ m}$$

$$c_2 = \frac{1}{2} d_2 = 0.0375 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.0375^4 - 0.015^4)$$

$$J = 3.0268 \times 10^{-6} \text{ m}^4, \qquad L = 4 \text{ m}$$

$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L} = \frac{(77.2 \times 10^9)(3.0268 \times 10^{-6})(34.907 \times 10^{-3})}{4}$$

$$T = 2.0392 \times 10^3 \text{ N} \cdot \text{m}$$

$$f = 120 \text{ rpm} = \frac{120}{60} \text{ Hz} = 2 \text{ Hz}$$

$$P = (2\pi f)T = 2\pi(2)(2.0392 \times 10^3) = 25.6 \times 10^3 \text{ W} = 25.6 \text{ kW}$$ ◀

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PROBLEM 3.69

Determine the required thickness of the 50-mm tubular shaft of Concept Application 3.7 if it is to transmit the same power while rotating at a frequency of 30 Hz.

SOLUTION

From Example 3.07, $P = 100\,\text{kW} = 100 \times 10^3\,\text{W}$

$$\tau_{\text{all}} = 60\,\text{MPa} = 60 \times 10^6\,\text{Pa} \qquad c_2 = \frac{1}{2}d = 25\,\text{mm} = 0.025\,\text{m}$$

$$f = 30\,\text{Hz}$$

$$T = \frac{P}{2\pi f} = 530.52\,\text{N} \cdot \text{m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) \qquad \tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 0.025^4 - \frac{(2)(530.52)(0.025)}{\pi(60 \times 10^6)} = 249.90 \times 10^{-9}\,\text{m}^4$$

$$c_1 = 22.358 \times 10^{-3}\,\text{m} = 22.358\,\text{mm}$$

$$t = c_2 - c_1 = 25\,\text{mm} - 22.358\,\text{mm} = 2.642\,\text{mm} \qquad t = 2.64\,\text{mm} \blacktriangleleft$$

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PROBLEM 3.70

A steel drive shaft is 6 ft long and its outer and inner diameters are respectively equal to 2.25 in. and 1.75 in. Knowing that the shaft transmits 240 hp while rotating at 1800 rpm, determine (*a*) the maximum shearing stress, (*b*) the angle of twist of the shaft ($G = 11.2 \times 10^6$ psi).

SOLUTION

$$L = 6 \text{ ft} = 72 \text{ in.}$$

$$G = 11 \times 10^6 \text{ psi}$$

$$\tau_{\text{all}} = 7500 \text{ psi}$$

$$P = 240 \text{ hp} \frac{6600 \text{ lb} \cdot \text{in./s}}{\text{hp}} = 1.584 \times 10^6 \text{ lb} \cdot \text{in./s}$$

$$f = 1800 \text{ rpm} \frac{1 \text{ Hz}}{60 \text{ rpm}} = 30 \text{ Hz}$$

$$P = T(2\pi f): \quad T = \frac{P}{2\pi f} = \frac{1.584 \times 10^6 \text{ lb} \cdot \text{in./s}}{2\pi(30 \text{ Hz})} = 8403.4 \text{ lb} \cdot \text{in.}$$

(*a*) $$\tau_m = \frac{Tc}{J} = \frac{(8403.4 \text{ lb} \cdot \text{in.})(1.125 \text{ in.})}{\frac{\pi}{2}[(1.125 \text{ in.})^4 - (0.875 \text{ in.})^4]}$$

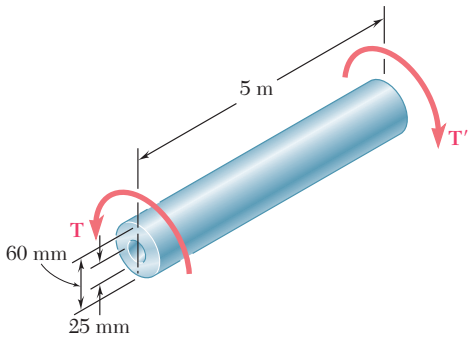
$$\tau_m = 5930 \text{ psi} \blacktriangleleft$$

(*b*) $$\varphi = \frac{TL}{GJ} = \frac{(8403.4 \text{ lb} \cdot \text{in.})(72 \text{ in.})}{(11 \times 10^6 \text{ psi})\frac{\pi}{2}[(1.125 \text{ in.})^4 - (0.875 \text{ in}^4)]}$$

$$= 34.48 \times 10^{-3} \text{ radians} \left(\frac{180^\circ}{\pi \text{ radians}} \right)$$

$$\varphi = 1.98^\circ \blacktriangleleft$$

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PROBLEM 3.71

The hollow steel shaft shown ($G = 77.2$ GPa, $\tau_{all} = 50$ MPa) rotates at 240 rpm. Determine (*a*) the maximum power that can be transmitted, (*b*) the corresponding angle of twist of the shaft.

SOLUTION

$$c_2 = \frac{1}{2}d_2 = 30 \text{ mm}$$

$$c_1 = \frac{1}{2}d_1 = 12.5 \text{ mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4]$$

$$= 1.234 \times 10^6 \text{mm}^4 = 1.234 \times 10^{-6} \text{m}^4$$

$$\tau_m = 50 \times 10^6 \text{ Pa}$$

$$\tau_m = \frac{Tc}{J} \quad T = \frac{\tau_m J}{c} = \frac{(50 \times 10^6)(1.234 \times 10^{-6})}{30 \times 10^{-3}} = 2056.7 \text{ N} \cdot \text{m}$$

Angular speed: $f = 240 \text{ rpm} = 4 \text{ rev/sec} = 4 \text{ Hz}$

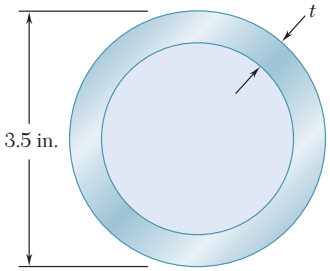
(*a*) Power being transmitted. $P = 2\pi f T = 2\pi(4)(2056.7) = 51.7 \times 10^3 \text{ W}$

$$P = 51.7 \text{ kW} \blacktriangleleft$$

(*b*) Angle of twist. $\varphi = \frac{TL}{GJ} = \frac{(2056.7)(5)}{(77.2 \times 10^9)(1.234 \times 10^{-6})} = 0.1078 \text{ rad}$

$$\varphi = 6.17° \blacktriangleleft$$

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PROBLEM 3.72

A steel pipe of 3.5-in. outer diameter is to be used to transmit a torque of 3000 lb · ft without exceeding an allowable shearing stress of 8 ksi. A series of 3.5-in.-outer-diameter pipes is available for use. Knowing that the wall thickness of the available pipes varies from 0.25 in. to 0.50 in. in 0.0625-in. increments, choose the lightest pipe that can be used.

SOLUTION

$$T = 3000 \text{ lb} \cdot \text{ft} = 36 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$c_2 = \frac{1}{2} d_o = 1.75 \text{ in.}$$

$$\tau = \frac{Tc_2}{J} = \frac{2\text{T}c_2}{\pi\left(c_2^4 - c_1^4\right)}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 1.75^4 - \frac{(2)(36 \times 10^3)(1.75)}{\pi(8 \times 10^3)} = 4.3655 \text{ in}^4$$

$$c_1 = 1.4455 \text{ in.}$$

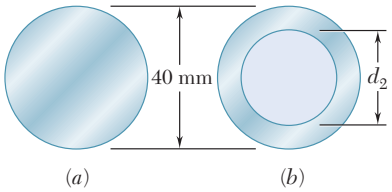
Required minimum thickness: $t = c_2 - c_1$

$$t = 1.75 - 1.4455 = 0.3045 \text{ in.}$$

Available thicknesses: 0.25 in., 0.3125 in., 0.375 in., etc.

Use $t = 0.3125$ in. ◀

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PROBLEM 3.73

The design of a machine element calls for a 40-mm-outer-diameter shaft to transmit 45 kW. (*a*) If the speed of rotation is 720 rpm, determine the maximum shearing stress in shaft *a*. (*b*) If the speed of rotation can be increased 50% to 1080 rpm, determine the largest inner diameter of shaft *b* for which the maximum shearing stress will be the same in each shaft.

SOLUTION

(*a*) $f = \dfrac{720}{60} = 12 \text{ Hz}$

$P = 45 \text{ kW} = 45 \times 10^3 \text{ W}$

$T = \dfrac{P}{2\pi f} = \dfrac{45 \times 10^3}{2\pi(12)} = 596.83 \text{ N} \cdot \text{m}$

$c = \dfrac{1}{2} d = 20 \text{ mm} = 0.020 \text{ m}$

$\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3} = \dfrac{(2)(596.83)}{\pi(0.020)^3} = 47.494 \times 10^6 \text{ Pa}$ $\qquad \tau_{\max} = 47.5 \text{ MPa}$ ◀

(*b*) $f = \dfrac{1080}{60} = 18 \text{ Hz}$

$T = \dfrac{45 \times 10^3}{2\pi(18)} = 397.89 \text{ N} \cdot \text{m}$

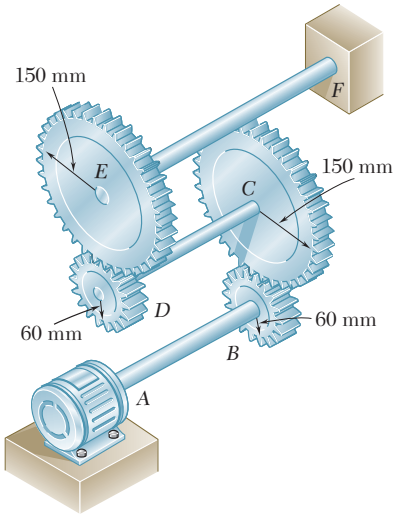
$\tau = \dfrac{Tc_2}{J} = \dfrac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$

$c_1^4 = c_2^4 - \dfrac{2Tc_2}{\pi\tau}$

$c_1^4 = 0.020^4 - \dfrac{(2)(397.89)(0.020)}{\pi(47.494 \times 10^6)} = 53.333 \times 10^{-9}$

$c_1 = 15.20 \times 10^{-3} \text{m} = 15.20 \text{ mm}$ $\qquad d_2 = 2c_1 = 30.4 \text{ mm}$ ◀

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PROBLEM 3.74

Three shafts and four gears are used to form a gear train that will transmit power from the motor at *A* to a machine tool at *F*. (Bearings for the shafts are omitted in the sketch.) The diameter of each shaft is as follows: $d_{AB}=16$ mm, $d_{CD}=20$ mm, $d_{EF}=28$ mm. Knowing that the frequency of the motor is 24 Hz and that the allowable shearing stress for each shaft is 75 MPa, determine the maximum power that can be transmitted.

SOLUTION

$$\tau_{\text{all}}=75\text{ MPa}=75\times10^{6}\text{ Pa}$$

Shaft *AB*:

$$c_{AB}=\frac{1}{2}d_{AB}=0.008\text{ m}\qquad \tau=\frac{Tc_{AB}}{J_{AB}}=\frac{2T}{\pi c_{AB}^{3}}$$

$$T_{\text{all}}=\frac{\pi}{2}c_{AB}^{3}\tau_{\text{all}}=\frac{\pi}{2}(0.008)^{3}(75\times10^{6})=60.319\text{ N}\cdot\text{m}$$

$$f_{AB}=24\text{ Hz}\qquad P_{\text{all}}=2\pi f_{AB}T_{\text{all}}=2\pi(24)(60.319)=9.10\times10^{3}\text{ W}$$

Shaft *CD*:

$$c_{CD}=\frac{1}{2}d_{CD}=0.010\text{ m}$$

$$\tau=\frac{Tc_{CD}}{J_{CD}}=\frac{2T}{\pi c_{CD}^{3}}\quad\therefore\quad T_{\text{all}}=\frac{\pi}{2}c_{CD}^{3}\tau_{\text{all}}=\frac{\pi}{2}(0.010)^{3}(75\times10^{6})=117.81\text{ N}\cdot\text{m}$$

$$f_{CD}=\frac{r_B}{r_C}f_{AB}=\frac{60}{150}(24)=9.6\text{ Hz}\qquad P_{\text{all}}=2\pi f_{CD}T_{\text{all}}=2\pi(9.6)(117.81)=7.11\times10^{3}\text{ W}$$

Shaft *EF*:

$$c_{EF}=\frac{1}{2}d_{EF}=0.014\text{ m}$$

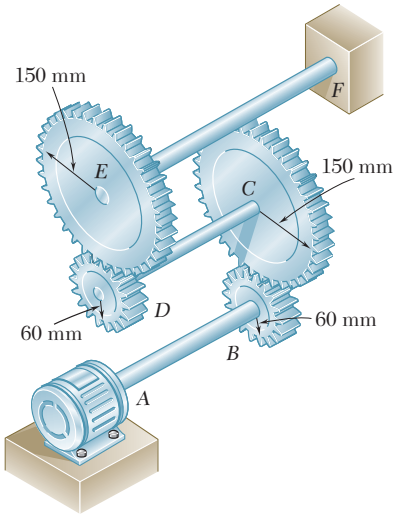
$$T_{\text{all}}=\frac{\pi}{2}c_{EF}^{3}\tau_{\text{all}}=\frac{\pi}{2}(0.014)^{3}(75\times10^{6})=323.27\text{ N}\cdot\text{m}$$

$$f_{EF}=\frac{r_D}{r_E}f_{CD}=\frac{60}{150}(9.6)=3.84\text{ Hz}$$

$$P_{\text{all}}=2\pi f_{EF}T_{\text{all}}=2\pi(3.84)(323.27)=7.80\times10^{3}\text{ W}$$

Maximum allowable power is the smallest value. $P_{\text{all}}=7.11\times10^{3}\text{ W}=7.11\text{ kW}$ ◀

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PROBLEM 3.75

Three shafts and four gears are used to form a gear train that will transmit 7.5 kW from the motor at *A* to a machine tool at *F*. (Bearings for the shafts are omitted in the sketch.) Knowing that the frequency of the motor is 30 Hz and that the allowable stress for each shaft is 60 MPa, determine the required diameter of each shaft.

SOLUTION

$$P = 7.5\text{ kW} = 7.5\times10^{3}\text{ W} \qquad \tau_{\text{all}} = 60\text{ MPa} = 60\times10^{6}\text{ Pa}$$

Shaft *AB*:

$$f_{AB} = 30\text{ Hz} \qquad T_{AB} = \frac{P}{2\pi f_{AB}} = \frac{7.5\times10^{3}}{2\pi(30)} = 39.789\text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc_{AB}}{J_{AB}} = \frac{2T}{\pi c_{AB}^{3}} \quad\therefore\quad c_{AB}^{3} = \frac{2T}{\pi\tau}$$

$$c_{AB}^{3} = \frac{(2)(39.789)}{\pi(60\times10^{6})} = 422.17\times10^{-9}\text{ m}^{3}$$

$$c_{AB} = 7.50\times10^{-3}\text{ m} = 7.50\text{ mm} \qquad d_{AB} = 2c_{AB} = 15.00\text{ mm}\ \blacktriangleleft$$

Shaft *CD*:

$$f_{CD} = \frac{r_B}{r_C}f_{AB} = \frac{60}{150}(30) = 12\text{ Hz} \qquad T_{CD} = \frac{P}{2\pi f_{CD}} = \frac{7.5\times10^{3}}{2\pi(12)} = 99.472\text{ N}\cdot\text{m}$$

$$\tau_{CD} = \frac{Tc_{CD}}{J_{CD}} = \frac{2T}{\pi c_{CD}^{3}} \quad\therefore\quad c_{CD}^{3} = \frac{2T_{CD}}{\pi\tau} = \frac{2(99.472)}{\pi(60\times10^{6})} = 1.05543\times10^{-6}\text{ m}^{3}$$

$$c_{CD} = 10.18\times10^{-3}\text{ m} = 10.18\text{ mm} \qquad d_{CD} = 2c_{CD} = 20.4\text{ mm}\ \blacktriangleleft$$

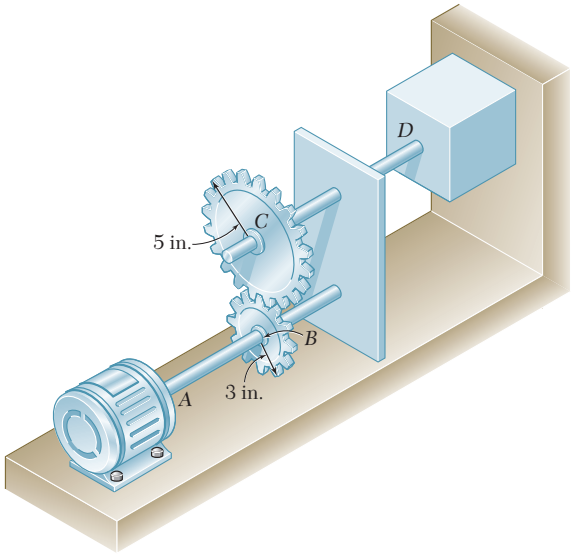
Shaft *EF*:

$$f_{EF} = \frac{r_D}{r_E}f_{CD} = \frac{60}{150}(12) = 4.8\text{ Hz} \qquad T_{EF} = \frac{P}{2\pi f_{EF}} = \frac{7.5\times10^{3}}{2\pi(4.8)} = 248.68\text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc_{EF}}{J_{EF}} = \frac{2T}{\pi c_{EF}^{3}} \quad\therefore\quad c_{EF}^{3} = \frac{(2)(248.68)}{\pi(60\times10^{6})} = 2.6886\times10^{-6}\text{ m}^{3}$$

$$c_{EF} = 13.82\times10^{-3}\text{ m} = 13.82\text{ mm} \qquad d_{EF} = 2c_{EF} = 27.6\text{ mm}\ \blacktriangleleft$$

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PROBLEM 3.76

The two solid shafts and gears shown are used to transmit 16 hp from the motor at *A* operating at a speed of 1260 rpm to a machine tool at *D*. Knowing that each shaft has a diameter of 1 in., determine the maximum shearing stress (*a*) in shaft *AB*, (*b*) in shaft *CD*.

SOLUTION

(*a*) Shaft *AB*:

$$P = 16 \text{ hp} = (16)(6600) = 105.6 \times 10^3 \text{ lb} \cdot \text{in./sec}$$

$$f = \frac{1260}{60} = 21 \text{ Hz}$$

$$T_{AB} = \frac{P}{2\pi f} = \frac{105.6 \times 10^3}{2\pi(21)} = 800.32 \text{ lb} \cdot \text{in.}$$

$$c = \frac{1}{2}d = 0.5 \text{ in.}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

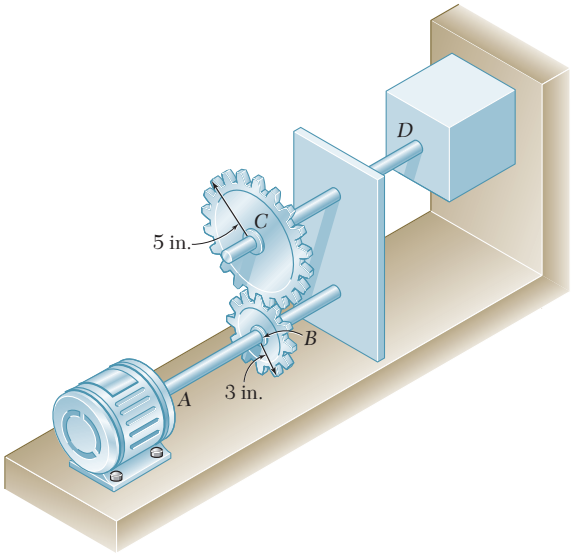
$$= \frac{(2)(800.32)}{\pi(0.5)^3} = 4.08 \times 10^3 \text{ psi} \qquad \tau_{AB} = 4.08 \text{ ksi} \blacktriangleleft$$

(*b*) Shaft *CD*:

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{5}{3}(800.32) = 1.33387 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\tau = \frac{2T}{\pi c^3} = \frac{(2)(1.33387 \times 10^3)}{\pi(0.5)^3} = 6.79 \times 10^3 \text{ psi} \qquad \tau_{CD} = 6.79 \text{ ksi} \blacktriangleleft$$

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PROBLEM 3.77

The two solid shafts and gears shown are used to transmit 16 hp from the motor at *A*, operating at a speed of 1260 rpm, to a machine tool at *D*. Knowing that the maximum allowable shearing stress is 8 ksi, determine the required diameter (*a*) of shaft *AB*, (*b*) of shaft *CD*.

SOLUTION

(*a*) Shaft *AB*:

$$P = 16 \text{ hp} = (16)(6600) = 105.6 \times 10^3 \text{ lb} \cdot \text{in./sec}$$

$$f = \frac{1260}{60} = 21 \text{ Hz}$$

$$\tau = 8 \text{ ksi} = 8 \times 10^3 \text{ psi}$$

$$T_{AB} = \frac{P}{2\pi f} = \frac{105.6 \times 10^3}{2\pi(21)} = 800.32 \text{ lb} \cdot \text{in.}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi\tau}}$$

$$c = \sqrt[3]{\frac{(2)(800.32)}{\pi(8 \times 10^3)}} = 0.399 \text{ in.}$$

$$d_{AB} = 2c = 0.799 \text{ in.} \qquad d_{AB} = 0.799 \text{ in.} \blacktriangleleft$$

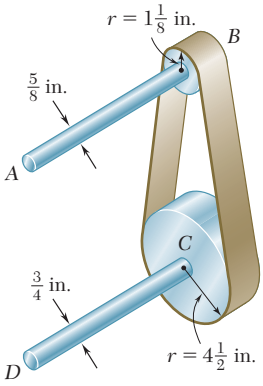
(*b*) Shaft *CD*:

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{5}{3}(800.32) = 1.33387 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(1.33387 \times 10^3)}{\pi(8 \times 10^3)}} = 0.473 \text{ in.}$$

$$d_{CD} = 2c = 0.947 \text{ in.} \qquad d_{CD} = 0.947 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.78

The shaft-disk-belt arrangement shown is used to transmit 3 hp from point *A* to point *D*. (*a*) Using an allowable shearing stress of 9500 psi, determine the required speed of shaft *AB*. (*b*) Solve part *a*, assuming that the diameters of shafts *AB* and *CD* are, respectively, 0.75 in. and 0.625 in.

SOLUTION

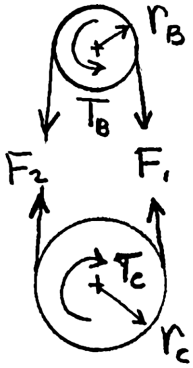
$\tau = 9500$ psi $\quad P = 3\,\text{hp} = (3)(6600) = 19{,}800\,\text{lb}\cdot\text{in./s}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad T = \frac{\pi}{2}c^3\tau$$

Allowable torques.

$\frac{5}{8}$-in. diameter shaft: $\quad c = \frac{5}{16}\,\text{in.}, \quad T_{\text{all}} = \frac{\pi}{2}\left(\frac{5}{16}\right)^3(9500) = 455.4\,\text{lb}\cdot\text{in.}$

$\frac{3}{4}$-in. diameter shaft: $\quad c = \frac{3}{8}\,\text{in.}, \quad T_{\text{all}} = \frac{\pi}{2}\left(\frac{3}{8}\right)^3(9500) = 786.9\,\text{lb}\cdot\text{in.}$



Statics: $\quad T_B = r_B(F_1 - F_2) \qquad T_C = r_C(F_1 - F_2)$

$$T_B = \frac{r_B}{r_C}T_C = \frac{1.125}{4.5}T_C = 0.25T_C$$

(*a*) Allowable torques. $\quad T_{B,\text{all}} = 455.4\,\text{lb}\cdot\text{in.} \qquad T_{C,\text{all}} = 786.9\,\text{lb}\cdot\text{in.}$

Assume $\quad T_C = 786.9\,\text{lb}\cdot\text{in.}$

Then $\quad T_B = (0.25)(786.9) = 196.73\,\text{lb}\cdot\text{in.} < 455.4\,\text{lb}\cdot\text{in.}$ (okay)

$$P = 2\pi f T \qquad f_{AB} = \frac{P}{2\pi T_B} = \frac{19{,}800}{2\pi(196.73)} \qquad f_{AB} = 16.02\ \text{Hz} \blacktriangleleft$$

(*b*) Allowable torques. $\quad T_{B,\text{all}} = 786.9\,\text{lb}\cdot\text{in.} \qquad T_{C,\text{all}} = 455.4\,\text{lb}\cdot\text{in.}$

Assume $\quad T_C = 455.4\,\text{lb}\cdot\text{in.}$

Then $\quad T_B = (0.25)(455.4) = 113.85\,\text{lb}\cdot\text{in.} < 786.9\,\text{lb}\cdot\text{in.}$

$$P = 2\pi f T \qquad f_{AB} = \frac{P}{2\pi T_B} = \frac{19{,}800}{2\pi(113.85)} \qquad f_{AB} = 27.2\ \text{Hz} \blacktriangleleft$$

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PROBLEM 3.79

A 5-ft-long solid steel shaft of 0.875-in. diameter is to transmit 18 hp between a motor and a machine tool. Determine the lowest speed at which the shaft can rotate, knowing that $G = 11.2 \times 10^6$ psi, that the maximum shearing stress must not exceed 4.5 ksi, and the angle of twist must not exceed 3.5°.

SOLUTION

$$L = 5 \text{ ft} = 60 \text{ in.} \quad c = \frac{1}{2}d = 0.4375 \text{ in.} \quad P = 18 \text{ hp} = 118.8 \times 10^3 \text{ lb} \cdot \text{in./s}$$

Stress requirement. $\tau = 4.5 \text{ ksi} = 4.5 \times 10^3 \text{ psi} \qquad \tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$$T = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(4.5 \times 10^3)(0.4375)^3 = 591.92 \text{ lb} \cdot \text{in.}$$

Twist angle requirement. $\varphi = 3.5° = 61.087 \times 10^{-3} \text{ rad} \qquad \varphi = \dfrac{TL}{GJ}$

$$T = \frac{GJ\varphi}{L} = \frac{\pi G c^4 \varphi}{2L} = \frac{\pi (11.2 \times 10^6)(0.4375)^4 (61.087 \times 10^{-3})}{(2)(60)}$$

$$= 656.21 \text{ lb} \cdot \text{in.}$$

Maximum allowable torque is the smaller value. $T = 591.92 \text{ lb} \cdot \text{in.}$

$$P = 2\pi f T \qquad f = \frac{P}{2\pi T} = \frac{118.8 \times 10^3}{2\pi (591.92)} = 31.943 \text{ Hz} \qquad f = 1917 \text{ rpm} \blacktriangleleft$$

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PROBLEM 3.80

A 2.5-m-long steel shaft of 30-mm diameter rotates at a frequency of 30 Hz. Determine the maximum power that the shaft can transmit, knowing that $G = 77.2\ \text{GPa}$, that the allowable shearing stress is 50 MPa, and that the angle of twist must not exceed 7.5°.

SOLUTION

$$c = \frac{1}{2}d = 15\ \text{mm} = 0.015\ \text{m} \quad L = 2.5\ \text{m}$$

Stress requirement.

$$\tau = 50 \times 10^6\ \text{Pa} \quad \tau = \frac{Tc}{J}$$

$$T = \frac{\tau J}{c} = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(50 \times 10^6)(0.015)^3 = 265.07\ \text{N} \cdot \text{m}$$

Twist angle requirement.

$$\varphi = 7.5° = 130.90 \times 10^{-3}\ \text{rad} \quad G = 77.2 \times 10^9\ \text{Pa}$$

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$T = \frac{\pi}{2}Gc^4\varphi = \frac{\pi}{2}(77.2 \times 10^9)(0.015)^4(130.90 \times 10^{-3}) = 803.60\ \text{N} \cdot \text{m}$$

Smaller value of T is the maximum allowable torque.

$$T = 265.07\ \text{N} \cdot \text{m}$$

Power transmitted at $f = 30$ Hz.

$$P = 2\pi f T = 2\pi(30)(265.07) = 49.96 \times 10^3\ \text{W} \qquad P = 50.0\ \text{kW} \blacktriangleleft$$

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PROBLEM 3.81

The design specifications of a 1.2-m-long solid transmission shaft require that the angle of twist of the shaft not exceed 4° when a torque of 750 N · m is applied. Determine the required diameter of the shaft, knowing that the shaft is made of a steel with an allowable shearing stress of 90 MPa and a modulus of rigidity of 77.2 GPa.

SOLUTION

$$T = 750 \text{ N} \cdot \text{m}, \quad \varphi = 4^\circ = 69.813 \times 10^{-3} \text{ rad},$$

$$L = 1.2 \text{ m}, \quad J = \frac{\pi}{2} c^4$$

$$\tau = 90 \text{ MPa} = 90 \times 10^6 \text{ Pa} \quad G = 77.2 \text{ GPa} = 77.2 \times 10^9 \text{ Pa}$$

Based on angle of twist. $\varphi = \dfrac{TL}{GJ} = \dfrac{2TL}{\pi G c^4}$

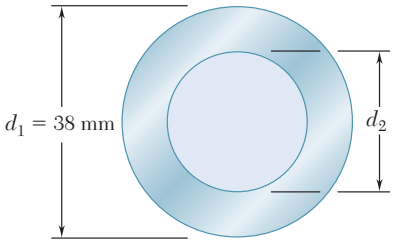
$$c = \sqrt[4]{\frac{2TL}{\pi G \varphi}} = \sqrt[4]{\frac{(2)(750)(1.2)}{\pi (77.2 \times 10^9)(69.813 \times 10^{-3})}} = 18.06 \times 10^{-3} \text{ m}$$

Based on shearing stress. $\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(750)}{\pi (90 \times 10^6)}} = 17.44 \times 10^{-3} \text{ m}$$

Use larger value. $c = 18.06 \times 10^{-3} \text{ m} = 18.06 \text{ mm}$ $\quad d = 2c = 36.1 \text{ mm}$ ◀

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PROBLEM 3.82

A 1.5-m-long tubular steel shaft (G = 77.2 GPa) of 38-mm outer diameter d_1 and 30-mm inner diameter d_2 is to transmit 100 kW between a turbine and a generator. Knowing that the allowable shearing stress is 60 MPa and that the angle of twist must not exceed 3°, determine the minimum frequency at which the shaft can rotate.

SOLUTION

$$L = 1.5 \text{ m}, \quad \varphi = 3° = 52.360 \times 10^{-3} \text{ rad}$$

$$c_2 = \frac{1}{2} d_o = 19 \text{ mm} = 0.019 \text{ m}, \quad c_1 = \frac{1}{2} d_i = 15 \text{ mm} = 0.015 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.019^4 - 0.015^4) = 125.186 \times 10^{-9} \text{ m}^4$$

Stress requirement. $\tau = 60 \times 10^6$ Pa $\quad \tau = \dfrac{Tc_2}{J}$

$$T = \frac{J\tau}{c_2} = \frac{(125.186 \times 10^{-9})(60 \times 10^6)}{0.019} = 395.32 \text{ N} \cdot \text{m}$$

Twist angle requirement. $\varphi = \dfrac{TL}{GJ}$

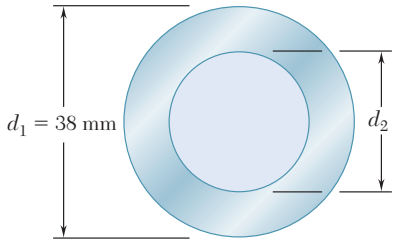
$$T = \frac{GJ\varphi}{L} = \frac{(77.2 \times 10^9)(125.186 \times 10^{-9})(52.360 \times 10^{-3})}{1.5} = 337.35 \text{ N} \cdot \text{m}$$

Maximum allowable torque is the smaller value. $T = 337.35 \text{ N} \cdot \text{m}$

Power transmitted: $P = 100 \text{ kW} = 100 \times 10^3 \text{ W} \quad P = 2\pi f T$

Frequency: $$f = \frac{P}{2\pi T} = \frac{100 \times 10^3}{2\pi(337.35)} = 47.2 \text{ Hz}$$ $f = 47.2$ Hz ◀

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PROBLEM 3.83

A 1.5-m-long tubular steel shaft of 38-mm outer diameter d_1 is to be made of a steel for which $\tau_{all} = 65$ MPa and $G = 77.2$ GPa. Knowing that the angle of twist must not exceed 4° when the shaft is subjected to a torque of 600 N · m, determine the largest inner diameter d_2 that can be specified in the design.

SOLUTION

$$L = 1.5 \text{ m} \quad c_2 = \frac{1}{2} d_o = 19 \text{ mm} = 0.019 \text{ m}$$

$$\tau = 65 \times 10^6 \text{ Pa} \quad \varphi = 4° = 69.813 \times 10^{-3} \text{ rad}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)$$

Stress requirement.

$$\tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$$

$$c_1 = \sqrt[4]{c_2^4 - \frac{2Tc_2}{\pi\tau}} = \sqrt[4]{0.019^4 - \frac{(2)(600)(0.019)}{\pi(65 \times 10^6)}}$$

$$= 11.6889 \times 10^{-3} \text{ m} = 11.6889 \text{ mm}$$

Twist angle requirement.

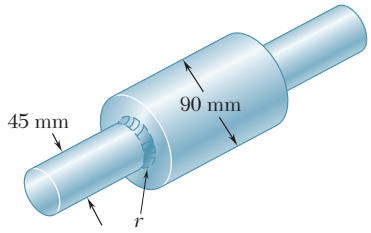
$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G\left(c_2^4 - c_1^4\right)}$$

$$c_1 = \sqrt[4]{c_2^4 - \frac{2TL}{\pi G\varphi}} = \sqrt[4]{0.019^4 - \frac{(2)(600)(1.5)}{\pi(77.2 \times 10^9)(69.813 \times 10^{-3})}}$$

$$c_1 = 12.448 \times 10^3 \text{ m} = 12.4482 \text{ mm}$$

Use smaller value of c_1. $c_1 = 11.6889$ mm $\quad d_i = 2c_1 = 23.4$ mm ◀

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PROBLEM 3.84

The stepped shaft shown must transmit 40 kW at a speed of 720 rpm. Determine the minimum radius r of the fillet if an allowable stress of 36 MPa is not to be exceeded.

SOLUTION

Angular speed: $f = (720 \text{ rpm})\left(\frac{1 \text{ Hz}}{60 \text{ rpm}}\right) = 12 \text{ Hz}$

Power: $P = 40 \times 10^3 \text{ W}$

Torque: $T = \frac{P}{2\pi f} = \frac{40 \times 10^3}{2\pi(12)} = 530.52 \text{ N} \cdot \text{m}$

In the smaller shaft, $d = 45 \text{ mm}, \quad c = 22.5 \text{ mm} = 0.0225 \text{ m}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(530.52)}{\pi(0.0225)^3} = 29.65 \times 10^6 \text{ Pa}$$

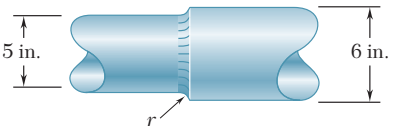
Using $\tau_{\max} = 36 \text{ MPa} = 36 \times 10^6 \text{ Pa}$ results in

$$K = \frac{\tau_{\max}}{\tau} = \frac{36 \times 10^6}{29.65 \times 10^6} = 1.214$$

From Fig 3.32 with $\frac{D}{d} = \frac{90 \text{ mm}}{45 \text{ mm}} = 2, \quad \frac{r}{d} = 0.24$

$r = 0.24d = (0.24)(45 \text{ mm})$ $\qquad r = 10.8 \text{ mm}$ ◀

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PROBLEM 3.85

The stepped shaft shown rotates at 450 rpm. Knowing that $r = 0.5$ in., determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 7500 psi.

SOLUTION

$$d = 5 \text{ in.}$$
$$D = 6 \text{ in.}$$
$$r = 0.5 \text{ in.}$$
$$\frac{D}{d} = \frac{6}{5} = 1.20$$
$$\frac{r}{d} = \frac{0.5}{5} = 0.10$$

From Fig. 3.32, $K = 1.33$

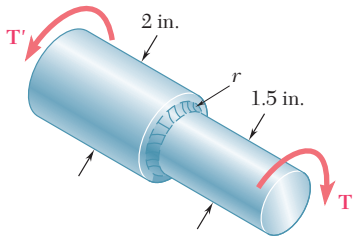
For smaller side, $c = \frac{1}{2}d = 2.5$ in.

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$
$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(2.5)^3(7500)}{(2)(1.33)} = 138.404 \times 10^3 \text{ lb} \cdot \text{in.}$$
$$f = 450 \text{ rpm} = 7.5 \text{ Hz}$$

Power. $P = 2\pi f T = 2\pi(7.5)(138.404 \times 10^3) = 6.52 \times 10^6 \text{ in.} \cdot \text{lb/s}$

Recalling that $1 \text{ hp} = 6600 \text{ in.} \cdot \text{lb/s}$, $P = 988 \text{ hp}$ ◀

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PROBLEM 3.86

Knowing that the stepped shaft shown transmits a torque of magnitude $T = 2.50$ kip · in., determine the maximum shearing stress in the shaft when the radius of the fillet is (*a*) $r = \frac{1}{8}$ in., (*b*) $r = \frac{3}{16}$ in.

SOLUTION

$$D = 2 \text{ in.} \quad d = 1.5 \text{ in.} \qquad \frac{D}{d} = \frac{2}{1.5} = 1.33$$

$$c = \frac{1}{2}d = 0.75 \text{ in.} \qquad T = 2.5 \text{ kip} \cdot \text{in.}$$

$$\frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2.5)}{\pi(0.75)^3} = 3.773 \text{ ksi}$$

(*a*) $r = \frac{1}{8}$ in. $\quad r = 0.125$ in.

$$\frac{r}{d} = \frac{0.125}{1.5} = 0.0833$$

From Fig. 3.32, $\quad K = 1.42$

$$\tau_{\max} = K\frac{Tc}{J} = (1.42)(3.773) \qquad \tau_{\max} = 5.36 \text{ ksi} \blacktriangleleft$$

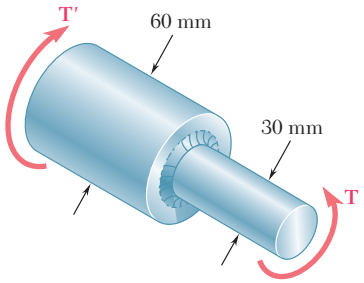
(*b*) $r = \frac{3}{16}$ in. $\qquad r = 0.1875$ in.

$$\frac{r}{d} = \frac{0.1875}{1.5} = 0.125$$

From Fig. 3.32, $\quad K = 1.33$

$$\tau_{\max} = K\frac{Tc}{J} = (1.33)(3.773) \qquad \tau_{\max} = 5.02 \text{ ksi} \blacktriangleleft$$

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PROBLEM 3.87

The stepped shaft shown must rotate at a frequency of 50 Hz. Knowing that the radius of the fillet is $r = 8$ mm and the allowable shearing stress is 45 MPa, determine the maximum power that can be transmitted.

SOLUTION

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} \qquad T = \frac{\pi c^3 \tau}{2K}$$

$$d = 30 \text{ mm} \qquad c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

$$D = 60 \text{ mm}, \qquad r = 8 \text{ mm}$$

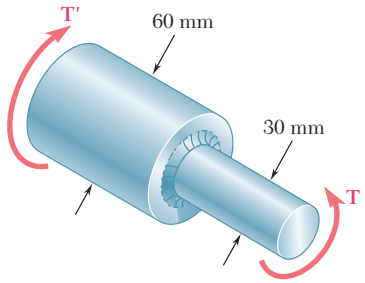
$$\frac{D}{d} = \frac{60}{30} = 2, \qquad \frac{r}{d} = \frac{8}{30} = 0.26667$$

From Fig. 3.32, $K = 1.18$

Allowable torque. $$T = \frac{\pi(15 \times 10^{-3})^3(45 \times 10^6)}{(2)(1.18)} = 202.17 \text{ N} \cdot \text{m}$$

Maximum power. $$P = 2\pi f T = (2\pi)(50)(202.17) = 63.5 \times 10^3 \text{ W}$$ $P = 63.5$ kW ◀

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PROBLEM 3.88

The stepped shaft shown must transmit 45 kW. Knowing that the allowable shearing stress in the shaft is 40 MPa and that the radius of the fillet is $r = 6$ mm, determine the smallest permissible speed of the shaft.

SOLUTION

$$\frac{r}{d} = \frac{6}{30} = 0.2$$

$$\frac{D}{d} = \frac{60}{30} = 2$$

From Fig. 3.32, $K = 1.26$

For smaller side, $c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m}$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

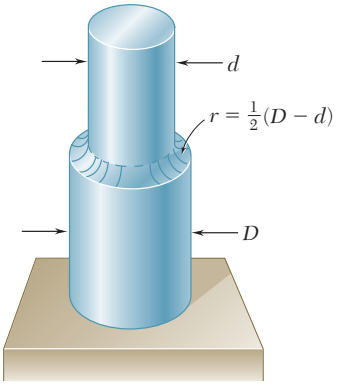
$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.015)^3(40 \times 10^6)}{(2)(1.26)} = 168.30 \text{ N} \cdot \text{m}$$

$$P = 45 \text{ kW} = 45 \times 10^3 \qquad P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{45 \times 10^3}{2\pi(168.30 \times 10^3)} = 42.6 \text{ Hz}$$

$f = 42.6$ Hz ◀

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Full quarter-circular fillet extends to edge of larger shaft.

PROBLEM 3.89

A torque of magnitude $T = 200\ \text{lb} \cdot \text{in.}$ is applied to the stepped shaft shown, which has a full quarter-circular fillet. Knowing that $D = 1\ \text{in.}$, determine the maximum shearing stress in the shaft when (*a*) $d = 0.8\ \text{in.}$, (*b*) $d = 0.9\ \text{in.}$

SOLUTION

(*a*) $\dfrac{D}{d} = \dfrac{1.0}{0.8} = 1.25$

$r = \dfrac{1}{2}(D - d) = 0.1\ \text{in.}$

$\dfrac{r}{d} = \dfrac{0.1}{0.8} = 0.125$

From Fig. 3.32, $K = 1.31$

For smaller side, $c = \dfrac{1}{2}d = 0.4\ \text{in.}$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$= \frac{(2)(1.31)(200)}{\pi(0.4)^3} = 2.61 \times 10^3\ \text{psi} \qquad \tau = 2.61\ \text{ksi} \blacktriangleleft$$

(*b*) $\dfrac{D}{d} = \dfrac{1.0}{0.9} = 1.111$

$r = \dfrac{1}{2}(D - d) = 0.05$

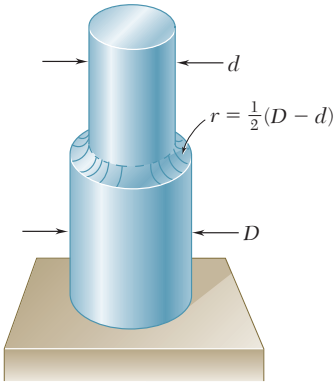
$\dfrac{r}{d} = \dfrac{0.05}{1.0} = 0.05$

From Fig. 3.32, $K = 1.44$

For smaller side, $c = \dfrac{1}{2}d = 0.45\ \text{in.}$

$$\tau = \frac{2KT}{\pi c^3} = \frac{(2)(1.44)(200)}{\pi(0.45)^3} = 2.01 \times 10^3\ \text{psi} \qquad \tau = 2.01\ \text{ksi} \blacktriangleleft$$

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Full quarter-circular fillet extends to edge of larger shaft.

PROBLEM 3.90

In the stepped shaft shown, which has a full quarter-circular fillet, the allowable shearing stress is 80 MPa. Knowing that $D = 30$ mm, determine the largest allowable torque that can be applied to the shaft if (*a*) $d = 26$ mm, (*b*) $d = 24$ mm.

SOLUTION

$$\tau = 80 \times 10^6 \text{ Pa}$$

(*a*) $\dfrac{D}{d} = \dfrac{30}{26} = 1.154 \qquad r = \dfrac{1}{2}(D - d) = 2 \text{ mm} \qquad \dfrac{r}{d} = \dfrac{2}{26} = 0.0768$

From Fig. 3.32, $K = 1.36$

Smaller side, $c = \dfrac{1}{2}d = 13 \text{ mm} = 0.013 \text{ m}$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.013)^3(80 \times 10^6)}{(2)(1.36)} = 203 \text{ N} \cdot \text{m} \qquad T = 203 \text{ N} \cdot \text{m} \blacktriangleleft$$

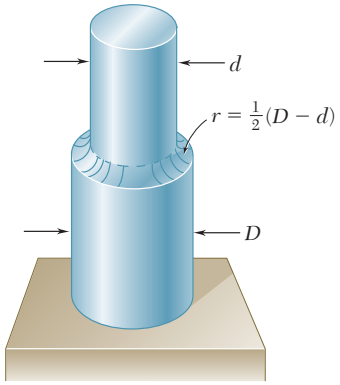
(*b*) $\dfrac{D}{d} = \dfrac{30}{24} = 1.25 \qquad r = \dfrac{1}{2}(D - d) = 3 \text{ mm} \qquad \dfrac{r}{d} = \dfrac{3}{24} = 0.125$

From Fig. 3.32, $K = 1.31$

$$c = \frac{1}{2}d = 12 \text{ mm} = 0.012 \text{ m}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.012)^3(80 \times 10^6)}{(2)(1.31)} = 165.8 \text{ N} \cdot \text{m} \qquad T = 165.8 \text{ N} \cdot \text{m} \blacktriangleleft$$

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Full quarter-circular fillet extends to edge of larger shaft.

PROBLEM 3.91

In the stepped shaft shown, which has a full quarter-circular fillet, $D = 1.25$ in. and $d = 1$ in. Knowing that the speed of the shaft is 2400 rpm and that the allowable shearing stress is 7500 psi, determine the maximum power that can be transmitted by the shaft.

SOLUTION

$$\frac{D}{d} = \frac{1.25}{1.0} = 1.25$$

$$r = \frac{1}{2}(D - d) = 0.15 \text{ in.}$$

$$\frac{r}{d} = \frac{0.15}{1.0} = 0.15$$

From Fig. 3.32, $K = 1.31$

For smaller side, $c = \frac{1}{2}d = 0.5$ in.

$$\tau = \frac{KTc}{J} \quad T = \frac{J\tau}{Kc} = \frac{\pi c^3 \tau}{2K}$$

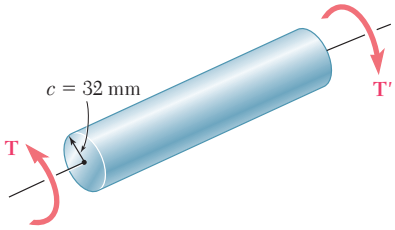
$$T = \frac{\pi(0.5)^3(7500)}{(2)(1.31)} = 1.1241 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$f = 2400 \text{ rpm} = 40 \text{ Hz}$$

$$P = 2\pi f T = 2\pi(40)(1.1241 \times 10^3)$$

$$= 282.5 \times 10^3 \text{ lb} \cdot \text{in./s} \qquad P = 42.8 \text{ hp} \blacktriangleleft$$

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PROBLEM 3.92

The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa. Determine the magnitude T of the applied torques when the plastic zone is (*a*) 16 mm deep, (*b*) 24 mm deep.

SOLUTION

$$c = 32 \text{ mm} = 0.32 \text{ m} \qquad \tau_Y = 145 \times 10^6 \text{ Pa}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.032)^3(145 \times 10^6)$$

$$= 7.4634 \times 10^3 \text{ N} \cdot \text{m}$$

(*a*) $t_p = 16 \text{ mm} = 0.016 \text{ m} \qquad \rho_Y = c - t_p = 0.032 - 0.016 = 0.016 \text{ m}$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(7.4634 \times 10^3)\left(1 - \frac{1}{4}\frac{0.016^3}{0.032^3}\right)$$

$$= 9.6402 \times 10^3 \text{ N} \cdot \text{m} \qquad T = 9.64 \text{ kN} \cdot \text{m} \blacktriangleleft$$

(*b*) $t_p = 24 \text{ mm} = 0.024 \text{ m} \qquad \rho_Y = c - t_p = 0.032 - 0.024 = 0.008 \text{ m}$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(7.4634 \times 10^3)\left(1 - \frac{1}{4}\frac{0.008^3}{0.032^3}\right)$$

$$= 9.9123 \times 10^3 \text{ N} \cdot \text{m} \qquad T = 9.91 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 3.93

A 1.25-in.-diameter solid rod is made of an elastoplastic material with $\tau_Y = 5$ ksi. Knowing that the elastic core of the rod is 1 in. in diameter, determine the magnitude of the applied torque **T**.

SOLUTION

$$c = \frac{1}{2}d = 0.625 \text{ in.}$$

$$\tau_Y = 5 \times 10^3 \text{ psi}$$

$$\rho_Y = \frac{1}{2}d_Y = 0.5 \text{ in.}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.625)^3(5 \times 10^3)$$

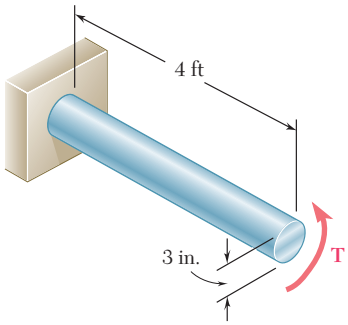
$$= 1.91747 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(1.91747 \times 10^3)\left(1 - \frac{1}{4}\frac{0.5^3}{0.625^3}\right)$$

$$= 2.23 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$T = 2230 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

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PROBLEM 3.94

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 11.2 \times 10^6$ psi and $\tau_Y = 21$ ksi. Determine the maximum shearing stress and the radius of the elastic core caused by the application of torque of magnitude (*a*) $T = 100$ kip · in., (*b*) $T = 140$ kip · in.

SOLUTION

$$c = 1.5 \text{ in.}, \quad J = \frac{\pi}{2} c^4 = 7.9522 \text{ in}^4, \quad \tau_Y = 21 \text{ ksi}$$

(*a*) $T = 100$ kip · in.

$$\tau_m = \frac{Tc}{J} = \frac{(100 \text{ kip} \cdot \text{in.})(1.5 \text{ in.})}{7.9522 \text{ in}^4} \qquad \tau_m = 18.86 \text{ ksi} \blacktriangleleft$$

Since $\tau_m < \tau_Y$, shaft remains elastic.

Radius of elastic core: $c = 1.500$ in. ◀

(*b*) $T = 140$ kip · in.

$$\tau_m = \frac{(140)(1.5)}{7.9522} = 26.4 \text{ ksi.}$$

Impossible: $\tau_m = \tau_Y = 21.0$ ksi ◀

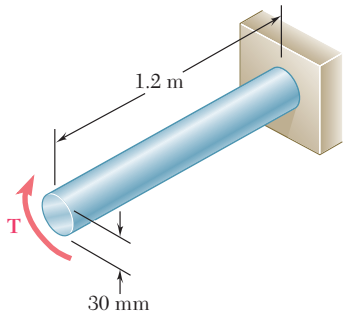
Plastic zone has developed. Torque at onset of yield is $T_Y = \frac{J}{c}\tau_Y = \frac{7.9522}{1.5}(21 \text{ ksi}) = 111.33 \text{ kip} \cdot \text{in.}$

Eq. (3.32): $T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\rho_Y^3}{c^3}\right)$

$$\left(\frac{\rho_Y}{c}\right)^3 = 4 - 3\frac{T}{T_Y} = 4 - 3\frac{140}{111.33} = 0.22743 \qquad \frac{\rho_Y}{c} = 0.6104$$

$$\rho_Y = 0.6104c = 0.6104(1.5 \text{ in.}) \qquad \rho_Y = 0.916 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.95

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2$ GPa and $\tau_Y = 145$ MPa. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (a) $T = 600$ N · m, (b) $T = 1000$ N · m.

SOLUTION

(*a*) $T = 600$ N · m: Since $T < T_Y$, the shaft is elastic

Maximum shearing stress: $\dfrac{\tau_m}{\tau_Y} = \dfrac{T}{T_Y}$

$$\tau_m = \frac{\tau_Y T}{T_Y} = \frac{(145 \times 10^6)(600)}{768.11} = 113.3 \times 10^6 \text{ Pa} \qquad \tau_m = 113.3 \text{ MPa} \blacktriangleleft$$

Radius of elastic core: $\rho = c$ $\qquad \rho_E = 15.00$ mm ◀

(*b*) $T = 1000$ N · m: Since $T > T_Y$, there is a plastic zone.

Maximum shearing stress: $\tau_m = \tau_Y$ $\qquad \tau_m = 145.0$ MPa ◀

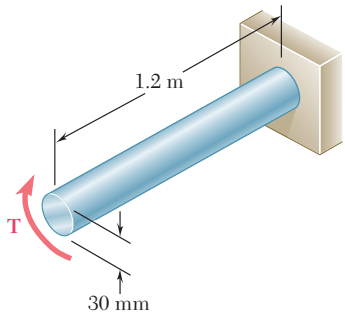
Radius of elastic core: $T = \dfrac{4}{3} T_Y \left[1 - \left(\dfrac{\rho_Y}{\rho} \right)^3 \right]$

$$\frac{\rho_Y}{c} = \sqrt[3]{4 - 3\frac{T}{T_Y}} = \sqrt[3]{4 - \frac{(3)(1000)}{768.71}} = 0.46003$$

$$\rho_Y = 0.46003c = (0.46003)(15 \text{ mm})$$

$\rho_Y = 6.90$ mm ◀

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PROBLEM 3.96

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa. Determine the radius of the elastic core caused by the application of a torque equal to 1.1 T_Y, where T_Y is the magnitude of the torque at the onset of yield.

SOLUTION

$$c = \frac{1}{2}d = 15 \text{ mm} \qquad T = \frac{4}{3}T_Y\left[1 - \left(\frac{\rho_Y}{c}\right)^3\right]$$

$$\frac{\rho_Y}{c} = \sqrt[3]{4 - 3\frac{T}{T_Y}} = \sqrt{4 - (3)(1.1)} = 0.88790$$

$$\rho_Y = 0.88790\,c = (0.88790)(15 \text{ mm}) \qquad \rho_Y = 13.32 \text{ mm} \blacktriangleleft$$

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PROBLEM 3.97

It is observed that a straightened paper clip can be twisted through several revolutions by the application of a torque of approximately 60 mN · m. Knowing that the diameter of the wire in the paper clip is 0.9 mm, determine the approximate value of the yield stress of the steel.

SOLUTION

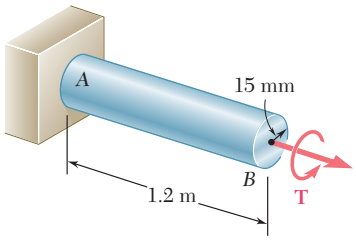
$$c = \frac{1}{2}d = 0.45 \text{ mm} = 0.45 \times 10^{-3} \text{ m}$$

$$T_P = 60 \text{ mN} \cdot \text{m} = 60 \times 10^{-3} \text{ N} \cdot \text{m}$$

$$T_P = \frac{4}{3}T_Y = \frac{4}{3}\frac{J\tau_Y}{c} = \frac{4}{3} \cdot \frac{\pi}{2}c^3 \; T_Y = \frac{2\pi}{3}c^3\tau_Y$$

$$\tau_Y = \frac{3T_P}{2\pi c^3} = \frac{(3)(60 \times 10^{-3})}{2\pi(0.45 \times 10^{-3})^3} = 314 \times 10^6 \text{ Pa} \qquad \tau_Y = 314 \text{ MPa} \blacktriangleleft$$

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PROBLEM 3.98

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2$ GPa and $\tau_Y = 145$ MPa. Determine the angle of twist caused by the application of a torque of magnitude (*a*) $T = 600$ N · m, (*b*) $T = 1000$ N · m.

SOLUTION

$$c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

Torque at onset of yielding: $\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$$T_Y = \frac{\pi c^3 \tau_Y}{2} = \frac{\pi (15 \times 10^{-3})^3 (145 \times 10^6)}{2} = 768.71 \text{ N} \cdot \text{m}$$

(*a*) $T = 600$ N · m. Since $T \le T_Y$, the shaft is elastic.

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi c^4 G} = \frac{(2)(600)(1.2)}{\pi (15 \times 10^{-3})^4 (77.2 \times 10^9)} = 0.11728 \text{ rad} \quad \varphi = 6.72° \blacktriangleleft$$

(*b*) $T = 1000$ N · m. $T > T_Y$ A plastic zone has developed.

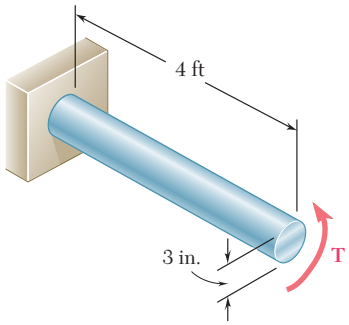
$$T = \frac{4}{3} T_Y \left[1 - \left(\frac{\varphi_Y}{\varphi} \right)^3 \right] \qquad \frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - 3\left(\frac{T}{T_Y} \right)}$$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(768.71)(2.1)}{\pi (15 \times 10^{-3})^4 (77.2 \times 10^9)} = 0.15026 \text{ rad}$$

$$\frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - \frac{(3)(1000)}{768.71}} = 0.46003$$

$$\varphi = \frac{\varphi_Y}{0.46003} = \frac{0.15026}{0.46003} = 0.32663 \text{ rad} \qquad \varphi = 18.71° \blacktriangleleft$$

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PROBLEM 3.99

For the solid circular shaft of Prob. 3.94, determine the angle of twist caused by the application of a torque of magnitude (*a*) $T = 80$ kip · in., (*b*) $T = 130$ kip · in.

SOLUTION

$$c = \frac{1}{2}d = \frac{1}{2}(3) = 1.5 \text{ in.} \quad \tau_Y = 21 \times 10^3 \text{ psi}$$

$$L = 4 \text{ ft} = 48 \text{ in.} \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.5)^4 = 7.9522 \text{ in}^4$$

Torque at onset of yielding: $\tau = \dfrac{Tc}{J} \qquad T = \dfrac{\tau J}{c}$

$$T_Y = \frac{\tau_Y J}{c} = \frac{(21 \times 10^3)(7.9522)}{1.5} = 111.330 \times 10^3 \text{ lb} \cdot \text{in.}$$

(*a*) $T = 80 \text{ kip} \cdot \text{in.} = 80 \times 10^3 \text{ lb} \cdot \text{in.}$

Since $T < T_Y$, the shaft is fully elastic. $\varphi = \dfrac{TL}{GJ}$

$$\varphi = \frac{(80 \times 10^3)(48)}{(11.2 \times 10^6)(7.9522)} = 43.115 \times 10^{-3} \text{ rad} \qquad \varphi = 2.47° \blacktriangleleft$$

(*b*) $T = 130 \text{ kip} \cdot \text{in.} = 130 \times 10^3 \text{ lb} \cdot \text{in.} \qquad T > T_Y \qquad T = \dfrac{4}{3}T_Y\left[1 - \left(\dfrac{\varphi_Y}{\varphi}\right)^3\right]$

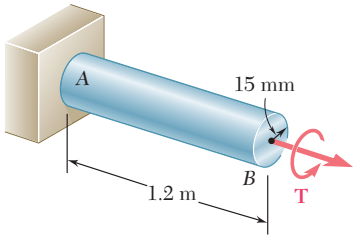
$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{(111.330 \times 10^3)(48)}{(11.2 \times 10^6)(7.9522)} = 60.000 \times 10^{-3} \text{ rad}$$

$$\frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - 3\frac{T}{T_Y}} = \sqrt[3]{4 - \frac{(3)(130 \times 10^3)}{111.330 \times 10^3}} = 0.79205$$

$$\varphi = \frac{\varphi_Y}{0.79205} = \frac{60.000 \times 10^{-3}}{0.79205} = 75.75 \times 10^{-3} \text{ rad} \qquad \varphi = 4.34° \blacktriangleleft$$

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PROBLEM 3.100



For the solid shaft of Prob. 3.98, determine (*a*) the magnitude of the torque **T** required to twist the shaft through an angle of 15°, (*b*) the radius of the corresponding elastic core.

PROBLEM 3.98 The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2$ GPa and $\tau_Y = 145$ MPa. Determine the angle of twist caused by the application of a torque of magnitude (*a*) $T = 600\ \text{N} \cdot \text{m}$, (*b*) $T = 1000\ \text{N} \cdot \text{m}$.

SOLUTION

$$c = \frac{1}{2}d = 15\,\text{mm} = 15 \times 10^{-3}\,\text{m}$$

$$\varphi = 15° = 0.2618\ \text{rad}$$

$$\varphi_Y = \frac{L\gamma_Y}{c} = \frac{L\tau_Y}{cG} = \frac{(1.2)(145 \times 10^6)}{(15 \times 10^{-3})(77.2 \times 10^9)} = 0.15026\ \text{rad}$$

(*a*) Since $\varphi > \varphi_Y$, there is a plastic zone.

$$\tau_Y = \frac{T_Y c}{J} = \frac{2T}{\pi c^3}$$

$$T_Y = \frac{\pi c^3 \tau_Y}{2} = \frac{\pi(15 \times 10^{-3})^3(145 \times 10^6)}{2} = 768.71\ \text{N} \cdot \text{m}$$

$$T = \frac{4}{3}T_Y\left[1 - \frac{1}{4}\left(\frac{\varphi_Y}{\varphi}\right)^3\right] = \frac{4}{3}(768.71)\left[1 - \frac{1}{4}\left(\frac{0.15026}{0.2618}\right)^3\right]$$

$$= 976.5\ \text{N} \cdot \text{m} \qquad T = 977\ \text{N} \cdot \text{m} \blacktriangleleft$$

(*b*) $L\gamma_Y = \rho_Y\varphi = c\varphi_Y$

$$\rho_Y = \frac{c\varphi_Y}{\varphi} = \frac{(15 \times 10^{-3})(0.15026)}{0.2618} = 8.61 \times 10^{-3}\,\text{m} \qquad \rho_Y = 8.61\ \text{mm} \blacktriangleleft$$

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PROBLEM 3.101

A 3-ft-long solid shaft has a diameter of 2.5 in. and is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 21$ ksi and $G = 11.2 \times 10^6$ psi. Determine the torque required to twist the shaft through an angle of (*a*) 2.5°, (*b*) 5°.

SOLUTION

$$L = 3 \text{ ft} = 36 \text{ in.}, \quad c = \frac{1}{2}d = 1.25 \text{ in.}, \quad \tau_Y = 21 \times 10^3 \text{ psi}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.25)^4 = 3.835 \text{ in}^4$$

$$\tau_Y = \frac{T_Y c}{J} \qquad T_Y = \frac{J\tau_Y}{c} = \frac{(3.835)(21 \times 10^3)}{1.25} = 64.427 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{(64.427 \times 10^3)(36)}{(11.2 \times 10^6)(3.835)} = 53.999 \times 10^{-3} \text{ rad} = 3.0939°$$

(*a*) $\varphi = 2.5° = 43.633 \times 10^{-3}$ rad $\qquad \varphi < \varphi_Y$ The shaft remains elastic.

$$\varphi = \frac{TL}{GJ}$$

$$T = \frac{GJ\varphi}{L} = \frac{(11.2 \times 10^6)(3.835)(43.633 \times 10^{-3})}{36}$$

$$= 52.059 \times 10^3 \text{ lb} \cdot \text{in.} \qquad T = 52.1 \text{ kip} \cdot \text{in.} \blacktriangleleft$$

(*b*) $\varphi = 5° = 87.266 \times 10^{-3}$ rad $\qquad \varphi > \varphi_Y$ A plastic zone occurs.

$$T = \frac{4}{3}T_Y\left[1 - \frac{1}{4}\left(\frac{\varphi_Y}{\varphi}\right)^3\right]$$

$$= \frac{4}{3}(64.427 \times 10^3)\left[1 - \frac{1}{4}\left(\frac{53.999 \times 10^{-3}}{87.266 \times 10^{-3}}\right)^3\right]$$

$$= 80.814 \times 10^3 \text{ lb} \cdot \text{in.} \qquad T = 80.8 \text{ kip} \cdot \text{in.} \blacktriangleleft$$

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PROBLEM 3.102

A 18-mm-diameter solid circular shaft is made of a material that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. For a 1.2-m length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 200-N · m torque.

SOLUTION

$$\tau_Y = 145 \times 10^6 \text{ Pa}, \quad c = \frac{1}{2}d = 0.009 \text{ m}, \quad L = 1.2 \text{ m}, \quad T = 200 \text{ N} \cdot \text{m}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.009)^3(145 \times 10^6) = 166.04 \text{ N} \cdot \text{m}$$

$T > T_Y$ (plastic region with elastic core) $\qquad \tau_{\max} = \tau_Y = 145.0 \text{ MPa}$ ◀

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(166.04)(1.2)}{\pi(0.009)^4(77.2 \times 10^9)} = 250.43 \times 10^{-3} \text{ radians}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\varphi^3}{\varphi_Y^3}\right)$$

$$\left(\frac{\varphi_Y}{\varphi}\right)^3 = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(200)}{166.04} = 0.38641 \qquad \frac{\varphi_Y}{\varphi} = 0.72837$$

$$\varphi = \frac{\varphi}{0.72837} = \frac{250.43 \times 10^{-3}}{0.72837} = 343.82 \times 10^{-3} \text{ radians}$$

$\varphi = 19.70°$ ◀

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PROBLEM 3.103

A 0.75-in.-diameter solid circular shaft is made of a material that is assumed to be elastoplastic with $\tau_Y = 20$ ksi and $G = 11.2 \times 10^6$ psi. For a 4-ft length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 1800-lb · in. torque.

SOLUTION

$$\tau_Y = 20 \text{ ksi} = 20 \times 10^3 \text{ psi}, \quad c = \frac{1}{2} d = 0.375 \text{ in.},$$

$$L = 4 \text{ ft} = 48 \text{ in.}, \quad T = 1800 \text{ lb} \cdot \text{in.}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2}(0.375)^3(20 \times 10^6)$$

$$= 1656.70 \text{ lb} \cdot \text{in.}$$

$T > T_Y$ (Plastic region with elastic core) $\qquad \tau_{\max} = \tau_Y = 20.0 \text{ ksi} \blacktriangleleft$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(1656.70)(48)}{\pi(0.375)^4(11.2 \times 10^6)} = 0.22857 \text{ rad}$$

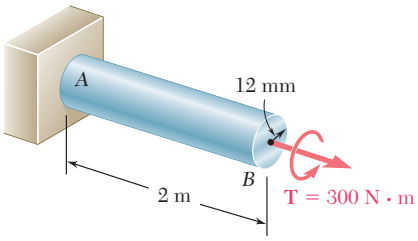
$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\varphi^3}{\varphi_Y^3}\right)$$

$$\left(\frac{\varphi_Y}{\varphi}\right)^3 = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(1800)}{1656.70} = 0.7451 \qquad \frac{\varphi_Y}{\varphi} = 0.90471$$

$$\varphi = \frac{\varphi_Y}{0.90471} = \frac{0.22857}{0.9047} = 0.25264 \text{ rad}$$

$$\varphi_Y = 14.48° \blacktriangleleft$$

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PROBLEM 3.104

The shaft *AB* is made of a material that is elastoplastic with $\tau_Y = 90$ MPa and $G = 30$ GPa. For the loading shown, determine (*a*) the radius of the elastic core of the shaft, (*b*) the angle of twist at end *B*.

SOLUTION

(*a*) $c = 12 \text{ mm} = 0.012 \text{ m}$

$\tau_Y = 90 \times 10^6 \text{ Pa}$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y$$

$$= \frac{\pi}{2}(0.012)^3(90 \times 10^6) = 244.29 \text{ N} \cdot \text{m}$$

$T = 300 \text{ N} \cdot \text{m} > T_Y$ Plastic region with elastic core

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) \qquad \frac{\rho_Y^3}{c^3} = 1 - \frac{3T}{T_Y} = 1 - \frac{(3)(300)}{244.29} = 0.31585$$

$$\frac{\rho_Y}{c} = 0.68102 \qquad \rho_Y = (0.68102)(0.012) = 8.17 \times 10^{-3} \text{ m}$$

$\rho_Y = 8.17 \text{ mm}$ ◀

(*b*) $L = 2 \text{ m}$

$G = 30 \times 10^9 \text{ Pa}$

$$\varphi_Y = \frac{T_Y L}{JG} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(244.29)(2)}{\pi(0.012)^4(30 \times 10^9)} = 0.5000 \text{ rad}$$

$$\frac{\varphi_Y}{\varphi} = \frac{\rho_Y}{c} \qquad \varphi = \frac{\varphi_Y}{\rho_Y/c} = \frac{0.5000}{0.68120} = 0.734 \text{ rad}$$

$\varphi = 42.1°$ ◀

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PROBLEM 3.105

A solid circular rod is made of a material that is assumed to be elastoplastic. Denoting by T_Y and φ_Y, respectively, the torque and the angle of twist at the onset of yield, determine the angle of twist if the torque is increased to (*a*) $T = 1.1\,T_Y$, (*b*) $T = 1.25\,T_Y$, (*c*) $T = 1.3\,T_Y$.

SOLUTION

$$T = \frac{4}{3}\,T_Y\left(1 - \frac{1}{4}\frac{{\varphi_Y}^3}{\varphi^3}\right)$$

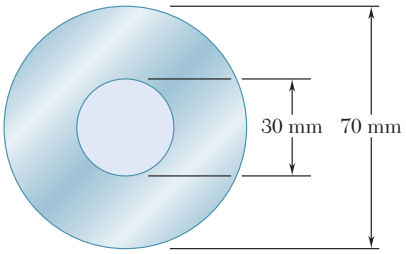
$$\frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - \frac{3T}{T_Y}} \quad \text{or} \quad \frac{\varphi}{\varphi_Y} = \frac{1}{\sqrt[3]{4 - \frac{3T}{T_Y}}}$$

(*a*) $\dfrac{T}{T_Y} = 1.10 \qquad \dfrac{\varphi}{\varphi_Y} = \dfrac{1}{\sqrt[3]{4 - (3)(1.10)}} = 1.126 \qquad \varphi = 1.126\varphi_Y$ ◀

(*b*) $\dfrac{T}{T_Y} = 1.25 \qquad \dfrac{\varphi}{\varphi_Y} = \dfrac{1}{\sqrt[3]{4 - (3)(1.25)}} = 1.587 \qquad \varphi = 1.587\varphi_Y$ ◀

(*c*) $\dfrac{T}{T_Y} = 1.3 \qquad \dfrac{\varphi}{\varphi_Y} = \dfrac{1}{\sqrt[3]{4 - (3)(1.3)}} = 2.15 \qquad \varphi = 2.15\varphi_Y$ ◀

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PROBLEM 3.106

A hollow shaft is 0.9 m long and has the cross section shown. The steel is assumed to be elastoplastic with $\tau_Y = 180$ MPa and $G = 77.2$ GPa. Determine (*a*) the angle of twist at which the section first becomes fully plastic, (*b*) the corresponding magnitude of the applied torque.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 0.015\text{ m} \qquad c_2 = \frac{1}{2}d_2 = 0.035\text{ m}$$

(*a*) For onset of fully plastic yielding, $\rho_Y = c_1$

$$\tau = \tau_Y \quad \therefore \quad \gamma = \frac{\tau_Y}{G} = \frac{\rho_Y\varphi}{L} = \frac{c_1\varphi}{L}$$

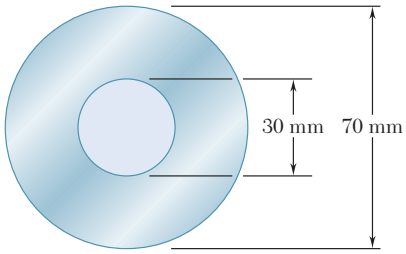
$$\varphi = \frac{L\tau_Y}{c_1 G} = \frac{(0.9)(180\times10^6)}{(0.015)(77.2\times10^9)} = 139.90\times10^{-3}\text{ rad} \qquad \varphi = 8.02^\circ \blacktriangleleft$$

(*b*) $$T_p = 2\pi\int_{c_1}^{c_2}\tau_Y\rho^2 d\rho = 2\pi\tau_Y\frac{\rho^3}{3}\Big|_{c_1}^{c_2} = \frac{2\pi}{3}\tau_Y\left(c_2^3 - c_1^3\right)$$

$$= \frac{2\pi}{3}(180\times10^6)(0.035^3 - 0.015^3) = 14.89\times10^3\text{ N}\cdot\text{m}$$

$$= 14.89\text{ kN}\cdot\text{m} \qquad T_p = 14.89\text{ kN}\cdot\text{m} \blacktriangleleft$$

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PROBLEM 3.107

A hollow shaft is 0.9 m long and has the cross section shown. The steel is assumed to be elastoplastic with $\tau_Y = 180$ MPa and $G = 77.2$ GPa. Determine the applied torque and the corresponding angle of twist (*a*) at the onset of yield, (*b*) when the plastic zone is 10 mm deep.

SOLUTION

(*a*) At the onset of yield, the stress distribution is the elastic distribution with $\tau_{\max} = \tau_Y$

$$c_2 = \frac{1}{2}d_2 = 0.035\text{ m}, \quad c_1 = \frac{1}{2}d_1 = 0.015\text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.035^4 - 0.015^4) = 2.2777 \times 10^{-6}\text{ m}$$

$$\tau_{\max} = \tau_Y = \frac{T_Y c_2}{J} \quad \therefore \quad T_Y = \frac{J\tau_Y}{c_2} = \frac{(2.2777 \times 10^{-6})(180 \times 10^6)}{0.035} = 11.7139 \times 10^3\text{ N} \cdot \text{m}$$

$$T_Y = 11.71\text{ kN} \cdot \text{m} \blacktriangleleft$$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{(11.7139 \times 10^3)(0.9)}{(77.2 \times 10^9)(2.2777 \times 10^{-6})} = 59.956 \times 10^{-3}\text{ rad} \qquad \varphi_Y = 3.44° \blacktriangleleft$$

(*b*) $t = 0.010$ m $\quad \rho_Y = c_2 - t = 0.035 - 0.010 = 0.025$ m

$$\gamma = \frac{\rho\varphi}{L} = \frac{\rho_Y\varphi}{L} = \gamma_Y = \frac{\tau_Y}{G}$$

$$\varphi = \frac{\tau_Y L}{G\rho_Y} = \frac{(180 \times 10^6)(0.9)}{(77.2 \times 10^9)(0.025)} = 83.938 \times 10^{-3}\text{ rad} \qquad \varphi = 4.81° \blacktriangleleft$$

Torque T_1 carried by elastic portion: $\quad c_1 \le \rho \le \rho_Y$

$\tau = \tau_Y$ at $\rho = \rho_Y$. $\quad \tau_Y = \dfrac{T_1\rho_Y}{J_1}$ $\quad$ where $\quad J_1 = \dfrac{\pi}{2}\left(\rho_Y^4 - c_1^4\right)$

$$J_1 = \frac{\pi}{2}(0.025^4 - 0.015^4) = 534.07 \times 10^{-9}\text{ m}^4$$

$$T_1 = \frac{J_1\tau_Y}{\rho_Y} = \frac{(534.07 \times 10^{-9})(180 \times 10^6)}{0.025} = 3.8453 \times 10^3\text{ N} \cdot \text{m}$$

Torque T_2 carried by plastic portion:

$$T_2 = 2\pi\int_{\rho_Y}^{c_2} \tau_Y \rho^2 d\rho = 2\pi\tau_Y \frac{\rho^3}{3}\bigg|_{\rho_Y}^{c_2} = \frac{2\pi}{3}\tau_Y\left(c_2^3 - \rho_Y^3\right)$$

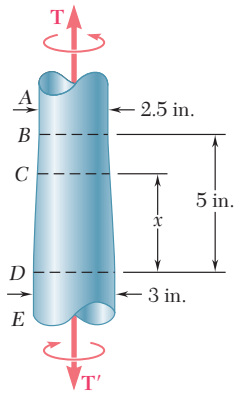
$$= \frac{2\pi}{3}(180 \times 10^6)(0.035^3 - 0.025^3) = 10.2730 \times 10^3\text{ N} \cdot \text{m}$$

Total torque:

$$T = T_1 + T_2 = 3.8453 \times 10^3 + 10.2730 \times 10^3$$

$$= 14.1183 \times 10^3\text{ N} \cdot \text{m} \qquad 14.12\text{ kN} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 3.108

A steel rod is machined to the shape shown to form a tapered solid shaft to which a torque of magnitude $T = 75$ kip · in. is applied. Assuming the steel to be elastoplastic with $\tau_Y = 21$ ksi and $G = 11.2 \times 10^6$ psi, determine (*a*) the radius of the elastic core in portion *AB* of the shaft, (*b*) the length of portion *CD* that remains fully elastic.

SOLUTION

(*a*) In portion *AB*. $c = \dfrac{1}{2}d = 1.25$ in.

$$T_Y = \frac{J_{AB}\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(1.25)^3(21 \times 10^3) = 64.427 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{\rho_Y^3}{c^3}\right)$$

$$\frac{\rho_Y}{c} = \sqrt[3]{4 - \frac{3T}{T_Y}} = \sqrt[3]{4 - \frac{(3)(75 \times 10^3)}{64.427 \times 10^3}} = 0.79775$$

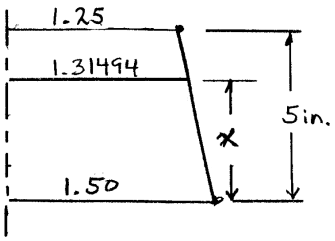
$$\rho_Y = 0.79775c = (0.79775)(1.25) = 0.99718 \text{ in.} \qquad \rho_Y = 0.997 \text{ in.} \blacktriangleleft$$

(*b*) For yielding at point *C*. $\tau = \tau_Y, \quad c = c_x, \quad T = 75 \times 10^3$ lb · in.

$$T = \frac{J_C\tau_Y}{c_x} = \frac{\pi}{2}c_x^3\tau_Y$$

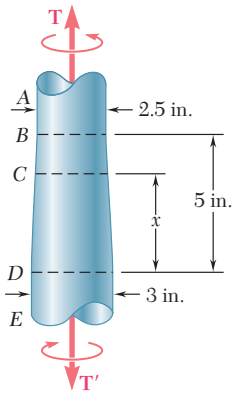
$$c_x = \sqrt[3]{\frac{2T}{\pi\tau_Y}} = \sqrt[3]{\frac{(2)(75 \times 10^3)}{\pi(21 \times 10^3)}} = 1.31494 \text{ in.}$$

Using proportions from the sketch,



$$\frac{1.50 - 1.31494}{1.50 - 1.25} = \frac{x}{5} \qquad x = 3.70 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.109

If the torque applied to the tapered shaft of Prob. 3.108 is slowly increased, determine (*a*) the magnitude T of the largest torques that can be applied to the shaft, (*b*) the length of the portion *CD* that remains fully elastic.

PROBLEM 3.108 A steel rod is machined to the shape shown to form a tapered solid shaft to which a torque of magnitude $T = 75$ kip · in. is applied. Assuming the steel to be elastoplastic with $\tau_Y = 21$ ksi and $G = 11.2 \times 10^6$ psi, determine (*a*) the radius of the elastic core in portion *AB* of the shaft, (*b*) the length of portion *CD* that remains fully elastic.

SOLUTION

(*a*) The largest torque that may be applied is that which makes portion *AB* fully plastic.

In portion *AB*, $c = \frac{1}{2}d = 1.25$ in.

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(1.25)^3(21\times10^3) = 64.427\times10^3 \text{ lb}\cdot\text{in.}$$

For fully plastic shaft, $\rho_Y = 0$ $\qquad T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}T$

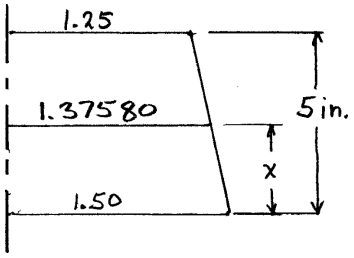
$$T = \frac{4}{3}(64.427\times10^3) = 85.903\times10^3 \text{ lb}\cdot\text{in.} \qquad T = 85.9 \text{ kip}\cdot\text{in.} \blacktriangleleft$$

(*b*) For yielding at point *C*, $\tau = \tau_Y$, $\quad c = c_x$, $\quad T = 85.903\times10^3$ lb · in.

$$\tau_Y = \frac{Tc_x}{J_x} = \frac{2T}{\pi c_x^3}$$

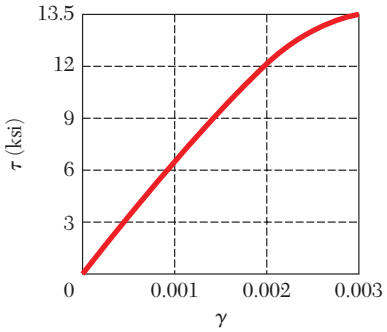
$$c_x = \sqrt[3]{\frac{2T}{\pi\tau_Y}} = \sqrt[3]{\frac{(2)(85.903\times10^3)}{\pi(21\times10^3)}} = 1.37580 \text{ in.}$$

Using proportions from the sketch,



$$\frac{1.50 - 1.37580}{1.50 - 1.25} = \frac{x}{5} \qquad x = 2.48 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.110

A solid brass rod of 1.2-in. diameter is subjected to a torque that causes a maximum shearing stress of 13.5 ksi in the rod. Using the τ-γ diagram shown for the brass rod used, determine (*a*) the magnitude of the torque, (*b*) the angle of twist in a 24-in. length of the rod.

SOLUTION

(*a*) $\tau_{\max} = 13.5$ ksi $\quad c = \frac{1}{2}d = 0.600$ in.

From the stress-strain diagram, $\gamma_{\max} = 0.003$

Let $z = \dfrac{\gamma}{\gamma_{\max}} = \dfrac{\rho}{c}$

$$T = 2\pi\int_0^c \rho^2\tau\, d\rho = 2\pi c^3\int_0^1 z^2\tau\, dz = 2\pi c^3 I \quad \text{where the integral } I \text{ is given by } I = \int_0^1 z^2\tau\, dz$$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3}\sum wz^2\tau$$

where w is a weighting factor. Using $\Delta z = 0.25$, we get the values given in the table below:

z	γ	τ, ksi	$z^2\tau$, ksi	w	$wz^2\tau$, ksi
0	0	0	0.000	1	0
0.25	0.00075	4.5	0.281	4	1.125
0.5	0.0015	8.6	2.15	2	4.30
0.75	0.00225	12.2	6.86	4	27.45
1.0	0.003	13.5	13.5	1	13.5
					46.375

$\leftarrow \sum wz^2\tau$

$$I = \frac{(0.25)(46.375)}{3} = 3.865\times10^{-3}\text{ ksi}$$

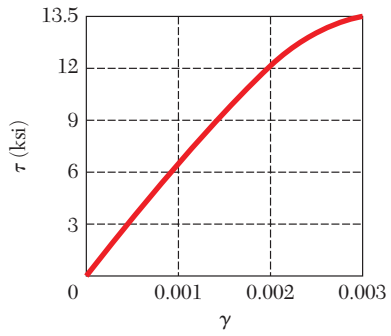
(*a*) $T = 2\pi c^3 I = 2\pi(0.600)^3(3.865) = 5.24$ kip · in. $\qquad T = 5.24$ kip · in. ◀

(*b*) $\gamma_{\max} = \dfrac{c\varphi}{L}$

$$\varphi = \frac{L\gamma_m}{c} = \frac{(24)(0.003)}{0.800} = 120\times10^{-3}\text{ rad} \qquad \varphi = 6.88° \blacktriangleleft$$

Note: Answer may differ slightly due to reading of graph and choice of numerical integration formula.

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PROBLEM 3.111

A solid brass rod of 0.8-in. diameter and 30-in. length is twisted through an angle of 10°. Using the τ-γ diagram shown for the brass rod used, determine (*a*) the magnitude of the torque applied to the rod, (*b*) the maximum shearing stress in the rod.

SOLUTION

(*a*) $\varphi = 10° = 174.53 \times 10^{-3}$ rad

$c = \frac{1}{2}d = 0.400$ in. $\quad L = 30$ in.

$$\gamma_{\max} = \frac{c\varphi}{L} = \frac{(0.4)(174.53 \times 10^{-3})}{30} = 0.00233$$

Let $z = \dfrac{\gamma}{\gamma_{\max}} = \dfrac{\rho}{c_2}$

$$T = 2\pi \int_0^c \rho^2 \tau d\rho = 2\pi c_2^3 \int_0^1 z^2 \tau dz = 2\pi c_2^3 I \quad \text{where the integral } I \text{ is given by } I = \int_{1/3}^1 z^2 \tau dz$$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = \frac{1}{6}$, we get the values given in the table below:

z	γ	τ, ksi	$z^2\tau$, ksi	w	$wz^2\tau$, ksi	
0	0	0	0	1	0	
0.25	0.000583	3.5	0.219	4	0.88	
0.5	0.001165	7.0	1.75	2	3.50	
0.75	0.001748	10.0	5.625	4	22.50	
1.0	0.00233	12.2	12.6	1	12.60	
					39.48	$\leftarrow \sum wz^2\tau$

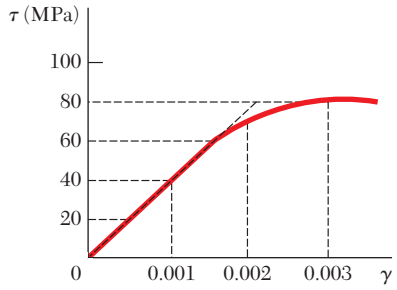
$$I = \frac{(0.25)(39.48)}{3} = 3.29 \text{ ksi}$$

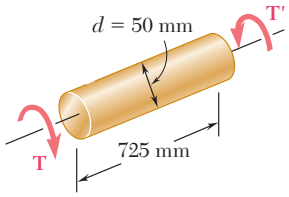
$T = 2\pi c_2^3 I = 2\pi(0.400)^3(3.29)$ $\qquad T = 1.322$ kip · in. ◀

Note: Answer may differ slightly due to differences of opinion in reading the stress-strain curve.

(*b*) From the graph, $\qquad \tau_{\max} = 12.60$ ksi ◀

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PROBLEM 3.112

A 50-mm-diameter cylinder is made of a brass for which the stress-strain diagram is as shown. Knowing that the angle of twist is 5° in a 725-mm length, determine by approximate means the magnitude T of torque applied to the shaft

SOLUTION

(*a*) $\varphi = 5° = 87.266 \times 10^{-3}$ rad

$$c = \frac{1}{2}d = 0.025 \text{ m} \qquad L = 0.725 \text{ m}$$

$$\gamma_{\max} = \frac{c\varphi}{L} = \frac{(0.025)(87.266 \times 10^{-3})}{0.725} = 0.00301$$

Let $$z = \frac{\gamma}{\gamma_{\max}} = \frac{\rho}{c}$$

$$T = 2\pi \int_0^c \rho^2 \tau d\rho = 2\pi c_2^3 \int_0^1 z^2 \tau dz = 2\pi c_2^3 I \qquad \text{where the integral } I \text{ is given by } I = \int_{1/3}^1 z^2 \tau dz$$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = 0.25$, we get the values given in the table below.

z	γ	τ, MPa	$z^2\tau$, MPa	w	$wz^2\tau$, MPa
0	0	0	0	1	0
0.25	0.00075	30	1.875	4	7.5
0.5	0.0015	55	13.75	2	27.5
0.75	0.00226	75	42.19	4	168.75
1.0	0.00301	80	80	1	80
					283.75

$\leftarrow \sum wz^2\tau = 283.75 \times 10^6$ Pa

$$I = \frac{(0.25)(283.75 \times 10^6)}{3} = 23.65 \times 10^6 \text{ Pa}$$

$$T = 2\pi c^3 I = 2\pi (0.025)^3 (23.65 \times 10^6) = 2.32 \times 10^3 \text{ N} \cdot \text{m} \qquad T = 2.32 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 3.113

Three points on the nonlinear stress-strain diagram used in Prob. 3.112 are (0, 0), (0.0015, 55 MPa), and (0.003, 80 MPa). By fitting the polynomial $T = A + B\gamma + C\gamma^2$ through these points, the following approximate relation has been obtained.

$$T = 46.7 \times 10^9 \gamma - 6.67 \times 10^{12} \gamma^2$$

Solve Prob. 3.113 using this relation, Eq. (3.2), and Eq. (3.26).

PROBLEM 3.112 A 50-mm diameter cylinder is made of a brass for which the stress-strain diagram is as shown. Knowing that the angle of twist is 5° in a 725-mm length, determine by approximate means the magnitude T of torque applied to the shaft.

SOLUTION

$$\varphi = 5° = 87.266 \times 10^{-3} \text{ rad}, \quad c = \frac{1}{2} d = 0.025 \text{ m}, \quad L = 0.725 \text{ m}$$

$$\gamma_{\max} = \frac{c\varphi}{L} = \frac{(0.025)(87.266 \times 10^{-3})}{0.725} = 3.009 \times 10^{-3}$$

Let $z = \dfrac{\gamma}{\gamma_{\max}} = \dfrac{\rho}{c}$

$$T = 2\pi \int_0^c \rho^2 \tau \, d\rho = 2\pi c^3 \int_0^1 z^2 \tau \, dz$$

The given stress-strain curve is

$$\tau = A + B\gamma + C\gamma^2 = A + B\gamma_{\max} z + C\gamma_{\max}^2 z^2$$

$$T = 2\pi c^3 \int_0^1 z^2 \left(A + B\gamma_{\max} z + C\gamma_{\max}^2 z^2 \right) dz$$

$$= 2\pi c^3 \left\{ A \int_0^1 z^2 \, dz + B\gamma_{\max} \int_0^1 z^3 dz + C\gamma_{\max}^2 \int_0^1 z^4 dz \right\}$$

$$= 2\pi c^2 \left\{ \frac{1}{3} A + \frac{1}{4} B\gamma_{\max} + \frac{1}{5} C\gamma_{\max}^2 \right\}$$

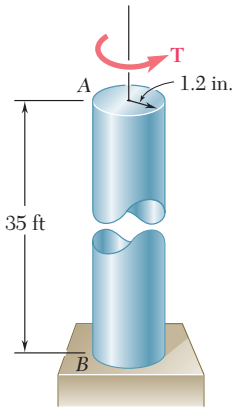
Data: $A = 0, \quad B = 46.7 \times 10^9, \quad C = -6.67 \times 10^{12}$

$$\frac{1}{3} A = 0, \quad \frac{1}{4} B\gamma_{\max} = \frac{1}{4}(46.7 \times 10^9)(3.009 \times 10^{-3}) = 35.13 \times 10^3$$

$$\frac{1}{5} C\gamma_{\max}^2 = -\frac{1}{5}(6.67 \times 10^{12})(3.009 \times 10^{-3})^2 = -12.08 \times 10^3$$

$$T = 2\pi(0.025)^3(0 + (35.13 \times 10^3 - 12.08 \times 10^3)) = 2.26 \times 10^3 \text{ N} \cdot \text{m} \qquad T = 2.26 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 3.114

The solid circular drill rod *AB* is made of a steel that is assumed to be elastoplastic with $\tau_Y = 22$ ksi and $G = 11.2 \times 10^6$ psi. Knowing that a torque $T = 75$ kip · in. is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.

SOLUTION

$$c = 1.2 \text{ in.} \qquad L = 35 \text{ ft} = 420 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.2)^4 = 3.2572 \text{ in}^4$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{(3.2572)(22)}{1.2} = 59.715 \text{ kip} \cdot \text{in.}$$

Loading: $T = 75$ kip · in.

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right)$$

$$\frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(75)}{59.715} = 0.23213$$

$$\frac{\rho_Y}{c} = 0.61458, \qquad \rho_Y = 0.61458c = 0.73749 \text{ in.}$$

Unloading: $\tau' = \dfrac{T\rho}{J}$ where $T = 75$ kip · in.

At $\rho = c$, $\tau' = \dfrac{(75)(1.2)}{3.2572} = 27.63$ ksi

At $\rho = \rho_Y$, $\tau' = \dfrac{(75)(0.73749)}{3.2572} = 16.98$ ksi

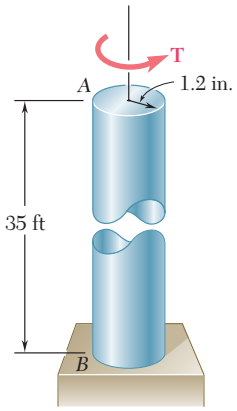
Residual: $\tau_{\text{res}} = \tau_{\text{load}} - \tau'$

At $\rho = c$, $\tau_{\text{res}} = 22 - 27.63 = -5.63$ ksi

At $\rho = \rho_Y$, $\tau_{\text{res}} = 22 - 16.98 = 5.02$ ksi

maximum $\tau_{\text{res}} = 5.63$ ksi ◀

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PROBLEM 3.115

In Prob. 3.114, determine the permanent angle of twist of the rod.

PROBLEM 3.114 The solid circular drill rod *AB* is made of steel that is assumed to be elastoplastic with $\tau_Y = 22$ ksi and $G = 11.2 \times 10^6$ psi. Knowing that a torque $T = 75$ kip · in. is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.

SOLUTION

From the solution to Prob. 3.114,

$$c = 1.2 \text{ in.}$$

$$J = 3.2572 \text{ in}^4$$

$$\frac{\rho_Y}{c} = 0.61458$$

$$\rho_Y = 0.73749 \text{ in.}$$

After loading, $\gamma = \dfrac{\rho\varphi}{L} \quad \therefore \quad \varphi = \dfrac{L\gamma}{\rho} = \dfrac{L\gamma_Y}{\rho_Y} = \dfrac{L\tau_Y}{\rho_Y G} \qquad L = 35 \text{ ft} = 420 \text{ in.}$

$$\varphi_{\text{load}} = \frac{(420)(22 \times 10^3)}{(0.73749)(11.2 \times 10^6)} = 1.11865 \text{ rad} = 64.09°$$

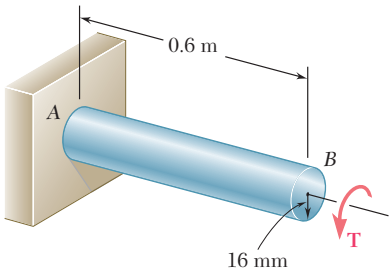
During unloading, $\varphi' = \dfrac{TL}{GJ}$ (elastic) $\qquad T = 75 \text{ kip} \cdot \text{in.}$

$$\varphi' = \frac{(75 \times 10^3)(420)}{(11.2 \times 10^6)(3.2572)} = 0.86347 \text{ rad} = 49.47°$$

Permanent angle of twist.

$$\varphi_{\text{perm}} = \varphi_{\text{load}} - \varphi' = 1.11865 - 0.86347 = 0.25518 \qquad \varphi = 14.62° \blacktriangleleft$$

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PROBLEM 3.116

The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The torque is increased in magnitude until the shaft has been twisted through 6°; the torque is then removed. Determine (*a*) the magnitude and location of the maximum residual shearing stress, (*b*) the permanent angle of twist.***

SOLUTION

$$c = 0.016 \text{ m} \qquad \varphi = 6° = 104.712 \times 10^{-3} \text{rad}$$

$$\gamma_{\max} = \frac{c\varphi}{L} = \frac{(0.016)(104.72 \times 10^{-3})}{0.6} = 0.0027925$$

$$\gamma_Y = \frac{\tau_Y}{G} = \frac{145 \times 10^6}{77.2 \times 10^9} = 0.0018782$$

$$\frac{\rho_Y}{c} = \frac{\gamma_Y}{\gamma_{\max}} = \frac{0.0018}{0.0027925} = 0.67260$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.016)^4 = 102.944 \times 10^{-9} \text{m}^4$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2}(0.016)^3(145 \times 10^6) = 932.93 \text{ N} \cdot \text{m}$$

At end of loading. $T_{\text{load}} = \frac{4}{3} T_Y \left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(932.93)\left[1 - \frac{1}{4}(0.67433)^3\right] = 1.14855 \times 10^3 \text{ N} \cdot \text{m}$

Unloading: elastic $T' = 1.14855 \times 10^3 \text{ N} \cdot \text{m}$

At $\rho = c$, $\tau' = \frac{T'c}{J} = \frac{(1.14855 \times 10^3)(0.016)}{102.944 \times 10^{-9}} = 178.513 \times 10^6 \text{ Pa}$

At $\rho = \rho_Y$, $\tau' = \frac{T'c}{J}\frac{\rho_Y}{c} = (178.513 \times 10^6)(0.67433) = 120.376 \times 10^6 \text{ Pa}$

$$\varphi' = \frac{T'L}{GJ} = \frac{(1.14855 \times 10^3)(0.6)}{(77.2 \times 10^9)(102.944 \times 10^{-9})} = 86.713 \times 10^{-3} \text{ rad} = 4.97°$$

Residual: $\tau_{\text{res}} = \tau_{\text{load}} - \tau' \qquad \varphi_{\text{perm}} = \varphi_{\text{load}} - \varphi'$

(*a*) At $\rho = c$, $\tau_{\text{res}} = 145 \times 10^6 - 178.513 \times 10^6 = -33.513 \times 10^6 \text{ Pa}$ $\tau_{\text{res}} = -33.5 \text{ MPa}$

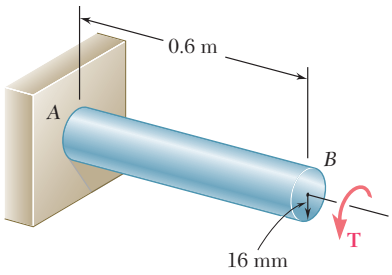
At $\rho = \rho_Y$, $\tau_{\text{res}} = 145 \times 10^6 - 120.376 \times 10^6 = 24.624 \times 10^6 \text{ Pa}$ $\tau_{\text{res}} = 24.6 \text{ MPa}$

Maximum residual stress: 33.5 MPa at $\rho = 16.00$ mm ◀

(*b*) $\varphi_{\text{perm}} = 104.712 \times 10^{-3} - 86.713 \times 10^{-3} = 17.9990 \times 10^{-3} \text{rad}$ $\varphi_{\text{perm}} = 1.03°$ ◀

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PROBLEM 3.117



After the solid shaft of Prob. 3.116 has been loaded and unloaded as described in that problem, a torque $\mathbf{T}_1$ of sense opposite to the original torque $\mathbf{T}$ is applied to the shaft. Assuming no change in the value of φ_Y, determine the angle of twist φ_1 for which yield is initiated in this second loading and compare it with the angle φ_Y for which the shaft started to yield in the original loading.

PROBLEM 3.116 The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The torque is increased in magnitude until the shaft has been twisted through 6°; the torque is then removed. Determine (*a*) the magnitude and location of the maximum residual shearing stress, (*b*) the permanent angle of twist.

SOLUTION

From the solution to Prob. 3.116,

$$c = 0.016 \text{ m}, \quad L = 0.6 \text{ m}$$

$$\tau_Y = 145 \times 10^6 \text{ Pa},$$

$$J = 102.944 \times 10^{-9} \text{ m}^4$$

The residual stress at $\rho = c$ is $\tau_{\text{res}} = 33.5$ MPa

For loading in the opposite sense, the change in stress to produce reversed yielding is

$$\tau_1 = \tau_Y - \tau_{\text{res}} = 145 \times 10^6 - 33.5 \times 10^6 = 111.5 \times 10^6 \text{ Pa}$$

$$\tau_1 = \frac{T_1 c}{J} \quad \therefore \quad T_1 = \frac{J\tau_1}{c} = \frac{(102.944 \times 10^{-9})(111.5 \times 10^6)}{0.016}$$

$$= 717 \text{ N} \cdot \text{m}$$

Angle of twist at yielding under reversed torque.

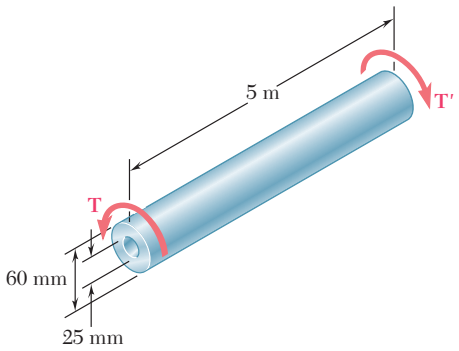
$$\varphi_1 = \frac{T_1 L}{GJ} = \frac{(717 \times 10^3)(0.6)}{(77.2 \times 10^9)(102.944 \times 10^{-9})} = 54.16 \times 10^{-3} \text{ rad} \qquad \varphi_1 = 3.10° \blacktriangleleft$$

Angle of twist for yielding in original loading.

$$\gamma = \frac{\tau_Y}{G} = \frac{c\varphi_Y}{L}$$

$$\varphi_Y = \frac{L\tau_Y}{cG} = \frac{(0.6)(145 \times 10^6)}{(0.016)(77.2 \times 10^9)} = 70.434 \times 10^{-3} \text{ rad} \qquad \varphi_Y = 4.04° \blacktriangleleft$$

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PROBLEM 3.118

The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The magnitude T of the torques is slowly increased until the plastic zone first reaches the inner surface of the shaft; the torques are then removed. Determine the magnitude and location of the maximum residual shearing stress in the rod.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 12.5 \text{ mm}$$

$$c_2 = \frac{1}{2}d_2 = 30 \text{ mm}$$

When the plastic zone reaches the inner surface, the stress is equal to τ_Y. The corresponding torque is calculated by integration.

$$dT = \rho\tau\, dA = \rho\tau_Y(2\pi\rho\, d\rho) = 2\pi\tau_Y\rho^2 d\rho$$

$$T = 2\pi\tau_Y\int_{c_1}^{c_2}\rho^2\, d\rho = \frac{2\pi}{3}\tau_Y\left(c_2^3 - c_1^3\right)$$

$$= \frac{2\pi}{3}(145\times10^6)[(30\times10^{-3})^3 - (12.5\times10^{-3})^3] = 7.6064\times10^3 \text{ N}\cdot\text{m}$$

Unloading. $T' = 7.6064\times10^3 \text{ N}\cdot\text{m}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4] = 1.234\times10^6 \text{ mm}^4 = 1.234\times10^{-6} \text{ m}^4$$

$$\tau_1' = \frac{T'c_1}{J} = \frac{(7.6064\times10^3)(12.5\times10^{-3})}{1.234\times10^{-6}} = 77.050\times10^6 \text{ Pa} = 77.05 \text{ MPa}$$

$$\tau_2' = \frac{T'c_2}{J} = \frac{(7.6064\times10^3)(30\times10^{-3})}{1.234\times10^{-6}} = 192.63\times10^6 \text{ Pa} = 192.63 \text{ MPa}$$

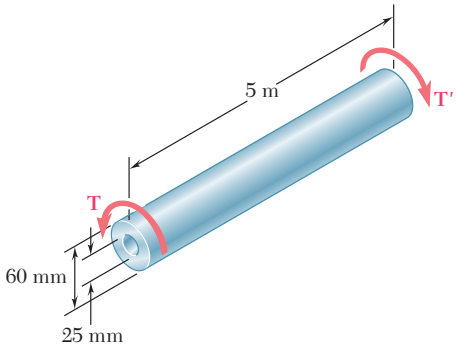
Residual stress.

Inner surface: $\tau_{\text{res}} = \tau_Y - \tau_1' = 145 - 77.05 = 67.95$ MPa

Outer surface: $\tau_{\text{res}} = \tau_Y - \tau_2' = 145 - 192.63 = -47.63$ MPa

Maximum residual stress: 68.0 MPa at inner surface ◀

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PROBLEM 3.119

In Prob. 3.118, determine the permanent angle of twist of the rod.

PROBLEM 3.118 The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The magnitude T of the torques is slowly increased until the plastic zone first reaches the inner surface of the shaft; the torques are then removed. Determine the magnitude and location of the maximum residual shearing stress in the rod.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 12.5 \text{ mm}$$

$$c_2 = \frac{1}{2}d_2 = 30 \text{ mm}$$

When the plastic zone reaches the inner surface, the stress is equal to τ_Y. The corresponding torque is calculated by integration.

$$dT = \rho\tau\, dA = \rho\tau_Y(2\pi\rho\, d\rho) = 2\pi\tau_Y\, \rho^2 d\rho$$

$$T = 2\pi\tau_Y \int_{c_1}^{c_2} \rho^2\, d\rho = \frac{2\pi}{3}\tau_Y\left(c_2^3 - c_1^3\right)$$

$$= \frac{2\pi}{3}(145\times10^6)[(30\times10^{-3})^3 - (12.5\times10^{-3})^3] = 7.6064\times10^3 \text{ N}\cdot\text{m}$$

Rotation angle at maximum torque.

$$\frac{c_1\varphi_{\max}}{L} = \gamma_Y = \frac{\tau_Y}{G}$$

$$\varphi_{\max} = \frac{\tau_Y L}{Gc_1} = \frac{(145\times10^6)(5)}{(77.2\times10^9)(12.5\times10^{-3})} = 0.75130 \text{ rad}$$

Unloading. $T' = 7.6064\times10^3 \text{ N}\cdot\text{m}$

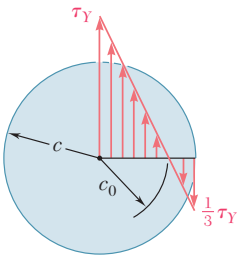
$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4] = 1.234\times10^6 \text{ mm}^4 = 1.234\times10^{-6} \text{ m}^4$$

$$\varphi' = \frac{T'L}{GJ} = \frac{(7.6064\times10^3)(5)}{(77.2\times10^9)(1.234\times10^{-6})} = 0.39922 \text{ rad}$$

Permanent angle of twist.

$$\varphi_{\text{perm}} = \varphi_{\max} - \varphi' = 0.75130 - 0.39922 = 0.35208 \text{ rad} \qquad \varphi_{\text{perm}} = 20.2° \blacktriangleleft$$

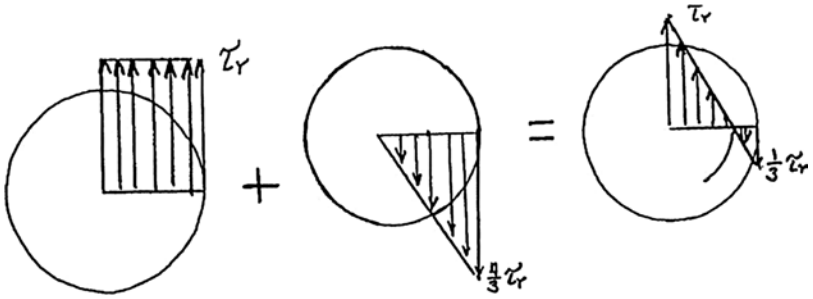
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PROBLEM 3.120

A torque **T** applied to a solid rod made of an elastoplastic material is increased until the rod is fully plastic and them removed. (*a*) Show that the distribution of residual shearing stresses is as represented in the figure. (*b*) Determine the magnitude of the torque due to the stresses acting on the portion of the rod located within a circle of radius c_0.

SOLUTION



(*a*)

After loading: $\rho_Y = 0, \quad T_{\text{load}} = \frac{4}{3}T_Y = \frac{4}{3}\frac{\pi}{2}c^3\tau_Y = \frac{2\pi}{3}c^3\tau_Y$

Unloading: $\tau' = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(T_{\text{load}})}{\pi c^3} = \frac{4}{3}\tau_Y \quad \text{at } \rho = c$

$$\tau' = \frac{4}{3}\tau_Y\frac{\rho}{c}$$

Residual: $\tau_{\text{res}} = \tau_Y - \frac{4}{3}\tau_Y\frac{\rho}{c} = \tau_Y\left(1 - \frac{4\rho}{3c}\right)$

To find c_0 set, $\tau_{\text{res}} = 0 \quad \text{and} \quad \rho = c_0$

$$0 = 1 - \frac{4c_0}{3c} \quad \therefore \quad c_0 = \frac{3}{4}c \qquad c_0 = 0.150c \blacktriangleleft$$

(*b*) $$T_0 = 2\pi\int_0^{c_0}\rho^2\tau\,d\rho = 2\pi\int_0^{(3/4)c}\rho^2\tau_Y\left(1 - \frac{4}{3}\frac{\rho}{c}\right)d\rho$$

$$= 2\pi\tau_Y\left(\frac{\rho^3}{3} - \frac{4}{3}\frac{\rho^4}{4c}\right)\Bigg|_0^{(3/4)c} = 2\pi\tau_Y c^3\left\{\frac{1}{3}\left(\frac{3}{4}\right)^3 - \left(\frac{4}{3}\right)\frac{1}{4}\left(\frac{3}{4}\right)^4\right\}$$

$$= 2\pi\tau_Y c^3\left\{\frac{9}{64} - \frac{27}{256}\right\} = \frac{9\pi}{128}\tau_Y c^3 = 0.2209\,\tau_Y c^3 \qquad T_0 = 0.221\tau_Y c^3 \blacktriangleleft$$

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PROBLEM 3.121

Determine the largest allowable square cross section of a steel shaft of length 20 ft if the maximum shearing stress is not to exceed 10 ksi when the shaft is twisted through one complete revolution. Use $G = 11.2 \times 10^6$ psi.

SOLUTION

$$L = 20 \text{ ft} = 240 \text{ in.}$$

$$\tau_{\max} = 10 \text{ ksi} = 10 \times 10^3 \text{ psi}$$

$$\varphi = 1 \text{ rev} = 2\pi \text{ radians}$$

$$\tau_{\max} = \frac{T}{c_1 a b^2} \tag{1}$$

$$\varphi = \frac{TL}{c_2 a b^3 G} \tag{2}$$

Divide (2) by (1) to eliminate T.

$$\frac{\varphi}{\tau_{\max}} = \frac{c_1 a b^2 L}{c_2 a b^3 G} = \frac{c_1 L}{c_2 b G}$$

Solve for b.

$$b = \frac{c_1 L \tau_{\max}}{c_2 G \varphi}$$

For a square section,

$$\frac{a}{b} = 1.0$$

From Table 3.1,

$$c_1 = 0.208,$$

$$c_2 = 0.1406$$

$$b = \frac{(0.208)(240)(10 \times 10^3)}{(0.1406)(11.2 \times 10^6)(2\pi)}$$

$$b = 0.0505 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.122

Determine the largest allowable length of a stainless steel shaft of $\frac{3}{8} \times \frac{3}{4}$-in. cross section if the shearing stress is not to exceed 15 ksi when the shaft is twisted through 15°. Use $G = 11.2 \times 10^6$ psi.

SOLUTION

$$a = \frac{3}{4} \text{ in.} = 0.75 \text{ in.}$$

$$b = \frac{3}{8} \text{ in.} = 0.375 \text{ in.}$$

$$\tau_{\max} = 15 \text{ ksi} = 15 \times 10^3 \text{ psi}$$

$$\varphi = 15° = \frac{15\pi}{180} \text{ rad} = 0.26180 \text{ rad}$$

$$\tau_{\max} = \frac{T}{c_1 a b^2} \tag{1}$$

$$\varphi = \frac{TL}{c_2 a b^3 G} \tag{2}$$

Divide (2) by (1) to eliminate T. $\dfrac{\varphi}{\tau_{\max}} = \dfrac{c_1 a b^2 L}{c_2 a b^3 G} = \dfrac{c_1 L}{c_2 b G}$

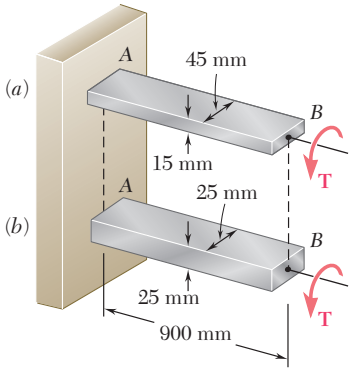
Solve for L. $L = \dfrac{c_2 b G \varphi}{c_1 \tau_{\max}}$

$$\frac{a}{b} = \frac{0.75}{0.375} = 2$$

Table 3.1 gives $c_1 = 0.246, \quad c_2 = 0.229$

$$L = \frac{(0.229)(0.375)(11.2 \times 10^6)(0.26180)}{(0.246)(15 \times 10^3)} = 68.2 \text{ in.} \qquad L = 68.2 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.123

Using $\tau_{all} = 70$ MPa and $G = 27$ GPa, determine for each of the aluminum bars shown the largest torque **T** that can be applied and the corresponding angle of twist at end *B*.

SOLUTION

$$\tau_{all} = 70 \times 10^6 \text{ Pa} \quad G = 27 \times 10^9 \text{ Pa} \quad L = 0.900 \text{ m}$$

(*a*) $a = 45$ mm $b = 15$ mm $\dfrac{a}{b} = 3.0$ From Table 3.1, $c_1 = 0.267$, $c_2 = 0.263$

$$\tau_{max} = \frac{T}{c_1 a b^2} \qquad T = c_1 a b^2 \tau_{max}$$

$$T = (0.267)(0.045)(0.015)^2(70 \times 10^6) = 189.236 \text{ N} \cdot \text{m} \qquad T = 189.2 \text{ N} \cdot \text{m} \blacktriangleleft$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(189.236)(0.900)}{(0.263)(0.045)(0.015)^3(27 \times 10^9)} = 157.921 \times 10^{-3} \text{ rad} \qquad \varphi = 9.05° \blacktriangleleft$$

(*b*) $a = 25$ mm $b = 25$ mm, $\dfrac{a}{b} = 1.0$ From Table 3.1, $c_1 = 0.208$, $c_2 = 0.1406$

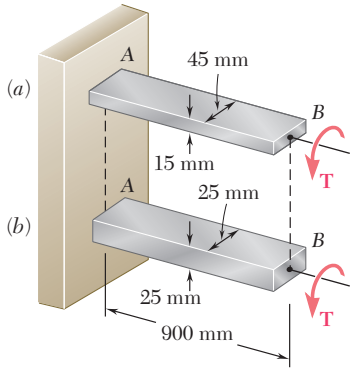
$$\tau_{max} = \frac{T}{a b^2} \qquad T = c_1 a b^2 \tau_{max}$$

$$= (0.208)(0.025)(0.025)^2(70 \times 10^6)$$

$$= 227.5 \text{ N} \cdot \text{m} \qquad T = 228 \text{ N} \cdot \text{m} \blacktriangleleft$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(227.5)(0.900)}{(0.1406)(0.025)(0.025)^3(27 \times 10^9)} = 138.075 \times 10^{-3} \text{ rad} \qquad \varphi = 7.91° \blacktriangleleft$$

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PROBLEM 3.124

Knowing that the magnitude of the torque **T** is 200 N · m and that $G = 27$ GPa, determine for each of the aluminum bars shown the maximum shearing stress and the angle of twist at end *B*.

SOLUTION

$$T = 200 \text{ N} \cdot \text{m} \quad L = 0.900 \text{ m} \quad G = 27 \times 10^9 \text{ Pa}$$

(*a*) $a = 45$ mm, $b = 15$ mm, $\dfrac{a}{b} = 3.0$ From Table 3.1, $c_1 = 0.267$ $c_2 = 0.263$

$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{200}{(0.267)(0.045)(0.015)^2}$$

$$= 74.0 \times 10^6 \text{ Pa} \qquad \tau_{\max} = 74.0 \text{ MPa} \blacktriangleleft$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(200)(0.900)}{(0.263)(0.045)(0.015)^3(27 \times 10^9)}$$

$$= 166.904 \times 10^{-3} \text{ rad} \qquad \varphi = 9.56° \blacktriangleleft$$

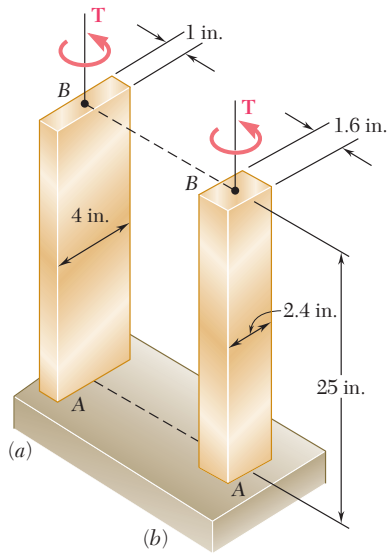
(*b*) $a = 25$ mm, $b = 25$ mm, $\dfrac{a}{b} = 1.0$ From Table 3.1, $c_1 = 0.208$, $c_2 = 0.1406$

$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{200}{(0.208)(0.025)(0.025)^2} = 61.539 \times 10^6 \text{ Pa} \qquad \tau_{\max} = 61.5 \text{ MPa} \blacktriangleleft$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(200)(0.900)}{(0.1406)(0.025)(0.025)^3(27 \times 10^9)}$$

$$= 121.385 \times 10^{-3} \text{ rad} \qquad \varphi = 6.95° \blacktriangleleft$$

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PROBLEM 3.125

Determine the largest torque **T** that can be applied to each of the two brass bars shown and the corresponding angle of twist at *B*, knowing that $\tau_{all} = 12$ ksi and $G = 5.6 \times 10^6$ psi.

SOLUTION

$$L = 25\text{ in.}, \quad G = 5.6 \times 10^6\text{ psi}, \quad \tau_{all} = 12 \times 10^3\text{ psi}$$

$$\tau_{max} = \frac{T}{c_1 ab^2} \quad \text{or} \quad T = c_1 ab^2 \tau_{max} \tag{1}$$

$$\varphi = \frac{TL}{c_2 ab^3 G} \quad \text{or} \quad \varphi = \frac{c_1 L \tau_{max}}{c_2 bG} \tag{2}$$

(*a*) $a = 4\text{ in.}, \quad b = 1\text{ in.}, \quad \frac{a}{b} = 4.0$ From Table 3.1, $c_1 = 0.282, \quad c_2 = 0.281$

From (1), $T = (0.282)(4)(1)^2(12 \times 10^3) = 13.54 \times 10^3$ $T = 13.54\text{ kip} \cdot \text{in.}$ ◀

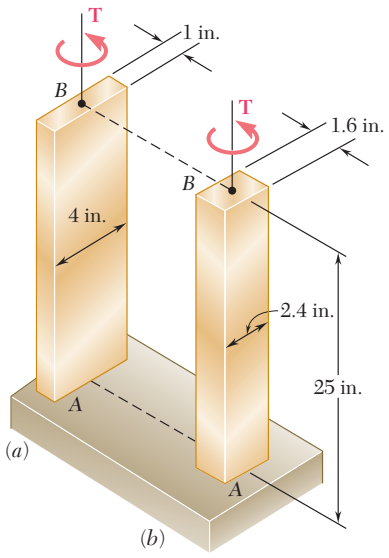
From (2), $\varphi = \dfrac{(0.282)(25)(12 \times 10^3)}{(0.281)(1)(5.6 \times 10^6)} = 0.05376\text{ radians}$ $\varphi = 3.08°$ ◀

(*b*) $a = 2.4\text{ in.}, \quad b = 1.6\text{ in.}, \quad \frac{a}{b} = 1.5$ From Table 3.1, $c_1 = 0.231, \quad c_2 = 0.1958$

From (1), $T = (0.231)(2.4)(1.6)^2(12 \times 10^3) = 17.03 \times 10^3$ $T = 17.03\text{ kip} \cdot \text{in.}$ ◀

From (2), $\varphi = \dfrac{(0.231)(25)(12 \times 10^3)}{(0.1958)(1.6)(5.6 \times 10^6)} = 0.0395\text{ radians}$ $\varphi = 2.26°$ ◀

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PROBLEM 3.126

Each of the two brass bars shown is subjected to a torque of magnitude $T = 12.5\,\text{kip} \cdot \text{in.}$ Knowing that $G = 5.6 \times 10^6\,\text{psi}$, determine for each bar the maximum shearing stress and the angle of twist at *B*.

SOLUTION

$$L = 25\,\text{in.}, \qquad G = 5.6 \times 10^6\,\text{psi}, \qquad T = 12.5 \times 10^3\,\text{lb} \cdot \text{in.}$$

(*a*) $a = 4\,\text{in.}, \qquad b = 1\,\text{in.}, \qquad \dfrac{a}{b} = 4.0$

From Table 3.1, $\quad c_1 = 0.282, \quad c_2 = 0.281$

$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{12.5 \times 10^3}{(0.282)(4)(1)^2} = 11.08 \times 10^3\,\text{psi}$$

$$\tau_{\max} = 11.08\ \text{ksi} \blacktriangleleft$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(12.5 \times 10^3)(25)}{(0.282)(4)(1)^3(5.6 \times 10^6)} = 0.04965\ \text{radians}$$

$$\varphi = 2.84° \blacktriangleleft$$

(*b*) $a = 2.4\,\text{in.}, \qquad b = 1.6\,\text{in.}, \qquad \dfrac{a}{b} = 1.5$

From Table 3.1, $\quad c_1 = 0.231, \quad c_2 = 0.1958$

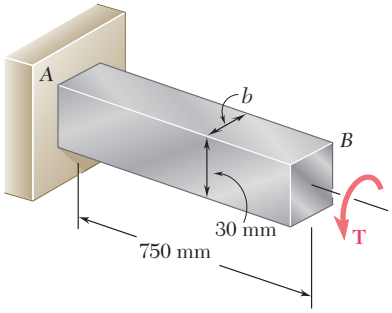
$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{12.5 \times 10^3}{(0.231)(2.4)(1.6)^2} = 8.81 \times 10^3\,\text{psi}$$

$$\tau_{\max} = 8.81\ \text{ksi} \blacktriangleleft$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(12.5 \times 10^6)(25)}{(0.1958)(2.4)(1.6)^3(5.6 \times 10^6)} = 0.02899\ \text{radians}$$

$$\varphi = 1.661° \blacktriangleleft$$

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PROBLEM 3.127

The torque **T** causes a rotation of $0.6°$ at end B of the aluminum bar shown. Knowing that $b = 15$ mm and $G = 26$ GPa, determine the maximum shearing stress in the bar.

SOLUTION

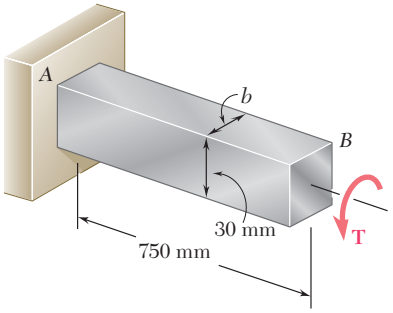
$$a = 30 \text{ mm} = 0.030 \text{ m}$$
$$b = 15 \text{ mm} = 0.015 \text{ m}$$
$$\varphi = 0.6° = 10.472 \times 10^{-3} \text{ rad}$$
$$\varphi = \frac{TL}{c_2 a b^3 G} \quad \therefore \quad T = \frac{c_2 a b^3 G \varphi}{c_1 L}$$
$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{c_2 a b^3 G \varphi}{c_1 a b^2 L} = \frac{c_2 b G \varphi}{c_1 L}$$
$$\frac{a}{b} = \frac{30}{15} = 2.0$$

From Table 3.1,

$$c_1 = 0.246$$
$$c_2 = 0.229$$
$$\tau_{\max} = \frac{(0.229)(15 \times 10^{-3})(26 \times 10^{9})(10.472 \times 10^{-3})}{(0.246)(750 \times 10^{-3})}$$
$$= 5.07 \times 10^{6} \text{ Pa}$$

$\tau_{\max} = 5.07$ MPa ◀

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PROBLEM 3.128

The torque **T** causes a rotation of $2°$ at end B of the stainless steel bar shown. Knowing that $b = 20$ mm and $G = 75$ GPa, determine the maximum shearing stress in the bar.

SOLUTION

$$a = 30 \text{ mm} = 0.030 \text{ m}$$

$$b = 20 \text{ mm} = 0.020 \text{ m}$$

$$\varphi = 2° = 34.907 \times 10^{-3} \text{ rad}$$

$$\varphi = \frac{TL}{c_2 a b^3 G} \quad \therefore \quad T = \frac{c_2 a b^3 G \varphi}{L}$$

$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{c_2 a b^3 G \varphi}{c_1 a b^2 L} = \frac{c_2 b G \varphi}{c_1 L}$$

$$\frac{a}{b} = \frac{30}{20} = 1.5$$

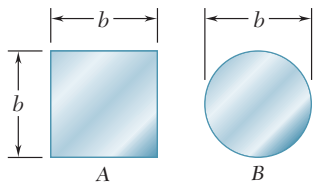
From Table 3.1,

$$c_1 = 0.231$$

$$c_2 = 0.1958$$

$$\tau_{\max} = \frac{(0.1958)(20 \times 10^{-3})(75 \times 10^{9})(34.907 \times 10^{-3})}{(0.231)(750 \times 10^{-3})} = 59.2 \times 10^{6} \text{ Pa} \qquad \tau_{\max} = 59.2 \text{ MPa} \blacktriangleleft$$

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PROBLEM 3.129

Two shafts are made of the same material. The cross section of shaft *A* is a square of side *b* and that of shaft *B* is a circle of diameter *b*. Knowing that the shafts are subjected to the same torque, determine the ratio τ_A/τ_B of maximum shearing stresses occurring in the shafts.

SOLUTION

A. Square:

$$\frac{a}{b} = 1, \quad c_1 = 0.208 \quad \text{(Table 3.1)}$$

$$\tau_A = \frac{T}{c_1 a b^2} = \frac{T}{0.208 b^3}$$

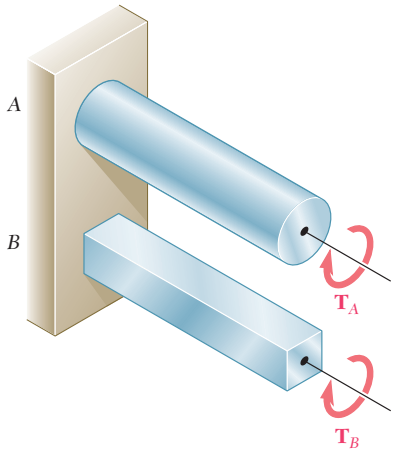
B. Circle:

$$c = \frac{1}{2}b \qquad \tau_B = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{16T}{\pi b^3}$$

Ratio:

$$\frac{\tau_A}{\tau_B} = \frac{1}{0.208} \cdot \frac{\pi}{16} = 0.3005\pi \qquad \frac{\tau_A}{\tau_B} = 0.944 \blacktriangleleft$$

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PROBLEM 3.130

Shafts A and B are made of the same material and have the same cross-sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum torques T_A and T_B when the two shafts are subjected to the same maximum shearing stress ($\tau_A = \tau_B$). Assume both deformations to be elastic.

SOLUTION

Let c = radius of circular section A and b = side of square section B.

For equal areas $\pi c^2 = b^2$,

$$c = \frac{b}{\sqrt{\pi}}$$

Circle: $$\tau_A = \frac{T_A c}{J} = \frac{2T_A}{\pi c^3} \quad \therefore \quad T_A = \frac{\pi}{2} c^3 \tau_A$$

Square:

From Table 3.1, $c_1 = 0.208$

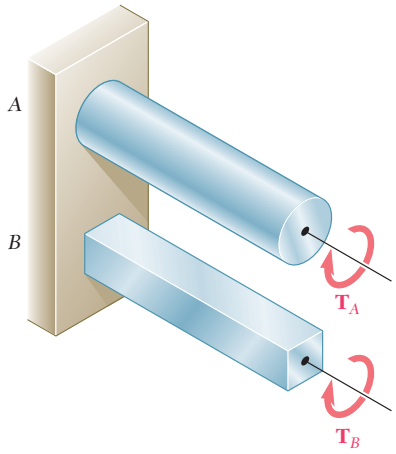
$$\tau_B = \frac{T_A}{c_1 a b^2} = \frac{T_B}{c_1 b^3} \quad \therefore \quad T_B = c_1 b^3 \tau_B$$

Ratio: $$\frac{T_A}{T_B} = \frac{\frac{\pi}{2} c^3 \tau_B}{c_1 b^3 \tau_B} = \frac{\frac{\pi}{2} \cdot \frac{b^3}{\pi^{3/2}} \tau_B}{c_1 b^3 \tau_B} = \frac{1}{2c_1\sqrt{\pi}} \frac{\tau_A}{\tau_B}$$

For the same stresses, $\tau_B = \tau_A \quad \therefore \quad \frac{T_A}{T_B} = \frac{1}{(2)(0.208)\sqrt{\pi}}$

$$\frac{T_A}{T_B} = 1.356 \blacktriangleleft$$

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PROBLEM 3.131

Shafts A and B are made of the same material and have the same length and cross-sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum values of the angles φ_A and φ_B when the two shafts are subjected to the same maximum shearing stress ($\tau_A = \tau_B$). Assume both deformations to be elastic.

SOLUTION

Let c = radius of circular section A and b = side of square section B.

For equal areas, $\pi c^2 = b^2 \quad \therefore \quad b = \sqrt{\pi}c$

Circle: $\gamma_{\max} = \dfrac{\tau_A}{G} = \dfrac{c\varphi_A}{L} \quad \therefore \quad \varphi_A = \dfrac{L\tau_A}{cG}$

Square: From Table 3.1, $c_1 = 0.208, \quad c_2 = 0.1406$

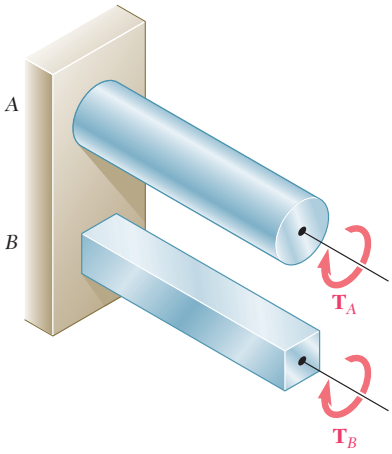
$$\tau_B = \frac{T_B}{c_1 a b^2} = \frac{T_B}{0.208 b^3} \quad \therefore \quad T_B = 0.208 b^3 \tau_B$$

$$\varphi_B = \frac{T_B L}{c_2 a b^3 G} = \frac{0.208 b^3 \tau_B L}{0.1406 b^4 G} = \frac{1.4794 L \tau_B}{bG}$$

Ratio: $$\frac{\varphi_A}{\varphi_B} = \frac{L\tau_A}{cG} \cdot \frac{bG}{1.4794 L \tau_B} = 0.676 \frac{b\tau_A}{c\tau_B} = 0.676\sqrt{\pi}\,\frac{\tau_A}{\tau_B}$$

For equal stresses, $\tau_A = \tau_B$ $\quad \dfrac{\varphi_B}{\varphi_A} = 0.676\sqrt{\pi}$ $\quad \dfrac{\varphi_B}{\varphi_A} = 1.198$ ◀

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PROBLEM 3.132

Shafts A and B are made of the same material and have the same cross-sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the angles ϕ_A and ϕ_B through which shafts A and B are respectively twisted when the two shafts are subjected to the same torque ($T_A = T_B$). Assume both deformations to be elastic.

SOLUTION

Let c = radius of circle section A and b = side of square section B.

For equal areas, $\pi c^2 = b^2 \quad \therefore \quad b = c\sqrt{\pi}$

Circle: $$\phi_A = \frac{TL}{JG} = \frac{T_A L}{\frac{\pi}{2} c^4 G}$$

Square: $$\phi_B = \frac{TL}{c_2 a b^3 G} = \frac{T_B L}{c_2 b^4 G}$$

From Table 3.1, $c_2 = 0.1406$

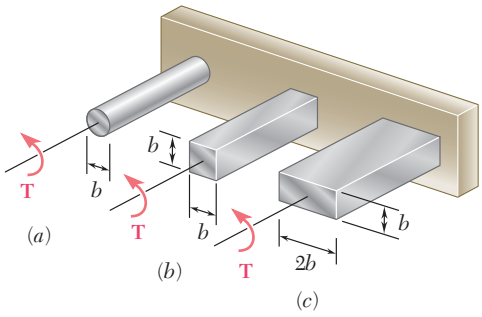
Ratio: $$\frac{\phi_A}{\phi_B} = \frac{T_A L}{\frac{\pi}{2} c^4 G} \cdot \frac{(0.1406) b^4 G}{T_B L}$$

For $T_A = T_B$,

$$\frac{\phi_A}{\phi_B} = \frac{(0.1406) c^4 \pi^2}{\frac{\pi}{2} c^4} = 0.883$$

◀

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PROBLEM 3.133

A torque of magnitude $T = 2$ kip · in. is applied to each of the steel bars shown. Knowing that $\tau_{all} = 6$ ksi, determine the required dimension b for each bar.

SOLUTION

$T = 2$ kip · in. $\quad \tau_{max} = 6$ ksi

(*a*) Circle: $c = \frac{1}{2}b$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{16T}{\pi b^3}$$

$$b^3 = \frac{16T}{\pi \tau_{max}} = \frac{(16)(2)}{\pi(6)} = 1.698 \text{ in}^3 \qquad b = 1.193 \text{ in.} \blacktriangleleft$$

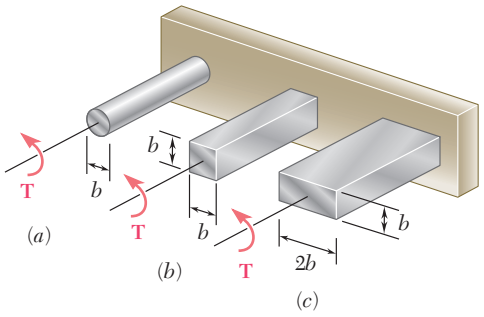
(*b*) Square: $a = b, \quad \frac{a}{b} = 1.0$ From Table 3.1, $c_1 = 0.208$

$$\tau_{max} = \frac{T}{c_1 ab^2} = \frac{T}{c_1 b^3} \quad \therefore \quad b^3 = \frac{T}{c_1 \tau_{max}} = \frac{2}{(0.208)(6)} = 1.603 \text{ in}^3 \qquad b = 1.170 \text{ in.} \blacktriangleleft$$

(*c*) Rectangle: $a = 2b \quad \frac{a}{b} = 2.0, \quad c_1 = 0.246$

$$\tau_{max} = \frac{T}{c_1 ab^2} = \frac{T}{2c_1 b^3} \quad \therefore \quad b^3 = \frac{T}{2c_1 \tau_{max}} = \frac{2}{(2)(0.246)(6)} = 0.668 \text{ in}^3 \qquad b = 0.878 \text{ in.} \blacktriangleleft$$

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PROBLEM 3.134

A torque of magnitude $T = 300\ \text{N} \cdot \text{m}$ is applied to each of the aluminum bars shown. Knowing that $\tau_{all} = 60$ MPa, determine the required dimension b for each bar.

SOLUTION

$$T = 300\ \text{N} \cdot \text{m} \quad \tau_{\max} = 60 \times 10^6\ \text{Pa}$$

(*a*) Circle: $c = \dfrac{1}{2}b$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{16T}{\pi b^3}$$

$$b^3 = \frac{16T}{\pi \tau_{\max}} = \frac{(16)(300)}{\pi(60 \times 10^6)} = 25.46 \times 10^{-6}\ \text{m}^3$$

$$b = 29.4 \times 10^{-3}\ \text{m} = 29.4\ \text{mm} \blacktriangleleft$$

(*b*) Square: $a = b, \quad \dfrac{a}{b} = 1.0$ From Table 3.1, $c_1 = 0.208$

$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{T}{c_1 b^3} \quad \therefore \quad b^3 = \frac{T}{c_1 \tau_{\max}} = \frac{300}{(0.208)(60 \times 10^6)}$$

$$= 24.04 \times 10^{-6}\ \text{m}^3$$

$$b = 28.9 \times 10^{-3}\ \text{m} = 28.9\ \text{mm} \blacktriangleleft$$

(*c*) Rectangle: $a = 2b, \quad \dfrac{a}{b} = 2.0, \quad c_1 = 0.246$

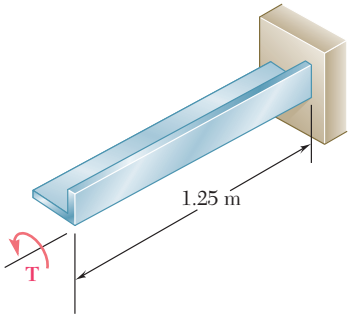
$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{T}{2c_1 b^3} \quad \therefore \quad b^3 = \frac{T}{2c_1 \tau_{\max}}$$

$$= \frac{300}{(2)(0.246)(60 \times 10^6)}$$

$$= 10.16 \times 10^{-3}\ \text{m}^3$$

$$b = 21.7 \times 10^{-3}\ \text{m} = 21.7\ \text{mm} \blacktriangleleft$$

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PROBLEM 3.135

A 1.25-m-long steel angle has an L127 × 76 × 6.4 cross section. From Appendix C we find that the thickness of the section is 6.4 mm and that its area is 1250 mm^2. Knowing that $\tau_{all} = 60$ MPa and that $G = 77.2$ GPa, and ignoring the effect of stress concentrations, determine (*a*) the largest torque **T** that can be applied, (*b*) the corresponding angle of twist.

SOLUTION

$$A = 1250\ \text{mm}^2 \quad b = 6.4\ \text{mm} = 0.0064\ \text{m}$$

$$a = \frac{A}{b} = \frac{1250}{6.4} = 195.313\ \text{mm} = 0.195313\ \text{m}$$

$$\frac{a}{b} = \frac{195.313}{6.4} = 30.518, \quad \frac{b}{a} = 0.032768$$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.32645$$

$$\tau_{\max} = \frac{T}{c_1 a b^2} \quad \therefore \quad T = c_1 a b^2 \tau_{\max}$$

(*a*) $$T = (0.32645)(0.195313)(0.0064)^2(60 \times 10^6) = 156.696 \times 10^3\ \text{N} \cdot \text{m}$$

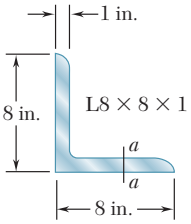
$$T = 157.0\ \text{kN} \cdot \text{m} \blacktriangleleft$$

(*b*) $$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{c_1 a b^2 \tau_{\max} L}{c_2 a b^3 G} = \frac{c_1 \tau_{\max} L}{c_2 b G} = \frac{\tau_{\max} L}{bG}$$

$$\varphi = \frac{(60 \times 10^6)(1.25)}{(0.0064)(77.2 \times 10^9)} = 151.797 \times 10^{-3}\ \text{rad}$$

$$\varphi = 8.70° \blacktriangleleft$$

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PROBLEM 3.136

A 36-kip · in. torque is applied to a 10-ft-long steel angle with an $L8 \times 8 \times 1$ cross section. From Appendix C, we find that the thickness of the section is 1 in. and that its area is 15 in^2. Knowing that $G = 11.2 \times 10^6$ psi, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the angle of twist.

SOLUTION

$$a = \frac{A}{t} = \frac{15\text{ in}^2}{1\text{ in.}} = 15\text{ in.}, \quad b = 1\text{ in.}, \quad \frac{a}{b} = 15$$

Since $\dfrac{a}{b} > 5, \quad c_1 = c_2 = \dfrac{1}{3}\left(1 - 0.630\dfrac{b}{a}\right)$

or $c_1 = c_2 = \dfrac{1}{3}\left(1 - \dfrac{0.630}{15}\right) = 0.3193$

$$T = 36 \times 10^3\text{ lb} \cdot \text{in.}; \quad L = 120\text{ in.}; \quad G = 11.2 \times 10^6\text{ psi}$$

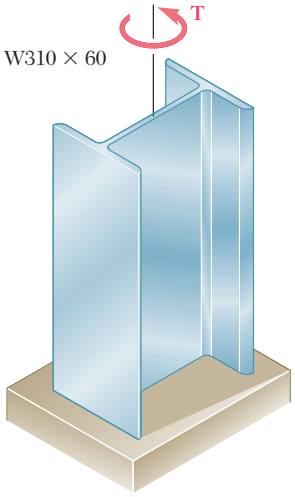
(*a*) Maximum shearing stress: $\tau_{\max} = \dfrac{T}{c_1 a b^2}$

$$\tau_{\max} = \frac{36 \times 10^3}{(0.3193)(15)(1)^2} = 7.52 \times 10^3\text{ psi} \qquad \tau_{\max} = 7.52\text{ ksi} \blacktriangleleft$$

(*b*) Angle of twist: $\varphi = \dfrac{TL}{c_2 a b^2 G}$

$$\varphi = \frac{(36 \times 10^3)(120)}{(0.3193)(15)(1)^3(11.2 \times 10^6)} = 0.08052\text{ radians} \qquad \varphi = 4.61° \blacktriangleleft$$

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PROBLEM 3.137

A 4-m-long steel member has a W310 × 60 cross section. Knowing that $G = 77.2$ GPa and that the allowable shearing stress is 40 MPa, determine (*a*) the largest torque **T** that can be applied, (*b*) the corresponding angle of twist. Refer to Appendix C for the dimensions of the cross section and neglect the effect of stress concentrations. (*Hint*: consider the web and flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

SOLUTION

$$\text{W310} \times 60, \quad L = 4\text{ m}, \quad G = 77.2\text{ GPa}, \quad \tau_{\text{all}} = 40\text{ MPa}$$

For one flange: From App. C, $a = 203$ mm, $b = 13.1$ mm, $a/b = 15.50$

Eq. (3.45): $$c_1 = c_2 = \frac{1}{3}\left(1 - \frac{0.630}{15.50}\right) = 0.31979$$

Eq. (3.44): $$\phi_f = \frac{T_f L}{c_2 a b^3 G} = \frac{T_f(4)}{0.31979(0.203)(0.0131)^3(77.2\times10^9)} \tag{1}$$

$$\phi_f = 355.04\times10^{-6}T_f$$

For web: From App. C, $a = 302 - 2(13.1) = 275.8$ mm, $b = 7.49$ mm, $a/b = 36.822$

Eq. (3.45): $$c_1 = c_2 = \frac{1}{3}\left(1 - \frac{0.630}{36.822}\right) = 0.32763$$

Eq. (3.44): $$\phi_w = \frac{T_w(4)}{0.32763(0.2758)(0.00749)^3(77.2\times10^9)}$$

$$\phi_w = 1364.64\times10^{-6}T_w \tag{2}$$

Since angle of twist is the same for flanges and web,

$$\phi_f = \phi_w: \quad 355.04\times10^{-6}T_f = 1364.64\times10^{-6}T_w$$

$$T_f = 3.8436T_w \tag{3}$$

But the sum of the torques exerted on the two flanges and on the web is equal to the torque **T** applied to the member:

$$2T_f + T_w = T \tag{4}$$

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PROBLEM 3.137 (*Continued*)

Substituting for T_f from (3) into (4),

$$2(3.8436T_w) + T_w = T \qquad T_w = 0.115112T \tag{5}$$

From (3), $T_f = 3.8436(0.115112T) \qquad T_f = 0.44244T$ (6)

For one flange:

From Eq. (3.43), $T_f = c_1 ab^2 \tau_{\max} = 0.31979(0.203)(0.0131)^2(40 \times 10^6)$

$= 445.62 \text{ N} \cdot \text{m}$

Eq. (6): $445.62 = 0.44244T$ $\qquad T = 1007 \text{ N} \cdot \text{m}$

For web: $T_w = c_1 ab^2 \tau_{\max} = 0.32763(0.2758)(0.00749)^2(40 \times 10^6)$

$= 202.77 \text{ N} \cdot \text{m}$

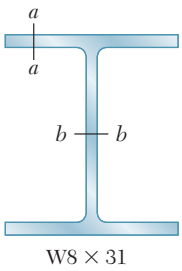
Eq. (5): $202.77 = 0.115112T$ $\qquad T = 1762 \text{ N} \cdot \text{m}$

(*a*) Largest allowable torque: Use the smaller value. $T = 1007 \text{ N} \cdot \text{m}$ ◀

(*b*) Angle of twist: Use T_f, which is critical.

Eq. (1): $\phi = \phi_f = (355.04 \times 10^{-6})(445.62) = 0.161763 \text{ rad}$ $\qquad \phi = 9.27°$ ◀

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PROBLEM 3.138

An 8-ft-long steel member with a W8 × 31 cross section is subjected to a 5-kip · in. torque. The properties of the rolled-steel section are given in Appendix C. Knowing that $G = 11.2 \times 10^6$ psi, determine (*a*) the maximum shearing stress along line *a*-*a*, (*b*) the maximum shearing stress along line *b*-*b*, (*c*) the angle of twist. (See hint of Prob. 3.137.)

SOLUTION

Flange: $a = 8.00$ in., $b = 0.435$ in., $\dfrac{a}{b} = \dfrac{8.00}{0.435} = 18.3908$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.32192 \qquad \varphi_f = \frac{T_f L}{c_2 a b^3 G}$$

$$T_f = c_2 a b^3 \frac{G\varphi_f}{L} = K_f \frac{G\varphi}{L} \quad \text{where} \quad K_f = c_2 a b^3$$

$$K_f = (0.32192)(8.00)(0.435)^3 = 0.21199 \text{ in}^3$$

Web: $a = 8.0 - (2)(0.435) = 7.13$ in., $b = 0.285$ in., $\dfrac{a}{b} = \dfrac{7.13}{0.285} = 25.018$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.32494 \qquad \varphi_w = \frac{T_w L}{c_2 a b^3 G}$$

$$T_w = c_2 a b^3 \frac{G\varphi_w}{L} = K_w \frac{G\varphi}{L} \quad \text{where} \quad K_w = c_2 a b^3$$

$$K_w = (0.32494)(7.13)(0.285)^3 = 0.053632 \text{ in}^4$$

For matching twist angles, $\varphi_f = \varphi_w = \varphi$

Total torque. $T = 2T_f + T_w = (2K_f + K_w)\dfrac{G\varphi}{L}$

$$\frac{G\varphi}{L} = \frac{T}{2K_p + K_w}, \quad T_f = \frac{K_f T}{2K_f + K_w}, \quad T_w = \frac{K_w T}{2K_f + K_w}$$

$$T_f = \frac{(0.21199)(5000)}{(2)(0.21199) + 0.053632} = 2219.3 \text{ lb} \cdot \text{in.}; \quad T_w = \frac{(0.053632)(5000)}{(2)(0.21199) + 0.053632} = 561.46 \text{ lb} \cdot \text{in.}$$

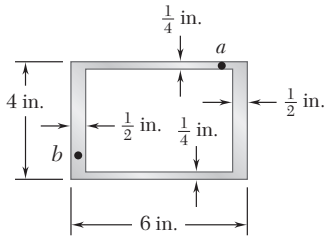
(*a*) $\tau_f = \dfrac{T_f}{c_1 a b^2} = \dfrac{2219.3}{(0.32192)(8.00)(0.435)^2} = 4550$ psi $\qquad \tau_f = 4.55$ ksi ◀

(*b*) $\tau_w = \dfrac{T_w}{c_1 a b^2} = \dfrac{561.46}{(0.32494)(7.13)(0.285)^2} = 2980$ psi $\qquad \tau_w = 2.98$ ksi ◀

(*c*) $\dfrac{G\varphi}{L} = \dfrac{T}{2K_f + K_w} \quad \therefore \quad \varphi = \dfrac{TL}{G(2K_f + K_w)}$ where $L = 8$ ft $= 96$ in.

$$\varphi = \frac{(5000)(96)}{(11.2 \times 10^6)[(2)(0.21199) + 0.53632]} = 44.629 \times 10^{-3} \text{ rad} \qquad \varphi = 2.56° \blacktriangleleft$$

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PROBLEM 3.139

A 5-kip · ft torque is applied to a hollow aluminum shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.

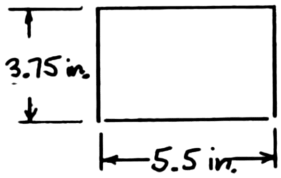
SOLUTION

$$T = (5)(10^3)(12) = 60 \times 10^3 \text{ lb} \cdot \text{in.}$$

Area bounded by center line:

$$a = bh = (5.5)(3.75) = 20.625 \text{ in}^2$$

Area bounded by center line.



At Point *a*, $t = 0.25$ in.

$$\tau = \frac{T}{2ta}$$

$$= \frac{60 \times 10^3}{(2)(0.25)(20.625)}$$

$$= 5.82 \times 10^3 \text{ psi}$$

$$\tau = 5.82 \text{ ksi} \blacktriangleleft$$

At Point *b*, $t = 0.50$ in.

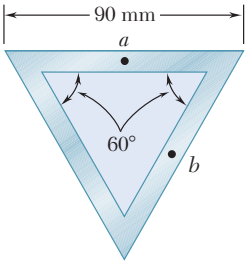
$$\tau = \frac{T}{2ta}$$

$$= \frac{60 \times 10^3}{(2)(0.50)(20.625)}$$

$$= 2.91 \times 10^3 \text{ psi}$$

$$\tau = 2.91 \text{ ksi} \blacktriangleleft$$

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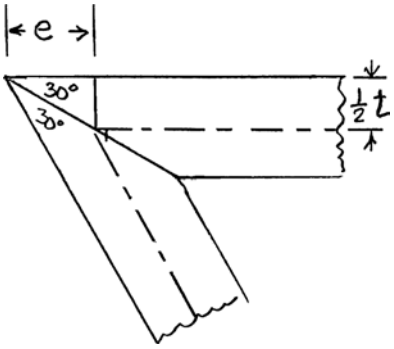


PROBLEM 3.140

A torque $T = 750$ kN · m is applied to a hollow shaft shown that has a uniform 8-mm wall thickness. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.

SOLUTION

Detail of corner.



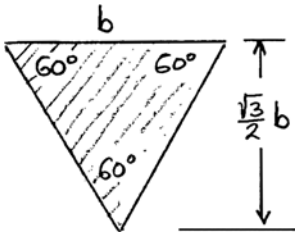
$$\frac{1}{2}t = e \tan 30°$$

$$e = \frac{t}{2 \tan 30°}$$

$$= \frac{8}{2 \tan 30°} = 6.928 \text{ mm}$$

$$b = 90 - 2e = 76.144 \text{ mm}$$

Area bounded by center line.



$$a = \frac{1}{2}b\frac{\sqrt{3}}{2}b = \frac{\sqrt{3}}{4}b^2 = \frac{\sqrt{3}}{4}(76.144)^2$$

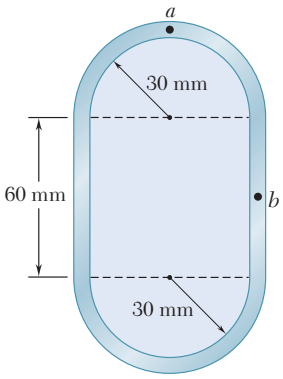
$$= 2510.6 \text{ mm}^2 = 2510.6 \times 10^{-6} \text{ m}^2$$

$$t = 0.008 \text{ m}$$

$$\tau = \frac{T}{2ta} = \frac{750}{(2)(0.008)(2510 \times 10^{-6})} = 18.67 \times 10^6 \text{ Pa}$$

$$\tau = 18.67 \text{ MPa} \blacktriangleleft$$

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PROBLEM 3.141

A 750-N · m torque is applied to a hollow shaft having the cross section shown and a uniform 6-mm wall thickness. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.

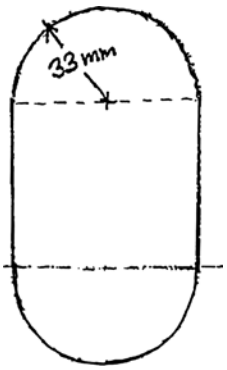
SOLUTION

Area bounded by center line.

$$a = 2\frac{\pi}{2}(33)^2 + (60)(66) = 7381\ \text{mm}^2$$

$$= 7381 \times 10^{-6}\ \text{m}^2$$

$$t = 0.006\ \text{m at both } a \text{ and } b$$

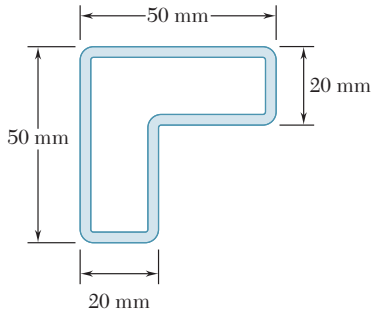


Then at points *a* and *b*,

$$\tau = \frac{T}{2ta} = \frac{750}{(2)(0.006)(7381 \times 10^{-6})} = 8.47 \times 10^6\ \text{Pa}$$

$\tau = 8.47$ MPa ◀

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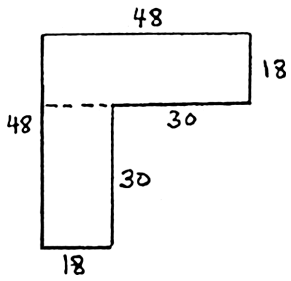


PROBLEM 3.142

A hollow member having the cross section shown is formed from sheet metal of 2-mm thickness. Knowing that the shearing stress must not exceed 3 MPa, determine the largest torque that can be applied to the member.

SOLUTION

Area bounded by center line.



$$a = (48)(18) + (30)(18)$$

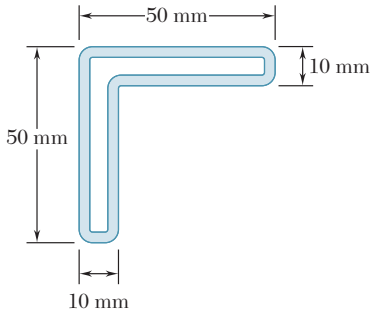
$$= 1404\ \text{mm}^2 = 1404 \times 10^{-6}\ \text{m}^2$$

$$t = 0.002\ \text{m}$$

$$\tau = \frac{T}{2ta} \qquad \text{or} \qquad T = 2ta\tau = (2)(0.002)(1404 \times 10^{-6})(3 \times 10^{6})$$

$$T = 16.85\ \text{N} \cdot \text{m} \blacktriangleleft$$

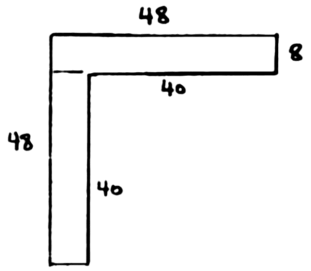
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PROBLEM 3.143

A hollow member having the cross section shown is formed from sheet metal of 2-mm thickness. Knowing that the shearing stress must not exceed 3 MPa, determine the largest torque that can be applied to the member.

SOLUTION



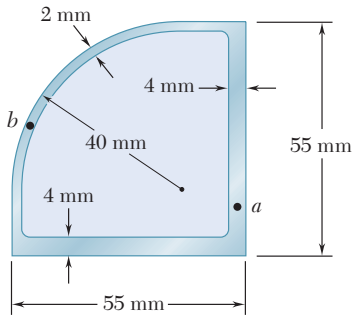
Area bounded by center line.

$$a = (48)(8) + (40)(8)$$
$$= 704 \text{ mm}^2 = 704 \times 10^{-6} \text{ m}^2$$
$$t = 0.002 \text{ m}$$

$$\tau = \frac{T}{2ta} \quad \therefore \quad T = 2ta\tau = (2)(0.002)(704 \times 10^{-6})(3 \times 10^{6})$$
$$= 8.45 \text{ N} \cdot \text{m}$$

◀

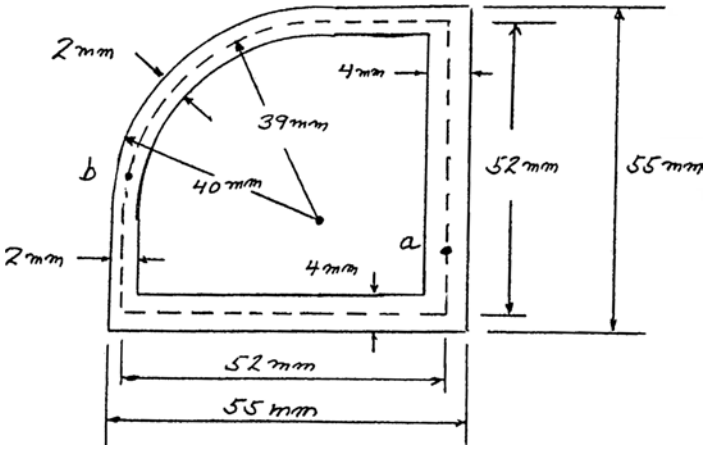
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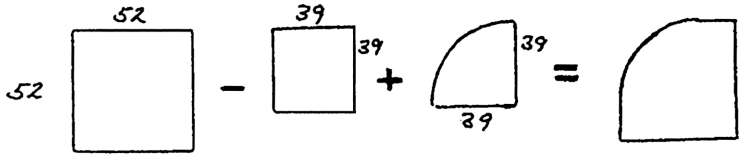
PROBLEM 3.144

A 90-N · m torque is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.

SOLUTION



Area bounded by center line.



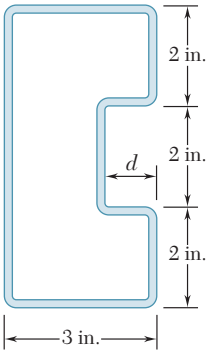
$$a = 52 \times 52 - 39 \times 39 + \frac{\pi}{4}(39)^2 = 2377.6 \text{ mm}^2 = 2.3776 \times 10^{-3} \text{ m}^2$$

$$T = 90 \text{ N} \cdot \text{m}$$

$$\tau_a = \frac{T}{2ta} = \frac{90 \text{ N} \cdot \text{m}}{2(4 \times 10^{-3} \text{ m})(2.3776 \times 10^{-3} \text{ m}^2)} \qquad \tau_a = 4.73 \text{ MPa} \blacktriangleleft$$

$$\tau_b = \frac{T}{2ta} = \frac{90 \text{ N} \cdot \text{m}}{2(2 \times 10^{-3} \text{ m})(2.3776 \times 10^{-3} \text{ m}^2)} \qquad \tau_b = 9.46 \text{ MPa} \blacktriangleleft$$

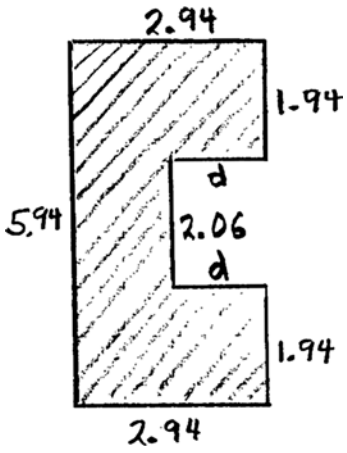
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PROBLEM 3.145

A hollow member having the cross section shown is to be formed from sheet metal of 0.06-in. thickness. Knowing that a 1250-lb · in. torque will be applied to the member, determine the smallest dimension d that can be used if the shearing stress is not to exceed 750 psi.

SOLUTION



Area bounded by center line.

$$a = (5.94)(2.94) - 2.06d = 17.4636 - 2.06d$$

$$t = 0.06 \text{ in.}, \quad \tau = 750 \text{ psi}, \quad T = 1250 \text{ lb} \cdot \text{in.}$$

$$\tau = \frac{T}{2ta}$$

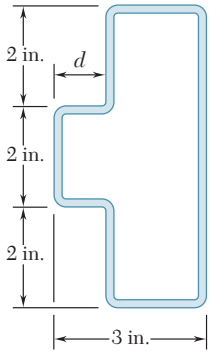
$$a = \frac{T}{2t\tau}$$

$$17.4636 - 2.06d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{2.06} = 1.735 \text{ in.}$$

$d = 1.735$ in. ◀

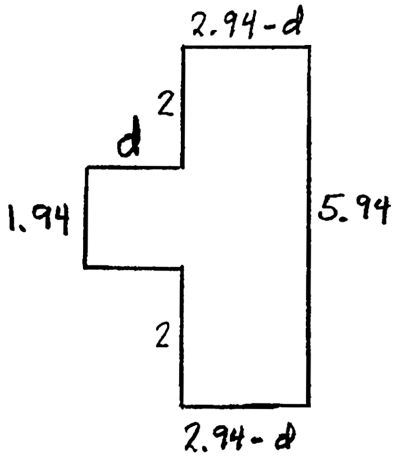
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PROBLEM 3.146

A hollow member having the cross section shown is to be formed from sheet metal of 0.06-in. thickness. Knowing that a 1250-lb · in. torque will be applied to the member, determine the smallest dimension d that can be used if the shearing stress is not to exceed 750 psi.

SOLUTION



Area bounded by center line.

$$a = (5.94)(2.94 - d) + 1.94d = 17.4636 - 4.00d$$

$$t = 0.06 \text{ in.}, \quad \tau = 750 \text{ psi}, \quad T = 1250 \text{ lb} \cdot \text{in.}$$

$$\tau = \frac{T}{2ta}$$

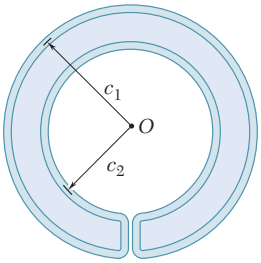
$$a = \frac{T}{2t\tau}$$

$$17.4636 - 4.00d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{4.00} = 0.894 \text{ in.}$$

$d = 0.894$ in. ◀

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PROBLEM 3.147

A cooling tube having the cross section shown is formed from a sheet of stainless steel of 3-mm thickness. The radii $c_1 = 150$ mm and $c_2 = 100$ mm are measured to the center line of the sheet metal. Knowing that a torque of magnitude $T = 3$ kN · m is applied to the tube, determine (*a*) the maximum shearing stress in the tube, (*b*) the magnitude of the torque carried by the outer circular shell. Neglect the dimension of the small opening where the outer and inner shells are connected.

SOLUTION

Area bounded by center line.

$$a = \pi\left(c_1^2 - c_2^2\right) = \pi(150^2 - 100^2) = 39.27 \times 10^3 \text{ mm}^2$$

$$= 39.27 \times 10^{-3} \text{ m}^2$$

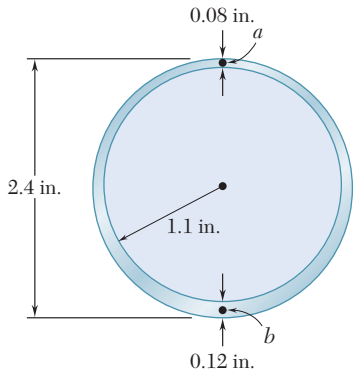
$$t = 0.003 \text{ m}$$

(*a*) $$\tau = \frac{T}{2ta} = \frac{3 \times 10^3}{(2)(0.003)(39.27 \times 10^{-3})} = 12.73 \times 10^6 \text{ Pa}$$ $\tau = 12.76$ MPa ◀

(*b*) $$T_1 = (2\pi c_1 t \tau c_1) = 2\pi c_1^2 t \tau$$

$$= 2\pi(0.150)^2(0.003)(12.73 \times 10^6) = 5.40 \times 10^3 \text{ N} \cdot \text{m}$$ $T_1 = 5.40$ kN · m ◀

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PROBLEM 3.148

A hollow cylindrical shaft was designed to have a uniform wall thickness of 0.1 in. Defective fabrication, however, resulted in the shaft having the cross section shown. Knowing that a 15-kip · in. torque is applied to the shaft, determine the shearing stresses at points *a* and *b*.

SOLUTION

Radius of outer circle $= 1.2$ in.

Radius of inner circle $= 1.1$ in.

Mean radius $= 1.15$ in.

Area bounded by center line.

$$a = \pi r_m^2 = \pi(1.15)^2 = 4.155 \text{ in}^2$$

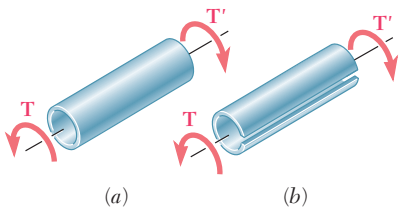
At point *a*, $t = 0.08$ in.

$$\tau = \frac{T}{2ta} = \frac{15}{(2)(0.08)(4.155)} \qquad \tau = 22.6 \text{ ksi} \blacktriangleleft$$

At point *b*, $t = 0.12$ in.

$$\tau = \frac{T}{2ta} = \frac{15}{(2)(0.12)(4.155)} \qquad \tau = 15.04 \text{ ksi} \blacktriangleleft$$

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PROBLEM 3.149

Equal torques are applied to thin-walled tubes of the same length L, same thickness t, and same radius c. One of the tubes has been slit lengthwise as shown. Determine (*a*) the ratio τ_b / τ_a of the maximum shearing stresses in the tubes, (*b*) the ratio φ_b / φ_a of the angles of twist of the shafts.

SOLUTION

Without slit:

Area bounded by center line. $a = \pi c^2$

$$\tau_a = \frac{T}{2ta} = \frac{T}{2\pi c^2 t}$$

$$J \approx 2\pi c^3 t \qquad \varphi_a = \frac{TL}{GJ} = \frac{TL}{2\pi c^3 tG}$$

With slit: $a = 2\pi c, \quad b = t, \quad \frac{a}{b} = \frac{2\pi c}{t} >> 1$

$$c_1 = c_2 = \frac{1}{3}$$

$$\tau_b = \frac{T}{c_1 a b^2} = \frac{3T}{2\pi c t^2}$$

$$\varphi_b = \frac{T}{c_2 a b^3 G} = \frac{3TL}{2\pi c t^3 G}$$

(*a*) Stress ratio:

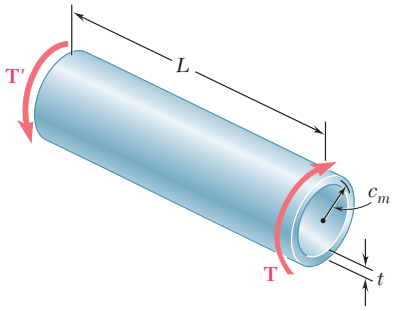
$$\frac{\tau_b}{\tau_a} = \frac{3T}{2\pi c t^2} \cdot \frac{2\pi c^2 t}{T} = \frac{3c}{t} \qquad \boxed{\frac{\tau_b}{\tau_a} = \frac{3c}{t}} \blacktriangleleft$$

(*b*) Twist ratio:

$$\frac{\varphi_b}{\varphi_a} = \frac{3TL}{2\pi c t^3 G} \cdot \frac{2\pi c^3 tG}{TL} = \frac{3c^2}{t^2} \qquad \boxed{\frac{\varphi_b}{\varphi_a} = \frac{3c^2}{t^2}} \blacktriangleleft$$

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PROBLEM 3.150



A hollow cylindrical shaft of length L, mean radius c_m, and uniform thickness t is subjected to a torque of magnitude T. Consider, on the one hand, the values of the average shearing stress τ_{ave} and the angle of twist φ obtained from the elastic torsion formulas developed in Secs. 3.1C and 3.2 and, on the other hand, the corresponding values obtained from the formulas developed in Sec. 3.10 for thin-walled shafts. (*a*) Show that the relative error introduced by using the thin-walled-shaft formulas rather than the elastic torsion formulas is the same for τ_{ave} and φ and that the relative error is positive and proportional to the ratio t/c_m. (*b*) Compare the percent error corresponding to values of the ratio t/c_m of 0.1, 0.2, and 0.4.

SOLUTION

Let c_2 = outer radius = $c_m + \frac{1}{2}t$ and c_1 = inner radius = $c_m - \frac{1}{2}t$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}\left(c_2^2 + c_1^2\right)(c_2 + c_1)(c_2 - c_1)$$

$$= \frac{\pi}{2}\left(c_m^2 + c_m t + \frac{1}{4}t^2 + c_m^2 - c_m t + \frac{1}{4}t^2\right)(2c_m)t$$

$$= 2\pi\left(c_m^2 + \frac{1}{4}t^2\right)c_m t$$

$$\tau_m = \frac{Tc_m}{J} = \frac{T}{2\pi\left(c_m^2 + \frac{1}{4}t^2\right)t}$$

$$\varphi_1 = \frac{TL}{JG} = \frac{TL}{2\pi\left(c_m^2 + \frac{1}{4}t^2\right)c_m t\,G}$$

Area bounded by center line. $a = \pi c_m^2$

$$\tau_{ave} = \frac{T}{2ta} = \frac{T}{2\pi c_m^2 t}$$

$$\varphi_2 = \frac{TL}{4a^2G}\oint\frac{ds}{t} = \frac{TL(2\pi c_m/t)}{4(\pi c_m^2)^2G} = \frac{TL}{2\pi c_m^3 tG}$$

(*a*) Ratios:

$$\frac{\tau_{ave}}{\tau_m} = \frac{T}{2\pi c_m^2 t} \times \frac{2\pi\left(c_m^2 + \frac{1}{4}t^2\right)t}{T} = 1 + \frac{1}{4}\frac{t^2}{c_m^2}$$ ◀

$$\frac{\varphi_2}{\varphi_1} = \frac{TL}{2\pi c_m^3 tG} \times \frac{2\pi\left(c_m^2 + \frac{1}{4}t^2\right)c_m tG}{TL} = 1 + \frac{1}{4}\frac{t^2}{c_m}$$ ◀

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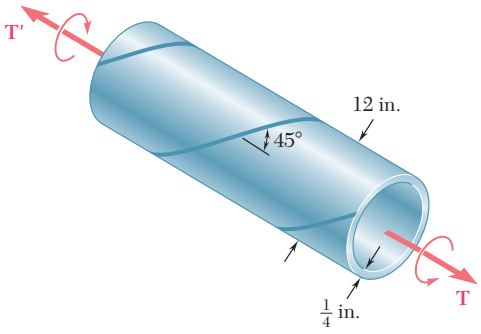
PROBLEM 3.150 (*Continued*)

(*b*) $$\frac{\tau_{\text{ave}}}{\tau_m} - 1 = \frac{\varphi_2}{\varphi_1} - 1 = \frac{1}{4}\frac{t^2}{c_m^2}$$

$\frac{t}{c_m}$	0.1	0.2	0.4
$\frac{1}{4}\frac{t^2}{c_m^2}$	0.0025	0.01	0.04
%	0.25%	1.000%	4.00%

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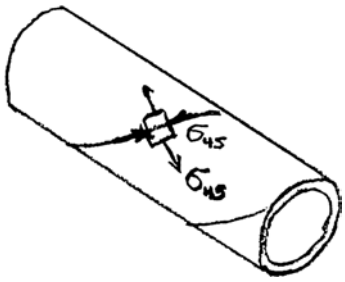
PROBLEM 3.151

A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix which forms an angle of 45° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable tensile stress in the weld is 12 ksi, determine the largest torque that can be applied to the pipe.

SOLUTION

From Eq. (3.14) of the textbook,

$$\sigma_{45} = \tau_{\max}$$



hence, $\tau_{\max} = 12 \text{ ksi} = 12 \times 10^3 \text{ psi}$

$$c_2 = \frac{1}{2} d_o = \frac{1}{2}(12) = 6.00 \text{ in.}$$

$$c_1 = c_2 - t = 6.00 - 0.25 = 5.75 \text{ in.}$$

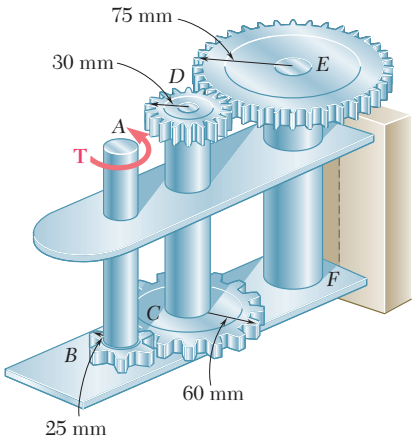
$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(6.00)^4 - (5.75)^4] = 318.67 \text{ in.}$$

$$\tau_{\max} = \frac{Tc}{J} \qquad T = \frac{\tau_{\max} J}{c}$$

$$T = \frac{(12 \times 10^3)(318.67)}{6.00} = 637 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$T = 637 \text{ kip} \cdot \text{in.} \blacktriangleleft$$

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PROBLEM 3.152

A torque of magnitude $T = 120$ N · m is applied to shaft *AB* of the gear train shown. Knowing that the allowable shearing stress is 75 MPa in each of the three solid shafts, determine the required diameter of (*a*) shaft *AB*, (*b*) shaft *CD*, (*c*) shaft *EF*.

SOLUTION

Statics:

Shaft *AB*: $T_{AB} = T_A = T_B = T$

Gears *B* and *C*: $r_B = 25$ mm, $r_C = 60$ mm

Force on gear circles. $F_{BC} = \dfrac{T_B}{r_B} = \dfrac{T_C}{r_C}$

$$T_C = \frac{r_C}{r_B} T_B = \frac{60}{25} T = 2.4T$$

Shaft *CD*: $T_{CD} = T_C = T_D = 2.4T$

Gears *D* and *E*: $r_D = 30$ mm, $r_E = 75$ mm

Force on gear circles. $F_{DE} = \dfrac{T_D}{r_D} = \dfrac{T_E}{r_E}$

$$T_E = \frac{r_E}{r_D} T_D = \frac{75}{30}(2.4T) = 6T$$

Shaft *EF*: $T_{EF} = T_E = T_F = 6T$

Required diameters.

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}}$$

$$d = 2c = 2\sqrt[3]{\frac{2T}{\pi\tau_{\max}}}$$

$$\tau_{\max} = 75 \times 10^6 \text{ Pa}$$

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PROBLEM 3.152 (*Continued*)

(*a*) Shaft *AB*:

$$T_{AB} = T = 120 \text{ N} \cdot \text{m}$$

$$d_{AB} = 2\sqrt[3]{\frac{2(120)}{\pi(75 \times 10^6)}} = 20.1 \times 10^{-3} \text{ m} \qquad d_{AB} = 20.1 \text{ mm} \blacktriangleleft$$

(*b*) Shaft *CD*:

$$T_{CD} = (2.4)(120) = 288 \text{ N} \cdot \text{m}$$

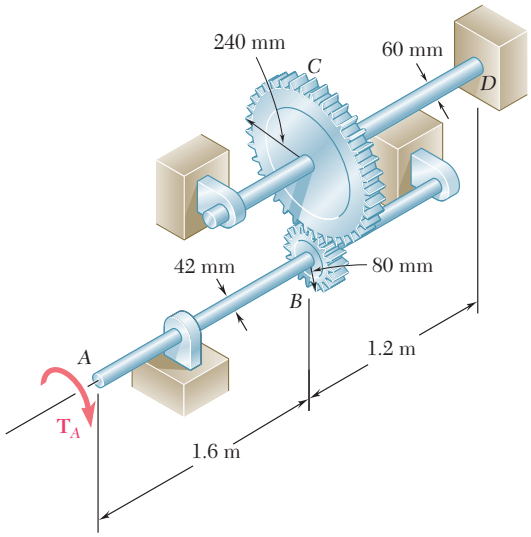
$$d_{CD} = 2\sqrt[3]{\frac{(2)(288)}{\pi(75 \times 10^6)}} = 26.9 \times 10^{-3} \text{ m} \qquad d_{CD} = 26.9 \text{ mm} \blacktriangleleft$$

(*c*) Shaft *EF*:

$$T_{EF} = (6)(120) = 720 \text{ N} \cdot \text{m}$$

$$d_{EF} = 2\sqrt[3]{\frac{(2)(720)}{\pi(75 \times 10^3)}} = 36.6 \times 10^{-3} \text{ m} \qquad d_{EF} = 36.6 \text{ mm} \blacktriangleleft$$

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PROBLEM 3.153

Two solid shafts are connected by gears as shown. Knowing that $G = 77.2$ GPa for each shaft, determine the angle through which end A rotates when $T_A = 1200\ \text{N} \cdot \text{m}$.

SOLUTION

Calculation of torques:

Circumferential contact force between gears B and C: $F = \dfrac{T_{AB}}{r_B} = \dfrac{T_{CD}}{r_C} \quad T_{CD} = \dfrac{r_C}{r_B} T_{AB}$

$$T_{AB} = 1200\ \text{N} \cdot \text{m} \qquad T_{CD} = \frac{240}{80}(1200) = 3600\ \text{N} \cdot \text{m}$$

Twist in shaft CD: $c = \dfrac{1}{2}d = 0.030\ \text{m}, \quad L = 1.2\ \text{m}, \quad G = 77.2 \times 10^9\ \text{Pa}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6}\ \text{m}^4$$

$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(3600)(1.2)}{(77.2 \times 10^9)(1.27234 \times 10^{-9})} = 43.981 \times 10^{-3}\ \text{rad}$$

Rotation angle at C. $\varphi_C = \varphi_{C/D} = 43.981 \times 10^{-3}\ \text{rad}$

Circumferential displacement at contact points of gears B and C: $\delta = r_C\varphi_C = r_B\varphi_B$

Rotation angle at B. $\varphi_B = \dfrac{r_C}{r_B}\varphi_C = \dfrac{240}{80}(43.981 \times 10^{-3}) = 131.942 \times 10^{-3}\ \text{rad}$

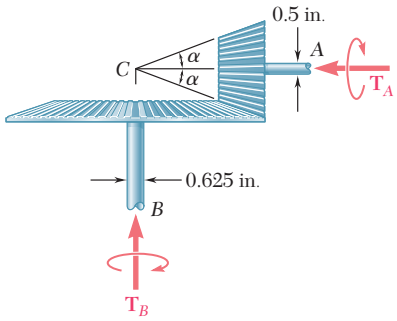
Twist in shaft AB: $c = \dfrac{1}{2}d = 0.021\ \text{m}, \quad L = 1.6\ \text{m}, \quad G = 77.2 \times 10^9\ \text{Pa}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.021)^4 = 305.49 \times 10^9\ \text{m}^4$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(1.6)}{(77.2 \times 10^9)(305.49 \times 10^{-9})} = 81.412 \times 10^{-3}\ \text{rad}$$

Rotation angle at A. $\varphi_A = \varphi_B + \varphi_{A/B} = 213.354 \times 10^{-3}\ \text{rad}$ $\qquad \varphi_A = 12.22°$ ◀

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PROBLEM 3.154

In the bevel-gear system shown, $\alpha = 18.43°$. Knowing that the allowable shearing stress is 8 ksi in each shaft and that the system is in equilibrium, determine the largest torque $\mathbf{T}_A$ that can be applied at A.

SOLUTION

Using stress limit for shaft A,

$$\tau = 8 \text{ ksi}, \qquad c = \frac{1}{2}d = 0.25 \text{ in.}$$

$$T_A = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(8)(0.25)^3 = 0.196350 \text{ kip} \cdot \text{in.}$$

Using stress limit for shaft B,

$$\tau = 8 \text{ ksi}, \quad c = \frac{1}{2}d = 0.3125 \text{ in.}$$

$$T_B = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(8)(0.3125)^3 = 0.3835 \text{ kip} \cdot \text{in.}$$

From statics,

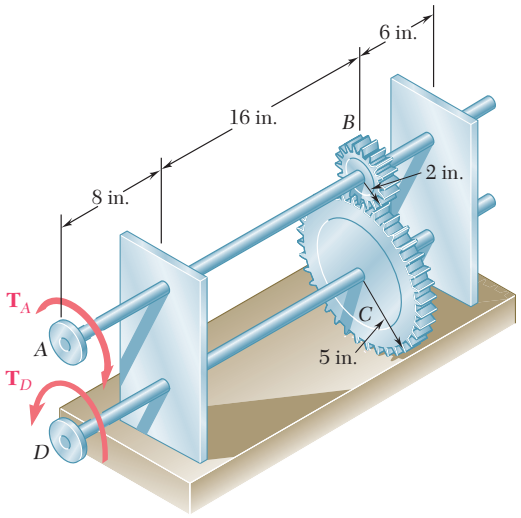
$$T_A = \frac{r_A}{r_B}T_B = (\tan\alpha)T_B$$

$$T_A = (\tan 18.43°)(0.3835) = 0.1278 \text{ kip} \cdot \text{in.}$$

The allowable value of T_A is the smaller.

$$T_A = 0.1278 \text{ kip} \cdot \text{in.} \qquad T_A = 127.8 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

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PROBLEM 3.155

The design specifications for the gear-and-shaft system shown require that the same diameter be used for both shafts, and that the angle through which pulley *A* will rotate when subjected to a 2-kip · in. torque $\mathbf{T}_A$ while pulley *D* is held fixed will not exceed 7.5°. Determine the required diameter of the shafts if both shafts are made of a steel with $G = 11.2 \times 10^6$ psi and $\tau_{\text{all}} = 12$ ksi.

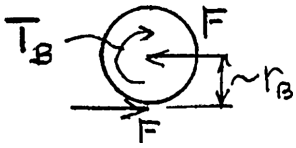
SOLUTION

Statics:

Gear *B*.

$$+\circlearrowleft \Sigma M_B = 0:$$

$$r_B F - T_A = 0 \quad F = T_B/r_B$$

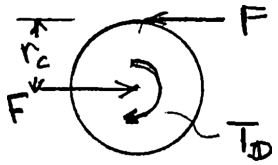


Gear *C*.

$$+\circlearrowleft \Sigma M_C = 0:$$

$$r_C F - T_D = 0$$

$$T_D = r_C F = \frac{r_C}{r_B} T_A = n T_B$$



$$n = \frac{r_C}{r_B} = \frac{5}{2} = 2.5$$

Torques in shafts.

$$T_{AB} = T_A = T_B \quad T_{CD} = T_C = nT_B = nT_A$$

Deformations:

$$\varphi_{C/D} = \frac{T_{CD}L}{GJ} \circlearrowleft = \frac{nT_AL}{GJ} \circlearrowleft$$

$$\varphi_{A/B} = \frac{T_{AB}L}{GJ} \circlearrowleft = \frac{T_AL}{GJ} \circlearrowleft$$

Kinematics:

$$\varphi_D = 0 \quad \varphi_C = \varphi_D + \varphi_{C/D} = 0 + \frac{nT_AL}{GJ} \circlearrowleft$$

$$r_B\varphi_B = -r_C\varphi_B \quad \varphi_B = -\frac{r_C}{r_B}\varphi_C = -n\varphi_C \quad \varphi_B \frac{n^2T_AL}{GJ} \circlearrowleft$$

$$\varphi_A = \varphi_C + \varphi_{B/C} = \frac{n^2T_AL}{GJ} + \frac{T_AL}{GJ} = \frac{(n^2+1)T_AL}{GJ} \circlearrowleft$$

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PROBLEM 3.155 *(Continued)*

Diameter based on stress.

Largest torque: $T_m = T_{CD} = nT_A$

$$\tau_m = \frac{T_m c}{J} = \frac{2nT_A}{\pi c^3} \quad \tau_m = \tau_{\text{all}} = 12 \times 10^3 \text{ psi}, \quad T_A = 2 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$c = \sqrt[3]{\frac{2nT_A}{\pi \tau_m}} = \sqrt[3]{\frac{(2)(2.5)(2 \times 10^3)}{\pi (12 \times 10^3)}} = 0.6425 \text{ in.}, \quad d = 2c = 1.285 \text{ in.}$$

Diameter based on rotation limit.

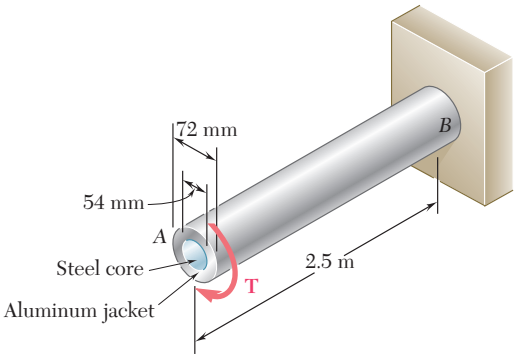
$$\varphi = 7.5^\circ = 0.1309 \text{ rad}$$

$$\varphi = \frac{(n^2 + 1)T_A L}{GJ} = \frac{(2)(7.25)T_A L}{\pi c^4 G} \quad L = 8 + 16 = 24 \text{ in.}$$

$$c = \sqrt[4]{\frac{(2)(7.25)T_A L}{\pi G \varphi}} = \sqrt[4]{\frac{(2)(7.25)(2 \times 10^3)(24)}{\pi (11.2 \times 10^6)(0.1309)}} = 0.62348 \text{ in.}, \quad d = 2c = 1.247 \text{ in.}$$

Choose the larger diameter. $d = 1.285$ in. ◀

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PROBLEM 3.156

A torque of magnitude $T = 4$ kN · m is applied at end A of the composite shaft shown. Knowing that the modulus of rigidity is 77.2 GPa for the steel and 27 GPa for the aluminum, determine (*a*) the maximum shearing stress in the steel core, (*b*) the maximum shearing stress in the aluminum jacket, (*c*) the angle of twist at A.

SOLUTION

Steel core: $c_1 = \frac{1}{2} d_1 = 0.027 \text{ m} \qquad J_1 = \frac{\pi}{2} c_1^4 = \frac{\pi}{2}(0.027)^4 = 834.79 \times 10^{-9}$

$$G_1 J_1 = (77.2 \times 10^9)(834.79 \times 10^{-9}) = 64.446 \times 10^3 \text{ N} \cdot \text{m}^2$$

Torque carried by steel core. $T_1 = G_1 J_1 \varphi / L$

Aluminum jacket: $c_1 = \frac{1}{2} d_1 = 0.027 \text{ m}, \qquad c_2 = \frac{1}{2} d_2 = 0.036 \text{ m}$

$$J_2 = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.036^4 - 0.027^4) = 1.80355 \times 10^{-6} \text{ m}^4$$

$$G_2 J_2 = (27 \times 10^9)(1.80355 \times 10^{-6}) = 48.70 \times 10^3 \text{ N} \cdot \text{m}^2$$

Torque carried by aluminum jacket. $T_2 = G_2 J_2 \varphi / L$

Total torque: $T = T_1 + T_2 = (G_1 J_1 + G_2 J_2)\, \varphi / L$

$$\frac{\varphi}{L} = \frac{T}{G_1 J_1 + G_2 J_2} = \frac{4 \times 10^3}{64.446 \times 10^3 + 48.70 \times 10^3} = 35.353 \times 10^{-3} \text{ rad/m}$$

(*a*) Maximum shearing stress in steel core.

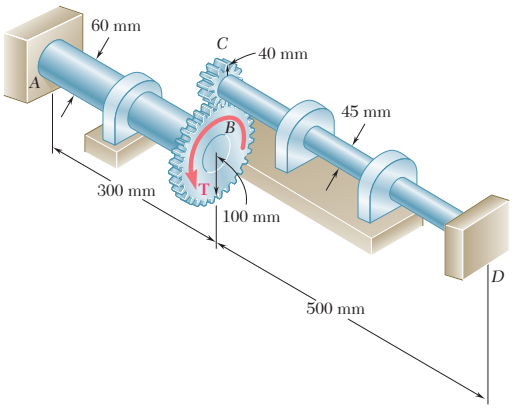
$$\tau = G_1 \gamma = G_1 c_1 \frac{\varphi}{L} = (77.2 \times 10^9)(0.027)(35.353 \times 10^{-3}) = 73.7 \times 10^6 \text{ Pa} \qquad 73.7 \text{ MPa} \blacktriangleleft$$

(*b*) Maximum shearing stress in aluminum jacket.

$$\tau = G_2 \gamma = G_2 c_2 \frac{\varphi}{L} = (27 \times 10^9)(0.036)(35.353 \times 10^{-3}) = 34.4 \times 10^6 \text{ Pa} \qquad 34.4 \text{ MPa} \blacktriangleleft$$

(*c*) Angle of twist. $\varphi = L \frac{\varphi}{L} = (2.5)(35.353 \times 10^{-3}) = 88.383 \times 10^{-3} \text{ rad} \qquad \varphi = 5.06° \blacktriangleleft$

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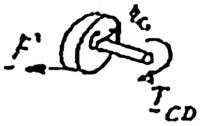
PROBLEM 3.157

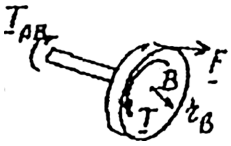
Ends *A* and *D* of the two solid steel shafts *AB* and *CD* are fixed, while ends *B* and *C* are connected to gears as shown. Knowing that the allowable shearing stress is 50 MPa in each shaft, determine the largest torque **T** that may be applied to gear *B*.

SOLUTION

Gears *B* and *C*:

$$\phi_B = \frac{r_C}{r_B}\phi_C = \frac{40}{100}\phi_C \qquad \phi_B = 0.4\,\phi_C \tag{1}$$





$$\Sigma M_C = 0: \; T_{CD} = r_C F \tag{2}$$

$$\Sigma M_B = 0: \; T - T_{AB} = r_B F \tag{3}$$

Solve (2) for *F* and substitute into (3).

$$T - T_{AB} = \frac{r_B}{r_C}T_{CD} \qquad T = T_{AB} + \frac{100}{40}T_{CD}$$

$$T = T_{AB} + 2.5\,T_{CD} \tag{4}$$

Shaft *AB*: $L = 0.3$ m, $c = 0.030$ m

$$\phi_B = \phi_{B/A} = \frac{T_{AB}L}{JG} = \frac{T_{AB}(0.3)}{\frac{\pi}{2}(0.030)^4 G} = 235.77 \times 10^3 \frac{T_{AB}}{G} \tag{5}$$

Shaft *CD*: $L = 0.5$ m, $c = 0.0225$ m

$$\phi_C = \phi_{C/D} = \frac{T_{CD}L}{JG} = \frac{T_{CD}(0.5)}{\frac{\pi}{2}(0.0225)^4 G} = 1242 \times 10^3 \frac{T_{CD}}{G} \tag{6}$$

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PROBLEM 3.157 *(Continued)*

Substitute from (5) and (6) into (1).

$$\phi_B = 0.4\,\phi_C: \quad 235.79 \times 10^3 \frac{T_{AB}}{G} = 0.4 \times 1242 \times 10^3 \frac{T_{CD}}{G}$$

$$T_{CD} = 0.47462 T_{AB} \tag{7}$$

Substitute for T_{CD} from (7) into (4).

$$T = T_{AB} + 2.5\,(0.47462 T_{AB}) \qquad T = 2.1865 T_{AB} \tag{8}$$

Solving (7) for T_{AB} and substituting into (8),

$$T = 2.1865\left(\frac{T_{CD}}{0.47462}\right) \qquad T = 4.6068 T_{CD} \tag{9}$$

Stress criterion for shaft *AB*:

$$\tau_{AB} = \tau_{\text{all}} = 50 \text{ MPa}$$

$$\tau_{AB} = \frac{T_{AB} c}{J} \quad T_{AB} = \frac{J}{c}\tau_{AB} = \frac{\pi}{2} c^3 \tau_{AB}$$

$$= \frac{\pi}{2}(0.030 \text{ m})^3 (50 \times 10^6 \text{ Pa}) = 2120.6 \text{ N} \cdot \text{m}$$

From (8), $T = 2.1865(2120.6 \text{ N} \cdot \text{m}) = 4.64 \text{ kN} \cdot \text{m}$

Stress criterion for shaft *CD*:

$$\tau_{CD} = \tau_{\text{all}} = 50 \text{ MPa:}$$

$$\tau_{CD} = \frac{T_{CD} c}{J} \quad T_{CD} = \frac{\pi}{2} c^3 \tau_{CD} = \frac{\pi}{2}(0.0225 \text{ m})^3 (50 \times 10^6 \text{ Pa})$$

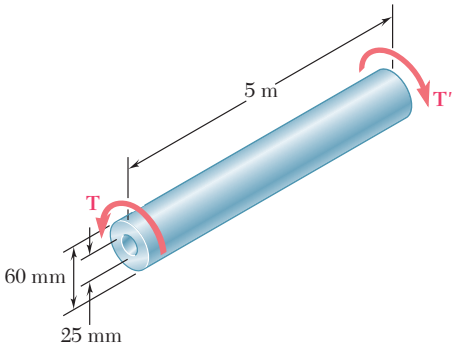
$$= 894.62 \text{ N} \cdot \text{m}$$

From (7), $T = 4.6068(894.62 \text{ N} \cdot \text{m}) = 4.12 \text{ kN} \cdot \text{m}$

The smaller value for *T* governs.

$$T = 4.12 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 3.158

As the hollow steel shaft shown rotates at 180 rpm, a stroboscopic measurement indicates that the angle of twist of the shaft is 3°. Knowing that $G = 77.2\,\text{GPa}$, determine (*a*) the power being transmitted, (*b*) the maximum shearing stress in the shaft.

SOLUTION

$$c_2 = \frac{1}{2}d_2 = 30\ \text{mm}$$

$$c_1 = \frac{1}{2}d_1 = 12.5\ \text{mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4]$$

$$= 1.234 \times 10^6\ \text{mm}^4 = 1.234 \times 10^{-6}\ \text{m}^4$$

$$\varphi = 3° = 0.05236\ \text{rad}$$

$$\varphi = \frac{TL}{GJ}$$

$$T = \frac{GJ\varphi}{L} = \frac{(77.2 \times 10^9)(1.234 \times 10^{-6})(0.0536)}{5} = 997.61\ \text{N} \cdot \text{m}$$

Angular speed: $f = 180\ \text{rpm} = 3\ \text{rev/sec} = 3\ \text{Hz}$

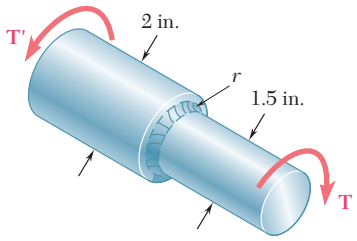
(*a*) Power being transmitted. $P = 2\pi f T = 2\pi(3)(997.61) = 18.80 \times 10^3\ \text{W}$

$$P = 18.80\ \text{kW} \blacktriangleleft$$

(*b*) Maximum shearing stress. $\tau_m = \frac{Tc_2}{J} = \frac{(997.61)(30 \times 10^{-3})}{1.234 \times 10^{-6}}$

$$= 24.3 \times 10^6\ \text{Pa} \qquad \tau_m = 24.3\ \text{MPa} \blacktriangleleft$$

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PROBLEM 3.159

Knowing that the allowable shearing stress is 8 ksi for the stepped shaft shown, determine the magnitude T of the largest torque that can be transmitted by the shaft when the radius of the fillet is (*a*) $r = \frac{3}{16}$ in., (*b*) $r = \frac{1}{4}$ in.

SOLUTION

$$D = 2 \text{ in.} \qquad d = 1.5 \text{ in.} \qquad \frac{D}{d} = 1.33$$

$$c = \frac{1}{2}d = 0.75 \text{ in.} \qquad \tau_{\max} = 8 \text{ ksi}$$

$$\tau_{\max} = K\frac{Tc}{J} \qquad \text{or} \qquad T = \frac{J\tau_{\max}}{Kc} = \frac{\pi\tau_{\max}c^3}{2K}$$

(*a*) $r = \frac{3}{16}$ in. $\qquad r = 0.1875$ in.

$$\frac{r}{d} = \frac{0.1875}{1.5} = 0.125$$

From Fig. 3.32, $\qquad K = 1.33$

$$T = \frac{\pi(8)(0.75)^3}{(2)(1.33)}$$

$T = 3.99$ kip · in. ◀

(*b*) $r = \frac{1}{4}$ in. $\qquad r = 0.25$ in.

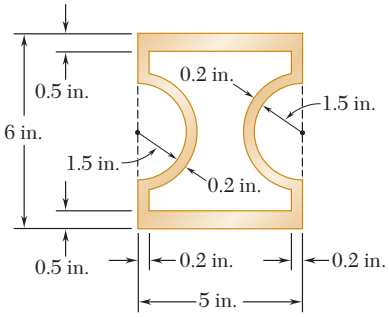
$$\frac{r}{d} = \frac{0.25}{1.5} = 0.1667$$

From Fig. 3.32, $\qquad K = 1.27$

$$T = \frac{\pi(8)(0.75)^3}{(2)(1.27)}$$

$T = 4.17$ kip · in. ◀

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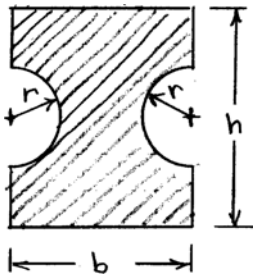


PROBLEM 3.160

A hollow brass shaft has the cross section shown. Knowing that the shearing stress must not exceed 12 ksi and neglecting the effect of stress concentrations, determine the largest torque that can be applied to the shaft.

SOLUTION

Calculate the area bounded by the center line of the wall cross section. The area is a rectangle with two semi-circular cutouts.



$$b = 5 - 0.2 = 4.8 \text{ in.}$$

$$h = 6 - 0.5 = 5.5 \text{ in.}$$

$$r = 1.5 + 0.1 = 1.6 \text{ in.}$$

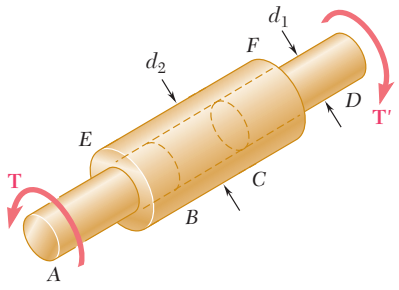
$$a = bh - 2\left(\frac{\pi}{2}r^2\right) = (4.8)(5.5) - \pi(1.6)^2 = 18.3575 \text{ in}^2$$

$$\tau_{\max} = \frac{T}{2at_{\min}} \qquad \tau_{\max} = 12 \times 10^3 \text{ psi} \qquad t_{\min} = 0.2 \text{ in.}$$

$$T = 2at_{\min}\tau_{\max} = (2)(18.3575)(0.2)(12 \times 10^3) = 88.116 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$T = 88.1 \text{ kip} \cdot \text{in.} = 7.34 \text{ kip} \cdot \text{ft} \blacktriangleleft$$

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PROBLEM 3.161

Two solid brass rods *AB* and *CD* are brazed to a brass sleeve *EF*. Determine the ratio d_2/d_1 for which the same maximum shearing stress occurs in the rods and in the sleeve.

SOLUTION

Let $c_1 = \frac{1}{2}d_1$ and $c_2 = \frac{1}{2}d_2$

Shaft *AB*: $\tau_1 = \frac{Tc_1}{J_1} = \frac{2T}{\pi c_1^3}$

Sleeve *EF*: $\tau_2 = \frac{Tc_2}{J_2} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$

For equal stresses, $\frac{2T}{\pi c_1^3} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$

$$c_2^4 - c_1^4 = c_1^3 c_2$$

Let $x = \frac{c_2}{c_1}$ $\qquad x^4 - 1 = x$ or $x = \sqrt[4]{1 + x}$

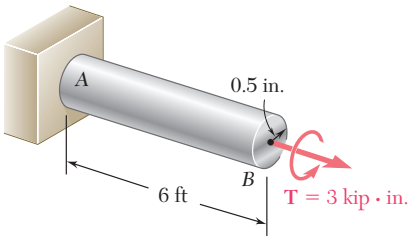
Solve by successive approximations starting with $x_0 = 1.0$.

$$x_1 = \sqrt[4]{2} = 1.189, \quad x_2 = \sqrt[4]{2.189} = 1.216, \quad x_3 = \sqrt[4]{2.216} = 1.220$$

$$x_4 = \sqrt[4]{2.220} = 1.221, \quad x_5 = \sqrt[4]{2.221} = 1.221 \quad \text{(converged)}.$$

$x = 1.221$ $\qquad \frac{c_2}{c_1} = 1.221$ $\qquad \frac{d_2}{d_1} = 1.221$ ◀

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PROBLEM 3.162

The shaft AB is made of a material that is elastoplastic with $\tau_Y = 12.5$ ksi and $G = 4 \times 10^6$ psi. For the loading shown, determine (*a*) the radius of the elastic core of the shaft, (*b*) the angle of twist of the shaft.

SOLUTION

$$c = 0.5 \text{ in.}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.5)^3(1.25) = 2.454 \text{ kip} \cdot \text{in.}$$

$$T = 3 \text{ kip} \cdot \text{in.} > T_Y \quad \text{plastic region with elastic core}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) \quad \therefore \quad \frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(2.454)}{3} = 0.33307$$

$$\frac{\rho_Y}{c} = 0.69318 \qquad \rho_Y = (0.69318)(0.5) = 0.347 \text{ in.} \quad \blacktriangleleft$$

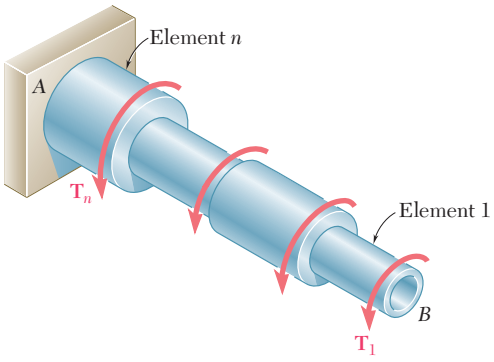
$$L = 6 \text{ ft} = 72 \text{ in.} \qquad G = 4 \times 10^6 \text{ psi} = 4 \times 10^3 \text{ ksi}$$

$$\varphi_Y = \frac{T_Y L}{JG} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(2.454)(72)}{\pi(0.5)^9(4 \times 10^3)} = 0.4499 \text{ rad}$$

$$\frac{\varphi_Y}{\varphi} = \frac{\rho_Y}{c} \quad \therefore \quad \varphi = \frac{\varphi_Y}{\rho_Y/c} = \frac{0.4499}{0.69318} = 0.64904 \text{ rad} = 37.2° \quad \blacktriangleleft$$

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PROBLEM 3.C1



Shaft AB consists of n homogeneous cylindrical elements, which can be solid or hollow. Its end A is fixed, while its end B is free, and it is subjected to the loading shown. The length of element i is denoted by L_i, its outer diameter by OD_i, its inner diameter by ID_i, its modulus of rigidity by G_i, and the torque applied to its right end by $\mathbf{T}_i$, the magnitude T_i of this torque being assumed to be positive if $\mathbf{T}_i$ is counterclockwise from end B and negative otherwise. (Note that $ID_i = 0$ if the element is solid.) (*a*) Write a computer program that can be used to determine the maximum shearing stress in each element, the angle of twist of each element, and the angle of twist of the entire shaft. (*b*) Use this program to solve Probs. 3.35, 3.36, and 3.38.

SOLUTION

For each cylindrical element, enter

$$L_i, \quad OD_i, \quad ID_i, \quad G_i, \quad T_i$$

and compute

$$J_i = (\pi/32)\left(OD_i^4 - ID_i^4\right)$$

Outline of program.

Update torque $T = T + T_i$

and compute

$$\tau_i = T(OD_i/2)/J_i$$
$$\phi_i = TL_i/G_iJ_i$$

Angle of twist of entire shaft, starting with $\theta = 0$, update through nth element

$$\theta = \theta + \phi_i$$

Program Outputs

Problem 3.35

Element	Maximum Stress (MPa)	Angle of Twist (degrees)
1.0000	11.9575	1.3841
2.0000	23.0259	1.8323

Angle of twist for entire shaft $= 3.2164°$

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PROBLEM 3.C1 *(Continued)*

Program Outputs ***(Continued)***

Problem 3.36

Element	Maximum Stress (MPa)	Angle of Twist (degrees)
1.0000	56.5884	2.5199
2.0000	36.6264	0.8864

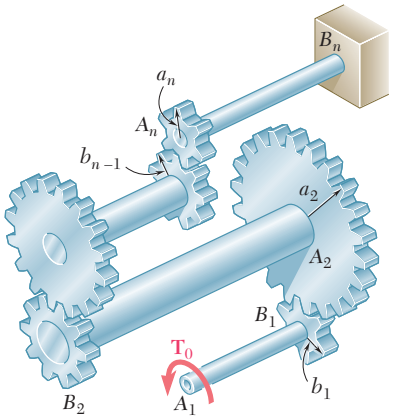
Angle of twist for entire shaft = 3.4063°

Problem 3.38

Element	Maximum Stress (MPa)	Angle of Twist (degrees)
1.0000	87.3278	4.1181
2.0000	56.5884	1.0392
3.0000	70.5179	0.8633

Angle of twist for entire shaft = 6.0206°

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PROBLEM 3.C2

The assembly shown consists of n cylindrical shafts, which can be solid or hollow, connected by gears and supported by brackets (not shown). End A_1 of the first shaft is free and is subjected to a torque $\mathbf{T}_0$, while end B_n of the last shaft is fixed. The length of shaft A_iB_i is L_i, its outer diameter OD_i, its inner diameter ID_i, and its modulus of rigidity G_i. (Note that $ID_i = 0$ if the element is solid.) The radius of gear A_i is a_i, and the radius of gear B_i is b_i. (*a*) Write a computer program that can be used to determine the maximum shearing stress in each shaft, the angle of twist of each shaft, and the angle through which end A_i rotates. (*b*) Use this program to solve Probs. 3.41 and 3.44.

SOLUTION

Torque in shafts. Enter

$$T_i = T_0$$
$$T_{i+1} = T_i(A_{i+1}/B_i)$$

For each shaft, enter

$$L_i \quad OD_i \quad ID_i \quad G_i$$

Compute:

$$J_i = (\pi/32)\left(OD_i^4 - ID_i^4\right)$$
$$\tau_i = T_i(OD_i/2)J_i$$
$$\phi_i = T_iL_iG_iJ_i$$

Angle of rotation at end A_1:

Compute rotation at the "A" end of each shaft.

Start with angle $= \phi_n$ and update from n to 1, and add ϕ_i.

$$\text{Angle} = \text{Angle}(A_i)/B_{i-1} + \phi_{i-1}$$

Program Output

Problem 3.41

Shaft No.	Max Stress (ksi)	Angle of Twist (degrees)
1	9.29	1.493
2	12.16	1.707

Angle through which A_1 rotates $= 3.769°$

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PROBLEM 3.C2 *(Continued)*

Program Output ***(Continued)***

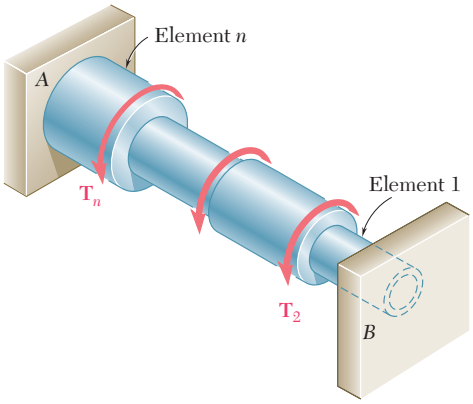
Problem 3.44

Shaft No.	Max Stress (ksi)	Angle of Twist (degrees)
1	104.31	40.979
2	52.15	20.490
3	26.08	10.245

Angle through which A_1 rotates $= 53.785°$

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PROBLEM 3.C3



Shaft AB consists of n homogeneous cylindrical elements, which can be solid or hollow. Both of its ends are fixed, and it is subjected to the loading shown. The length of element i is denoted by L_i, its outer diameter by OD_i, its inner diameter by ID_i, its modulus of rigidity by G_i, and the torque applied to its right end by $\mathbf{T}_i$, the magnitude T_i of this torque being assumed to be positive if $\mathbf{T}_i$ is observed as counterclockwise from end B and negative otherwise. Note that $ID_i = 0$ if the element is solid and also that $T_1 = 0$. Write a computer program that can be used to determine the reactions at A and B, the maximum shearing stress in each element, and the angle of twist of each element. Use this program (*a*) to solve Prob. 3.55 and (*b*) to determine the maximum shearing stress in the shaft of Sample Problem 3.7.

SOLUTION

We consider the reaction at B as redundant and release the shaft at B.

Compute θ_B with $T_B = 0$:

For each element, enter

$$L_i,\quad OD_i,\quad ID_i,\quad G_i,\quad T_i \qquad (\text{Note: } T_1 = T_B = 0)$$

Compute

$$J_i = (\pi/32)(OD_i^4 - ID_i^4)$$

Update torque

$$T = T + T_i$$

And compute for each element

$$\tau_i = T(OD_i/2)J_i$$
$$\phi_i = TL_i/G_iJ_i$$

Compute θ_B : Starting with $\theta = 0$ and updating through n elements,

$$\theta_i = \theta_i + \phi_i: \quad \theta_B = \theta_n$$

Compute θ_B due to unit torque at B.

$$\text{Unit}\quad \tau_i = OD_i/2J_i$$
$$\text{Unit}\quad \phi_i = L_i/G_iJ_i$$

For n elements,

$$\text{Unit}\quad \theta_B(i) = \text{Unit}\ \theta_B(i) + \text{Unit}\ \phi_i$$

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PROBLEM 3.C3 *(Continued)*

Superposition:

For total angle at B to be zero, $\theta_B + T_B(\text{Unit } \theta_B(n)) = 0$

$$T_B = -\theta_B/(\text{Unit } \theta_B(n))$$

Then $T_A = \Sigma T(i) + T_B$

For each element:

Max stress: Total $\tau_i = \tau_i + T_B \ (\text{Unit } \tau_i)$

Angle of twist: Total $\phi_i = \phi_i + T_B \ (\text{Unit } \phi_i)$

Program Outputs Problem 3.55

$T_A = -0.295 \text{ kN} \cdot \text{m}$

$T_B = -1.105 \text{ kN} \cdot \text{m}$

Element	$\tau_{\max}$ (MPa)	Angle of Twist (degrees)
1	–45.024	–0.267
2	27.375	–0.267

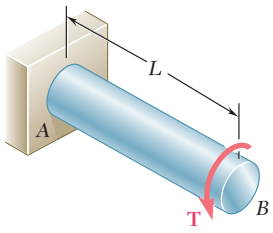
Problem 3.05

$T_A = -51.733 \text{ lb} \cdot \text{ft}$

$T_B = -38.267 \text{ lb} \cdot \text{ft}$

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PROBLEM 3.C4



The homogeneous, solid cylindrical shaft *AB* has a length *L*, a diameter *d*, a modulus of rigidity *G*, and a yield strength τ_Y. It is subjected to a torque **T** that is gradually increased from zero until the angle of twist of the shaft has reached a maximum value ϕ_m and then decreased back to zero. (*a*) Write a computer program that, for each of 16 values of ϕ_m equally spaced over a range extending from 0 to a value 3 times as large as the angle of twist at the onset of yield, can be used to determine the maximum value T_m of the torque, the radius of the elastic core, the maximum shearing stress, the permanent twist, and the residual shearing stress both at the surface of the shaft and at the interface of the elastic core and the plastic region. (*b*) Use this program to obtain approximate answers to Probs. 3.114, 3.115, 3.116.

SOLUTION

At onset of yield:

$$T_Y = \tau_Y \frac{J}{c} = \frac{\pi}{2}\tau_Y c^3$$

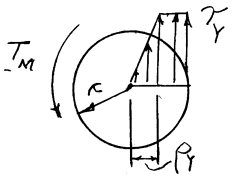
$$\phi_Y = \frac{T_Y L}{GJ} = \left(\frac{T_Y J}{c}\right)\frac{L}{GJ} = \frac{\tau_Y L}{cG}$$



Loading: $T_m > T_Y$.

$$T_m = \frac{4}{3}T_m\left[1 - \frac{1}{4}\left(\frac{\phi_Y}{\phi_m}\right)^3\right] \qquad \text{Eq. (1)}$$

$$\rho_Y = c\frac{\phi_Y}{\phi_m} \qquad \text{Eq. (2)}$$

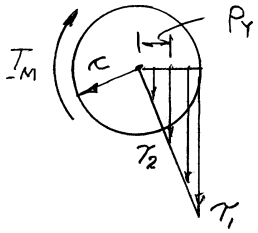


Unloading (elastic):

$$\phi_u = \frac{T_m L}{GJ} \qquad \phi_u = \text{Angle of twist for unloading}$$

$$\tau_1 = T_m\frac{c}{J} \qquad \tau_1 = \tau \quad \text{at} \quad \rho = c$$

$$\tau_2 = \tau_1\frac{\rho_Y}{c} \qquad \tau_2 = \tau \quad \text{at} \quad \rho = \rho_Y$$

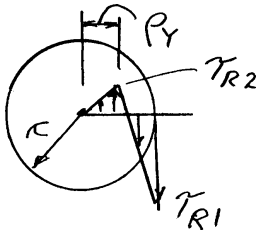


Superpose loading and unloading for $\phi = 0$ to $\phi = 3\phi_Y$ using $0.2\phi_Y$ increments.

When $\phi < \phi_Y$: $T_m = T_Y\frac{\phi}{\phi_Y}$ $\rho_Y = \frac{1}{2}d$ $\phi_m = \phi_Y\frac{\phi}{\phi_Y}$

When $\phi > \phi_Y$: T_m, use Eq. (1). ρ_Y, use Eq. (2).

Residual: $\phi_\rho = \phi_m - \phi_u$ $\tau_{R1} = \tau_1 - \tau_Y$ $\tau_{R2} = \tau_2 - \tau_Y$



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PROBLEM 3.C4 *(Continued)*

Interpolate between values at the values of T_{max} or ϕ_{max} indicated,

Problems 3.114 and 3.115

PHIM deg	TM kip · in.	RY in.	TAUM ksi	PHIP deg	TAUR1 ksi	TAUR2 ksi	
0.000	0.000	1.200	0.000	0.000	0.000	0.000	
7.878	11.943	1.200	4.400	0.000	0.000	0.000	
15.756	23.886	1.200	8.800	0.000	0.000	0.000	
23.635	35.829	1.200	13.200	0.000	0.000	0.000	
31.513	47.772	1.200	17.600	0.000	0.000	0.000	
39.391	59.715	1.200	22.000	0.000	0.000	0.000	
47.269	68.101	1.000	22.000	2.346	1.092	–3.090	
55.147	72.366	0.857	22.000	7.411	2.957	–4.661	
63.025	74.761	0.750	22.000	13.710	4.786	–5.543	
70.904	76.207	0.667	22.000	20.634	6.402	–6.076	$\leftarrow T_{max} = 75$ kip · in.
78.782	77.132	0.600	22.000	27.902	7.792	–6.417	
86.660	77.751	0.545	22.000	35.372	8.980	–6.645	
94.538	78.181	0.500	22.000	42.967	9.999	–6.803	
102.416	78.488	0.462	22.000	50.642	10.878	–6.916	
110.294	78.714	0.429	22.000	58.371	11.643	–6.999	
118.173	78.883	0.400	22.000	66.138	12.313	–7.062	

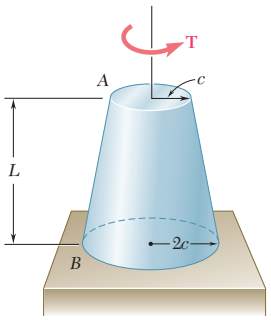
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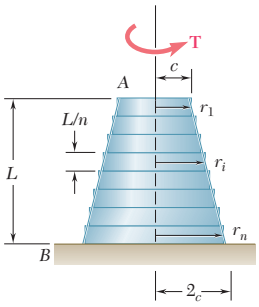
PROBLEM 3.C4 *(Continued)*

Problem 3.116

PHIM deg	TM kN · m	RY mm	TAUM MPa	PHIP deg	TAUR1 MPa	TAUR2 MPa	
0.000	0.000	16.000	0.000	0.000	0.000	0.000	
0.807	0.187	16.000	29.000	0.000	0.000	0.000	
1.614	0.373	16.000	58.000	0.000	0.000	0.000	
2.421	0.560	16.000	87.000	0.000	0.000	0.000	
3.228	0.746	16.000	116.000	0.000	0.000	0.000	
4.036	0.933	16.000	145.000	0.000	0.000	0.000	
4.843	1.064	13.333	145.000	0.240	7.198	–20.363	
5.650	1.131	11.429	145.000	0.759	19.486	–30.719	
6.457	1.168	10.000	145.000	1.405	31.542	–36.533	$\leftarrow \phi_{\max} = 6°$
7.264	1.191	8.889	145.000	2.114	42.197	–40.046	
8.071	1.205	8.000	145.000	2.859	51.354	–42.292	
8.878	1.215	7.273	145.000	3.624	59.184	–43.794	
9.685	1.221	6.667	145.000	4.402	65.901	–44.837	
10.492	1.226	6.154	145.000	5.188	71.699	–45.583	
11.300	1.230	5.714	145.000	5.980	76.739	–46.132	
12.107	1.232	5.333	145.000	6.776	81.152	–46.543	

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PROBLEM 3.C5

The exact expression is given in Prob. 3.158 for the angle of twist of the solid tapered shaft *AB* when a torque **T** is applied as shown. Derive an approximate expression for the angle of twist by replacing the tapered shaft by *n* cylindrical shafts of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \ldots, n$. Using for *T*, *L*, *G*, and *c* values of your choice, determine the percentage error in the approximate expression when (*a*) $n = 4$, (*b*) $n = 8$, (*c*) $n = 20$, (*d*) $n = 100$.

SOLUTION

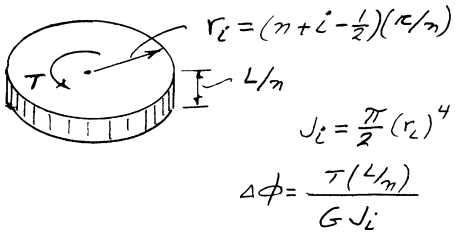
From Problem 3.158, exact expression:

$$\phi = \frac{7TL}{12\pi Gc^4}$$

or

$$\phi = \left(\frac{7}{12\pi}\right)\left(\frac{TL}{Gc^4}\right) = 0.18568\frac{TL}{Gc^4}$$

Consider typical *i*th shaft:



Enter unit values of *T*, *L*, *G*, and *c*.

(Note: Specific values can be entered).

Enter initial value of zero for ϕ.

Enter *n* = number cylindrical shafts.

For $i = 1$ to *n*, update ϕ.

$$\phi = \phi + \Delta\phi$$

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PROBLEM 3.C5 *(Continued)*

Program Output

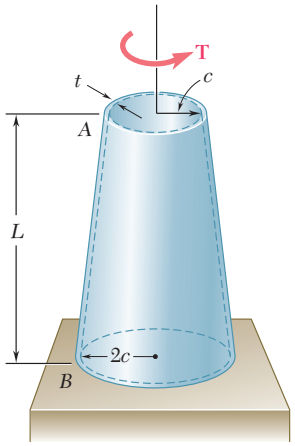
Coefficient of TL/Gc^4:

Exact coefficient from Problem 3.158 is 0.18568.

Number of elemental disks = n.

n	Approximate	Exact	Percent Error
4	0.17959	0.18568	–3.28185
8	0.18410	0.18568	–0.85311
20	0.18542	0.18568	–0.13810
100	0.18567	0.18568	–0.00554

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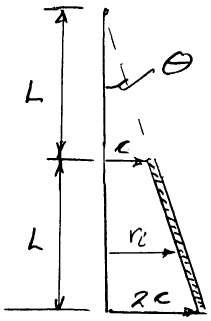


PROBLEM 3.C6

A torque **T** is applied as shown to the long, hollow, tapered shaft *AB* of uniform thickness *t*. Derive an approximate expression for the angle of twist by replacing the tapered shaft by *n* cylindrical rings of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \ldots, n$. Using for *T*, *L*, *G*, *c* and *t* values of your choice, determine the percentage error in the approximate expression when (*a*) $n = 4$, (*b*) $n = 8$, (*c*) $n = 20$, (*d*) $n = 100$.

SOLUTION

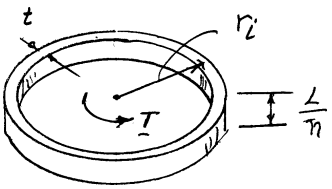
Since the shaft is long, $c << L$, the angle θ is small, and we can use *t* as the thickness of the *n* cylindrical rings.



For $c << L$,

$$\theta = \tan\theta = \frac{zc - c}{L} = \frac{c}{L}$$

$$r_i = \left(n + i - \frac{1}{2}\right)\left(\frac{c}{n}\right)$$



$$J_i \approx (\text{Area})r_i^2 = (2\pi r_i t)r_i^2 = 2\pi t r_i^3$$

$$\Delta\phi = \frac{T(L/n)}{GJ_i}$$

Enter unit values for *T*, *L*, *G*, *t*, and *c*.

(Note: Specific values can be entered is desired).

Enter initial value of zero for ϕ.

Enter n = number of cylindrical rings.

For $i = 1$ to n, update ϕ.

$$\phi = \phi + \Delta\phi$$

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PROBLEM 3.C6 *(Continued)*

Program Output

Coefficient of TL/Gtc^3:

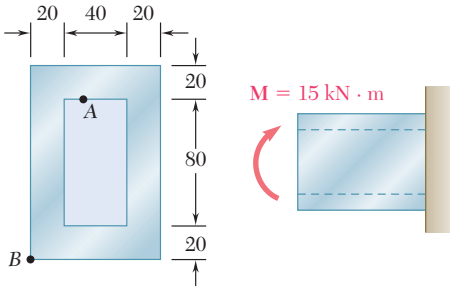
Exact coefficient from Problem 3.153 is 0.05968.

Number of elemental disks = n.

n	Approximate	Exact	Percent Error
4	0.058559	0.059683	–1.883078
8	0.059394	0.059683	–0.483688
20	0.059637	0.059683	–0.078022
100	0.059681	0.059683	–0.003127

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CHAPTER 4



Dimensions in mm

PROBLEM 4.1

Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

For rectangle: $I = \frac{1}{12}bh^3$

Outside rectangle: $I_1 = \frac{1}{12}(80)(120)^3$

$$I_1 = 11.52 \times 10^6 \text{ mm}^4 = 11.52 \times 10^{-6} \text{ m}^4$$

Cutout: $I_2 = \frac{1}{12}(40)(80)^3$

$$I_2 = 1.70667 \times 10^6 \text{ mm}^4 = 1.70667 \times 10^{-6} \text{ m}^4$$

Section: $I = I_1 - I_2 = 9.81333 \times 10^{-6} \text{ m}^4$

(*a*) $y_A = 40 \text{ mm} = 0.040 \text{ m}$

$$\sigma_A = -\frac{My_A}{I} = -\frac{(15 \times 10^3)(0.040)}{9.81333 \times 10^{-6}} = -61.6 \times 10^6 \text{ Pa}$$

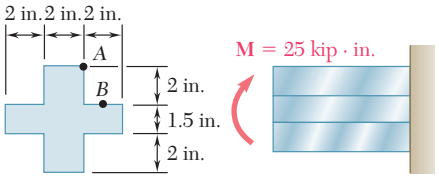
$$\sigma_A = -61.6 \text{ MPa} \blacktriangleleft$$

(*b*) $y_B = -60 \text{ mm} = -0.060 \text{ m}$

$$\sigma_B = -\frac{My_B}{I} = -\frac{(15 \times 10^3)(-0.060)}{9.81333 \times 10^{-6}} = 91.7 \times 10^6 \text{ Pa}$$

$$\sigma_B = 91.7 \text{ MPa} \blacktriangleleft$$

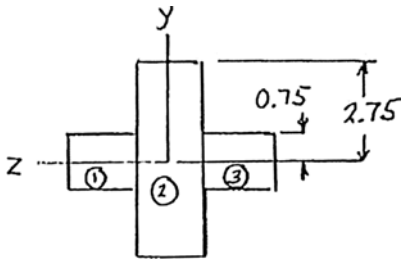
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PROBLEM 4.2

Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION



For rectangle: $I = \frac{1}{12}bh^3$

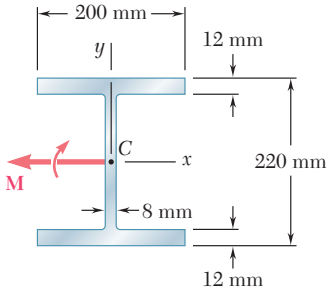
For cross sectional area:

$$I = I_1 + I_2 + I_3 = \frac{1}{12}(2)(1.5)^3 + \frac{1}{12}(2)(5.5)^3 + \frac{1}{12}(2)(1.5)^3 = 28.854 \text{ in}^4$$

(*a*) $y_A = 2.75$ in. $\quad \sigma_A = -\frac{My_A}{I} = -\frac{(25)(2.75)}{28.854} \quad \sigma_A = -2.38$ ksi ◀

(*b*) $y_B = 0.75$ in. $\quad \sigma_B = -\frac{My_B}{I} = -\frac{(25)(0.75)}{28.854} \quad \sigma_B = -0.650$ ksi ◀

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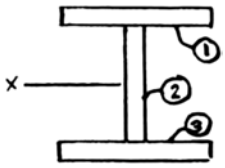


PROBLEM 4.3

Using an allowable stress of 155 MPa, determine the largest bending moment **M** that can be applied to the wide-flange beam shown. Neglect the effect of fillets.

SOLUTION

Moment of inertia about *x*-axis:



$$I_1 = \frac{1}{12}(200)(12)^3 + (200)(12)(104)^2$$

$$= 25.9872 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(8)(196)^3 = 5.0197 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 25.9872 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 56.944 \times 10^6 \text{ mm}^4 = 56.944 \times 10^{-6} \text{ m}^4$$

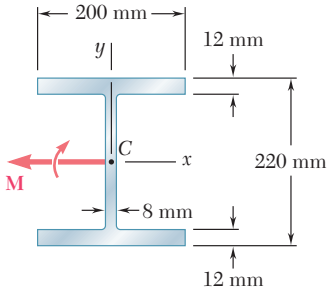
$$\sigma = \frac{Mc}{I} \quad \text{with} \quad c = \frac{1}{2}(220) = 110 \text{ mm} = 0.110 \text{ m}$$

$$M = \frac{I\sigma}{c} \quad \text{with} \quad \sigma = 155 \times 10^6 \text{ Pa}$$

$$M_x = \frac{(56.944 \times 10^{-6})(155 \times 10^6)}{0.110} = 80.2 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_x = 80.2 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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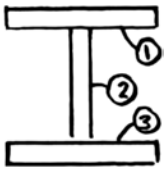
PROBLEM 4.4

Solve Prob. 4.3, assuming that the wide-flange beam is bent about the y axis by a couple of moment M_y.

PROBLEM 4.3. Using an allowable stress of 155 MPa, determine the largest bending moment **M** that can be applied to the wide-flange beam shown. Neglect the effect of fillets.

SOLUTION

Moment of inertia about y axis:



$$I_1 = \frac{1}{12}(12)(200)^3 = 8 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(196)(8)^3 = 8.3627 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 8 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 16.0084 \times 10^6 \text{ mm}^4 = 16.0084 \times 10^{-6} \text{ m}^4$$

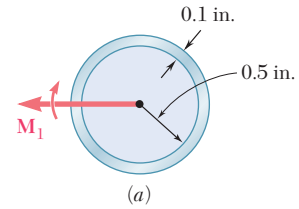
$$\sigma = \frac{Mc}{I} \quad \text{with} \quad c = \frac{1}{2}(200) = 100 \text{ mm} = 0.100 \text{ m}$$

$$M_y = \frac{I\sigma}{c} \quad \text{with} \quad \sigma = 155 \times 10^6 \text{ Pa}$$

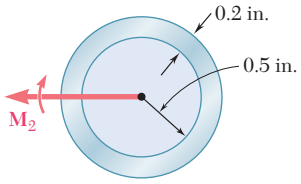
$$M_y = \frac{(16.0084 \times 10^{-6})(155 \times 10^6)}{0.100} = 24.8 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_y = 24.8 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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(a)



(b)

PROBLEM 4.5

Using an allowable stress of 16 ksi, determine the largest couple that can be applied to each pipe.

SOLUTION

(a) $I = \frac{\pi}{4}\left(r_o^4 - r_i^4\right) = \frac{\pi}{4}(0.6^4 - 0.5^4) = 52.7 \times 10^{-3}\ \text{in}^4$

$c = 0.6$ in.

$\sigma = \frac{Mc}{I}: \qquad M = \frac{\sigma I}{c} = \frac{(16)(52.7 \times 10^{-3})}{0.6}$

$M = 1.405\ \text{kip} \cdot \text{in.}$ ◀

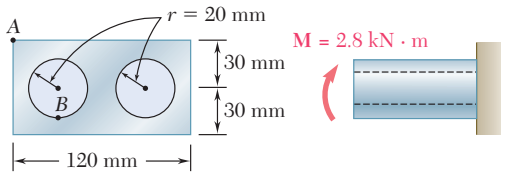
(b) $I = \frac{\pi}{4}(0.7^4 - 0.5^4) = 139.49 \times 10^{-3}\ \text{in}^4$

$c = 0.7$ in.

$\sigma = \frac{Mc}{I}: \qquad M = \frac{\sigma I}{c} = \frac{(16)(139.49 \times 10^{-3})}{0.7}$

$M = 3.19\ \text{kip} \cdot \text{in.}$ ◀

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PROBLEM 4.6

Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$I = \frac{1}{12}(0.120\text{ m})(0.06\text{ m})^3 - 2\left[\frac{1}{12}\cdot\frac{\pi}{4}(0.02\text{ m})^4\right]$$

$$= 2.1391\times10^{-6}\text{ mm}^4$$

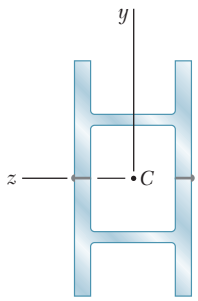
(*a*) $$\sigma_A = -\frac{My_A}{I} = -\frac{(2.8\times10^3\text{ N}\cdot\text{m})(0.03\text{ m})}{2.1391\times10^{-6}\text{ mm}^4}$$

$$\sigma_A = -39.3\text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma_B = \frac{My_B}{I} = \frac{(2.8\times10^3\text{ N}\cdot\text{m})(0.02\text{ m})}{2.1391\times10^{-6}\text{ m}^4}$$

$$\sigma_B = 26.2\text{ MPa} \blacktriangleleft$$

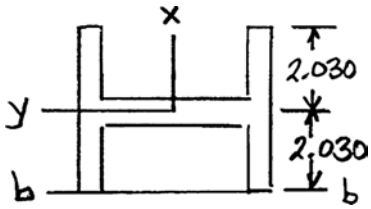
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PROBLEM 4.7

Two W4×13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the *z* axis.

SOLUTION



Properties of W4 × 13 rolled section.

(See Appendix C.)

$\text{Area} = 3.83 \text{ in}^2$

$\text{Width} = 4.060 \text{ in.}$

$I_y = 3.86 \text{ in}^4$

For one rolled section, moment of inertia about axis *b*-*b* is

$$I_b = I_y + Ad^2 = 3.86 + (3.83)(2.030)^2 = 19.643 \text{ in}^4$$

For both sections, $I_z = 2I_b = 39.286 \text{ in}^4$

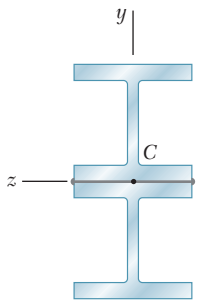
$c = \text{width} = 4.060 \text{ in.}$

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

$$M_{\text{all}} = \frac{\sigma_{\text{all}} I}{c} = \frac{(19.333)(39.286)}{4.060}$$

$M_{\text{all}} = 187.1 \text{ kip} \cdot \text{in.}$ ◀

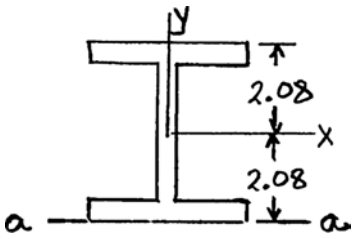
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PROBLEM 4.8

Two $W4\times13$ rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.

SOLUTION



Properties of W4 × 13 rolled section.

(See Appendix C.)

$\text{Area} = 3.83 \text{ in}^2$

$\text{Depth} = 4.16 \text{ in.}$

$I_x = 11.3 \text{ in}^4$

For one rolled section, moment of inertia about axis a-a is

$$I_a = I_x + Ad^2 = 11.3 + (3.83)(2.08)^2 = 27.87 \text{ in}^4$$

For both sections, $I_z = 2I_a = 55.74 \text{ in}^4$

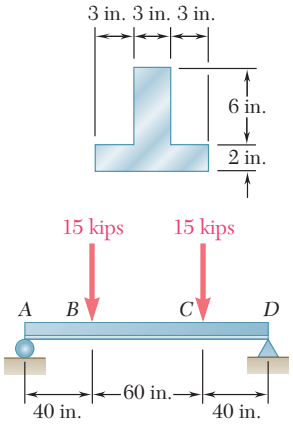
$c = \text{depth} = 4.16 \text{ in.}$

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

$$M_{\text{all}} = \frac{\sigma_{\text{all}} I}{c} = \frac{(19.333)(55.74)}{4.16}$$

$M_{\text{all}} = 259 \text{ kip} \cdot \text{in.}$ ◀

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PROBLEM 4.9

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

SOLUTION

	A	$\bar{y}_0$	$A\,\bar{y}_0$
①	18	5	90
②	18	1	18
Σ	36		108

$$\bar{Y}_0 = \frac{108}{36} = 3 \text{ in.}$$

Neutral axis lies 3 in. above the base.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(6)^3 + (18)(2)^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(9)(2)^3 + (18)(2)^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 + 78 = 204 \text{ in}^4$$

$$y_{\text{top}} = 5 \text{ in.} \quad y_{\text{bot}} = -3 \text{ in.}$$

$$M - Pa = 0$$

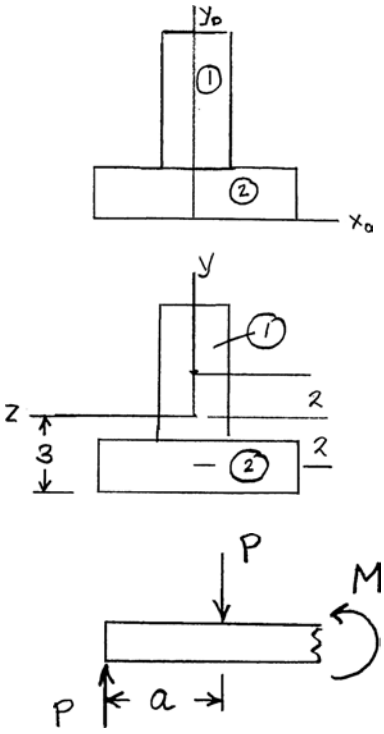
$$M = Pa = (15)(40) = 600 \text{ kip} \cdot \text{in.}$$

$$\sigma_{\text{top}} = -\frac{M y_{\text{top}}}{I} = -\frac{(600)(5)}{204}$$

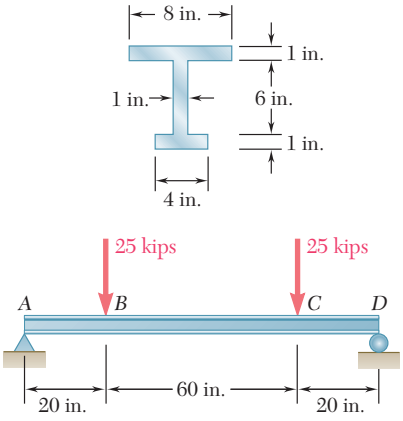
$\sigma_{\text{top}} = -14.71$ ksi (compression) ◀

$$\sigma_{\text{bot}} = -\frac{M y_{\text{bot}}}{I} = -\frac{(600)(-3)}{204}$$

$\sigma_{\text{bot}} = 8.82$ ksi (tension) ◀



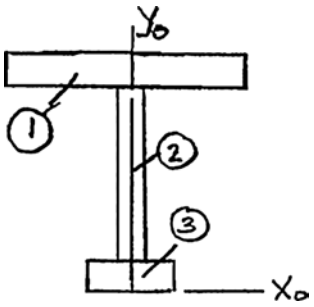
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PROBLEM 4.10

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

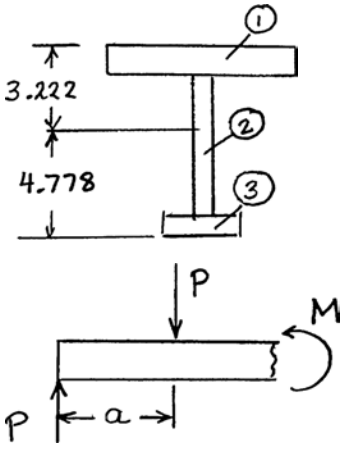
SOLUTION



	A	$\bar{y}_0$	$A\bar{y}_0$
①	8	7.5	60
②	6	4	24
③	4	0.5	2
Σ	18		86

$$\bar{Y}_o = \frac{86}{18} = 4.778 \text{ in.}$$

Neutral axis lies 4.778 in. above the base.



$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(8)(1)^3 + (8)(2.772)^2 = 59.94 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(1)(6)^3 + (6)(0.778)^2 = 21.63 \text{ in}^4$$

$$I_3 = \frac{1}{12}b_3h_3^3 + A_3d_3^2 = \frac{1}{12}(4)(1)^3 + (4)(4.278)^2 = 73.54 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 59.94 + 21.63 + 73.57 = 155.16 \text{ in}^4$$

$$y_{\text{top}} = 3.222 \text{ in.} \quad y_{\text{bot}} = -4.778 \text{ in.}$$

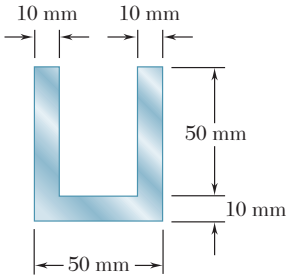
$$M - Pa = 0$$

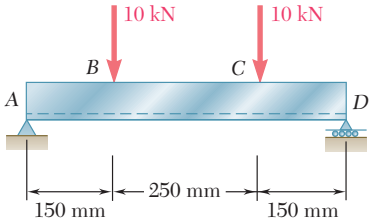
$$M = Pa = (25)(20) = 500 \text{ kip} \cdot \text{in.}$$

$$\sigma_{\text{top}} = -\frac{My_{\text{top}}}{I} = -\frac{(500)(3.222)}{155.16} \qquad \sigma_{\text{top}} = -10.38 \text{ ksi (compression)} \blacktriangleleft$$

$$\sigma_{\text{bot}} = -\frac{My_{\text{bot}}}{I} = -\frac{(500)(-4.778)}{155.16} \qquad \sigma_{\text{bot}} = 15.40 \text{ ksi (tension)} \blacktriangleleft$$

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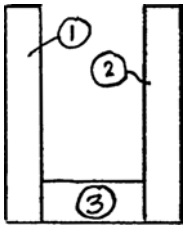




PROBLEM 4.11

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

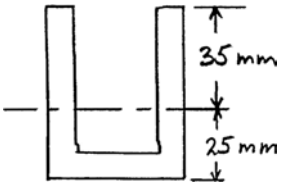
SOLUTION



	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	30	18×10^3
②	600	30	18×10^3
③	300	5	1.5×10^3
	1500		37.5×10^3

$$\bar{Y}_0 = \frac{37.5\times10^3}{1500} = 25\,\text{mm}$$

Neutral axis lies 25 mm above the base.



$$I_1 = \frac{1}{12}(10)(60)^3 + (600)(5)^2 = 195\times10^3\,\text{mm}^4 \quad I_2 = I_1 = 195\ \text{mm}^4$$

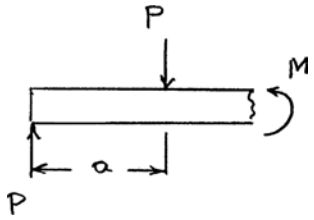
$$I_3 = \frac{1}{12}(30)(10)^3 + (300)(20)^2 = 122.5\times10^3\,\text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 512.5\times10^3\,\text{mm}^4 = 512.5\times10^{-9}\,\text{m}^4$$

$$y_{\text{top}} = 35\ \text{mm} = 0.035\ \text{m} \quad y_{\text{bot}} = -25\ \text{mm} = -0.025\ \text{m}$$

$$a = 150\ \text{mm} = 0.150\ \text{m} \quad P = 10\times10^3\,\text{N}$$

$$M = Pa = (10\times10^3)(0.150) = 1.5\times10^3\,\text{N}\cdot\text{m}$$



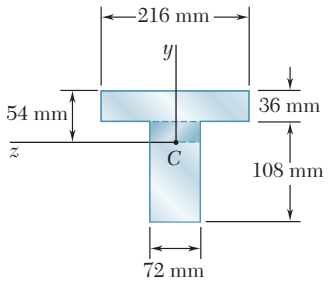
$$\sigma_{\text{top}} = -\frac{My_{\text{top}}}{I} = -\frac{(1.5\times10^3)(0.035)}{512.5\times10^{-9}} = -102.4\times10^6\,\text{Pa}$$

$\sigma_{\text{top}} = -102.4$ MPa (compression) ◀

$$\sigma_{\text{bot}} = -\frac{My_{\text{bot}}}{I} = -\frac{(1.5\times10^3)(-0.025)}{512.5\times10^{-9}} = 73.2\times10^6\,\text{Pa}$$

$\sigma_{\text{bot}} = 73.2$ MPa (tension) ◀

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PROBLEM 4.12

Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kN · m, determine the total force acting on the shaded portion of the web.

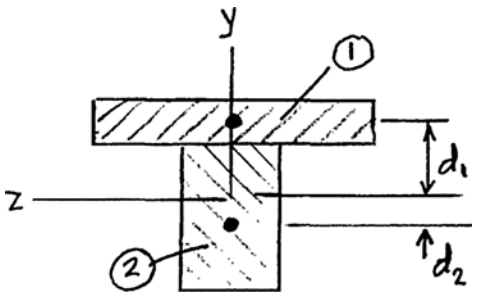
SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula:

$$\sigma_x = -\frac{My}{I}$$

where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded portion is calculated from this stress distribution. Over an area element dA, the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$



The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$d_1 = 54 - 18 = 36 \text{ mm}$$
$$d_2 = 54 + 36 - 54 = 36 \text{ mm}$$

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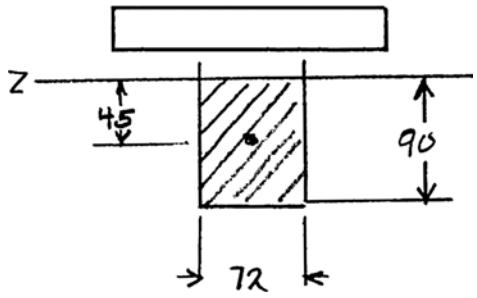
PROBLEM 4.12 ***(Continued)***

Moment of inertia of entire cross section:

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(216)(36)^3 + (216)(36)(36)^2 = 10.9175 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(72)(108)^3 + (72)(108)(36)^2 = 17.6360 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 28.5535 \times 10^6 \text{ mm}^4 = 28.5535 \times 10^{-6} \text{ m}^4$$



For the shaded area,

$$A^* = (72)(90) = 6480 \text{ mm}^2$$

$$\bar{y}^* = 45 \text{ mm}$$

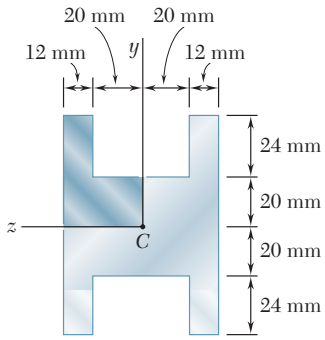
$$A^* \bar{y}^* = 291.6 \times 10^3 \text{ mm}^3 = 291.6 \times 10^{-6} \text{ m}$$

$$F = \left| \frac{MA^* \bar{y}^*}{I} \right| = \frac{(6 \times 10^3)(291.6 \times 10^{-6})}{28.5535 \times 10^{-6}}$$

$$= 61.3 \times 10^3 \text{ N}$$

$$F = 61.3 \text{ kN} \blacktriangleleft$$

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PROBLEM 4.13

Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is $4\ \text{kN}\cdot\text{m}$, determine the total force acting on the shaded portion of the beam.

SOLUTION

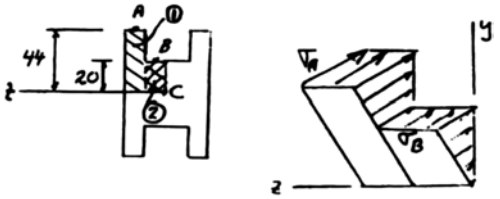
Dimensions in mm:

$$I_z = \frac{1}{12}(12+12)(88)^3 + \frac{1}{12}(40)(40)^3$$
$$= 1.3629\times10^6 + 0.213\times10^6$$
$$= 1.5763\times10^6\,\text{mm}^4 = 1.5763\times10^{-6}\,\text{m}^4$$

For use in Prob. 4.14,

$$I_y = \frac{1}{12}(88)(64)^3 - \frac{1}{12}(24+24)(40)^3$$
$$= 1.9224\times10^6 - 0.256\times10^6$$
$$= 1.6664\times10^6\,\text{mm}^4 = 1.6664\times10^{-6}\,\text{m}^4$$

Bending about horizontal axis. $M_z = 4\ \text{kN}\cdot\text{m}$



$$\sigma_A = \frac{M_z c}{I_z} = \frac{(4\ \text{kN}\cdot\text{m})(0.044\ \text{m})}{1.5763\times10^{-6}\,\text{m}^4} = 111.654\ \text{MPa}$$

$$\sigma_B = \frac{M_z c}{I_z} = \frac{(4\ \text{kN}\cdot\text{m})(0.020\ \text{m})}{1.5763\times10^{-6}\ \text{m}^4} = 50.752\ \text{MPa}$$

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PROBLEM 4.13 *(Continued)*

Portion (1): $A = (44)(12) = 528 \text{ mm}^2 = 528 \times 10^{-6} \text{ m}^2$

$$\sigma_{\text{avg}} = \frac{1}{2}\sigma_A = \frac{1}{2}(111.654) = 55.83 \text{ MPa}$$

$$\text{Force}_1 = \sigma_{\text{avg}} A = (55.83 \text{ MPa})(528 \times 10^{-6} \text{ m}^2) = 29.477 \text{ kN}$$

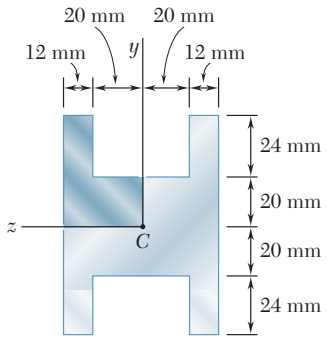
Portion (2): $A = (20)(20) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$

$$\sigma_{\text{avg}} = \frac{1}{2}\sigma_B = \frac{1}{2}(50.752) = 25.376 \text{ MPa}$$

$$\text{Force}_2 = \sigma_{\text{avg}} A = (25.376 \text{ MPa})(400 \times 10^{-6} \text{ m}^2) = 10.150 \text{ kN}$$

Total force on shaded area $= 29.477 + 10.150 = 39.6 \text{ kN}$ ◀

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PROBLEM 4.14

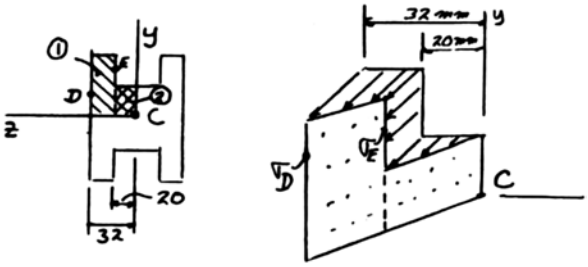
Solve Prob. 4.13, assuming that the beam is bent about a vertical axis by a couple of moment $4\ \text{kN}\cdot\text{m}$.

PROBLEM 4.13. Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is $4\ \text{kN}\cdot\text{m}$, determine the total force acting on the shaded portion of the beam.

SOLUTION

Bending about vertical axis. $M_y = 4\ \text{kN}\cdot\text{m}$

See Prob. 4.13 for sketch and $I_y = 1.6664\times10^{-6}\ \text{m}^4$



$$\sigma_D = \frac{Mc}{I_y} = \frac{(4\ \text{kN}\cdot\text{m})(0.032\ \text{m})}{1.6664\times10^{-6}\,\text{m}^4} = 76.81\ \text{MPa}$$

$$\sigma_E = \frac{Mc}{I_y} = \frac{(4\ \text{kN}\cdot\text{m})(0.020\ \text{m})}{1.6664\times10^{-6}\ \text{m}^4} = 48.01\ \text{MPa}$$

Portion (1): $A = (44)(12) = 528\ \text{mm}^2 = 528\times10^{-6}\ \text{m}^2$

$$\sigma_{\text{avg}} = \frac{1}{2}(\sigma_D + \sigma_E) = \frac{1}{2}(76.81 + 48.01) = 62.41\ \text{MPa}$$

$$\text{Force}_1 = \sigma_{\text{avg}} A = (62.41\ \text{MPa})(528\times10^{-6}\ \text{m}^2) = 32.952\ \text{kN}$$

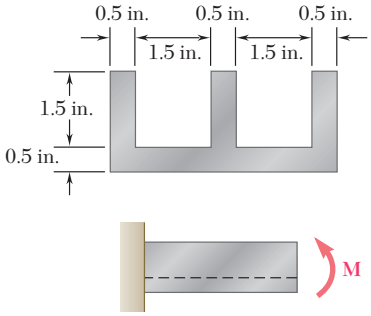
Portion (2): $A = (20)(20) = 400\ \text{mm}^2 = 400\times10^{-6}\ \text{m}^2$

$$\sigma_{\text{avg}} = \frac{1}{2}\sigma_E = \frac{1}{2}(48.01) = 24.005\ \text{MPa}$$

$$\text{Force}_2 = \sigma_{\text{avg}} A = (24.005\ \text{MPa})(400\times10^{-6}\ \text{m}^2) = 9.602\ \text{kN}$$

Total force on shaded area $= 32.952 + 9.602 = 42.6\ \text{kN}$ ◀

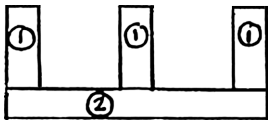
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PROBLEM 4.15

Knowing that for the extruded beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.

SOLUTION



	A	$\bar{y}_0$	$A\bar{y}_0$
①	2.25	1.25	2.8125
②	2.25	0.25	0.5625
	4.50		3.375

$$\bar{Y} = \frac{3.375}{4.50} = 0.75 \text{ in.}$$

The neutral axis lies 0.75 in. above bottom.

$$y_{top} = 2.0 - 0.75 = 1.25 \text{ in.}, \qquad y_{bot} = -0.75 \text{ in.}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(1.5)(1.5)^3 + (2.25)(0.5)^2 = 0.984375 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^2 + A_2 d_2^2 = \frac{1}{12}(4.5)(0.5)^3 + (2.25)(0.5)^2 = 0.609375 \text{ in}^4$$

$$I = I_1 + I_2 = 1.59375 \text{ in}^4$$

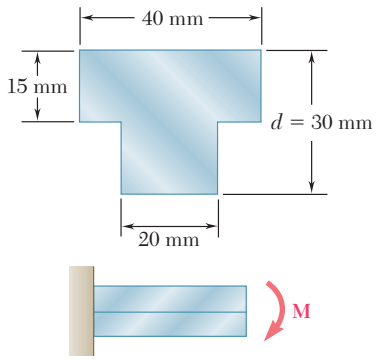
$$|\sigma| = \left|\frac{My}{I}\right| \qquad |M| = \left|\frac{\sigma I}{y}\right|$$

Top: (compression) $$M = \frac{(16)(1.59375)}{1.25} = 20.4 \text{ kip} \cdot \text{in.}$$

Bottom: (tension) $$M = \frac{(12)(1.59375)}{0.75} = 25.5 \text{ kip} \cdot \text{in.}$$

Choose the smaller as M_{all}. $M_{all} = 20.4 \text{ kip} \cdot \text{in.}$ ◀

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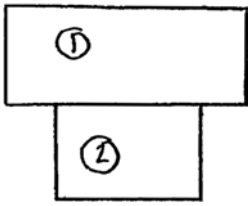


PROBLEM 4.16

The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	22.5	13.5×10^3
②	300	7.5	2.25×10^3
Σ	900		15.75×10^3



$$\bar{Y}_0 = \frac{15.5\times10^3}{900} = 17.5 \text{ mm}$$ The neutral axis lies 17.5 mm above the bottom.

$$y_{top} = 30 - 17.5 = 12.5 \text{ mm} = 0.0125 \text{ m}$$

$$y_{bot} = -17.5 \text{ mm} = -0.0175 \text{ m}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(5)^2 = 26.25\times10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(20)(15)^3 + (300)(10)^2 = 35.625\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 61.875\times10^3 \text{ mm}^4 = 61.875\times10^{-9} \text{ m}^4$$

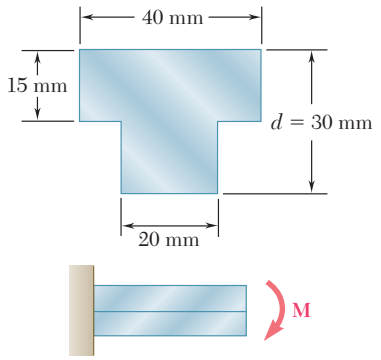
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $$M = \frac{(24\times10^6)(61.875\times10^{-9})}{0.0125} = 118.8 \text{ N}\cdot\text{m}$$

Bottom: (compression) $$M = \frac{(30\times10^6)(61.875\times10^{-9})}{0.0175} = 106.1 \text{ N}\cdot\text{m}$$

Choose smaller value. $M = 106.1 \text{ N}\cdot\text{m}$ ◀

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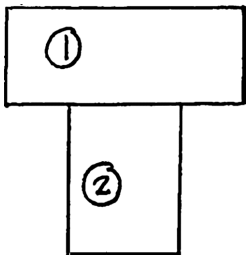
PROBLEM 4.17

Solve Prob. 4.16, assuming that $d = 40$ mm.

PROBLEM 4.16 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	32.5	19.5×10^3
②	500	12.5	6.25×10^3
Σ	1100		25.75×10^3



$$\bar{Y}_0 = \frac{25.75\times10^3}{1100} = 23.41 \text{ mm}$$ The neutral axis lies 23.41 mm above the bottom.

$$y_{top} = 40 - 23.41 = 16.59 \text{ mm} = 0.01659 \text{ m}$$

$$y_{bot} = -23.41 \text{ mm} = -0.02341 \text{ m}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(9.09)^2 = 60.827\times10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^2 + A_2d_2^2 = \frac{1}{12}(20)(25)^3 + (500)(10.91)^2 = 85.556\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 146.383\times10^3 \text{ mm}^4 = 146.383\times10^{-9} \text{ m}^4$$

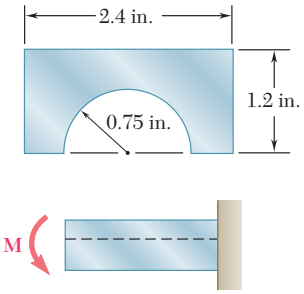
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $$M = \frac{(24\times10^6)(146.383\times10^{-9})}{0.01659} = 212 \text{ N}\cdot\text{m}$$

Bottom: (compression) $$M = \frac{(30\times10^6)(146.383\times10^{-9})}{0.02341} = 187.6 \text{ N}\cdot\text{m}$$

Choose smaller value. $M = 187.6 \text{ N}\cdot\text{m}$ ◀

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PROBLEM 4.18

Knowing that for the beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.

SOLUTION

① = rectangle ② = semi-circular cutout

$$A_1 = (2.4)(1.2) = 2.88 \text{ in}^2$$

$$A_2 = \frac{\pi}{2}(0.75)^2 = 0.8836 \text{ in}^2$$

$$A = 2.88 - 0.8836 = 1.9964 \text{ in}^2$$

$$\bar{y}_1 = 0.6 \text{ in.}$$

$$\bar{y}_2 = \frac{4r}{3\pi} = \frac{(4)(0.75)}{3\pi} = 0.3183 \text{ in.}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{(2.88)(0.6) - (0.8836)(0.3183)}{1.9964} = 0.7247 \text{ in.}$$

Neutral axis lies 0.7247 in. above the bottom.

Moment of inertia about the base:

$$I_b = \frac{1}{3}bh^3 - \frac{\pi}{8}r^4 = \frac{1}{3}(2.4)(1.2)^3 - \frac{\pi}{8}(0.75)^4 = 1.25815 \text{ in}^4$$

Centroidal moment of inertia:

$$\bar{I} = I_b - A\bar{Y}^2 = 1.25815 - (1.9964)(0.7247)^2 = 0.2097 \text{ in}^4$$

$$y_{\text{top}} = 1.2 - 0.7247 = 0.4753 \text{ in.},$$

$$y_{\text{bot}} = -0.7247 \text{ in.}$$

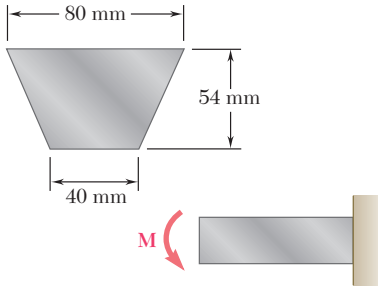
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side)
$$M = \frac{(12)(0.2097)}{0.4753} = 5.29 \text{ kip} \cdot \text{in.}$$

Bottom: (compression)
$$M = \frac{(16)(0.2097)}{0.7247} = 4.63 \text{ kip} \cdot \text{in.}$$

Choose the smaller value. $M = 4.63 \text{ kip} \cdot \text{in.}$ ◀

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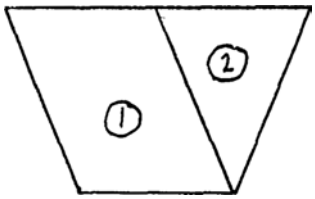


PROBLEM 4.19

Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3	d, mm
①	2160	27	58,320	3
②	1080	36	38,880	3
Σ	3240		97,200	



$$\bar{Y} = \frac{97,200}{3240} = 30 \text{ mm}$$ The neutral axis lies 30 mm above the bottom.

$$y_{top} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m} \qquad y_{bot} = -30 \text{ mm} = -0.030 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(40)(54)^3 + (40)(54)(3)^2 = 544.32 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{36} b_2 h_2^2 + A_2 d_2^2 = \frac{1}{36}(40)(54)^3 + \frac{1}{2}(40)(54)(6)^2 = 213.84 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 758.16 \times 10^3 \text{ mm}^4 = 758.16 \times 10^{-9} \text{ m}^4$$

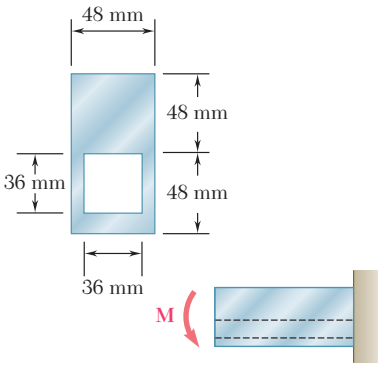
$$|\sigma| = \left|\frac{My}{I}\right| \qquad |M| = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $$M = \frac{(120 \times 10^6)(758.16 \times 10^{-9})}{0.024} = 3.7908 \times 10^3 \text{ N} \cdot \text{m}$$

Bottom: (compression) $$M = \frac{(150 \times 10^6)(758.16 \times 10^{-9})}{0.030} = 3.7908 \times 10^3 \text{ N} \cdot \text{m}$$

Choose the smaller as M_{all}. $M_{all} = 3.7908 \times 10^3 \text{ N} \cdot \text{m}$ $M_{all} = 3.79 \text{ kN} \cdot \text{m}$ ◀

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PROBLEM 4.20

Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
① Solid rectangle	4608	48	221,184
② Square cutout	–1296	30	–38,880
Σ	3312		182,304

$$\bar{Y} = \frac{182{,}304}{3312} = 55.04 \text{ mm}$$ Neutral axis lies 55.04 mm above bottom.

$$y_{top} = 96 - 55.04 = 40.96 \text{ mm} = 0.04096 \text{ m}$$

$$y_{bot} = -55.04 \text{ mm} = -0.05504 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(48)(96)^3 + (48)(96)(7.04)^2 = 3.7673 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(36)(36)^3 + (36)(36)(25.04)^2 = 0.9526 \times 10^6 \text{ mm}^4$$

$$I = I_1 - I_2 = 2.8147 \times 10^6 \text{ mm}^4 = 2.8147 \times 10^{-6} \text{ m}^4$$

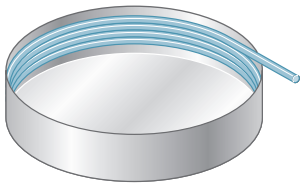
$$|\sigma| = \left|\frac{My}{I}\right| \quad \therefore \quad M = +\left|\frac{\sigma I}{y}\right|$$

Top: (tension side)
$$M = \frac{(120 \times 10^6)(2.8147 \times 10^{-6})}{0.04096} = 8.25 \times 10^3 \text{ N} \cdot \text{m}$$

Bottom: (compression)
$$M = \frac{(150 \times 10^6)(2.8147 \times 10^{6})}{0.05504} = 7.67 \times 10^3 \text{ N} \cdot \text{m}$$

M_{all} is the smaller value.
$$M = 7.67 \times 10^3 \text{ N} \cdot \text{m}$$
7.67 kN · m ◀

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PROBLEM 4.21

Straight rods of 6-mm diameter and 30-m length are stored by coiling the rods inside a drum of 1.25-m inside diameter. Assuming that the yield strength is not exceeded, determine (*a*) the maximum stress in a coiled rod, (*b*) the corresponding bending moment in the rod. Use $E = 200$ GPa.

SOLUTION

Let $D =$ inside diameter of the drum,

$d =$ diameter of rod, $\quad c = \dfrac{1}{2}d,$

$\rho =$ radius of curvature of center line of rods when bent.

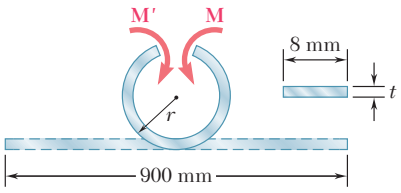
$$\rho = \frac{1}{2}D - \frac{1}{2}d = \frac{1}{2}(1.25) - \frac{1}{2}(6\times10^{-3}) = 0.622 \text{ m}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.003)^4 = 63.617\times10^{-12}\,\text{m}^4$$

(*a*) $$\sigma_{\max} = \frac{Ec}{\rho} = \frac{(200\times10^9)(0.003)}{0.622} = 965\times10^6\,\text{Pa}$$ $\sigma = 965$ MPa ◀

(*b*) $$M = \frac{EI}{\rho} = \frac{(200\times10^9)(63.617\times10^{-12})}{0.622} = 20.5\ \text{N}\cdot\text{m}$$ $M = 20.5\ \text{N}\cdot\text{m}$ ◀

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PROBLEM 4.22

A 900-mm strip of steel is bent into a full circle by two couples applied as shown. Determine (*a*) the maximum thickness t of the strip if the allowable stress of the steel is 420 MPa, (*b*) the corresponding moment M of the couples. Use $E = 200$ GPa.

SOLUTION

When the rod is bent into a full circle, the circumference is 900 mm. Since the circumference is equal to 2π times ρ, the radius of curvature, we get

$$\rho = \frac{900 \text{ mm}}{2\pi} = 143.24 \text{ mm} = 0.14324 \text{ m}$$

Stress: $\sigma = E\varepsilon = \dfrac{Ec}{\rho}$ or $c = \dfrac{\rho\sigma}{E}$

For $\sigma = 420$ MPa and $E = 200$ GPa,

$$c = \frac{(0.14324)(420 \times 10^6)}{200 \times 10^9} = 0.3008 \times 10^{-3} \text{ m}$$

(*a*) Maximum thickness: $t = 2c = 0.6016 \times 10^{-3}$ m

$$t = 0.602 \text{ mm} \blacktriangleleft$$

Moment of inertia for a rectangular section.

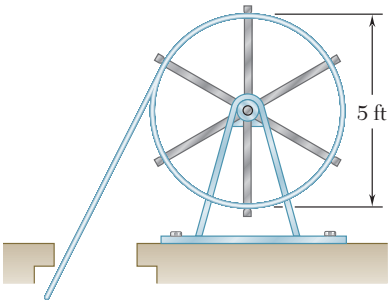
$$I = \frac{bt^3}{12} = \frac{(8 \times 10^{-3})(0.6016 \times 10^{-3})^3}{12} = 145.16 \times 10^{-15} \text{ m}^4$$

(*b*) Bending moment: $M = \dfrac{EI}{\rho}$

$$M = \frac{(200 \times 10^9)(145.16 \times 10^{-15})}{0.14324} = 0.203 \text{ N} \cdot \text{m}$$

$$M = 0.203 \text{ N} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 4.23

Straight rods of 0.30-in. diameter and 200-ft length are sometimes used to clear underground conduits of obstructions or to thread wires through a new conduit. The rods are made of high-strength steel and, for storage and transportation, are wrapped on spools of 5-ft diameter. Assuming that the yield strength is not exceeded, determine (*a*) the maximum stress in a rod, when the rod, which is initially straight, is wrapped on a spool, (*b*) the corresponding bending moment in the rod. Use $E = 29 \times 10^6$ psi .

SOLUTION

Radius of cross section: $r = \frac{1}{2}d = \frac{1}{2}(0.30) = 0.15$ in.

Moment of inertia: $I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(0.15)^4 = 397.61 \times 10^{-6}$ in^4

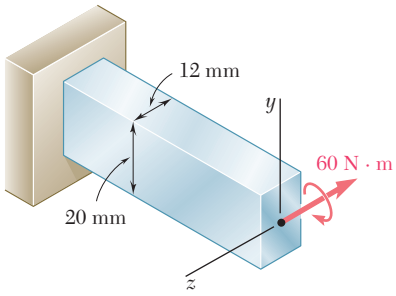
$D = 5$ ft $= 60$ in. $\quad \rho = \frac{1}{2}D = 30$ in.

$c = r = 0.15$ in.

(*a*) $\sigma_{\max} = \frac{Ec}{\rho} = \frac{(29 \times 10^6)(0.15)}{30} = 145.0 \times 10^3$ psi $\quad \sigma_{\max} = 145.0$ ksi ◀

(*b*) $M = \frac{EI}{\rho} = \frac{(29 \times 10^6)(397.61 \times 10^{-6})}{30} \quad M = 384$ lb · in. ◀

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PROBLEM 4.24

A 60-N · m couple is applied to the steel bar shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the bar. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use $E = 200$ GPa.

SOLUTION

(*a*) Bending about *z*-axis.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(20)^3 = 8\times10^3\ \text{mm}^4 = 8\times10^{-9}\ \text{m}^4$$

$$c = \frac{20}{2} = 10\ \text{mm} = 0.010\ \text{m}$$

$$\sigma = \frac{Mc}{I} = \frac{(60)(0.010)}{8\times10^{-9}} = 75.0\times10^6\ \text{Pa} \qquad \sigma = 75.0\ \text{MPa} \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200\times10^9)(8\times10^{-9})} = 37.5\times10^{-3}\ \text{m}^{-1} \qquad \rho = 26.7\ \text{m} \blacktriangleleft$$

(*b*) Bending about *y*-axis.

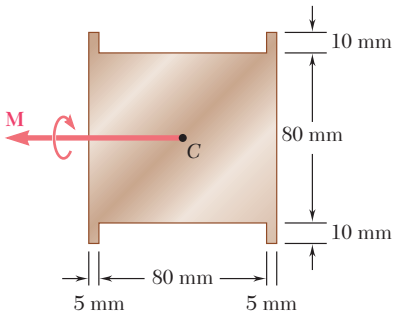
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(20)(12)^3 = 2.88\times10^3\ \text{mm}^4 = 2.88\times10^{-9}\ \text{m}^4$$

$$c = \frac{12}{2} = 6\ \text{mm} = 0.006\ \text{m}$$

$$\sigma = \frac{Mc}{I} = \frac{(60)(0.006)}{2.88\times10^{-9}} = 125.0\times10^6\ \text{Pa} \qquad \sigma = 125.0\ \text{MPa} \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200\times10^9)(2.88\times10^{-9})} = 104.17\times10^{-3}\ \text{m}^{-1} \qquad \rho = 9.60\ \text{m} \blacktriangleleft$$

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PROBLEM 4.25

(*a*) Using an allowable stress of 120 MPa, determine the largest couple **M** that can be applied to a beam of the cross section shown. (*b*) Solve part *a*, assuming that the cross section of the beam is an 80-mm square.

SOLUTION

(*a*) $I = I_1 + 4I_2$, where I_1 is the moment of inertia of an 80-mm square and I_2 is the moment of inertia of one of the four protruding ears.

$$I_1 = \frac{1}{12}bh^3 = \frac{1}{12}(80)(80)^3 = 3.4133\times10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(5)(10)^3 + (5)(10)(45)^2 = 101.667\times10^3 \text{ mm}^4$$

$$I = I_1 + 4I_2 = 3.82\times10^6 \text{ mm}^4 = 3.82\times10^{-6} \text{ mm}^4, \qquad c = 50 \text{ mm} = 0.050 \text{ m}$$

$$\sigma = \frac{Mc}{I} \qquad \therefore\ M = \frac{\sigma I}{c} = \frac{(120\times10^6)(3.82\times10^{-6})}{0.050}$$

$$= 9.168\times10^3 \text{ N}\cdot\text{m}$$

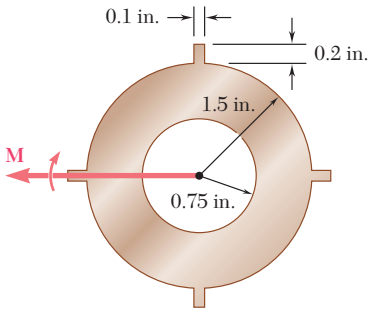
$$= 9.17 \text{ kN}\cdot\text{m} \blacktriangleleft$$

(*b*) Without the ears:

$$I = I_1 = 3.4133\times10^{-6} \text{ m}^2, \qquad c = 40 \text{ mm} = 0.040 \text{ m}$$

$$M = \frac{\sigma I}{c} = \frac{(120\times10^6)(3.4133\times10^{-6})}{0.040} = 10.24\times10^3 \text{ N}\cdot\text{m} \qquad = 10.24 \text{ kN}\cdot\text{m} \blacktriangleleft$$

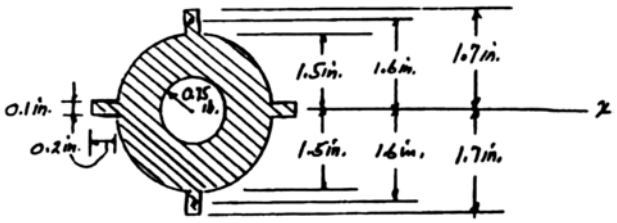
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PROBLEM 4.26

A thick-walled pipe is bent about a horizontal axis by a couple **M**. The pipe may be designed with or without four fins. (*a*) Using an allowable stress of 20 ksi, determine the largest couple that may be applied if the pipe is designed with four fins as shown. (*b*) Solve part *a*, assuming that the pipe is designed with no fins.

SOLUTION



I_x of hollow pipe: $$I_x = \frac{\pi}{4}\left[(1.5\text{ in.})^4 - (0.75\text{ in.})^4\right] = 3.7276\text{ in}^4$$

I_x of fins: $$I_x = 2\left[\frac{1}{12}(0.1)(0.2)^3 + (0.1\times 0.2)(1.6)^2\right] + 2\left[\frac{1}{12}(0.2)(0.1)^3\right]$$

$$= 0.1026\text{ in}^4$$

(*a*) Pipe as designed, with fins:

$$I_x = 3.8302\text{ in}^4, \qquad c = 1.7\text{ in.}$$

$$\sigma_{\text{all}} = 20\text{ ksi}, \qquad M = \sigma_{\text{all}}\frac{I_x}{c} = (20\text{ ksi})\frac{3.8302\text{ in}^4}{1.7\text{ in.}}$$

$$M = 45.1\text{ kip}\cdot\text{in.} \blacktriangleleft$$

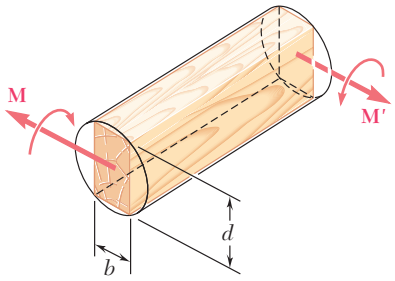
(*b*) Pipe with no fins:

$$\sigma_{\text{all}} = 20\text{ ksi}, \qquad I_x = 3.7276\text{ in}^4, \qquad c = 1.5\text{ in.}$$

$$M = \sigma_{\text{all}}\frac{I_x}{c} = (20\text{ ksi})\frac{3.7276\text{ in}^4}{1.5\text{ in.}}$$

$$M = 49.7\text{ kip}\cdot\text{in.} \blacktriangleleft$$

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PROBLEM 4.27

A couple **M** will be applied to a beam of rectangular cross section that is to be sawed from a log of circular cross section. Determine the ratio d/b for which (a) the maximum stress σ_m will be as small as possible, (b) the radius of curvature of the beam will be maximum.

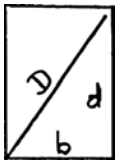
SOLUTION

Let D be the diameter of the log.

$$D^2 = b^2 + d^2 \qquad d^2 = D^2 - b^2$$

$$I = \frac{1}{12}bd^3 \qquad c = \frac{1}{2}d \qquad \frac{I}{c} = \frac{1}{6}bd^2$$

(a) σ_m is the minimum when $\dfrac{I}{c}$ is maximum.



$$\frac{I}{c} = \frac{1}{6}b(D^2 - b^2) = \frac{1}{6}D^2b - \frac{1}{6}b^3$$

$$\frac{d}{db}\left(\frac{I}{c}\right) = \frac{1}{6}D^2 - \frac{3}{6}b^2 = 0 \qquad b = \frac{1}{\sqrt{3}}D$$

$$d = \sqrt{D^2 - \frac{1}{3}D^2} = \sqrt{\frac{2}{3}}D \qquad \frac{d}{b} = \sqrt{2} \blacktriangleleft$$

(b) $\rho = \dfrac{EI}{M}$

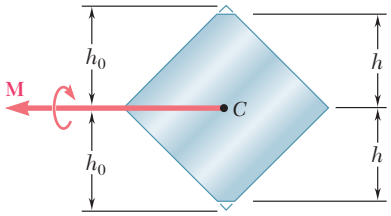
ρ is maximum when I is maximum, $\dfrac{1}{12}bd^3$ is maximum, or b^2d^6 is maximum.

$(D^2 - d^2)d^6$ is maximum.

$$6D^2d^5 - 8d^7 = 0 \qquad d = \frac{\sqrt{3}}{2}D$$

$$b = \sqrt{D^2 - \frac{3}{4}D^2} = \frac{1}{2}D \qquad \frac{d}{b} = \sqrt{3} \blacktriangleleft$$

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PROBLEM 4.28

A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$, where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.

SOLUTION

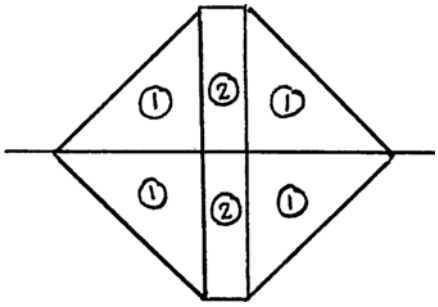
$$I = 4I_1 + 2I_2$$

$$= (4)\left(\frac{1}{12}\right)h\,h^3 + (2)\left(\frac{1}{3}\right)(2h_0 - 2h)(h^3)$$

$$= \frac{1}{3}h^4 + \frac{4}{3}h_0h^3 - \frac{4}{3}h\,h^3 = \frac{4}{3}h_0h^3 - h^4$$

$$c = h$$

$$\sigma = \frac{Mc}{I} = \frac{Mh}{\frac{4}{3}h_0h^3 - h^4} = \frac{3M}{(4h_0 - 3h)h^2}$$



For the original square, $h = h_0,\ c = h_0.$

$$\sigma_0 = \frac{3M}{(4h_0 - 3h_0)h_0^2} = \frac{3M}{h_0^3}$$

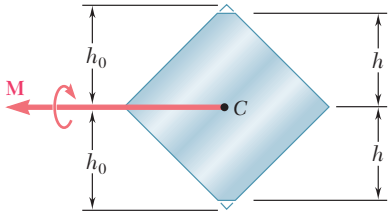
$$\frac{\sigma}{\sigma_0} = \frac{h_0^3}{(4h_0 - 3h)h^2} = \frac{h_0^3}{(4h_0 - (3)(0.9)h_0)(0.9\,h_0^2)} = 0.950$$

$$\sigma = 0.950\,\sigma_0$$

$$k = 0.950 \blacktriangleleft$$

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PROBLEM 4.29



In Prob. 4.28, determine (*a*) the value of *h* for which the maximum stress σ_m is as small as possible, (*b*) the corresponding value of *k*.

PROBLEM 4.28 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$, where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of *k*.

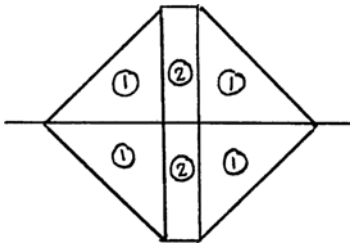
SOLUTION

$$I = 4I_1 + 2I_2$$

$$= (4)\left(\frac{1}{12}\right)hh^3 + (2)\left(\frac{1}{3}\right)(2h_0 - 2h)h^3$$

$$= \frac{1}{3}h^4 - \frac{4}{3}h_0h^3 - \frac{4}{3}h^3 = \frac{4}{3}h_0h^3 - h^4$$

$$c = h \qquad \frac{I}{c} = \frac{4}{3}h_0h^2 - h^3$$



$\frac{I}{c}$ is maximum at $\frac{d}{dh}\left[\frac{4}{3}h_0h^2 - h^3\right] = 0.$

$$\frac{8}{3}h_0h - 3h^2 = 0 \qquad h = \frac{8}{9}h_0 \blacktriangleleft$$

$$\frac{I}{c} = \frac{4}{3}h_0\left(\frac{8}{9}h_0\right)^2 - \left(\frac{8}{9}h_0\right)^3 = \frac{256}{729}h_0^3 \qquad \sigma = \frac{Mc}{I} = \frac{729M}{256h_0^3}$$

For the original square, $h = h_0$ $c = h_0$ $\frac{I_0}{c_0} = \frac{1}{3}h_0^3$

$$\sigma_0 = \frac{Mc_0}{I_0} = \frac{3M}{h_0^2}$$

$$\frac{\sigma}{\sigma_0} = \frac{729}{256}\cdot\frac{1}{3} = \frac{729}{768} = 0.949 \qquad k = 0.949 \blacktriangleleft$$

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PROBLEM 4.30

For the bar and loading of Concept Application 4.1, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section, (*c*) the angle between the sides of the bar that were originally vertical. Use $E = 29 \times 10^6$ psi and $v = 0.29$.

SOLUTION

From Example 4.01, $M = 30 \text{ kip} \cdot \text{in.}$ $I = 1.042 \text{ in}^4$

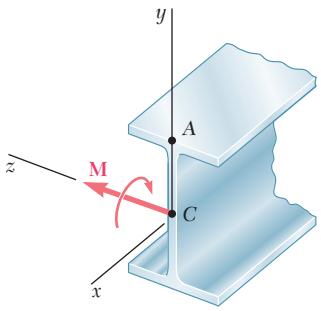
(*a*) $$\frac{1}{\rho} = \frac{M}{EI} = \frac{(30 \times 10^3)}{(29 \times 10^6)(1.042)} = 993 \times 10^{-6} \text{ in.}^{-1}$$ $\rho = 1007 \text{ in.}$ ◀

(*b*) $$\varepsilon^1 = v\varepsilon = \frac{vc}{\rho} = v\frac{c}{\rho'}$$

$$\frac{1}{\rho'} = v\frac{1}{\rho} = (0.29)(993 \times 10^{-6}) \text{ in.}^{-1} = 288 \times 10^{-6} \text{ in.}^{-1}$$ $\rho' = 3470 \text{ in.}$ ◀

(*c*) $$\theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{0.8}{3470} = 230 \times 10^{-6} \text{ rad}$$ $\theta = 0.01320°$ ◀

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PROBLEM 4.31

A $W200 \times 31.3$ rolled-steel beam is subjected to a couple **M** of moment 45 kN · m. Knowing that $E = 200$ GPa and $\nu = 0.29$, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section.

SOLUTION

For $W\,200 \times 31.3$ rolled steel section,

$$I = 31.3 \times 10^{6}\ \text{mm}^4$$
$$= 31.3 \times 10^{-6}\ \text{m}^4$$

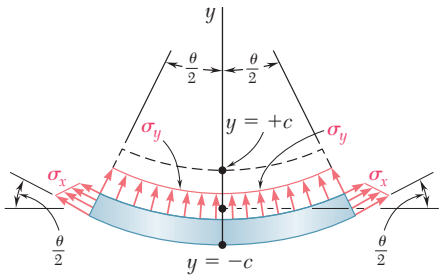
(*a*) $$\frac{1}{\rho} = \frac{M}{EI} = \frac{45 \times 10^{3}}{(200 \times 10^{9})(31.3 \times 10^{-6})} = 7.1885 \times 10^{-3}\ \text{m}^{-1}$$

$\rho = 139.1$ m ◀

(*b*) $$\frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.29)(7.1885 \times 10^{-3}) = 2.0847 \times 10^{-3}\ \text{m}^{-1}$$

$\rho' = 480$ m ◀

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PROBLEM 4.32

It was assumed in Sec. 4.1B that the normal stresses σ_y in a member in pure bending are negligible. For an initially straight elastic member of rectangular cross section, (*a*) derive an approximate expression for σ_y as a function of *y*, (*b*) show that $(\sigma_y)_{\max} = -(c/2\rho)(\sigma_x)_{\max}$ and, thus, that σ_y can be neglected in all practical situations. (*Hint:* Consider the free-body diagram of the portion of beam located below the surface of ordinate *y* and assume that the distribution of the stress σ_x is still linear.)

SOLUTION

Denote the width of the beam by *b* and the length by *L*.

$$\theta = \frac{L}{\rho}$$

Using the free body diagram above, with $\cos\frac{\theta}{2} \approx 1$

$$\Sigma F_y = 0: \quad \sigma_y bL + 2\int_{-c}^{y} \sigma_x b\, dy \sin\frac{\theta}{2} = 0$$

$$\sigma_y = -\frac{2}{L}\sin\frac{\theta}{2}\int_{-c}^{y}\sigma_x\, dy \approx -\frac{\theta}{L}\int_{-c}^{y}\sigma_x\, dy = -\frac{1}{\rho}\int_{-c}^{y}\sigma_x\, dy$$

But, $$\sigma_x = -(\sigma_x)_{\max}\frac{y}{c}$$

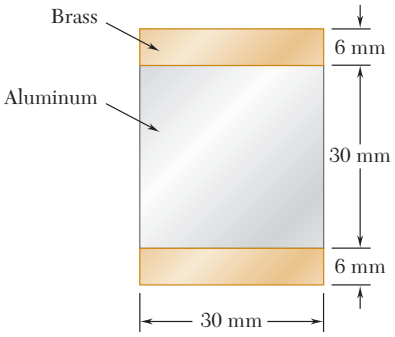
(*a*) $$\sigma_y = \frac{(\sigma_x)_{\max}}{\rho c}\int_{-c}^{y} y\, dy = \frac{(\sigma_x)_{\max}}{\rho c}\frac{y^2}{2}\Big|_{-c}^{y}$$

$$\sigma_y = \frac{(\sigma_x)_{\max}}{2\rho c}(y^2 - c^2) \blacktriangleleft$$

The maximum value σ_y occurs at $y = 0$.

(*b*) $$(\sigma_y)_{\max} = -\frac{(\sigma_x)_{\max} c^2}{2\rho c} = -\frac{(\sigma_x)_{\max} c}{2\rho} \blacktriangleleft$$

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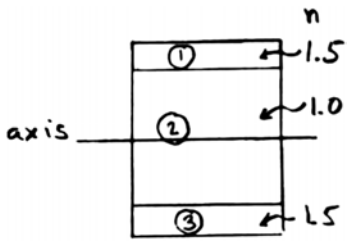


PROBLEM 4.33

A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

$$n = 1.0 \text{ in aluminum}$$
$$n = E_b/E_a = 105/70 = 1.5 \text{ in brass}$$

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$
$$= \frac{1.5}{12}(30)(6)^3 + (1.5)(30)(6)(18)^3 = 88.29 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.0}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$$
$$I_3 = I_1 = 88.29 \times 10^3 \text{ mm}^4$$
$$I = I_1 + I_2 + I_3 = 244.08 \times 10^3 \text{ mm}^4$$
$$= 244.08 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \frac{\sigma I}{ny}$$

Aluminum: $n = 1.0, \quad y = 15 \text{ mm} = 0.015 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

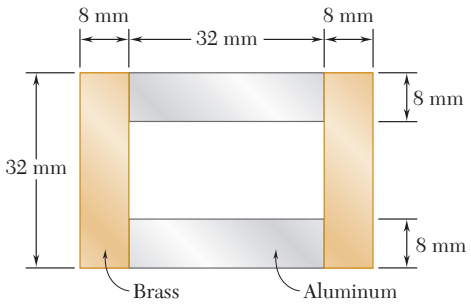
$$M = \frac{(100 \times 10^6)(244.08 \times 10^{-9})}{(1.0)(0.015)} = 1.627 \times 10^3 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5, \quad y = 21 \text{ mm} = 0.021 \text{ m}, \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(244.08 \times 10^{-9})}{(1.5)(0.021)} = 1.240 \times 10^3 \text{ N} \cdot \text{m}$$

Choose the smaller value $M = 1.240 \times 10^3 \text{ N} \cdot \text{m}$ 1.240 kN · m ◀

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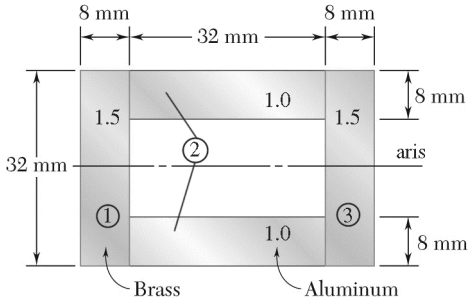


PROBLEM 4.34

A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.5}{12}(8)(32)^3 = 32.768 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 \left(H_2^3 - h_2^3 \right) = \frac{1.0}{12}(32)(32^3 - 16^3) = 76.459 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 32.768 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 141.995 \times 10^3 \text{ mm}^4 = 141.995 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \quad M = \left| \frac{\sigma I}{ny} \right|$$

Aluminum: $n = 1.0, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

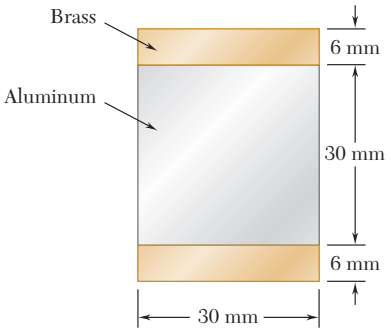
$$M = \frac{(100 \times 10^6)(141.995 \times 10^{-9})}{(1.0)(0.016)} = 887.47 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(141.995 \times 10^{-9})}{(1.5)(0.016)} = 946.63 \text{ N} \cdot \text{m}$$

Choose the smaller value. $M = 887 \text{ N} \cdot \text{m}$ ◀

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PROBLEM 4.35

For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

PROBLEM 4.35. Bar of Prob. 4.33.

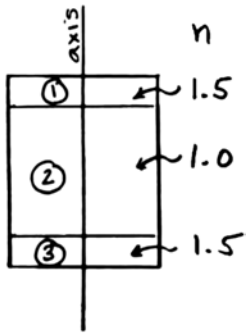
	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION

Use aluminum as reference material.

$n = 1.0$ in aluminum

$n = E_b/E_a = 105/70 = 1.5$ in brass



For transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3$$

$$= \frac{1.5}{12}(6)(30)^3 = 20.25 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3$$

$$= \frac{1.0}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 20.25 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 108 \times 10^3 \text{ mm}^4 = 108 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Aluminum: $n = 1.0, \quad y = 15 \text{ mm} = 0.015 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

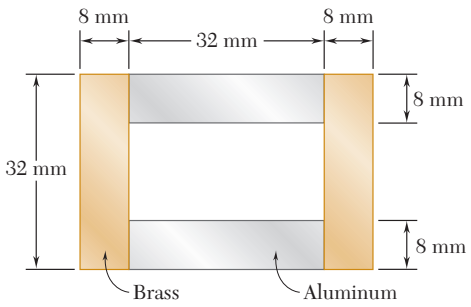
$$M = \frac{(100 \times 10^6)(108 \times 10^{-9})}{(1.0)(0.015)} = 720 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5, \quad y = 15 \text{ mm} = 0.015 \text{ m}, \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(108 \times 10^{-9})}{(1.5)(0.015)} = 768 \text{ N} \cdot \text{m}$$

Choose the smaller value. $M = 720 \text{ N} \cdot \text{m}$ ◀

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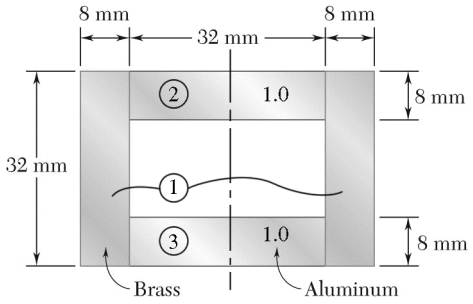
PROBLEM 4.36

For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

PROBLEM 4.36 Bar of Prob. 4.34.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the transformed section,

$$I_1 = \frac{n_1}{12} h_1 \left(B_1^3 - b_1^3 \right) = \frac{1.5}{12}(32)(48^3 - 32^3) = 311.296 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.0}{12}(8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_3 = I_2 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 354.99 \times 10^3 \text{ mm}^4 = 354.99 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \quad M = \left| \frac{\sigma I}{ny} \right|$$

Aluminum: $n = 1.0, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(354.99 \times 10^{-9})}{(1.0)(0.016)} = 2.2187 \times 10^3 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5 \quad |y| = 24 \text{ mm} = 0.024 \text{ m} \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(354.99 \times 10^{-9})}{(1.5)(0.024)} = 1.57773 \times 10^3 \text{ N} \cdot \text{m}$$

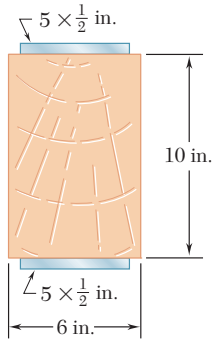
Choose the smaller value.

$$M = 1.57773 \times 10^3 \text{ N} \cdot \text{m}$$

$$M = 1.578 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 4.37



Wooden beams and steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

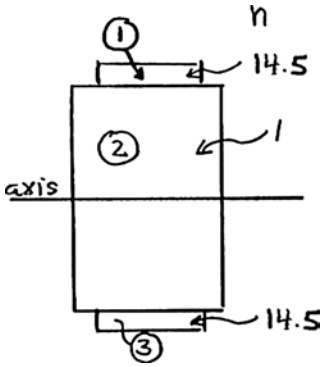
	Wood	Steel
Modulus of elasticity:	2×10^6 psi	29×10^6 psi
Allowable stress:	2000 psi	22 ksi

SOLUTION

Use wood as the reference material.

$n = 1.0$ in wood

$n = E_s/E_w = 29/2 = 14.5$ in steel



For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{14.5}{12}(5)\left(\frac{1}{2}\right)^3 + (14.5)(5)\left(\frac{1}{2}\right)(5.25)^2 = 999.36 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^2 = \frac{1.0}{12}(6)(10)^3 = 500 \text{ in}^4$$

$$I_3 = I_1 = 999.36 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 2498.7 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

Wood: $n = 1.0, \quad y = 5$ in., $\quad \sigma = 2000$ psi

$$M = \frac{(2000)(2499)}{(1.0)(5)} = 999.5\times10^3 \text{ lb}\cdot\text{in.}$$

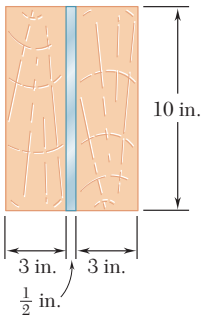
Steel: $n = 14.5, \quad y = 5.5$ in., $\quad \sigma = 22$ ksi $= 22\times10^3$ psi

$$M = \frac{(22\times10^3)(2499)}{(14.5)(5.5)} = 689.3\times10^3 \text{ lb}\cdot\text{in.}$$

Choose the smaller value. $M = 689\times10^3$ lb · in. $M = 689$ kip · in. ◀

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PROBLEM 4.38



Wooden beams and steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity:	2×10^6 psi	29×10^6 psi
Allowable stress:	2000 psi	22 ksi

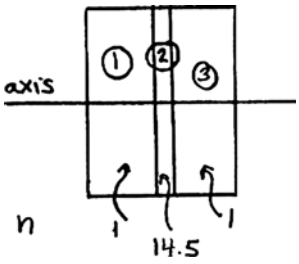
SOLUTION

Use wood as the reference material.

$n = 1.0$ in wood

$n = E_s/E_w = 29/2 = 14.5$ in steel

For the transformed section,



$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12}(3)(10)^3 = 250 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{14.5}{12}\left(\frac{1}{2}\right)(10)^3 = 604.17 \text{ in}^4$$

$$I_3 = I_1 = 250 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 1104.2 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

Wood: $n = 1.0, \quad y = 5$ in., $\quad \sigma = 2000$ psi

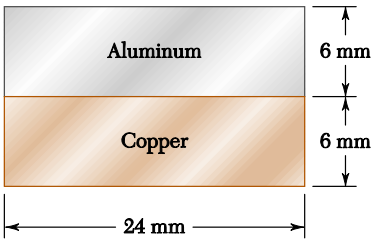
$$M = \frac{(2000)(1104.2)}{(1.0)(5)} = 441.7\times10^3 \text{ lb}\cdot\text{in.}$$

Steel: $n = 14.5, \quad y = 5$ in., $\quad \sigma = 22$ ksi $= 22\times10^3$ psi

$$M = \frac{(22\times10^3)(1104.2)}{(14.5)(5)} = 335.1\times10^3 \text{ lb}\cdot\text{in.}$$

Choose the smaller value. $M = 335\times10^3$ lb · in. $M = 335$ kip · in. ◀

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PROBLEM 4.39

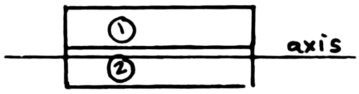
A copper strip (E_c = 105 GPa) and an aluminum strip (E_a = 75 GPa) are bonded together to form the composite beam shown. Knowing that the beam is bent about a horizontal axis by a couple of moment M = 35 N · m, determine the maximum stress in (*a*) the aluminum strip, (*b*) the copper strip.

SOLUTION

Use aluminum as the reference material.

$n = 1.0$ in aluminum

$n = E_c/E_a = 105/75 = 1.4$ in copper



Transformed section:

	A, mm^2	nA, mm^2	$\bar{y}_0$, mm	$nA\bar{y}_0$, mm^3
①	144	144	9	1296
②	144	201.6	3	604.8
Σ		345.6		1900.8

$$\bar{Y}_0 = \frac{1900.8}{345.6} = 5.50 \text{ mm}$$

The neutral axis lies 5.50 mm above the bottom.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(24)(6)^3 + (1.0)(24)(6)(3.5)^2 = 2196 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.4}{12}(24)(6)^3 + (1.4)(24)(6)(2.5)^2 = 1864.8 \text{ mm}^4$$

$$I = I_1 + I_2 = 4060.8 \text{ mm}^4 = 4.0608 \times 10^{-9} \text{ m}^4$$

(*a*) Aluminum: $n = 1.0, \quad y = 12 - 5.5 = 6.5 \text{ mm} = 0.0065 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(35)(0.0065)}{4.0608 \times 10^{-9}} = -56.0 \times 10^6 \text{ Pa} = -56.0 \text{ MPa}$$

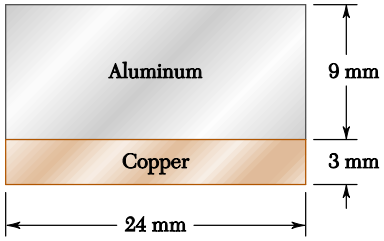
$\sigma = -56.0$ MPa ◀

(*b*) Copper: $n = 1.4, \quad y = -5.5 \text{ mm} = -0.0055 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.4)(35)(-0.0055)}{4.0608 \times 10^{-9}} = 66.4 \times 10^6 \text{ Pa} = 66.4 \text{ MPa}$$

$\sigma = 66.4$ MPa ◀

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PROBLEM 4.40

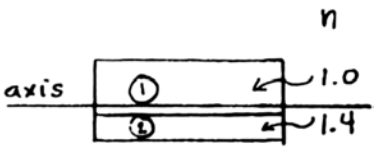
A copper strip (E_c = 105 GPa) and an aluminum strip (E_a = 75 GPa) are bonded together to form the composite beam shown. Knowing that the beam is bent about a horizontal axis by a couple of moment M = 35 N · m, determine the maximum stress in (*a*) the aluminum strip, (*b*) the copper strip.

SOLUTION

Use aluminum as the reference material.

$n = 1.0$ in aluminum

$n = E_c/E_a = 105/75 = 1.4$ in copper



Transformed section:

	A, mm^2	nA, mm^2	$A\bar{y}_0$, mm	$nA\bar{y}_0$, mm^3
①	216	216	7.5	1620
②	72	100.8	1.5	151.8
Σ		316.8		1771.2

$$\bar{Y}_0 = \frac{1771.2}{316.8} = 5.5909 \text{ mm}$$

The neutral axis lies 5.5909 mm above the bottom.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(24)(9)^3 + (1.0)(24)(9)(1.9091)^2 = 2245.2 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.4}{12}(24)(3)^3 + (1.4)(24)(3)(4.0909)^2 = 1762.5 \text{ mm}^4$$

$$I = I_1 + I_2 = 4839 \text{ mm}^4 = 4.008 \times 10^{-9} \text{ m}^4$$

(*a*) Aluminum: $n = 1.0, \quad y = 12 - 5.5909 = 6.4091 \text{ mm} = 0.0064091$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(35)(0.0064091)}{4.008 \times 10^{-9}} = -56.0 \times 10^{-6} \text{ Pa}$$

$$= -56.0 \text{ MPa}$$

$\sigma = -56.0$ MPa ◀

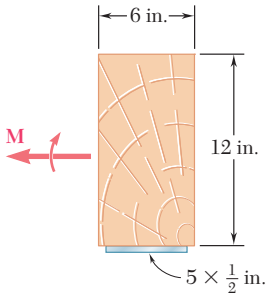
(*b*) Copper: $n = 1.4, \quad y = -5.5909 \text{ mm} = -0.0055909 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.4)(35)(-0.0055909)}{4.008 \times 10^{-9}} = 68.4 \times 10^{6} \text{ Pa}$$

$$= 68.4 \text{ MPa}$$

$\sigma = 68.4$ MPa ◀

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PROBLEM 4.41

The 6×12-in. timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 1.8×10^6 psi and for steel, 29×10^6 psi. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450$ kip·in., determine the maximum stress in (*a*) the wood, (*b*) the steel.

SOLUTION

Use wood as the reference material.

For wood, $n = 1$

For steel, $n = E_s/E_w = 29/1.8 = 16.1111$

Transformed section: ① = wood ② = steel

$$\bar{Y}_o = \frac{421.931}{112.278} = 3.758 \text{ in.}$$

	A, in^2	nA, in^2	$\bar{y}_0$	$nA\bar{y}_0$, in^3
①	72	72	6	432
②	2.5	40.278	–0.25	–10.069
		112.278		421.931

The neutral axis lies 3.758 in. above the wood-steel interface.

$$I_1 = \frac{n_1}{12}b_1h_1^3 + n_1A_1d_1^2 = \frac{1}{12}(6)(12)^3 + (72)(6-3.758)^2 = 1225.91 \text{ in}^4$$

$$I_2 = \frac{n_2}{12}b_2h_2^3 + n_2A_2d_2^2 = \frac{16.1111}{12}(5)(0.5)^3 + (40.278)(3.578+0.25)^2 = 647.87 \text{ in}^4$$

$$I = I_1 + I_2 = 1873.77 \text{ in}^4$$

$M = 450$ kip·in. $\qquad \sigma = -\dfrac{nMy}{I}$

(*a*) Wood: $n = 1$, $y = 12 - 3.758 = 8.242$ in.

$$\sigma_w = -\frac{(1)(450)(8.242)}{1873.77} = -1.979 \text{ ksi}$$

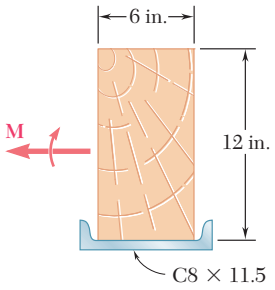
$\sigma_w = -1.979$ ksi ◀

(*b*) Steel: $n = 16.1111$, $y = -3.758 - 0.5 = -4.258$ in.

$$\sigma_s = -\frac{(16.1111)(450)(-4.258)}{1873.77} = 16.48 \text{ ksi}$$

$\sigma_s = 16.48$ ksi ◀

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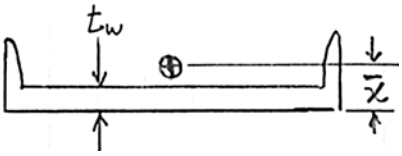


PROBLEM 4.42

The 6×12-in. timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 1.8×10^6 psi and for steel, 29×10^6 psi. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450$ kip·in., determine the maximum stress in (*a*) the wood, (*b*) the steel.

SOLUTION

Use wood as the reference material.



For wood, $n = 1$

For steel, $n = \dfrac{E_s}{E_w} = \dfrac{29\times10^6}{1.8\times10^6} = 16.1111$

For C8 ×11.5 channel section,

$A = 3.38$ in^2, $t_w = 0.220$ in., $\bar{x} = 0.571$ in., $I_y = 1.32$ in^4

For the composite section, the centroid of the channel (part 1) lies 0.571 in. above the bottom of the section. The centroid of the wood (part 2) lies 0.220 + 6.00 = 6.22 in. above the bottom.

Transformed section:

Part	A, in^2	nA, in^2	$\bar{y}$, in.	$nA\bar{y}$, in^3	d, in.
1	3.38	54.456	0.571	31.091	3.216
2	72	72	6.22	447.84	2.433
Σ		126.456		478.93	

$$\bar{Y}_0 = \frac{478.93 \text{ in}^3}{126.456 \text{ in}^2} = 3.787 \text{ in.} \qquad d = \left|\bar{y}_0 - \bar{Y}_0\right|$$

The neutral axis lies 3.787 in. above the bottom of the section.

$$I_1 = n_1\bar{I}_1 + n_1A_1d_1^2 = (16.1111)(1.32) + (54.456)(3.216)^2 = 584.49 \text{ in}^4$$

$$I_2 = \frac{n_2}{12}b_2h_2^3 + n_2A_2d_2^2 = \frac{1}{12}(6)(12)^3 + (72)(2.433)^2 = 1290.20 \text{ in}^4$$

$$I = I_1 + I_2 = 1874.69 \text{ in}^4$$

$$M = 450 \text{ kip·in} \qquad \sigma = -\frac{nMy}{I}$$

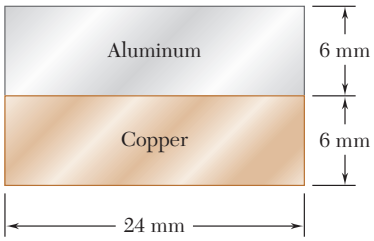
(*a*) Wood: $n = 1$, $y = 12 + 0.220 - 3.787 = 8.433$ in.

$$\sigma_w = -\frac{(1)(450)(8.433)}{1874.69} = -2.02 \text{ ksi} \qquad \sigma_w = -2.02 \text{ ksi} \blacktriangleleft$$

(*b*) Steel: $n = 16.1111$, $y = -3.787$ in.

$$\sigma_s = -\frac{(16.1111)(450)(-3.787)}{1874.67} = 14.65 \text{ ksi} \qquad \sigma_s = 14.65 \text{ ksi} \blacktriangleleft$$

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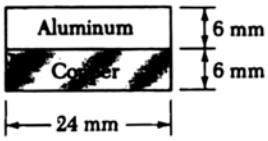
PROBLEM 4.43

For the composite beam indicated, determine the radius of curvature caused by the couple of moment 35 N · m.

Beam of Prob. 4.39.

SOLUTION

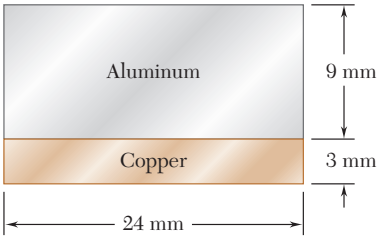
See solution to Prob. 4.39 for the calculation of *I*.



$$\frac{1}{\rho} = \frac{M}{E_a I} = \frac{35}{(75\times10^{9})(4.0608\times10^{-9})} = 0.1149 \text{ m}^{-1}$$

$$\rho = 8.70 \text{ m} \blacktriangleleft$$

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PROBLEM 4.44

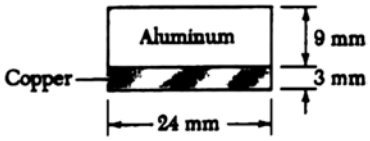
For the composite beam indicated, determine the radius of curvature caused by the couple of moment $35\ \text{N}\cdot\text{m}$.

Beam of Prob. 4.40.

SOLUTION

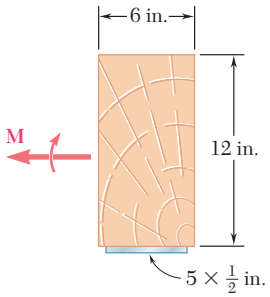
See solution to Prob. 4.40 for the calculation of I.

$$\frac{1}{\rho} = \frac{M}{E_a I} = \frac{35}{(75\times10^{9})(4.008\times10^{-9})} = 0.1164\ \text{m}^{-1}$$



$\rho = 8.59\ \text{m}$ ◀

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PROBLEM 4.45

For the composite beam indicated, determine the radius of curvature caused by the couple of moment 450 kip · in.

Beam of Prob. 4.41.

SOLUTION

See solution to Prob. 4.41 for calculation of I.

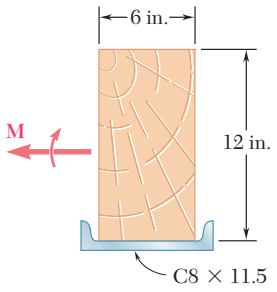
$$I = 1873.77 \text{ in}^4 \qquad E_w = 1.8 \times 10^6 \text{ psi}$$

$$M = 450 \text{ kip} \cdot \text{in} = 450 \times 10^3 \text{ lb} \cdot \text{in}.$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{450 \times 10^3}{(1.8 \times 10^6)(1873.77)} = 133.421 \times 10^{-6} \text{ in.}^{-1}$$

$$\rho = 7495 \text{ in.} = 625 \text{ ft} \blacktriangleleft$$

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PROBLEM 4.46

For the composite beam indicated, determine the radius of curvature caused by the couple of moment 450 kip · in.

Beam of Prob. 4.42.

SOLUTION

See solution to Prob. 4.42 for calculation of I.

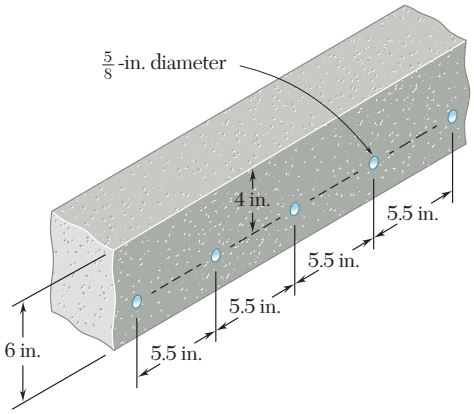
$$I = 1874.69 \text{ in}^4 \qquad E_w = 1.8 \times 10^6 \text{ psi}$$

$$M = 450 \text{ kip} \cdot \text{in.} = 450 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{450 \times 10^3}{(1.8 \times 10^6)(1874.69)} = 133.355 \times 10^{-6} \text{ in.}^{-1}$$

$$\rho = 7499 \text{ in.} = 625 \text{ ft} \blacktriangleleft$$

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PROBLEM 4.47

A concrete slab is reinforced by $\frac{5}{8}$-in.-diameter steel rods placed on 5.5-in. centers as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 29×10^6 psi for the steel. Using an allowable stress of 1400 psi for the concrete and 20 ksi for the steel, determine the largest bending moment in a portion of slab 1 ft wide.

SOLUTION

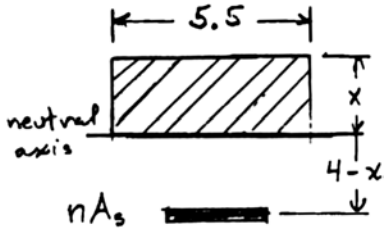
$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3 \times 10^6} = 9.6667$$

Consider a section 5.5 in. wide.

$$A_s = \frac{\pi}{4} d_s^2 = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.3068 \text{ in}^2$$

$$nA_s = 2.9657 \text{ in}^2$$

Locate the natural axis.



$$5.5x\frac{x}{2} - (4-x)(2.9657) = 0$$

$$2.75x^2 + 2.9657x - 11.8628 = 0$$

Solve for x.

$$x = 1.6066 \text{ in.} \qquad 4 - x = 2.3934 \text{ in.}$$

$$I = \frac{1}{3}(5.5)x^3 + (2.9657)(4-x)^2$$

$$= \frac{1}{3}(5.5)(1.6066)^3 + (2.9657)(2.3934)^2 = 24.591 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{I\sigma}{ny}\right|$$

Concrete: $n = 1$, $y = 1.6066$ in., $\sigma = 1400$ psi

$$M = \frac{(24.591)(1400)}{(1.0)(1.6066)} = 21.429 \times 10^3 \text{ lb} \cdot \text{in.}$$

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PROBLEM 4.47 ***(Continued)***

Steel: $n = 9.6667, \quad y = 2.3934 \text{ in.}, \quad \sigma = 20 \text{ ksi} = 20 \times 10^3 \text{ psi}$

$$M = \frac{(24.591)(20 \times 10^3)}{(9.6667)(2.3934)} = 21.258 \times 10^3 \text{ lb} \cdot \text{in.}$$

Choose the smaller value as the allowable moment for a 5.5 in. width.

$$M = 21.258 \times 10^3 \text{ lb} \cdot \text{in.}$$

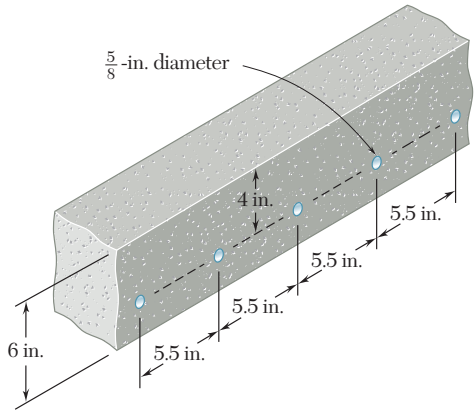
For a 1 ft = 12 in. width,

$$M = \frac{12}{5.5}(21.258 \times 10^3) = 46.38 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$M = 46.38 \text{ kip} \cdot \text{in.}$$

$3.87 \text{ kip} \cdot \text{ft}$ ◀

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PROBLEM 4.48

Solve Prob. 4.47, assuming that the spacing of the $\frac{5}{8}$-in.-diameter steel rods is increased to 7.5 in.

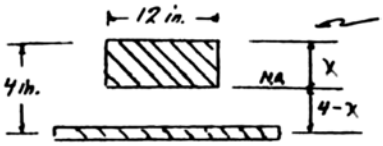
PROBLEM 4.47 A concrete slab is reinforced by $\frac{5}{8}$-in.-diameter steel rods placed on 5.5-in. centers as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 29×10^6 psi for the steel. Using an allowable stress of 1400 psi for the concrete and 20 ksi for the steel, determine the largest bending moment in a portion of slab 1 ft wide.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6 \text{ psi}}{3 \times 10^6 \text{ psi}} = 9.667$$

Number of rails per foot:

$$= \frac{12 \text{ in.}}{7.5 \text{ in.}} = 1.6$$



Area of $\frac{5}{8}$-in.-diameter bars per foot: $1.6\frac{\pi}{4}\left(\frac{5}{8}\right)^2$ $\quad A_s = 0.4909 \text{ in}^2$

Transformed section, all concrete.

First moment of area:

$$12x\left(\frac{x}{2}\right) - 4.745(4 - x) = 0$$

$$x = 1.4266 \text{ in.}$$

$$nA_s = 9.667(0.4909) = 4.745 \text{ in}^2$$

$$I_{NA} = \frac{1}{3}(12)(1.4266)^3 + 4.745(4 - 1.4266)^2 = 43.037 \text{ in}^4$$

For concrete: $\sigma_{\text{all}} = 1400 \text{ psi} \quad c = x = 1.4266 \text{ in.}$

$$M = \sigma\frac{I}{c} = (1400 \text{ psi}) \quad \frac{43.037 \text{ in}^4}{1.4266 \text{ in.}} \qquad M = 42.24 \text{ kip.in.} \blacktriangleleft$$

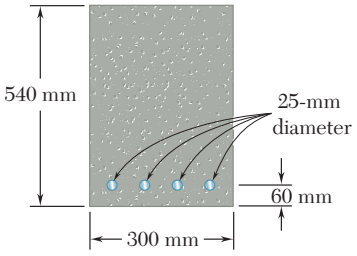
For steel: $\sigma_{\text{all}} = 20 \text{ ksi} \quad c = 4 - x = 4 - 1.4266 = 2.5734 \text{ in.}$

$$M = \frac{\sigma_{\text{steel}}}{n} \cdot \frac{I}{c} = \frac{20 \text{ ksi}}{9.667} \cdot \frac{43.042 \text{ in}^4}{2.5734 \text{ in.}} \qquad M = 34.60 \text{ kip} \cdot \text{in.} \blacktriangleleft$$

We choose the smaller M. $\quad M = 34.60 \text{ kip} \cdot \text{in.}$

Steel controls. $\qquad M = 2.88 \text{ kip} \cdot \text{ft} \blacktriangleleft$

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PROBLEM 4.49

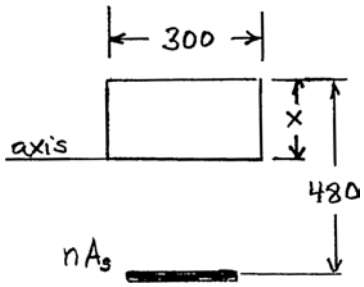
The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4)\left(\frac{\pi}{4}\right)(25)^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$nA_s = 15.708 \times 10^3 \text{ mm}^2$$



Locate the neutral axis.

$$300\, x\frac{x}{2} - (15.708 \times 10^3)(480 - x) = 0$$

$$150x^2 + 15.708 \times 10^3 x - 7.5398 \times 10^6 = 0$$

Solve for x.

$$x = \frac{-15.708 \times 10^3 + \sqrt{(15.708 \times 10^3)^2 + (4)(150)(7.5398 \times 10^6)}}{(2)(150)}$$

$$x = 177.87 \text{ mm}, \qquad 480 - x = 302.13 \text{ mm}$$

$$I = \frac{1}{3}(300)x^3 + (15.708 \times 10^3)(480 - x)^2$$

$$= \frac{1}{3}(300)(177.87)^3 + (15.708 \times 10^3)(302.13)^2$$

$$= 1.9966 \times 10^9 \text{ mm}^4 = 1.9966 \times 10^{-3} \text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

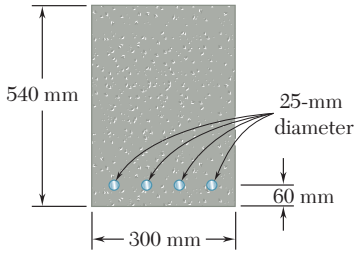
(*a*) Steel: $y = -302.45 \text{ mm} = -0.30245 \text{ m}$

$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.30245)}{1.9966 \times 10^{-3}} = 212 \times 10^6 \text{ Pa} \qquad \sigma = 212 \text{ MPa} \blacktriangleleft$$

(*b*) Concrete: $y = 177.87 \text{ mm} = 0.17787 \text{ m}$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.17787)}{1.9966 \times 10^{-3}} = -15.59 \times 10^6 \text{ Pa} \qquad \sigma = -15.59 \text{ MPa} \blacktriangleleft$$

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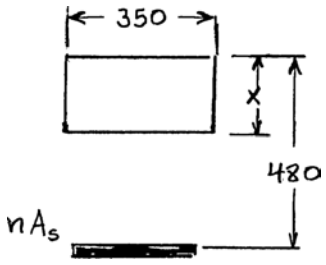


PROBLEM 4.50

Solve Prob. 4.49, assuming that the 300-mm width is increased to 350 mm.

PROBLEM 4.49 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{200\text{ GPa}}{25\text{ GPa}} = 8.0$$

$$A_s = 4\frac{\pi}{4}d^2 = (4)\left(\frac{\pi}{4}\right)(25)^2$$

$$= 1.9635\times10^{3}\text{ mm}^2$$

$$nA_s = 15.708\times10^{3}\text{ mm}^2$$

Locate the neutral axis.

$$350x\frac{x}{2} - (15.708\times10^{3})(480 - x) = 0$$

$$175x^2 + 15.708\times10^{3}x - 7.5398\times10^{6} = 0$$

Solve for x.

$$x = \frac{-15.708\times10^{3} + \sqrt{(15.708\times10^{3})^2 + (4)(175)(7.5398\times10^{6})}}{(2)(175)}$$

$$x = 167.48\text{ mm}, \quad 480 - x = 312.52\text{ mm}$$

$$I = \frac{1}{3}(350)x^3 + (15.708\times10^{3})(480 - x)^2$$

$$= \frac{1}{3}(350)(167.48)^3 + (15.708\times10^{3})(312.52)^2$$

$$= 2.0823\times10^{9}\text{ mm}^4 = 2.0823\times10^{-3}\text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

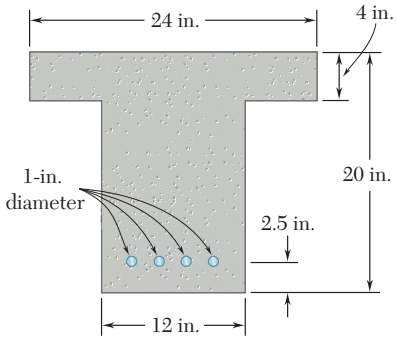
(*a*) Steel: $y = -312.52\text{ mm} = -0.31252\text{ m}$

$$\sigma = -\frac{(8.0)(175\times10^{3})(-0.31252)}{2.0823\times10^{-3}} = 210\times10^{6}\text{ Pa} \qquad \sigma = 210\text{ MPa} \blacktriangleleft$$

(*b*) Concrete: $y = 167.48\text{ mm} = 0.16748\text{ m}$

$$\sigma = -\frac{(1.0)(175\times10^{3})(0.16748)}{2.0823\times10^{-3}} = -14.08\times10^{6}\text{ Pa} \qquad \sigma = -14.08\text{ MPa} \blacktriangleleft$$

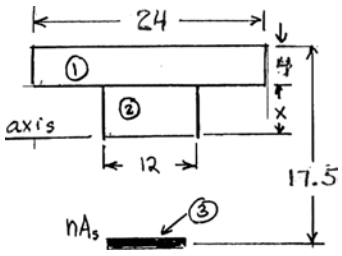
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PROBLEM 4.51

Knowing that the bending moment in the reinforced concrete beam is +100 kip · ft and that the modulus of elasticity is 3.625×10^6 psi for the concrete and 29×10^6 psi for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.625 \times 10^6} = 8.0$$

$$A_s = (4)\left(\frac{\pi}{4}\right)(1)^2 = 3.1416 \text{ in}^2 \qquad nA_s = 25.133 \text{ in}^2$$

Locate the neutral axis.

$$(24)(4)(x + 2) + (12x)\left(\frac{x}{2}\right) - (25.133)(17.5 - 4 - x) = 0$$

$$96x + 192 + 6x^2 - 339.3 + 25.133x = 0 \quad \text{or} \quad 6x^2 + 121.133x - 147.3 = 0$$

Solve for x.
$$x = \frac{-121.133 + \sqrt{(121.133)^2 + (4)(6)(147.3)}}{(2)(6)} = 1.150 \text{ in.}$$

$$d_3 = 17.5 - 4 - x = 12.350 \text{ in.}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(24)(4)^3 + (24)(4)(3.150)^2 = 1080.6 \text{ in}^4$$

$$I_2 = \frac{1}{3} b_2 x^3 = \frac{1}{3}(12)(1.150)^3 = 6.1 \text{ in}^4$$

$$I_3 = nA_3 d_3^2 = (25.133)(12.350)^2 = 3833.3 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 4920 \text{ in}^4$$

$$\sigma = -\frac{nMy}{I} \qquad \text{where } M = 100 \text{ kip} \cdot \text{ft} = 1200 \text{ kip} \cdot \text{in.}$$

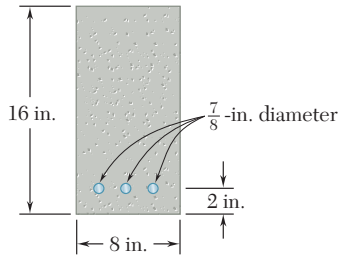
(*a*) Steel: $n = 8.0$ $\qquad y = -12.350$ in.

$$\sigma_s = -\frac{(8.0)(1200)(-12.350)}{4920} \qquad \sigma_s = 24.1 \text{ ksi} \blacktriangleleft$$

(*b*) Concrete: $n = 1.0, \quad y = 4 + 1.150 = 5.150$ in.

$$\sigma_c = -\frac{(1.0)(1200)(5.150)}{4920} \qquad \sigma_c = -1.256 \text{ ksi} \blacktriangleleft$$

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PROBLEM 4.52

A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 29×10^6 psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.

SOLUTION

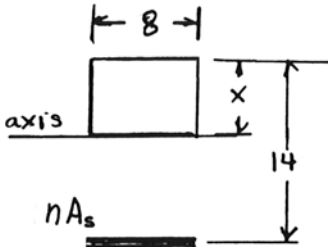
$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3 \times 10^6} = 9.67$$

$$A_s = 3\frac{\pi}{4}d^2 = (3)\left(\frac{\pi}{4}\right)\left(\frac{7}{8}\right)^2 = 1.8040 \text{ in}^2 \qquad nA_s = 17.438 \text{ in}^2$$

Locate the neutral axis:

$$8x\frac{x}{2} - (17.438)(14 - x) = 0$$

$$4x^2 + 17.438x - 244.14 = 0$$



Solve for x. $$x = \frac{-17.438 + \sqrt{17.438^2 + (4)(4)(244.14)}}{(2)(4)} = 5.6326 \text{ in.}$$

$$14 - x = 8.3674 \text{ in.}$$

$$I = \frac{1}{3}8x^3 + nA_s(14 - x)^2 = \frac{1}{3}(8)(5.6326)^3 + (17.438)(8.3674)^2 = 1697.45 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $n = 1.0, \quad |y| = 5.6326 \text{ in.}, \quad |\sigma| = 1350 \text{ psi}$

$$M = \frac{(1350)(1697.45)}{(1.0)(5.6326)} = 406.835 \times 10^3 \text{ lb} \cdot \text{in.} = 407 \text{ kip} \cdot \text{in.}$$

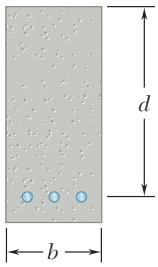
Steel: $n = 9.67, \quad |y| = 8.3674 \text{ in.}, \quad \sigma = 20 \times 10^3 \text{ psi}$

$$M = \frac{(20 \times 10^3)(1697.45)}{(9.67)(8.3674)} = 419.72 \text{ lb} \cdot \text{in.} = 420 \text{ kip} \cdot \text{in.}$$

Choose the smaller value. $M = 407 \text{ kip} \cdot \text{in.}$ $M = 33.9 \text{ kip} \cdot \text{ft}$ ◀

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PROBLEM 4.53



The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c. Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \dfrac{\sigma_s E_c}{\sigma_c E_s}}$$

where E_c and E_s are the moduli of elasticity of concrete and steel, respectively, and d is the distance from the top of the beam to the reinforcing steel.

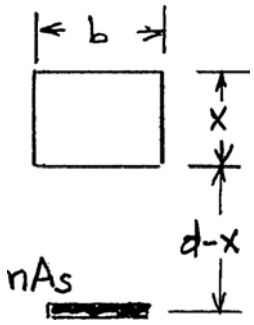
SOLUTION

$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

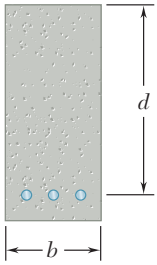
$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{E_c \sigma_s}{E_s \sigma_c}$$

$$x = \frac{d}{1 + \dfrac{E_c \sigma_s}{E_s \sigma_c}}$$



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PROBLEM 4.54

For the concrete beam shown, the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel. Knowing that b = 200 mm and d = 450 mm, and using an allowable stress of 12.5 MPa for the concrete and 140 MPa for the steel, determine (*a*) the required area A_s of the steel reinforcement if the beam is to be balanced, (*b*) the largest allowable bending moment. (See Prob. 4.53 for definition of a balanced beam.)

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200\times10^9}{25\times10^9} = 8.0$$

$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.0}\cdot\frac{140\times10^6}{12.5\times10^6} = 2.40$$

$$x = 0.41667d = (0.41667)(450) = 187.5 \text{ mm}$$

Locate neutral axis. $bx\frac{x}{2} - nA_s(d-x)$

(*a*) $$A_s = \frac{bx^2}{2n(d-x)} = \frac{(200)(187.5)^2}{(2)(8.0)(262.5)} = 1674 \text{ mm}^2$$ $A_s = 1674 \text{ mm}^2$ ◀

$$I = \frac{1}{3}bx^3 + nA_s(d-x)^2 = \frac{1}{3}(200)(187.5)^3 + (8.0)(1674)(262.5)^2$$

$$= 1.3623\times10^9 \text{ mm}^4 = 1.3623\times10^{-3} \text{ m}^4$$

$$\sigma = \frac{nMy}{I} \qquad M = \frac{I\sigma}{ny}$$

(b) Concrete: $n = 1.0$ $y = 187.5 \text{ mm} = 0.1875 \text{ m}$ $\sigma = 12.5\times10^6 \text{ Pa}$

$$M = \frac{(1.3623\times10^{-3})(12.5\times10^6)}{(1.0)(0.1875)} = 90.8\times10^3 \text{ N}\cdot\text{m}$$

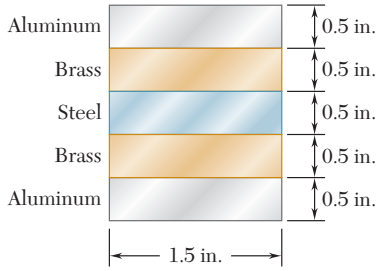
Steel: $n = 8.0$ $y = 262.5 \text{ mm} = 0.2625 \text{ m}$ $\sigma = 140\times10^6 \text{ Pa}$

$$M = \frac{(1.3623\times10^{-3})(140\times10^6)}{(8.0)(0.2625)} = 90.8\times10^3 \text{ N}\cdot\text{m}$$

Note that both values are the same for balanced design.

$M = 90.8 \text{ kN}\cdot\text{m}$ ◀

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PROBLEM 4.55

Five metal strips, each a 0.5 × 1.5-in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 30×10^6 psi for the steel, 15×10^6 psi for the brass, and 10×10^6 psi for the aluminum. Knowing that the beam is bent about a horizontal axis by a couple of moment $12\ \text{kip}\cdot\text{in.}$, determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

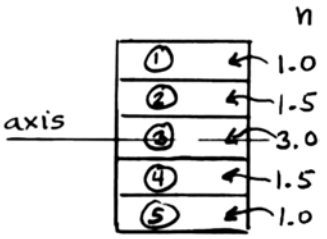
SOLUTION

Use aluminum as the reference material.

$$n = \frac{E_s}{E_a} = \frac{30\times10^6}{10\times10^6} = 3.0 \text{ in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15\times10^6}{10\times10^6} = 1.5 \text{ in brass}$$

$$n = 1.0 \text{ in aluminum}$$



For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(1.5)(0.5)^3 + (0.75)(1.0)^2$$

$$= 0.7656 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.5}{12}(1.5)(0.5)^3 + (1.5)(0.75)(0.5)^2 = 0.3047 \text{ in}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{3.0}{12}(1.5)(0.5)^3 = 0.0469 \text{ in}^4$$

$$I_4 = I_2 = 0.3047 \text{ in}^4 \qquad I_5 = I_1 = 0.7656 \text{ in}^4$$

$$I = \sum_1^5 I_i = 2.1875 \text{ in}^4$$

(*a*) Aluminum:
$$\sigma = \frac{nMy}{I} = \frac{(1.0)(12)(1.25)}{2.1875} = 6.86 \text{ ksi}$$ ◀

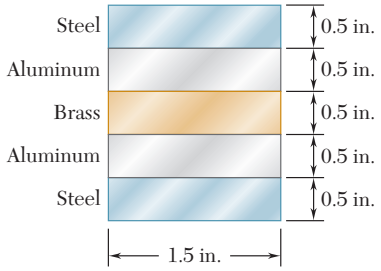
Brass:
$$\sigma = \frac{nMy}{I} = \frac{(1.5)(12)(0.75)}{2.1875} = 6.17 \text{ ksi}$$ ◀

Steel:
$$\sigma = \frac{nMy}{I} = \frac{(3.0)(12)(0.25)}{2.1875} = 4.11 \text{ ksi}$$ ◀

(*b*)
$$\frac{1}{\rho} = \frac{M}{E_a I} = \frac{12\times10^3}{(10\times10^6)(2.1875)} = 548.57\times10^{-6} \text{ in.}^{-1}$$

$$\rho = 1823 \text{ in.} = 151.9 \text{ ft}$$ ◀

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PROBLEM 4.56

Five metal strips, each a 0.5 × 1.5-in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 30×10^6 psi for the steel, 15×10^6 psi for the brass, and 10×10^6 psi for the aluminum. Knowing that the beam is bent about a horizontal axis by a couple of moment 12 kip · in., determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

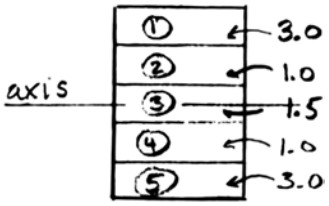
SOLUTION

Use aluminum as the reference material.

$$n = \frac{E_s}{E_a} = \frac{30\times10^6}{10\times10^6} = 3.0 \text{ in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15\times10^6}{10\times10^6} = 1.5 \text{ in brass}$$

$$n = 1.0 \text{ in aluminum}$$



For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{3.0}{12}(1.5)(0.5)^3 + (3.0)(0.75)(1.0)^2$$

$$= 2.2969 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.0}{12}(1.5)(0.5)^3 + (1.0)(0.75)(0.5)^2 = 0.2031 \text{ in}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{1.5}{12}(1.5)(0.5)^3 = 0.0234 \text{ in}^4$$

$$I_4 = I_2 = 0.2031 \text{ in}^4 \qquad I_5 = I_1 = 2.2969 \text{ in}^4$$

$$I = \sum_1^5 I_i = 5.0234 \text{ in}^4$$

(*a*) Steel: $$\sigma = \frac{nMy}{I} = \frac{(3.0)(12)(1.25)}{5.0234} = 8.96 \text{ ksi}$$ ◀

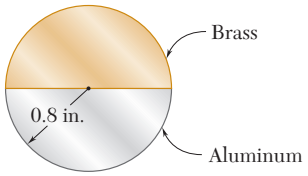
Aluminum: $$\sigma = \frac{nMy}{I} = \frac{(1.0)(12)(0.75)}{5.0234} = 1.792 \text{ ksi}$$ ◀

Brass: $$\sigma = \frac{nMy}{I} = \frac{(1.5)(12)(0.25)}{5.0234} = 0.896 \text{ ksi}$$ ◀

(*b*) $$\frac{1}{\rho} = \frac{M}{E_a I} = \frac{12\times10^3}{(10\times10^6)(5.0234)} = 238.89\times10^{-6} \text{ in.}^{-1}$$

$$\rho = 4186 \text{ in.} = 349 \text{ ft}$$ ◀

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PROBLEM 4.57

The composite beam shown is formed by bonding together a brass rod and an aluminum rod of semicircular cross sections. The modulus of elasticity is 15×10^6 psi for the brass and 10×10^6 psi for the aluminum. Knowing that the composite beam is bent about a horizontal axis by couples of moment 8 kip · in., determine the maximum stress (*a*) in the brass, (*b*) in the aluminum.

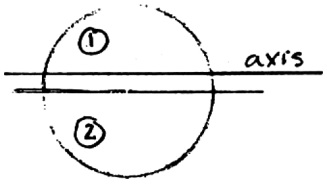
SOLUTION

For each semicircle, $r = 0.8$ in. $A = \frac{\pi}{2} r^2 = 1.00531 \text{ in}^2$,

$$\bar{y}_0 = \frac{4r}{3\pi} = \frac{(4)(0.8)}{3\pi} = 0.33953 \text{ in.} \quad I_{\text{base}} = \frac{\pi}{8} r^4 = 0.160850 \text{ in}^4$$

$$\bar{I} = I_{\text{base}} - A\bar{y}_0^2 = 0.160850 - (1.00531)(0.33953)^2 = 0.044953 \text{ in}^4$$

Use aluminum as the reference material.



$n = 1.0$ in aluminum

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \text{ in brass}$$

Locate the neutral axis.

	A, in^2	nA, in^2	$\bar{y}_0$, in.	$nA\bar{y}_0$, in^3
①	1.00531	1.50796	0.33953	0.51200
②	1.00531	1.00531	–0.33953	–0.34133
Σ		2.51327		0.17067

$$\bar{Y}_0 = \frac{0.17067}{2.51327} = 0.06791 \text{ in.}$$

The neutral axis lies 0.06791 in. above the material interface.

$$d_1 = 0.33953 - 0.06791 = 0.27162 \text{ in.}, \quad d_2 = 0.33953 + 0.06791 = 0.40744 \text{ in.}$$

$$I_1 = n_1\bar{I} + n_1 A d_1^2 = (1.5)(0.044957) + (1.5)(1.00531)(0.27162)^2 = 0.17869 \text{ in}^4$$

$$I_2 = n_2\bar{I} + n_2 A d_2^2 = (1.0)(0.044957) + (1.0)(1.00531)(0.40744)^2 = 0.21185 \text{ in}^4$$

$$I = I_1 + I_2 = 0.39054 \text{ in}^4$$

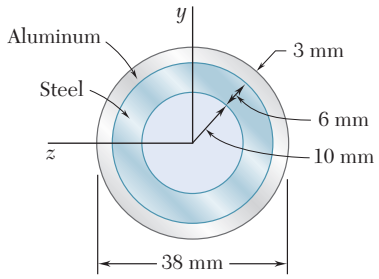
(*a*) Brass: $n = 1.5$, $y = 0.8 - 0.06791 = 0.73209$ in.

$$\sigma = -\frac{nMy}{I} = -\frac{(1.5)(8)(0.73209)}{0.39054} \qquad \sigma = -22.5 \text{ ksi} \blacktriangleleft$$

(*b*) Aluminium: $n = 1.0$, $y = -0.8 - 0.06791 = -0.86791$ in.

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(8)(-0.86791)}{0.39054} \qquad \sigma = 17.78 \text{ ksi} \blacktriangleleft$$

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PROBLEM 4.58

A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 200 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by a couple of moment 500 N · m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

SOLUTION

Use aluminum as the reference material.

$$n = 1.0 \text{ in aluminum}$$
$$n = E_s / E_a = 200/70 = 2.857 \text{ in steel}$$

For the transformed section,

Steel: $$I_s = n_s \frac{\pi}{4}\left(r_o^4 - r_i^4\right) = (2.857)\left(\frac{\pi}{4}\right)(16^4 - 10^4) = 124.62 \times 10^3 \text{ mm}^4$$

Aluminium: $$I_a = n_a \frac{\pi}{4}\left(r_o^4 - r_i^4\right) = (1.0)\left(\frac{\pi}{4}\right)(19^4 - 16^4) = 50.88 \times 10^3 \text{ mm}^4$$

$$I = I_s + I_a = 175.50 \times 10^3 \text{ mm}^4 = 175.5 \times 10^{-9} \text{ m}^4$$

(*a*) Aluminum: $c = 19 \text{ mm} = 0.019 \text{ m}$

$$\sigma_a = \frac{n_a Mc}{I} = \frac{(1.0)(500)(0.019)}{175.5 \times 10^{-9}} = 54.1 \times 10^6 \text{ Pa}$$

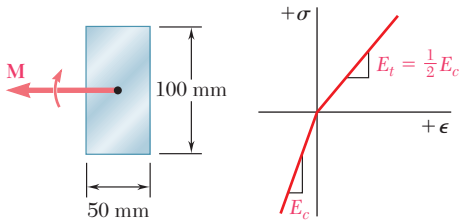
$$\sigma_a = 54.1 \text{ MPa} \blacktriangleleft$$

(*b*) Steel: $c = 16 \text{ mm} = 0.016 \text{ m}$

$$\sigma_s = \frac{n_s Mc}{I} = \frac{(2.857)(500)(0.016)}{175.5 \times 10^{-9}} = 130.2 \times 10^6 \text{ Pa}$$

$$\sigma_s = 130.2 \text{ MPa} \blacktriangleleft$$

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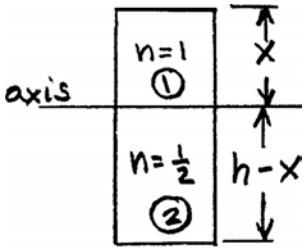
PROBLEM 4.59

The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one-half of its value in compression. For a bending moment $M = 600\ \text{N}\cdot\text{m}$, determine the maximum (*a*) tensile stress, (*b*) compressive stress.

SOLUTION

$n = \frac{1}{2}$ on the tension side of neutral axis

$n = 1$ on the compression side



Locate neutral axis.

$$n_1 bx\frac{x}{2} - n_2 b(h-x)\frac{h-x}{2} = 0$$

$$\frac{1}{2}bx^2 - \frac{1}{4}b(h-x)^2 = 0$$

$$x^2 = \frac{1}{2}(h-x)^2 \quad x = \frac{1}{\sqrt{2}}(h-x)$$

$$x = \frac{1}{\sqrt{2}+1}h = 0.41421h = 41.421\ \text{mm}$$

$$h - x = 58.579\ \text{mm}$$

$$I_1 = n_1\frac{1}{3}bx^3 = (1)\left(\frac{1}{3}\right)(50)(41.421)^3 = 1.1844\times10^6\,\text{mm}^4$$

$$I_2 = n_2\frac{1}{3}b(h-x)^3 = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(50)(58.579)^3 = 1.6751\times10^6\,\text{mm}^4$$

$$I = I_1 + I_2 = 2.8595\times10^6\,\text{mm}^4 = 2.8595\times10^{-6}\,\text{m}^4$$

(*a*) Tensile stress: $n = \frac{1}{2}$, $y = -58.579\ \text{mm} = -0.058579\ \text{m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(0.5)(600)(-0.058579)}{2.8595\times10^{-6}} = 6.15\times10^6\,\text{Pa} \qquad \sigma_t = 6.15\ \text{MPa} \blacktriangleleft$$

(*b*) Compressive stress: $n = 1$, $y = 41.421\ \text{mm} = 0.041421\ \text{m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(600)(0.041421)}{2.8595\times10^{-6}} = -8.69\times10^6\,\text{Pa} \qquad \sigma_c = -8.69\ \text{MPa} \blacktriangleleft$$

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PROBLEM 4.60*

A rectangular beam is made of material for which the modulus of elasticity is E_t in tension and E_c in compression. Show that the curvature of the beam in pure bending is

$$\frac{1}{\rho} = \frac{M}{E_r I}$$

where

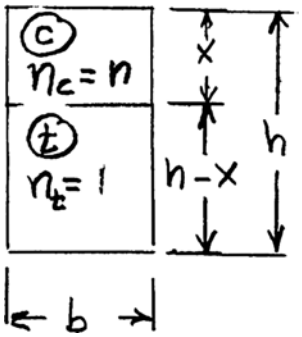
$$E_r = \frac{4E_t E_c}{(\sqrt{E_t} + \sqrt{E_c})^2}$$

SOLUTION

Use E_t as the reference modulus.

Then $E_c = nE_t$.

Locate neutral axis.



$$nbx\frac{x}{2} - b(h-x)\frac{h-x}{2} = 0$$

$$nx^2 - (h-x)^2 = 0 \quad \sqrt{n}x = (h-x)$$

$$x = \frac{h}{\sqrt{n}+1} \quad h - x = \frac{\sqrt{n}h}{\sqrt{n}+1}$$

$$I_{\text{trans}} = \frac{n}{3}bx^3 + \frac{1}{3}b(h-x)^3 = \left[\frac{h}{3}\left(\frac{1}{\sqrt{n}+1}\right)^3 + \left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)^3\right]bh^3$$

$$= \frac{1}{3}\frac{n+n^{3/2}}{\left(\sqrt{n}+1\right)^3}bh^3 = \frac{1}{3}\frac{n\left(1+\sqrt{n}\right)}{\left(\sqrt{n}+1\right)^3}bh^3 = \frac{1}{3}\times\frac{n}{\left(\sqrt{n}+1\right)^2}bh^3$$

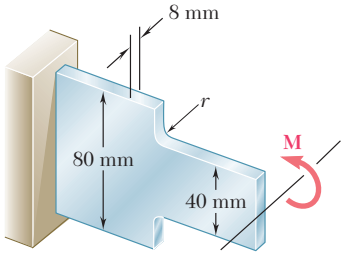
$$\frac{1}{\rho} = \frac{M}{E_t I_{\text{trans}}} = \frac{M}{E_r I} \quad \text{where} \quad I = \frac{1}{12}bh^3$$

$$E_r I = E_t I_{\text{trans}}$$

$$E_r = \frac{E_t I_{\text{trans}}}{I} = \frac{12}{bh^3}\times E_t \times \frac{n}{3\left(\sqrt{n}+1\right)^2}bh^3$$

$$= \frac{4E_t E_c/E_t}{\left(\sqrt{E_c/E_t}+1\right)^2} = \frac{4E_t E_c}{\left(\sqrt{E_c}+\sqrt{E_t}\right)^2}$$

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PROBLEM 4.61

Knowing that $M = 250\ \text{N}\cdot\text{m}$, determine the maximum stress in the beam shown when the radius r of the fillets is (*a*) 4 mm, (*b*) 8 mm.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667 \times 10^3\ \text{mm}^4 = 42.667 \times 10^{-9}\ \text{m}^4$$

$$c = 20\ \text{mm} = 0.020\ \text{m}$$

$$\frac{D}{d} = \frac{80\ \text{mm}}{40\ \text{mm}} = 2.00$$

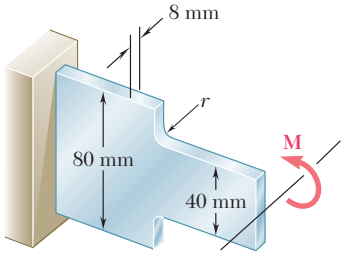
(*a*) $\dfrac{r}{d} = \dfrac{4\ \text{mm}}{40\ \text{mm}} = 0.10$ From Fig. 4.27, $K = 1.87$

$$\sigma_{\max} = K\frac{Mc}{I} = \frac{(1.87)(250)(0.020)}{42.667 \times 10^{-9}} = 219 \times 10^6\ \text{Pa} \qquad \sigma_{\max} = 219\ \text{MPa} \blacktriangleleft$$

(*b*) $\dfrac{r}{d} = \dfrac{8\ \text{mm}}{40\ \text{mm}} = 0.20$ From Fig. 4.27, $K = 1.50$

$$\sigma_{\max} = K\frac{Mc}{I} = \frac{(1.50)(250)(0.020)}{42.667 \times 10^{-9}} = 176.0 \times 10^6\ \text{Pa} \qquad \sigma_{\max} = 176.0\ \text{MPa} \blacktriangleleft$$

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PROBLEM 4.62

Knowing that the allowable stress for the beam shown is 90 MPa, determine the allowable bending moment M when the radius r of the fillets is (*a*) 8 mm, (*b*) 12 mm.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667 \times 10^3 \text{ mm}^4 = 42.667 \times 10^{-9} \text{ m}^4$$

$$c = 20 \text{ mm} = 0.020 \text{ m}$$

$$\frac{D}{d} = \frac{80 \text{ mm}}{40 \text{ mm}} = 2.00$$

(*a*) $\dfrac{r}{d} = \dfrac{8 \text{ mm}}{40 \text{ mm}} = 0.2$ From Fig. 4.27, $K = 1.50$

$$\sigma_{\max} = K\frac{Mc}{I} \qquad M = \frac{\sigma_{\max} I}{Kc} = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.50)(0.020)}$$

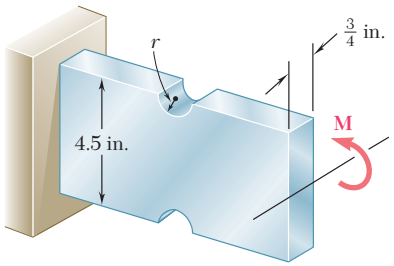
$$M = 128.0 \text{ N} \cdot \text{m} \blacktriangleleft$$

(*b*) $\dfrac{r}{d} = \dfrac{12 \text{ mm}}{40 \text{ mm}} = 0.3$ From Fig. 4.27, $K = 1.35$

$$M = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.35)(0.020)}$$

$$M = 142.0 \text{ N} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 4.63

Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Using an allowable stress of 8 ksi, determine the largest bending moment that can be applied to the member when (*a*) $r = \frac{3}{8}$ in., (*b*) $r = \frac{3}{4}$ in.

SOLUTION

(*a*) $d = D - 2r = 4.5 - (2)\left(\frac{3}{8}\right) = 3.75$ in.

$$\frac{D}{d} = \frac{4.5}{3.75} = 1.20 \qquad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$$

From Fig. 4.28, $K = 2.07$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}\left(\frac{3}{4}\right)(3.75)^3 = 3.296 \text{ in}^4 \qquad c = \frac{1}{2} = 1.875 \text{ in.}$$

$$\sigma = K\frac{Mc}{I} \qquad \therefore \quad M = \frac{\sigma I}{Kc} = \frac{(8)(3.296)}{(2.07)(1.875)} = 6.79 \text{ kip} \cdot \text{in.}$$

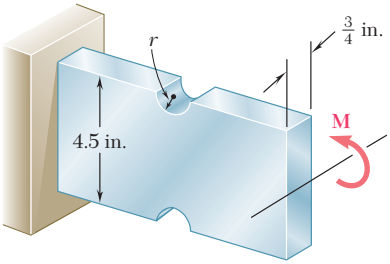
$\sigma = 6.79$ kip · in. ◀

(*b*) $d = D - 2r = 4.5 - (2)\left(\frac{3}{4}\right) = 3.0 \qquad \frac{D}{d} = \frac{4.5}{3.0} = 1.5 \qquad \frac{r}{d} = \frac{0.75}{3.0} = 0.25$

From Fig. 4.28, $K = 1.61 \qquad I = \frac{1}{12}bh^3 = \frac{1}{12}\left(\frac{3}{4}\right)(3.0)^3 = 1.6875 \text{ in}^4$

$$c = \frac{1}{2}d = 1.5 \text{ in.} \qquad M = \frac{\sigma I}{Kc} = \frac{(8)(1.6875)}{(1.61)(1.5)} = 5.59 \text{ kip} \cdot \text{in.}$$ ◀

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PROBLEM 4.64

Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Knowing that $M = 4$ kip · in., determine the maximum stress in the member when the radius r of the semicircular grooves is (*a*) $r = \frac{3}{8}$ in., (*b*) $r = \frac{3}{4}$ in.

SOLUTION

(*a*) $$d = D - 2r = 4.5 - (2)\left(\frac{3}{8}\right) = 3.75 \text{ in.}$$

$$\frac{D}{d} = \frac{4.5}{3.75} = 1.20 \qquad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$$

From Fig. 4.28, $K = 2.07$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}\left(\frac{3}{4}\right)(3.75)^3 = 3.296 \text{ in}^4 \qquad c = \frac{1}{2} = 1.875 \text{ in.}$$

$$\sigma = K\frac{Mc}{I} = \frac{(2.07)(4)(1.875)}{3.296} = 4.71 \text{ ksi}$$

$\sigma = 4.71$ ksi ◀

(*b*) $d = D - 2r = 4.5 - (2)\left(\frac{3}{4}\right) = 3.00$ in. $\quad \frac{D}{d} = \frac{4.5}{3.00} = 1.50 \quad \frac{r}{d} = \frac{0.75}{3.0} = 0.25$

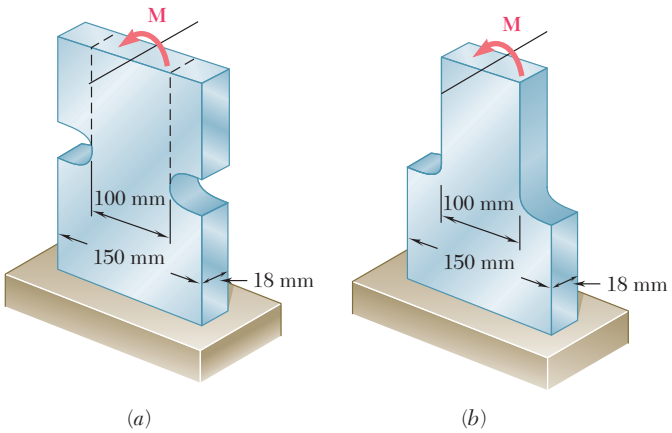
From Fig. 4.28,

$K = 1.61 \quad I = \frac{1}{12}bh^3 = \frac{1}{12}\left(\frac{3}{4}\right)(3.00)^3 = 1.6875 \text{ in}^4 \quad c = \frac{1}{2}d = 1.5 \text{ in.}$

$$\sigma = K\frac{Mc}{I} = \frac{(1.61)(4)(1.5)}{1.6875} = 5.72 \text{ ksi}$$

$\sigma = 5.72$ ksi ◀

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(*a*) (*b*)

PROBLEM 4.65

A couple of moment $M = 2$ kN · m is to be applied to the end of a steel bar. Determine the maximum stress in the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 10$ mm, as shown in Fig. *a*, (*b*) if the bar is redesigned by removing the material to the left and right of the dashed lines as shown in Fig. *b*.

SOLUTION

For both configurations,

$$D = 150 \text{ mm}$$
$$d = 100 \text{ mm}$$
$$r = 10 \text{ mm}$$
$$\frac{D}{d} = \frac{150}{100} = 1.50$$
$$\frac{r}{d} = \frac{10}{100} = 0.10$$

For configuration (*a*),

Fig. 4.28 gives $K_a = 2.21$.

For configuration (*b*), Fig. 4.27 gives $K_b = 1.79$.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$
$$c = \frac{1}{2}d = 50 \text{ mm} = 0.05 \text{ m}$$

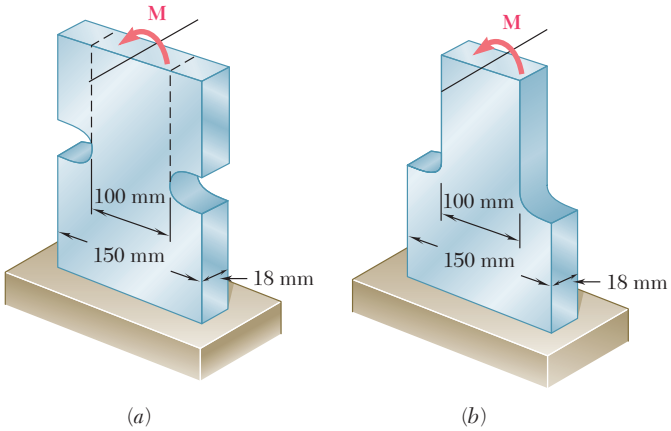
(*a*) $$\sigma = \frac{KMc}{I} = \frac{(2.21)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 147.0 \times 10^6 \text{ Pa} = 147.0 \text{ MPa}$$

$$\sigma = 147.0 \text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma = \frac{KMc}{I} = \frac{(1.79)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 119.0 \times 10^6 \text{ Pa} = 119.0 \text{ MPa}$$

$$\sigma = 119.0 \text{ MPa} \blacktriangleleft$$

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(*a*) (*b*)

PROBLEM 4.66

The allowable stress used in the design of a steel bar is 80 MPa. Determine the largest couple **M** that can be applied to the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 15$ mm, as shown in Fig. *a*, (*b*) if the bar is redesigned by removing the material to the left and right of the dashed lines as shown in Fig. *b*.

SOLUTION

For both configurations,

$$D = 150 \text{ mm} \qquad d = 100 \text{ mm}$$

$$r = 15 \text{ mm}$$

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{15}{100} = 0.15$$

For configuration (*a*), Fig. 4.28 gives $K_a = 1.92$.

For configuration (*b*), Fig. 4.27 gives $K_b = 1.57$.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

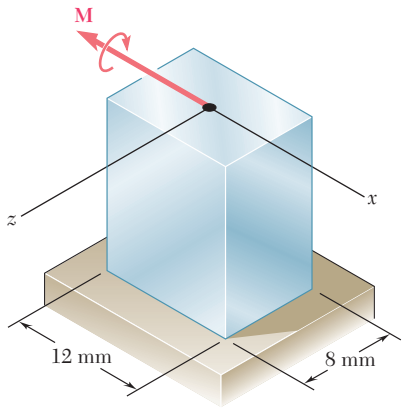
$$c = \frac{1}{2}d = 50 \text{ mm} = 0.050 \text{ m}$$

(*a*) $$\sigma = \frac{KMc}{I} \quad \therefore \quad M = \frac{\sigma I}{Kc} = \frac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.92) \times (0.05)} = 1.250 \times 10^3 \text{ N} \cdot \text{m}$$

$$= 1.250 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

(*a*) $$M = \frac{\sigma I}{Kc} = \frac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.57)(0.050)} = 1.530 \times 10^3 \text{ N} \cdot \text{m} = 1.530 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

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PROBLEM 4.67

The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the x axis. Determine the moment M of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 4 mm thick.

SOLUTION

(*a*) $$I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(8)^3 = 512 \text{ mm}^4$$

$$= 512 \times 10^{-12} \text{ m}^4$$

$$c = \frac{1}{2}h = 4 \text{ mm} = 0.004 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300 \times 10^6)(512 \times 10^{-12})}{0.004}$$

$$= 38.4 \text{ N} \cdot \text{m}$$

$$M_Y = 38.4 \text{ N} \cdot \text{m} \blacktriangleleft$$

(*b*) $$y_Y = \frac{1}{2}(4) = 2 \text{ mm} \qquad \frac{y_Y}{c} = \frac{2}{4} = 0.5$$

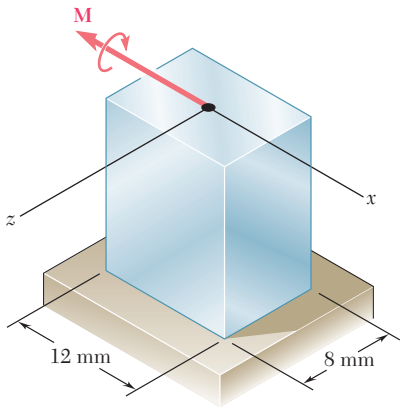
$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$= \frac{3}{2}(38.4)\left[1 - \frac{1}{3}(0.5)^2\right]$$

$$= 52.8 \text{ N} \cdot \text{m}$$

$$M = 52.8 \text{ N} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 4.68

Solve Prob. 4.67, assuming that the couple **M** is parallel to the *z* axis.

PROBLEM 4.67 The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the *x* axis. Determine the moment *M* of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 4 mm thick.

SOLUTION

(*a*) $$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(12)^3 = 1.152\times10^3 \text{mm}^4$$

$$= 1.152\times10^{-9}\ \text{m}^4$$

$$c = \frac{1}{2}h = 6\ \text{mm} = 0.006\ \text{m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times10^6)(1.152\times10^{-9})}{0.006}$$

$$= 57.6\ \text{N}\cdot\text{m}$$

$M_Y = 57.6\ \text{N}\cdot\text{m}$ ◀

(*b*) $y_Y = \frac{1}{2}(4) = 2$ mm $\quad \frac{y_Y}{c} = \frac{2}{6} = \frac{1}{3}$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$= \frac{3}{2}(57.6)\left[1 - \frac{1}{3}\left(\frac{1}{3}\right)^2\right]$$

$$= 83.2\ \text{N}\cdot\text{m}$$

$M = 83.2\ \text{N}\cdot\text{m}$ ◀

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PROBLEM 4.69

A solid square rod of side 0.6 in. is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 48$ ksi. Knowing that a couple **M** is applied and maintained about an axis parallel to a side of the cross section, determine the moment M of the couple for which the radius of curvature is 6 ft.

SOLUTION

$$I = \frac{1}{12}(0.6)(0.6)^3 = 10.8 \times 10^{-3} \text{ in}^4 \qquad c = \frac{1}{2}h = 0.3 \text{ in.}$$

$$M_Y = \frac{I\sigma_Y}{c} = \frac{(10.8 \times 10^{-3})(48 \times 10^3)}{0.3} = 1728 \text{ lb} \cdot \text{in.} \qquad \rho = 6 \text{ ft} = 72 \text{ in.}$$

$$\varepsilon_{\max} = \frac{c}{\rho} = \frac{0.3}{72} = 4.16667 \times 10^{-3} \qquad \varepsilon_Y = \frac{\sigma_Y}{E} = \frac{48 \times 10^3}{29 \times 10^6} = 1.65517 \times 10^{-3}$$

$$\frac{y_Y}{c} = \frac{\varepsilon_Y}{\varepsilon_M} = \frac{1.65517 \times 10^{-3}}{4.16667 \times 10^{-3}} = 0.39724$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{Y_Y}{c}\right)^2\right] = \frac{3}{2}(1728)\left[1 - \frac{1}{3}(0.39724)^2\right]$$

$$M = 2460 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

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PROBLEM 4.70

For the solid square rod of Prob. 4.69, determine the moment M for which the radius of curvature is 3 ft.

PROBLEM 4.69. A solid square rod of side 0.6 in. is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 48$ ksi. Knowing that a couple **M** is applied and maintained about an axis parallel to a side of the cross section, determine the moment M of the couple for which the radius of curvature is 6 ft.

SOLUTION

$$I = \frac{1}{12}(0.6)(0.6)^3 = 10.8 \times 10^{-3}\ \text{in}^4 \qquad c = \frac{1}{2}h = 0.3\ \text{in.}$$

$$M_Y = \frac{I\sigma_Y}{c} = \frac{(10.8 \times 10^{-3})(48 \times 10^3)}{0.3} = 1728\ \text{lb} \cdot \text{in.} \qquad \rho = 3\ \text{ft} = 36\ \text{in.}$$

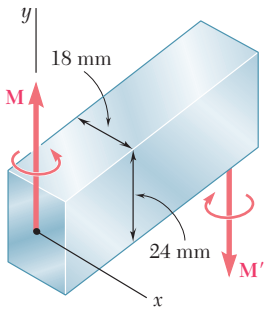
$$\varepsilon_{\max} = \frac{c}{\rho} = \frac{0.3}{36} = 8.3333 \times 10^{-3} \qquad \varepsilon_Y = \frac{\sigma_Y}{E} = \frac{48 \times 10^3}{29 \times 10^6} = 1.65517 \times 10^{-3}$$

$$\frac{y_Y}{c} = \frac{\varepsilon_Y}{\varepsilon_{\max}} = \frac{1.65517 \times 10^{-3}}{8.3333 \times 10^{-3}} = 0.19862$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{Y_Y}{c}\right)^2\right] = \frac{3}{2}(1728)\left[1 - \frac{1}{3}(0.19862)^2\right]$$

$$M = 2560\ \text{lb} \cdot \text{in.} \blacktriangleleft$$

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PROBLEM 4.71

The prismatic rod shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 280$ MPa. Knowing that couples **M** and **M′** of moment 525 N · m are applied and maintained about axes parallel to the y axis, determine (*a*) the thickness of the elastic core, (*b*) the radius of curvature of the bar.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(24)(18)^3 = 11.664 \times 10^3 \text{ mm}^4 = 11.664 \times 10^{-3} \text{ m}^4$$

$$c = \frac{1}{2}h = 9 \text{ mm} = 0.009 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(280 \times 10^6)(11.664 \times 10^{-9})}{0.009} = 362.88 \text{ N} \cdot \text{m}$$

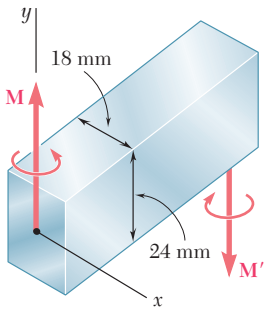
$$M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right) \quad \text{or} \quad \frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}}$$

$$\frac{y_Y}{c} = \sqrt{3 - \frac{(2)(525)}{362.88}} = 0.32632 \qquad y_Y = 0.32632c = 2.9368 \text{ mm}$$

(*a*) $t_{\text{core}} = 2y_Y = 5.87$ mm ◀

(*b*) $\varepsilon_Y = \frac{y_Y}{\rho} = \frac{\sigma_Y}{E} \quad \therefore \quad \rho = \frac{Ey_Y}{\sigma_Y} = \frac{(200 \times 10^9)(2.9368 \times 10^{-3})}{280 \times 10^6} = 2.09 \text{ m}$ ◀

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PROBLEM 4.72

Solve Prob. 4.71, assuming that the couples **M** and **M′** are applied and maintained about axes parallel to the *x* axis.

PROBLEM 4.71 The prismatic rod shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 280$ MPa. Knowing that couples **M** and **M′** of moment 525 N · m are applied and maintained about axes parallel to the *y* axis, determine (*a*) the thickness of the elastic core, (*b*) the radius of curvature of the bar.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(24)^3 = 20.736 \times 10^3 \text{ mm}^4 = 20.736 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}h = 12 \text{ mm} = 0.012 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(280 \times 10^6)(20.736 \times 10^{-9})}{0.012} = 483.84 \text{ N} \cdot \text{m}$$

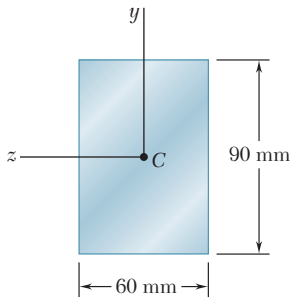
$$M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right) \quad \text{or} \quad \frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}}$$

$$\frac{y_Y}{c} = \sqrt{3 - \frac{(2)(525)}{483.84}} = 0.91097 \qquad y_Y = 0.91097 \qquad c = 10.932 \text{ mm}$$

(*a*) $t_{\text{core}} = 2y_Y = 21.9 \text{ mm}$ ◀

(*b*) $\varepsilon_Y = \frac{y_Y}{\rho} = \frac{\sigma_Y}{E} \quad \therefore \quad \rho = \frac{Ey_Y}{\sigma_Y} = \frac{(200 \times 10^9)(10.932 \times 10^{-3})}{280 \times 10^6} = 7.81 \text{ m}$ ◀

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PROBLEM 4.73

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 30 mm thick.

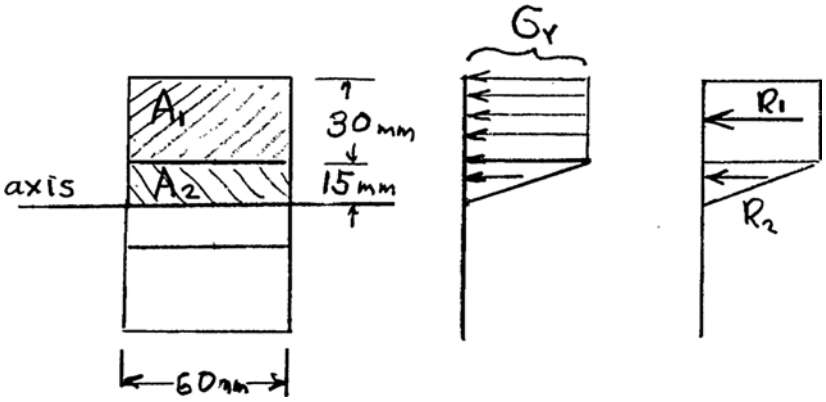
SOLUTION

(a) $$I = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645\times10^6\ \text{mm}^4 = 3.645\times10^{-6}\ \text{m}^4$$

$$c = \frac{1}{2}h = 45\ \text{mm} = 0.045\ \text{m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(3.645\times10^{-6})}{0.045} = 19.44\times10\ \text{N}\cdot\text{m}$$

$$M_Y = 19.44\ \text{kN}\cdot\text{m} \blacktriangleleft$$



$$R_1 = \sigma_Y A_1 = (240\times10^6)(0.060)(0.030)$$
$$= 432\times10^3\ \text{N}$$

$$y_1 = 15\ \text{mm} + 15\ \text{mm} = 0.030\ \text{m}$$

$$R_2 = \frac{1}{2}\sigma_Y A_2 = \left(\frac{1}{2}\right)(240\times10^6)(0.060)(0.015)$$
$$= 108\times10^3\ \text{N}$$

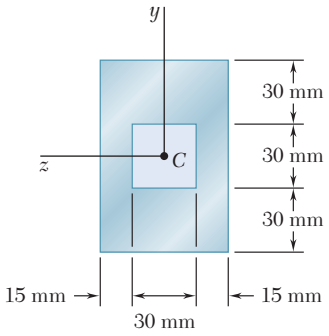
$$y_2 = \frac{2}{3}(15\ \text{mm}) = 10\ \text{mm} = 0.010\ \text{m}$$

(b) $$M = 2(R_1y_1 + R_2y_2) = 2[(432\times10^3)(0.030) + (108\times10^3)(0.010)]$$

$$= 28.08\times10^3\ \text{N}\cdot\text{m}$$

$$M = 28.1\ \text{kN}\cdot\text{m} \blacktriangleleft$$

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PROBLEM 4.74

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 30 mm thick.

SOLUTION

(*a*) $$I_{\text{rect}} = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645\times10^6 \text{ mm}^4$$

$$I_{\text{cutout}} = \frac{1}{12}bh^3 = \frac{1}{12}(30)(30)^3 = 67.5\times10^3 \text{ mm}^4$$

$$I = 3.645\times10^6 - 67.5\times10^3 = 3.5775\times10^6 \text{ mm}^4$$

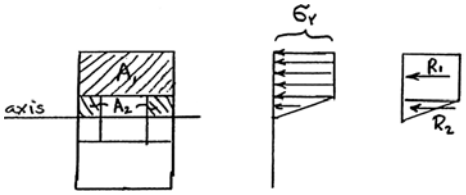
$$= 3.5775\times10^{-6} \text{ mm}^4$$

$$c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(3.5775\times10^{-6})}{0.045}$$

$$= 19.08\times10^3 \text{ N}\cdot\text{m}$$

$M_Y = 19.08 \text{ kN}\cdot\text{m}$ ◀



$$R_1 = \sigma_Y A_1 = (240\times10^6)(0.060)(0.030) = 432\times10^3 \text{ N}$$

$$y_1 = 15 \text{ mm} + 15 \text{ mm} = 30 \text{ mm} = 0.030 \text{ m}$$

$$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(240\times10^6)(0.030)(0.015) = 54\times10^3 \text{ N}$$

$$y_2 = \frac{2}{3}(15 \text{ mm}) = 10 \text{ mm} = 0.010 \text{ m}$$

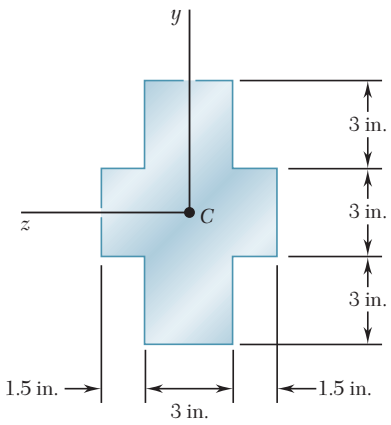
(*b*) $$M = 2(R_1y_1 + R_2y_2)$$

$$= 2[(432\times10^3)(0.030) + (54\times10^3)(0.010)]$$

$$= 27.00\times10^3 \text{ N}\cdot\text{m}$$

$M = 27.0 \text{ kN}\cdot\text{m}$ ◀

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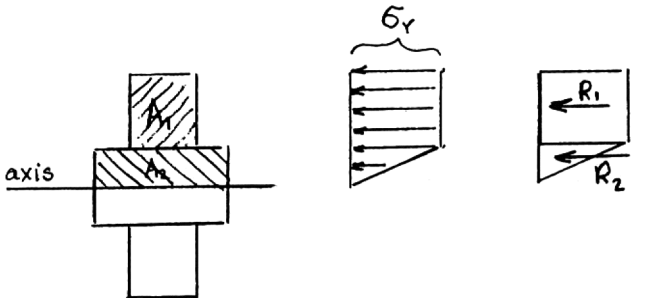


PROBLEM 4.75

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 3 in. thick.

SOLUTION

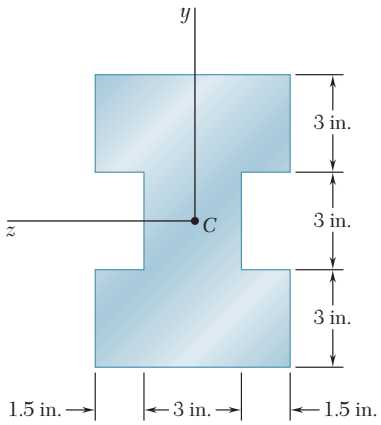
(*a*)
$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(3)(3)^3 + (3)(3)(3)^2 = 87.75 \text{ in}^4$$
$$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(6)(3)^3 = 13.5 \text{ in}^4$$
$$I_3 = I_1 = 87.75 \text{ in}^4$$
$$I = I_1 + I_2 + I_3 = 188.5 \text{ in}^4$$
$$c = 4.5 \text{ in.}$$
$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5} \qquad M_Y = 1759 \text{ kip}\cdot\text{in.} \blacktriangleleft$$



$$R_1 = \sigma_Y A_1 = (42)(3)(3) = 378 \text{ kip}$$
$$y_1 = 1.5 + 1.5 = 3.0 \text{ in.}$$
$$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(6)(1.5) = 189 \text{ kip}$$
$$y_2 = \frac{2}{3}(1.5) = 1.0 \text{ in.}$$

(*b*)
$$M = 2(R_1y_1 + R_2y_2) = 2[(378)(3.0) + (189)(1.0)] = 2646 \text{ kip}\cdot\text{in.} \qquad M = 2650 \text{ kip}\cdot\text{in.} \blacktriangleleft$$

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PROBLEM 4.76

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 3 in. thick.

SOLUTION

(*a*) $$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(6)(3)^3 + (6)(3)(3)^2 = 175.5 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(3)(3)^3 = 6.75 \text{ in}^4$$

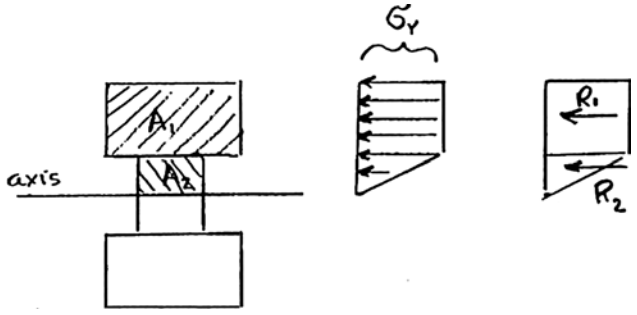
$$I_3 = I_1 = 175.5 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 357.75 \text{ in}^4$$

$$c = 4.5 \text{ in.}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(357.75)}{4.5} = 3339 \text{ kip}\cdot\text{in.}$$

$M_Y = 3340$ kip · in. ◀



$$R_1 = \sigma_Y A_1 = (42)(6)(3) = 756 \text{ kip}$$

$$y_1 = 1.5 + 1.5 = 3 \text{ in.}$$

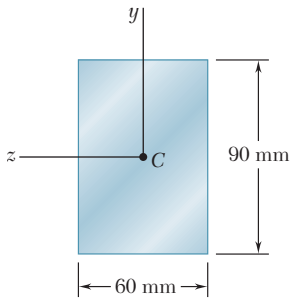
$$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(3)(1.5) = 94.5 \text{ kip}$$

$$y_2 = \frac{2}{3}(1.5) = 1.0 \text{ in.}$$

(*b*) $$M = 2(R_1y_1 + R_2y_2) = 2[(756)(3) + (94.5)(1.0)] = 4725 \text{ kip}\cdot\text{in.}$$

$M = 4730$ kip · in. ◀

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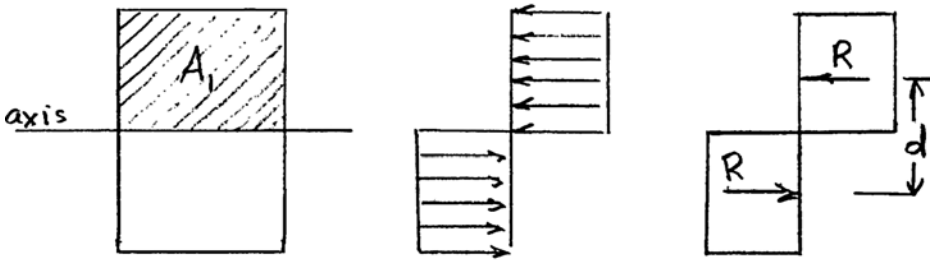
PROBLEM 4.77

For the beam indicated (of Prob. 4.73), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

From Problem 4.73, $E = 200$ GPa and $\sigma_Y = 240$ MPa

$$A_1 = (60)(45) = 2700 \text{ mm}^2$$
$$= 2700 \times 10^{-6} \text{ m}^2$$
$$R = \sigma_Y A_1$$
$$= (240 \times 10^6)(2700 \times 10^{-6})$$
$$= 648 \times 10^3 \text{ N}$$
$$d = 45 \text{ mm} = 0.045 \text{ m}$$



(*a*) $M_p = Rd = (648 \times 10^3)(0.045) = 29.16 \times 10^3 \text{ N} \cdot \text{m}$ $\quad M_p = 29.2 \text{ kN} \cdot \text{m}$ ◀

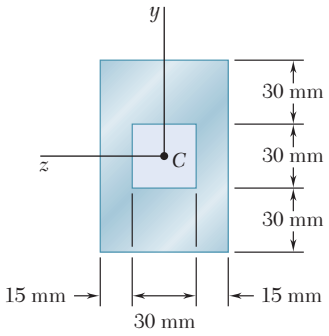
(*b*) $I = \dfrac{1}{12} bh^3 = \dfrac{1}{12}(60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4 = 3.645 \times 10^{-6} \text{ m}^4$

$c = 45 \text{ mm} = 0.045 \text{ m}$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(3.645 \times 10^{-6})}{0.045} = 19.44 \times 10^3 \text{ N} \cdot \text{m}$$

$$k = \frac{M_p}{M_Y} = \frac{29.16}{19.44} \qquad k = 1.500 \text{ ◀}$$

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PROBLEM 4.78

For the beam indicated (of Prob. 4.74), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

From Problem 4.74, $E = 200$ GPa and $\sigma_Y = 240$ MPa

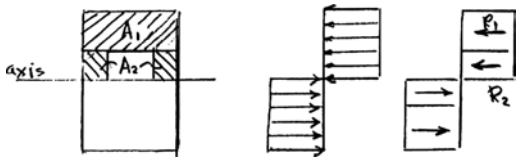
(*a*) $R_1 = \sigma_Y A_1$

$= (240\times10^6)(0.060)(0.030)$

$= 432\times10^3$ N

$y_1 = 15$ mm $+ 15$ mm $= 30$ mm

$= 0.030$ m



$R_2 = \sigma_Y A_2$

$= (240\times10^6)(0.030)(0.015)$

$= 108\times10^3$ N

$y_2 = \frac{1}{2}(15) = 7.5$ mm $= 0.0075$ m

$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(432\times10^3)(0.030) + (108\times10^3)(0.0075)]$

$= 27.54\times10^3$ N·m

$M_p = 27.5$ kN·m ◀

(*b*) $I_{\text{rect}} = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645\times10^6$ mm^4

$I_{\text{cutout}} = \frac{1}{12}bh^3 = \frac{1}{12}(30)(30)^3 = 67.5\times10^3$ mm^4

$I = I_{\text{rect}} - I_{\text{cutout}} = 3.645\times10^6 - 67.5\times10^3 = 3.5775\times10^3$ mm^4

$= 3.5775\times10^{-9}$ m^4

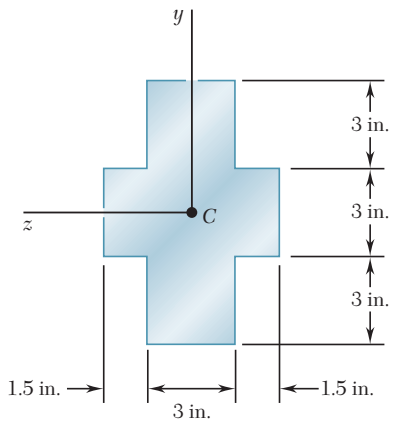
$c = \frac{1}{2}h = 45$ mm $= 0.045$ m

$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(3.5775\times10^{-9})}{0.045} = 19.08\times10^3$ N·m

$k = \frac{M_p}{M_Y} = \frac{27.54}{19.08}$

$k = 1.443$ ◀

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PROBLEM 4.79

For the beam indicated (of Prob. 4.75), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

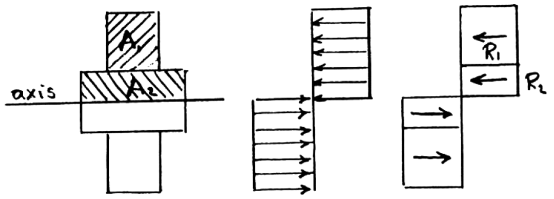
From Problem 4.75, $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi.

(*a*) $R_1 = \sigma_Y A_1 = (42)(3)(3) = 378$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in.

$R_2 = \sigma_Y A_2 = (42)(6)(1.5) = 378$ kip

$y_2 = \frac{1}{2}(1.5) = 0.75$ in.



$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(378)(3.0) + (378)(0.75)]$

$M_p = 2840$ kip·in. ◀

(*b*) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(3)^3 + (3)(3)(3)^2 = 87.75$ in^4

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(6)(3)^3 = 13.5$ in^4

$I_3 = I_1 = 87.75$ in^4

$I = I_1 + I_2 + I_3 = 188.5$ in^4

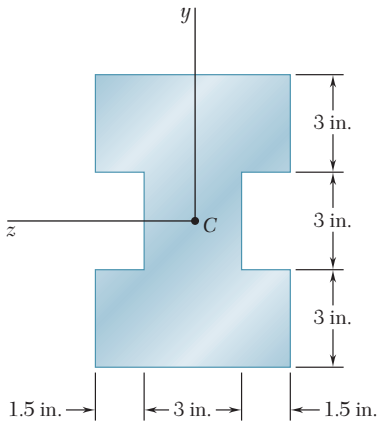
$c = 4.5$ in.

$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5} = 1759.3$ kip·in

$k = \frac{M_p}{M_Y} = \frac{2835}{1759.3}$

$k = 1.611$ ◀

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PROBLEM 4.80

For the beam indicated (of Prob. 4.76), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

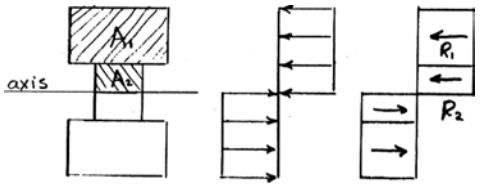
From Problem 4.76, $E = 29\times10^6$ and $\sigma_Y = 42$ ksi

(*a*) $R_1 = \sigma_Y A_1 = (42)(6)(3) = 756$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in.

$R_2 = \sigma_Y A_2 = (42)(3)(1.5) = 189$ kip

$y_2 = \frac{1}{2}(1.5) = 0.75$ in.



$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(756)(3.0) + (189)(0.75)]$

$M_p = 4820$ kip·in. ◀

(*b*) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(6)(3)^3 + (6)(3)(3)^2 = 175.5$ in^4

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(3)(3)^3 = 6.75$ in^4

$I_3 = I_1 = 175.5$ in^4

$I = I_1 + I_2 + I_3 = 357.75$ in^4

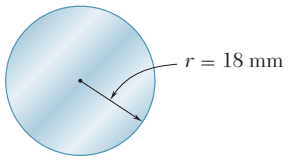
$c = 4.5$ in.

$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(357.75)}{4.5} = 3339$ kip·in.

$k = \frac{M_p}{M_Y} = \frac{4819.5}{3339}$

$k = 1.443$ ◀

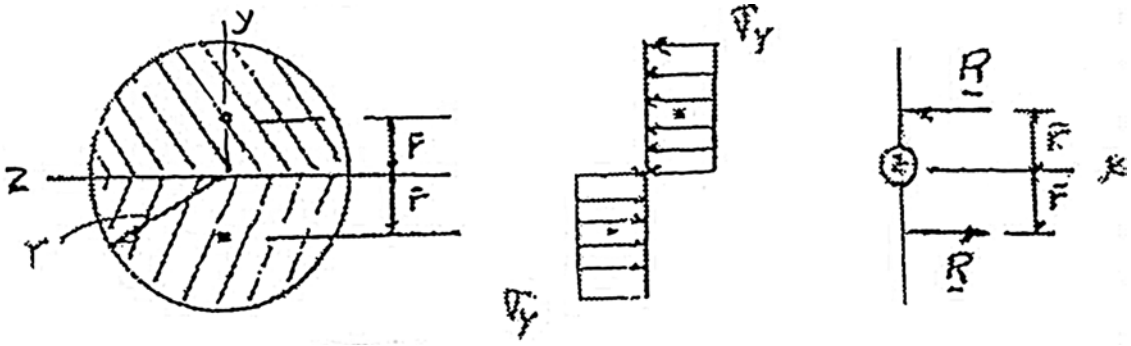
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PROBLEM 4.81

Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION



For a semicircle, $A = \frac{\pi}{2} r^2; \qquad \bar{r} = \frac{4r}{3\pi}$

Resultant force on semicircular section: $R = \sigma_Y A$

Resultant moment on entire cross section:

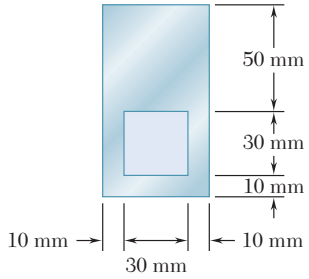
$$M_p = 2R\bar{r} = \frac{4}{3}\sigma_Y r^3$$

Data: $\sigma_Y = 240 \text{ MPa} = 240 \times 10^6 \text{ Pa}, \quad r = 18 \text{ mm} = 0.018 \text{ m}$

$$M_p = \frac{4}{3}(240 \times 10^6)(0.018)^3 = 1866 \text{ N} \cdot \text{m}$$

$$M_p = 1.866 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 4.82

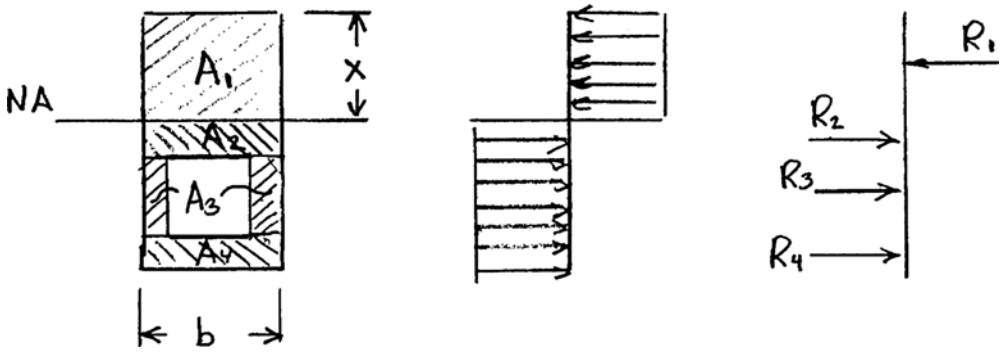
Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Total area: $A = (50)(90) - (30)(30) = 3600 \text{ mm}^2$

$$\frac{1}{2}A = 1800 \text{ mm}^2$$

$$x = \frac{\frac{1}{2}A}{b} = \frac{1800}{50} = 36 \text{ mm}$$



$A_1 = (50)(36) = 1800 \text{ mm}^2, \quad \bar{y}_1 = 18 \text{ mm}, \quad A_1\bar{y}_1 = 32.4 \times 10^3 \text{ mm}^3$

$A_2 = (50)(14) = 700 \text{ mm}^2, \quad \bar{y}_2 = 7 \text{ mm}, \quad A_2\bar{y}_2 = 4.9 \times 10^3 \text{ mm}^3$

$A_3 = (20)(30) = 600 \text{ mm}^2, \quad \bar{y}_3 = 29 \text{ mm}, \quad A_3\bar{y}_3 = 17.4 \times 10^3 \text{ mm}^3$

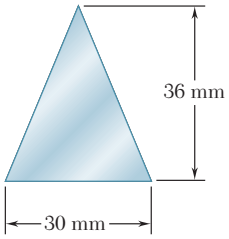
$A_4 = (50)(10) = 500 \text{ mm}^2, \quad \bar{y}_4 = 49 \text{ mm}, \quad A_4\bar{y}_4 = 24.5 \times 10^3 \text{ mm}^3$

$$A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4 = 79.2 \times 10^3 \text{ mm}^3 = 79.2 \times 10^{-6} \text{ m}^3$$

$$M_p = \sigma_Y \Sigma A_i \bar{y}_i = (240 \times 10^6)(79.2 \times 10^{-6}) = 19.008 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_p = 19.01 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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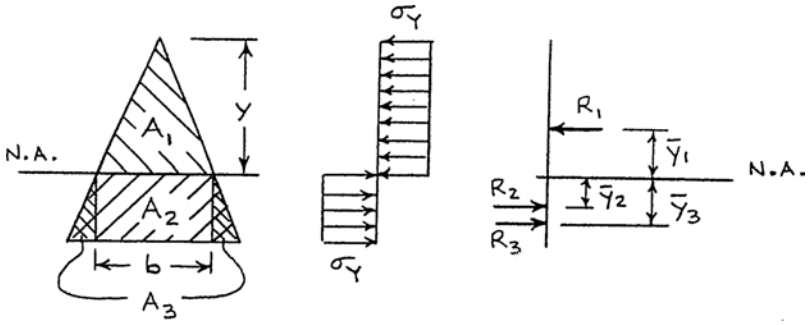


PROBLEM 4.83

Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Total area: $A = \frac{1}{2}(30)(36) = 540 \text{ mm}^2$ Half area: $\frac{1}{2}A = 270 \text{ mm}^2 = A_1$



By similar triangles, $\frac{b}{y} = \frac{30}{36}$ $b = \frac{5}{6}y$

Since $A_1 = \frac{1}{2}by = \frac{5}{12}y^2$, $y^2 = \frac{12}{5}A_1$

$$y = \sqrt{\frac{12}{5}(270)} = 25.4558 \text{ mm}$$

$$b = 21.2132 \text{ mm}$$

$$A_1 = \frac{1}{2}(21.2132)(25.4558) = 270 \text{ mm}^2 = 270 \times 10^{-6} \text{ m}^2$$

$$A_2 = (21.2132)(36 - 25.4558) = 223.676 \text{ mm}^2 = 223.676 \times 10^{-6} \text{ m}^2$$

$$A_3 = A - A_1 - A_2 = 46.324 \text{ mm}^2 = 46.324 \times 10^{-6} \text{ m}^2$$

$$R_i = \sigma_Y A_i = 240 \times 10^6 A_i$$

$$R_1 = 64.8 \times 10^3 \text{ N}, \quad R_2 = 53.6822 \times 10^3 \text{ N}, \quad R_3 = 11.1178 \times 10^3 \text{ N}$$

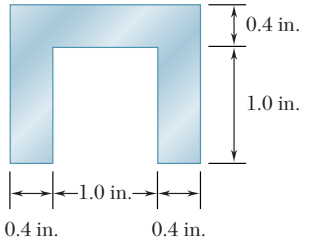
$$\bar{y}_1 = \frac{1}{3}y = 8.4853 \text{ mm} = 8.4853 \times 10^{-3} \text{ m}$$

$$\bar{y}_2 = \frac{1}{2}(36 - 25.4558) = 5.2721 \text{ mm} = 5.2721 \times 10^{-3} \text{ m}$$

$$\bar{y}_3 = \frac{2}{3}(36 - 25.4558) = 7.0295 \text{ mm} = 7.0295 \times 10^{-3} \text{ m}$$

$$M_p = R_1\bar{y}_1 + R_2\bar{y}_2 + R_3\bar{y}_3 = 911 \text{ N} \cdot \text{m} \qquad M_p = 911 \text{ N} \cdot \text{m} \blacktriangleleft$$

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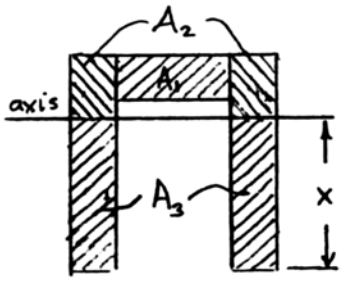
PROBLEM 4.84

Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 42 ksi.

SOLUTION

$$A = (1.0\text{ in.})(0.4\text{ in.}) + 2(1.4\text{ in.})(0.4\text{ in.}) = 1.52\text{ in}^2$$

$$x = \frac{\frac{1}{2}(1.52)}{2(0.4)} = 0.95\text{ in.}$$



$$R_1 = (1.0)(0.4) = 0.4\text{ in}^2$$

$$\bar{y}_1 = 1.2 - 0.95 = 0.25\text{ in.}$$

$$R_2 = 2(0.4)(1.4 - 0.95) = 0.36\text{ in}^2$$

$$\bar{y}_2 = \frac{1}{2}(1.4 - 0.95) = 0.225\text{ in.}$$

$$R_3 = 2(0.4)(0.95) = 0.760\text{ in}^2$$

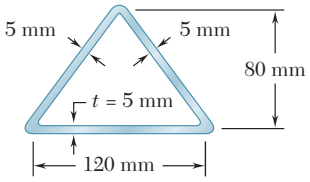
$$\bar{y}_3 = \frac{1}{2}(0.95) = 0.475\text{ in.}$$

$$M_p = (R_1\bar{y}_1 + R_2\bar{y}_2 + R_3\bar{y}_3)(\sigma_y)$$

$$= [(0.4)(0.25) + (0.36)(0.225) + (0.760)(0.475)](42)$$

$$M_p = 22.8\text{ kip}\cdot\text{in.} \blacktriangleleft$$

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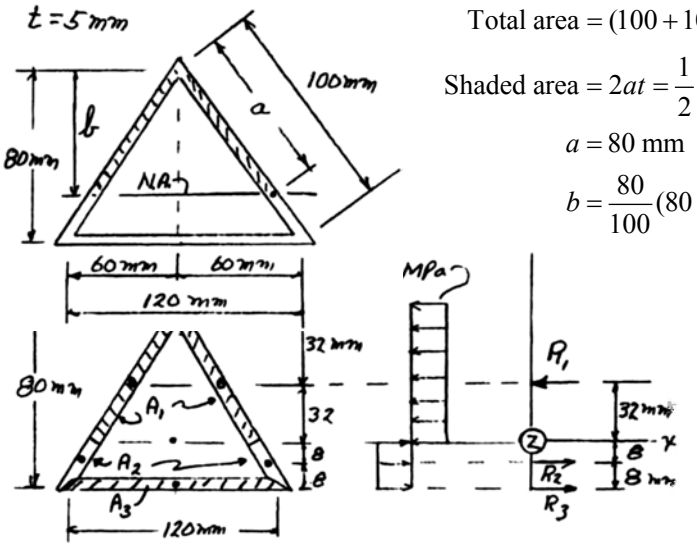


PROBLEM 4.85

Determine the plastic moment M_p of the cross section shown when the beam is bent about a horizontal axis. Assume the material to be elastoplastic with a yield strength of 175 MPa.

SOLUTION

For M_p, the neutral axis divides the area into two equal parts.



$$\text{Total area} = (100 + 100 + 120)t = 320t$$

$$\text{Shaded area} = 2at = \frac{1}{2}(320)t$$

$$a = 80 \text{ mm}$$

$$b = \frac{80}{100}(80 \text{ mm}) = 64 \text{ mm}$$

$$A_1 = 2at = 2(0.08 \text{ m})(0.005 \text{ m}) = 800 \times 10^{-6} \text{ m}^2$$

$$A_2 = 2(100 - a)t = 2(0.02 \text{ m})(0.005 \text{ m}) = 200 \times 10^{-6} \text{ m}^2$$

$$A_3 = (120 \text{ mm})t = (0.120 \text{ m})(0.005 \text{ m}) = 600 \times 10^{-6} \text{ m}^2$$

$$R_1 = \sigma_Y A_1 = (175 \text{ MPa})(800 \times 10^{-6} \text{ m}^2) = 140 \text{ kN}$$

$$R_2 = \sigma_Y A_2 = (175 \text{ MPa})(200 \times 10^{-6} \text{ m}^2) = 35 \text{ kN}$$

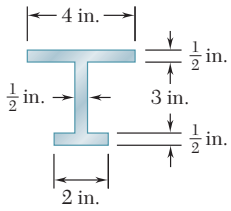
$$R_3 = \sigma_Y A_3 = (175 \text{ MPa})(600 \times 10^{-6} \text{ m}^2) = 105 \text{ kN}$$

$$\sum M_z: M_p = R_1(32 \text{ mm}) + R_2(8 \text{ mm}) + R_3(16 \text{ mm})$$

$$= (140 \text{ kN})(0.032 \text{ m}) + (35 \text{ kN})(0.008 \text{ m}) + (105 \text{ kN})(0.016 \text{ m})$$

$$M_p = 6.44 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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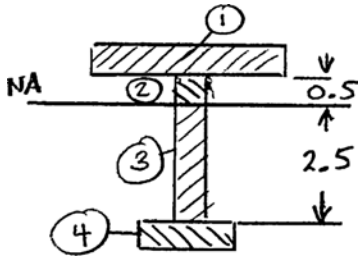
PROBLEM 4.86

Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 36 ksi.

SOLUTION

Total area:
$$A = (4)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)(3) + (2)\left(\frac{1}{2}\right) = 4.5 \text{ in}^2$$

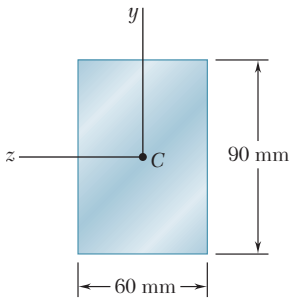
$$\frac{1}{2}A = 2.25 \text{ in}^2$$



$$A_1 = 2.00 \text{ in}^2, \quad \bar{y}_1 = 0.75, \quad A_1\bar{y}_1 = 1.50 \text{ in}^3$$
$$A_2 = 0.25 \text{ in}^2, \quad \bar{y}_2 = 0.25, \quad A_2\bar{y}_2 = 0.0625 \text{ in}^3$$
$$A_3 = 1.25 \text{ in}^2, \quad \bar{y}_3 = 1.25, \quad A_3\bar{y}_3 = 1.5625 \text{ in}^3$$
$$A_4 = 1.00 \text{ in}^2, \quad \bar{y}_4 = 2.75, \quad A_4\bar{y}_4 = 2.75 \text{ in}^3$$

$$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4)$$
$$= (36)(1.50 + 0.0625 + 1.5625 + 2.75)$$
$$M_p = 212 \text{ kip} \cdot \text{in}. \blacktriangleleft$$

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PROBLEM 4.87

For the beam indicated (of Prob. 4.73), a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45\,\text{mm}$.

SOLUTION

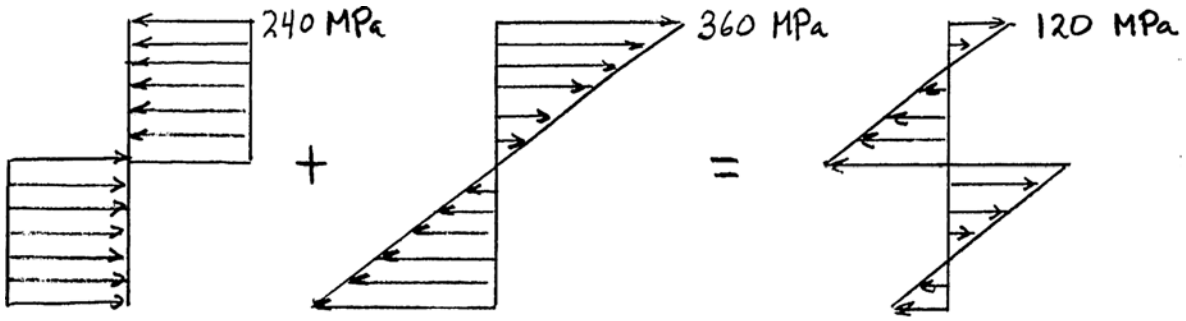
$$M_p = 29.16 \times 10^3\ \text{N} \cdot \text{m}$$

See solutions to Problems 4.73 and 4.77.

$$I = 3.645 \times 10^{-6}\ \text{m}^4$$

$$c = 0.045\ \text{m}$$

$$\sigma' = \frac{M_{\max} y}{I} = \frac{M_p c}{I} \qquad \text{at } y = c = 45\ \text{mm}$$



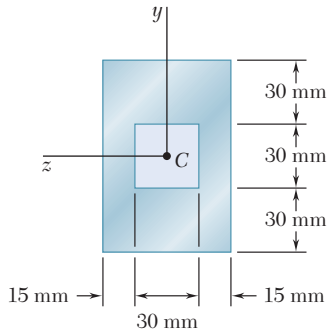
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$$\sigma' = \frac{(29.16 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 360 \times 10^6\ \text{Pa}$$

$$\sigma_{\text{res}} = \sigma' - \sigma_Y = 360 \times 10^6 - 240 \times 10^6$$

$$= 120 \times 10^6\ \text{Pa} \qquad \sigma_{\text{res}} = 120.0\ \text{MPa} \blacktriangleleft$$

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PROBLEM 4.88

For the beam indicated (of Prob. 4.74), a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45\,\text{mm}$.

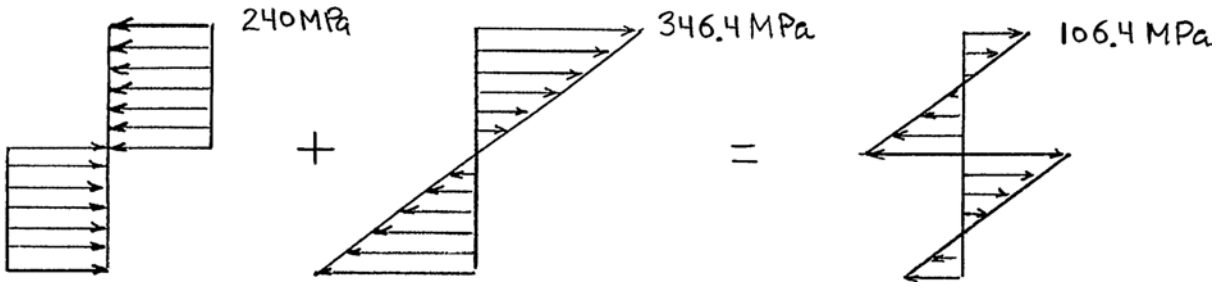
SOLUTION

$M_p = 27.54 \times 10^3\ \text{N} \cdot \text{m}$ (See solutions to Problems 4.74 and 4.78.)

$I = 3.5775 \times 10^{-6}\ \text{m}^4, \qquad c = 0.045\ \text{m}$

$$\sigma' = \frac{M_{\max} y}{I} = \frac{M_p c}{I} \quad \text{at} \quad y = c$$

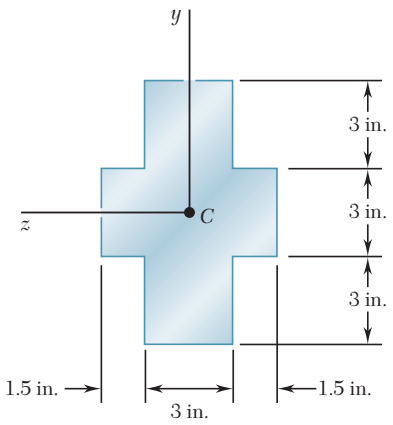
$$\sigma' = \frac{(27.54 \times 10^3)(0.045)}{3.5775 \times 10^{-6}} = 346.4 \times 10^6\ \text{Pa}$$



$$\sigma_{\text{res}} = \sigma' - \sigma_Y = 346.4 \times 10^6 - 240 \times 10^6 = 106.4 \times 10^6\ \text{Pa}$$

$$\sigma_{\text{res}} = 106.4\ \text{MPa} \blacktriangleleft$$

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PROBLEM 4.89

A bending couple is applied to the bar indicated, causing plastic zones 3 in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 4.5$ in., (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

Beam of Prob. 4.75.

SOLUTION

See solution to Problem 4.75 for bending couple and stress distribution during loading.

$$M = 2646 \text{ kip} \cdot \text{in.} \quad y_Y = 1.5 \text{ in.} \quad E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

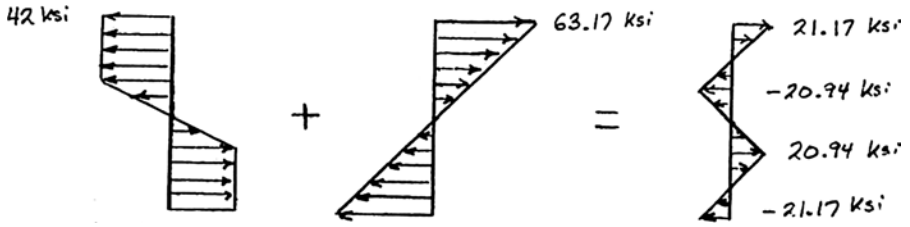
$$\sigma_Y = 42 \text{ ksi} \quad I = 188.5 \text{ in}^4 \quad c = 4.5 \text{ in.}$$

(*a*) $$\sigma' = \frac{Mc}{I} = \frac{(2646)(4.5)}{188.5} = 63.167 \text{ ksi}$$

$$\sigma'' = \frac{My_Y}{I} = \frac{(2646)(1.5)}{188.5} = 21.056 \text{ ksi}$$

At $y = c$, $\sigma_{\text{res}} = \sigma' - \sigma_Y = 63.167 - 42 = 21.167$ ksi

At $y = y_Y$, $\sigma_{\text{res}} = \sigma'' - \sigma_Y = 21.056 - 42 = -20.944$ ksi $\qquad \sigma_{\text{res}} = -20.9$ ksi ◀



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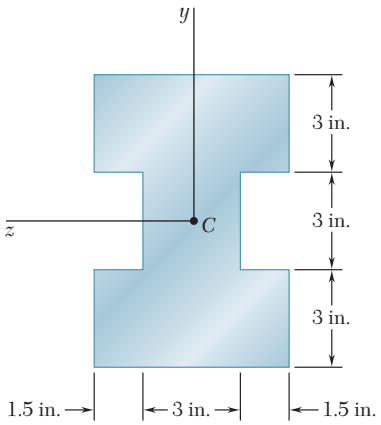
(*b*) $$\sigma_{\text{res}} = 0 \quad \therefore \quad \frac{My_0}{I} = \sigma_Y$$

$$y_0 = \frac{I\sigma_Y}{M} = \frac{(188.5)(42)}{2646} = 2.99 \text{ in.}$$ **Answer:** $y_0 = -2.99$ in., 0, 2.99 in. ◀

(*c*) At $y = y_Y$, $\sigma_{\text{res}} = -20.944$ ksi

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(29 \times 10^3)(1.5)}{20.944} = 2077 \text{ in.}$$ $\rho = 173.1$ ft ◀

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PROBLEM 4.90

A bending couple is applied to the bar indicated, causing plastic zones 3 in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 4.5$ in., (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

Beam of Prob. 4.76.

SOLUTION

See solution to Problem 4.76 for bending couple and stress distribution.

$$M = 4725 \text{ kip} \cdot \text{in.} \qquad y_Y = 1.5 \text{ in.} \qquad E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

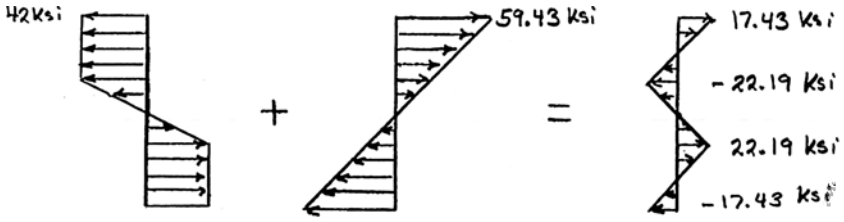
$$\sigma_Y = 42 \text{ ksi} \qquad I = 357.75 \text{ in}^4 \qquad c = 4.5 \text{ in.}$$

(*a*) $$\sigma' = \frac{Mc}{I} = \frac{(4725)(4.5)}{357.75} = 59.434 \text{ ksi}$$ $\sigma' = 59.4$ ksi ◀

$$\sigma'' = \frac{My_Y}{I} = \frac{(4725)(1.5)}{357.75} = 19.8113 \text{ ksi}$$

At $y = c$, $\sigma_{\text{res}} = \sigma' - \sigma_Y = 59.434 - 42 = 17.4340$ ksi

At $y = y_Y$, $\sigma_{\text{res}} = \sigma'' - \sigma_Y = 19.8113 - 42 = -22.189$ ksi



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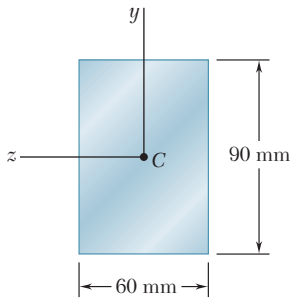
(*b*) $\sigma_{\text{res}} = 0 \quad \therefore \quad \frac{My_0}{I} - \sigma_Y = 0$

$$y_0 = \frac{I\sigma_Y}{M} = \frac{(357.75)(42)}{4725} = 3.18 \text{ in.}$$ **Answer:** $y_0 = -3.18$ in., 0, 3.18 in. ◀

(*c*) At $y = y_Y$, $\sigma_{\text{res}} = -22.189$ ksi

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(29 \times 10^3)(1.5)}{22.189} = 1960.43 \text{ in.}$$ $\rho = 163.4$ ft. ◀

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PROBLEM 4.91

A bending couple is applied to the beam of Prob. 4.73, causing plastic zones 30 mm thick to develop at the top and bottom of the beam. After the couple has been removed, determine (*a*) the residual stress at $y = 45$ mm, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the beam.

SOLUTION

See solution to Problem 4.73 for bending couple and stress distribution during loading.

$$M = 28.08 \times 10^3 \text{ N} \cdot \text{m} \quad y_Y = 15 \text{ mm} = 0.015 \text{ m} \quad E = 200 \text{ GPa}$$

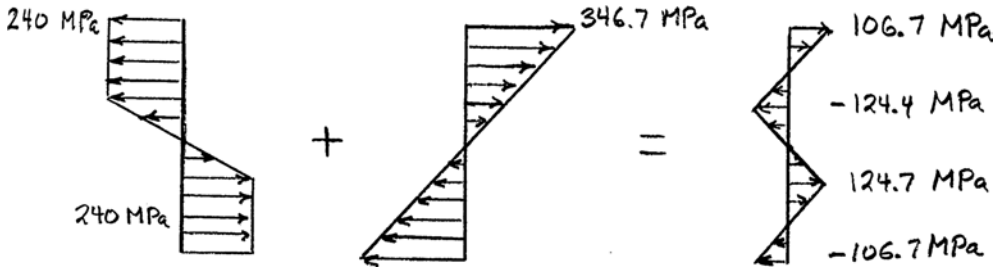
$$\sigma_Y = 240 \text{ MPa} \quad I = 3.645 \times 10^{-6} \text{ m}^4 \quad c = 0.045 \text{ m}$$

(*a*) $$\sigma' = \frac{Mc}{I} = \frac{(28.08 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 346.67 \times 10^6 \text{ Pa} = 346.67 \text{ MPa}$$

$$\sigma'' = \frac{My_Y}{I} = \frac{(28.08 \times 10^3)(0.015)}{3.645 \times 10^{-6}} = 115.556 \times 10^6 \text{ Pa} = 115.556 \text{ MPa}$$

At $y = c$, $\sigma_{\text{res}} = \sigma' - \sigma_Y = 346.67 - 240 = 106.670$ MPa $\quad \sigma_{\text{res}} = 106.7$ MPa ◀

At $y = y_Y$, $\sigma_{\text{res}} = \sigma'' - \sigma_Y = 115.556 - 240 = -124.444$ MPa $\quad \sigma_{\text{res}} = -124.4$ MPa



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(*b*) $$\sigma_{\text{res}} = 0 \quad \therefore \quad \frac{My_0}{I} - \sigma_Y = 0$$

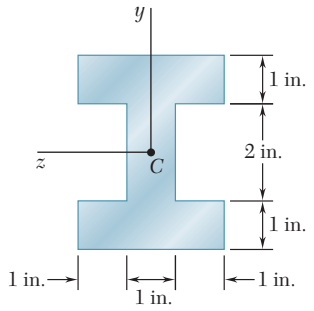
$$y_0 = \frac{I\sigma_Y}{M} = \frac{(3.645 \times 10^{-6})(240 \times 10^6)}{28.08 \times 10^3} = 31.15 \times 10^{-3} \text{ m} = 31.15 \text{ mm}$$

Answer: $y_0 = -31.2$ mm, 0, 31.2 mm ◀

(*c*) At $y = y_Y$, $\sigma_{\text{res}} = -124.444 \times 10^6$ Pa

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(200 \times 10^9)(0.015)}{-124.444 \times 10^6} \qquad \rho = 24.1 \text{ m} \blacktriangleleft$$

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PROBLEM 4.92

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi. A bending couple is applied to the beam about the z axis, causing plastic zones 2 in. thick to develop at the top and bottom of the beam. After the couple has been removed, determine (*a*) the residual stress at $y = 2$ in., (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the beam.

SOLUTION

Flange: $$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(1)^3 + (3)(1)(1.5)^2 = 7.0 \text{ in}^4$$

Web: $$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(1)(2)^3 = 0.6667 \text{ in}^4$$

$$I_3 = I_1 = 7.0 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 14.6667 \text{ in}^4$$

$$c = 2 \text{ in.}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(14.6667)}{2}$$

$$M_Y = 308 \text{ kip} \cdot \text{in.} \blacktriangleleft$$

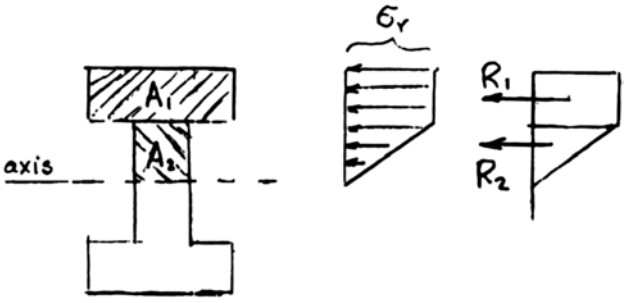
$$R_1 = \sigma_Y A_1 = (42)(3)(1) = 126 \text{ kips}$$

$$\bar{y}_1 = 1.0 + 0.5 = 1.5 \text{ in.}$$

$$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(1)(1)$$

$$= 21 \text{ kips}$$

$$\bar{y}_2 = \frac{2}{3}(1.0) = 0.6667 \text{ in.}$$



$$M = 2(R_1 \bar{y}_1 + R_2 \bar{y}_2) = 2[(126)(1.5) + (21)(0.6667)]$$

$$M = 406 \text{ kip} \cdot \text{in.} \blacktriangleleft$$

$M = 406$ kip · in. $y_Y = 1.0$ in. $E = 29 \times 10^6 = 29 \times 10^3$ ksi

$\sigma_Y = 42$ ksi $I = 14.6667$ in^4 $c = 2$ in.

(*a*) $$\sigma' = \frac{Mc}{I} \frac{(406)(2)}{14.6667} = 55.364$$

$$\sigma'' = \frac{My_Y}{I} \frac{(406)(1.0)}{14.662} = 27.682$$

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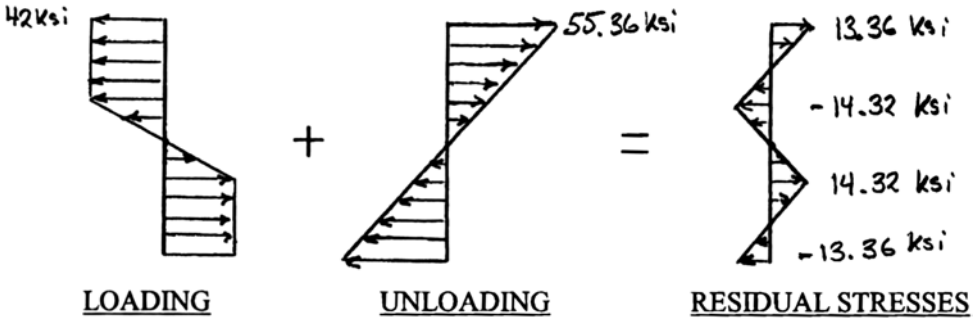
PROBLEM 4.92 ***(Continued)***

At $y = c$, $\sigma_{\text{res}} = \sigma' - \sigma_Y = 55.364 - 42 = 13.3640$ ksi

$$\sigma_{\text{res}} = 13.36\,\text{ksi} \blacktriangleleft$$

At $y = y_Y$, $\sigma_{\text{res}} = \sigma' - \sigma_Y = 27.682 - 42 = -14.3180$ ksi

$$\sigma_{\text{res}} = -14.32\,\text{ksi} \blacktriangleleft$$



(*b*) $\sigma_{\text{res}} = 0 \quad \therefore \quad \dfrac{My_0}{I} - \sigma_Y = 0$

$$y_0 = \frac{I\sigma_Y}{M} = \frac{(14.667)(42)}{406} = 1.517 \text{ in}$$

Answer : $y_0 = -1.517$ in., $0, 1.517$ in. ◀

(*c*) At $y = y_Y$, $\sigma_{\text{res}} = -14.3180$ ksi

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(29 \times 10^3)(1.0)}{14.3180} = 2025.42 \text{ in.}$$

$$\rho = 168.8 \text{ ft} \blacktriangleleft$$

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PROBLEM 4.93*

A rectangular bar that is straight and unstressed is bent into an arc of circle of radius by two couples of moment M. After the couples are removed, it is observed that the radius of curvature of the bar is ρ_R. Denoting by ρ_Y the radius of curvature of the bar at the onset of yield, show that the radii of curvature satisfy the following relation:

$$\frac{1}{\rho_R} = \frac{1}{\rho}\left\{1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left[1 - \frac{1}{3}\left(\frac{\rho}{\rho_Y}\right)^2\right]\right\}$$

SOLUTION

$$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \qquad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right),$$

Let m denote $\dfrac{M}{M_Y}$.

$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{\rho^2}{\rho_Y^2}\right) \quad \therefore \quad \frac{\rho^2}{\rho_Y^2} = 3 - 2\,m$$

$$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y}$$

$$= \frac{1}{\rho}\left\{1 - \frac{\rho}{\rho_Y}m\right\} = \frac{1}{\rho}\left\{1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)\right\} \quad \blacktriangleleft$$

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PROBLEM 4.94

A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by M_Y and ρ_Y, respectively, the bending moment and radius of curvature at the onset of yield, determine (*a*) the radius of curvature when a couple of moment $M = 1.25M_Y$ is applied to the bar, (*b*) the radius of curvature after the couple is removed. Check the results obtained by using the relation derived in Prob. 4.93.

SOLUTION

(*a*) $$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \qquad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \text{Let } m = \frac{M}{M_Y} = 1.25$$

$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \frac{\rho}{\rho_Y} = \sqrt{3 - 2m} = 0.70711$$

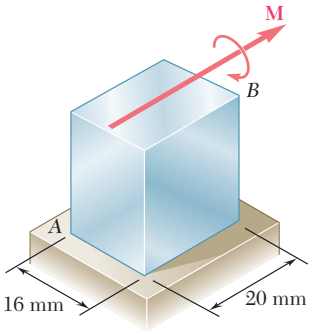
$$\rho = 0.707\rho_Y \blacktriangleleft$$

(*b*) $$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y} = \frac{1}{0.70711\rho_Y} - \frac{1.25}{\rho_Y}$$

$$= \frac{0.16421}{\rho_Y}$$

$$\rho_R = 6.09\rho_Y \blacktriangleleft$$

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PROBLEM 4.95

The prismatic bar AB is made of a steel that is assumed to be elastoplastic and for which $E = 200\text{ GPa}$. Knowing that the radius of curvature of the bar is 2.4 m when a couple of moment $M = 350\text{ N}\cdot\text{m}$ is applied as shown, determine (*a*) the yield strength of the steel, (*b*) the thickness of the elastic core of the bar.

SOLUTION

$$M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)$$

$$= \frac{3}{2}\frac{\sigma_Y I}{c}\left(1 - \frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$

$$= \frac{3}{2}\frac{\sigma_Y b(2c)^3}{12c}\left(1 - \frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$

$$= \sigma_Y bc^2\left(1 - \frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$

(*a*) $bc^2\sigma_Y\left(1 - \dfrac{\rho^2\sigma_Y^2}{3E^2c^2}\right) = M$ Cubic equation for σ_Y

Data:

$$E = 200\times10^9\text{ Pa}$$
$$M = 420\text{ N}\cdot\text{m}$$
$$\rho = 2.4\text{ m}$$
$$b = 20\text{ mm} = 0.020\text{ m}$$
$$c = \frac{1}{2}h = 8\text{ mm} = 0.008\text{ m}$$

$$(1.28\times10^{-6})\,\sigma_Y\left[1 - 750\times10^{-21}\sigma_Y^2\right] = 350$$

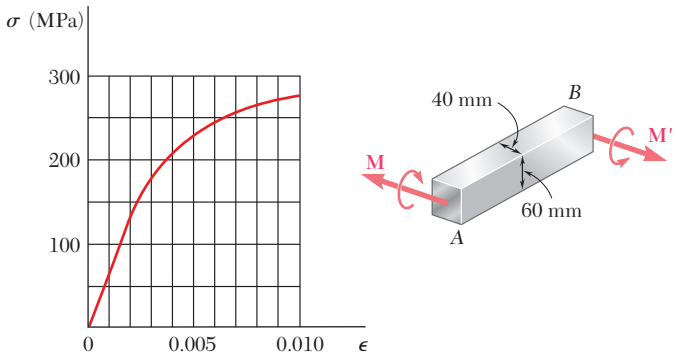
$$\sigma_Y\left[1 - 750\times10^{-21}\sigma_Y^2\right] = 273.44\times10^6$$

Solving by trial, $\sigma_Y = 292\times10^6\text{ Pa}$ $\sigma_Y = 292\text{ MPa}$ ◀

(*b*) $y_Y = \dfrac{\sigma_Y\rho}{E} = \dfrac{(292\times10^6)(2.4)}{200\times10^9} = 3.504\times10^{-3}\text{ m} = 3.504\text{ mm}$

thickness of elastic core $= 2y_Y = 7.01\text{ mm}$ ◀

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PROBLEM 4.96

The prismatic bar AB is made of an aluminum alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ε diagram is the same in compression as in tension, determine (*a*) the radius of curvature of the bar when the maximum stress is 250 MPa, (*b*) the corresponding value of the bending moment. (*Hint:* For part *b*, plot σ versus y and use an approximate method of integration.)

SOLUTION

(*a*) $\sigma_m = 250 \text{ MPa} = 250 \times 10^6 \text{ Pa}$

$\varepsilon_m = 0.0064$ from curve

$c = \frac{1}{2}\text{h} = 30 \text{ mm} = 0.030 \text{ m}$

$b = 40 \text{ mm} = 0.040 \text{ m}$

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{0.0064}{0.030} = 0.21333 \text{ m}^{-1} \qquad \rho = 4.69 \text{ m} \blacktriangleleft$$

(*b*) Strain distribution: $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{\varepsilon}$

Bending couple: $M = -\int_{-c}^{c} y\sigma b\, dy = 2b\int_0^c y\,|\sigma|\, dy = 2bc^2\int_0^1 u|\sigma|\, du = 2bc^2 J$

where the integral J is given by $\int_0^1 u|\sigma|\, du$

Evaluate J using a method of numerial integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\Sigma wu|\sigma|$$

where w is a weighting factor.

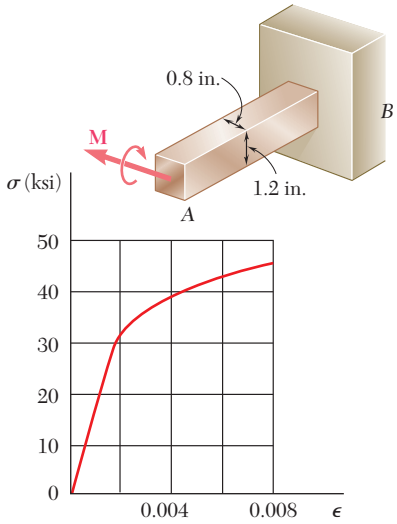
Using $\Delta u = 0.25$, we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, (MPa)	$u\,\lvert\sigma\rvert$, (MPa)	w	$wu\,\lvert\sigma\rvert$, (MPa)
0	0	0	0	1	0
0.25	0.0016	110	27.5	4	110
0.5	0.0032	180	90	2	180
0.75	0.0048	225	168.75	4	675
1.00	0.0064	250	250	1	250
					1215 ← $\Sigma wu\,\lvert\sigma\rvert$

$$J = \frac{(0.25)(1215)}{3} = 101.25 \text{ MPa} = 101.25 \times 10^6 \text{ Pa}$$

$$M = (2)(0.040)(0.030)^2(101.25 \times 10^6) = 7.29 \times 10^3 \text{ N} \cdot \text{m} \qquad M = 7.29 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 4.97

The prismatic bar AB is made of a bronze alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ε diagram is the same in compression as in tension, determine (*a*) the maximum stress in the bar when the radius of curvature of the bar is 100 in., (*b*) the corresponding value of the bending moment. (See hint given in Prob. 4.96.)

SOLUTION

(*a*) $\rho = 100$ in., $b = 0.8$ in., $c = 0.6$ in.

$$\varepsilon_m = \frac{c}{\rho} = \frac{0.6}{100} = 0.006$$

From the curve, $\sigma_m = 43.0$ ksi ◀

(*b*) Strain distribution: $\varepsilon = -\varepsilon_m \dfrac{y}{c} = -\varepsilon_m u$ where $u = \dfrac{y}{\varepsilon}$

Bending couple: $M = -\int_{-c}^{c} y\,\sigma\, b\, dy = 2b\int_0^c y\,|\sigma|dy = 2bc^2\int_0^1 u\,|\sigma|\,du = 2bc^2 J$

where the integral J is given by $\int_0^1 u\,|\sigma|\,du$

Evaluate J using a method of numerial integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\Sigma wu|\sigma|$$

where w is a weighting factor.

Using $\Delta u = 0.25$, we get the values given the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, ksi	$u\,\lvert\sigma\rvert$, ksi	w	$wu\,\lvert\sigma\rvert$, ksi
0	0	0	0	1	0
0.25	0.0015	25	6.25	4	25
0.5	0.003	36	18	2	36
0.75	0.0045	40	30	4	120
1.00	0.006	43	43	1	43
					224 ← $\Sigma wu\,\lvert\sigma\rvert$

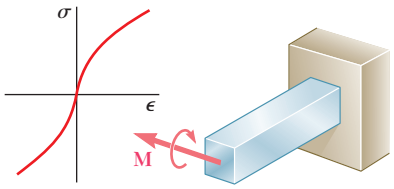
$$J = \frac{(0.25)(224)}{3} = 18.67 \text{ ksi}$$

$$M = (2)(0.8)(0.6)^2(18.67)$$

$M = 10.75$ kip · in. ◀

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PROBLEM 4.98



A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation $\varepsilon = k\sigma^n$ for $\sigma > 0$ and $\varepsilon = -|k\sigma^n|$ for $\sigma < 0$. If a couple **M** is applied to the bar, show that the maximum stress is

$$\sigma_m = \frac{1+2n}{3n}\frac{Mc}{I}$$

SOLUTION

Strain distribution: $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u \quad \text{where} \quad u = \frac{y}{c}$

Bending couple:

$$M = -\int_{-c}^{c} y\,\sigma\, b\, dy = 2b\int_0^c y\,|\sigma|\,dy = 2bc^2\int_0^c \frac{y}{c}|\sigma|\frac{dy}{c}$$

$$= 2bc^2\int_0^1 u\,|\sigma|\,du$$

For $\varepsilon = K\sigma^n, \quad \varepsilon_m = K\sigma_m$

$$\frac{\varepsilon}{\varepsilon_m} = u = \left(\frac{\sigma}{\sigma_m}\right)^n \quad \therefore \quad |\sigma| = \sigma_m u^{\frac{1}{n}}$$

Then $M = 2bc^2\int_0^1 u\sigma_m u^{\frac{1}{n}}\,du = 2bc^2\sigma_m\int_0^1 u^{1+\frac{1}{n}}du$

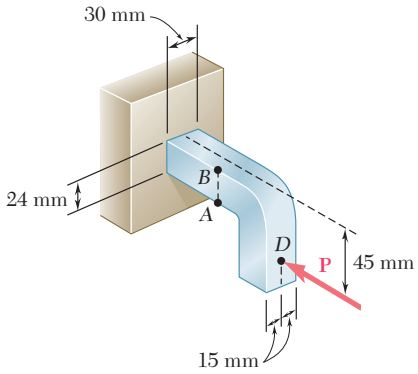
$$= 2bc^2\sigma_m \frac{u^{2+\frac{1}{n}}}{2+\frac{1}{n}}\Big|_0^1 = \frac{2n}{2n+1}bc^2\sigma_m$$

$$\sigma_m = \frac{2n+1}{2}\frac{M}{bc^2}$$

Recall that $\frac{I}{c} = \frac{1}{12}\frac{b(2c)^3}{c} = \frac{2}{3}bc^2 \quad \therefore \quad \frac{1}{bc^2} = \frac{2}{3}\frac{c}{I}$

Then $\sigma_m = \frac{2n+1}{3n}\frac{Mc}{I}$

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PROBLEM 4.99

Knowing that the magnitude of the horizontal force **P** is 8 kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$A = (30)(24) = 720\ \text{mm}^2 = 720 \times 10^{-6}\ \text{m}^2$$

$$e = 45 - 12 = 33\ \text{mm} = 0.033\ \text{m}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(30)(24)^3 = 34.56 \times 10^3\ \text{mm}^4 = 34.56 \times 10^{-9}\ \text{m}^4$$

$$c = \frac{1}{2}(24\ \text{mm}) = 12\ \text{mm} = 0.012\ \text{m} \qquad P = 8 \times 10^3\ \text{N}$$

$$M = Pe = (8 \times 10^3)(0.033) = 264\ \text{N} \cdot \text{m}$$

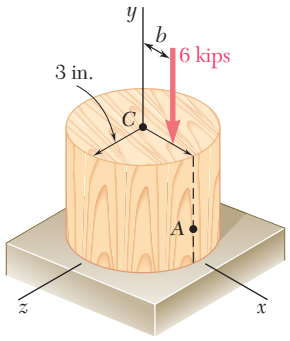
(*a*) $$\sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} - \frac{(264)(0.012)}{34.56 \times 10^{-9}} = -102.8 \times 10^6\ \text{Pa}$$

$$\sigma_A = -102.8\ \text{MPa} \blacktriangleleft$$

(*b*) $$\sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} + \frac{(264)(0.012)}{34.56 \times 10^{-9}} = 80.6 \times 10^6\ \text{Pa}$$

$$\sigma_B = 80.6\ \text{MPa} \blacktriangleleft$$

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PROBLEM 4.100

A short wooden post supports a 6-kip axial load as shown. Determine the stress at point A when (a) $b = 0$, (b) $b = 1.5$ in., (c) $b = 3$ in.

SOLUTION

$$A = \pi r^2 = \pi(3)^2 = 28.27 \text{ in}^2$$

$$I = \frac{\pi}{4} r^4 = \frac{\pi}{4}(3)^4 = 63.62 \text{ in}^4$$

$$S = \frac{I}{c} = \frac{63.62}{3} = 21.206 \text{ in}^3$$

$$P = 6 \text{ kips} \qquad M = Pb$$

(a) $b = 0$ $\quad M = 0$

$$\sigma = -\frac{P}{A} = -\frac{6}{28.27} = -0.212 \text{ ksi}$$

$\sigma = -212$ psi ◀

(b) $b = 1.5$ in. $\quad M = (6)(1.5) = 9$ kip · in.

$$\sigma = -\frac{P}{A} - \frac{M}{S} = -\frac{6}{28.27} - \frac{9}{21.206} = -0.637 \text{ ksi}$$

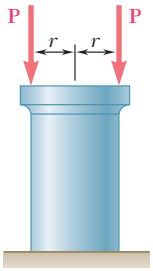
$\sigma = -637$ psi ◀

(c) $b = 3$ in. $\quad M = (6)(3) = 18$ kip · in.

$$\sigma = -\frac{P}{A} - \frac{M}{S} = -\frac{6}{28.27} - \frac{18}{21.206} = -1.061 \text{ ksi}$$

$\sigma = -1061$ psi ◀

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PROBLEM 4.101

Two forces **P** can be applied separately or at the same time to a plate that is welded to a solid circular bar of radius *r*. Determine the largest compressive stress in the circular bar, (*a*) when both forces are applied, (*b*) when only one of the forces is applied.

SOLUTION

For a solid section, $A = \pi r^2, \quad I = \frac{\pi}{4} r^4, \quad c = r$

Compressive stress $\sigma = -\frac{F}{A} - \frac{Mc}{I}$

$$= -\frac{F}{\pi r^2} - \frac{4M}{\pi r^3}$$

(*a*) Both forces applied. $F = 2P, \quad M = 0$

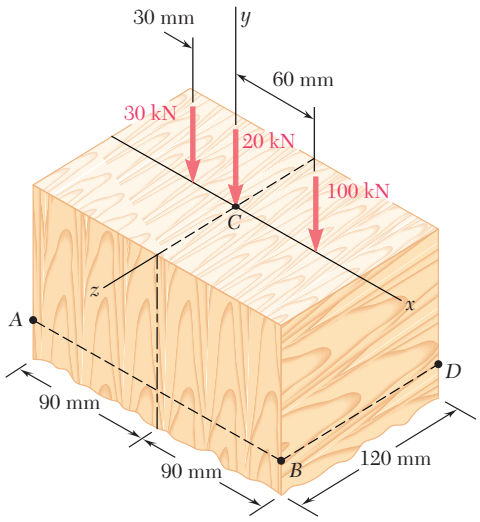
$$\sigma = \frac{-2P}{\pi r^2} \blacktriangleleft$$

(*b*) One force applied. $F = P, \quad M = Pr$

$$\sigma = -\frac{F}{\pi r^2} - \frac{4P_r}{\pi r^2}$$

$$\sigma = \frac{-5P}{\pi r^2} \blacktriangleleft$$

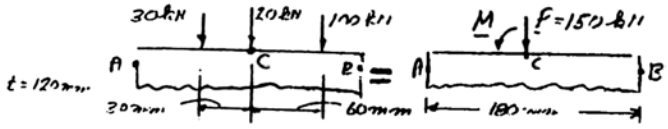
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PROBLEM 4.102

A short 120 × 180-mm column supports the three axial loads shown. Knowing that section *ABD* is sufficiently far from the loads to remain plane, determine the stress at (*a*) corner *A*, (*b*) corner *B*.

SOLUTION



$$A = (0.120\ \text{m})(0.180\ \text{m}) = 21.6 \times 10^{-3}\ \text{m}^2$$

$$S = \frac{1}{6}(0.120\ \text{m})(0.180\ \text{m})^2 = 6.48 \times 10^{-4}\ \text{m}^2$$

$$M = (30\ \text{kN})(0.03\ \text{m}) - (100\ \text{kN})(0.06\ \text{m}) = -5.10\ \text{kN} \cdot \text{m}$$

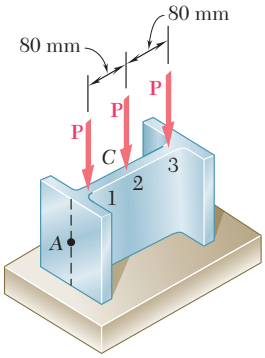
(*a*) $$\sigma_A = -\frac{P}{A} - \frac{M}{S} = \frac{-150 \times 10^3\ \text{N}}{21.6 \times 10^{-3}\ \text{m}^2} - \frac{-5.10 \times 10^3\ \text{N} \cdot \text{m}}{6.48 \times 10^{-4}\ \text{m}^3}$$

$$\sigma_A = 0.926\ \text{MPa} \blacktriangleleft$$

(*b*) $$\sigma_B = -\frac{P}{A} + \frac{M}{S} = \frac{-150 \times 10^3\ \text{N}}{21.6 \times 10^{-3}\ \text{m}^2} + \frac{-5.10 \times 10^3\ \text{N} \cdot \text{m}}{6.48 \times 10^{-4}\ \text{m}^3}$$

$$\sigma_B = -14.81\ \text{MPa} \blacktriangleleft$$

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PROBLEM 4.103

As many as three axial loads, each of magnitude $P = 50$ kN, can be applied to the end of a W200 × 31.1 rolled-steel shape. Determine the stress at point A (*a*) for the loading shown, (*b*) if loads are applied at points 1 and 2 only.

SOLUTION

For W200 × 31.3 rolled-steel shape.

$$A = 3970 \text{ mm}^2 = 4.000 \times 10^{-3} \text{ m}^2$$

$$c = \frac{1}{2}d = \frac{1}{2}(210) = 105 \text{ mm} = 0.105 \text{ m}$$

$$I = 31.3 \times 10^{6} \text{ mm}^4 = 31.3 \times 10^{-6} \text{ m}^4$$

(*a*) Centric load: $3P = 50 + 50 + 50 = 150 \text{ kN} = 150 \times 10^{3} \text{ N}$

$$\sigma = -\frac{3P}{A} = -\frac{150 \times 10^{3}}{3.970 \times 10^{-3}} = -37.783 \times 10^{6} \text{ Pa} = -37.8 \text{ MPa} \quad \blacktriangleleft$$

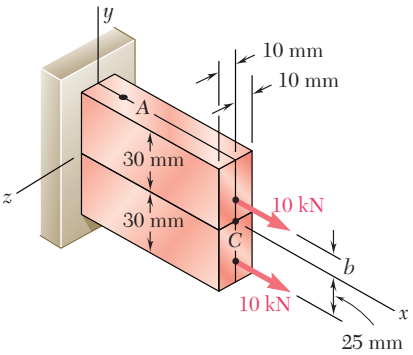
(*b*) Ececentric loading: $e = 80 \text{ mm} = 0.080 \text{ m}$

$$2P = 50 + 50 = 100 \text{ kN} = 100 \times 10^{3} \text{ N}$$

$$M = Pe = (50 \times 10^{3})(0.080) = 4.0 \times 10^{3} \text{ N} \cdot \text{m}$$

$$\sigma_A = -\frac{2P}{A} - \frac{Mc}{I} = -\frac{100 \times 10^{3}}{3.970 \times 10^{-3}} - \frac{(4.0 \times 10^{3})(0.105)}{31.3 \times 10^{-6}} = -38.607 \times 10^{6} \text{ Pa} = -38.6 \text{ MPa} \quad \blacktriangleleft$$

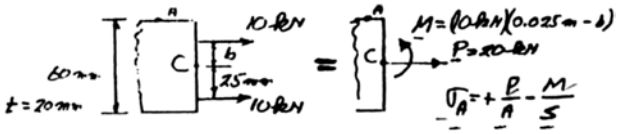
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PROBLEM 4.104

Two 10-kN forces are applied to a 20 × 60-mm rectangular bar as shown. Determine the stress at point A when (a) $b = 0$, (b) $b = 15$ mm, (c) $b = 25$ mm.

SOLUTION



$$A = (0.060\ \text{m})(0.020\ \text{m}) = 1.2 \times 10^{-3}\ \text{m}^2$$

$$S = \frac{1}{6}(0.020\ \text{m})(0.060\ \text{m})^2 = 12 \times 10^{-6}\ \text{m}^3$$

(a) $b = 0,\ M = (10\ \text{kN})(0.025\ \text{m}) = 250\ \text{N} \cdot \text{m}$

$$\sigma_A = (20\ \text{kN})/(1.2 \times 10^{-3}\ \text{m}^2) - (250\ \text{N} \cdot \text{m})/(12 \times 10^{-6}\ \text{m}^3)$$
$$= 16.667\ \text{MPa} - 20.833\ \text{MPa}$$

$$\sigma_A = -4.17\ \text{MPa} \blacktriangleleft$$

(b) $b = 15\ \text{mm},\ M = (10\ \text{kN})(0.025\ \text{m} - 0.015\ \text{m}) = 100\ \text{N} \cdot \text{m}$

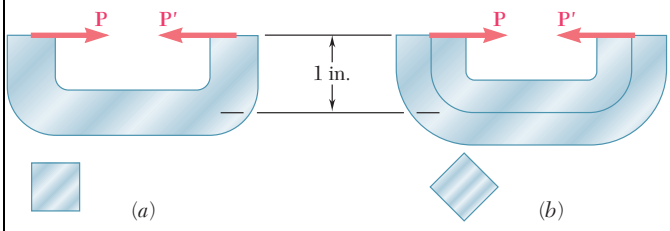
$$\sigma_A = (20\ \text{kN})/(1.2 \times 10^{-3}\ \text{m}^2) - (100\ \text{N} \cdot \text{m})/(12 \times 10^{-6}\ \text{m}^3)$$
$$= 16.667\ \text{MPa} - 8.333\ \text{MPa}$$

$$\sigma_A = 8.33\ \text{MPa} \blacktriangleleft$$

(c) $b = 25\ \text{mm},\ M = 0$: $\sigma_A = (20\ \text{kN})/(1.2 \times 10^{-3}\ \text{m}^2)$

$$\sigma_A = 16.67\ \text{MPa} \blacktriangleleft$$

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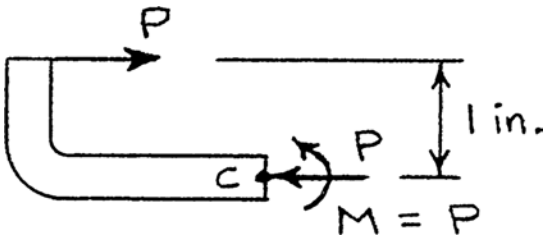


PROBLEM 4.105

Portions of a $\frac{1}{2} \times \frac{1}{2}$-in. square bar have been bent to form the two machine components shown. Knowing that the allowable stress is 15 ksi, determine the maximum load that can be applied to each component.

SOLUTION

The maximum stress occurs at point B.



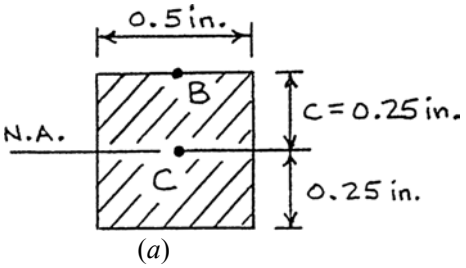
$$\sigma_B = -15 \text{ ksi} = -15 \times 10^3 \text{ psi}$$

$$\sigma_B = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pec}{I} = -KP$$

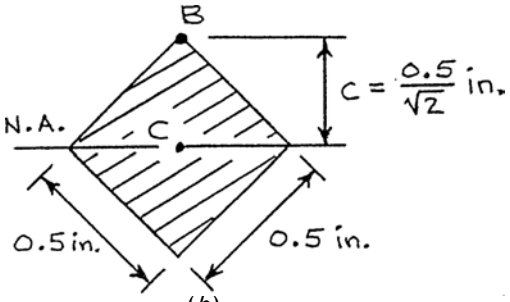
where $K = \dfrac{1}{A} + \dfrac{ec}{I}$ $\quad e = 1.0$ in.

$$A = (0.5)(0.5) = 0.25 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)(0.5)^3 = 5.2083 \times 10^{-3} \text{ in}^4 \text{ for all centroidal axes.}$$



(a)



(b)

(a) $c = 0.25$ in.

$$K = \frac{1}{0.25} + \frac{(1.0)(0.25)}{5.2083 \times 10^{-3}} = 52 \text{ in}^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-15 \times 10^3)}{52}$$

$P = 288$ lb ◀

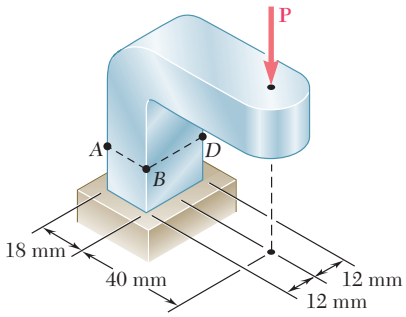
(b) $c = \dfrac{0.5}{\sqrt{2}} = 0.35355$ in.

$$K = \frac{1}{0.25} + \frac{(1.0)(0.35355)}{5.2083 \times 10^{-3}} = 71.882 \text{ in}^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-15 \times 10^3)}{71.882}$$

$P = 209$ lb ◀

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PROBLEM 4.106

Knowing that the allowable stress in section *ABD* is 80 MPa, determine the largest force **P** that can be applied to the bracket shown.

SOLUTION

$$A = (24)(18) = 432 \text{ mm}^2 = 432 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(24)(18)^3 - 11.664 \times 10^3 \text{ mm}^4 = 11.664 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}(18) = 9 \text{ mm} = 0.009 \text{ m}$$

$$e = \frac{1}{2}(18) + 40 = 49 \text{ mm} = 0.049 \text{ m}$$

On line *BD*, both axial and bending stresses are compressive.

Therefore $$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

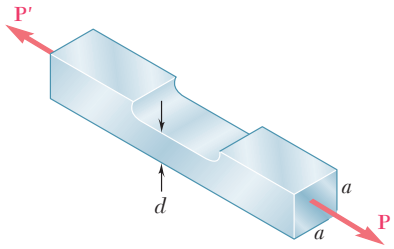
$$= \frac{P}{A} + \frac{Pec}{I}$$

Solving for *P* gives $$P = \frac{\sigma_{\max}}{\left(\frac{1}{A} + \frac{ec}{I}\right)}$$

$$P = \frac{80 \times 10^6 \text{ Pa}}{\left(\frac{1}{432 \times 10^{-6} \text{ m}^2} + \frac{(0.049 \text{ m})(0.009 \text{ m})}{11.664 \times 10^{-9} \text{ m}^4}\right)}$$

$$P = 1.994 \text{ kN} \blacktriangleleft$$

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PROBLEM 4.107

A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that $a = 30$ mm, $d = 20$ mm, and $\sigma_{all} = 60$ MPa, determine the magnitude P of the largest forces that can be safely applied at the centers of the ends of the bar.

SOLUTION

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{6Ped}{ad^3}$$

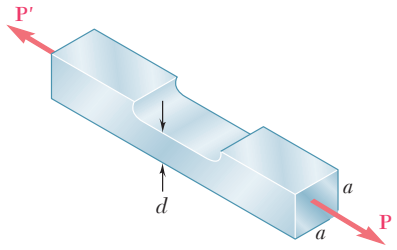
$$\sigma = \frac{P}{ad} + \frac{3P(a-d)}{ad^2} = KP \quad \text{where} \quad K = \frac{1}{ad} + \frac{3(a-d)}{ad^2}$$

Data: $a = 30 \text{ mm} = 0.030 \text{ m} \quad d = 20 \text{ mm} = 0.020 \text{ m}$

$$K = \frac{1}{(0.030)(0.020)} + \frac{(3)(0.010)}{(0.030)(0.020)^2} = 4.1667 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{60 \times 10^6}{4.1667 \times 10^3} = 14.40 \times 10^3 \text{ N} \qquad P = 14.40 \text{ kN} \blacktriangleleft$$

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PROBLEM 4.108

A milling operation was used to remove a portion of a solid bar of square cross section. Forces of magnitude $P = 18$ kN are applied at the centers of the ends of the bar. Knowing that $a = 30$ mm and $\sigma_{all} = 135$ MPa, determine the smallest allowable depth d of the milled portion of the bar.

SOLUTION

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{Pec}{I} = \frac{P}{ad} + \frac{P\frac{1}{2}(a-d)\frac{1}{2}d}{\frac{1}{12}ad^3} = \frac{P}{ad} + \frac{3P(a-d)}{ad^2}$$

$$\sigma = \frac{3P}{d^2} - \frac{2P}{ad} \quad \text{or} \quad \sigma d^2 + \frac{2P}{a}d - 3P = 0$$

Solving for d,

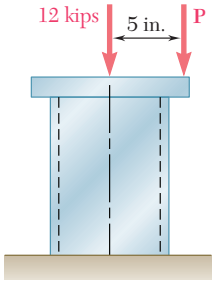
$$d = \frac{1}{2\sigma}\left\{\sqrt{\left(\frac{2P}{a}\right)^2 + 12P\sigma} - \frac{2P}{a}\right\}$$

Data: $a = 0.030$ m, $P = 18 \times 10^3$ N, $\sigma = 135 \times 10^6$ Pa

$$d = \frac{1}{(2)(135 \times 10^6)}\left\{\sqrt{\left[\frac{(2)(18 \times 10^3)}{0.030}\right]^2 + 12(18 \times 10^3)(135 \times 10^6)} - \frac{(2)(18 \times 10^3)}{0.030}\right\}$$

$$= 16.04 \times 10^{-3} \text{ m} \qquad d = 16.04 \text{ mm} \blacktriangleleft$$

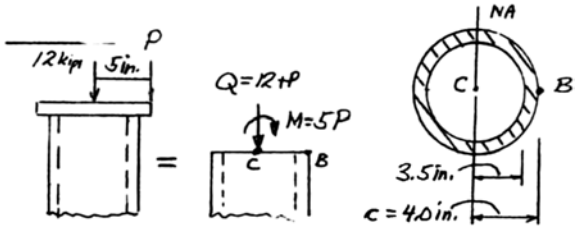
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PROBLEM 4.109

The two forces shown are applied to a rigid plate supported by a steel pipe of 8-in. outer diameter and 7-in. inner diameter. Determine the value of **P** for which the maximum compressive stress in the pipe is 15 ksi.

SOLUTION



$$\sigma_{\text{all}} = -15 \text{ ksi} \qquad I_{NA} = \frac{\pi}{4}(4 \text{ in.})^4 - \frac{\pi}{4}(3.5 \text{ in.})^4 = 83.2 \text{ in}^4$$

$$A = \pi(4 \text{ in.})^2 - \pi(3.5 \text{ in.})^2 = 11.78 \text{ in}^2$$

Max. compressive stress is at point B.

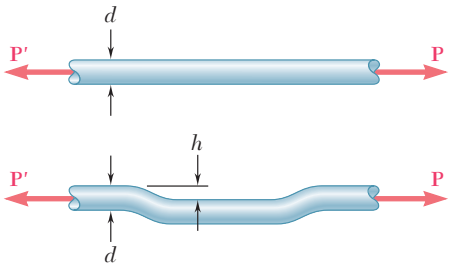
$$\sigma_B = -\frac{Q}{A} - \frac{Mc}{I} = -\frac{12 + P}{11.78 \text{ in}^2} - \frac{(5P)(4.0 \text{ in.})}{83.2 \text{ in}^4}$$

$$-15 \text{ ksi} = -1.019 - 0.085P - 0.240P$$

$$-13.981 = -0.325P$$

$$P = 43.0 \text{ kips} \blacktriangleleft$$

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PROBLEM 4.110

An offset h must be introduced into a solid circular rod of diameter d. Knowing that the maximum stress after the offset is introduced must not exceed 5 times the stress in the rod when it is straight, determine the largest offset that can be used.

SOLUTION

For centric loading, $\sigma_c = \dfrac{P}{A}$

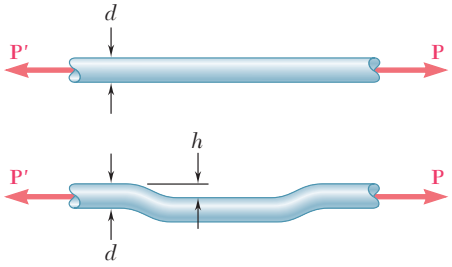
For eccentric loading, $\sigma_e = \dfrac{P}{A} + \dfrac{Phc}{I}$

Given $\sigma_e = 5\sigma_c$

$$\frac{P}{A} + \frac{Phc}{I} = 5\frac{P}{A}$$

$$\frac{Phc}{I} = 4\frac{P}{A} \quad \therefore \quad h = \frac{4I}{cA} = \frac{(4)\left(\frac{\pi}{64}d^4\right)}{\left(\frac{d}{2}\right)\left(\frac{\pi}{4}d^2\right)} = \frac{1}{2}d \qquad h = 0.500\,d \blacktriangleleft$$

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PROBLEM 4.111

An offset h must be introduced into a metal tube of 0.75-in. outer diameter and 0.08-in. wall thickness. Knowing that the maximum stress after the offset is introduced must not exceed 4 times the stress in the tube when it is straight, determine the largest offset that can be used.

SOLUTION

$$c = \frac{1}{2}d = 0.375 \text{ in.}$$

$$c_1 = c - t = 0.375 - 0.08 = 0.295 \text{ in.}$$

$$A = \pi\left(c^2 - c_1^2\right) = \pi(0.375^2 - 0.295^2)$$

$$= 0.168389 \text{ in}^2$$

$$I = \frac{\pi}{4}\left(c^4 - c_1^4\right) = \frac{\pi}{4}(0.375^4 - 0.295^4)$$

$$= 9.5835 \times 10^{-3} \text{ in}^4$$

For centric loading, $\sigma_{\text{cen}} = \dfrac{P}{A}$

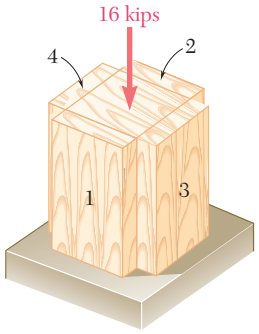
For eccentric loading, $\sigma_{\text{ecc}} = \dfrac{P}{A} + \dfrac{Phc}{I}$

$$\sigma_{\text{ecc}} = 4\sigma_{\text{cen}} \quad \text{or} \quad \frac{P}{A} + \frac{Phc}{I} = 4\frac{P}{A}$$

$$\frac{hc}{I} = \frac{3}{A} \qquad h = \frac{3I}{Ac} = \frac{(3)(9.5835 \times 10^{-3})}{(0.168389)(0.375)}$$

$$h = 0.455 \text{ in.} \blacktriangleleft$$

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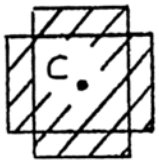


PROBLEM 4.112

A short column is made by nailing four 1 × 4-in. planks to a 4 × 4-in. timber. Using an allowable stress of 600 psi, determine the largest compressive load **P** that can be applied at the center of the top section of the timber column as shown if (*a*) the column is as described, (*b*) plank 1 is removed, (*c*) planks 1 and 2 are removed, (*d*) planks 1, 2, and 3 are removed, (*e*) all planks are removed.

SOLUTION

(*a*) Centric loading: $M = 0 \qquad \sigma = \dfrac{P}{A}$



$$A = (4 \times 4) + 4(1)(4) = 32 \text{ in}^2$$

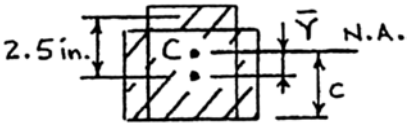
$$\sigma = \frac{P}{A} \qquad \therefore \qquad P = (0.600 \text{ ksi})(32 \text{ in}^2)$$

$$= 19.20 \text{ kips} \quad \blacktriangleleft$$

(*b*) Eccentric loading:

$$M = Pe \qquad \sigma = +\frac{P}{A} + \frac{Pec}{I}$$

$$A = (4)(4) + (3)(1)(3) = 28 \text{ in}^2 \qquad e = \bar{y}$$



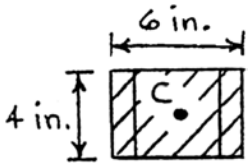
$$\bar{y} = \frac{\Sigma A\bar{y}}{A} = \frac{(1)(4)(2.5)}{28} = 0.35714 \text{ in.}$$

$$I = \Sigma(\bar{I} + Ad^2)$$

$$= \frac{1}{12}(6)(4)^3 + (6)(4)(0.35714)^2 + \frac{1}{12}(4)(1)^3 + (4)(1)(2.14286)^2 = 53.762 \text{ in}^4$$

$$\sigma = P\left(\frac{1}{A} + \frac{ec}{I}\right) \qquad \therefore \quad P = \frac{(0.600 \text{ ksi})}{\left[\dfrac{1}{28} + \dfrac{(0.35714)(2.35714)}{53.762}\right]} = 11.68 \text{ kips} \quad \blacktriangleleft$$

(*c*) Centric loading: $M = 0 \qquad \sigma = \dfrac{P}{A}$



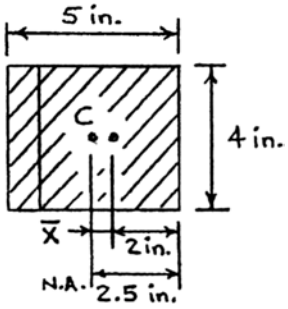
$$A = (6)(4) = 24 \text{ in}^2$$

$$P = (0.600 \text{ ksi})(24) = 14.40 \text{ kips} \quad \blacktriangleleft$$

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PROBLEM 4.112 *(Continued)*

(*d*) Eccentric loading: $M = Pe \qquad \sigma = -\dfrac{P}{A} - \dfrac{Pec}{I}$



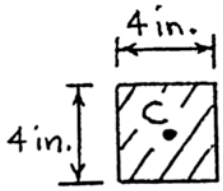
$A = (4)(4) + (1)(4)(1) = 20 \text{ in}^2 \qquad e = \bar{x}$

$\bar{x} = 2.5 - 2 = 0.5 \text{ in.}$

$I = \dfrac{1}{12}(4)(5)^3 = 41.667 \text{ in}^4$

$$P = \frac{(0.600 \text{ ksi})}{\left[\dfrac{1}{20} + \dfrac{(0.5)(2.5)}{41.667}\right]} = 7.50 \text{ kips}$$ ◀

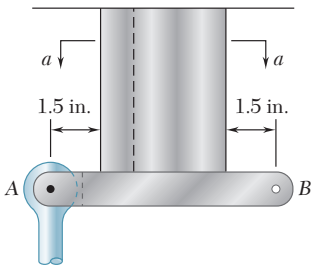
(*e*) Centric loading: $M = 0 \qquad \sigma = -\dfrac{P}{A}$

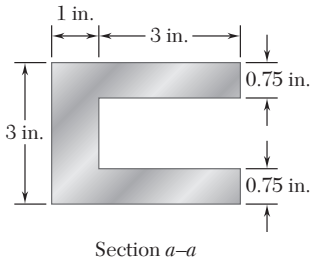


$A = (4)(4) = 16 \text{ in}^2$

$P = (0.600 \text{ ksi})(16.0 \text{ in}^2) = 9.60 \text{ kips}$ ◀

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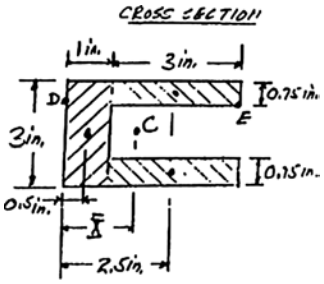




PROBLEM 4.113

A vertical rod is attached at point A to the cast iron hanger shown. Knowing that the allowable stresses in the hanger are $\sigma_{all} = +5$ ksi and $\sigma_{all} = -12$ ksi, determine the largest downward force and the largest upward force that can be exerted by the rod.

SOLUTION



$$\bar{X} = \frac{\sum A\bar{y}}{\sum A} = \frac{(1 \times 3)(0.5) + 2(3 \times 0.25)(2.5)}{(1 \times 3) + 2(3 \times 0.75)}$$

$$\bar{X} = \frac{12.75 \text{ in}^3}{7.5 \text{ in}^2} = 1.700 \text{ in.}$$

$$A = 7.5 \text{ in}^2$$

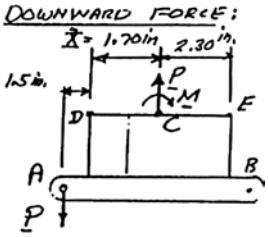
$$\sigma_{all} = +5 \text{ ksi} \qquad \sigma_{all} = -12 \text{ ksi}$$

$$I_c = \sum\left(\frac{1}{12}bh^3 + Ad^2\right)$$

$$= \frac{1}{12}(3)(1)^3 + (3 \times 1)(1.70 - 0.5)^2 + \frac{1}{12}(1.5)(3)^3 + (1.5 \times 3)(2.5 - 1.70)^2$$

$$I_c = 10.825 \text{ in}^4$$

Downward force.



$$M = P(1.5 \text{ in.} + 1.70 \text{ in.}) = (3.20 \text{ in.})P$$

At D: $\sigma_D = +\frac{P}{A} + \frac{Mc}{I}$

$$+5 \text{ ksi} = \frac{P}{7.5} + \frac{(3.20)P(1.70)}{10.825}$$

$$+5 = P(+0.6359) \qquad \mathbf{P} = 7.86 \text{ kips} \downarrow$$

At E: $\sigma_E = +\frac{P}{A} - \frac{Mc}{I}$

$$-12 \text{ ksi} = \frac{P}{7.5} - \frac{(3.20)P(2.30)}{10.825}$$

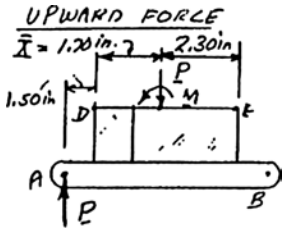
$$-12 = P(-0.5466) \qquad \mathbf{P} = 21.95 \text{ kips} \downarrow$$

We choose the smaller value. $\mathbf{P} = 7.86 \text{ kips} \downarrow$ ◀

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PROBLEM 4.113 ***(Continued)***

Upward force.



$$M = P(1.5 \text{ in.} + 1.70 \text{ in.}) = (3.20 \text{ in.})P$$

At D: $\sigma_D = +\dfrac{P}{A} - \dfrac{Mc}{I}$

$$-12 \text{ ksi} = -\frac{P}{7.5} - \frac{(3.20)P(1.70)}{10.825}$$

$$-12 = P(-0.6359) \qquad \mathbf{P} = 18.87 \text{ kips} \uparrow$$

At E: $\sigma_E = +\dfrac{P}{A} + \dfrac{Mc}{I}$

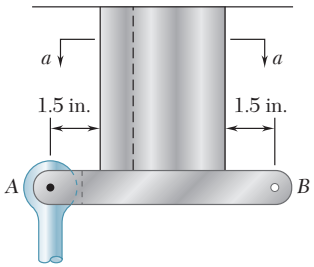
$$+5 \text{ ksi} = \frac{P}{7.5} + \frac{(3.20)P(2.30)}{10.825}$$

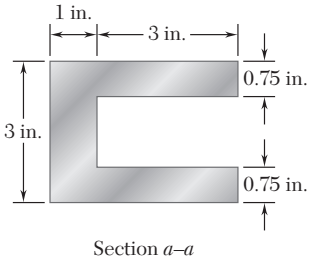
$$+5 = P(+0.5466) \qquad \mathbf{P} = 9.15 \text{ kips} \uparrow$$

We choose the smaller value. $\mathbf{P} = 9.15 \text{ kips} \uparrow \blacktriangleleft$

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PROBLEM 4.114

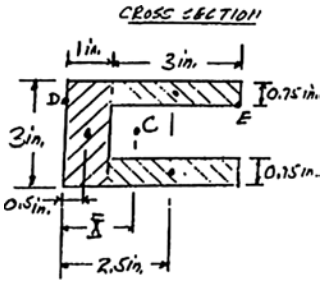




Solve Prob. 4.113, assuming that the vertical rod is attached at point *B* instead of point *A*.

PROBLEM 4.113 A vertical rod is attached at point *A* to the cast iron hanger shown. Knowing that the allowable stresses in the hanger are $\sigma_{all} = +5$ ksi and $\sigma_{all} = -12$ ksi, determine the largest downward force and the largest upward force that can be exerted by the rod.

SOLUTION



$$\bar{X} = \frac{\sum A\bar{y}}{\sum A} = \frac{(1\times 3)(0.5) + 2(3\times 0.25)(2.5)}{(1\times 3) + 2(3\times 0.75)}$$

$$\bar{X} = \frac{12.75\ \text{in}^3}{7.5\ \text{in}^2} = 1.700\ \text{in.}$$

$$A = 7.5\ \text{in}^2$$

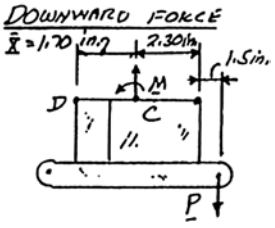
$$\sigma_{all} = +5\ \text{ksi} \qquad \sigma_{all} = -12\ \text{ksi}$$

$$I_c = \sum\left(\frac{1}{12}bh^3 + Ad^2\right)$$

$$= \frac{1}{12}(3)(1)^3 + (3\times 1)(1.70 - 0.5)^2 + \frac{1}{12}(1.5)(3)^3 + (1.5\times 3)(2.5 - 1.70)^2$$

$$I_c = 10.825\ \text{in}^4$$

Downward force.



$$\sigma_{all} = +5\ \text{ksi} \qquad \sigma_{all} = -12\ \text{ksi}$$

$$M = (2.30\ \text{in.} + 1.5\ \text{in.}) = (3.80\ \text{in.})P$$

At *D*: $\sigma_D = +\frac{P}{A} - \frac{Mc}{I}$

$$-12\ \text{ksi} = +\frac{P}{7.5} - \frac{(3.80)P(1.70)}{10.825}$$

$$-12 = P(-0.4634) \qquad \mathbf{P} = 25.9\ \text{kips} \downarrow$$

At *E*: $\sigma_E = +\frac{P}{A} + \frac{Mc}{I}$

$$+5\ \text{ksi} = +\frac{P}{7.5} + \frac{(3.80)P(2.30)}{10.825}$$

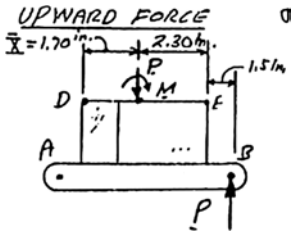
$$+5 = P(+0.9407) \qquad \mathbf{P} = 5.32\ \text{kips} \downarrow$$

We choose the smaller value. $\mathbf{P} = 5.32\ \text{kips} \downarrow \blacktriangleleft$

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PROBLEM 4.114 ***(Continued)***

Upward force.



$$\sigma_{\text{all}} = +5 \text{ ksi} \qquad \sigma_{\text{all}} = -12 \text{ ksi}$$

$$M = (2.30 \text{ in.} + 1.5 \text{ in.})P = (3.80 \text{ in.})P$$

At D: $\sigma_D = -\dfrac{P}{A} + \dfrac{Mc}{I}$

$$5 \text{ ksi} = -\frac{P}{7.5} + \frac{(3.80)P(1.70)}{10.825}$$

$$5 = P(+0.4634) \qquad \mathbf{P} = 10.79 \text{ kips} \uparrow$$

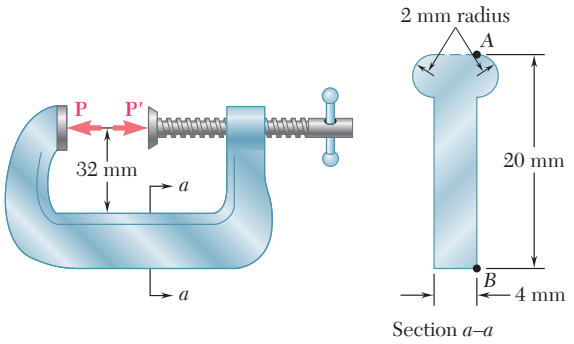
At E: $\sigma_E = -\dfrac{P}{A} - \dfrac{Mc}{I}$

$$-12 \text{ ksi} = -\frac{P}{7.5} - \frac{(3.80)P(2.30)}{10.825}$$

$$-12 = P(-0.9407) \qquad \mathbf{P} = 12.76 \text{ kips} \uparrow$$

We choose the smaller value. $\mathbf{P} = 10.79 \text{ kips} \uparrow \blacktriangleleft$

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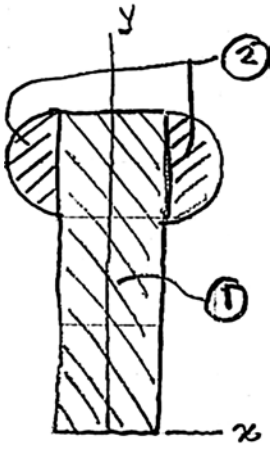


PROBLEM 4.115

Knowing that the clamp shown has been tightened until $P = 400$ N, determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the location of the neutral axis of section $a - a$.

SOLUTION

Cross section: Rectangle ① + Circle ②



$$A_1 = (20\text{ mm})(4\text{ mm}) = 80\text{ mm}^2$$

$$\bar{y}_1 = \frac{1}{2}(20\text{ mm}) = 10\text{ mm}$$

$$A_2 = \pi(2\text{ mm})^2 = 4\pi\text{ mm}^2$$

$$\bar{y}_2 = 20 - 2 = 18\text{ mm}$$

$$c_B = \bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{(80)(10) + (4\pi)(18)}{80 + 4\pi} = 11.086\text{ mm}$$

$$c_A = 20 - \bar{y} = 8.914\text{ mm}$$

$$d_1 = 11.086 - 10 = 1.086\text{ mm}$$

$$d_2 = 18 - 11.086 = 6.914\text{ mm}$$

$$I_1 = \bar{I}_1 + A_1 d_1^2 = \frac{1}{12}(4)(20)^3 + (80)(1.086)^2 = 2.761 \times 10^3\text{ mm}^4$$

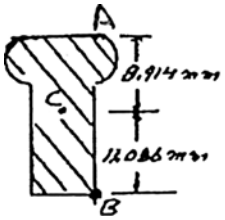
$$I_2 = \bar{I}_2 + A_2 d_2^2 = \frac{\pi}{4}(2)^4 + (4\pi)(6.914)^2 = 0.613 \times 10^3\text{ mm}^4$$

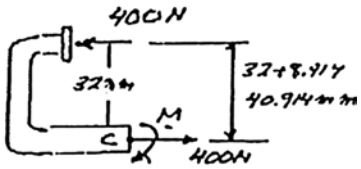
$$I = I_1 + I_2 = 3.374 \times 10^3\text{ mm}^4 = 3.374 \times 10^{-9}\text{ m}^4$$

$$A = A_1 + A_2 = 92.566\text{ mm}^2 = 92.566 \times 10^{-6}\text{ m}^2$$

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PROBLEM 4.115 ***(Continued)***





$$e = 32 + 8.914 = 40.914 \text{ mm} = 0.040914 \text{ m}$$

$$M = Pe = (400 \text{ N})(0.040914 \text{ m}) = 16.3656 \text{ N} \cdot \text{m}$$

(*a*) Point *A*:

$$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{400}{92.566 \times 10^{-6}} + \frac{(16.3656)(8.914 \times 10^{-3})}{3.374 \times 10^{-9}}$$

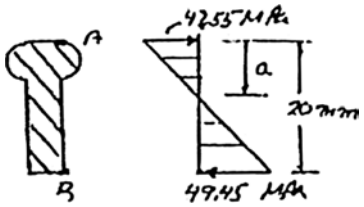
$$= 4.321 \times 10^6 + 43.23 \times 10^6 = 47.55 \times 10^6 \text{ Pa} \qquad \sigma_A = 47.6 \text{ MPa} \blacktriangleleft$$

(*b*) Point *B*:

$$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{400}{92.566 \times 10^{-6}} - \frac{(16.3656)(11.086)}{3.374 \times 10^{-9}}$$

$$= 4.321 \times 10^6 - 53.72 \times 10^6 = -49.45 \times 10^6 \text{ Pa} \qquad \sigma_B = -49.4 \text{ MPa} \blacktriangleleft$$

(*c*) Neutral axis:

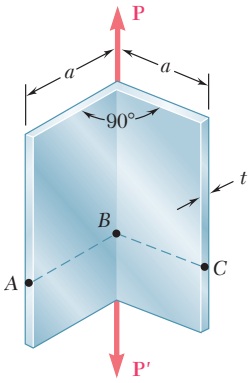


By proportions,

$$\frac{a}{47.55} = \frac{20}{47.55 + 49.45}$$

$$a = 9.80 \text{ mm} \qquad \text{9.80 mm below top of section} \blacktriangleleft$$

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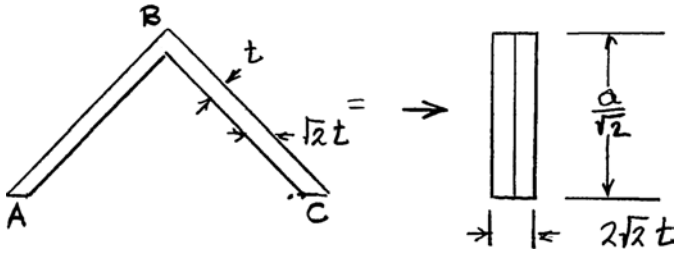


PROBLEM 4.116

The shape shown was formed by bending a thin steel plate. Assuming that the thickness t is small compared to the length a of a side of the shape, determine the stress (a) at A, (b) at B, (c) at C.

SOLUTION

Moment of inertia about centroid:



$$I = \frac{1}{12}\left(2\sqrt{2}t\right)\left(\frac{a}{\sqrt{2}}\right)^3$$

$$= \frac{1}{12}ta^3$$

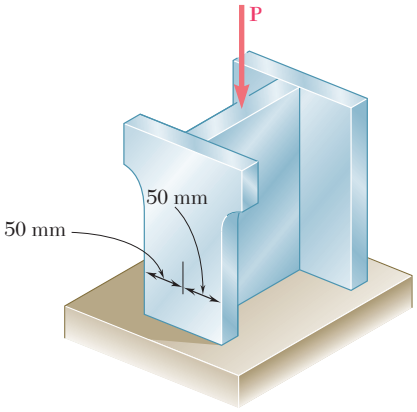
Area: $A = \left(2\sqrt{2}t\right)\left(\frac{a}{\sqrt{2}}\right) = 2at, \quad c = \frac{a}{2\sqrt{2}}$

(a) $\sigma_A = \frac{P}{A} - \frac{Pec}{I} = \frac{P}{2at} - \frac{P\left(\frac{a}{2\sqrt{2}}\right)\left(\frac{a}{2\sqrt{2}}\right)}{\frac{1}{12}ta^3}$ $\qquad \sigma_A = -\frac{P}{2at}$ ◀

(b) $\sigma_B = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{2at} + \frac{P\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right)}{\frac{1}{12}ta^3}$ $\qquad \sigma_B = -\frac{2P}{at}$ ◀

(c) $\sigma_C = \sigma_A$ $\qquad \sigma_C = -\frac{P}{2at}$ ◀

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PROBLEM 4.117

Three steel plates, each of 25×150-mm cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 25-mm strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section, and that the allowable stress is 100 MPa, determine the largest force **P** (*a*) that could be applied to the original column, (*b*) that can be applied to the modified column.

SOLUTION

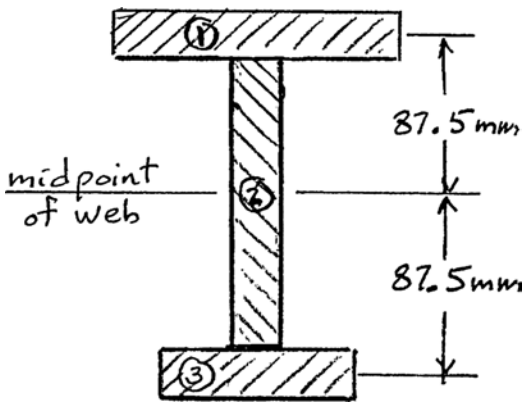
(*a*) Centric loading: $\sigma = -\dfrac{P}{A}$

$$A = (3)(150)(25) = 11.25 \times 10^{3}\ \text{mm}^2 = 11.25 \times 10^{-3}\ \text{m}^2$$

$$P = -\sigma A = -(-100 \times 10^{6})(11.25 \times 10^{-3})$$

$$= 1.125 \times 10^{6}\ \text{N} \qquad P = 1125\ \text{kN} \blacktriangleleft$$

(*b*) Eccentric loading (reduced cross section):



	A, $10^3\ \text{mm}^2$	$\bar{y}$, mm	$A\bar{y}$ $(10^3\ \text{mm}^3)$	d, mm
①	3.75	87.5	328.125	76.5625
②	3.75	0	0	10.9375
③	2.50	−87.5	−218.75	98.4375
Σ	10.00		109.375	

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{109.375 \times 10^{3}}{10.00 \times 10^{3}} = 10.9375\ \text{mm}$$

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PROBLEM 4.117 *(Continued)*

The centroid lies 10.9375 mm from the midpoint of the web.

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(150)(25)^3 + (3.75\times10^3)(76.5625)^2 = 22.177\times10^6\ \text{mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(25)(150)^3 + (3.75\times10^3)(10.9375)^2 = 7.480\times10^6\ \text{mm}^4$$

$$I_3 = \frac{1}{12}b_3h_3^3 + A_3d_3^2 = \frac{1}{12}(100)(25)^3 + (2.50\times10^3)(98.4375)^2 = 24.355\times10^6\ \text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 54.012\times10^6\ \text{mm}^4 = 54.012\times10^{-6}\ \text{m}^4$$

$$c = 10.9375 + 75 + 25 = 110.9375\ \text{mm} = 0.1109375\ \text{m}$$

$$M = Pe \qquad \text{where} \qquad e = 10.4375\ \text{mm} = 10.4375\times10^{-3}\ \text{m}$$

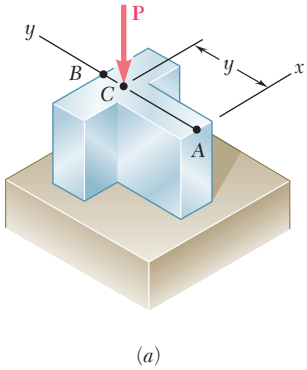
$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pec}{I} = -KP \qquad A = 10.00\times10^{-3}\ \text{m}^2$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{10.00\times10^{-3}} + \frac{(101.9375\times10^{-3})(0.1109375)}{54.012\times10^{-6}} = 122.465\ \text{m}^{-2}$$

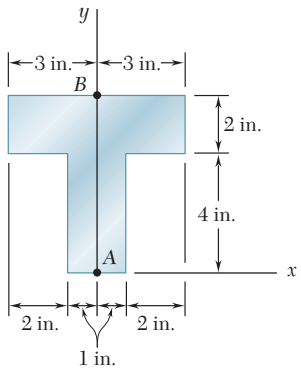
$$P = -\frac{\sigma}{K} = -\frac{(-100\times10^6)}{122.465} = 817\times10^3\ \text{N}$$

$P = 817\ \text{kN}$ ◀

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(a)



(b)

PROBLEM 4.118

A vertical force **P** of magnitude 20 kips is applied at point *C* located on the axis of symmetry of the cross section of a short column. Knowing that $y = 5$ in., determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the location of the neutral axis.

SOLUTION

Locate centroid.

Part	A, in^2	$\bar{y}$, in.	$A\bar{y}$, in^3
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{76}{20} = 3.8 \text{ in.}$$

Eccentricity of load: $e = 5 - 3.8 = 1.2$ in.

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4 \qquad I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

(*a*) Stress at *A*: $c_A = 3.8$ in.

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = -\frac{20}{20} + \frac{20(1.2)(3.8)}{57.867} \qquad \sigma_A = 0.576 \text{ ksi} \blacktriangleleft$$

(*b*) Stress at *B*: $c_B = 6 - 3.8 = 2.2$ in.

$$\sigma_B = -\frac{P}{A} + \frac{Pec_B}{I} = -\frac{20}{20} - \frac{20(1.2)(2.2)}{57.867} \qquad \sigma_B = -1.912 \text{ ksi} \blacktriangleleft$$

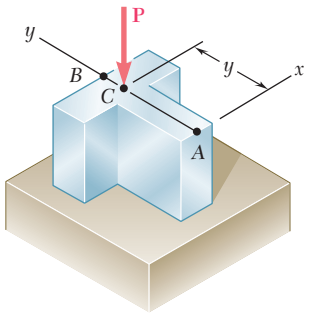
(*c*) Location of neutral axis: $\sigma = 0$ $\quad \sigma = -\frac{P}{A} + \frac{Pea}{I} = 0 \quad \therefore \quad \frac{ea}{I} = \frac{1}{A}$

$$a = \frac{I}{Ae} = \frac{57.867}{(20)(1.2)} = 2.411 \text{ in.}$$

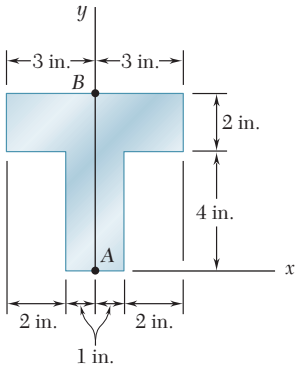
Neutral axis lies 2.411 in. below centroid or $3.8 - 2.411 = 1.389$ in. above point *A*.

Answer: 1.389 in. from point *A* ◀

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(a)



(b)

PROBLEM 4.119

A vertical force **P** is applied at point *C* located on the axis of symmetry of the cross section of a short column. Determine the range of values of *y* for which tensile stresses do not occur in the column.

SOLUTION

Locate centroid.

	A, in^2	$\bar{y}$, in.	$A\bar{y}$, in^3
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{76}{20} = 3.8 \text{ in.}$$

Eccentricity of load: $e = y - 3.8$ in. $\quad y = e + 3.8$ in.

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4 \qquad I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

If stress at A equals zero, $c_A = 3.8$ in.

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = 0 \qquad \therefore \qquad \frac{ec_A}{I} = \frac{1}{A}$$

$$e = \frac{I}{Ac_A} = \frac{57.867}{(20)(3.8)} = 0.761 \text{ in.} \qquad y = 0.761 + 3.8 = 4.561 \text{ in.}$$

If stress at B equals zero, $c_B = 6 - 3.8 = 2.2$ in.

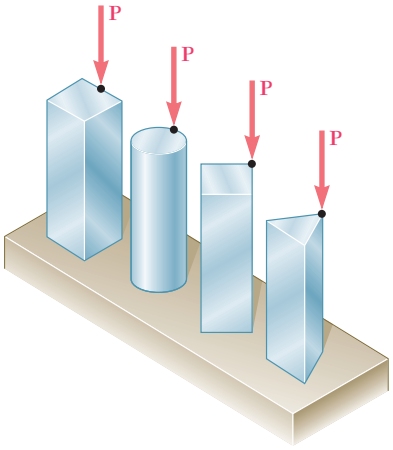
$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = 0 \qquad \therefore \qquad \frac{ec_B}{I} = -\frac{1}{A}$$

$$e = -\frac{I}{Ac_B} = -\frac{57.867}{(20)(2.2)} = -1.315 \text{ in.}$$

$$y = -1.315 + 3.8 = 2.485 \text{ in.}$$

Answer: 2.49 in. $< y <$ 4.56 in. ◀

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PROBLEM 4.120

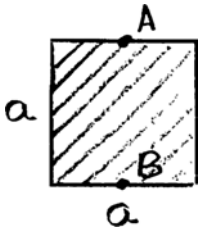
The four bars shown have the same cross-sectional area. For the given loadings, show that (*a*) the maximum compressive stresses are in the ratio 4:5:7:9, (*b*) the maximum tensile stresses are in the ratio 2:3:5:3. (*Note:* the cross section of the triangular bar is an equilateral triangle.)

SOLUTION

Stresses:

At A,
$$\sigma_A = -\frac{P}{A} - \frac{Pec_A}{I} = -\frac{P}{A}\left(1 + \frac{Aec_A}{I}\right)$$

At B,
$$\sigma_B = -\frac{P}{A} + \frac{Pec_B}{I} = \frac{P}{A}\left(\frac{Aec_B}{I} - 1\right)$$



$$A_1 = a^2, \quad I_1 = \frac{1}{12}a^4, \quad c_A = c_B = \frac{1}{2}a, \quad e = \frac{1}{2}a$$

$$\sigma_A = -\frac{P}{A}\left(1 + \frac{(a^2)\left(\frac{1}{2}a\right)\left(\frac{1}{2}a\right)}{\frac{1}{12}a^2}\right) \qquad \sigma_A = -4\frac{P}{A_1} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A}\left(\frac{(a^2)\left(\frac{1}{2}a\right)\left(\frac{1}{2}a\right)}{\frac{1}{12}a^2} - 1\right) \qquad \sigma_B = 2\frac{P}{A_1} \blacktriangleleft$$



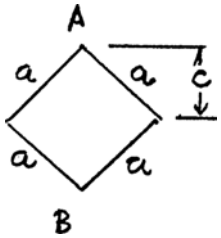
$$A_2 = \pi c^2 = a^2 \quad \therefore \quad c = \frac{a}{\sqrt{\pi}}, \quad I_2 = \frac{\pi}{4}c^4, \quad e = c$$

$$\sigma_A = -\frac{P}{A_2}\left(1 + \frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4}\right) \qquad \sigma_A = -5\frac{P}{A_2} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A_2}\left(\frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4} - 1\right) \qquad \sigma_B = 3\frac{P}{A_2} \blacktriangleleft$$

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PROBLEM 4.120 ***(Continued)***



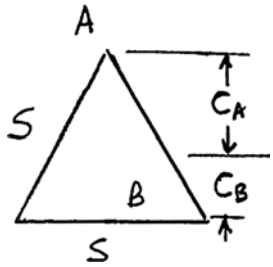
$$A_3 = a^2 \quad c = \frac{\sqrt{2}}{2}a \quad I_3 = \frac{1}{12}a^4 \quad e = c$$

$$\sigma_A = -\frac{P}{A_3}\left(1 + \frac{(a^2)\left(\frac{\sqrt{2}}{2}a\right)\left(\frac{\sqrt{2}}{2}a\right)}{\frac{1}{12}a^4}\right)$$

$$\sigma_A = -7\frac{P}{A_3} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A_3}\left(\frac{(a^2)\left(\frac{\sqrt{2}}{2}a\right)\left(\frac{\sqrt{2}}{2}a\right)}{\frac{1}{12}a^4} - 1\right)$$

$$\sigma_B = 5\frac{P}{A_3} \blacktriangleleft$$



$$A_4 = \frac{1}{2}(s)\left(\frac{\sqrt{3}}{2}s\right) = \frac{\sqrt{3}}{4}s^2$$

$$I_4 = \frac{1}{36}s\left(\frac{\sqrt{3}}{2}s\right)^3 = \frac{\sqrt{3}}{96}s^4$$

$$c_A = \frac{2}{3}\frac{\sqrt{3}}{2}s = \frac{s}{\sqrt{3}} = e \qquad c_B = s$$

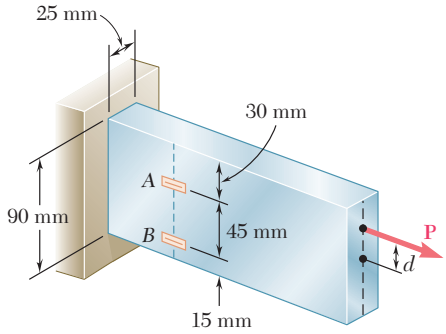
$$\sigma_A = -\frac{P}{A_4}\left(1 + \frac{\left(\frac{\sqrt{3}}{4}s^2\right)\left(\frac{s}{\sqrt{3}}\right)\left(\frac{s}{\sqrt{3}}\right)}{\frac{\sqrt{3}}{96}s^4}\right)$$

$$\sigma_A = -9\frac{P}{A_4} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A_4}\left(\frac{\left(\frac{\sqrt{3}}{4}s^2\right)\left(\frac{s}{\sqrt{3}}\right)\left(\frac{s}{2\sqrt{3}}\right)}{\frac{\sqrt{3}}{96}s^4} - 1\right)$$

$$\sigma_B = 3\frac{P}{A_4} \blacktriangleleft$$

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PROBLEM 4.121

An eccentric force **P** is applied as shown to a steel bar of 25×90-mm cross section. The strains at A and B have been measured and found to be

$$\varepsilon_A = +350\,\mu \qquad \varepsilon_B = -70\,\mu$$

Knowing that $E = 200$ GPa, determine (*a*) the distance d, (*b*) the magnitude of the force **P**.

SOLUTION

$$h = 15 + 45 + 30 = 90 \text{ mm} \qquad b = 25 \text{ mm} \qquad c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh - (25)(90) = 2.25 \times 10^3 \text{ mm}^2 = 2.25 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875 \times 10^6 \text{ mm}^4 = 1.51875 \times 10^{-6} \text{ m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m} \qquad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gages at A and B: $\sigma_A = E\varepsilon_A = (200 \times 10^9)(350 \times 10^{-6}) = 70 \times 10^6 \text{ Pa}$

$$\sigma_B = E\varepsilon_B = (200 \times 10^9)(-70 \times 10^{-6}) = -14 \times 10^6 \text{ Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \tag{1}$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \tag{2}$$

Subtracting, $\sigma_A - \sigma_B = -\dfrac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875 \times 10^{-6})(84 \times 10^6)}{0.045} = -2835 \text{ N} \cdot \text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting, $y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\dfrac{P}{A}$

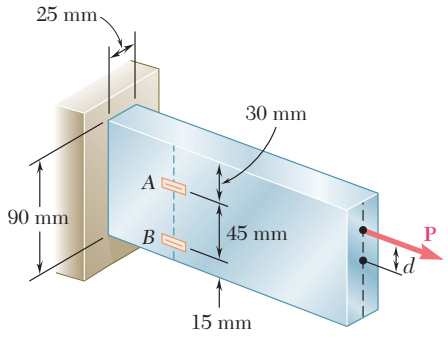
$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25 \times 10^{-3})[(0.015)(-14 \times 10^6) - (-0.030)(70 \times 10^6)]}{0.045} = 94.5 \times 10^3 \text{ N}$$

(*a*) $M = -Pd \quad \therefore \quad d = -\dfrac{M}{P} = -\dfrac{-2835}{94.5 \times 10^3} = 0.030 \text{ m}$ $\qquad d = 30.0 \text{ mm}$ ◀

(*b*) $\qquad P = 94.5 \text{ kN}$ ◀

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PROBLEM 4.122



Solve Prob. 4.121, assuming that the measured strains are

$$\varepsilon_A = +600\,\mu \qquad \varepsilon_B = +420\,\mu$$

PROBLEM 4.121 An eccentric force **P** is applied as shown to a steel bar of 25×90-mm cross section. The strains at A and B have been measured and found to be

$$\varepsilon_A = +350\mu \qquad \varepsilon_B = -70\mu$$

Knowing that $E = 200$ GPa, determine (*a*) the distance d, (*b*) the magnitude of the force **P**.

SOLUTION

$$h = 15 + 45 + 30 = 90 \text{ mm} \qquad b = 25 \text{ mm} \qquad c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh = (25)(90) = 2.25 \times 10^3 \text{ mm}^2 = 2.25 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875 \times 10^6 \text{ mm}^4 = 1.51875 \times 10^{-6} \text{ m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m} \qquad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gages at A and B: $\sigma_A = E\varepsilon_A = (200 \times 10^9)(600 \times 10^{-6}) = 120 \times 10^6 \text{ Pa}$

$$\sigma_B = E\varepsilon_B = (200 \times 10^9)(420 \times 10^{-6}) = 84 \times 10^6 \text{ Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \tag{1}$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \tag{2}$$

Subtracting, $\sigma_A - \sigma_B = -\dfrac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875 \times 10^{-6})(36 \times 10^6)}{0.045} = -1215 \text{ N} \cdot \text{m}$$

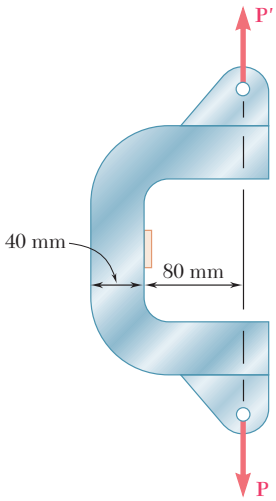
Multiplying (2) by y_A and (1) by y_B and subtracting, $y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\dfrac{P}{A}$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25 \times 10^{-3})[(0.015)(84 \times 10^6) - (-0.030)(120 \times 10^6)]}{0.045} = 243 \times 10^3 \text{ N}$$

(*a*) $M = -Pd \quad \therefore \quad d = -\dfrac{M}{P} = -\dfrac{-1215}{243 \times 10^3} = 5 \times 10^{-3} \text{ m}$ $\qquad d = 5.00 \text{ mm}$ ◀

(*b*) $\qquad P = 243 \text{ kN}$ ◀

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PROBLEM 4.123

The C-shaped steel bar is used as a dynamometer to determine the magnitude P of the forces shown. Knowing that the cross section of the bar is a square of side 40 mm and that strain on the inner edge was measured and found to be 450μ, determine the magnitude P of the forces. Use $E = 200$ GPa.

SOLUTION

At the strain gage location,

$$\sigma = E\varepsilon = (200 \times 10^9)(450 \times 10^{-6}) = 90 \times 10^6 \text{ Pa}$$

$$A = (40)(40) = 1600 \text{ mm}^2 = 1600 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(40)(40)^3 = 213.33 \times 10^3 \text{ mm}^4 = 213.33 \times 10^{-9} \text{ m}^4$$

$$e = 80 + 20 = 100 \text{ mm} = 0.100 \text{ m}$$

$$c = 20 \text{ mm} = 0.020 \text{ m}$$

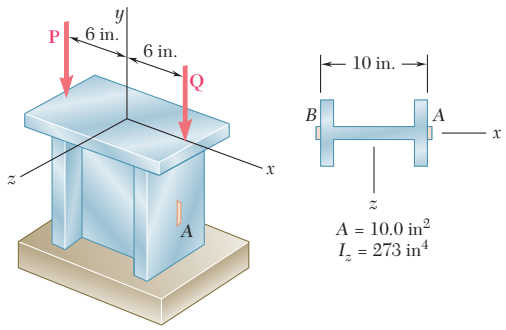
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{1600 \times 10^{-6}} + \frac{(0.100)(0.020)}{213.33 \times 10^{-9}} = 10.00 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{90 \times 10^6}{10.00 \times 10^3} = 9.00 \times 10^3 \text{ N}$$

$P = 9.00$ kN ◀

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PROBLEM 4.124

A short length of a rolled-steel column supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points *A* and *B* on the centerline of the outer faces of the flanges have been measured and found to be

$$\varepsilon_A = -400\times10^{-6} \text{ in./in.} \qquad \varepsilon_B = -300\times10^{-6} \text{ in./in.}$$

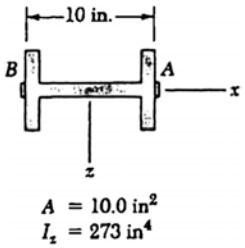
Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.

SOLUTION

Stresses at *A* and *B* from strain gages:

$$\sigma_A = E\varepsilon_A = (29\times10^6)(-400\times10^{-6}) = -11.6\times10^3 \text{ psi}$$

$$\sigma_B = E\varepsilon_B = (29\times10^6)(-300\times10^{-6}) = -8.7\times10^3 \text{ psi}$$



Centric force: $F = P + Q$

Bending couple: $M = 6P - 6Q$

$c = 5$ in.

$$\sigma_A = -\frac{F}{A} + \frac{Mc}{I} = -\frac{P+Q}{10.0} + \frac{(6P-6Q)(5)}{273}$$

$$-11.6\times10^3 = +0.00989P - 0.20989Q \qquad (1)$$

$$\sigma_B = -\frac{F}{A} - \frac{Mc}{I} = -\frac{P+Q}{10.0} - \frac{(6P-6Q)(5)}{273}$$

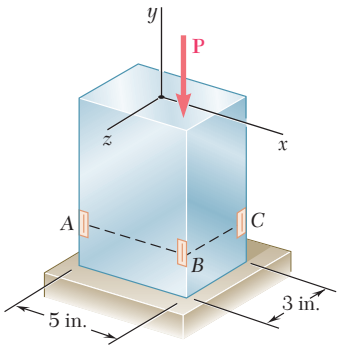
$$-8.7\times10^3 = -0.20989P - 0.00989Q \qquad (2)$$

Solving (1) and (2) simultaneously,

$$P = 44.2\times10^3 \text{ lb} = 44.2 \text{ kips} \quad \blacktriangleleft$$

$$Q = 57.3\times10^3 \text{ lb} = 57.3 \text{ kips} \quad \blacktriangleleft$$

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PROBLEM 4.125

A single vertical force **P** is applied to a short steel post as shown. Gages located at *A, B,* and *C* indicate the following strains:

$$\varepsilon_A = -500\mu \qquad \varepsilon_B = -1000\mu \qquad \varepsilon_C = -200\mu$$

Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the magnitude of **P**, (*b*) the line of action of **P**, (*c*) the corresponding strain at the hidden edge of the post, where $x = -2.5$ in. and $z = -1.5$ in.

SOLUTION

$$I_x = \frac{1}{12}(5)(3)^3 = 11.25 \text{ in}^4 \qquad I_z = \frac{1}{12}(3)(5)^3 = 31.25 \text{ in}^4 \qquad A = (5)(3) = 15 \text{ in}^2$$

$$M_x = Pz \qquad M_z = -Px$$

$$x_A = -2.5 \text{ in.}, \quad x_B = 2.5 \text{ in.}, \quad x_C = 2.5 \text{ in.}, \quad x_D = -2.5 \text{ in.}$$

$$z_A = 1.5 \text{ in.}, \quad z_B = 1.5 \text{ in.}, \quad z_C = -1.5 \text{ in.}, \quad z_D = -1.5 \text{ in.}$$

$$\sigma_A = E\varepsilon_A = (29 \times 10^6)(-500 \times 10^{-6}) = -14{,}500 \text{ psi} = -14.5 \text{ ksi}$$

$$\sigma_B = E\varepsilon_B = (29 \times 10^6)(-1000 \times 10^{-6}) = -29{,}000 \text{ psi} = -29 \text{ ksi}$$

$$\sigma_C = E\varepsilon_C = (29 \times 10^6)(-200 \times 10^{-6}) = -5800 \text{ psi} = -5.8 \text{ ksi}$$

$$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -0.06667P - 0.13333M_x - 0.08M_z \tag{1}$$

$$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -0.06667P - 0.13333M_x + 0.08M_z \tag{2}$$

$$\sigma_C = -\frac{P}{A} - \frac{M_x z_C}{I_x} + \frac{M_z x_C}{I_z} = -0.06667P + 0.13333M_x + 0.08M_z \tag{3}$$

Substituting the values for σ_A, σ_B, and σ_C into (1), (2), and (3) and solving the simultaneous equations gives

$$M_x = 87 \text{ kip} \cdot \text{in.} \qquad M_z = -90.625 \text{ kip} \cdot \text{in.}$$

(*a*) $P = 152.3$ kips ◀

$$x = -\frac{M_z}{P} = -\frac{-90.625}{152.25}$$

(*b*) $x = 0.595$ in. ◀

$$z = \frac{M_x}{P} = \frac{87}{152.25}$$

$z = 0.571$ in. ◀

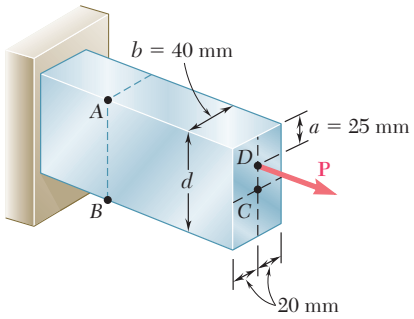
$$\sigma_D = -\frac{P}{A} - \frac{M_x z_D}{I_x} + \frac{M_z x_D}{I_z} = -0.06667P + 0.13333M_x - 0.08M_z$$

$$= -(0.06667)(152.25) + (0.13333)(87) - (0.08)(-90.625) = 8.70 \text{ ksi}$$

(*c*) Strain at hidden edge: $\varepsilon = \frac{\sigma_D}{E} = \frac{8.70 \times 10^3}{29 \times 10^6}$

$\varepsilon = 300\mu$ ◀

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PROBLEM 4.126

The eccentric axial force **P** acts at point *D*, which must be located 25 mm below the top surface of the steel bar shown. For $P = 60$ kN, determine (*a*) the depth *d* of the bar for which the tensile stress at point *A* is maximum, (*b*) the corresponding stress at point *A*.

SOLUTION

$$A = bd \qquad I = \frac{1}{12}bd^3$$

$$c = \frac{1}{2}d \qquad e = \frac{1}{2}d - a$$

$$\sigma_A = \frac{P}{A} + \frac{Pec}{I}$$

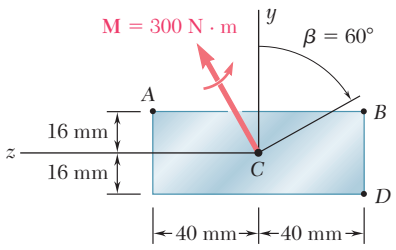
$$\sigma_A = \frac{P}{b}\left\{\frac{1}{d} + \frac{12\left(\frac{1}{2}d - a\right)\left(\frac{1}{2}d\right)}{d^3}\right\} = \frac{P}{b}\left\{\frac{4}{d} - \frac{6a}{d^2}\right\}$$

(*a*) Depth *d* for maximum σ_A: Differentiate with respect to *d*.

$$\frac{d\sigma_A}{dd} = \frac{P}{b}\left\{-\frac{4}{d^2} + \frac{12a}{d^3}\right\} = 0 \qquad d = 3a \qquad d = 75 \text{ mm} \blacktriangleleft$$

(*b*) $$\sigma_A = \frac{60\times10^3}{40\times10^{-3}}\left\{\frac{4}{75\times10^{-3}} - \frac{(6)(25\times10^{-3})}{(75\times10^{-3})^2}\right\} = 40\times10^6 \text{ Pa} \qquad \sigma_A = 40 \text{ MPa} \blacktriangleleft$$

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PROBLEM 4.127

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{1}{12}(80)(32)^3 = 218.45\times10^3 \text{ mm}^4 = 218.45\times10^{-9} \text{ m}^4$$

$$I_y = \frac{1}{12}(32)(80)^3 = 1.36533\times10^6 \text{ mm}^4 = 1.36533\times10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = 16 \text{ mm}$$

$$z_A = -z_B = -z_D = 40 \text{ mm}$$

$$M_y = 300\cos 30^\circ = 259.81 \text{ N}\cdot\text{m} \qquad M_z = 300\sin 30^\circ = 150 \text{ N}\cdot\text{m}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(150)(16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(40\times10^{-3})}{1.36533\times10^{-6}}$$

$$= -3.37\times10^6 \text{ Pa} \qquad \sigma_A = -3.37 \text{ MPa} \blacktriangleleft$$

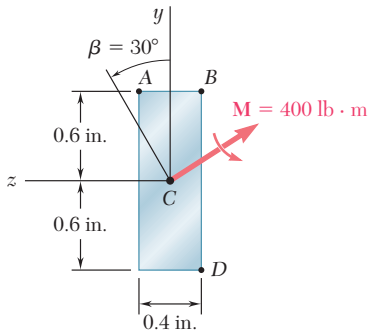
(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(150)(16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(-40\times10^{-3})}{1.36533\times10^{-6}}$$

$$= -18.60\times10^6 \text{ Pa} \qquad \sigma_B = -18.60 \text{ MPa} \blacktriangleleft$$

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(150)(-16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(-40\times10^{-3})}{1.36533\times10^{-6}}$$

$$= 3.37\times10^6 \text{ Pa} \qquad \sigma_D = 3.37 \text{ MPa} \blacktriangleleft$$

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PROBLEM 4.128

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{1}{12}(0.4)(1.2)^3 = 57.6\times10^{-3}\ \text{in}^4$$

$$I_y = \frac{1}{12}(1.2)(0.4)^3 = 6.40\times10^{-3}\ \text{in}^4$$

$$y_A = y_B = -y_D = 0.6\ \text{in.}$$

$$z_A = -z_B = -z_D = \left(\frac{1}{2}\right)(0.4) = 0.2\ \text{in.}$$

$$M_y = 400\cos 60^\circ = 200\ \text{lb}\cdot\text{in.}, \qquad M_z = -400\sin 60^\circ = -346.41\ \text{lb}\cdot\text{in.}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-346.41)(0.6)}{57.6\times10^{-3}} + \frac{(200)(0.2)}{6.40\times10^{-3}}$$

$$= 9.86\times10^3\ \text{psi} = 9.86\ \text{ksi}$$ ◀

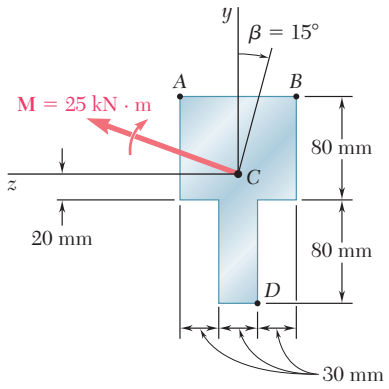
(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-346.41)(0.6)}{57.6\times10^{-6}} + \frac{(200)(-0.2)}{6.4\times10^{-3}}$$

$$= -2.64\times10^3\ \text{psi} = -2.64\ \text{ksi}$$ ◀

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-346.41)(-0.6)}{57.6\times10^{-3}} + \frac{(200)(-0.2)}{6.40\times10^{-3}}$$

$$= -9.86\times10^3\ \text{psi} = -9.86\ \text{ksi}$$ ◀

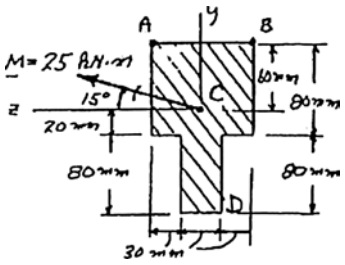
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PROBLEM 4.129

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION



$$M_y = 25\sin 15° = 6.4705 \text{ kN}\cdot\text{m}$$

$$M_z = 25\cos 15° = 24.148 \text{ kN}\cdot\text{m}$$

$$I_y = \frac{1}{12}(80)(90)^3 + \frac{1}{12}(80)(30)^3 = 5.04\times10^6 \text{ mm}^4$$

$$I_y = 5.04\times10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{3}(90)(60)^3 + \frac{1}{3}(60)(20)^3 + \frac{1}{3}(30)(100)^3 = 16.64\times10^6 \text{ mm}^4 = 16.64\times10^{-6} \text{ m}^4$$

Stress: $\sigma = \dfrac{M_y z}{I_y} - \dfrac{M_z y}{I_z}$

(*a*) $$\sigma_A = \frac{(6.4705 \text{ kN}\cdot\text{m})(0.045 \text{ m})}{5.04\times10^{-6} \text{ m}^4} - \frac{(24.148 \text{ kN}\cdot\text{m})(0.060 \text{ m})}{16.64\times10^{-6} \text{ m}^4}$$

$$= 57.772 \text{ MPa} - 87.072 \text{ MPa} \qquad \sigma_A = -29.3 \text{ MPa} \blacktriangleleft$$

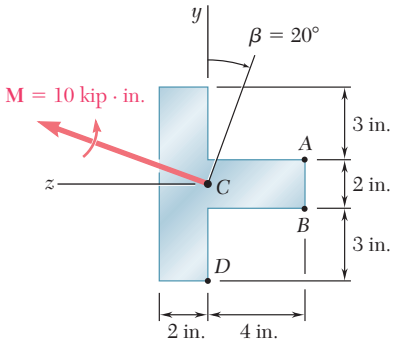
(*b*) $$\sigma_B = \frac{(6.4705 \text{ kN}\cdot\text{m})(-0.045 \text{ m})}{5.04\times10^{-6} \text{ m}^4} - \frac{(24.148 \text{ kN}\cdot\text{m})(0.060 \text{ m})}{16.64\times10^{-6} \text{ m}^4}$$

$$= -57.772 \text{ MPa} - 87.072 \text{ MPa} \qquad \sigma_B = -144.8 \text{ MPa} \blacktriangleleft$$

(*c*) $$\sigma_D = \frac{(6.4705 \text{ kN}\cdot\text{m})(-0.015 \text{ m})}{5.04\times10^{-6} \text{ m}^4} - \frac{(24.148 \text{ kN}\cdot\text{m})(-0.100 \text{ m})}{16.64\times10^{-6} \text{ m}^4}$$

$$= -19.257 \text{ MPa} + 145.12 \text{ MPa} \qquad \sigma_D = -125.9 \text{ MPa} \blacktriangleleft$$

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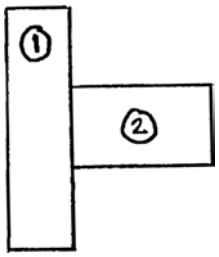


PROBLEM 4.130

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

Locate centroid.



	A, in^2	$\bar{z}$, in.	$A\bar{z}$, in^3
①	16	−1	−16
②	8	2	16
Σ	24		0

The centroid lies at point *C*.

$$I_z = \frac{1}{12}(2)(8)^3 + \frac{1}{12}(4)(2)^3 = 88 \text{ in}^4$$

$$I_y = \frac{1}{3}(8)(2)^3 + \frac{1}{3}(2)(4)^3 = 64 \text{ in}^4$$

$$y_A = -y_B = 1 \text{ in.}, \qquad y_D = -4 \text{ in.}$$

$$z_A = z_B = -4 \text{ in.}, \qquad z_D = 0$$

$$M_z = 10 \cos 20° = 9.3969 \text{ kip} \cdot \text{in.}$$

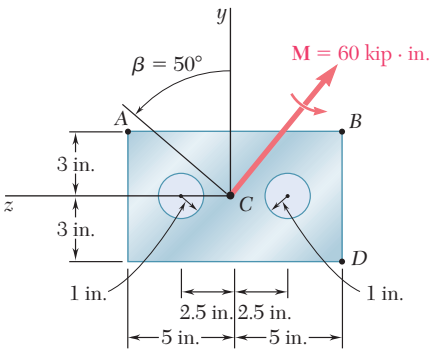
$$M_y = 10 \sin 20° = 3.4202 \text{ kip} \cdot \text{in.}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(9.3969)(1)}{88} + \frac{(3.4202)(-4)}{64}$$ $\sigma_A = 0.321$ ksi ◀

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(9.3969)(-1)}{88} + \frac{(3.4202)(-4)}{64}$$ $\sigma_B = -0.107$ ksi ◀

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(9.3969)(-4)}{88} + \frac{(3.4202)(0)}{64}$$ $\sigma_D = 0.427$ ksi ◀

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PROBLEM 4.131

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$M_z = -60 \sin 40^\circ = -38.567 \text{ kip} \cdot \text{in.}$$
$$M_y = 60 \cos 40^\circ = 45.963 \text{ kip} \cdot \text{in.}$$
$$y_A = y_B = -y_D = 3 \text{ in.}$$
$$z_A = -z_B = -z_D = 5 \text{ in.}$$
$$I_z = \frac{1}{12}(10)(6)^3 - 2\left[\frac{\pi}{4}(1)^2\right] = 178.429 \text{ in}^4$$
$$I_y = \frac{1}{12}(6)(10)^3 - 2\left[\frac{\pi}{4}(1)^4 + \pi(1)^2(2.5)^2\right] = 459.16 \text{ in}^4$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-38.567)(3)}{178.429} + \frac{(45.963)(5)}{459.16}$$

$$= 1.149 \text{ ksi} \qquad \sigma_A = 1.149 \text{ ksi} \blacktriangleleft$$

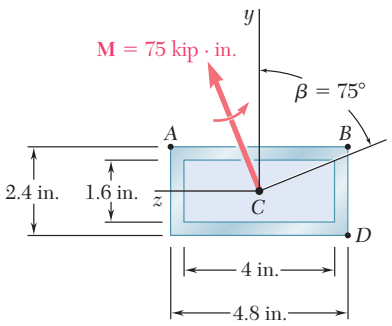
(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-38.567)(3)}{178.429} + \frac{(45.963)(-5)}{459.16}$$

$$= 0.1479 \qquad \sigma_B = 0.1479 \text{ ksi} \blacktriangleleft$$

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-38.567)(-3)}{178.429} + \frac{(45.963)(-5)}{459.16}$$

$$= -1.149 \text{ ksi} \qquad \sigma_D = -1.149 \text{ ksi} \blacktriangleleft$$

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PROBLEM 4.132

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{1}{12}(4.8)(2.4)^3 - \frac{1}{12}(4)(1.6)^3 = 4.1643 \text{ in}^4$$

$$I_y = \frac{1}{12}(2.4)(4.8)^3 - \frac{1}{12}(1.6)(4)^3 = 13.5851 \text{ in}^4$$

$$y_A = y_B = -y_D = 1.2 \text{ in.}$$

$$z_A = -z_B = -z_D = 2.4 \text{ in.}$$

$$M_z = 75\sin 15° = 19.4114 \text{ kip} \cdot \text{in.}$$

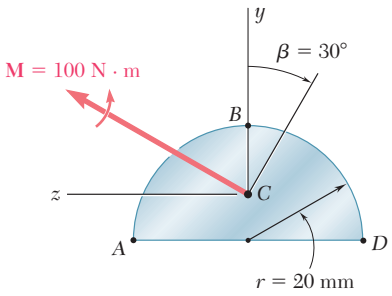
$$M_y = 75\cos 15° = 72.444 \text{ kip} \cdot \text{in.}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(19.4114)(1.2)}{4.1643} + \frac{(72.444)(2.4)}{13.5851}$$ $\sigma_A = 7.20$ ksi ◀

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(19.4114)(1.2)}{4.1643} + \frac{(72.444)(-2.4)}{13.5851}$$ $\sigma_B = -18.39$ ksi ◀

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(19.4114)(-1.2)}{4.1643} + \frac{(72.444)(-2.4)}{13.5851}$$ $\sigma_D = -7.20$ ksi ◀

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PROBLEM 4.133

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{\pi}{8}r^4 - \left(\frac{\pi}{2}r^2\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$$

$$= (0.109757)(20)^4 = 17.5611\times10^{-3}\ \text{mm}^4 = 17.5611\times10^{-9}\ \text{m}^4$$

$$I_y = \frac{\pi}{8}r^4 = \frac{\pi(20)^4}{8} = 62.832\times10^{-3}\ \text{mm}^4 = 62.832\times10^{-9}\ \text{m}^4$$

$$y_A = y_D = -\frac{4r}{3\pi} = -\frac{(4)(20)}{3\pi} = -8.4883\ \text{mm}$$

$$y_B = 20 - 8.4883 = 11.5117\ \text{mm}$$

$$z_A = -z_D = 20\ \text{mm} \qquad z_B = 0$$

$$M_z = 100\cos 30° = 86.603\ \text{N}\cdot\text{m}$$

$$M_y = 100\sin 30° = 50\ \text{N}\cdot\text{m}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(86.603)(-8.4883\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(20\times10^{-3})}{62.832\times10^{-9}}$$

$$= 57.8\times10^6\ \text{Pa} \qquad \sigma_A = 57.8\ \text{MPa} \blacktriangleleft$$

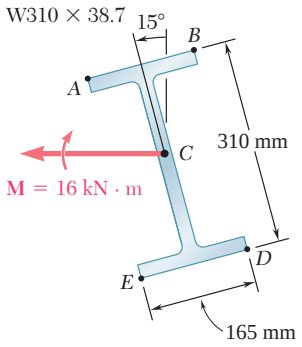
(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(86.603)(11.5117\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(0)}{62.832\times10^{-9}}$$

$$= -56.8\times10^6\ \text{Pa} \qquad \sigma_B = -56.8\ \text{MPa} \blacktriangleleft$$

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(86.603)(-8.4883\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(-20\times10^{3})}{62.832\times10^{-9}}$$

$$= 25.9\times10^6\ \text{Pa} \qquad \sigma_D = 25.9\ \text{MPa} \blacktriangleleft$$

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PROBLEM 4.134

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

For W310×38.7 rolled steel shape,

$$I_z = 84.9\times10^{6}\ \text{mm}^4 = 84.9\times10^{-6}\ \text{m}^4$$
$$I_y = 7.20\times10^{6}\ \text{mm}^4 = 7.20\times10^{-6}\ \text{m}^4$$
$$y_A = y_B = \frac{1}{2}y_D = -y_E = \left(\frac{1}{2}\right)(310) = 155\ \text{mm}$$
$$z_A = z_E = -z_B = -z_D = \left(\frac{1}{2}\right)(165) = 82.5\ \text{mm}$$

$$M_z = (16\times10^{3})\cos 15^\circ = 15.455\times10^{3}\ \text{N}\cdot\text{m}$$
$$M_y = (16\times10^{3})\sin 15^\circ = 4.1411\times10^{3}\ \text{N}\cdot\text{m}$$



(*a*) $$\tan\phi = \frac{I_z}{I}\tan\theta = \frac{84.9\times10^{-6}}{7.20\times10^{-6}}\tan 15^\circ = 3.1596$$

$$\phi = 72.4^\circ$$

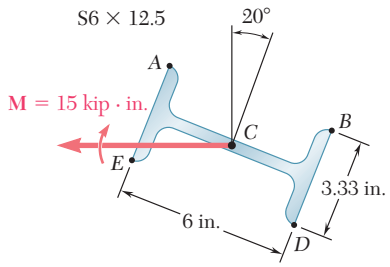
$$\alpha - 72.4 - 15 = 57.4^\circ$$ ◀

(*b*) Maximum tensile stress occurs at point *E*.

$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(15.455\times10^{3})(-155\times10^{-3})}{84.9\times10^{-6}} + \frac{(4.1411\times10^{3})(82.5\times10^{-3})}{7.20\times10^{-6}}$$

$$= 75.7\times10^{6}\ \text{Pa} = 75.7\ \text{MPa}$$ ◀

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PROBLEM 4.135

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

For S6×12.5 rolled steel shape,

$$I_z = 22.0 \text{ in}^4$$

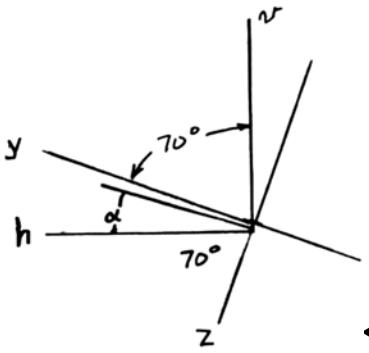
$$I_y = 1.80 \text{ in}^4$$

$$z_E = -z_A = -z_B = z_D = \frac{1}{2}(3.33) = 1.665 \text{ in.}$$

$$y_A = y_B = -y_D = -y_E = \frac{1}{2}(6) = 3 \text{ in.}$$

$$M_z = 15 \sin 20° = 5.1303 \text{ kip} \cdot \text{in.}$$

$$M_y = 15 \cos 20° = 14.095 \text{ kip} \cdot \text{in.}$$



(*a*) $$\tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{22.0}{1.80} \tan (90° - 20°) = 33.58°$$

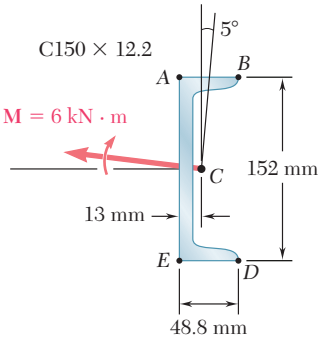
$$\phi = 88.29°$$

$$\alpha = 88.29° - 70° = 18.29°$$ ◀

(*b*) Maximum tensile stress occurs at point *D*.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(5.1303)(-3)}{22.0} + \frac{(14.095)(1.665)}{1.80} = 13.74 \text{ ksi}$$ ◀

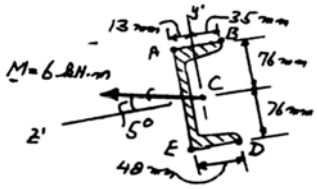
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PROBLEM 4.136

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION



$$M_{y'} = 6 \sin 5° = 0.52293 \text{ kN} \cdot \text{m}$$

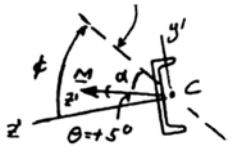
$$M_{z'} = 6 \cos 5° = 5.9772 \text{ kN} \cdot \text{m}$$

C150×12.2

$$I_{y'} = 0.286 \times 10^{-6} \text{ m}^4$$

$$I_{z'} = 5.45 \times 10^{-6} \text{ m}^4$$

(*a*) Neutral axis:



$$\tan \phi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{5.45 \times 10^{-6} \text{ m}^4}{0.286 \times 10^{-6} \text{ m}^4} \tan 5°$$

$$\tan \phi = 1.66718 \qquad \phi = 59.044°$$

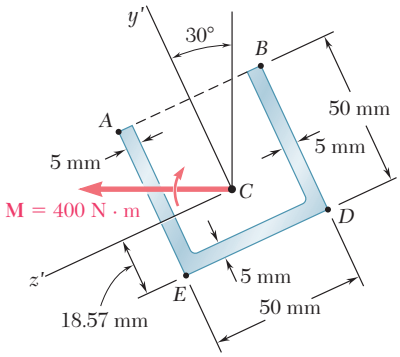
$$\alpha = \phi - 5° \qquad \alpha = 54.0°$$ ◀

(*b*) Maximum tensile stress at E: $y'_E = -76$ mm, $z'_E = 13$ mm

$$\sigma_E = -\frac{M_{y'} z'_E}{I_{y'}} - \frac{M_{z'} y'_E}{I_{z'}} = \frac{(0.52293 \text{ kN} \cdot \text{m})(0.013 \text{ m})}{0.286 \times 10^{-6} \text{ m}^4} - \frac{(5.9772 \text{ kN} \cdot \text{m})(-0.076 \text{ m})}{5.45 \times 10^{-6} \text{ m}^4}$$

$$\sigma_E = 107.1 \text{ MPa}$$ ◀

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$I_{y'} = 281 \times 10^3 \text{ mm}^4$
$I_{z'} = 176.9 \times 10^3 \text{ mm}^4$

PROBLEM 4.137

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

$$I_{z'} = 176.9 \times 10^3 \text{ mm}^4 = 176.9 \times 10^{-9} \text{ m}^4$$
$$I_{y'} = 281 \times 10^3 \text{ mm}^4 = 281 \times 10^{-9} \text{ m}^4$$
$$y'_E = -18.57 \text{ mm}, \quad z_E = 25 \text{ mm}$$
$$M_{z'} = 400 \cos 30° = 346.41 \text{ N} \cdot \text{m}$$
$$M_{y'} = 400 \sin 30° = 200 \text{ N} \cdot \text{m}$$

(*a*) $$\tan\varphi = \frac{I_{z'}}{I_{y'}} \tan\theta = \frac{176.9 \times 10^{-9}}{281 \times 10^{-9}} \cdot \tan 30° = 0.36346$$

$$\varphi = 19.97°$$

$$\alpha = 30° - 19.97° \qquad \alpha = 10.03° \blacktriangleleft$$

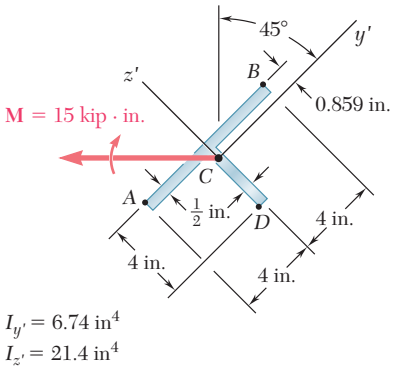
(*b*) Maximum tensile stress occurs at point *E*.

$$\sigma_E = -\frac{M_{z'} y'_E}{I_{z'}} + \frac{M_{y'} z'_E}{I_{y'}} = -\frac{(346.41)(-18.57 \times 10^{-3})}{176.9 \times 10^{-9}} + \frac{(200)(25 \times 10^{-3})}{281 \times 10^{-9}}$$

$$= 36.36 \times 10^6 + 17.79 \times 10^6 = 54.2 \times 10^6 \text{ Pa}$$

$$\sigma_E = 54.2 \text{ MPa} \blacktriangleleft$$

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$I_{y'} = 6.74 \text{ in}^4$
$I_{z'} = 21.4 \text{ in}^4$

PROBLEM 4.138

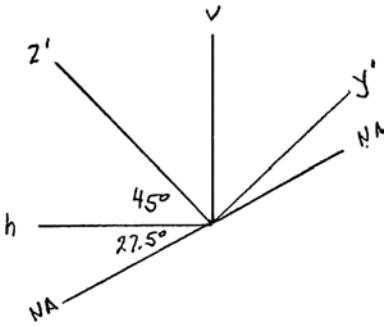
The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

$$I_{z'} = 21.4 \text{ in}^4 \qquad I_{y'} = 6.74 \text{ in}^4$$
$$z'_A = z'_B = 0.859 \text{ in.} \qquad z'_D = -4 + 0.859 \text{ in.} = -3.141 \text{ in.}$$
$$y_A = -4 \text{ in.} \qquad y_{B'} = 4 \text{ in.} \qquad y_{D'} = -0.25 \text{ in.}$$
$$M_{y'} = -15 \sin 45° = -10.6066 \text{ kip} \cdot \text{in.}$$
$$M_{z'} = 15 \cos 45° = 10.6066 \text{ kip} \cdot \text{in.}$$

(*a*) Angle of neutral axis:

$$\tan\varphi = \frac{I_{z'}}{I_{y'}} \tan\theta = \frac{21.4}{6.74} \tan(-45°) = 3.1751$$

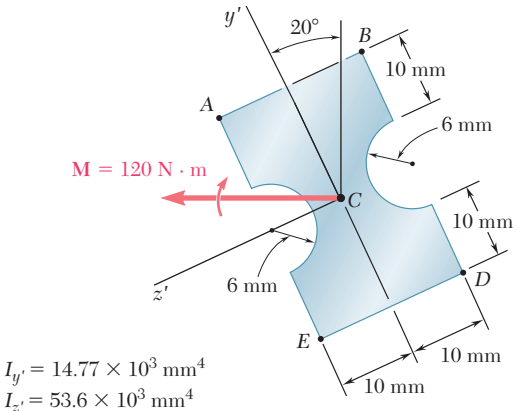


$$\varphi = -72.5°$$
$$\alpha = 72.5° - 45° \qquad \alpha = 27.5° \blacktriangleleft$$

(*b*) The maximum tensile stress occurs at point *D*.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(10.6066)(-0.25)}{21.4} + \frac{(-10.6066)(-3.141)}{6.74}$$
$$= 0.12391 + 4.9429 \qquad \sigma_D = 5.07 \text{ ksi} \blacktriangleleft$$

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$I_{y'} = 14.77 \times 10^3 \text{ mm}^4$
$I_{z'} = 53.6 \times 10^3 \text{ mm}^4$

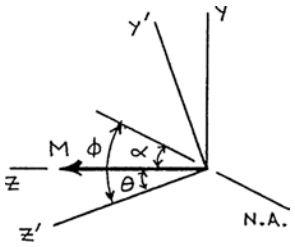
PROBLEM 4.139

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

$$I_{z'} = 53.6 \times 10^3 \text{ mm}^4 = 53.6 \times 10^{-9} \text{ m}^4$$
$$I_{y'} = 14.77 \times 10^3 \text{ mm}^4 = 14.77 \times 10^{-9} \text{ m}^4$$
$$M_{z'} = 120 \sin 70° = 112.763 \text{ N} \cdot \text{m}$$
$$M_{y'} = 120 \cos 70° = 41.042 \text{ N} \cdot \text{m}$$

(*a*) Angle of neutral axis: $\theta = 20°$

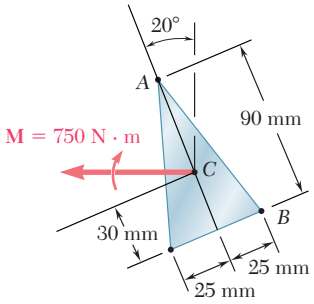


$$\tan\varphi = \frac{I_{z'}}{I_{y'}} \tan\theta = \frac{53.6 \times 10^{-9}}{14.77 \times 10^{-9}} \tan 20° = 1.32084$$
$$\varphi = 52.871°$$
$$\alpha = 52.871° - 20°$$
$$\alpha = 32.9° \blacktriangleleft$$

(*b*) The maximum tensile stress occurs at point *E*.

$$y'_E = -16 \text{ mm} = -0.016 \text{ m}$$
$$z_E = 10 \text{ mm} = 0.010 \text{ m}$$
$$\sigma_E = -\frac{M_{z'} y'_E}{I_{z'}} + \frac{M_{y'} z'_E}{I_{y'}}$$
$$= -\frac{(112.763)(-0.016)}{53.6 \times 10^{-9}} + \frac{(41.042)(0.010)}{14.77 \times 10^{-9}}$$
$$= 61.448 \times 10^6 \text{ Pa}$$
$$\sigma_E = 61.4 \text{ MPa} \blacktriangleleft$$

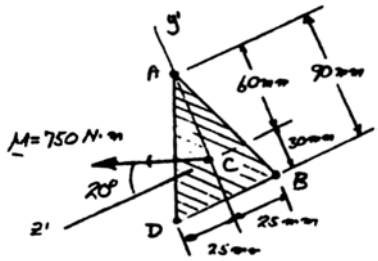
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PROBLEM 4.140

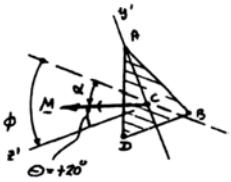
The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION



$$M_{y'} = 750 \sin 20°$$
$$M_{y'} = 256.5 \text{ N} \cdot \text{m}$$
$$M_{z'} = 750 \cos 20°$$
$$M_{z'} = 704.8 \text{ N} \cdot \text{m}$$
$$I_{y'} = 2\left[\frac{1}{12}(90)(25)^3\right] = 0.2344 \times 10^6 \text{ mm}^4$$
$$I_{y'} = 0.2344 \times 10^{-6} \text{ m}^4$$
$$I_{z'} = \frac{1}{36}(50)(90)^3 = 1.0125 \times 10^6 \text{ mm}^4 = 1.0125 \times 10^{-6} \text{ m}^4$$

(*a*) Neutral axis:



$$\tan\phi = \frac{I_{z'}}{I_{y'}}\tan\theta = \frac{1.0125 \times 10^{-6} \text{ m}^4}{0.2344 \times 10^6 \text{ m}^4}\tan 20°$$
$$\tan\phi = 1.5724 \qquad \phi = 57.5°$$
$$\alpha = \phi - 20° = 57.5° - 30° = 37.5°$$

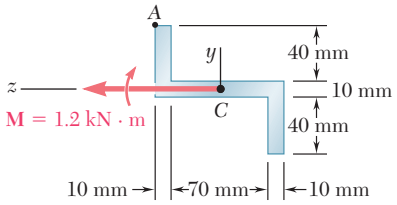
$\alpha = 37.5°$ ◀

(*b*) Maximum tensile stress, at *D*: $y'_D = -30$ mm $\quad z' = 25$ mm

$$\sigma_D = \frac{M_{y'}z'_D}{I_{y'}} - \frac{M_{z'}y'_D}{I_{z'}} = \frac{(256.5 \text{ N} \cdot \text{m})(-0.030 \text{ m})}{0.2344 \times 10^{-6} \text{ m}^4} - \frac{(704.8 \text{ N} \cdot \text{m})(0.025 \text{ m})}{1.0125 \times 10^{-6} \text{ m}^4}$$
$$= 32.83 \text{ MPa} + 17.40 \text{ MPa} = 50.23 \text{ MPa}$$

$\sigma_D = 50.2$ MPa ◀

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$I_y = 1.894 \times 10^6 \text{ mm}^4$
$I_z = 0.614 \times 10^6 \text{ mm}^4$
$I_{yz} = +0.800 \times 10^6 \text{ mm}^4$

PROBLEM 4.141

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point *A*.

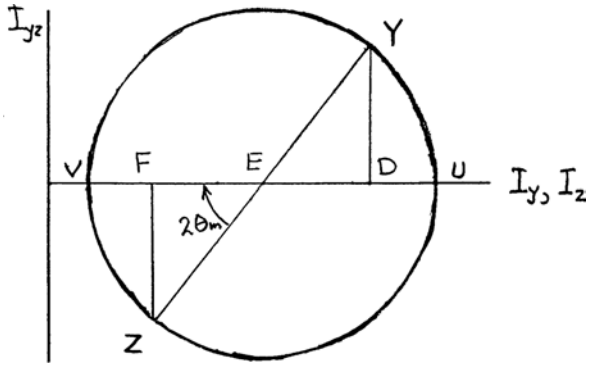
SOLUTION

Using Mohr's circle, determine the principal axes and principal moments of inertia.

$Y: (1.894, 0.800) \times 10^6 \text{ mm}^4$

$Z: (0.614, 0.800) \times 10^6 \text{ mm}^4$

$E: (1.254, 0) \times 10^6 \text{ mm}^4$



$$R = \sqrt{\overline{EF}^2 + \overline{FZ}^2} = \sqrt{0.640^2 + 0.800^2} \times 10^{-6} = 1.0245 \times 10^6 \text{ mm}^4$$

$$I_v = (1.254 - 1.0245) \times 10^6 \text{ mm}^4 = 0.2295 \times 10^6 \text{ mm}^4 = 0.2295 \times 10^{-6} \text{ m}^4$$

$$I_u = (1.254 + 1.0245) \times 10^6 \text{ mm}^4 = 2.2785 \times 10^6 \text{ mm}^4 = 2.2785 \times 10^{-6} \text{ m}^4$$

$$\tan 2\theta_m = \frac{FZ}{FE} = \frac{0.800 \times 10^6}{0.640 \times 10^6} = 1.25 \qquad \theta_m = 25.67°$$

$$M_v = M \cos \theta_m = (1.2 \times 10^3) \cos 25.67° = 1.0816 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_u = -M \sin \theta_m = -(1.2 \times 10^3) \sin 25.67° = -0.5198 \times 10^3 \text{ N} \cdot \text{m}$$

$$u_A = y_A \cos \theta_m - z_A \sin \theta_m = 45 \cos 25.67° - 45 \sin 25.67° = 21.07 \text{ mm}$$

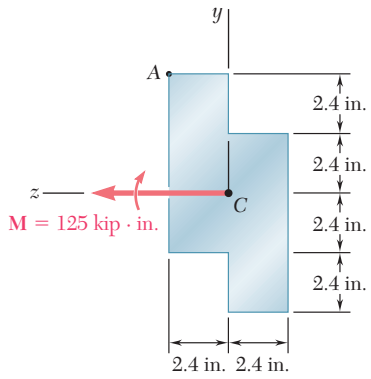
$$v_A = z_A \cos \theta_m + y_A \sin \theta_m = 45 \cos 25.67° + 45 \sin 25.67° = 60.05 \text{ mm}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(1.0816 \times 10^3)(21.07 \times 10^{-3})}{0.2295 \times 10^{-6}} + \frac{(-0.5198 \times 10^3)(60.05 \times 10^{-3})}{2.2785 \times 10^{-6}}$$

$$= 113.0 \times 10^6 \text{ Pa}$$

$$\sigma_A = 113.0 \text{ MPa} \blacktriangleleft$$

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PROBLEM 4.142

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point *A*.

SOLUTION

$$I_y = 2\left\{\frac{1}{3}(7.2)(2.4)^3\right\} = 66.355 \text{ in}^4$$

$$I_z = 2\left\{\frac{1}{12}(2.4)(7.2)^3 + (2.4)(7.2)(1.2)^2\right\} = 199.066 \text{ in}^4$$

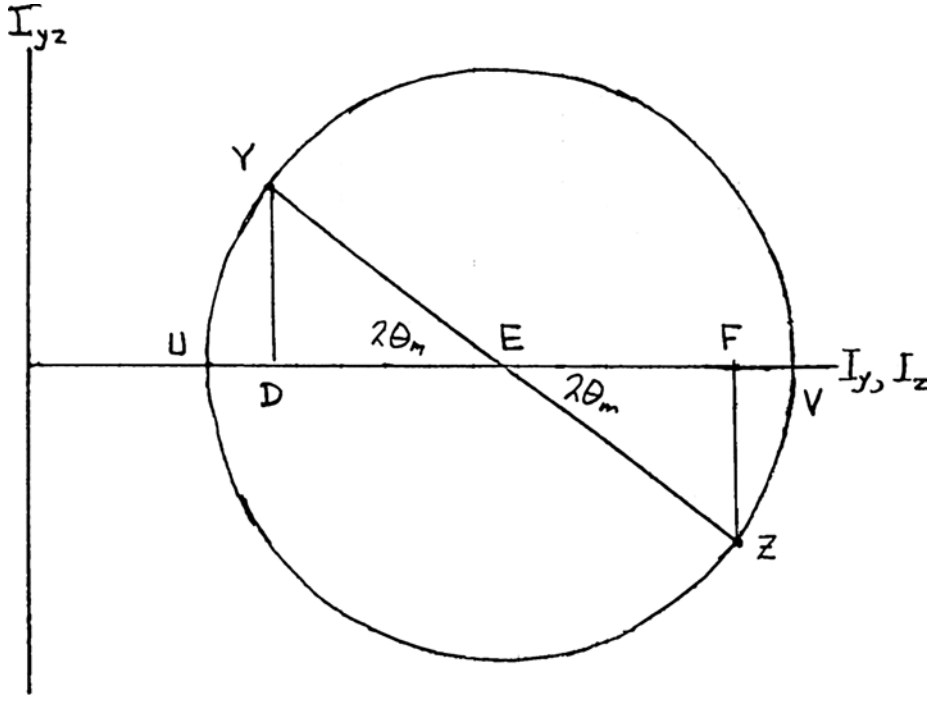
$$I_{yz} = 2\{(2.4)(7.2)(1.2)(1.2)\} = 49.766 \text{ in}^4$$

Using Mohr's circle, determine the principal axes and principal moments of inertia.

$$Y: (66.355 \text{ in}^4,\ 49.766 \text{ in}^4)$$

$$Z: (199.066 \text{ in}^4,\ -49.766 \text{ in}^4)$$

$$E: (132.710 \text{ in}^4,\ 0)$$



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PROBLEM 4.142 ***(Continued)***

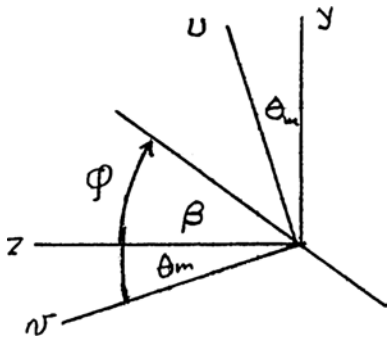
$$\tan 2\theta_m = \frac{DY}{DE} = \frac{49.766}{66.355}$$

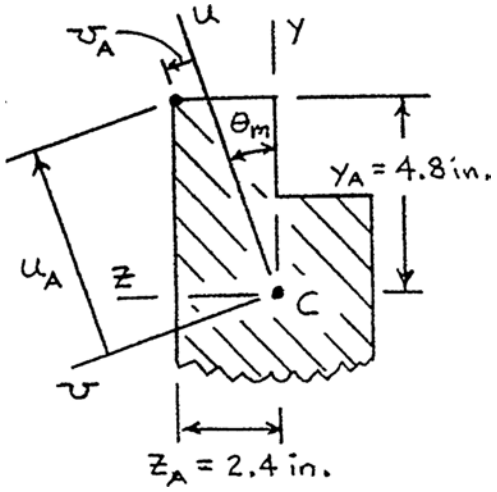
$$2\theta_m = 36.87^\circ \quad \theta_m = 18.435^\circ$$

$$R = \sqrt{\overline{DE}^2 + \overline{DY}^2} = 82.944 \text{ in}^4$$

$$I_u = 132.710 - 82.944 = 49.766 \text{ in}^4$$

$$I_v = 132.710 + 82.944 = 215.654 \text{ in}^4$$





$$M_u = 125 \sin 18.435^\circ = 39.529 \text{ kip} \cdot \text{in.}$$

$$M_v = 125 \cos 18.435^\circ = 118.585 \text{ kip} \cdot \text{in.}$$

$$u_A = 4.8 \cos 18.435^\circ + 2.4 \sin 18.435^\circ = 5.3126 \text{ in.}$$

$$v_A = -4.8 \sin 18.435^\circ + 2.4 \cos 18.435^\circ = 0.7589 \text{ in.}$$

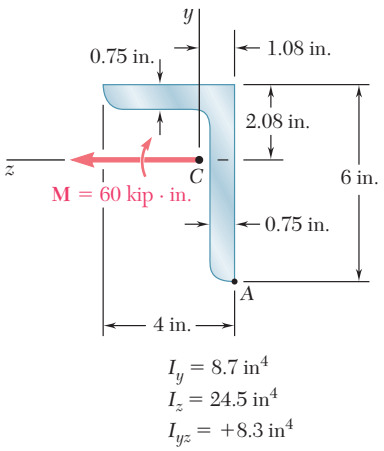
$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u}$$

$$= -\frac{(118.585)(5.3126)}{215.654} + \frac{(39.529)(0.7589)}{49.766}$$

$$= -2.32 \text{ ksi}$$

$$\sigma_A = -2.32 \text{ ksi} \blacktriangleleft$$

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$I_y = 8.7 \text{ in}^4$
$I_z = 24.5 \text{ in}^4$
$I_{yz} = +8.3 \text{ in}^4$

PROBLEM 4.143

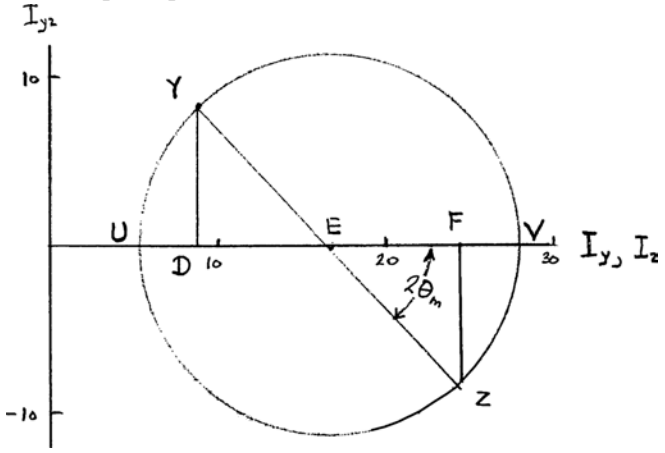
The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A.

SOLUTION

Using Mohr's circle, determine the principal axes and principal moments of inertia.

$Y: (8.7,\ 8.3) \text{ in}^4$
$Z: (24.5,\ -8.3) \text{ in}^4$
$E: (16.6,\ 0) \text{ in}^4$

$EF = 7.9 \text{ in}^4$
$FZ = 8.3 \text{ in}^4$



$$R = \sqrt{7.9^2 + 8.3^2} = 11.46 \text{ in}^4 \qquad \tan 2\theta_m = \frac{FZ}{EF} = \frac{8.3}{7.9} = 1.0506$$

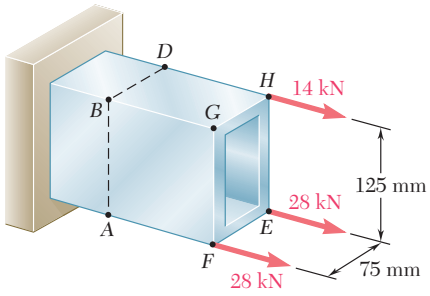
$$\theta_m = 23.2^\circ \quad I_u = 16.6 - 11.46 = 5.14 \text{ in}^4 \quad I_v = 16.6 + 11.46 = 28.06 \text{ in}^4$$

$$M_u = M \sin\theta_m = (60)\sin 23.2^\circ = 23.64 \text{ kip} \cdot \text{in.}$$
$$M_v = M \cos\theta_m = (60)\cos 23.2^\circ = 55.15 \text{ kip} \cdot \text{in.}$$
$$u_A = y_A \cos\theta_m + z_A \sin\theta_m = -3.92\cos 23.2^\circ - 1.08 \sin 23.2^\circ = -4.03 \text{ in.}$$
$$v_A = z_A \cos\theta_m - y_A \sin\theta_m = -1.08\cos 23.2^\circ + 3.92 \sin 23.2^\circ = 0.552 \text{ in.}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(55.15)(-4.03)}{28.06} + \frac{(23.64)(0.552)}{5.14}$$

$$\sigma_A = 10.46 \text{ ksi} \blacktriangleleft$$

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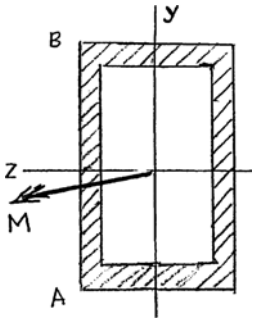


PROBLEM 4.144

The tube shown has a uniform wall thickness of 12 mm. For the loading given, determine (*a*) the stress at points *A* and *B*, (*b*) the point where the neutral axis intersects line *ABD*.

SOLUTION

Add *y*- and *z*-axes as shown. Cross section is a 75 mm × 125-mm rectangle with a 51 mm × 101-mm rectangular cutout.



$$I_z = \frac{1}{12}(75)(125)^3 - \frac{1}{12}(51)(101)^3 = 7.8283\times10^{6}\,\text{mm}^4 = 7.8283\times10^{-6}\,\text{m}^4$$

$$I_y = \frac{1}{12}(125)(75)^3 - \frac{1}{12}(101)(51)^3 = 3.2781\times10^{3}\,\text{mm}^4 = 3.2781\times10^{-6}\,\text{m}^4$$

$$A = (75)(125) - (51)(101) = 4.224\times10^{3}\,\text{mm}^2 = 4.224\times10^{-3}\,\text{m}^2$$

Resultant force and bending couples:

$$P = 14 + 28 + 28 = 70\ \text{kN} = 70\times10^{3}\ \text{N}$$

$$M_z = -(62.5\ \text{mm})(14\ \text{kN}) + (62.5\ \text{mm})(28\text{kN}) + (62.5\ \text{mm})(28\ \text{kN}) = 2625\ \text{N}\cdot\text{m}$$

$$M_y = -(37.5\ \text{mm})(14\ \text{kN}) + (37.5\ \text{mm})(28\ \text{kN}) + (37.5\ \text{mm})(28\ \text{kN}) = -525\ \text{N}\cdot\text{m}$$

(*a*) $$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{70\times10^{3}}{4.224\times10^{-3}} - \frac{(2625)(-0.0625)}{7.8283\times10^{-6}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}$$

$$= 31.524\times10^{6}\,\text{Pa} \qquad \sigma_A = 31.5\ \text{MPa} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{70\times10^{3}}{4.224\times10^{-3}} - \frac{(2625)(0.0625)}{7.8283\times10^{-6}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}$$

$$= -10.39\times10^{6}\,\text{Pa} \qquad \sigma_B = -10.39\ \text{MPa} \blacktriangleleft$$

(*b*) Let point *H* be the point where the neutral axis intersects *AB*.

$$z_H = 0.0375\ \text{m}, \quad y_H = ?, \quad \sigma_H = 0$$

$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$

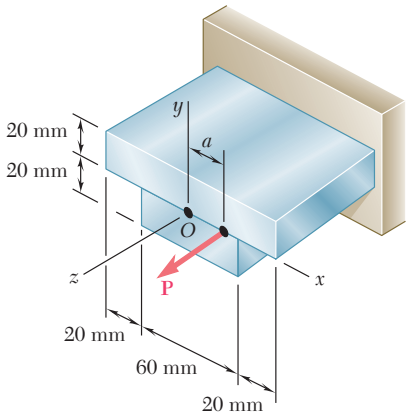
$$y_H = \frac{I_z}{M_z}\left(\frac{P}{A} + \frac{Mz_H}{I_y}\right) = \frac{7.8283\times10^{-6}}{2625}\left[\frac{70\times10^{3}}{4.224\times10^{-3}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}\right]$$

$$= 0.03151\ \text{m} = 31.51\ \text{mm}$$

$$31.51 + 62.5 = 94.0\ \text{mm}$$

Answer: 94.0 mm above point *A*. ◀

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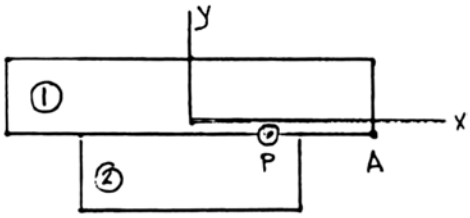


PROBLEM 4.145

A horizontal load **P** of magnitude 100 kN is applied to the beam shown. Determine the largest distance *a* for which the maximum tensile stress in the beam does not exceed 75 MPa.

SOLUTION

Locate the centroid.



	A, mm^2	$\bar{y}$, mm	$A\bar{y}$, mm^3
①	2000	10	20×10^3
②	1200	−10	-12×10^3
Σ	3200		8×10^3

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{8 \times 10^3}{3200} = 2.5 \text{ mm}$$

Move coordinate origin to the centroid.

Coordinates of load point: $X_P = a, \quad y_P = -2.5$ mm

Bending couples: $M_x = y_P P \quad M_y = -aP$

$$I_x = \frac{1}{12}(100)(20)^3 + (2000)(7.5)^2 + \frac{1}{12}(60)(20)^3 + (1200)(12.5)^2 = 0.40667 \times 10^6 \text{ mm}^4$$

$$= 0.40667 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(20)(100)^3 + \frac{1}{12}(20)(60)^3 = 2.0267 \times 10^6 \text{ mm}^4 = 2.0267 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y} \qquad \sigma_A = 75 \times 10^6 \text{ Pa}, \quad P = 100 \times 10^3 \text{ N}$$

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PROBLEM 4.145 ***(Continued)***

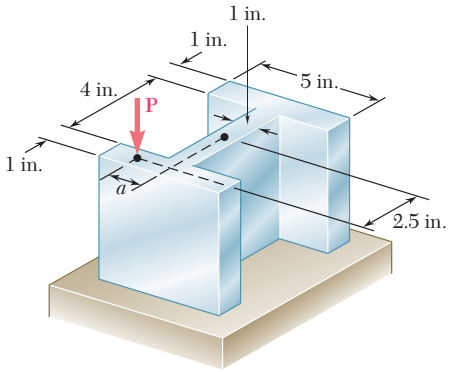
$$M_y = \frac{I_y}{x}\left\{\frac{P}{A} + \frac{M_x y}{I_x} - \sigma\right\} \qquad \text{For point } A, \quad x = 50 \text{ mm}, \; y = -2.5 \text{ mm}$$

$$M_y = \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}}\left\{\frac{100 \times 10^{3}}{3200 \times 10^{-6}} + \frac{(-2.5)(100 \times 10^{3})(-2.5 \times 10^{-3})}{0.40667 \times 10^{-6}} - 75 \times 10^{6}\right\}$$

$$= \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}}\{31.25 + 1.537 - 75\} \times 10^{6} = -1.7111 \times 10^{3} \text{ N} \cdot \text{m}$$

$$a = -\frac{M_y}{P} = -\frac{(1.7111 \times 10^{3})}{100 \times 10^{3}} = 17.11 \times 10^{3} \text{ m} \qquad a = 17.11 \text{ mm} \blacktriangleleft$$

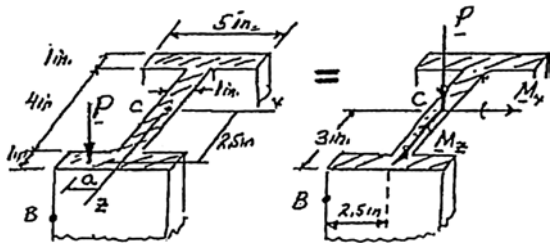
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PROBLEM 4.146

Knowing that $P = 90$ kips, determine the largest distance a for which the maximum compressive stress dose not exceed 18 ksi.

SOLUTION



$$A = (5\text{ in.})(6\text{ in.}) - 2(2\text{ in.})(4\text{ in.}) = 14\text{ in}^2$$

$$I_x = \frac{1}{12}(5\text{ in.})(6\text{ in.})^3 - 2\frac{1}{12}(2\text{ in.})(4\text{ in.})^3 = 68.67\text{ in}^4$$

$$I_z = 2\frac{1}{12}(1\text{ in.})(5\text{ in.})^3 + \frac{1}{12}(4\text{ in.})(1\text{ in.})^3 = 21.17\text{ in}^4$$

Force-couple system at C: $P = P \quad M_x = P(2.5\text{ in.}) \quad M_z = Pa$

For $P = 90$ kips: $P = 90\text{ kips} \quad M_x = (90\text{ kips})(2.5\text{ in.}) = 225\text{ kip}\cdot\text{in.} \quad M_z = (90\text{ kips})a$

Maximum compressive stress at B: $\sigma_B = -18\text{ ksi}$

$$\sigma_B = -\frac{P}{A} - \frac{M_x(3\text{ in.})}{I_x} - \frac{M_z(2.5\text{ in.})}{I_z}$$

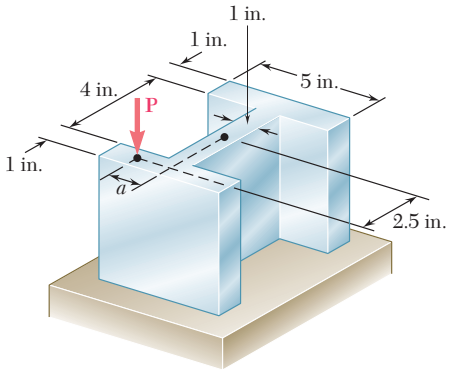
$$-18\text{ ksi} = -\frac{90\text{ kips}}{14\text{ in}^2} - \frac{(225\text{ kip}\cdot\text{in.})(3\text{ in.})}{68.67\text{ in}^4} - \frac{(90\text{ kips})a(2.5\text{ in.})}{21.17\text{ in}^4}$$

$$-18 = -6.429 - 9.830 - 10.628a$$

$$-1.741 = -10.628\,a$$

$$a = 0.1638\text{ in.} \blacktriangleleft$$

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PROBLEM 4.147

Knowing that $a = 1.25$ in., determine the largest value of **P** that can be applied without exceeding either of the following allowable stresses:

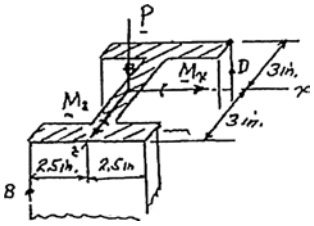
$$\sigma_{ten} = 10 \text{ ksi} \qquad \sigma_{comp} = 18 \text{ ksi}$$

SOLUTION

$$A = (5 \text{ in.})(6 \text{ in.}) - (2)(2 \text{ in.})(4 \text{ in.}) = 14 \text{ in}^2$$

$$I_x = \frac{1}{12}(5 \text{ in.})(6 \text{ in.})^3 - 2\frac{1}{12}(2 \text{ in.})(4 \text{ in.})^3 = 68.67 \text{ in}^4$$

$$I_z = 2\frac{1}{12}(1 \text{ in.})(5 \text{ in.})^3 + \frac{1}{12}(4 \text{ in.})(1 \text{ in.})^3 = 21.17 \text{ in}^4$$



Force-couple system at *C*: For $a = 1.25$ in.,

$$P = P \quad M_x = P(2.5 \text{ in.})$$

$$M_y = Pa = (1.25 \text{ in.})$$

Maximum compressive stress at *B*: $\sigma_B = -18$ ksi

$$\sigma_B = -\frac{P}{A} - \frac{M_x(3 \text{ in.})}{I_x} - \frac{M_z(2.5 \text{ in.})}{I_z}$$

$$-18 \text{ ksi} = -\frac{P}{14 \text{ in}^2} - \frac{P(2.5 \text{ in.})(3 \text{ in.})}{68.67 \text{ in}^4} - \frac{P(1.25 \text{ in.})(2.5 \text{ in.})}{21.17 \text{ in}^4}$$

$$-18 = -0.0714P - 0.1092P - 0.1476P$$

$$-18 = 0.3282P \qquad P = 54.8 \text{ kips}$$

Maximum tensile stress at *D*: $\sigma_D = +10$ ksi

$$\sigma_D = -\frac{P}{A} + \frac{M_x(3 \text{ in.})}{I_x} + \frac{M_z(2.5 \text{ in.})}{I_z}$$

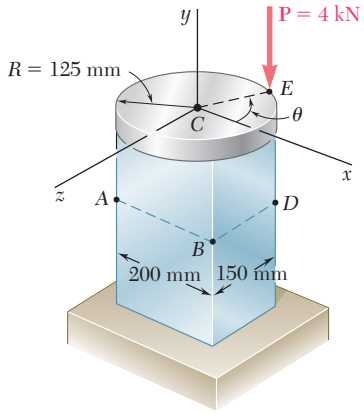
$$+10 \text{ ksi} = -0.0714P + 0.1092P + 0.1476P$$

$$10 = 0.1854P \qquad P = 53.9 \text{ kips}$$

The smaller value of **P** is the largest allowable value.

$$P = 53.9 \text{ kips} \blacktriangleleft$$

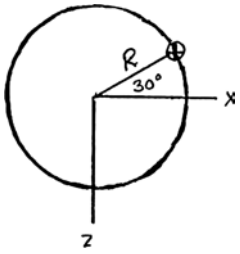
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PROBLEM 4.148

A rigid circular plate of 125-mm radius is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at E with $\theta = 30°$, determine (*a*) the stress at point A, (*b*) the stress at point B, (*c*) the point where the neutral axis intersects line ABD.

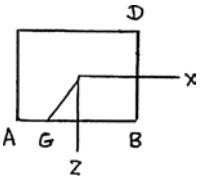
SOLUTION



$$P = 4 \times 10^3 \text{ N (compression)}$$

$$M_x = -PR\sin 30° = -(4 \times 10^3)(125 \times 10^{-3})\sin 30° = -250 \text{ N} \cdot \text{m}$$

$$M_z = -PR\cos 30° = -(4 \times 10^3)(125 \times 10^{-3})\cos 30° = -433 \text{ N} \cdot \text{m}$$



$$I_x = \frac{1}{12}(200)(150)^3 = 56.25 \times 10^6 \text{ mm}^4 = 56.25 \times 10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{12}(150)(200)^3 = 100 \times 10^6 \text{ mm}^4 = 100 \times 10^{-6} \text{ m}^4$$

$$-x_A = x_B = 100 \text{ mm} \qquad z_A = z_B = 75 \text{ mm}$$

$$A = (200)(150) = 30 \times 10^3 \text{ mm}^2 = 30 \times 10^{-3} \text{ m}^2$$

(*a*) $$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} - \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} + \frac{(-433)(-100 \times 10^{-3})}{100 \times 10^{-6}}$$

$$\sigma_A = 633 \times 10^3 \text{ Pa} = 633 \text{ kPa} \blacktriangleleft$$

(*b*) $$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} - \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} + \frac{(-433)(100 \times 10^{-3})}{100 \times 10^{-6}}$$

$$\sigma_B = -233 \times 10^3 \text{ Pa} = -233 \text{ kPa} \blacktriangleleft$$

(*c*) Let G be the point on AB where the neutral axis intersects.

$$\sigma_G = 0 \qquad z_G = 75 \text{ mm} \qquad x_G = ?$$

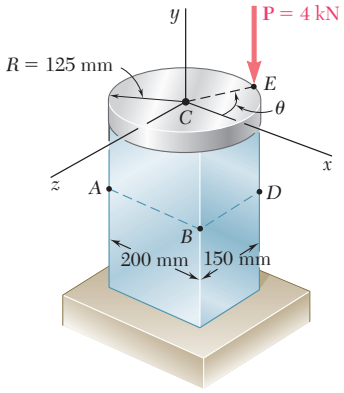
$$\sigma_G = -\frac{P}{A} - \frac{M_x z_G}{I_x} + \frac{M_z x_G}{I_z} = 0$$

$$x_G = \frac{I_z}{M_z}\left\{\frac{P}{A} + \frac{M_x z_G}{I_x}\right\} = \frac{100 \times 10^{-6}}{-433}\left\{\frac{4 \times 10^3}{30 \times 10^{-3}} + \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}}\right\}$$

$$= 46.2 \times 10^{-3} \text{ m} = 46.2 \text{ mm}$$

Point G lies 146.2 mm from point A. ◀

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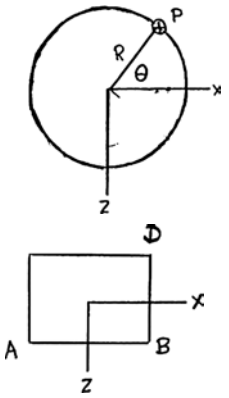
PROBLEM 4.149

In Prob. 4.148, determine (*a*) the value of θ for which the stress at *D* reaches it largest value, (*b*) the corresponding values of the stress at *A*, *B*, *C*, and *D*.

PROBLEM 4.148 A rigid circular plate of 125-mm radius is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at *E* with $\theta = 30°$, determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the point where the neutral axis intersects line *ABD*.

SOLUTION

(*a*)



$$P = 4 \times 10^3 \text{ N} \qquad PR = (4 \times 10^3)(125 \times 10^{-3}) = 500 \text{ N} \cdot \text{m}$$

$$M_x = -PR\sin\theta = -500\sin\theta \qquad M_x = -PR\cos\theta = -500\cos\theta$$

$$I_x = \frac{1}{2}(200)(150)^3 = 56.25 \times 10^6 \text{ mm}^4 = 56.25 \times 10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{2}(150)(200)^3 = 100 \times 10^6 \text{ mm}^4 = 100 \times 10^{-6} \text{ m}^4$$

$$x_D = 100 \text{ mm} \qquad z_D = -75 \text{ mm}$$

$$A = (200)(150) = 30 \times 10^3 \text{ mm}^2 = 30 \times 10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} - \frac{M_x z}{I_x} + \frac{M_z x}{I_z} = -P\left\{\frac{1}{A} - \frac{Rz\sin\theta}{I_x} + \frac{Rx\cos\theta}{I_z}\right\}$$

For σ to be a maximum, $\dfrac{d\sigma}{d\theta} = 0$ with $z = z_D$, $x = x_D$

$$\frac{d\sigma_D}{d\theta} = -P\left\{0 + \frac{Rz_D\cos\theta}{I_x} + \frac{Rx_D\sin\theta}{I_Z}\right\} = 0$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = -\frac{I_z z_D}{I_x x_D} = -\frac{(100 \times 10^{-6})(-75 \times 10^{-3})}{(56.25 \times 10^{-6})(100 \times 10^{-3})} = \frac{4}{3}$$

$$\sin\theta = 0.8, \qquad \cos\theta = 0.6 \qquad \theta = 53.1° \blacktriangleleft$$

(*b*) $$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} + \frac{(500)(0.8)(75 \times 10^{-3})}{56.25 \times 10^{-6}} - \frac{(500)(0.6)(-100 \times 10^{-3})}{100 \times 10^{-6}}$$

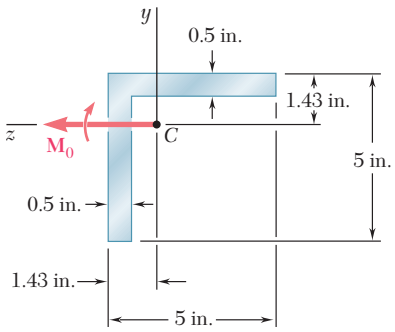
$$= (-0.13333 + 0.53333 + 0.300) \times 10^6 \text{ Pa} = 0.700 \times 10^6 \text{ Pa} \qquad \sigma_A = 700 \text{ kPa} \blacktriangleleft$$

$$\sigma_B = (-0.13333 + 0.53333 - 0.300) \times 10^6 \text{ Pa} = 0.100 \times 10^6 \text{ Pa} \qquad \sigma_B = 100 \text{ kPa} \blacktriangleleft$$

$$\sigma_C = (-0.13333 + 0 + 0) \times 10^6 \text{ Pa} \qquad \sigma_C = -133.3 \text{ kPa} \blacktriangleleft$$

$$\sigma_D = (-0.13333 - 0.53333 - 0.300) \times 10^6 \text{ Pa} = -0.967 \times 10^6 \text{ Pa} \qquad \sigma_D = -967 \text{ kPa} \blacktriangleleft$$

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PROBLEM 4.150

A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 12 ksi. *Given:* $I_y = I_z = 11.3\ \text{in}^4$, $A = 4.75\ \text{in}^2$, $k_{\min} = 0.983$ in. (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{\min} = Ak_{\min}^2$ and $I_{\min} + I_{\max} = I_y + I_z$.)

SOLUTION

$$M_u = M_0 \sin 45° = 0.70711 M_0$$

$$M_v = M_0 \cos 45° = 0.70711 M_0$$

$$I_{\min} = Ak_{\min}^2 = (4.75)(0.983)^2 = 4.59\ \text{in}^4$$

$$I_{\max} = I_y + I_z - I_{\min} = 11.3 + 11.3 - 4.59 = 18.01\ \text{in}^4$$

$$u_B = y_B \cos 45° + z_B \sin 45° = -3.57 \cos 45° + 0.93 \sin 45° = -1.866\ \text{in.}$$

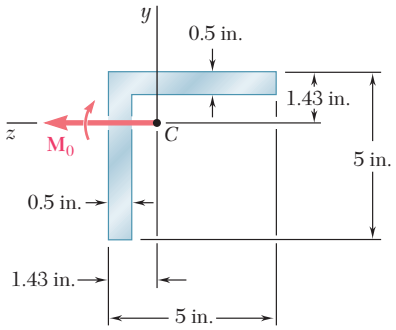
$$v_B = z_B \cos 45° - y_B \sin 45° = 0.93 \cos 45° - (-3.57) \sin 45° = 3.182\ \text{in.}$$

$$\sigma_B = -\frac{M_v u_B}{Iv} + \frac{M_u v_B}{Iu} = -0.70711 M_0 \left[-\frac{u_B}{I_{\min}} + \frac{v_B}{I_{\max}} \right]$$

$$= 0.70711 M_0 \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124 M_0$$

$$M_0 = \frac{\sigma_B}{0.4124} = \frac{12}{0.4124} \qquad M_0 = 29.1\ \text{kip} \cdot \text{in.} \blacktriangleleft$$

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PROBLEM 4.151

Solve Prob. 4.150, assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane.

PROBLEM 4.150 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 12 ksi. *Given:* $I_y = I_z = 11.3\ \text{in}^4$, $A = 4.75\ \text{in}^2$, $k_{\min} = 0.983\ \text{in.}$ (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{\min} = Ak_{\min}^2$ and $I_{\min} + I_{\max} = I_y + I_z$.)

SOLUTION

$$M_u = M_0 \cos 45° = 0.70711 M_0$$

$$M_v = -M_0 \sin 45° = -0.70711 M_0$$

$$I_{\min} = Ak_{\min}^2 = (4.75)(0.983)^2 = 4.59\ \text{in}^4$$

$$I_{\max} = I_y + I_z - I_{\min} = 11.3 + 11.3 - 4.59 = 18.01\ \text{in}^4$$

$$u_D = y_D \cos 45° + z_D \sin 45° = -0.93 \cos 45° + (-3.57 \sin 45°) = -1.866\ \text{in.}$$

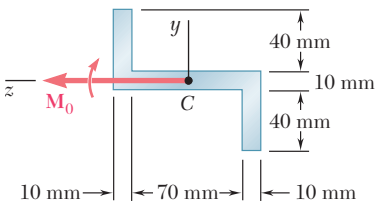
$$v_D = z_D \cos 45° - y_D \sin 45° = (-3.57) \cos 45° - (0.93) \sin 45° = 3.182\ \text{in.}$$

$$\sigma_D = -\frac{M_v u_D}{I_v} + \frac{M_u v_D}{I_u} = -0.70711 M_0 \left[-\frac{u_D}{I_{\min}} + \frac{v_D}{I_{\max}} \right]$$

$$= 0.70711 M_0 \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124 M_0$$

$$M_0 = \frac{\sigma_D}{0.4124} = \frac{12}{0.4124} \qquad M_0 = 29.1\ \text{kip} \cdot \text{in.} \blacktriangleleft$$

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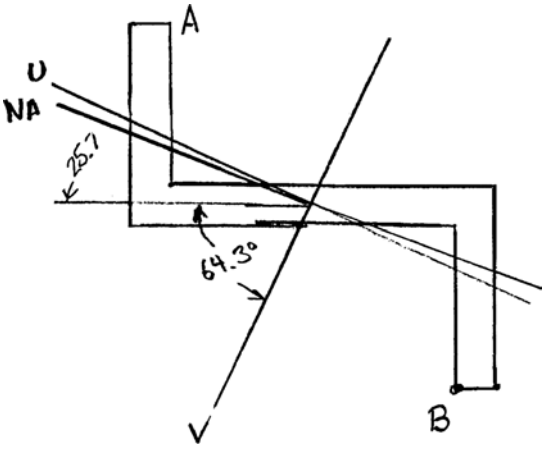


PROBLEM 4.152

The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given:* $I_{\max} = 2.28 \times 10^{-6}\ \text{mm}^4$, $I_{\min} = 0.23 \times 10^{-6}\ \text{mm}^4$, principal axes $25.7°$ and $64.3°$.

SOLUTION

$$I_v = I_{\max} = 2.28 \times 10^{6}\ \text{mm}^4 = 2.28 \times 10^{-6}\ \text{m}^4$$
$$I_u = I_{\min} = 0.23 \times 10^{6}\ \text{mm}^4 = 0.23 \times 10^{-6}\ \text{m}^4$$
$$M_v = M_0 \cos 64.3°$$
$$M_u = M_0 \sin 64.3°$$
$$\theta = 64.3°$$
$$\tan\varphi = \frac{I_v}{I_u}\tan\theta$$
$$= \frac{2.28 \times 10^{-6}}{0.23 \times 10^{-6}} \tan 64.3°$$
$$= 20.597$$
$$\varphi = 87.22°$$



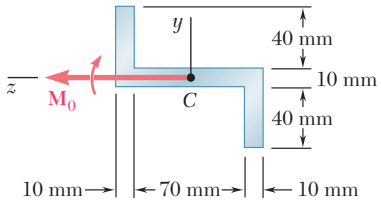
Points A and B are farthest from the neutral axis.

$$u_B = y_B \cos 64.3° + z_B \sin 64.3° = (-45)\cos 64.3° + (-35)\sin 64.3°$$
$$= -51.05\ \text{mm}$$
$$v_B = z_B \cos 64.3° - y_B \sin 64.3° = (-35)\cos 64.3° - (-45)\sin 64.3°$$
$$= +25.37\ \text{mm}$$
$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u}$$
$$80 \times 10^{6} = -\frac{(M_0 \cos 64.3°)(-51.05 \times 10^{-3})}{2.28 \times 10^{-6}} + \frac{(M_0 \sin 64.3°)(25.37 \times 10^{-3})}{0.23 \times 10^{-6}}$$
$$= 109.1 \times 10^{3} M_0$$
$$M_0 = \frac{80 \times 10^{6}}{109.1 \times 10^{3}}$$

$$M_0 = 733\ \text{N} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 4.153



Solve Prob. 4.152 assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane.

PROBLEM 4.152 The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given:* $I_{\max} = 2.28 \times 10^{-6}$ mm^4, $I_{\min} = 0.23 \times 10^{-6}$ mm^4, principal axes 25.7° and 64.3°.

SOLUTION

$$I_v = I_{\min} = 0.23 \times 10^6 \text{ mm}^4 = 0.23 \times 10^6 \text{ m}^4$$

$$I_u = I_{\max} = 2.28 \times 10^6 \text{ mm}^4 = 2.28 \times 10^6 \text{ m}^4$$

$$M_v = M_0 \cos 64.3°$$

$$M_u = M_0 \sin 64.3°$$

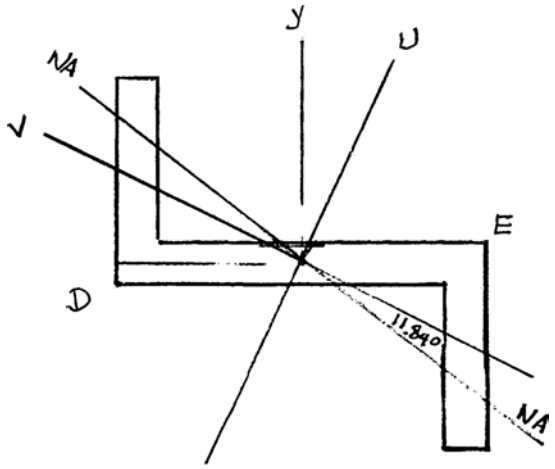
$$\theta = 64.3°$$

$$\tan \varphi = \frac{I_v}{I_u} \tan \theta$$

$$= \frac{0.23 \times 10^{-6}}{2.28 \times 10^{-6}} \tan 64.3°$$

$$= 0.20961$$

$$\varphi = 11.84°$$



Points D and E are farthest from the neutral axis.

$$u_D = y_D \cos 25.7° - z_D \sin 25.7° = (-5) \cos 25.7° - 45 \sin 25.7°$$

$$= -24.02 \text{ mm}$$

$$v_D = z_D \cos 25.7° + y_D \sin 25.7° = 45 \cos 25.7° + (-5) \sin 25.7°$$

$$= 38.38 \text{ mm}$$

$$\sigma_D = -\frac{M_v u_D}{I_v} + \frac{M_u v_D}{I_u} = -\frac{(M_D \cos 64.3°)(-24.02 \times 10^{-3})}{0.23 \times 10^{-6}}$$

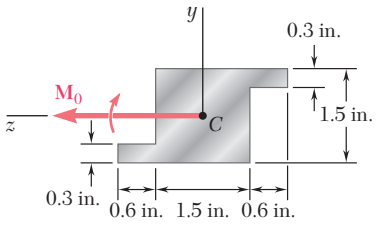
$$+ \frac{(M_0 \sin 64.3°)(38.38 \times 10^{-3})}{2.28 \times 10^{-6}}$$

$$80 \times 10^6 = 60.48 \times 10^3 M_0$$

$$M_0 = 1.323 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_0 = 1.323 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 4.154

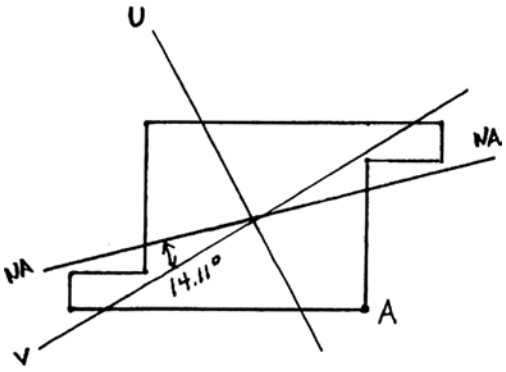
An extruded aluminum member having the cross section shown is subjected to a couple acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 12 ksi. *Given:* $I_{max} = 0.957\ \text{in}^4$, $I_{min} = 0.427\ \text{in}^4$, principal axes 29.4° ↗ and 60.6° ↘.

SOLUTION

$$I_u = I_{max} = 0.957\ \text{in}^4$$

$$I_v = I_{min} = 0.427\ \text{in}^4$$

$$M_u = M_0 \sin 29.4°, \quad M_v = M_0 \cos 29.4°$$



$$\theta = 29.4°$$

$$\tan\varphi = \frac{I_v}{I_u}\tan\theta = \frac{0.427}{0.957}\tan 29.4°$$

$$= 0.2514 \qquad \varphi = 14.11°$$

Point A is farthest from the neutral axis.

$$y_A = -0.75\ \text{in.}, \quad z_A = -0.75\ \text{in.}$$

$$u_A = y_A \cos 29.4° + z_A \sin 29.4° = -1.0216\ \text{in.}$$

$$v_A = z_A \cos 29.4° - y_A \sin 29.4° = -0.2852\ \text{in.}$$

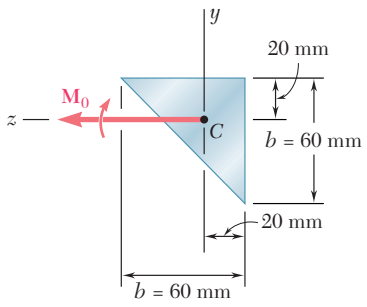
$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u V_A}{I_u} = -\frac{(M_0 \cos 29.4°)(-1.0216)}{0.427} + \frac{(M_0 \sin 29.4°)(-0.2852)}{0.957}$$

$$= 1.9381 M_0$$

$$M_0 = \frac{\sigma_A}{1.9381} = \frac{12}{1.9381}$$

$$M_0 = 6.19\ \text{kip} \cdot \text{in.} \blacktriangleleft$$

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PROBLEM 4.155

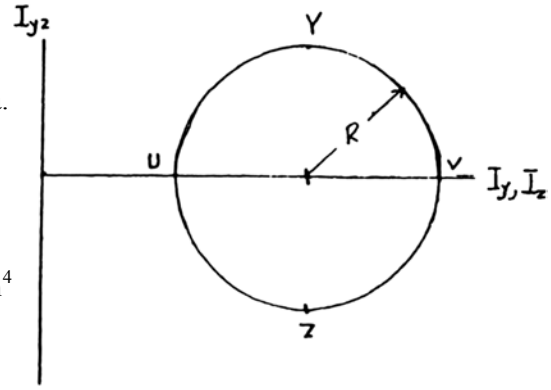
A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 100 MPa. *Given:* $I_y = I_z = b^4/36$ and $I_{yz} = b^4/72$.

SOLUTION

$$I_y = I_z = \frac{b^4}{36} = \frac{60^4}{36} = 0.360 \times 10^6 \text{ mm}^4$$

$$I_{yz} = \frac{b^4}{72} = \frac{60^4}{72} = 0.180 \times 10^6 \text{ mm}^4$$

Principal axes are symmetry axes.



Using Mohr's circle, determine the principal moments of inertia.

$$R = |I_{yz}| = 0.180 \times 10^6 \text{ mm}^4$$

$$I_v = \frac{I_y + I_z}{2} + R$$

$$= 0.540 \times 10^6 \text{ mm}^4 = 0.540 \times 10^{-6} \text{ m}^4$$

$$I_u = \frac{I_y + I_z}{2} - R$$

$$= 0.180 \times 10^6 \text{ mm}^4 = 0.180 \times 10^{-6} \text{ m}^4$$

$$M_u = M_0 \sin 45° = 0.70711 M_0, \qquad M_v = M_0 \cos 45° = 0.70711 M_0$$

$$\theta = 45° \qquad \tan\varphi = \frac{I_v}{I_u} \tan\theta = \frac{0.540 \times 10^{-6}}{0.180 \times 10^{-6}} \tan 45° = 3$$

$$\varphi = 71.56°$$

Point *A*: $u_A = 0 \qquad v_A = -20\sqrt{2} \text{ mm}$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = 0 + \frac{(0.70711 M_0)(-20\sqrt{2} \times 10^3)}{0.180 \times 10^{-6}} = -11.11 \times 10^{-3} M_0$$

$$M_0 = -\frac{\sigma_A}{111.11 \times 10^3} = -\frac{-100 \times 10^6}{111.11 \times 10^3} = 900 \text{ N} \cdot \text{m}$$

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PROBLEM 4.155 ***(Continued)***

Point *B*:

$$u_B = -\frac{60}{\sqrt{2}}\text{ mm}, \qquad v_B = \frac{20}{\sqrt{2}}\text{ mm}$$

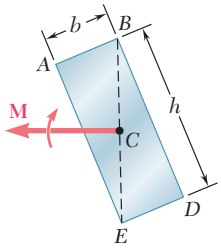
$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u} = -\frac{(0.70711M_0)\left(-\frac{60}{\sqrt{2}}\times 10^{-3}\right)}{0.540\times 10^{-6}} + \frac{(0.70711M_0)\left(\frac{20}{\sqrt{2}}\times 10^{-3}\right)}{0.180\times 10^{-6}}$$

$$= -111.11\times 10^{3} M_0$$

$$M_0 = \frac{\sigma_B}{111.11\times 10^{-3}} = \frac{100\times 10^{6}}{111.11\times 10^{-3}} = 900\text{ N}\cdot\text{m}$$

Choose the smaller value. $M_0 = 900\text{ N}\cdot\text{m}$ ◀

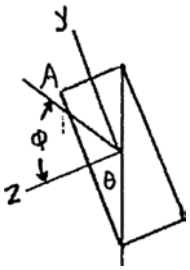
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PROBLEM 4.156

Show that, if a solid rectangular beam is bent by a couple applied in a plane containing one diagonal of a rectangular cross section, the neutral axis will lie along the other diagonal.

SOLUTION



$$\tan\theta = \frac{b}{h}$$

$$M_z = M\cos\theta, \quad M_z = M\sin\theta$$

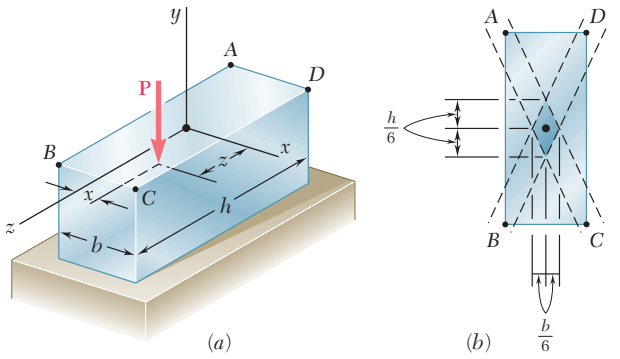
$$I_z = \frac{1}{12}bh^3 \qquad I_y = \frac{1}{12}hb^3$$

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{\frac{1}{12}bh^3}{\frac{1}{12}hb^3}\cdot\frac{b}{h} = \frac{h}{b}$$

The neutral axis passes through corner A of the diagonal AD. ◀

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PROBLEM 4.157



(*a*) Show that the stress at corner *A* of the prismatic member shown in part *a* of the figure will be zero if the vertical force **P** is applied at a point located on the line

$$\frac{x}{b/6} + \frac{z}{h/6} = 1$$

(*b*) Further show that, if no tensile stress is to occur in the member, the force **P** must be applied at a point located within the area bounded by the line found in part *a* and three similar lines corresponding to the condition of zero stress at *B*, *C*, and *D*, respectively. This area, shown in part *b* of the figure, is known as the *kern* of the cross section.

SOLUTION

$$I_z = \frac{1}{12}hb^3 \quad I_x = \frac{1}{12}bh^3 \quad A = bh$$

$$z_A = -\frac{h}{2} \quad x_A = -\frac{b}{2}$$

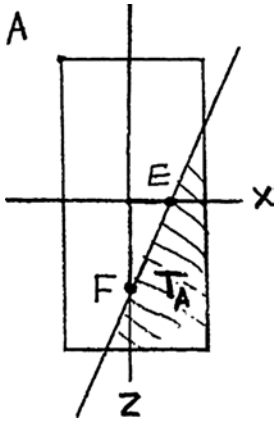
Let *P* be the load point.

$$M_z = -Px_P \quad M_x = Pz_P$$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x}$$

$$= -\frac{P}{bh} + \frac{(-Px_P)\left(-\frac{b}{2}\right)}{\frac{1}{12}hb^3} - \frac{(Pz_P - \frac{h}{2})}{\frac{1}{12}bh^3}$$

$$= -\frac{P}{bh}\left[1 - \frac{x_P}{b/6} - \frac{z_P}{h/6}\right]$$



(*a*) For $\sigma_A = 0$, $1 - \frac{x}{b/6} - \frac{z}{h/6} = 0$ $\frac{x}{b/6} + \frac{z}{h/6} = 1$

(*b*) At point *E*: $z = 0 \quad \therefore \quad x_E = b/6$

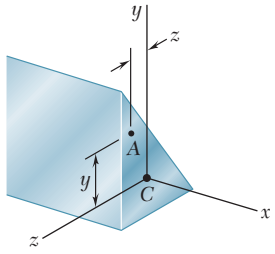
At point *F*: $x = 0 \quad \therefore \quad z_F = h/6$

If the line of action (x_P, z_P) lies within the portion marked T_A, a tensile stress will occur at corner *A*.

By considering $\sigma_B = 0$, $\sigma_C = 0$, and $\sigma_D = 0$, the other portions producing tensile stresses are identified.

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PROBLEM 4.158



A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the horizontal plane xz. Show that the stress at point A, of coordinates y and z, is

$$\sigma_A = \frac{zI_z - yI_{yz}}{I_yI_z - I_{yz}^2}M_y$$

where I_y, I_z, and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_y the moment of the couple.

SOLUTION

The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes:

$$\sigma_A = C_1y + C_2z$$

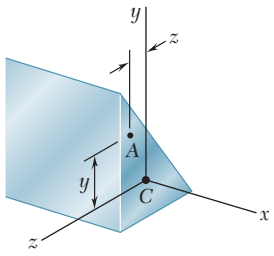
where C_1 and C_2 are constants.

$$\begin{aligned}
M_z &= -\int y\sigma_A dA = -C_1\int y^2dA - C_2\int yz\,dA \\
&= -I_zC_1 - I_{yz}C_2 = 0 \\
C_1 &= -\frac{I_{yz}}{I_z}C_2 \\
M_y &= \int z\sigma_A dA = C_1\int yz\,dA + C_2\int z^2dA \\
&= I_{yz}C_1 + I_yC_2 \\
&\quad - I_{yz}\frac{I_{yz}}{I_z}C_2 + I_yC_2 \\
I_zM_y &= \left(I_yI_z - I_{yz}^2\right)C_2 \\
C_2 &= \frac{I_zM_y}{I_yI_z - I_{yz}^2} \\
C_1 &= -\frac{I_{yz}M_y}{I_yI_z - I_{yz}^2} \\
\sigma_A &= \frac{I_zz - I_{yz}y}{I_yI_z - I_{yz}^2}M_y
\end{aligned}$$

◀

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PROBLEM 4.159



A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the vertical plane *xy*. Show that the stress at point *A*, of coordinates *y* and *z*, is

$$\sigma_A = \frac{yI_y - zI_{yz}}{I_yI_z - I_{yz}^2}M_z$$

where $I_y, I_z,$ and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_z the moment of the couple.

SOLUTION

The stress σ_A varies linearly with the coordinates *y* and *z*. Since the axial force is zero, the *y*- and *z*-axes are centroidal axes:

$$\sigma_A = C_1y + C_2z$$

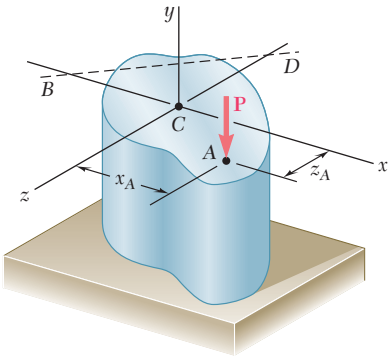
where C_1 and C_2 are constants.

$$M_y = \int z\sigma_A dA = C_1\int yz\,dA + C_2\int z^2dA$$
$$= I_{yz}C_1 + I_yC_2 = 0$$
$$C_2 = -\frac{I_{yz}}{I_y}C_1$$
$$M_z = -\int y\sigma_A dz = -C_1\int y^2dA + C_2\int yz\,dA$$
$$= -I_zC_1 - I_{yz}\frac{I_{yz}}{I_y}C_1$$
$$I_yM_z = -\left(I_yI_z - I_{yz}^2\right)C_1$$
$$C_1 = -\frac{I_yM_z}{I_yI_z - I_{yz}^2}$$
$$C_2 = +\frac{I_{yz}M_z}{I_yI_z - I_{yz}^2}$$
$$\sigma_A = -\frac{I_yy - I_{yz^2}}{I_yI_z - I_{yz}^2}M_z$$

◀

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PROBLEM 4.160



(*a*) Show that, if a vertical force **P** is applied at point *A* of the section shown, the equation of the neutral axis *BD* is

$$\left(\frac{x_A}{r_z^2}\right)x + \left(\frac{z_A}{r_x^2}\right)z = -1$$

where r_z and r_x denote the radius of gyration of the cross section with respect to the *z* axis and the *x* axis, respectively. (*b*) Further show that, if a vertical force **Q** is applied at any point located on line *BD*, the stress at point *A* will be zero.

SOLUTION

Definitions:

$$r_x^2 = \frac{I_x}{A}, \quad r_z^2 = \frac{I_z}{A}$$

(*a*) $M_x = Pz_A \qquad M_z = -Px_A$

$$\sigma_E = -\frac{P}{A} + \frac{M_z x_E}{I_z} - \frac{M_x z_E}{I_x} = -\frac{P}{A} - \frac{Px_A x_E}{Ar_z^2} - \frac{Pz_A z_E}{Ar_x^2}$$

$$= -\frac{P}{A}\left[1 + \left(\frac{x_A}{r_z^2}\right)x_E + \left(\frac{z_A}{r_x^2}\right)z_E\right] = 0$$

if *E* lies on neutral axis.

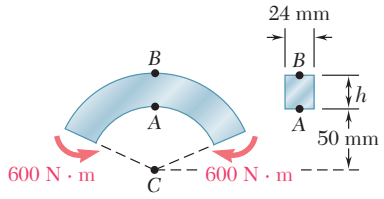
$$1 + \left(\frac{x_A}{r_z^2}\right)x + \left(\frac{z_A}{r_x^2}\right)z = 0, \qquad \left(\frac{x_A}{r_z^2}\right)x + \left(\frac{z_A}{r_x^2}\right)z = -1 \quad \blacktriangleleft$$

(*b*) $M_x = Pz_E \quad M_z = -Px_E$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_y} = -\frac{P}{A} - \frac{Px_E x_A}{Ar_z^2} - \frac{Pz_E z_A}{Ar_x^2}$$

$= 0$ by equation from part (*a*). $\blacktriangleleft$

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PROBLEM 4.161

For the curved bar shown, determine the stress at point A when (*a*) $h = 50$ mm, (*b*) $h = 60$ mm.

SOLUTION

(*a*) $h = 50$ mm, $r_1 = 50$ mm, $r_2 = 100$ mm

$$A = (24)(50) = 1.200 \times 10^3 \text{ mm}$$

$$\mathrm{R} = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{50}{\ln \frac{100}{50}} = 72.13475 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 75 \text{ mm}$$

$$e = \bar{r} - R = 2.8652 \text{ mm}$$

$$y_A = 72.13475 - 50 = 22.13475 \text{ mm} \qquad r_A = 50 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(22.13475 \times 10^{-3})}{(1.200 \times 10^{-3})(2.8652 \times 10^{-3})(50 \times 10^{-3})} = -77.3 \times 10^6 \text{ Pa}$$

-77.3 MPa ◀

(*b*) $h = 60$ mm, $r_1 = 50$ mm, $r_2 = 110$ mm, $A = (24)(60) = 1.440 \times 10^3 \text{ mm}^2$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{60}{\ln \frac{110}{50}} = 76.09796 \text{ mm}$$

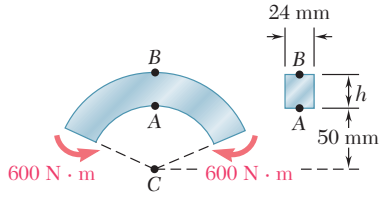
$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 80 \text{ mm} \qquad e = \bar{r} - R = 3.90204 \text{ mm}$$

$$y_A = 76.09796 - 50 = 26.09796 \text{ mm} \qquad r_A = 50 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(26.09796 \times 10^{-3})}{(1.440 \times 10^{-3})(3.90204 \times 10^{-3})(50 \times 10^{-3})} = -55.7 \times 10^6 \text{ Pa}$$

-55.7 MPa ◀

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PROBLEM 4.162

For the curved bar shown, determine the stress at points A and B when $h = 55$ mm.

SOLUTION

$$h = 55 \text{ mm}, \qquad r_1 = 50 \text{ mm}, \qquad r_2 = 105 \text{ mm}$$

$$A = (24)(55) = 1.320 \times 10^3 \text{ mm}^2$$

$$R = \frac{h}{\ln \dfrac{r_2}{r1}} = \frac{50}{\ln \dfrac{105}{50}} = 74.13025 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 77.5 \text{ mm}$$

$$e = \bar{r} - R = 3.36975 \text{ mm}$$

$$y_A = 74.13025 - 50 = 24.13025 \text{ mm} \qquad r_A = 50 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(24.13025 \times 10^{-3})}{(1.320 \times 10^{-3})(3.36975 \times 10^{-3})(50 \times 10^{-3})} = -65.1 \times 10^6 \text{ Pa}$$

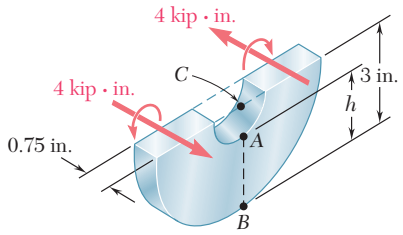
$$-65.1 \text{ MPa} \blacktriangleleft$$

$$y_B = 74.13025 - 105 = -30.86975 \text{ mm} \qquad r_B = 105 \text{ mm}$$

$$\sigma_B = \frac{My_B}{Aer_B} = -\frac{(600)(-30.8697 \times 10^{-3})}{(1.320 \times 10^{-3})(3.36975 \times 10^{-3})(105 \times 10^{-3})} = 39.7 \times 10^6 \text{ Pa}$$

$$39.7 \text{ MPa} \blacktriangleleft$$

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PROBLEM 4.163

For the machine component and loading shown, determine the stress at point A when (a) $h = 2$ in., (b) $h = 2.6$ in.

SOLUTION

$$M = -4 \text{ kip} \cdot \text{in.}$$

Rectangular cross section: $A = bh \quad r_2 = 3 \text{ in.} \quad r_1 = r_2 - h$

$$\bar{r} = \frac{1}{2}(r_1 + r_2), \quad R = \frac{h}{\ln \frac{r_2}{r_1}}, \quad e = \bar{r} - R$$

(a) $h = 2$ in. $\quad A = (0.75)(2) = 1.5 \text{ in}^2$

$$r_1 = 3 - 2 = 1 \text{ in.} \qquad \bar{r} = \frac{1}{2}(3 + 1) = 2 \text{ in.}$$

$$R = \frac{2}{\ln \frac{3}{1}} = 1.8205 \text{ in.} \quad e = 2 - 1.8205 = 0.1795 \text{ in.}$$

At point A: $\quad r = r_1 = 1$ in.

$$\sigma_A = \frac{M(r - R)}{Aer} = \frac{(-4)(1 - 1.8205)}{(1.5)(0.1795)(1)} = 12.19 \text{ ksi}$$

$$\sigma_A = 12.19 \text{ ksi} \blacktriangleleft$$

(b) $h = 2.6$ in. $\quad A = (0.75)(2.6) = 1.95 \text{ in}^2$

$$r_1 = 3 - 2.6 = 0.4 \text{ in.} \qquad \bar{r} = \frac{1}{2}(3 + 0.4) = 1.7 \text{ in.}$$

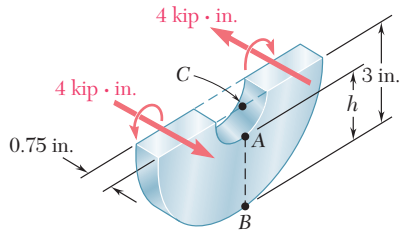
$$R = \frac{2.6}{\ln \frac{3}{0.4}} = 1.2904 \text{ in.} \quad e = 1.7 - 1.2904 = 0.4906 \text{ in.}$$

At point A: $\quad r = r_1 = 0.4$ in.

$$\sigma_A = \frac{M(r - R)}{Aer} = \frac{(-4)(0.4 - 1.2904)}{(1.95)(0.4096)(0.4)} = 11.15 \text{ ksi}$$

$$\sigma_A = 11.15 \text{ ksi} \blacktriangleleft$$

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PROBLEM 4.164

For the machine component and loading shown, determine the stress at points A and B when $h = 2.5$ in.

SOLUTION

$$M = -4 \text{ kip} \cdot \text{in.}$$

Rectangular cross section: $h = 2.5$ in. $b = 0.75$ in. $A = 1.875$ in.^{2}

$$r_2 = 3 \text{ in.} \quad r_1 = r_2 - h = 0.5 \text{ in.}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = \frac{1}{2}(0.5 + 3.0) = 1.75 \text{ in.}$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{2.5}{\ln \frac{3.0}{0.5}} = 1.3953 \text{ in.}$$

$$e = \bar{r} - R = 1.75 - 1.3953 = 0.3547 \text{ in.}$$

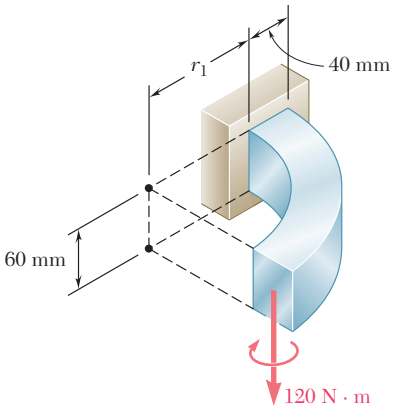
At point A: $r = r_1 = 0.5$ in.

$$\sigma_A = \frac{M(r - R)}{Aer} = \frac{(-4 \text{ kip} \cdot \text{in.})(0.5 \text{ in.} - 1.3953 \text{ in.})}{(0.75 \text{ in.})(2.5 \text{ in.})(0.3547 \text{ in.})(0.5 \text{ in.})} \qquad \sigma_A = 10.77 \text{ ksi} \blacktriangleleft$$

At point B: $r = r_2 = 3$ in.

$$\sigma_B = \frac{M(r - R)}{Aer} = \frac{(-4 \text{ kip} \cdot \text{in.})(3 \text{ in.} - 1.3953 \text{ in.})}{(0.75 \text{ in.} \times 2.5 \text{ in.})(0.3547 \text{ in.})(3 \text{ in.})} \qquad \sigma_B = -3.22 \text{ ksi} \blacktriangleleft$$

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PROBLEM 4.165

The curved bar shown has a cross section of 40×60 mm and an inner radius $r_1 = 15$ mm. For the loading shown, determine the largest tensile and compressive stresses.

SOLUTION

$$h = 40 \text{ mm}, \quad r_1 = 15 \text{ mm}, \quad r_2 = 55 \text{ mm}$$

$$A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{m}^2$$

$$R = \frac{h}{\ln \dfrac{r_2}{r_1}} = \frac{40}{\ln \dfrac{55}{40}} = 30.786 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 35 \text{ mm}$$

$$e = \bar{r} - R = 4.214 \text{ mm} \qquad \sigma = -\frac{My}{Aer}$$

At $r = 15$ mm, $y = 30.786 - 15 = 15.756$ mm

$$\sigma = -\frac{(120)(15.786 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(15 \times 10^{-3})} = -12.49 \times 10^{-6} \text{ Pa}$$

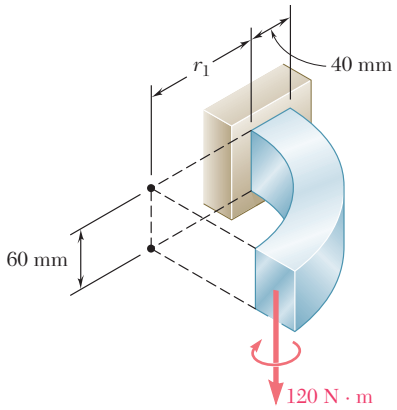
$$\sigma = -12.49 \text{ MPa} \blacktriangleleft$$

At $r = 55$ mm, $y = 30.786 - 55 = -24.214$ mm

$$\sigma = -\frac{(120)(-24.214 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(55 \times 10^{-3})} = 5.22 \times 10^{6} \text{ Pa}$$

$$\sigma = 5.22 \text{ MPa} \blacktriangleleft$$

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PROBLEM 4.166

For the curved bar and loading shown, determine the percent error introduced in the computation of the maximum stress by assuming that the bar is straight. Consider the case when (*a*) $r_1 = 20$ mm, (*b*) $r_1 = 200$ mm, (*c*) $r_1 = 2$ m.

SOLUTION

$$h = 40 \text{ mm}, \quad A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2, \quad M = 120 \text{ N} \cdot \text{m}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(60)(40)^3 = 0.32 \times 10^6 \text{ mm}^4 = 0.32 \times 10^{-6} \text{ mm}^4, \quad c = \frac{1}{2}h = 20 \text{ mm}$$

Assuming that the bar is straight,

$$\sigma_s = -\frac{Mc}{I} = -\frac{(120)(20 \times 10^{-8})}{(0.32 \times 10^{-6})} = 7.5 \times 10^6 \text{Pa} = 7.5 \text{ MPa}$$

(*a*) $r_1 = 20$ mm $\quad r_2 = 60$ mm

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{40}{\ln \frac{60}{20}} = 36.4096 \text{ mm} \qquad r_1 - R = -16.4096 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 40 \text{ mm} \qquad e = \bar{r} - R = 3.5904 \text{ mm}$$

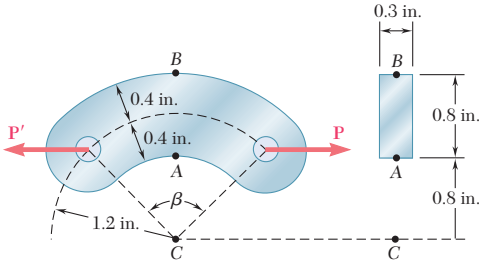
$$\sigma_a = \frac{M(r_1 - R)}{Aer} = \frac{(120)(-16.4096 \times 10^{-3})}{(2400 \times 10^{-6})(3.5904 \times 10^{-3})(20 \times 10^{-3})} = -11.426 \times 10^6 \text{ Pa} = -11.426 \text{ MPa}$$

$$\% \text{ error} = \frac{-11.426 - (-7.5)}{-11.426} \times 100\% = -34.4\% \quad \blacktriangleleft$$

For parts (*b*) and (*c*), we get the values in the table below:

	r_1, mm	r_2, mm	R, mm	$\bar{r}$, mm	e, mm	σ, MPa	% error
(*a*)	20	60	36.4096	40	3.5904	−11.426	−34.4 %
(*b*)	200	240	219.3926	220	0.6074	−7.982	6.0 % ◀
(*c*)	2000	2040	2019.9340	2020	0.0660	−7.546	0.6 % ◀

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PROBLEM 4.167

Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 12 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.

SOLUTION

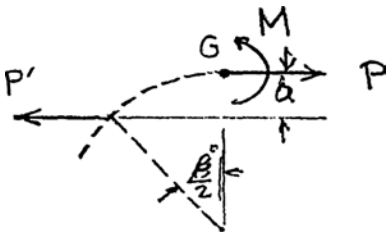
Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$

The bending couple is $M = -Pa$.

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln\frac{r_2}{r_1}}.$$



Also, $e = \bar{r} - R$

At point A, the tensile stress is $\sigma_A = \dfrac{P}{A} - \dfrac{My_A}{Aer_1} = \dfrac{P}{A} + \dfrac{Pay_A}{Aer_1} = \dfrac{P}{A}\left(1 + \dfrac{ay_A}{er_1}\right) = K\dfrac{P}{A}$

where $K = 1 + \dfrac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 1.2$ in., $r_1 = 0.8$ in., $r_2 = 1.6$ in., $h = 0.8$ in., $b = 0.3$ in.

$A = (0.3)(0.8) = 0.24$ in^2 $\qquad R = \dfrac{0.8}{\ln\frac{1.6}{0.8}} = 1.154156$ in.

$e = 1.2 - 1.154156 = 0.045844$ in., $\qquad y_A = 1.154156 - 0.8 = 0.35416$ in.

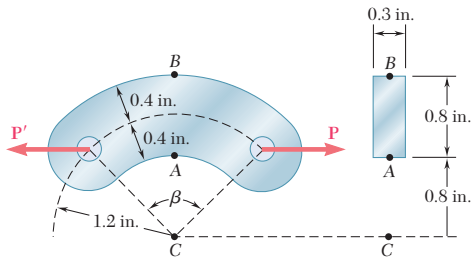
$a = 1.2(1 - \cos 45°) = 0.35147$ in.

$$K = 1 + \frac{(0.35147)(0.35416)}{(0.045844)(0.8)} = 4.3940$$

$$P = \frac{(0.24)(12)}{4.3940} = 0.65544 \text{ kips}$$

$P = 655$ lb ◀

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PROBLEM 4.168

Solve Prob. 4.167, assuming that $\beta = 60°$.

PROBLEM 4.167 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 12 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.

SOLUTION

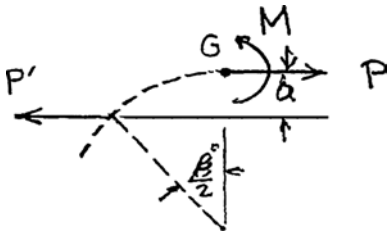
Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$

The bending couple is $M = -Pa$.

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln\frac{r_2}{r_1}}.$$



Also, $e = \bar{r} - R$

At point A, the tensile stress is $\sigma_A = \dfrac{P}{A} - \dfrac{My_A}{Aer_1} = \dfrac{P}{A} + \dfrac{Pay_A}{Aer_1} = \dfrac{P}{A}\left(1 + \dfrac{ay_A}{er_1}\right) = K\dfrac{P}{A}$

where $K = 1 + \dfrac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 1.2$ in., $r_1 = 0.8$ in., $r_2 = 1.6$ in., $h = 0.8$ in., $b = 0.3$ in.

$A = (0.3)(0.8) = 0.24$ in^2 $\qquad R = \dfrac{0.8}{\ln\frac{1.6}{0.8}} = 1.154156$ in.

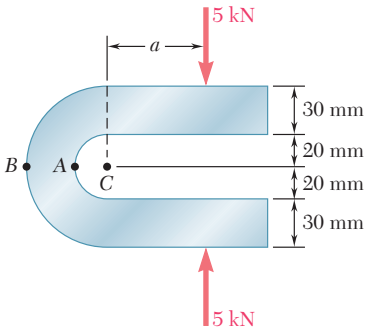
$e = 1.2 - 1.154156 = 0.045844$ in. $\qquad y_A = 1.154156 - 0.8 = 0.35416$ in.

$a = (1.2)(1 - \cos 30°) = 0.160770$ in.

$K = 1 + \dfrac{(0.160770)(0.35416)}{(0.045844)(0.8)} = 2.5525$

$P = \dfrac{(0.24)(12)}{2.5525} = 1.128$ kips $\qquad P = 1128$ lb ◀

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PROBLEM 4.169

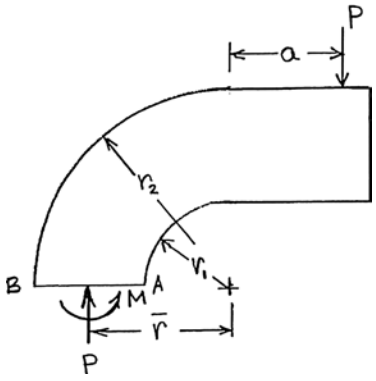
The curved bar shown has a cross section of 30×30 mm. Knowing that the allowable compressive stress is 175 MPa, determine the largest allowable distance a.

SOLUTION

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at $R = \dfrac{h}{\ln \dfrac{r_2}{r_1}}$.



Also, $e = \bar{r} - R$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a + \bar{r})y_A}{Aer_1}$$

$$= -K\frac{P}{A} \quad \text{with} \quad y_A = R - r_1$$

Thus, $K = 1 + \dfrac{(a + \bar{r})(R - r_1)}{er_1}$ (1)

Data: $h = 30$ mm, $r_1 = 20$ mm, $r_2 = 50$ mm, $\bar{r} = 35$ mm, $R = \dfrac{30}{\ln \frac{50}{20}} = 32.7407$ mm

$e = 35 - 32.7407 = 2.2593$ mm, $b = 30$ mm, $R - r_1 = 12.7407$ mm, $a = ?$

$\sigma_A = -175$ MPa $= -175 \times 10^6$ Pa, $P = 5$ kN $= 5 \times 10^3$ N

$$K = -\frac{A\sigma_A}{P} = -\frac{(900 \times 10^{-6})(-175 \times 10^6)}{5 \times 10^3} = 31.5$$

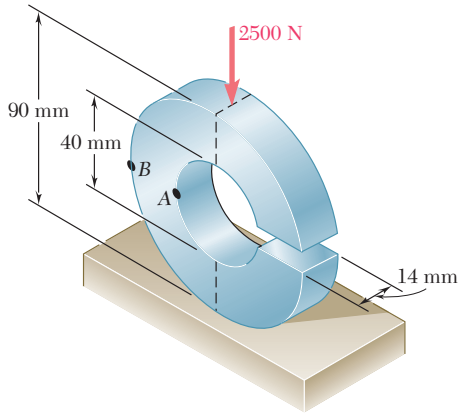
Solving (1) for $a + \bar{r}$, $a + \bar{r} = \dfrac{(K - 1)er_1}{R - r_1}$

$$a + \bar{r} = \frac{(30.5)(2.2593)(20)}{12.7407} = 108.17 \text{ mm}$$

$a = 108.17$ mm $- 35$ mm

$a = 73.2$ mm ◀

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PROBLEM 4.170

For the split ring shown, determine the stress at (*a*) point *A*, (*b*) point *B*.

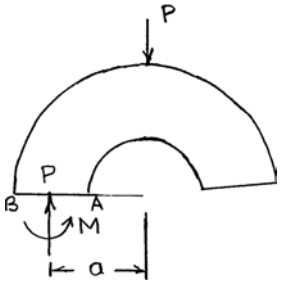
SOLUTION

$$r_1 = \frac{1}{2}40 = 20 \text{ mm}, \qquad r_2 = \frac{1}{2}(90) = 45 \text{ mm} \qquad h = r_2 - r_1 = 25 \text{ mm}$$

$$A = (14)(25) = 350 \text{ mm}^2 \qquad R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 32.5 \text{ mm} \qquad e = \bar{r} - R = 1.6712 \text{ mm}$$

Reduce the internal forces transmitted across section *AB* to a force-couple system at the centroid of the cross section. The bending couple is



$$M = Pa = P\bar{r} = (2500)(32.5 \times 10^{-3}) = 81.25 \text{ N} \cdot \text{m}$$

(*a*) Point *A*: $r_A = 20$ mm $\quad y_A = 30.8288 - 20 = 10.8288$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{AeR} = -\frac{2500}{350 \times 10^{-6}} - \frac{(81.25)(10.8288 \times 10^{-3})}{(350 \times 10^{-6})(1.6712 \times 10^{-3})(20 \times 10^{-3})}$$

$$= -82.4 \times 10^6 \text{ Pa}$$

$\sigma_A = -82.4$ MPa ◀

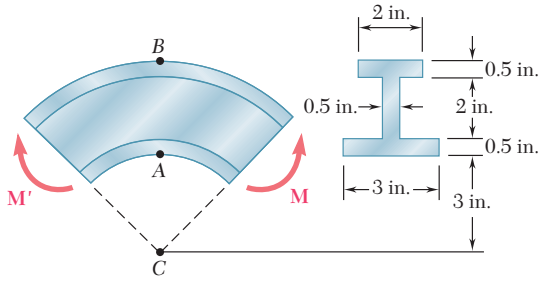
(*b*) Point *B*: $r_B = 45$ mm $\quad y_B = 30.8288 - 45 = -14.1712$ mm

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_B} = -\frac{2500}{350 \times 10^{-6}} - \frac{(81.25)(-14.1712 \times 10^{-3})}{(350 \times 10^{-6})(1.6712 \times 10^{-3})(45 \times 10^{-3})}$$

$$= 36.6 \times 10^6 \text{ Pa}$$

$\sigma_B = 36.6$ MPa ◀

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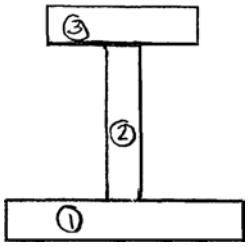
PROBLEM 4.171

Three plates are welded together to form the curved beam shown. For $M = 8$ kip · in., determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) the centroid of the cross section.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



r	Part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3	①	3	0.5	1.5	0.462452	3.25	4.875
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6	Σ			3.5	0.862468		15.125

$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in.}, \quad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.} \qquad M = -8 \text{ kip} \cdot \text{in.}$$

(*a*) $y_A = R - r_1 = 4.05812 - 3 = 1.05812$ in.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-8)(1.05812)}{(3.5)(0.26331)(3)} \qquad \sigma_A = 3.06 \text{ ksi} \blacktriangleleft$$

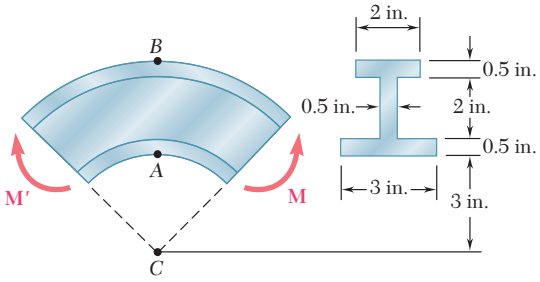
(*b*) $y_B = R - r_2 = 4.05812 - 6 = -1.94188$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-8)(-1.94188)}{(3.5)(0.26331)(6)} \qquad \sigma_B = -2.81 \text{ ksi} \blacktriangleleft$$

(*c*) $y_C = R - \bar{r} = -e$

$$\sigma_C = -\frac{My_C}{A\bar{r}} = -\frac{Me}{A\bar{r}} = -\frac{M}{A\bar{r}} = -\frac{-8}{(3.5)(4.32143)} \qquad \sigma_C = 0.529 \text{ ksi} \blacktriangleleft$$

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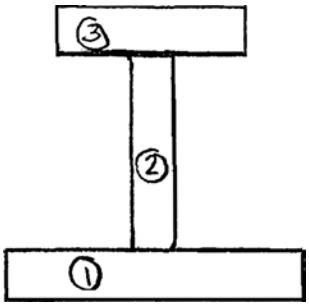
PROBLEM 4.172

Three plates are welded together to form the curved beam shown. For the given loading, determine the distance e between the neutral axis and the centroid of the cross section.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r}\,dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



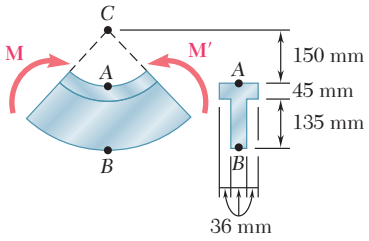
r	Part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3							
3.5	①	3	0.5	1.5	0.462452	3.25	4.875
5.5	②	0.5	2	1.0	0.225993	4.5	4.5
6	③	2	0.5	1.0	0.174023	5.75	5.75
	Σ			3.5	0.862468		15.125

$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in.}, \quad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.}$$

$e = 0.263$ in. ◀

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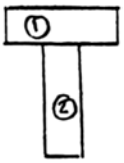
PROBLEM 4.173

Knowing that the maximum allowable stress is 45 MPa, determine the magnitude of the largest moment M that can be applied to the components shown.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i 2 \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$$



r, mm	Part	b_i, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}$, mm	$A\bar{r}$, mm^3
150							
195	①	108	45	4860	28.3353	172.5	838.35×10^3
330	②	36	135	4860	18.9394	262.5	1275.75×10^3
	Σ			9720	47.2747		2114.1×10^3

$$R = \frac{9720}{47.2747} = 205.606 \text{ mm} \qquad \bar{r} = \frac{2114.1 \times 10^3}{9720} = 217.5 \text{ mm}$$

$$e = \bar{r} - R = 11.894 \text{ mm}$$

$$y_A = R - r_1 = 205.606 - 150 = 55.606 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_1}$$

$$M = \frac{\sigma_A Aer_1}{y_A}$$

$$= \frac{(45 \times 10^6)(9720 \times 10^{-6})(11.894 \times 10^{-3})(150 \times 10^{-3})}{(55.606 \times 10^{-3})} = 14.03 \text{ kN} \cdot \text{m}$$

$$y_B = R - r_2 = 205.606 - 330 = -124.394 \text{ mm}$$

$$\sigma_B = -\frac{My_B}{Aer_2}$$

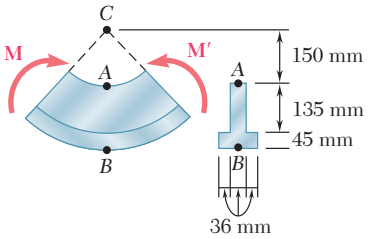
$$M = \frac{\sigma_B Aer_2}{y_B}$$

$$= \frac{(45 \times 10^6)(9720 \times 10^{-6})(11.894 \times 10^{-3})(330 \times 10^{-3})}{(124.394 \times 10^{-3})}$$

$$= 13.80 \text{ kN} \cdot \text{m}$$

$$M = 13.80 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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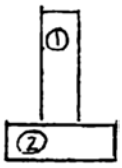
PROBLEM 4.174

Knowing that the maximum allowable stress is 45 MPa, determine the magnitude of the largest moment M that can be applied to the components shown.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$$



r, mm		b_i, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i^2}$, mm	$\bar{r}$, mm	$A\bar{r}$, mm^3
150	①	36	135	4860	23.1067	217.5	1.05705×10^6
285	②	108	45	4860	15.8332	307.5	1.49445×10^6
330	Σ			9720	38.9399		2.5515×10^6

$$R = \frac{9720}{38.9399} = 249.615 \text{ mm}, \qquad \bar{r} = \frac{2.5515 \times 10^6}{9720} = 262.5 \text{ mm}$$

$$e = \bar{r} - R = 12.885 \text{ mm}, \qquad M = 20 \times 10^3 \text{ N} \cdot \text{m}$$

$$y_A = R - r_1 = 249.615 - 150 = 99.615 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_1}$$

$$M = \frac{\sigma_A Aer_1}{y_A}$$

$$= \frac{(45 \times 10^6)(9720 \times 10^{-6})(12.885 \times 10^{-3})(150 \times 10^{-3})}{(99.615 \times 10^{-3})} = 8.49 \text{ kN} \cdot \text{m}$$

$$y_B = R - r_2 = 249.615 - 330 = -80.385 \text{ mm}$$

$$\sigma_B = -\frac{My_B}{Aer_2}$$

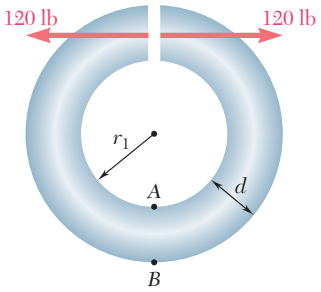
$$M = \frac{\sigma_B Aer_2}{y_B}$$

$$= \frac{(45 \times 10^6)(9720 \times 10^{-6})(12.885 \times 10^{-3})(330 \times 10^{-3})}{(80.385 \times 10^{-3})}$$

$$= 23.1 \text{ kN} \cdot \text{m}$$

$$M = 8.49 \text{ kN} \cdot \text{m} \blacktriangleleft$$

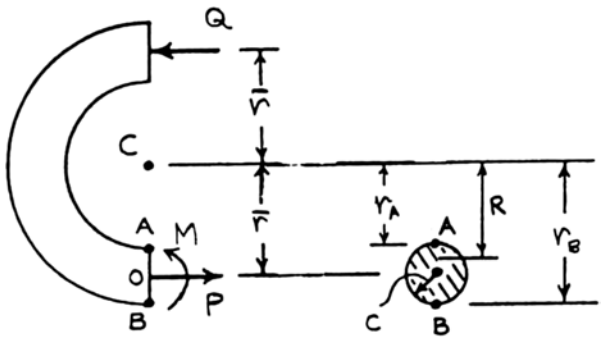
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PROBLEM 4.175

The split ring shown has an inner radius $r_1 = 0.8$ in. and a *circular* cross section of diameter $d = 0.6$ in. Knowing that each of the 120-lb forces is applied at the centroid of the cross section, determine the stress (*a*) at point *A*, (*b*) at point *B*.

SOLUTION



$r_A = r_1 = 0.8$ in.

$r_B = r_A + d = 0.8 + 0.6 = 1.4$ in.

$\bar{r} = \frac{1}{2}(r_A + r_B) = 1.1$ in. $\qquad c = \frac{1}{2}d = 0.3$ in.

$A = \pi c^2 = \pi(0.3)^2 = 0.28274 \text{ in}^2$ for solid circular section

$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[1.1 + \sqrt{(1.1)^2 - (0.3)^2}\right] = 1.079150$ in.

$e = \bar{r} - R = 1.1 - 1.0791503 = 0.020850$ in.

$Q = 120$ lb $\qquad \pm \sum F_x = 0: \quad P - Q = 0 \qquad P = 120$ lb

$+\circlearrowleft \sum M_0 = 0: \quad 2\bar{r}Q + M = 0 \qquad M = -2\bar{r}Q = -(2)(1.1)(120) = -264 \text{ lb} \cdot \text{in.}$

(*a*) $r = r_A = 0.8$ in.

$$\sigma_A = \frac{P}{A} + \frac{M(r_A - R)}{Aer_A} = \frac{120}{0.28274} + \frac{(-264)(0.8 - 1.079150)}{(0.28274)(0.020850)(0.8)} = 16.05 \times 10^3 \text{ psi}$$

$\sigma_A = 16.05$ ksi ◀

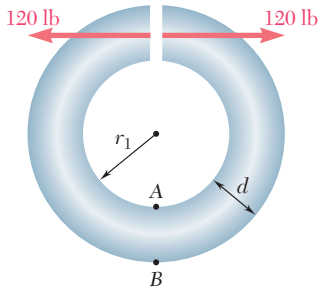
(*b*) $r = r_B = 1.4$ in.

$$\sigma_B = \frac{P}{A} + \frac{M(r_B - R)}{Aer_B} = \frac{120}{0.28274} + \frac{(-264)(1.4 - 1.079150)}{(0.28274)(0.020850)(1.4)}$$

$$= -9.84 \times 10^3 \text{ psi}$$

$\sigma_B = -9.84$ ksi ◀

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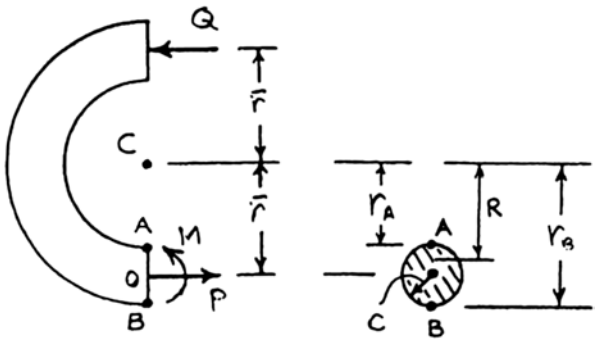


PROBLEM 4.176

Solve Prob. 4.175, assuming that the ring has an inner radius $r_1 = 0.6$ in. and a cross-sectional diameter $d = 0.8$ in.

PROBLEM 4.175 The split ring shown has an inner radius $r_1 = 0.8$ in. and a *circular* cross section of diameter $d = 0.6$ in. Knowing that each of the 120-lb forces is applied at the centroid of the cross section, determine the stress (*a*) at point *A*, (*b*) at point *B*.

SOLUTION



$$r_A = r_1 = 0.6 \text{ in.}$$

$$r_B = r_A + d = 0.6 + 0.8 = 1.4 \text{ in.}$$

$$\bar{r} = \frac{1}{2}(r_A + r_B) = 1.0 \text{ in.} \qquad c = \frac{1}{2}d = 0.4 \text{ in.}$$

$$A = \pi c^2 = \pi(0.4)^2 = 0.50265 \text{ in}^2 \qquad \text{for solid circular section.}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[1.0 + \sqrt{(1.0)^2 - (0.4)^2}\right] = 0.958258 \text{ in.}$$

$$e = \bar{r} - R = 1.0 - 0.958258 = 0.041742 \text{ in.}$$

$$Q = 120 \text{ lb} \qquad \pm \Sigma F_x = 0 \qquad P - Q = 0 \qquad P = 120 \text{ lb}$$

$$+\circlearrowleft \Sigma M_0 = 0: \quad 2\bar{r}Q + M = 0 \qquad M = -2\bar{r}Q = -(2)(1.0)(120) = -240 \text{ lb} \cdot \text{in.}$$

(*a*) $r = r_A = 0.6$ in.

$$\sigma_A = \frac{P}{A} + \frac{M(r_A - R)}{Aer_A} = \frac{120}{0.50265} + \frac{(-240)(0.6 - 0.958258)}{(0.50265)(0.041742)(0.6)}$$

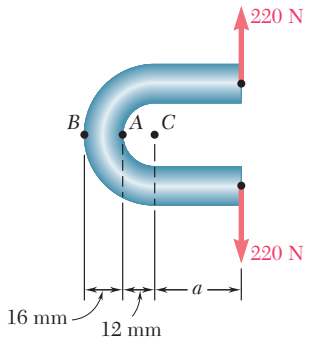
$$= 7.069 \times 10^3 \text{ psi} \qquad \sigma_A = 7.07 \text{ ksi} \blacktriangleleft$$

(*b*) $r = r_B = 1.4$ in.

$$\sigma_B = \frac{P}{A} + \frac{M(r_B - R)}{Aer_B} = \frac{120}{0.50265} + \frac{(-240)(1.4 - 0.958258)}{(0.50265)(0.041742)(1.4)}$$

$$= -3.37 \times 10^3 \text{ psi} \qquad \sigma_B = -3.37 \text{ ksi} \blacktriangleleft$$

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PROBLEM 4.177

The bar shown has a circular cross section of 14-mm diameter. Knowing that $a = 32$ mm, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$c = \frac{1}{2}d = 8 \text{ mm} \qquad \bar{r} = 12 + 8 = 20 \text{ mm}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[20 + \sqrt{20^2 - 8^2}\right] = 19.1652 \text{ mm}$$

$$e = \bar{r} - R = 20 - 19.1652 = 0.83485 \text{ mm}$$

$$A = \pi r^2 = \pi(8)^2 = 201.06 \text{ mm}^2$$

$$P = 220 \text{ N}$$

$$M = -P(a + \bar{r}) = 220(0.032 + 0.020) = 11.44 \text{ N} \cdot \text{m}$$

$$y_A = R - r_1 = 19.1652 - 12 = 7.1652 \text{ mm}$$

$$y_b = R - r_2 = 19.1652 - 28 = -8.8348 \text{ mm}$$

(*a*) $$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1}$$

$$= \frac{220}{201.06 \times 10^{-6}} - \frac{(-11.44)(7.1652 \times 10^{-3})}{(201.06 \times 10^{-6})(0.83485 \times 10^{-3})(12 \times 10^{-3})}$$

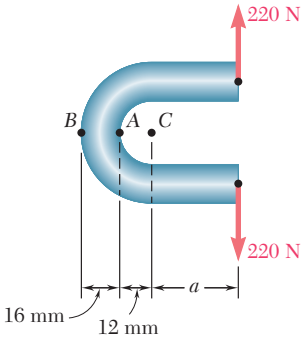
$$\sigma_A = 41.8 \text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma_B = \frac{P}{A} - \frac{My_B}{Aer_2}$$

$$= \frac{220}{201.06 \times 10^{-6}} - \frac{(-11.44)(8.8348 \times 10^{-3})}{(201.06 \times 10^{-6})(0.83485 \times 10^{-3})(28 \times 10^{-3})}$$

$$\sigma_B = -20.4 \text{ MPa} \blacktriangleleft$$

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PROBLEM 4.178

The bar shown has a circular cross section of 14-mm diameter. Knowing that the allowable stress is 38 MPa, determine the largest permissible distance a from the line of action of the 220-N forces to the plane containing the center of curvature of the bar.

SOLUTION

$$c = \frac{1}{2}d = 8 \text{ mm} \qquad \bar{r} = 12 + 8 = 20 \text{ mm}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[20 + \sqrt{20^2 - 8^2}\right] = 19.1652 \text{ mm}$$

$$e = \bar{r} - R = 20 - 19.1652 = 0.83485 \text{ mm}$$

$$A = \pi r^2 = \pi(8)^2 = 201.06 \text{ mm}^2$$

$$P = 220 \text{ N}$$

$$M = -P(a + \bar{r})$$

$$y_A = R - r_1 = 19.1652 - 12 = 7.1652 \text{ mm}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{P(a+\bar{r})y_A}{Aer_1} = \frac{P}{A}\left[1 + \frac{(a+\bar{r})y_A}{er_1}\right]$$

$$= \frac{KP}{A} \qquad \text{where} \qquad K = 1 + \frac{(a+\bar{r})y_A}{er_1}$$

$$K = \frac{\sigma_A A}{P} = \frac{(38 \times 10^6)(201.06 \times 10^{-6})}{220} = 34.729$$

$$a + \bar{r} = \frac{(K-1)er_1}{y_A}$$

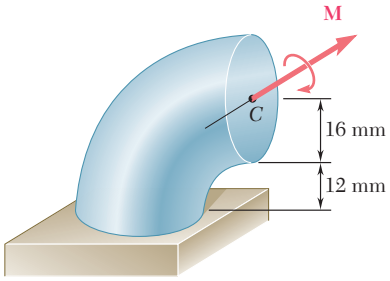
$$= \frac{(34.729 - 1)(0.83485 \times 10^{-3})(12 \times 10^{-3})}{(7.1652 \times 10^{-3})}$$

$$= 0.047158 \text{ m}$$

$$a = 0.047158 - 0.020 = 0.027158$$

$$a = 27.2 \text{ mm} \blacktriangleleft$$

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PROBLEM 4.179

The curved bar shown has a circular cross section of 32-mm diameter. Determine the largest couple **M** that can be applied to the bar about a horizontal axis if the maximum stress is not to exceed 60 MPa.

SOLUTION

$$c = 16\text{ mm} \qquad \bar{r} = 12 + 16 = 28\text{ mm}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right]$$

$$= \frac{1}{2}\left[28 + \sqrt{28^2 - 16^2}\right] = 25.4891\text{ mm}$$

$$e = \bar{r} - R = 28 - 25.4891 = 2.5109\text{ mm}$$

$\sigma_{\max}$ occurs at A, which lies at the inner radius.

It is given by $|\sigma_{\max}| = \left|\dfrac{My_A}{Aer_1}\right|$ from which $M = \dfrac{Aer_1|\sigma_{\max}|}{y_A}$.

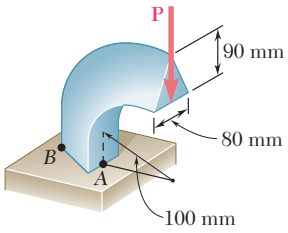
Also, $A = \pi c^2 = \pi(16)^2 = 804.25\text{ mm}^2$

Data: $y_A = R - r_1 = 25.4891 - 12 = 13.4891\text{ mm}$

$$M = \frac{(804.25 \times 10^{-6})(2.5109 \times 10^{-3})(12 \times 10^{-3})(60 \times 10^{6})}{13.4891 \times 10^{-3}}$$

$$M = 107.8\text{ N} \cdot \text{m} \blacktriangleleft$$

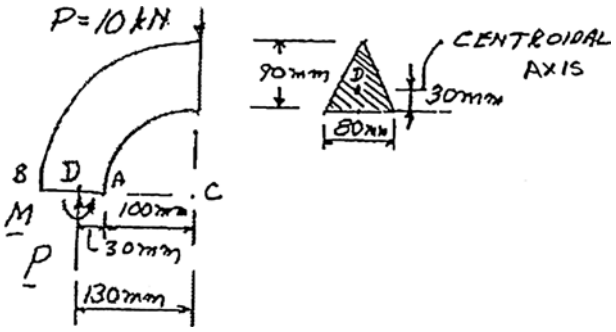
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PROBLEM 4.180

Knowing that $P = 10$ kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION



Locate the centroid *D* of the cross section.

$$\bar{r} = 100 \text{ mm} + \frac{90 \text{ mm}}{3} = 130 \text{ mm}$$

Force-couple system at *D*.

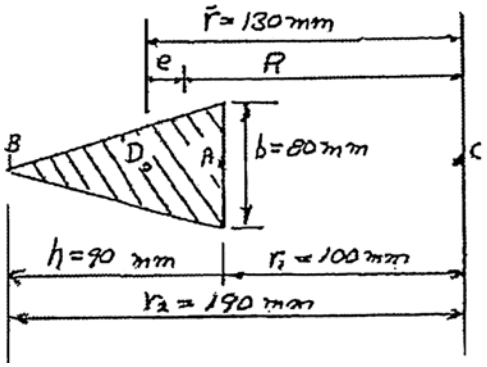
$$P = 10 \text{ kN}$$

$$M = P\bar{r} = (10 \text{ kN})(130 \text{ mm}) = 1300 \text{ N} \cdot \text{m}$$

Triangular cross section.

$$A = \frac{1}{2}bh = \frac{1}{2}(90 \text{ mm})(80 \text{ mm})$$

$$= 3600 \text{ mm}^2 = 3600 \times 10^{-6} \text{ m}^2$$



$$R = \frac{\frac{1}{2}h}{\frac{r_2}{h}\ln\frac{r_2}{r_1} - 1} = \frac{\frac{1}{2}(90)}{\frac{190}{90}\ln\frac{190}{100} - 1} = \frac{45 \text{ mm}}{0.355025}$$

$$R = 126.752 \text{ mm}$$

$$e = \bar{r} - R = 130 \text{ mm} - 126.752 \text{ mm} = 3.248 \text{ mm}$$

(*a*) Point *A*: $r_A = 100 \text{ mm} = 0.100 \text{ m}$

$$\sigma_A = -\frac{P}{A} + \frac{M(r_A - R)}{Aer_A} = -\frac{10 \text{ kN}}{3600 \times 10^{-6} \text{m}^2} + \frac{(1300 \text{ N} \cdot \text{m})(0.100 \text{ m} - 0.126752 \text{ m})}{(3600 \times 10^{-6} \text{ m}^2)(3248 \times 10^{-3} \text{ m})(0.100 \text{ m})}$$

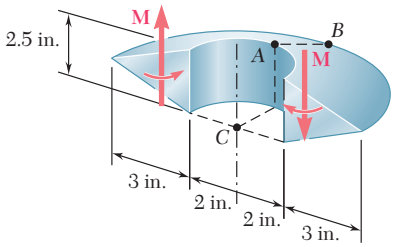
$$= -2.778 \text{ MPa} - 29.743 \text{ MPa} \qquad \sigma_A = -32.5 \text{ MPa} \blacktriangleleft$$

(*b*) Point *B*: $r_B = 190 \text{ mm} = 0.190 \text{ m}$

$$\sigma_B = -\frac{P}{A} + \frac{M(r_B - R)}{Aer_B} = -\frac{10 \text{ kN}}{3600 \times 10^{-6} \text{ m}^2} + \frac{(1300 \text{ N} \cdot \text{m})(0.190 \text{ m} - 0.126752 \text{ m})}{(3600 \times 10^{-6} \text{ m}^2)(3.248 \times 10^{-3} \text{ m})(0.190 \text{ m})}$$

$$= -2.778 \text{ MPa} + 37.01 \text{ MPa} \qquad \sigma_B = +34.2 \text{ MPa} \blacktriangleleft$$

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PROBLEM 4.181

Knowing that $M = 5$ kip · in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$A = \frac{1}{2}bh = \frac{1}{2}(2.5)(3) = 3.75 \text{ in}^2$$

$$\bar{r} = 2 + 1 = 3.00000 \text{ in.}$$

$$b_1 = 2.5 \text{ in.}, \quad r_1 = 2 \text{ in.}, \quad b_2 = 0, \quad r_2 = 5 \text{ in.}$$

Use formula for trapezoid with $b_2 = 0$.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(2.5 + 0)}{[(2.5)(5) - (0)(2)]\ln\frac{5}{2} - (3)(2.5 - 0)} = 2.84548 \text{ in.}$$

$$e = \bar{r} - R = 0.15452 \text{ in.} \qquad M = 5 \text{ kip} \cdot \text{in.}$$

(*a*) $y_A = R - r_1 = 0.84548$ in.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(0.84548)}{(3.75)(0.15452)(2)}$$

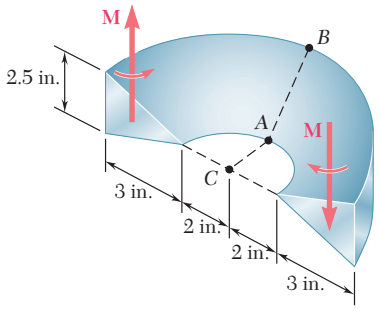
$\sigma_A = -3.65$ ksi ◀

(*b*) $y_B = R - r_2 = -2.15452$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-2.15452)}{(3.75)(0.15452)(5)}$$

$\sigma_B = 3.72$ ksi ◀

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PROBLEM 4.182

Knowing that $M = 5$ kip · in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$A = \frac{1}{2}(2.5)(3) = 3.75 \text{ in}^2$$

$$\bar{r} = 2 + 2 = 4.00000 \text{ in.}$$

$$b_1 = 0, \quad r_1 = 2 \text{ in.}, \quad b_2 = 2.5 \text{ in.}, \quad r_2 = 5 \text{ in.}$$

Use formula for trapezoid with $b_1 = 0$.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(0 + 2.5)}{[(0)(5) - (2.5)(2)]\ln\frac{5}{2} - (3)(0 - 2.5)} = 3.85466 \text{ in.}$$

$$e = \bar{r} - R = 0.14534 \text{ in.} \qquad M = 5 \text{ kip} \cdot \text{in.}$$

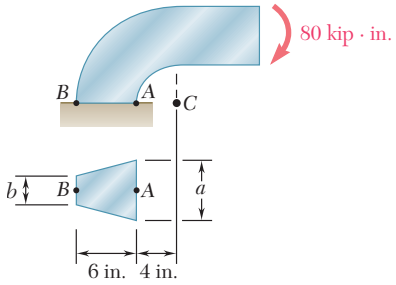
(*a*) $y_A = R - r_1 = 1.85466$ in.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(1.85466)}{(3.75)(0.14534)(2)} \qquad \sigma_A = -8.51 \text{ ksi} \blacktriangleleft$$

(*b*) $y_B = R - r_2 = -1.14534$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-1.14534)}{(3.75)(0.14534)(5)} \qquad \sigma_B = 2.10 \text{ ksi} \blacktriangleleft$$

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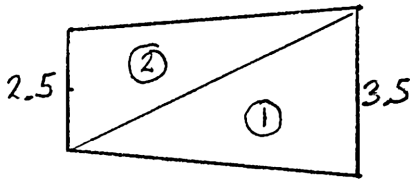


PROBLEM 4.183

Knowing that the machine component shown has a trapezoidal cross section with $a = 3.5$ in. and $b = 2.5$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Locate centroid.



	A, in^2	$\bar{r}$, in.	$A\bar{r}$, in^3
①	10.5	6	63
②	7.5	8	60
Σ	18		123

$$\bar{r} = \frac{123}{18} = 6.8333 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(3.5 + 2.5)}{[(3.5)(10) - (2.5)(4)]\ln\frac{10}{4} - (6)(3.5 - 2.5)} = 6.3878 \text{ in.}$$

$$e = \bar{r} - R = 0.4452 \text{ in.} \qquad M = 80 \text{ kip} \cdot \text{in.}$$

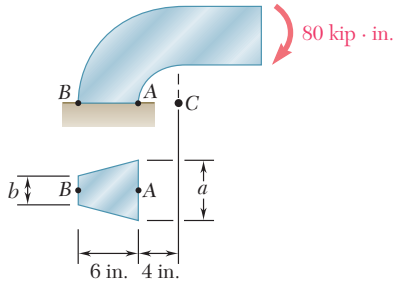
(*a*) $y_A = R - r_1 = 6.3878 - 4 = 2.3878$ in.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(80)(2.3878)}{(18)(0.4452)(4)} \qquad \sigma_A = -5.96 \text{ ksi} \blacktriangleleft$$

(*b*) $y_B = R - r_2 = 6.3878 - 10 = -3.6122$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(80)(-3.6122)}{(18)(0.4452)(10)} \qquad \sigma_B = 3.61 \text{ ksi} \blacktriangleleft$$

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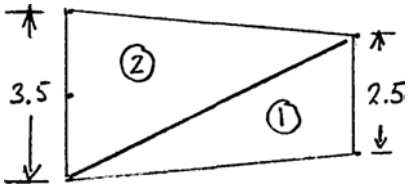


PROBLEM 4.184

Knowing that the machine component shown has a trapezoidal cross section with $a = 2.5$ in. and $b = 3.5$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Locate centroid.



	A, in^2	$\bar{r}$, in.	$A\bar{r}$, in^3
①	7.5	6	45
②	10.5	8	84
Σ	18		129

$$\bar{r} = \frac{129}{18} = 7.1667 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(2.5+3.5)}{[(2.5)(10) - (3.5)(4)]\ln\frac{10}{4} - (6)(2.5-3.5)} = 6.7168 \text{ in.}$$

$$e = \bar{r} - R = 0.4499 \text{ in.} \qquad M = 80 \text{ kip} \cdot \text{in.}$$

(*a*) $y_A = R - r_1 = 2.7168$ in.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(80)(2.7168)}{(18)(0.4499)(4)}$$

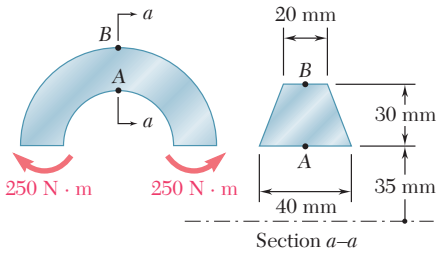
$\sigma_A = -6.71$ ksi ◀

(*b*) $y_B = R - r_2 = -3.2832$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(80)(-3.2832)}{(18)(0.4499)(10)}$$

$\sigma_B = 3.24$ ksi ◀

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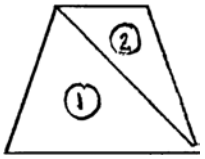


PROBLEM 4.185

For the curved beam and loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Locate centroid.



	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	600	45	27×10^3
②	300	55	16.5×10^3
Σ	900		43.5×10^3

$$\bar{r} = \frac{43.5 \times 10^3}{900} = 48.333 \text{ mm}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_2 - b_1)}$$

$$= \frac{(0.5)(30)^2(40 + 20)}{[(40)(65) - (20)(35)]\ln\frac{65}{35} - (30)(40 - 20)} = 46.8608 \text{ mm}$$

$$e = \bar{r} - R = 1.4725 \text{ mm} \qquad M = -250 \text{ N} \cdot \text{m}$$

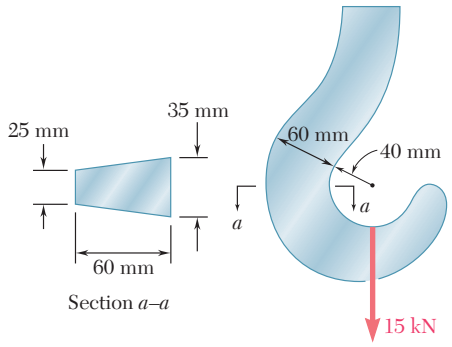
(*a*) $y_A = R - r_1 = 11.8608$ mm

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-250)(11.8608 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(35 \times 10^{-3})} = 63.9 \times 10^6 \text{ Pa} \qquad \sigma_A = 63.9 \text{ MPa} \blacktriangleleft$$

(*b*) $y_B = R - r_2 = -18.1392$ mm

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-250)(-18.1392 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(65 \times 10^{-3})} = -52.6 \times 10^6 \text{ Pa} \qquad \sigma_B = -52.6 \text{ MPa} \blacktriangleleft$$

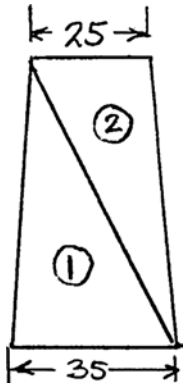
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PROBLEM 4.186

For the crane hook shown, determine the largest tensile stress in section *a-a*.

SOLUTION



Locate centroid.

	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	1050	60	63×10^3
②	750	80	60×10^3
Σ	1800		103×10^3

$$\bar{r}=\frac{103\times10^3}{1800}=63.333\text{ mm}$$

Force-couple system at centroid: $P=15\times10^3\text{ N}$

$$M=-P\bar{r}=-(15\times10^3)(68.333\times10^{-3})=-1.025\times10^3\text{ N}\cdot\text{m}$$

$$R=\frac{\frac{1}{2}h^2(b_1+b_2)}{(b_1r_2-b_2r_1)\ln\frac{r_2}{r_1}-h(b_1-b_2)}$$

$$=\frac{(0.5)(60)^2(35+25)}{[(35)(100)-(25)(40)]\ln\frac{100}{40}-(60)(35+25)}=63.878\text{ mm}$$

$$e=\bar{r}-R=4.452\text{ mm}$$

Maximum tensile stress occurs at point A.

$$y_A=R-r_1=23.878\text{ mm}$$

$$\sigma_A=\frac{P}{A}-\frac{My_A}{Aer_1}$$

$$=\frac{15\times10^3}{1800\times10^{-6}}-\frac{-(1.025\times10^3)(23.878\times10^{-3})}{(1800\times10^{-6})(4.452\times10^{-3})(40\times10^{-3})}$$

$$=84.7\times10^6\text{ Pa}$$

$$\sigma_A=84.7\text{ MPa}\ \blacktriangleleft$$

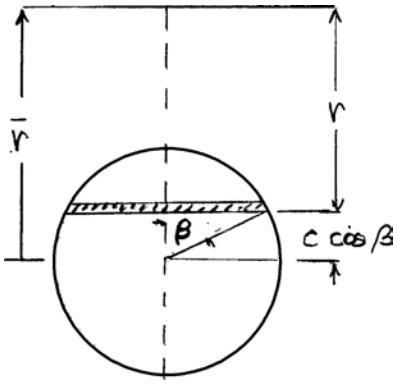
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PROBLEM 4.187

Using Eq. (4.66), derive the expression for R given in Fig. 4.61 for a circular cross section.

SOLUTION

Use polar coordinate β as shown. Let w be the width as a function of β



$$w = 2c\,\sin\beta$$

$$r = \bar{r} - c\cos\beta$$

$$dr = c\,\sin\beta\,d\beta$$

$$dA = w\,dr = 2c^2\sin^2\beta\,d\beta$$

$$\int\frac{dA}{r} = \int_0^{\pi}\frac{2c^2\sin\beta}{\bar{r}-c\cos\beta}\,d\beta$$

$$\int\frac{dA}{r} = \int_0^{\pi}\frac{c^2(1-\cos^2\beta)}{\bar{r}-c\cos\beta}d\beta$$

$$= 2\int_0^{\pi}\frac{\bar{r}^2 - c^2\cos^2\beta - (\bar{r}^2-c^2)}{\bar{r}-c\,\cos\beta}d\beta$$

$$= 2\int_0^{\pi}(\bar{r}+c\,\cos\beta)\,d\beta - 2(\bar{r}^2-c^2)\int_0^{\pi}\frac{dr}{\bar{r}-c\cos\beta}$$

$$= 2\bar{r}\,\beta\Big|_0^{\pi} + 2c\,\sin\beta\Big|_0^{\pi}$$

$$- 2(\bar{r}^2-c^2)\frac{2}{\sqrt{\bar{r}^2-c^2}}\tan^{-1}\frac{\sqrt{\bar{r}^2-c^2}\,\tan\left(\frac{1}{2}\beta\right)}{\bar{r}+c}\Big|_0^{\pi}$$

$$= 2\bar{r}(\pi-0) + 2c(0-0) - 4\sqrt{\bar{r}^2-c^2}\cdot\left(\frac{\pi}{2}-0\right)$$

$$= 2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2-c^2}$$

$$A = \pi c^2$$

$$R = \frac{A}{\int\frac{dA}{r}} = \frac{\pi c^2}{2\pi\bar{r}-2\pi\sqrt{\bar{r}^2-c^2}} = \frac{1}{2}\frac{c^2}{\bar{r}-\sqrt{r^2-c^2}}\times\frac{\bar{r}+\sqrt{\bar{r}^2-c^2}}{\bar{r}+\sqrt{\bar{r}^2-c^2}}$$

$$= \frac{1}{2}\frac{c^2\left(\bar{r}+\sqrt{\bar{r}^2-c^2}\right)}{\bar{r}^2-(\bar{r}^2-c^2)} = \frac{1}{2}\frac{c^2\left(\bar{r}+\sqrt{\bar{r}^2-c^2}\right)}{c^2} = \frac{1}{2}\left(\bar{r}+\sqrt{\bar{r}^2-c^2}\right)$$ ◀

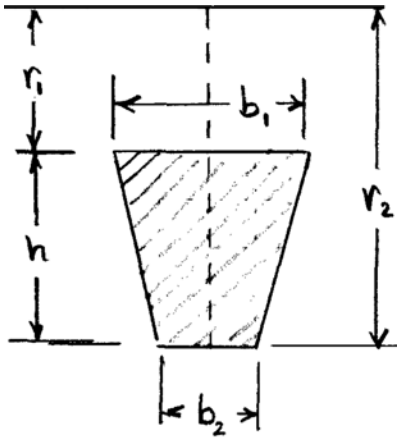
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PROBLEM 4.188

Using Eq. (4.66), derive the expression for R given in Fig. 4.73 for a trapezoidal cross section.

SOLUTION

The section width w varies linearly with r.



$$w = c_0 + c_1 r$$

$$w = b_1 \text{ at } r = r_1 \text{ and } w = b_2 \quad \text{at} \quad r = r_2$$

$$b_1 = c_0 + c_1 r_1$$

$$b_2 = c_0 + c_1 r_2$$

$$b_1 - b_2 = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b_1 - b_2}{h}$$

$$r_2 b_1 - r_1 b_2 = (r_2 - r_1)c_0 = h c_0$$

$$c_0 = \frac{r_2 b_1 - r_1 b_2}{h}$$

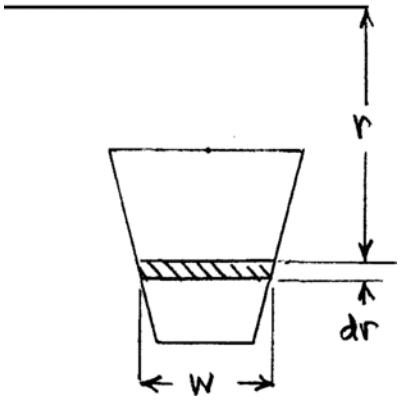
$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r} dr$$

$$= c_0 \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_0 \ln \frac{r_2}{r_1} + c_1(r_2 - r_1)$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - \frac{b_1 - b_2}{h} h$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - (b_1 - b_2)$$



$$A = \frac{1}{2}(b_1 + b_2)h$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(r_2 b_1 - r_1 b_2)\ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

◀

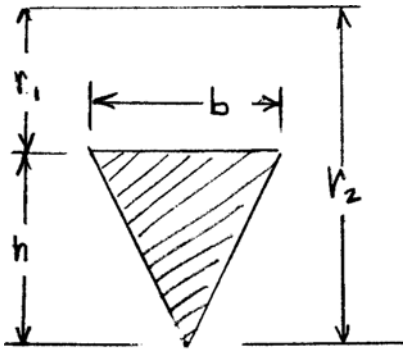
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PROBLEM 4.189

Using Equation (4.66), derive the expression for R given in Fig. 4.73 for a triangular cross section.

SOLUTION

The section width w varies linearly with r.



$$w = c_0 + c_1 r$$

$$w = b \text{ at } r = r_1 \quad \text{and} \quad w = 0 \text{ at } r = r_2$$

$$b = c_0 + c_1 r_1$$

$$0 = c_0 + c_1 r_2$$

$$b = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b}{h} \quad \text{and} \quad c_0 = -c_1 r_2 = \frac{b r_2}{h}$$

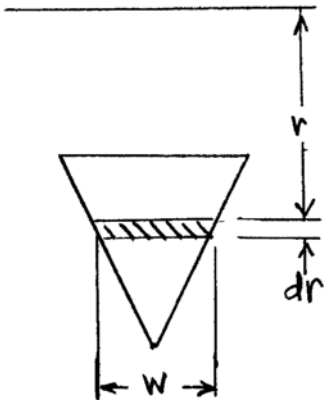
$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r} dr$$

$$= c_0 \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_0 \ln \frac{r_2}{r_1} + c_1 (r_2 - r_1)$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - \frac{b}{h} h$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - b = b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1 \right)$$

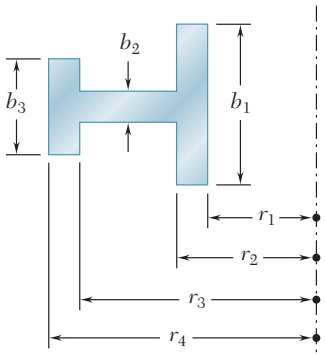


$$A = \frac{1}{2} bh$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} bh}{b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1 \right)} = \frac{\frac{1}{2} h}{\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1}$$

◀

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PROBLEM 4.190

Show that if the cross section of a curved beam consists of two or more rectangles, the radius R of the neutral surface can be expressed as

$$R = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$

where A is the total area of the cross section.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r}\,dA} = \frac{A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

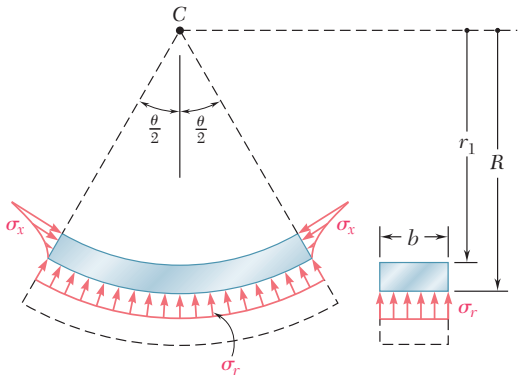
$$= \frac{A}{\Sigma \ln\left(\frac{r_{i+1}}{r_i}\right)^{b_i}} = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$ ◀

Note that for each rectangle,

$$\int \frac{1}{r}\,dA = \int_{r_i}^{r_{i+1}} b_i \frac{dr}{r}$$

$$= b_i \int_{r_2}^{r_{i+1}} \frac{dr}{r} = b_i \ln \frac{r_{i+1}}{r_i}$$

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PROBLEM 4.191

For a curved bar of rectangular cross section subjected to a bending couple **M**, show that the radial stress at the neutral surface is

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

and compute the value of σ_r for the curved bar of Concept Applications 4.10 and 4.11. (*Hint:* consider the free-body diagram of the portion of the beam located above the neutral surface.)

SOLUTION

At radial distance r, $\sigma_r = \dfrac{M(r-R)}{Aer} = \dfrac{M}{Ae} - \dfrac{MR}{Aer}$

For portion above the neutral axis, the resultant force is

$$H = \int \sigma_r dA = \int_{r_1}^{R} \sigma_r b\, dr$$

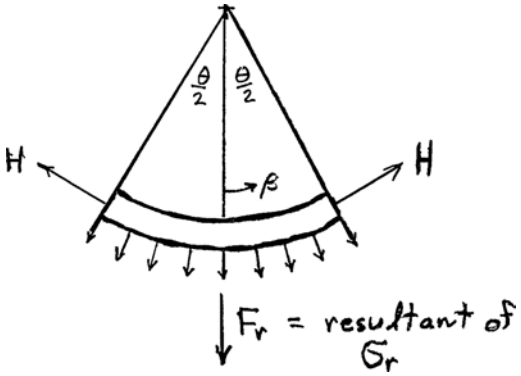
$$= \frac{Mb}{Ae}\int_{r_1}^{R} dr - \frac{MRb}{Ae}\int_{r_1}^{R}\frac{dr}{r}$$

$$= \frac{Mb}{Ae}(R - r_1) - \frac{MRb}{Ae}\ln\frac{R}{r_1} = \frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

Resultant of σ_n: $F_r = \int \sigma_r \cos\beta\, dA$

$$= \int_{-\theta/2}^{\theta/2} \sigma_r \cos\beta (bR\, d\beta) = \sigma_r bR\int_{-\theta/2}^{\theta/2}\cos\beta\, d\beta$$

$$= \sigma_r\, bR\ \sin\beta\Big|_{-\theta/2}^{\theta/2} = 2\sigma_r\, bR\sin\frac{\theta}{2}$$



For equilibrium: $F_r - 2H\sin\dfrac{\theta}{2} = 0$

$$2\sigma_r bR\sin\frac{\theta}{2} - 2\frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)\sin\frac{\theta}{2} = 0$$

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right) \quad \blacktriangleleft$$

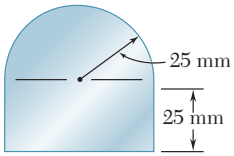
Using results of Examples 4.10 and 4.11 as data,

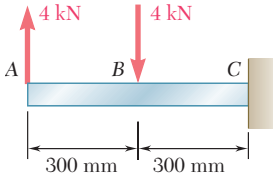
$M = 8$ kip · in., $A = 3.75$ in^2, $R = 5.9686$ in., $e = 0.0314$ in., $r_1 = 5.25$ in.

$$\sigma_r = \frac{8}{(3.75)(0.0314)}\left[1 - \frac{5.25}{5.9686} - \ln\frac{5.9686}{5.25}\right]$$

$\sigma_r = -0.536$ ksi ◀

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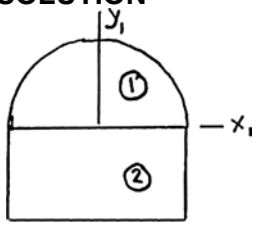




PROBLEM 4.192

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

SOLUTION



$$A_1 = \frac{\pi}{2}r^2 = \frac{\pi}{2}(25)^2 = 981.7 \text{ mm}^2 \qquad \bar{y}_1 = \frac{4r}{3\pi} = \frac{(4)(25)}{3\pi} = 10.610 \text{ mm}$$

$$A_2 = bh = (50)(25) = 1250 \text{ mm}^2 \qquad \bar{y}_2 = -\frac{h}{2} = -\frac{25}{2} = -12.5 \text{ mm}$$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2} = \frac{(981.7)(10.610) + (1250)(-12.5)}{981.7 + 1250} = -2.334 \text{ mm}$$

$$\bar{I}_1 = I_{x_1} - A_1\bar{y}_1^2 = \frac{\pi}{8}r^4 - A_1\bar{y}_1^2 = \frac{\pi}{8}(25)^4 - (981.7)(10.610)^2 = 42.886\times10^6 \text{ mm}^4$$

$$d_1 = \bar{y}_1 - \bar{y} = 10.610 - (-2.334) = 12.944 \text{ mm}$$

$$I_1 = \bar{I}_1 + A_1d_1^2 = 42.866\times10^3 + (981.7)(12.944)^2 = 207.35\times10^3 \text{ mm}^4$$

$$\bar{I}_2 = \frac{1}{12}bh^3 = \frac{1}{12}(50)(25)^3 = 65.104\times10^3 \text{ mm}^4$$

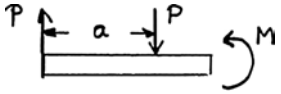
$$d_2 = |\bar{y}_2 - \bar{y}| = |-12.5 - (-2.334)| = 10.166 \text{ mm}$$

$$I_2 = \bar{I}_2 + A_2d_2^2 = 65.104\times10^3 + (1250)(10.166)^2 = 194.288\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 401.16\times10^3 \text{ mm}^4 = 401.16\times10^{-9} \text{ m}^4$$

$$y_{top} = 25 + 2.334 = 27.334 \text{ mm} = 0.027334 \text{ m}$$

$$y_{bot} = -25 + 2.334 = -22.666 \text{ mm} = -0.022666 \text{ m}$$

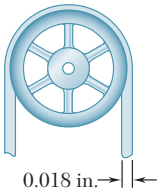


$$M - Pa = 0: \qquad M = Pa = (4\times10^3)(300\times10^{-3}) = 1200 \text{ N}\cdot\text{m}$$

$$\sigma_{top} = \frac{-My_{top}}{I} = -\frac{(1200)(0.027334)}{401.16\times10^{-9}} = -81.76\times10^6 \text{ Pa} \qquad \sigma_{top} = -81.8 \text{ MPa} \blacktriangleleft$$

$$\sigma_{bot} = \frac{-My_{bot}}{I} = -\frac{(1200)(-0.022666)}{401.16\times10^{-9}} = 67.80\times10^6 \text{ Pa} \qquad \sigma_{bot} = 67.8 \text{ MPa} \blacktriangleleft$$

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PROBLEM 4.193

A steel band saw blade that was originally straight passes over 8-in.-diameter pulleys when mounted on a band saw. Determine the maximum stress in the blade, knowing that it is 0.018 in. thick and 0.625 in. wide. Use $E = 29 \times 10^6$ psi.

SOLUTION

Band blade thickness: $t = 0.018$ in.

Radius of pulley: $r = \frac{1}{2} d = 4.000$ in.

Radius of curvature of centerline of blade:

$$\rho = r + \frac{1}{2} t = 4.009 \text{ in.}$$

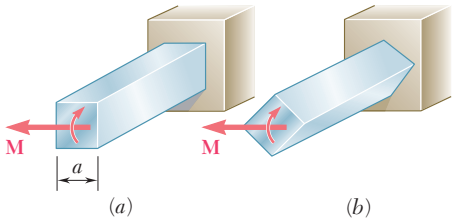
$$c = \frac{1}{2} t = 0.009 \text{ in.}$$

Maximum strain: $\varepsilon_m = \frac{c}{\rho} = \frac{0.009}{4.009} = 0.002245$

Maximum stress: $\sigma_m = E\varepsilon_m = (29 \times 10^6)(0.002245)$

$$\sigma_m = 65.1 \times 10^3 \text{ psi} \qquad \sigma_m = 65.1 \text{ ksi} \blacktriangleleft$$

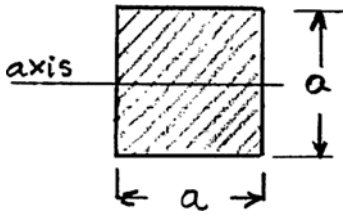
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PROBLEM 4.194

A couple of magnitude M is applied to a square bar of side a. For each of the orientations shown, determine the maximum stress and the curvature of the bar.

SOLUTION



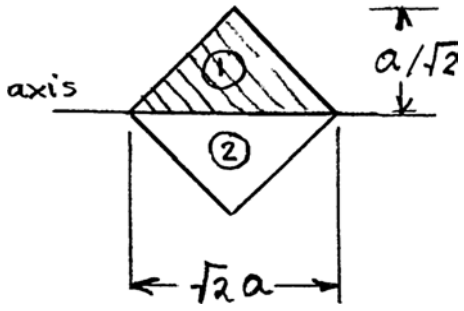
$$I = \frac{1}{12}bh^3 = \frac{1}{12}aa^3 = \frac{a^4}{12}$$

$$c = \frac{a}{2}$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M\frac{a}{2}}{\frac{a^4}{12}} \qquad \sigma_{\max} = \frac{6M}{a^3} \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} \qquad \frac{1}{\rho} = \frac{12M}{Ea^4} \blacktriangleleft$$

For one triangle, the moment of inertia about its base is



$$I_1 = \frac{1}{12}bh^3 = \frac{1}{12}\left(\sqrt{2}a\right)\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{a^4}{24}$$

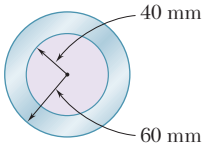
$$I_2 = I_1 = \frac{a^4}{24}$$

$$I = I_1 + I_2 = \frac{a^4}{12}$$

$$c = \frac{a}{\sqrt{2}} \qquad \sigma_{\max} = \frac{Mc}{I} = \frac{Ma/\sqrt{2}}{a^4/12} = \frac{6\sqrt{2}M}{a^3} \qquad \sigma_{\max} = \frac{8.49M}{a^3} \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} \qquad \frac{1}{\rho} = \frac{12M}{Ea^4} \blacktriangleleft$$

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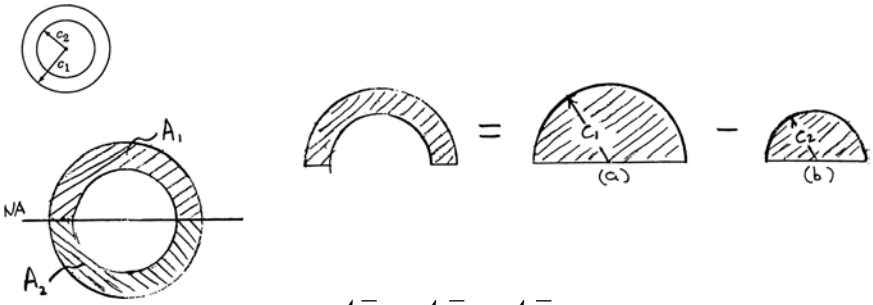


PROBLEM 4.195

Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Let c_1 be the outer radius and c_2 the inner radius.



$$A_1\bar{y}_1 = A_a\bar{y}_a - A_b\bar{y}_b$$

$$= \left(\frac{\pi}{2}c_1^2\right)\left(\frac{4c_1}{3\pi}\right) - \left(\frac{\pi}{2}c_2^2\right)\left(\frac{4c_2}{3\pi}\right)$$

$$= \frac{2}{3}\left(c_1^3 - c_2\right)^3$$

$$A_2\bar{y}_2 = A_1\bar{y}_1 = \frac{2}{3}\left(c_1^3 - c_2^2\right)$$

$$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2) = \frac{4}{3}\sigma_Y\left(c_1^3 - c_2^3\right)$$

Data:

$$\sigma_Y = 240 \text{ MPa} = 240\times10^6 \text{ Pa}$$

$$c_1 = 60 \text{ mm} = 0.060 \text{ m}$$

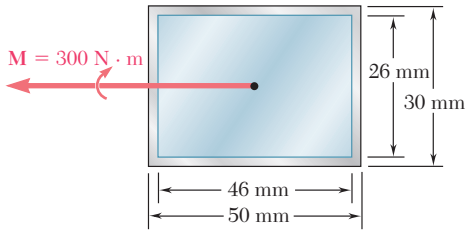
$$c_2 = 40 \text{ mm} = 0.040 \text{ m}$$

$$M_p = \frac{4}{3}(240\times10^6)(0.060^3 - 0.040^3)$$

$$= 48.64\times10^3 \text{ N}\cdot\text{m}$$

$$M_p = 48.6 \text{ kN}\cdot\text{m} \blacktriangleleft$$

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PROBLEM 4.196

In order to increase corrosion resistance, a 2-mm-thick cladding of aluminum has been added to a steel bar as shown. The modulus of elasticity is 200 GPa for steel and 70 GPa for aluminum. For a bending moment of 300 N · m, determine (*a*) the maximum stress in the steel, (*b*) the maximum stress in the aluminum, (*c*) the radius of curvature of the bar.

SOLUTION

Use aluminum as the reference material.

$n = 1$ in aluminum $\qquad n = \dfrac{E_s}{E_a} = \dfrac{200}{70} = 2.857$ in steel

Cross section geometry:

Steel: $A_s = (46\,\text{mm})(26\,\text{mm}) = 1196\ \text{mm}^2 \qquad I_s = \dfrac{1}{12}(46\ \text{mm})(26\ \text{mm})^3 = 67{,}375\ \text{mm}^4$

Aluminum: $A_a = (50\ \text{mm})(30\ \text{mm}) - 1196\ \text{mm}^2 = 304\ \text{mm}^2$

$$I_a = \frac{1}{12}(50\ \text{mm})(30\ \text{mm}^3) - 67{,}375\ \text{mm}^4 = 45{,}125\ \text{mm}^4$$

Transformed section.

$$I = n_a I_a + n_s I_s = (1)(45{,}125) + (2.857)(67{,}375) = 237{,}615\ \text{mm}^4 = 237.615 \times 10^{-9}\ \text{m}^4$$

Bending moment. $M = 300\ \text{N} \cdot \text{m}$

(*a*) Maximum stress in steel: $n_s = 2.857 \qquad y_s = 13\ \text{mm} = 0.013\ \text{m}$

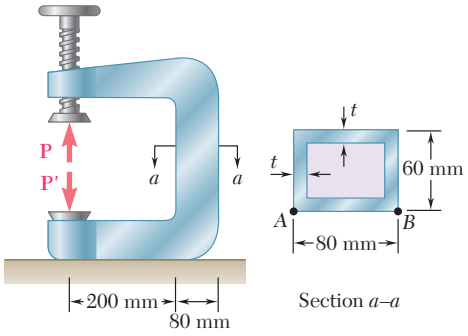
$$\sigma_s = \frac{n_s M y_s}{I} = \frac{(2.857)(300)(0.013)}{237.615 \times 10^{-9}} = 46.9 \times 10^6\ \text{Pa} \qquad \sigma_s = 46.9\ \text{MPa} \blacktriangleleft$$

(*b*) Maximum stress in aluminum: $n_a = 1, \qquad y_a = 15\ \text{mm} = 0.015\ \text{m}$

$$\sigma_a = \frac{n_a M y_a}{I} = \frac{(1)(300)(0.015)}{237.615 \times 10^{-9}} = 18.94 \times 10^6\ \text{Pa} \qquad \sigma_a = 18.94\ \text{MPa} \blacktriangleleft$$

(*c*) Radius of curvative: $\rho = \dfrac{EI}{M} \qquad \rho = \dfrac{(70 \times 10^9)(237.615 \times 10^{-9})}{300} \qquad \rho = 55.4\ \text{m} \blacktriangleleft$

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PROBLEM 4.197

The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Rectangular cutout is 60 mm × 40 mm.

$$A = (80)(60) - (60)(40) = 2.4 \times 10^{3}\ \text{mm}^2 = 2.4 \times 10^{-3}\ \text{m}^2$$

$$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(40)(60)^3 = 1.84 \times 10^{6}\ \text{mm}^4$$

$$= 1.84 \times 10^{-6}\ \text{m}^4$$

$$c = 40\ \text{mm} = 0.040\ \text{m} \quad e = 200 + 40 = 240\ \text{mm} = 0.240\ \text{m}$$

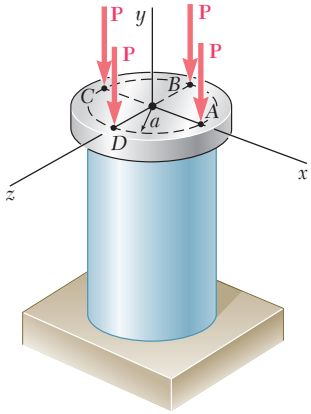
$$P = 20 \times 10^{3}\ \text{N}$$

$$M = Pe = (20 \times 10^{3})(0.240) = 4.8 \times 10^{3}\ \text{N} \cdot \text{m}$$

(*a*) $$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20 \times 10^{3}}{2.4 \times 10^{-3}} + \frac{(4.8 \times 10^{3})(0.040)}{1.84 \times 10^{-6}} = 112.7 \times 10^{6}\ \text{Pa}$$ $\sigma_A = 112.7$ MPa ◀

(*b*) $$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20 \times 10^{3}}{2.4 \times 10^{-3}} - \frac{(4.8 \times 10^{3})(0.040)}{1.84 \times 10^{-6}} = -96.0 \times 10^{6}\ \text{Pa}$$ $\sigma_B = -96.0$ MPa ◀

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PROBLEM 4.198

The four forces shown are applied to a rigid plate supported by a solid steel post of radius a. Knowing that $P = 24$ kips and $a = 1.6$ in., determine the maximum stress in the post when (a) the force at D is removed, (b) the forces at C and D are removed.

SOLUTION

For a solid circular section of radius a,

$$A = \pi a^2 \qquad I = \frac{\pi}{4}a^4$$

Centric force: $F = 4P, \quad M_x = M_z = 0 \qquad \sigma = -\frac{F}{A} = -\frac{4P}{\pi a^2}$

(a) Force at D is removed.

$$F = 3P, \qquad M_x = -Pa, \qquad M_z = 0$$

$$\sigma = -\frac{F}{A} - \frac{M_x z}{I} = -\frac{3P}{\pi a^2} - \frac{(-Pa)(-a)}{\frac{\pi}{4}a^2} = -\frac{7P}{\pi a^2}$$

(b) Forces at C and D are removed.

$$F = 2P, \qquad M_x = -Pa, \qquad M_z = -Pa$$

Resultant bending couple: $M = \sqrt{M_x^2 + M_z^2} = \sqrt{2}Pa$

$$\sigma = -\frac{F}{A} - \frac{Mc}{I} = -\frac{2P}{\pi a^2} - \frac{\sqrt{2}Paa}{\frac{\pi}{4}a^2} = -\frac{2 + 4\sqrt{2}}{\pi}\frac{P}{a^2} = -2.437P/a^2$$

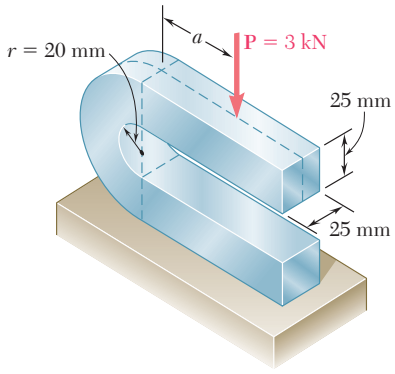
Numerical data: $P = 24.0$ kips, a $= 1.6$ in.

Answers:

(a) $\sigma = -\frac{(7)(24.0)}{\pi(1.6)^2}$ $\sigma = -20.9$ ksi ◀

(b) $\sigma = -\frac{(2.437)(24.0)}{(1.6)^2}$ $\sigma = -22.8$ ksi ◀

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PROBLEM 4.199

The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the allowable stress in the bar is 150 MPa, determine the largest permissible distance a from the line of action of the 3-kN force to the vertical plane containing the center of curvature of the bar.

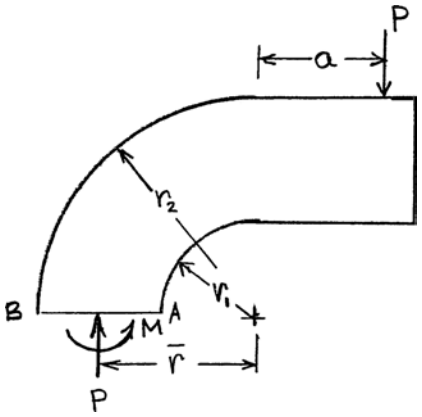
SOLUTION

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}$$



Also, $e = \bar{r} - R$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a + \bar{r})y_A}{Aer_1} = -K\frac{P}{A}$$

with $y_A = R - r_1$

Thus, $K = 1 + \dfrac{(a + \bar{r})(R - r_1)}{er_1}$

Data: $h = 25$ mm, $r_1 = 20$ mm, $r_2 = 45$ mm, $\bar{r} = 32.5$ mm

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$b = 25$ mm, $A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2$ $R - r_1 = 10.8288$ mm

$P = 3 \times 10^3 \text{ N} \cdot \text{m}$, $\sigma_A = -150 \times 10^6$ Pa

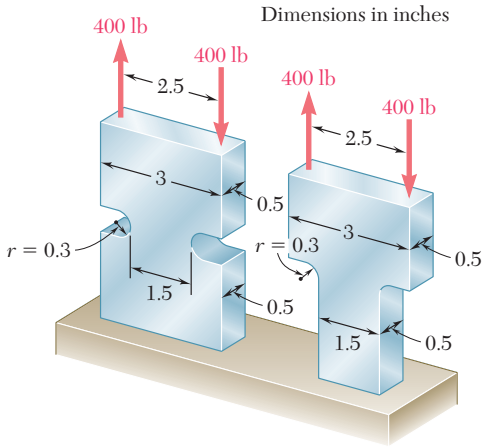
$$K = -\frac{\sigma_A A}{P} = -\frac{(-150 \times 10^6)(625 \times 10^{-6})}{3 \times 10^3} = 31.25$$

$$a + \bar{r} = \frac{(K - 1)er_1}{R - r_1} = \frac{(30.25)(1.6712)(20)}{10.8288} = 93.37 \text{ mm}$$

$a = 93.37 - 32.5$

$a = 60.9$ mm ◀

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PROBLEM 4.200

Determine the maximum stress in each of the two machine elements shown.

SOLUTION

For each case, $M = (400)(2.5) = 1000 \text{ lb} \cdot \text{in.}$

At the minimum section,

$$I = \frac{1}{12}(0.5)(1.5)^3 = 0.140625 \text{ in}^4$$

$$c = 0.75 \text{ in.}$$

(*a*) $D/d = 3/1.5 = 2$

$r/d = 0.3/1.5 = 0.2$

From Fig 4.32, $K = 1.75$

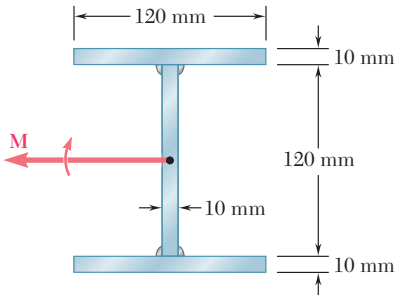
$$\sigma_{\max} = \frac{KMc}{I} = \frac{(1.75)(1000)(0.75)}{0.140625} = 9.33 \times 10^3 \text{ psi} \qquad \sigma_{\max} = 9.33 \text{ ksi} \blacktriangleleft$$

(*b*) $D/d = 3/1.5 = 2$ $r/d = 0.3/1.5 = 0.2$

From Fig. 4.31, $K = 1.50$

$$\sigma_{\max} = \frac{KMc}{I} = \frac{(1.50)(1000)(0.75)}{0.140625} = 8.00 \times 10^3 \text{ psi} \qquad \sigma_{\max} = 8.00 \text{ ksi} \blacktriangleleft$$

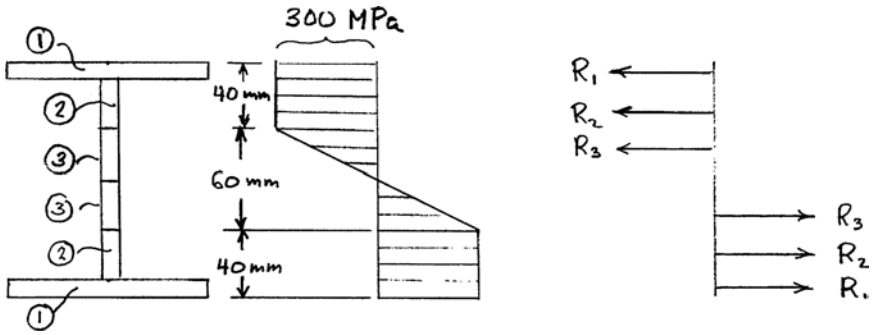
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PROBLEM 4.201

Three 120 × 10-mm steel plates have been welded together to form the beam shown. Assuming that the steel is elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa, determine (*a*) the bending moment for which the plastic zones at the top and bottom of the beam are 40 mm thick, (*b*) the corresponding radius of curvature of the beam.

SOLUTION

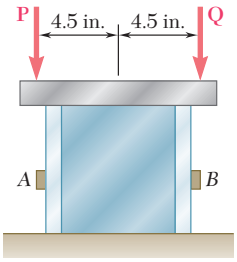


$$A_1 = (120)(10) = 1200 \text{ mm}^2$$
$$R_1 = \sigma_Y A_1 = (300\times10^6)(1200\times10^{-6}) = 360\times10^3 \text{ N}$$
$$A_2 = (30)(10) = 300 \text{ mm}^2$$
$$R_2 = \sigma_Y A_2 = (300\times10^6)(300\times10^{-6}) = 90\times10^3 \text{ N}$$
$$A_3 = (30)(10) = 300 \text{ mm}^2$$
$$R_3 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(300\times10^6)(300\times10^{-6}) = 45\times10^3 \text{ N}$$
$$y_1 = 65 \text{ mm} = 65\times10^{-3} \text{ m} \qquad y_2 = 45 \text{ mm} = 45\times10^{-3} \text{ m} \qquad y_3 = 20 \text{ mm} = 20\times10^{-3} \text{ m}$$

(*a*) $$M = 2(R_1y_1 + R_2y_2 + R_3y_3) = 2\{(360)(65) + (90)(45) + (45)(20)\}$$
$$= 56.7\times10^3 \text{ N}\cdot\text{m}$$
$$M = 56.7 \text{ kN}\cdot\text{m} \blacktriangleleft$$

(*b*) $$\frac{y_Y}{\rho} = \frac{\sigma_Y}{E} \qquad \rho = \frac{Ey_Y}{\sigma_Y} = \frac{(200\times10^9)(30\times10^{-3})}{300\times10^6}$$
$$\rho = 20.0 \text{ mm} \blacktriangleleft$$

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PROBLEM 4.202

A short length of a W8 × 31 rolled-steel shape supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points *A* and *B* on the centerline of the outer faces of the flanges have been measured and found to be

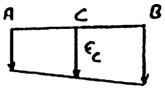
$$\varepsilon_A = -550 \times 10^{-6} \text{ in./in.} \qquad \varepsilon_B = -680 \times 10^{-6} \text{ in./in.}$$

Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.

SOLUTION

Strains:

$$\varepsilon_A = -550 \times 10^{-6} \text{ in./in.}$$

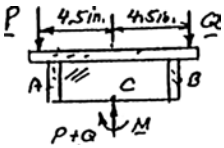


$$\varepsilon_B = -680 \times 10^{-6} \text{ in./in.}$$

$$\varepsilon_C = \frac{1}{2}(\varepsilon_A + \varepsilon_B) = \frac{1}{2}(-550 - 680)10^{-6} = -615 \times 10^{-6} \text{ in./in.}$$

Stresses: $\sigma_A = E\varepsilon_A = (29 \times 10^6 \text{ psi})(-550 \times 10^{-6} \text{ in./in.}) = -15.95 \text{ ksi}$

$$\sigma_C = E\varepsilon_C = (29 \times 10^6 \text{ psi})(-615 \times 10^{-6} \text{ in./in.}) = -17.835 \text{ ksi}$$



W8 × 31: $A = 9.12 \text{ in}^2$

$S = 27.5 \text{ in}^3$

$M = (4.5 \text{ in.})(P - Q)$

At point *C*: $\sigma_C = -\dfrac{P+Q}{A}; \quad -17.835 \text{ ksi} = -\dfrac{P+Q}{9.12 \text{ in}^2}$

$$P + Q = 162.655 \text{ kips} \quad (1)$$

At point *A*: $\sigma_A = -\dfrac{P+Q}{A} - \dfrac{M}{S}$

$$-15.95 \text{ ksi} = -17.835 \text{ ksi} - \frac{(4.5 \text{ in.})(P-Q)}{27.5 \text{ in}^3}; \qquad P - Q = -11.5194 \text{ kips} \quad (2)$$

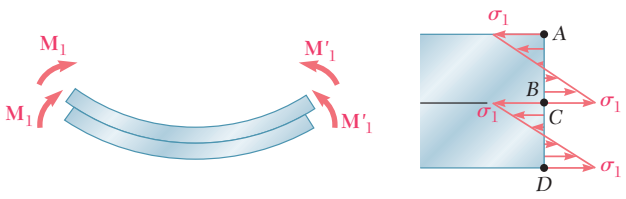
Solve simultaneously. $P = 75.6$ kips $Q = 87.1$ kips

P = 75.6 kips ↓ ◀

Q = 87.1 kips ↓ ◀

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PROBLEM 4.203



Two thin strips of the same material and same cross section are bent by couples of the same magnitude and glued together. After the two surfaces of contact have been securely bonded, the couples are removed. Denoting by σ_1 the maximum stress and by ρ_1 the radius of curvature of each strip while the couples were applied, determine (*a*) the final stresses at points *A, B, C,* and *D,* (*b*) the final radius of curvature.

SOLUTION

Let b = width and t = thickness of one strip.

Loading one strip, $M = M_1$

$$I_1 = \frac{1}{12}bt^3, \quad c = \frac{1}{2}t$$

$$\sigma_1 = \frac{M_1 c}{I} = \frac{\sigma M_1}{bt^2}$$

$$\frac{1}{\rho_1} = \frac{M_1}{EI_1} = \frac{12M_1}{Et^3}$$

After M_1 is applied to each of the strips, the stresses are those given in the sketch above. They are

$$\sigma_A = -\sigma_1, \quad \sigma_B = \sigma_1, \quad \sigma_C = -\sigma_1, \quad \sigma_D = \sigma_1$$

The total bending couple is $2M_1$.

After the strips are glued together, this couple is removed.

$$M' = 2M_1, \quad I' = \frac{1}{12}b(2t)^3 = \frac{2}{3}bt^3 \quad c = t$$

The stresses removed are

$$\sigma' = -\frac{M'y}{I} = -\frac{2M_1 y}{\frac{2}{3}bt^3} = -\frac{3M_1 y}{bt^2}$$

$$\sigma'_A = -\frac{3M_1}{bt^2} = -\frac{1}{2}\sigma_1, \quad \sigma'_B = \sigma'_C = 0, \quad \sigma'_D = \frac{3M_1}{bt^2} = \frac{1}{2}\sigma_1$$

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PROBLEM 4.203 ***(Continued)***

(*a*) Final stresses: $\sigma_A = -\sigma_1 - (-\frac{1}{2}\sigma_1)$ $\sigma_A = -\frac{1}{2}\sigma_1$ ◀

$\sigma_B = \sigma_1$ ◀

$\sigma_C = -\sigma_1$ ◀

$\sigma_D = \sigma_1 - \frac{1}{2}\sigma_1$ $\sigma_D = \frac{1}{2}\sigma_1$ ◀

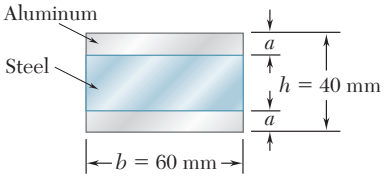
$$\frac{1}{\rho'} = \frac{M'}{EI'} = \frac{2M_1}{E\frac{2}{3}bt^3} = \frac{3M_1}{Et^3} = \frac{1}{4}\frac{1}{\rho'}$$

(*b*) Final radius: $\frac{1}{\rho} = \frac{1}{\rho_1} - \frac{1}{\rho'} = \frac{1}{\rho_1} - \frac{1}{4}\frac{1}{\rho_1} = \frac{3}{4}\frac{1}{\rho_1}$

$\rho = \frac{4}{3}\rho_1$ ◀

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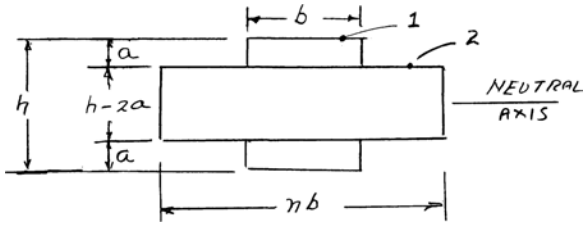
PROBLEM 4.C1



Two aluminum strips and a steel strip are to be bonded together to form a composite member of width $b = 60$ mm and depth $h = 40$ mm. The modulus of elasticity is 200 GPa for the steel and 75 GPa for the aluminum. Knowing that $M = 1500\ \text{N}\cdot\text{m}$, write a computer program to calculate the maximum stress in the aluminum and in the steel for values of a from 0 to 20 mm using 2-mm increments. Using appropriate smaller increments, determine (*a*) the largest stress that can occur in the steel, (*b*) the corresponding value of a.

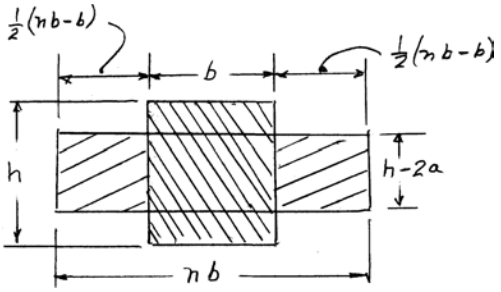
SOLUTION

Transformed section: (all steel) $n = \dfrac{E_{\text{steel}}}{E_{\text{alum}}}$



$$\bar{I} = \frac{1}{12}bh^3 + \frac{1}{12}\left[2\left(\frac{1}{2}\right)(nb-b)\right](h-2a)^3$$

At Point 1: $\sigma_{\text{alum}} = \dfrac{M\left(\frac{h}{2}\right)}{\bar{I}}$



At Point 2: $\sigma_{\text{steel}} = n\dfrac{M\left(\frac{h}{2}-a\right)}{\bar{I}}$

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PROBLEM 4.C1 *(Continued)*

For $a = 0$ to 20 mm using 2-mm intervals: compute: n, $\bar{I}$, σ_{alum}, σ_{steel}.

$b = 60$ mm $h = 40$ mm $M = 1500\ \text{N} \cdot \text{m}$

Moduli of elasticity: Steel = 200 GPa Aluminum = 75 GPa

Program Output

a mm	I $\text{m}^4/10^6$	Sigma Aluminum MPa	Sigma Steel MPa
0.000	0.8533	35.156	93.750
2.000	0.7088	42.325	101.580
4.000	0.5931	50.585	107.914
6.000	0.5029	59.650	111.347
8.000	0.4352	68.934	110.294
10.000	0.3867	77.586	103.448
12.000	0.3541	84.714	90.361
14.000	0.3344	89.713	71.770
16.000	0.3243	92.516	49.342
18.000	0.3205	93.594	24.958
20.000	0.3200	93.750	0.000

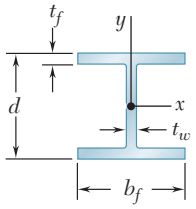
Find 'a' for max. steel stress and the corresponding aluminum stress.

6.600	0.4804	62.447	111.572083
6.610	0.4800	62.494	111.572159
6.620	0.4797	62.540	111.572113

Max. steel stress = 111.6 MPa occurs when $a = 6.61$ mm.

Corresponding aluminum stress = 62.5 MPa

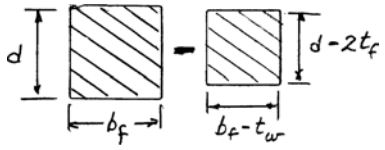
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PROBLEM 4.C2

A beam of the cross section shown, made of a steel that is assumed to be elastoplastic with a yield strength σ_Y and a modulus of elasticity E, is bent about the x axis. (*a*) Denoting by y_Y the half thickness of the elastic core, write a computer program to calculate the bending moment M and the radius of curvature ρ for values of y_Y from $\frac{1}{2}d$ to $\frac{1}{6}d$ using decrements equal to $\frac{1}{2}t_f$. Neglect the effect of fillets. (*b*) Use this program to solve Prob. 4.201.

SOLUTION



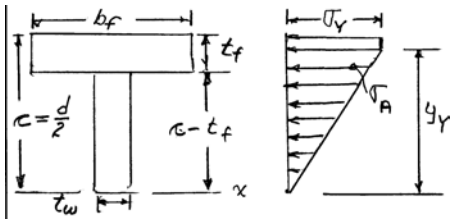
Compute moment of inertia I_x.

$$I_x = \frac{1}{12}b_f d^3 - \frac{1}{12}(b_f - t_w)(d - 2t_f)^3$$

Maximum elastic moment: $M_Y = \sigma_Y \dfrac{I_x}{(d/2)}$

For yielding in the flanges,

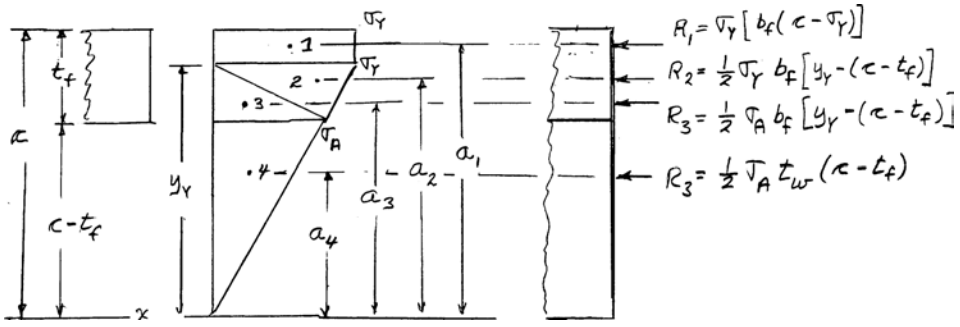
(Consider upper half of cross section.) $c = \dfrac{d}{2}$



Stress at junction of web and flange:

$$\sigma_A = \frac{(d/2) - t_f}{y_Y}\sigma_Y$$

Detail of stress diagram: Resultant forces:



$$a_1 = \frac{1}{2}(c + y_Y)$$

$$a_2 = y_Y - \frac{1}{3}\left[y_Y - (c - t_f)\right]$$

$$a_3 = y_Y - \frac{2}{3}[y_Y - (c - t_f)]$$

$$a_4 = \frac{2}{3}(c - t_f)$$

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PROBLEM 4.C2 *(Continued)*

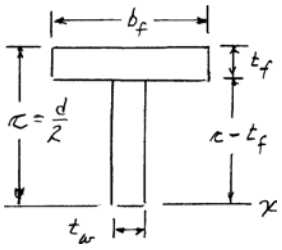
Bending moment.

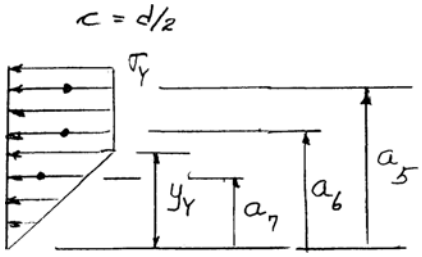
$$M = 2\sum_{n=1}^{4} R_n a_n$$

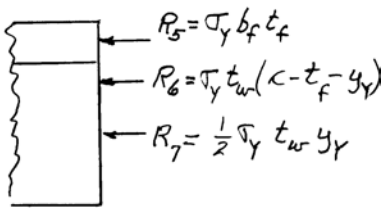
Radius of curvature.

$$y_Y = \varepsilon_Y \rho = \frac{\sigma_Y}{E}\rho; \quad \rho = \frac{y_Y E}{\sigma_Y}$$

For yielding in the web, (Consider upper half of cross section.)







$$a_5 = c - \frac{1}{2}t_f$$

$$a_6 = \frac{1}{2}[y_Y + (c - t_f)]$$

$$a_7 = \frac{2}{3}y_Y$$

Bending moment. $M = 2\sum_{n=5}^{7} R_n a_n$

Radius of curvature. $y_Y = \varepsilon_Y \rho = \frac{\sigma_Y}{E}\rho \qquad \rho = \frac{y_Y E}{\sigma_Y}$

Program: Key in expressions for a_n and R_n for $n = 1$ to 7.

For $y_Y = c$ to $(c - t_f)$ at $-t_f/2$ decrements, compute $M = 2\Sigma R_n a_n$ for $n = 1$ to 4 and $\rho = \frac{y_Y E}{\sigma_Y}$, then print.

For $y_Y = (c - t_w)$ to $c/3$ at $-t_f/2$ decrements, compute $M = 2\Sigma R_n a_n$ for $n = 5$ to 7 and $\rho = \frac{y_Y E}{\sigma_Y}$, then print.

Input numerical values and run program.

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PROBLEM 4.C2 *(Continued)*

Program Output

For a beam of Problem 4.201,

Depth $d = 140.00$ mm Width of flange $b_f = 120.00$ mm

Thickness of flange $t_f = 10.00$ mm Thickness of web $t_w = 10.00$ mm

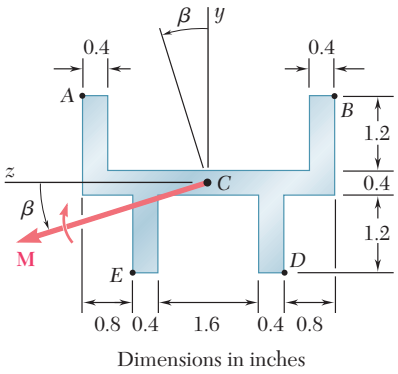
$I = 0.000011600$ m to the 4th

Yield strength of steel sigma$Y = 300$ MPa

Yield moment $M_Y = 49.71$ kip · in.

y_Y (mm)	M (kN · m)	ρ (m)
For yielding still in the flange,		
70.000	49.71	46.67
65.000	52.59	43.33
60.000	54.00	40.00
For yielding in the web,		
60.000	54.00	40.00
55.000	54.58	36.67
50.000	55.10	33.33
45.000	55.58	30.00
40.000	56.00	26.67
35.000	56.38	23.33
30.000	56.70	20.00
25.000	56.97	16.67

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Dimensions in inches

PROBLEM 4.C3

An 8 kip · in. couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Noting that the centroid of the cross section is located at *C* and that the *y* and *z* axes are principal axes, write a computer program to calculate the stress at *A*, *B*, *C*, and *D* for values of β from 0 to 180° using 10° increments. (*Given:* $I_y = 6.23 \text{ in}^4$ and $I_z = 1.481 \text{ in}^4$.)

SOLUTION

Input coordinates of *A*, *B*, *C*, *D*.

$$z_A = z(1) = 2 \qquad y_A = y(1) = 1.4$$
$$z_B = z(2) = -2 \qquad y_B = y(2) = 1.4$$
$$z_C = z(3) = -1 \qquad y_C = y(3) = -1.4$$
$$z_D = z(4) = 1 \qquad y_D = y(4) = -1.4$$

Components of *M*.

$$M_y = -M \sin \beta$$
$$M_z = M \cos \beta$$

Equation 4.55, Page 305: $$\sigma(n) = -\frac{M_z y(n)}{I_z} + \frac{M_y z(n)}{I_y}$$

Program: For $\beta = 0$ to 180° using 10° increments.

For $n = 1$ to 4 using unit increments.

Evaluate Equation 4.55 and print stresses.

Return

Return

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PROBLEM 4.C3 *(Continued)*

Program Output

Moment of couple: $M = 8.00 \text{ kip} \cdot \text{in.}$

Moments of inertia: $I_y = 6.23 \text{ in}^4 \quad I_z = 1.481 \text{ in}^4$

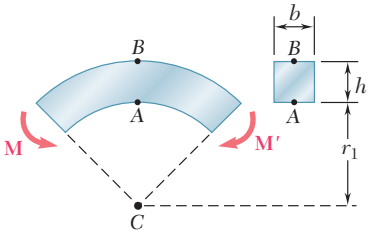
Coordinates of Points *A*, *B*, *D*, and *E*:

Point *A*: $z(1) = 2: \quad y(1) = 1.4$
Point *B*: $z(2) = -2: \quad y(2) = 1.4$
Point *D*: $z(3) = -1: \quad y(3) = -1.4$
Point *E*: $z(4) = 1: \quad y(4) = -1.4$

Beta °	---Stress at Points--- *A* ksi	*B* ksi	*D* ksi	*E* ksi
0	–7.565	–7.565	7.565	7.565
10	–7.896	–7.004	7.673	7.227
20	–7.987	–6.230	7.548	6.669
30	–7.836	–5.267	7.193	5.909
40	–7.446	–4.144	6.621	4.970
50	–6.830	–2.895	5.846	3.879
60	–6.007	–1.558	4.895	2.670
70	–5.001	–0.174	3.794	1.381
80	–3.843	1.216	2.578	0.049
90	–2.569	2.569	1.284	–1.284
100	–1.216	3.843	–0.049	–2.578
110	0.174	5.001	–1.381	–3.794
120	1.558	6.007	–2.670	–4.895
130	2.895	6.830	–3.879	–5.846
140	4.144	7.446	–4.970	–6.621
150	5.267	7.836	–5.909	–7.193
160	6.230	7.987	–6.669	–7.548
170	7.004	7.896	–7.227	–7.673
180	7.565	7.565	–7.565	–7.565

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PROBLEM 4.C4



Couples of moment $M = 2\ \text{kN}\cdot\text{m}$ are applied as shown to a curved bar having a rectangular cross section with $h = 100\ \text{mm}$ and $b = 25\ \text{mm}$. Write a computer program and use it to calculate the stresses at points A and B for values of the ratio r_1/h from 10 to 1 using decrements of 1, and from 1 to 0.1 using decrements of 0.1. Using appropriate smaller increments, determine the ratio r_1/h for which the maximum stress in the curved bar is 50% larger than the maximum stress in a straight bar of the same cross section.

SOLUTION

Input: $h = 100\ \text{mm}$,
$b = 25\ \text{mm}$,
$M = 2\ \text{kN}\cdot\text{m}$

For straight bar, $\sigma_{\text{straight}} = \dfrac{M}{S}$

$= \dfrac{6M}{h^2 b}$

$= 48\ \text{MPa}$

Following notation of Section 4.15, key in the following:

$$r_2 = h + r_1; \quad R = h/\ln(r_2 - r_1); \quad \bar{r} = r_1 + r_2: \quad e = \bar{r} - R; \quad A = bh = 2500 \qquad \text{(I)}$$

Stresses: $\sigma_A = \sigma_1 = M(r_1 - R)/(Aer_1) \qquad \sigma_B = \sigma_2 = M(r_2 - R)/(Aer_2)$ (II)

Since $h = 100\ \text{mm}$, for $r_1/h = 10$, $r_1 = 1000\ \text{mm}$. Also, $r_1/h = 10$, $r_1 = 100$

Program: For $r_1 = 1000$ to 100 at -100 decrements,

using equations of Lines I and II, evaluate r_2, R, $\bar{r}$, e, σ_1, and σ_2

Also evaluate ratio $= \sigma_1/\sigma_{\text{straight}}$

Return and repeat for $r_1 = 100$ to 10 at -10 decrements.

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PROBLEM 4.C4 *(Continued)*

Program Output

M = Bending moment = 2 kN · m h = 100.000 in. A = 2500.00 mm^2

Stress in straight beam = 48.00 MPa

r_1 mm	rbar mm	R mm	e mm	σ_1 MPa	σ_2 MPa	r_1/h –	ratio –
1000	1050	1049	0.794	–49.57	46.51	10.000	–1.033
900	950	949	0.878	–49.74	46.36	9.000	–1.036
800	850	849	0.981	–49.95	46.18	8.000	–1.041
700	750	749	1.112	–50.22	45.95	7.000	–1.046
600	650	649	1.284	–50.59	45.64	6.000	–1.054
500	550	548	1.518	–51.08	45.24	5.000	–1.064
400	450	448	1.858	–51.82	44.66	4.000	–1.080
300	350	348	2.394	–53.03	43.77	3.000	–1.105
200	250	247	3.370	–55.35	42.24	2.000	–1.153
100	150	144	5.730	–61.80	38.90	1.000	–1.288
100	150	144	5.730	–61.80	38.90	1.000	–1.288
90	140	134	6.170	–63.15	38.33	0.900	–1.316
80	130	123	6.685	–64.80	37.69	0.800	–1.350
70	120	113	7.299	–66.86	36.94	0.700	–1.393
60	110	102	8.045	–69.53	36.07	0.600	–1.449
50	100	91	8.976	–73.13	35.04	0.500	–1.523
40	90	80	10.176	–78.27	33.79	0.400	–1.631
30	80	68	11.803	–86.30	32.22	0.300	–1.798
20	70	56	14.189	–100.95	30.16	0.200	–2.103
10	60	42	18.297	–138.62	27.15	0.100	–2.888

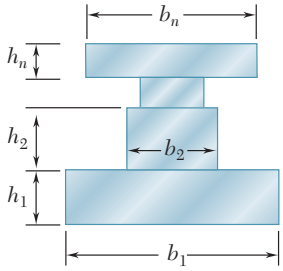
Find r_1/h for $(\sigma_{max})/(\sigma_{straight}) = 1.5$

52.70	103	94	8.703	–72.036	35.34	0.527	–1.501
52.80	103	94	8.693	–71.998	35.35	0.528	–1.500
52.90	103	94	8.683	–71.959	35.36	0.529	–1.499

Ratio of stresses is 1.5 for r_1 = 52.8 mm or r_1/h = 0.529.

[*Note:* The desired ratio r_1/h is valid for any beam having a rectangular cross section.]

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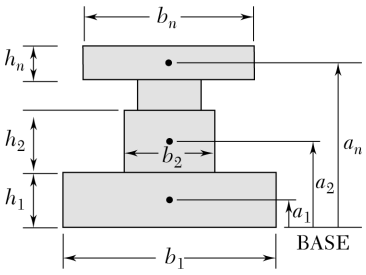




PROBLEM 4.C5

The couple **M** is applied to a beam of the cross section shown. (*a*) Write a computer program that, for loads expressed in either SI or U.S. customary units, can be used to calculate the maximum tensile and compressive stresses in the beam. (*b*) Use this program to solve Probs. 4.9, 4.10, and 4.11.

SOLUTION



Input: Bending moment M.

For $n = 1$ to n, Enter b_n and h_n

$$\Delta\text{Area} = b_n h_n \qquad \text{(Print)}$$

$$a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$$

[Moment of rectangle about base]

$$\Delta m = (\Delta\text{Area}) a_n$$

[For whole cross section]

$$m = m + \Delta m; \quad \text{Area} = \text{Area} + \Delta\text{Area}$$

Location of centroid above base.

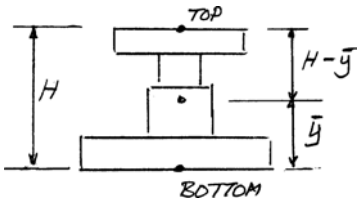
$$\bar{y} = m/\text{Area} \qquad \text{(Print)}$$

Moment of inertia about horizontal centroidal axis.

For $n = 1$ to n, $a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$

$$\Delta I = b_n h_n^3/12 + (b_n h_n)(\bar{y} - a_n)^2$$

$$I = I + \Delta I \qquad \text{(Print)}$$



Computation of stresses.

Total height: For $n = 1$ to n,

$$H = H + h_n$$

Stress at top:

$$M_{\text{top}} = -M\frac{H - \bar{y}}{I} \qquad \text{(Print)}$$

Stress at bottom:

$$M_{\text{bottom}} = M\frac{\bar{y}}{I} \qquad \text{(Print)}$$

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PROBLEM 4.C5 *(Continued)*

Problem 4.9

Summary of cross section dimensions:

Width (in.)	Height (in.)
9.00	2.00
3.00	6.00

Bending moment = 600.000 kip · in.

Centroid is 3.000 mm above lower edge.

Centroidal moment of inertia is 204.000 in^4.

Stress at top of beam = −14.706 ksi

Stress at bottom of beam = 8.824 ksi

Problem 4.10

Summary of cross section dimensions:

Width (in.)	Height (in.)
4.00	1.00
1.00	6.00
8.00	1.00

Bending moment = 500.000 kip · in.

Centroid is 4.778 in. above lower edge.

Centroidal moment of inertia is 155.111 in^4.

Stress at top of beam = −10.387 ksi

Stress at bottom of beam = 15.401 ksi

Problem 4.11

Summary of cross section dimensions:

Width (mm)	Height (mm)
50	10
20	50

Bending moment = 1500.0000 N · m

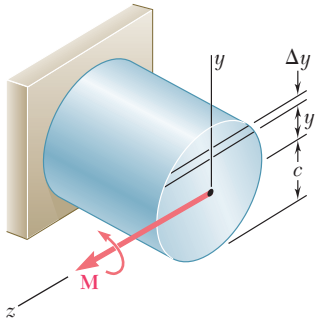
Centroid is 25.000 mm above lower edge.

Centroidal moment of inertia is 512,500 mm^4.

Stress at top of beam = −102.439 MPa

Stress at bottom of beam = 72.171 MPa

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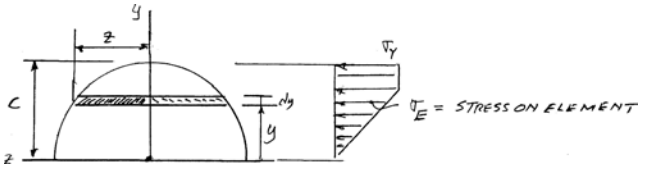
PROBLEM 4.C6

A solid rod of radius $c = 1.2$ in. is made of a steel that is assumed to be elastoplastic with $E = 29{,}000$ ksi and $\sigma_Y = 42$ ksi. The rod is subjected to a couple of moment M that increases from zero to the maximum elastic moment M_Y and then to the plastic moment M_p. Denoting by y_Y the half thickness of the elastic core, write a computer program and use it to calculate the bending moment M and the radius of curvature ρ for values of y_Y from 1.2 in. to 0 using 0.2-in. decrements. (*Hint:* Divide the cross section into 80 horizontal elements of 0.03-in. height.)

SOLUTION

$$M_Y = \sigma_Y \frac{\pi}{4} c^3 = (42 \text{ ksi}) \frac{\pi}{4} (1.2 \text{ in.})^3 = 57 \text{ kip} \cdot \text{in.}$$

$$M_p = \sigma_Y \frac{4}{3} c^3 = (42 \text{ ksi}) \frac{4}{3} (1.2 \text{ in.})^3 = 96.8 \text{ kip} \cdot \text{in.}$$



Consider top half of rod.

Let i = Number of elements in top half.

Δh = Height of each element: $\Delta h = \dfrac{c}{L}$

For $n = 0$ to $i - 1$, Step 1:

$y = n(\Delta h)$

$z = \left[c^2 - \{(n + 0.5)\Delta h\}^2 \right]$ ← z at midheight of element

If $y \geq y_Y$ go to 100

$\sigma_E = \sigma_Y \dfrac{(n + 0.5)\Delta h}{\sigma_y}$ ← Stress in elastic core

go to 200

100 $\sigma_E = \sigma_Y$ ← Stress in plastic zone

200 $\Delta\text{Area} = 2z(\Delta h)$

$\Delta\text{Force} = \sigma_E(\Delta\text{Area})$

$\Delta\text{Moment} = \Delta\text{Force}\ (n + 0.5)\Delta h$

$M = M + \Delta\text{Moment}$

$P = y_Y E / \sigma_Y$

Print y_Y, M, and ρ.

Next

Repeat for $y_Y = 1.2$ in. to $y_Y = 0$ At –0.2-in. decrements

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PROBLEM 4.C6 *(Continued)*

Program Output

Radius of rod = 1.2 in.

Yield point of steel = 42 ksi

Yield moment = 57.0 kip · in.

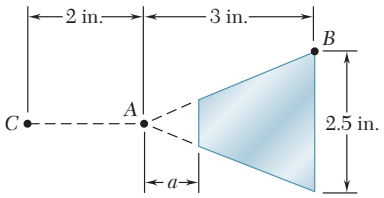
Plastic moment = 96.8 kip · in.

Number of elements in half of the rod = 40

For $y_Y = 1.20$ in.,	$M = 57.1$ kip · in.	Radius of curvature = 828.57 in.
For $y_Y = 1.00$ in.,	$M = 67.2$ kip · in.	Radius of curvature = 690.48 in.
For $y_Y = 0.80$ in.,	$M = 76.9$ kip · in.	Radius of curvature = 552.38 in.
For $y_Y = 0.60$ in.,	$M = 85.2$ kip · in.	Radius of curvature = 414.29 in.
For $y_Y = 0.40$ in.,	$M = 91.6$ kip · in.	Radius of curvature = 276.19 in.
For $y_Y = 0.20$ in.,	$M = 95.5$ kip · in.	Radius of curvature = 138.10 in.
For $y_Y = 0.00$ in.,	$M =$ infinite	Radius of curvature = zero

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PROBLEM 4.C7

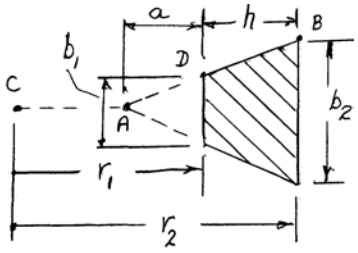


The machine element of Prob 4.178 is to be redesigned by removing part of the triangular cross section. It is believed that the removal of a small triangular area of width a will lower the maximum stress in the element. In order to verify this design concept, write a computer program to calculate the maximum stress in the element for values of a from 0 to 1 in. using 0.1-in. increments. Using appropriate smaller increments, determine the distance a for which the maximum stress is as small as possible and the corresponding value of the maximum stress.

SOLUTION

See Figure 4.79, Page 289.

$$M = 5 \text{ kip} \cdot \text{in.} \quad r_2 = 5 \text{ in.} \quad b_2 = 2.5 \text{ in.}$$



For $a = 0$ to 1.0 at 0.1 intervals,

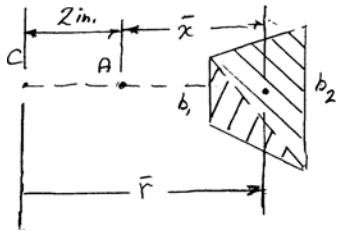
$$h = 3 - a$$

$$r_1 = 2 + a$$

$$b_1 = b_2(a/(h+a))$$

$$\text{Area} = (b_1 + b_2)(h/2)$$

$$\bar{x} = a + \left[\frac{1}{2}b_1 h(h/3) + \frac{1}{2}b_2 h(2h/3)\right] \Big/ \text{Area}$$



$$\bar{r} = r_2 - (h - \bar{x})$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$e = \bar{r} - R$$

$$\sigma_D = M(r_1 - R)/[\text{Area } (e)(r_1)]$$

$$\sigma_B = M(r_2 - R)/[\text{Area } (e)(r_2)]$$

Print and Return

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PROBLEM 4.C7 *(Continued)*

Program Output

a in.	R in.	σ_D ksi	σ_B ksi	b_1	$\bar{r}$	e
0.00	3.855	–8.5071	2.1014	0.00	4.00	0.145
0.10	3.858	–7.7736	2.1197	0.08	4.00	0.144
0.20	3.869	–7.2700	2.1689	0.17	4.01	0.140
0.30	3.884	–6.9260	2.2438	0.25	4.02	0.134
0.40	3.904	–6.7004	2.3423	0.33	4.03	0.127
0.50	3.928	–6.5683	2.4641	0.42	4.05	0.119
0.60	3.956	–6.5143	2.6102	0.50	4.07	0.111
0.70	3.985	–6.5296	2.7828	0.58	4.09	0.103
0.80	4.018	–6.6098	2.9852	0.67	4.11	0.094
0.90	4.052	–6.7541	3.2220	0.75	4.14	0.086
1.00	4.089	–6.9647	3.4992	0.83	4.17	0.078

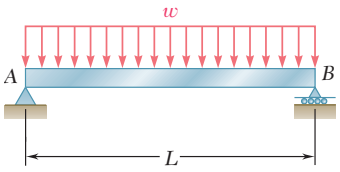
Determination of the maximum compressive stress that is as small as possible.

a in.	R in.	σ_D ksi	σ_B ksi	b_1	$\bar{r}$	e
0.620	3.961	–6.51198	2.6425	0.52	4.07	0.109
0.625	3.963	–6.51185	2.6507	0.52	4.07	0.109
0.630	3.964	–6.51188	2.6591	0.52	4.07	0.109

Answer: When $a = 625$ in., the compressive stress is 6.51 ksi.

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CHAPTER 5

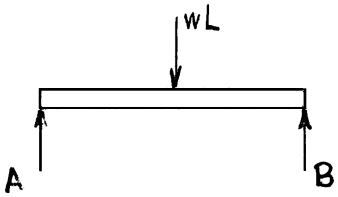


PROBLEM 5.1

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

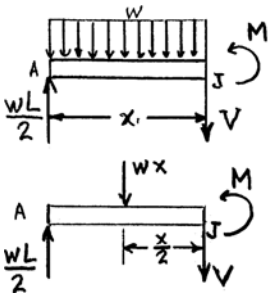
Reactions:



$$+\circlearrowleft \Sigma M_B = 0: \quad -AL + wL \cdot \frac{L}{2} = 0 \qquad A = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad BL - wL \cdot \frac{L}{2} = 0 \qquad B = \frac{wL}{2}$$

Free body diagram for determining reactions:



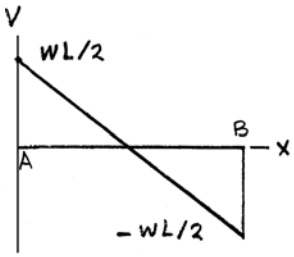
Over whole beam, $0 < x < L$

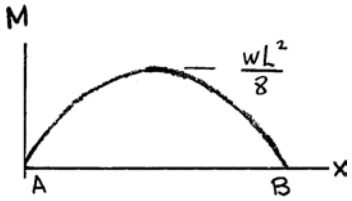
Place section at x.

Replace distributed load by equivalent concentrated load.

$$+\uparrow \Sigma F_y = 0: \quad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right) \blacktriangleleft$$





$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$$

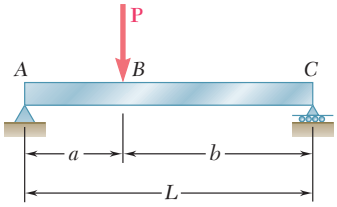
$$M = \frac{w}{2}(Lx - x^2)$$

$$M = \frac{w}{2}x(L - x) \blacktriangleleft$$

Maximum bending moment occurs at $x = \dfrac{L}{2}$.

$$M_{\max} = \frac{wL^2}{8} \blacktriangleleft$$

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PROBLEM 5.2

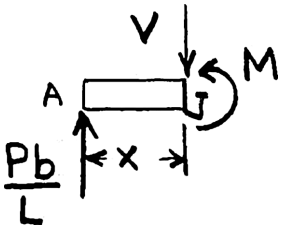
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Reactions:

$$(+\circlearrowleft)\ \Sigma M_C = 0: \quad LA - bP = 0 \qquad A = \frac{Pb}{L}$$

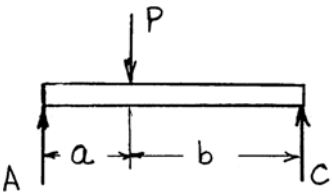
$$+\circlearrowright \Sigma M_A = 0: \quad LC - aP = 0 \qquad C = \frac{Pa}{L}$$



From *A* to *B*, $0 < x < a$

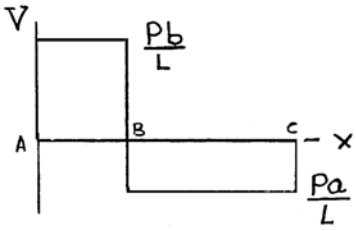
$$+\uparrow \Sigma F_y = 0: \quad \frac{Pb}{L} - V = 0$$

$$V = \frac{Pb}{L} \blacktriangleleft$$

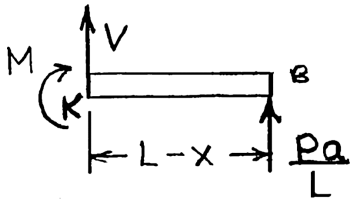


$$+\circlearrowright \Sigma M_J = 0: \quad M - \frac{Pb}{L}x = 0$$

$$M = \frac{Pbx}{L} \blacktriangleleft$$

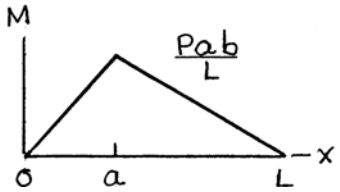


From *B* to *C*, $a < x < L$



$$+\uparrow \Sigma F_y = 0: \quad V + \frac{Pa}{L} = 0$$

$$V = -\frac{Pa}{L} \blacktriangleleft$$



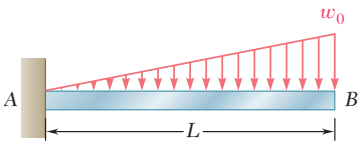
$$+\circlearrowright \Sigma M_K = 0: \quad -M + \frac{Pa}{L}(L - x) = 0$$

$$M = \frac{Pa(L - x)}{L} \blacktriangleleft$$

At section *B*,

$$M = \frac{Pab}{L^2} \blacktriangleleft$$

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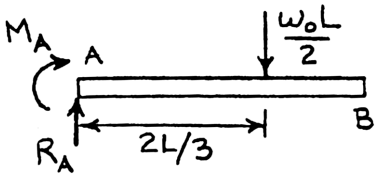


PROBLEM 5.3

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Free body diagram for determining reactions.

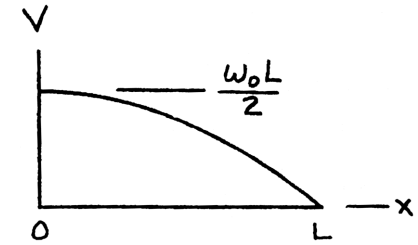


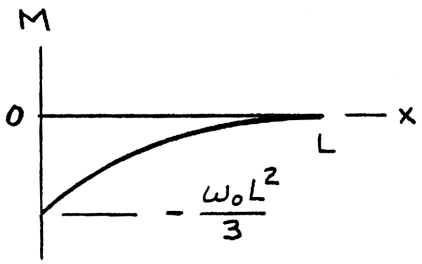
Reactions:

$$+\uparrow \Sigma F_y = 0: \quad R_A - \frac{w_0 L}{2} = 0 \qquad R_A = \frac{w_0 L}{2} \uparrow$$

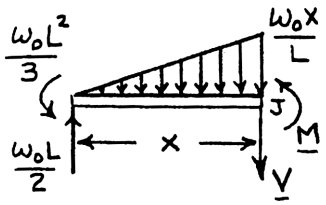
$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - \left(\frac{w_0 L}{2}\right)\left(\frac{2L}{3}\right) = 0$$

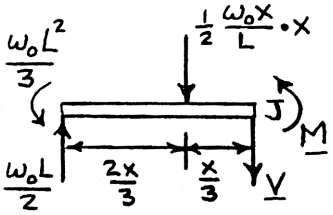
$$M_A = -\frac{w_0 L^2}{3} = \frac{w_0 L^2}{3} \circlearrowleft$$





Use portion to left of the section as the free body.





Replace distributed load with equivalent concentrated load.

$$+\uparrow \Sigma F_y = 0: \quad \frac{w_0 L}{2} - \frac{1}{2}\frac{w_0 x}{L} \cdot x - V = 0$$

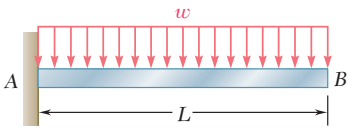
$$V = \frac{w_0 L}{2} - \frac{w_0 x^2}{2L} \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0:$$

$$\frac{w_0 L^2}{3} - \left(\frac{w_0 L}{2}\right)(x) + \left(\frac{1}{2}\frac{w_0 x}{L} \cdot x\right)\left(\frac{x}{3}\right) + M = 0$$

$$M = -\frac{w_0 L^2}{3} + \frac{w_0 L x}{2} - \frac{w_0 x^3}{6L} \blacktriangleleft$$

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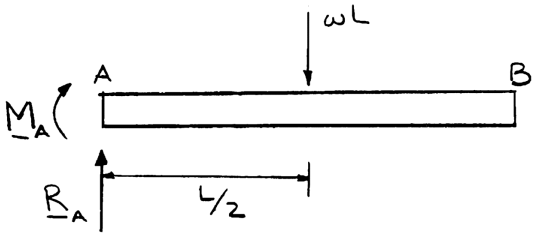


PROBLEM 5.4

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Free body diagram for determining reactions.



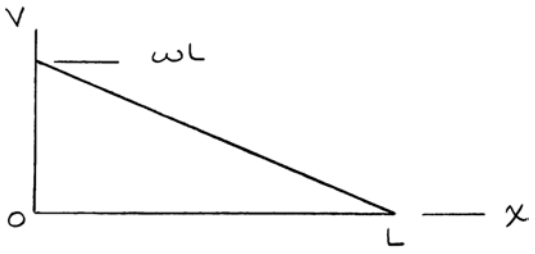
Reactions:

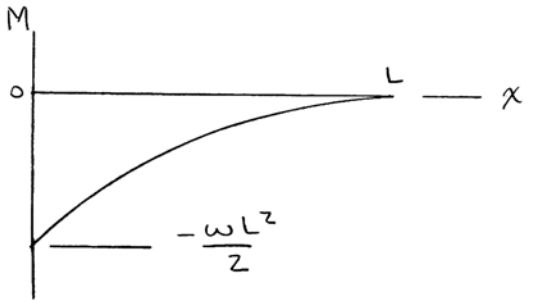
$$+\uparrow \Sigma F_y = 0: \quad R_A - wL = 0$$

$$R_A = wL \uparrow$$

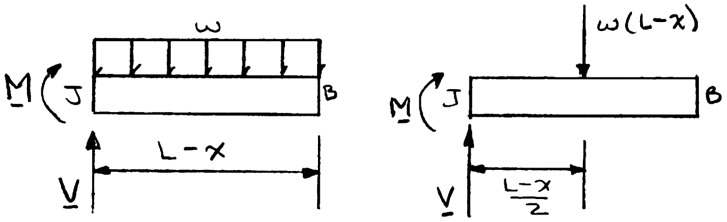
$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (wL)\left(\frac{L}{2}\right) = 0$$

$$M_A = \frac{w_0 L^2}{2} \circlearrowleft$$





Use portion to the right of the section as the free body.



Replace distributed load by equivalent concentrated load.

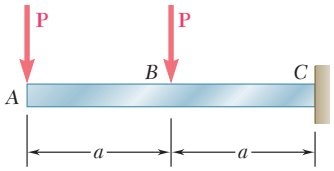
$$+\uparrow \Sigma F_y = 0: \quad V - w(L - x) = 0$$

$$V = w(L - x) \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -M - w(L - x)\left(\frac{L - x}{2}\right) = 0$$

$$M = -\frac{w}{2}(L - x)^2 \blacktriangleleft$$

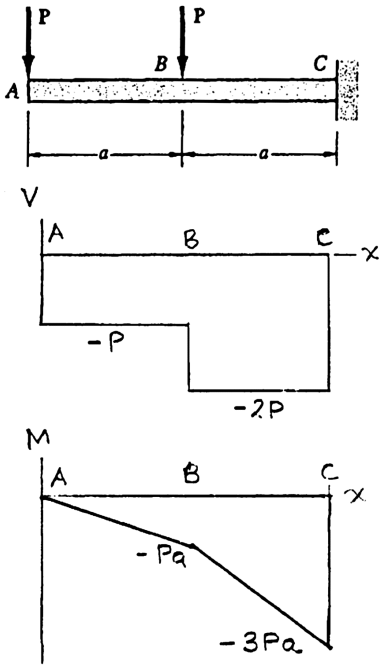
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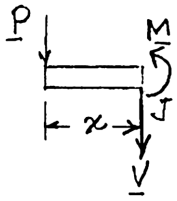
PROBLEM 5.5

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



From *A* to *B*: $0 < x < a$



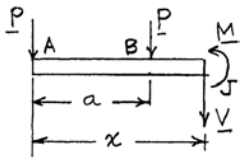
$+\uparrow \Sigma F_y = 0: \quad -P - V = 0$

$$V = -P \blacktriangleleft$$

$+\circlearrowleft \Sigma M_J = 0: \quad Px + M = 0$

$$M = -Px \blacktriangleleft$$

From *B* to *C*: $a < x \le 2a$



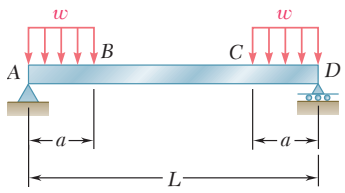
$+\uparrow \Sigma F_y = 0: \quad -P - P - V = 0$

$$V = -2P \blacktriangleleft$$

$+\circlearrowleft \Sigma M_J = 0: \quad Px + P(x - a) + M = 0$

$$M = -2Px + Pa \blacktriangleleft$$

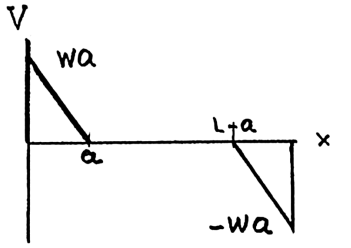
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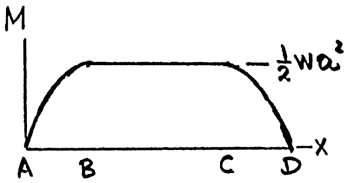


PROBLEM 5.6

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

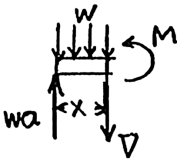
SOLUTION

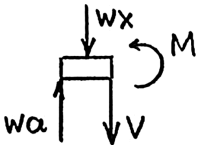




Reactions: $A = D = wa$

From A to B, $0 < x < a$



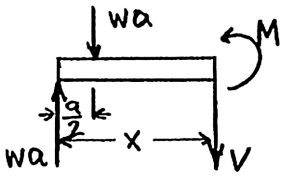


$+\uparrow \Sigma F_y = 0$: $wa - wx - V = 0$

$$V = w(a - x) \blacktriangleleft$$

$+\circlearrowleft \Sigma M_J = 0$: $-wax + (wx)\frac{x}{2} + M = 0$

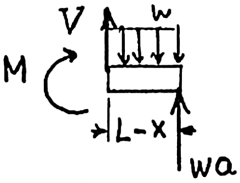
$$M = w\left(ax - \frac{x^2}{2}\right) \blacktriangleleft$$



From B to C, $a < x < L - a$

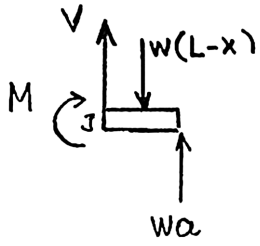
$\Sigma F_y = 0$: $wa - wa - V = 0$

$$V = 0 \blacktriangleleft$$



$+\circlearrowleft \Sigma M_J = 0$: $-wax + wa\left(x - \frac{a}{2}\right) + M = 0$ $\qquad M = \frac{1}{2}wa^2 \blacktriangleleft$

From C to D, $L - a < x < L$



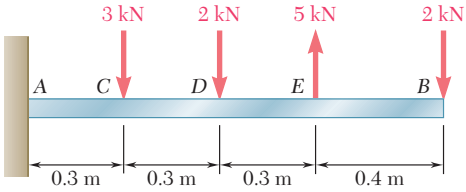
$+\uparrow \Sigma F_y = 0$: $V - w(L - x) + wa = 0$

$$V = w(L - x - a) \blacktriangleleft$$

$+\circlearrowleft \Sigma M_J = 0$: $-M - w(L - x)\left(\frac{L - x}{2}\right) + wa(L - x) = 0$

$$M = w\left[a(L - x) + \frac{1}{2}(L - x)^2\right] \blacktriangleleft$$

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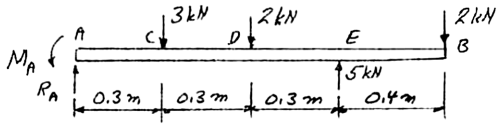


PROBLEM 5.7

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

Origin at *A*:



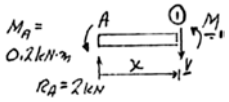
Reaction at *A*:

$$+\uparrow \Sigma F_y = 0: \quad R_A - 3 - 2 + 5 - 2 = 0 \qquad R_A = 2\ \text{kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (3\ \text{kN})(0.3\ \text{m}) - (2\ \text{kN})(0.6\ \text{m}) + (5\ \text{kN})(0.9\ \text{m}) - (2\ \text{kN})(1.3\ \text{m}) = 0$$

$$M_A = 0.2\ \text{kN} \cdot \text{m} \circlearrowleft \blacktriangleleft$$

From *A* to *C*:

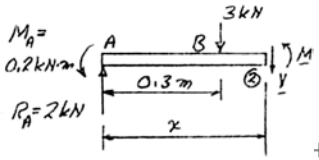


$$\Sigma F_y = 0: \quad V = 2\ \text{kN}$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad 0.2\ \text{kN} \cdot \text{m} - (2\ \text{kN})x + M = 0$$

$$M = -0.2 + 2x$$

From *C* to *D*:



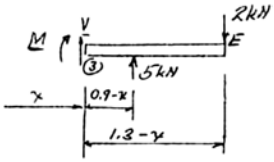
$$+\uparrow \Sigma F_y = 0: \quad 2 - 3 - V = 0$$

$$V = -1\ \text{kN}$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad +0.2\ \text{kN} \cdot \text{m} - (2\ \text{kN})x + (3\ \text{kN})(x - 0.3) + M = 0$$

$$M = 0.7 - x$$

From *D* to *E*:



$$\Sigma F_y = 0: \quad V + 5 - 2 = 0 \qquad V = -3\ \text{kN}$$

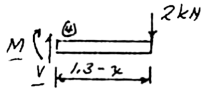
$$\circlearrowleft \Sigma M_3 = 0: \quad -M + 5(0.9 - x) - (2)(1.3 - x) = 0$$

$$M = +1.9 - 3x$$

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PROBLEM 5.7 *(Continued)*

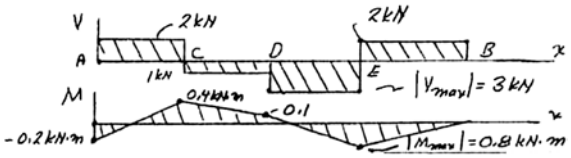
From *E* to *B*:



$$\Sigma F_y = 0: \quad V = 2 \text{ kN}$$

$$+\circlearrowleft \Sigma M_4 = 0: \quad -M - 2(1.3 - x) = 0$$

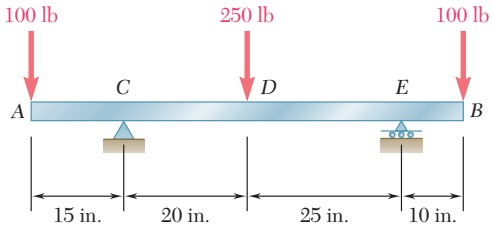
$$M = -2.6 + 2x$$



(*a*) $|V|_{\max} = 3.00 \text{ kN}$ ◀

(*b*) $|M|_{\max} = 0.800 \text{ kN} \cdot \text{m}$ ◀

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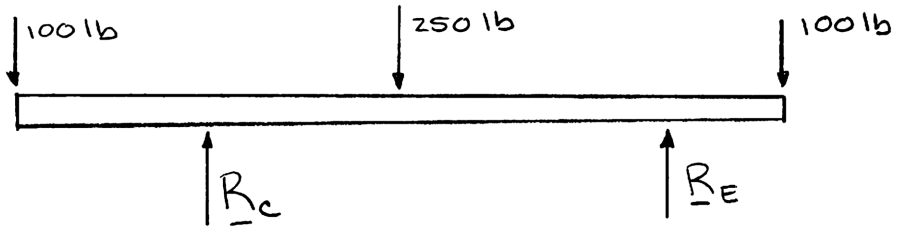


PROBLEM 5.8

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

Reactions:



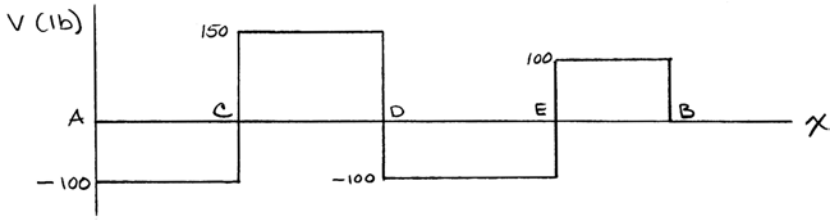
$$+\circlearrowleft \Sigma M_C = 0: \quad R_E(45 \text{ in.}) + 100 \text{ lb}(15 \text{ in.}) - 250 \text{ lb}(20 \text{ in.}) - 100 \text{ lb}(55 \text{ in.}) = 0$$

$$R_E = 200 \text{ lb}$$

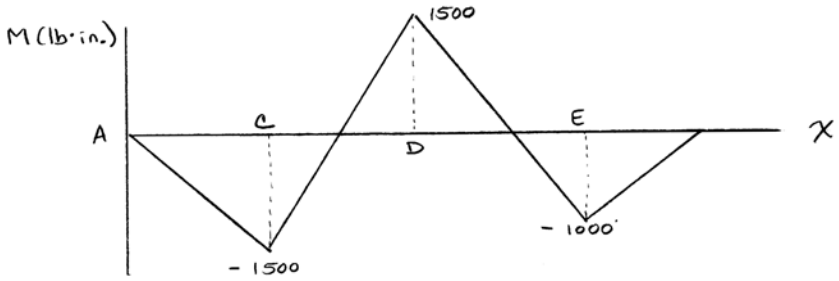
$$+\uparrow \Sigma F_y = 0: \quad R_C + 200 \text{ lb} - 100 \text{ lb} - 250 \text{ lb} - 100 \text{ lb} = 0$$

$$R_C = 250 \text{ lb}$$

At any point, V is the sum of the loads and reactions to the left (assuming $+\uparrow$) and M the sum of their moments about that point (assuming $+\circlearrowleft$).



(*a*) $V_{\max} = 150.0 \text{ lb}$ ◀



(*b*) $M_{\max} = 1500 \text{ lb} \cdot \text{in.}$ ◀

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PROBLEM 5.8 *(Continued)*

Detailed computations of moments:

$$M_A = 0$$

$$M_C = -(100\text{ lb})(15\text{ in.}) = -1500\text{ lb}\cdot\text{in.}$$

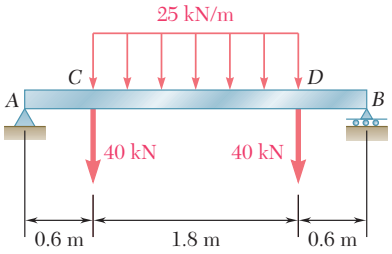
$$M_D = -(100\text{ lb})(35\text{ in.}) + (250\text{ lb})(20\text{ in.}) = +1500\text{ lb}\cdot\text{in.}$$

$$M_E = -(100\text{ lb})(60\text{ in.}) + (250\text{ lb})(45\text{ in.}) - (250\text{ lb})(25\text{ in.}) = -1000\text{ lb}\cdot\text{in.}$$

$$M_B = -(100\text{ lb})(70\text{ in.}) + (250\text{ lb})(55\text{ in.}) - (250\text{ lb})(35\text{ in.}) + (200\text{ lb})(10\text{ in.}) = 0$$

(Checks)

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PROBLEM 5.9

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

The distributed load is replaced with an equivalent concentrated load of 45 kN to compute the reactions.

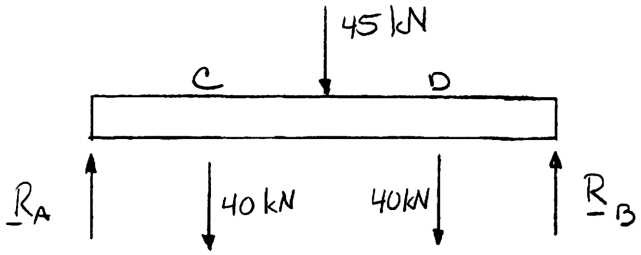
$(25\text{ kN/m})(1.8\text{ m}) = 45\text{ kN}$

$+\circlearrowleft \Sigma M_A = 0: \quad -(40\text{ kN})(0.6\text{ m}) - 45\text{ kN}(1.5\text{ m}) - 40\text{ kN}(2.4\text{ m}) + R_B(3.0\text{ m}) = 0$

$$R_B = 62.5\text{ kN}$$

$+\uparrow \Sigma F_y = 0: \quad R_A + 62.5\text{ kN} - 40\text{ kN} - 45\text{ kN} - 40\text{ kN} = 0$

$$R_A = 62.5\text{ kN}$$

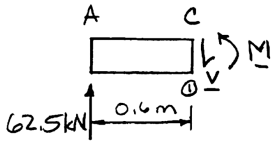


At *C*:

$+\uparrow \Sigma F_y = 0: \quad V = 62.5\text{ kN}$

$+\circlearrowleft \Sigma M_1 = 0: \quad M = (62.5\,\text{kN})(0.6\,\text{m}) = 37.5\,\text{kN}\cdot\text{m}$

At centerline of the beam:



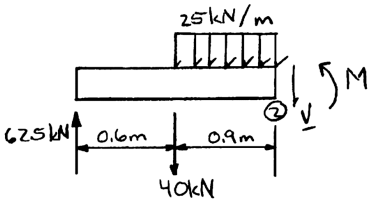
$+\uparrow \Sigma F_y = 0: \quad 62.5\text{ kN} - 40\text{ kN} - (25\text{ kN/m})(0.9\text{ m}) - V = 0$

$$V = 0$$

$+\circlearrowleft \Sigma M_2 = 0:$

$M - (62.5\text{ kN})(1.5\text{ m}) + (40\text{ kN})(0.9\text{ m}) + (25\text{ kN/m})(0.9\text{ m})(0.45\text{ m}) = 0$

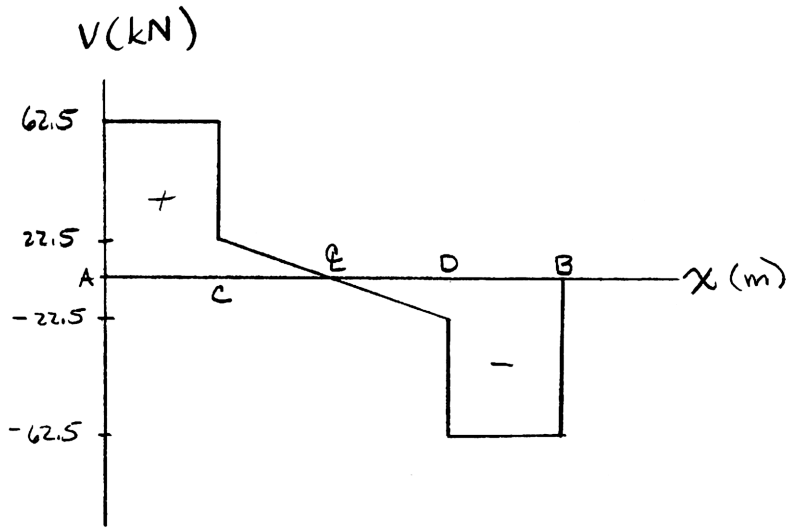
$$M = 47.625\text{ kN}\cdot\text{m}$$



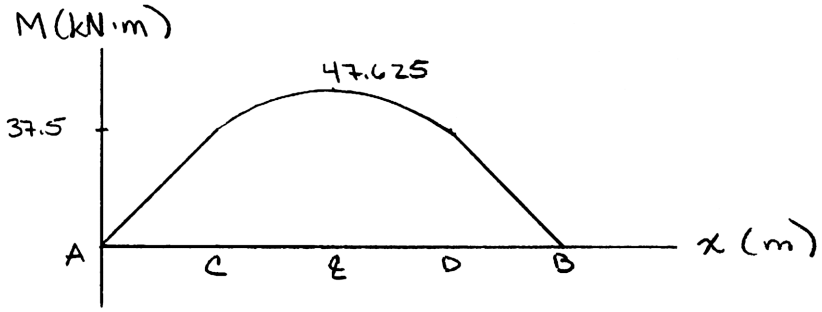
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PROBLEM 5.9 ***(Continued)***

Shear and bending-moment diagrams:



(*a*) $|V|_{\max} = 62.5 \text{ kN}$ ◀

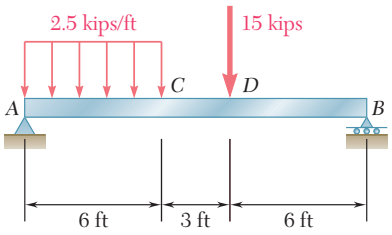


(*b*) $|M|_{\max} = 47.6 \text{ kN} \cdot \text{m}$ ◀

From *A* to *C* and *D* to *B*, *V* is uniform; therefore *M* is linear.

From *C* to *D*, *V* is linear; therefore *M* is parabolic.

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PROBLEM 5.10

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

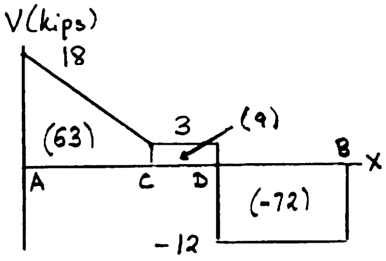
SOLUTION

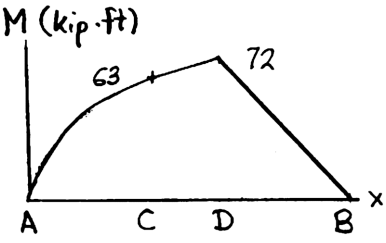
$+\circlearrowleft \Sigma M_B = 0: \quad -15R_A + (12)(6)(2.5) + (6)(15) = 0$

$$R_A = 18 \text{ kips}$$

$+\circlearrowleft \Sigma M_A = 0: \quad 15R_B - (3)(6)(2.5) - (9)(15) = 0$

$$R_B = 12 \text{ kips}$$





Shear:

$V_A = 18 \text{ kips}$

$V_C = 18 - (6)(2.5) = 3 \text{ kips}$

C to D: $V = 3 \text{ kips}$

D to B: $V = 3 - 15 = -12 \text{ kips}$

Areas under shear diagram:

A to C: $\int V\,dx = \left(\frac{1}{2}\right)(6)(18 + 3) = 63 \text{ kip} \cdot \text{ft}$

C to D: $\int V\,dx = (3)(3) = 9 \text{ kip} \cdot \text{ft}$

D to B: $\int V\,dx = (6)(-12) = -72 \text{ kip} \cdot \text{ft}$

Bending moments: $M_A = 0$

$M_C = 0 + 63 = 63 \text{ kip} \cdot \text{ft}$

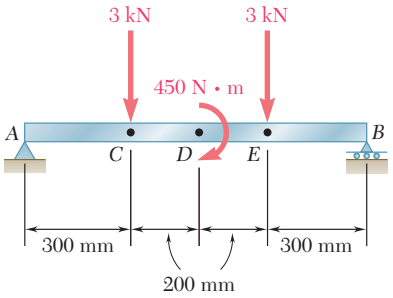
$M_D = 63 + 9 = 72 \text{ kip} \cdot \text{ft}$

$M_B = 72 - 72 = 0$

$$|V|_{\max} = 18.00 \text{ kips} \blacktriangleleft$$

$$|M|_{\max} = 72.0 \text{ kip} \cdot \text{ft} \blacktriangleleft$$

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PROBLEM 5.11

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

$+\circlearrowleft \Sigma M_B = 0: \quad (700)(3) - 450 + (300)(3) - 1000A = 0$

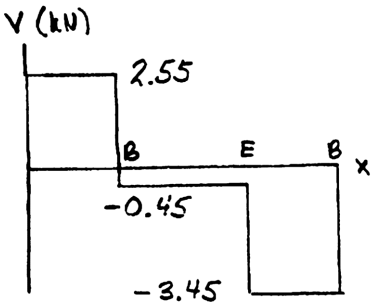
$A = 2.55 \text{ kN} \uparrow$

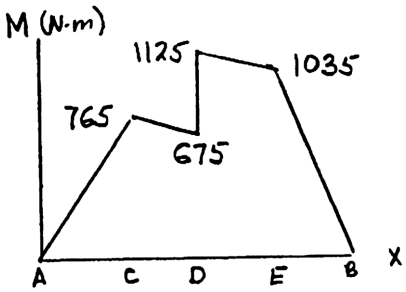
$+\circlearrowleft \Sigma M_A = 0: \quad -(300)(3) - 450 - (700)(3) + 1000B = 0$

$B = 3.45 \text{ kN} \uparrow$

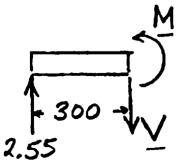
At *A*: $V = 2.55 \text{ kN} \quad M = 0$

A to *C*: $V = 2.55 \text{ kN}$





At *C*:



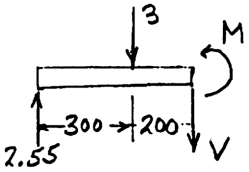
$+\circlearrowleft \Sigma M_C = 0:$

$-(300)(2.55) + M = 0$

$M = 765 \text{ N} \cdot \text{m}$

C to *E*: $V = -0.45 \text{ N} \cdot \text{m}$

At *D*:

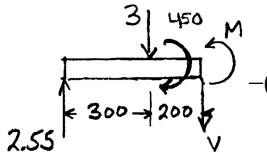


$+\circlearrowleft \Sigma M_D = 0:$

$-(500)(2.55) + (200)(3) + M = 0$

$M = 675 \text{ N} \cdot \text{m}$

At *D*:



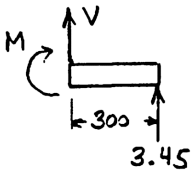
$+\circlearrowleft \Sigma M_D = 0:$

$-(500)(2.55) + (200)(3) - 450 + M = 0$

$M = 1125 \text{ N} \cdot \text{m}$

E to *B*: $V = -3.45 \text{ kN}$

At *E*:



$+\circlearrowleft \Sigma M_E = 0:$

$-M + (300)(3.45) = 0$

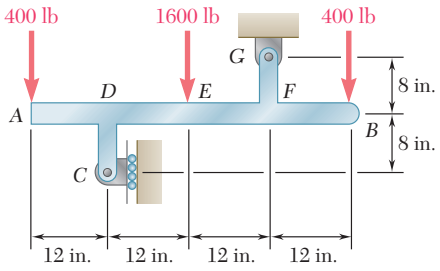
$M = 1035 \text{ N} \cdot \text{m}$

At *B*: $V = 3.45 \text{ kN}, \quad M = 0$

(*a*) $|V|_{\max} = 3.45 \text{ kN}$ ◀

(*b*) $|M|_{\max} = 1125 \text{ N} \cdot \text{m}$ ◀

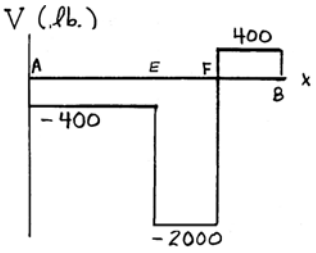
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PROBLEM 5.12

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

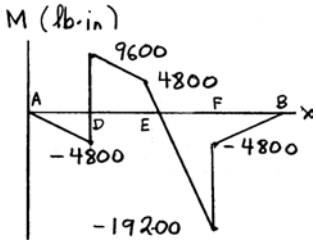


$+\circlearrowleft \Sigma M_G = 0: \quad -16C + (36)(400) + (12)(1600)$

$-(12)(400) = 0 \qquad C = 1800 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad -C + G_x = 0 \qquad G_x = 1800 \text{ lb}$

$+\uparrow \Sigma F_y = 0: \quad -400 - 1600 + G_y - 400 = 0 \qquad G_y = 2400 \text{ lb}$

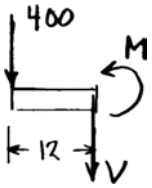


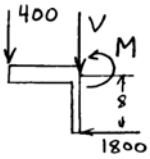
A to E: $V = -400$ lb

E to F: $V = -2000$ lb

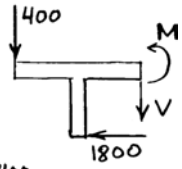
F to B: $V = 400$ lb

At A and B, $M = 0$





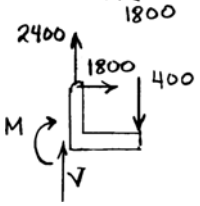
At D^-, $+\circlearrowleft \Sigma M_D = 0: \quad (12)(400) + M = 0 \qquad M = -4800 \text{ lb} \cdot \text{in.}$



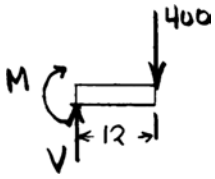
At D^+, $+\circlearrowleft \Sigma M_D = 0: \quad (12)(400) - (8)(1800) + M = 0 \qquad M = 9600 \text{ lb} \cdot \text{in.}$

At E, $+\circlearrowleft \Sigma M_E = 0: \quad (24)(400) - (8)(1800) + M = 0 \qquad M = 4800 \text{ lb} \cdot \text{in.}$

At F^-, $+\circlearrowleft \Sigma M_F = 0: \quad -M - (8)(1800) - (12)(400) = 0 \qquad M = -19,200 \text{ lb} \cdot \text{in.}$



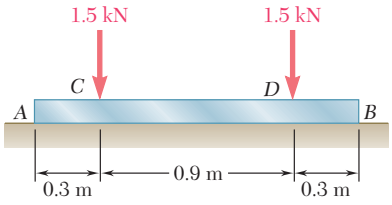
At F^+, $+\circlearrowleft \Sigma M_F = 0: \quad -M - (12)(400) = 0 \qquad M = -4800 \text{ lb} \cdot \text{in.}$



(*a*) Maximum $|V| = 2000$ lb ◀

(*b*) Maximum $|M| = 19,200 \text{ lb} \cdot \text{in.}$ ◀

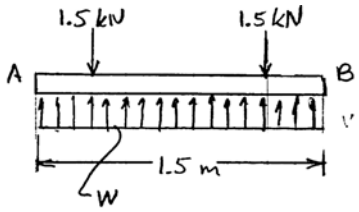
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PROBLEM 5.13

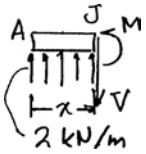
Assuming that the reaction of the ground is uniformly distributed, draw the shear and bending-moment diagrams for the beam *AB* and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

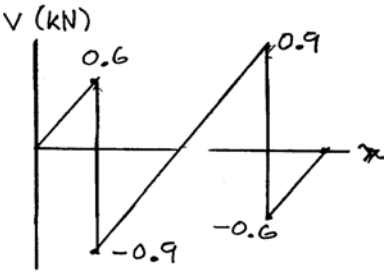
SOLUTION



Over the whole beam,

$$+\uparrow \Sigma F_y = 0: \quad 1.5w - 1.5 - 1.5 = 0 \qquad w = 2 \text{ kN/m}$$

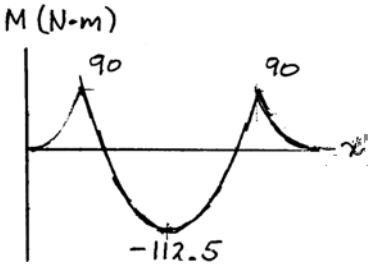




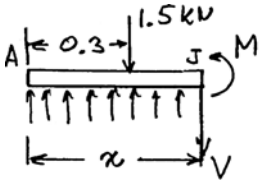
A to C: $\quad 0 \le x < 0.3$ m

$$+\uparrow \Sigma F_y = 0: \quad 2x - V = 0 \qquad V = (2x) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -(2x)\left(\frac{x}{2}\right) + M = 0 \qquad M = (x^2) \text{ kN} \cdot \text{m}$$



At C^-, $\quad x = 0.3$ m



$$V = 0.6 \text{ kN}, \quad M = 0.090 \text{ kN} \cdot \text{m} = 90 \text{ N} \cdot \text{m}$$

C to D: $\quad 0.3 \text{ m} < x < 1.2$ m

$$+\uparrow \Sigma F_y = 0: \quad 2x - 1.5 - V = 0 \qquad V = (2x - 1.5) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -(2x)\left(\frac{x}{2}\right) + (1.5)(x - 0.3) + M = 0$$

$$M = (x^2 - 1.5x + 0.45) \text{ kN} \cdot \text{m}$$

At the center of the beam, $x = 0.75$ m

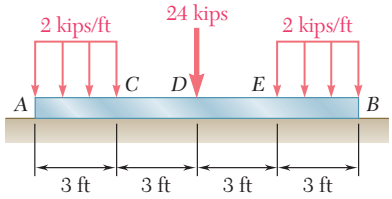
$$V = 0 \qquad M = -0.1125 \text{ kN} \cdot \text{m} = -112.5 \text{ N} \cdot \text{m}$$

At C^+, $\quad x = 0.3$ m, $\quad V = -0.9$ kN

(*a*) Maximum $|V| = 0.9$ kN $= 900$ N ◀

(*b*) Maximum $|M| = 112.5$ N · m ◀

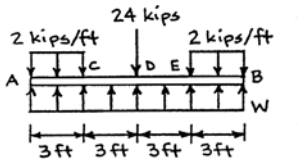
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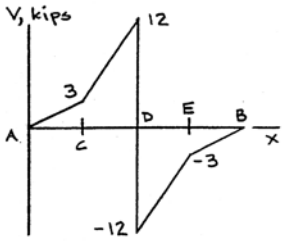


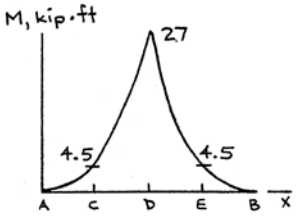
PROBLEM 5.14

Assuming that the reaction of the ground is uniformly distributed, draw the shear and bending-moment diagrams for the beam *AB* and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



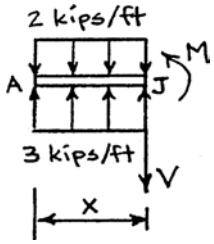




Over the whole beam,

$$+\uparrow \Sigma F_y = 0: \quad 12w - (3)(2) - 24 - (3)(2) = 0 \qquad w = 3 \text{ kips/ft}$$

A to *C*: $(0 \le x < 3 \text{ ft})$



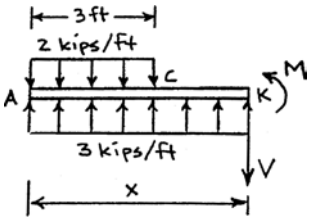
$$+\uparrow \Sigma F_y = 0: \quad 3x - 2x - V = 0 \qquad V = (x) \text{ kips}$$

$$+\Sigma M_J = 0: \quad -(3x)\frac{x}{2} + (2x)\frac{x}{2} + M = 0 \qquad M = (0.5x^2) \text{ kip} \cdot \text{ft}$$

At *C*, $x = 3$ ft

$V = 3$ kips, $M = 4.5$ kip · ft

C to *D*: $(3 \text{ ft} \le x < 6 \text{ ft})$



$$+\uparrow \Sigma F_y = 0: \quad 3x - (2)(3) - V = 0 \qquad V = (3x - 6) \text{ kips}$$

$$+\circlearrowleft \Sigma MK = 0: \quad -(3x)\left(\frac{x}{2}\right) + (2)(3)\left(x - \frac{3}{2}\right) + M = 0$$

$$M = (1.5x^2 - 6x + 9) \text{ kip} \cdot \text{ft}$$

At *D*, $x = 6$ ft

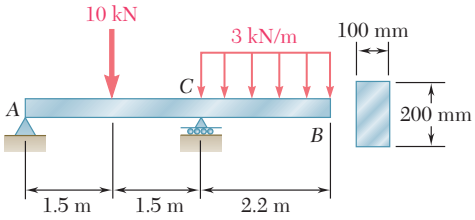
$V = 12$ kips, $M = 27$ kip · ft

D to *B*: Use symmetry to evaluate.

(*a*) $|V|_{\max} = 12.00$ kips ◀

(*b*) $|M|_{\max} = 27.0$ kip · ft ◀

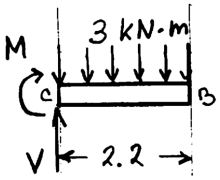
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PROBLEM 5.15

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION



Using *CB* as a free body,

$$+\circlearrowleft \Sigma M_C = 0: \quad -M + (2.2)(3\times10^3)(1.1) = 0$$

$$M = 7.26\times10^3 \text{ N}\cdot\text{m}$$

Section modulus for rectangle:

$$S = \frac{1}{6}bh^2$$

$$= \frac{1}{6}(100)(200)^2 = 666.7\times10^3 \text{ mm}^3$$

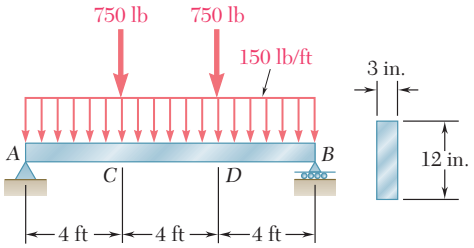
$$= 666.7\times10^{-6} \text{ m}^3$$

Normal stress:

$$\sigma = \frac{M}{S} = \frac{7.26\times10^3}{666.7\times10^{-6}} = 10.8895\times10^6 \text{ Pa}$$

$$\sigma = 10.89 \text{ MPa} \blacktriangleleft$$

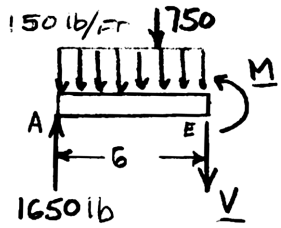
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PROBLEM 5.16

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION



Reactions: $C = A$ by symmetry.

$$+\uparrow \Sigma F_y = 0: \quad A + C - (2)(750) - (12)(150) = 0$$

$$A = C = 1650 \text{ lb}$$

Use left half of beam as free body.

$$+\circlearrowleft \Sigma M_E = 0:$$

$$-(1650)(6) + (750)(2) + (150)(6)(3) + M = 0$$

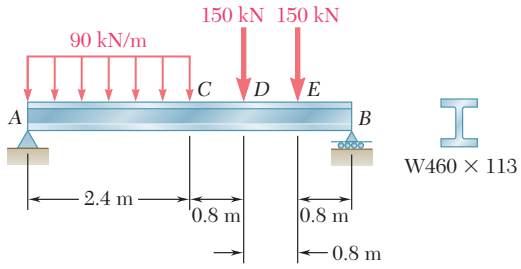
$$M = 5700 \text{ lb} \cdot \text{ft} = 68.4 \times 10^3 \text{ lb} \cdot \text{in.}$$

Section modulus: $$S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(3)(12)^2 = 72 \text{ in}^3$$

Normal stress: $$\sigma = \frac{M}{S} = \frac{68.4 \times 10^3}{72} = 950 \text{ psi}$$

$$\sigma = 950 \text{ psi} \blacktriangleleft$$

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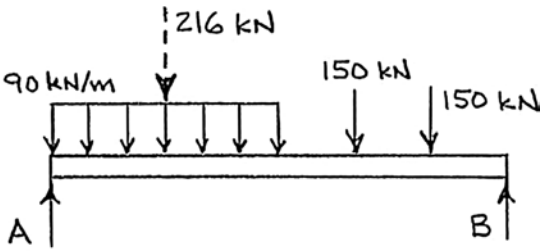


W460 × 113

PROBLEM 5.17

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION

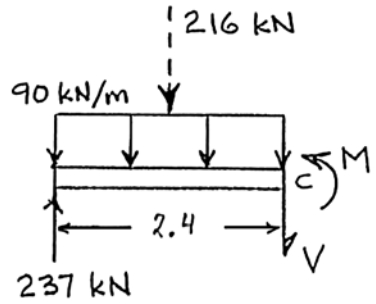


Use entire beam as free body.

$+\circlearrowleft \Sigma M_B = 0$:

$$-4.8A + (3.6)(216) + (1.6)(150) + (0.8)(150) = 0$$

$$\boldsymbol{A} = 237 \text{ kN} \uparrow$$



Use portion *AC* as free body.

$+\circlearrowleft \Sigma M_C = 0$:

$$M - (2.4)(237) + (1.2)(216) = 0$$

$$M = 309.6 \text{ kN} \cdot \text{m}$$

For W460 × 113, $S = 2390 \times 10^6 \text{mm}^3$

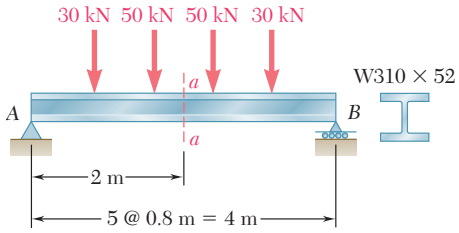
Normal stress:

$$\sigma = \frac{M}{S} = \frac{309.6 \times 10^3 \text{ N} \cdot \text{m}}{2390 \times 10^{-6} \text{ m}^3}$$

$$= 129.5 \times 10^6 \text{ Pa}$$

$$\sigma = 129.5 \text{ MPa} \blacktriangleleft$$

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PROBLEM 5.18

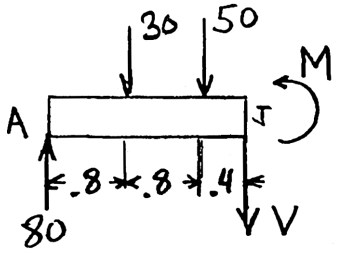
For the beam and loading shown, determine the maximum normal stress due to bending on section *a-a*.

SOLUTION

Reactions: By symmetry, $A = B$

$$+\uparrow \Sigma F_y = 0: \quad \mathbf{A} = \mathbf{B} = 80\ \text{kN}\uparrow$$

Using left half of beam as free body,



$$+\circlearrowleft \Sigma M_J = 0:$$

$$-(80)(2) + (30)(1.2) + (50)(0.4) + M = 0$$

$$M = 104\ \text{kN}\cdot\text{m} = 104\times10^3\ \text{N}\cdot\text{m}$$

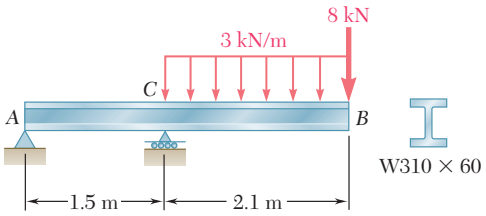
For W310 × 52, $S = 747\times10^3\ \text{mm}^3$

$$= 747\times10^{-6}\ \text{m}^3$$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{104\times10^3}{747\times10^{-6}} = 139.2\times10^6\ \text{Pa}$

$\sigma = 139.2\ \text{MPa}$ ◀

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PROBLEM 5.19

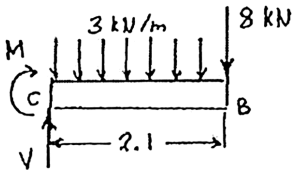
For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION

Use portion CB as free body.

$$+\circlearrowleft \Sigma M_C = 0: \quad -M + (3)(2.1)(1.05) + (8)(2.1) = 0$$

$$M = 23.415\ \text{kN} \cdot \text{m} = 23.415 \times 10^3\ \text{N} \cdot \text{m}$$



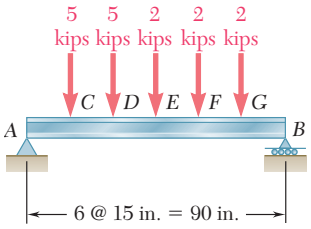
For W310 × 60, $S = 844 \times 10^3\ \text{mm}^3$

$$= 844 \times 10^{-6}\ \text{m}^3$$

Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{23.415 \times 10^3}{844 \times 10^{-6}} = 27.7 \times 10^6\ \text{Pa}$

$\sigma = 27.7\ \text{MPa}$ ◀

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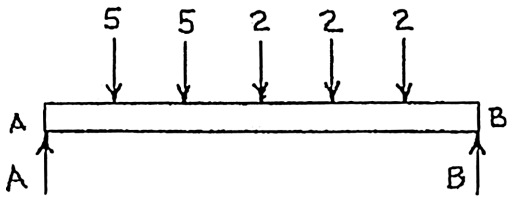




PROBLEM 5.20

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION



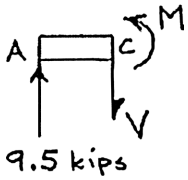
Use entire beam as free body.

$+\circlearrowleft \Sigma M_B = 0$:

$$-90A + (75)(5) + (60)(5) + (45)(2) + (30)(2) + (15)(2) = 0$$

$$A = 9.5 \text{ kips} \uparrow$$

Use portion AC as free body.



$+\circlearrowleft \Sigma M_C = 0$: $M - (15)(9.5) = 0$

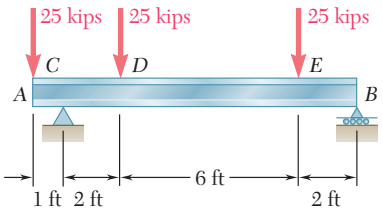
$$M = 142.5 \text{ kip} \cdot \text{in.}$$

For $S8 \times 18.4$, $S = 14.4 \text{ in}^3$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{142.5}{14.4}$

$$\sigma = 9.90 \text{ ksi} \blacktriangleleft$$

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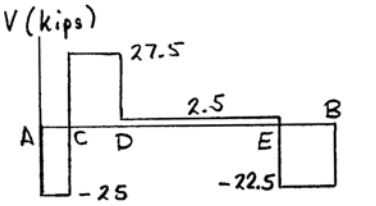


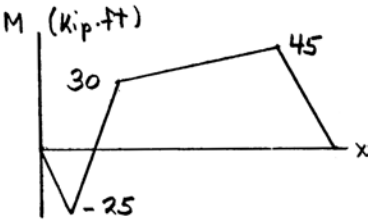


PROBLEM 5.21

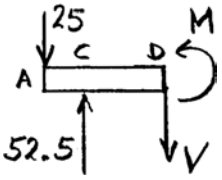
Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

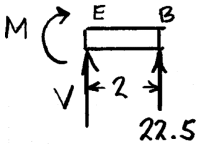
SOLUTION











$+\circlearrowleft \Sigma M_B = 0$:

$(11)(25) - 10C + (8)(25) + (2)(25) = 0 \quad \boldsymbol{C} = 52.5 \text{ kips} \uparrow$

$+\circlearrowleft \Sigma M_C = 0$:

$(1)(25) - (2)(25) - (8)(25) + 10B = 0 \quad \boldsymbol{B} = 22.5 \text{ kips}$

Shear:

A to C^-: $V = -25$ kips

C^+ to D^-: $V = 27.5$ kips

D^+ to E^-: $V = 2.5$ kips

E^+ to B: $V = -22.5$ kips

Bending moments:

At C, $+\circlearrowleft \Sigma M_C = 0$: $(1)(25) + M = 0$

$M = -25 \text{ kip} \cdot \text{ft}$

At D, $+\circlearrowleft \Sigma M_D = 0$: $(3)(25) - (2)(52.5) + M = 0$

$M = 30 \text{ kip} \cdot \text{ft}$

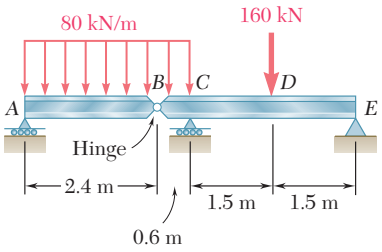
At E, $+\circlearrowleft \Sigma M_E = 0$: $-M + (2)(22.5) = 0 \quad M = 45 \text{ kip} \cdot \text{ft}$

$\max |M| = 45 \text{ kip} \cdot \text{ft} = 540 \text{ kip} \cdot \text{in.}$

For $S12 \times 35$ rolled steel section, $S = 38.1 \text{ in}^3$

Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{540}{38.1} = 14.17 \text{ ksi} \qquad \sigma = 14.17 \text{ ksi} \blacktriangleleft$

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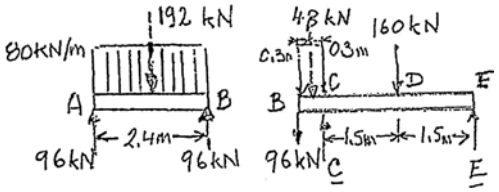


PROBLEM 5.22

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

Statics: Consider portion *AB* and *BE* separately.



Portion *BE*:

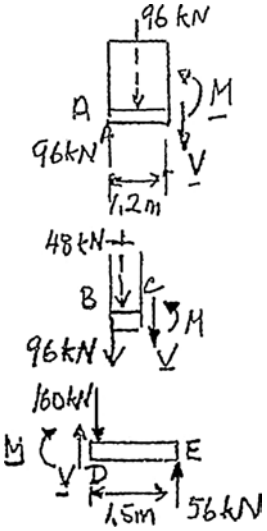
$+\circlearrowleft \sum M_E = 0$:

$(96)(3.6) + (48)(3.3) - C(3) + (160)(1.5) = 0$

$\mathbf{C} = 248\,\text{kN} \uparrow$

$\mathbf{E} = 56\,\text{kN} \uparrow$

$M_A = M_B = M_E = 0$



At midpoint of *AB*:

$\sum F_y = 0$: $V = 0$

$\sum M = 0$: $M = (96)(1.2) - (96)(0.6) = 57.6\ \text{kN} \cdot \text{m}$

Just to the left of *C*:

$\sum F_y = 0$: $V = -96 - 48 = -144\ \text{kN}$

$\sum M_C = 0$: $M = -(96)(0.6) - (48)(0.3) = -72\ \text{kN}$

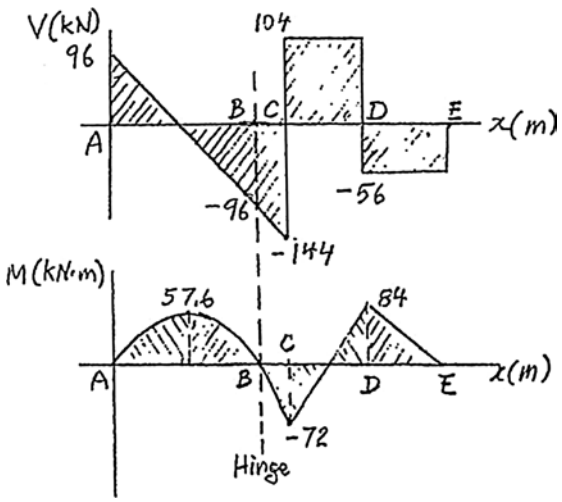
Just to the left of *D*:

$\sum F_y = 0$: $V = 160 - 56 = +104\ \text{kN}$

$\sum M_D = 0$: $M = (56)(1.5) = +84\ \text{kN} \cdot \text{m}$

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PROBLEM 5.22 *(Continued)*



From the diagram,

$$|M|_{\max} = 84\ \text{kN} \cdot \text{m} = 84 \times 10^3\ \text{N} \cdot \text{m}$$

For $W310 \times 60$ rolled-steel shape,

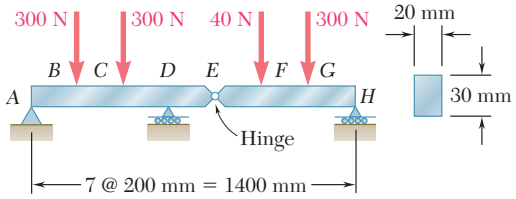
$$S_x = 844 \times 10^3\ \text{mm}^3$$
$$= 844 \times 10^{-6}\ \text{m}^3$$

Stress: $\sigma_m = \dfrac{|M|_{\max}}{S}$

$$\sigma_m = \frac{84 \times 10^3}{844 \times 10^{-6}} = 99.5 \times 10^6\ \text{Pa}$$

$$\sigma_m = 99.5\ \text{MPa} \blacktriangleleft$$

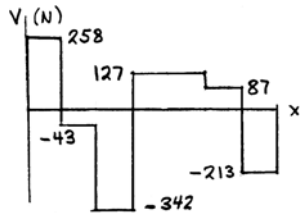
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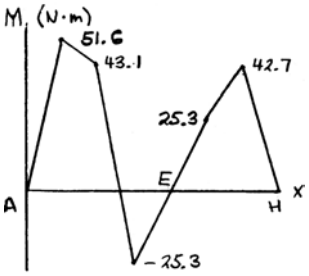


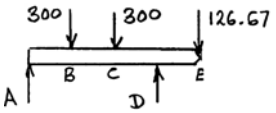
PROBLEM 5.23

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION



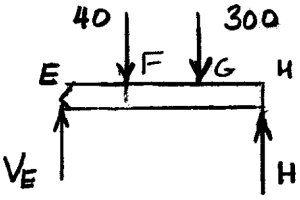




Free body *EFGH*. Note that $M_E = 0$ due to hinge.

$$+\circlearrowleft \Sigma M_E = 0: \quad 0.6\,H - (0.2)(40) - (0.40)(300) = 0$$

$$H = 213.33 \text{ N}$$



$$+\uparrow \Sigma F_y = 0: \quad V_E - 40 - 300 + 213.33 = 0$$

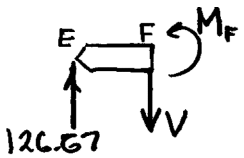
$$V_E = 126.67 \text{ N}$$

Shear:

E to F: $V = 126.67$ N · m

F to G: $V = 86.67$ N · m

G to H: $V = -213.33$ N · m



Bending moment at F:

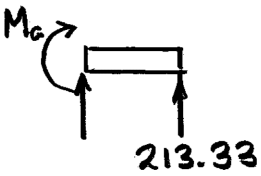
$$+\circlearrowleft \Sigma M_F = 0: \quad M_F - (0.2)(126.67) = 0$$

$$M_F = 25.33 \text{ N} \cdot \text{m}$$

Bending moment at G:

$$+\circlearrowleft \Sigma M_G = 0: \quad -M_G + (0.2)(213.33) = 0$$

$$M_G = 42.67 \text{ N} \cdot \text{m}$$



Free body *ABCDE*.

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.6A + (0.4)(300) + (0.2)(300) - (0.2)(126.63) = 0$$

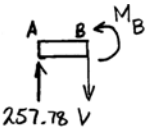
$$A = 257.78 \text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6D = 0$$

$$D = 468.89 \text{ N}$$

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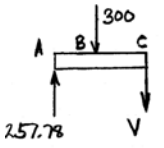
PROBLEM 5.23 *(Continued)*



Bending moment at *B*.

$$+\circlearrowleft \Sigma M_B = 0: \quad -(0.2)(257.78) + M_B = 0$$

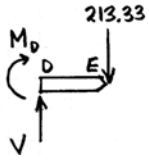
$$M_B = 51.56 \text{ N} \cdot \text{m}$$



Bending moment at *C*.

$$+\circlearrowleft \Sigma M_C = 0: \quad -(0.4)(257.78) + (0.2)(300) + M_C = 0$$

$$M_C = 43.11 \text{ N} \cdot \text{m}$$



Bending moment at *D*.

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - (0.2)(213.33) = 0$$

$$M_D = -25.33 \text{ N} \cdot \text{m}$$

$$\max|M| = 51.56 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

$$S = \frac{1}{6}bh^2 = \frac{1}{6}(20)(30)^2$$

$$= 3\times10^3 \text{ mm}^3 = 3\times10^{-6} \text{ m}^3$$

Normal stress:

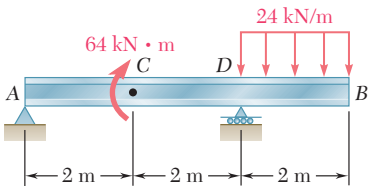
$$\sigma = \frac{51.56}{3\times10^{-6}} = 17.19\times10^6 \text{ Pa}$$

$$\sigma = 17.19 \text{ MPa} \quad \blacktriangleleft$$

$$|V|_{\max} = 342 \text{ N} \quad \blacktriangleleft$$

$$|M|_{\max} = 516 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

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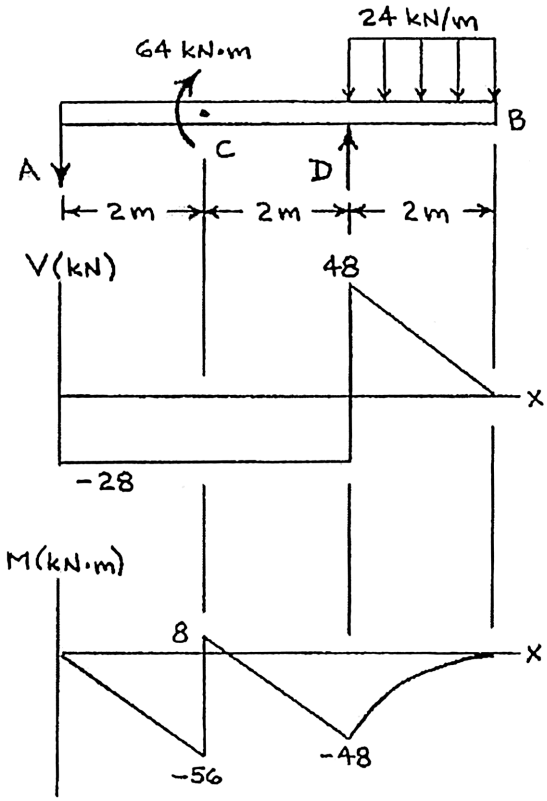




PROBLEM 5.24

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

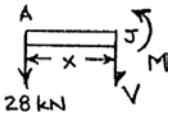


Reactions:

$+\circlearrowleft \sum M_D = 0: \quad 4A - 64 - (24)(2)(1) = 0 \quad \boldsymbol{A} = 28\ \text{kN}\downarrow$

$+\uparrow \sum F_y = 0: \quad -28 + D - (24)(2) = 0 \quad \boldsymbol{D} = 76\ \text{kN}\uparrow$

A to *C*: $0 < x < 2\text{m}$



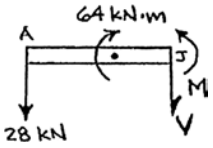
$+\uparrow \sum F_y = 0: \quad -V - 28 = 0$

$V = -28\ \text{kN}$

$+\circlearrowleft \sum M_J = 0: \quad M + 28x = 0$

$M = (-28x)\ \text{kN} \cdot \text{m}$

C to *D*: $2\text{m} < x < 4\text{m}$



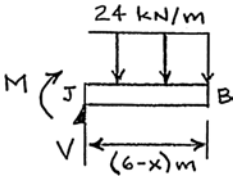
$+\uparrow \sum F_y = 0: \quad -V - 28 = 0$

$V = -28\ \text{kN}$

$+\circlearrowleft \sum M_J = 0: \quad M + 28x - 64 = 0$

$M = (-28x + 64)\ \text{kN} \cdot \text{m}$

D to *B*: $4\text{m} < x < 6\text{m}$



$+\uparrow \sum F_y = 0:$

$V - 24(6 - x) = 0$

$V = (-24x + 144)\ \text{kN}$

$+\circlearrowleft \sum M_J = 0:$

$$-M - 24(6 - x)\left(\frac{6 - x}{2}\right) = 0$$

$M = -12(6 - x)^2\ \text{kN} \cdot \text{m}$

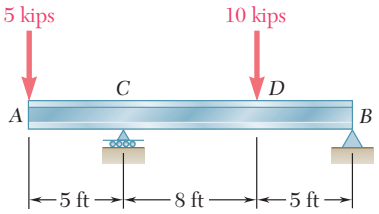
$\max|M| = 56\ \text{kN} \cdot \text{m} = 56 \times 10^3\ \text{N} \cdot \text{m}$

For $S250 \times 52$ section, $S = 482 \times 10^3\ \text{mm}^3$

Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{56 \times 10^3\ \text{N} \cdot \text{m}}{482 \times 10^{-6}\ \text{m}^3} = 116.2 \times 10^6\ \text{Pa}$

$\sigma = 116.2\ \text{MPa}$ ◀

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PROBLEM 5.25

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION

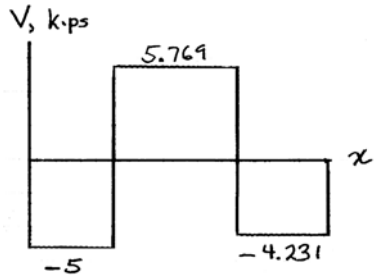
Reaction at C: $+\circlearrowleft \Sigma M_B = 0$: $(18)(5) - 13C + (5)(10) = 0$

$$C = 10.769 \text{ kips}$$

Reaction at B: $+\circlearrowleft M_C = 0$: $(5)(5) - (8)(10) + 13B = 0$

$$B = 4.231 \text{ kips}$$

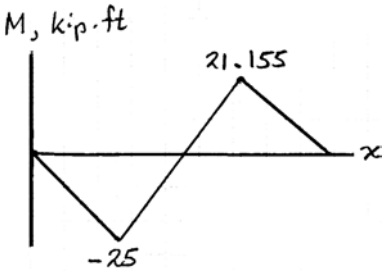
Shear diagram:



A to C^-: $V = -5$ kips

C^+ to D^-: $V = -5 + 10.769 = 5.769$ kips

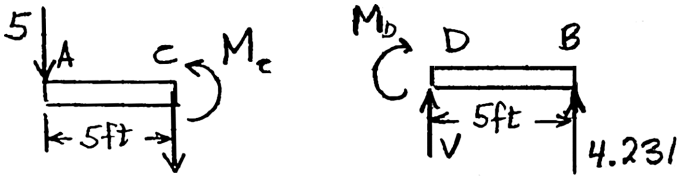
D^+ to B: $V = 5.769 - 10 = -4.231$ kips



At A and B, $M = 0$

At C, $+\circlearrowleft M_C = 0$: $(5)(5) + M_C = 0$

$$M_C = -25 \text{ kip} \cdot \text{ft}$$



At D, $+\circlearrowleft \Sigma M_D = 0$: $-M_D + (5)(4.231)$

$$M_D = 21.155 \text{ kip} \cdot \text{ft}$$

$|V|_{\max} = 5.77$ kips ◀

$|M|_{\max}$ occurs at C.

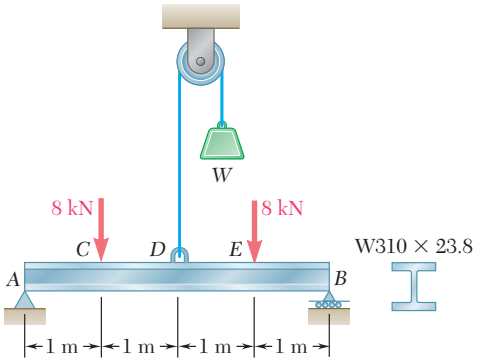
$|M|_{\max} = 25 \text{ kip} \cdot \text{ft} = 300 \text{ kip} \cdot \text{in.}$ ◀

For W14 × 22 rolled-steel section, $S = 29.0 \text{ in}^3$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{300}{29.0}$

$\sigma = 10.34$ ksi ◀

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PROBLEM 5.26

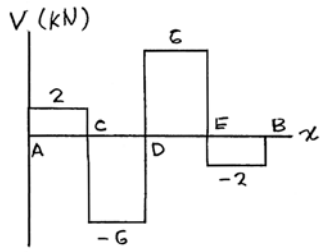
Knowing that $W = 12$ kN, draw the shear and bending-moment diagrams for beam AB and determine the maximum normal stress due to bending.

SOLUTION

By symmetry, $A = B$

$$+\uparrow \Sigma F_y = 0: \quad A - 8 + 12 - 8 + B = 0$$

$$A = B = 2 \text{ kN}$$



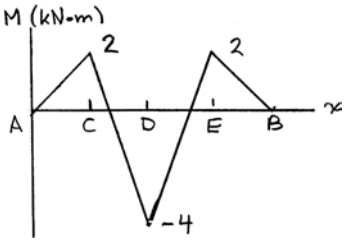
Shear: A to C^-: $V = 2$ kN

C^+ to D^-: $V = -6$ kN

D^+ to E^-: $V = 6$ kN

E^+ to B: $V = -2$ kN

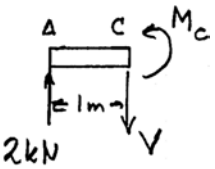
$$|V|_{\max} = 6.00 \text{ kN} \blacktriangleleft$$

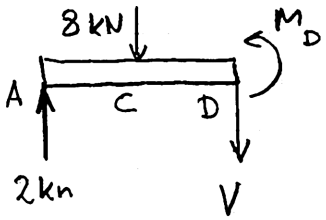


Bending moment:

At C, $+\circlearrowleft \Sigma M_C = 0: \quad M_C - (1)(2) = 0$

$$M_C = 2 \text{ kN} \cdot \text{m}$$





At D, $+\Sigma M_D = 0: \quad M_D - (2)(2) + (8)(1) = 0$

$$M_D = -4 \text{ kN} \cdot \text{m}$$

By symmetry, $M = 2$ kN · m at D. $M_E = 2$ kN · m

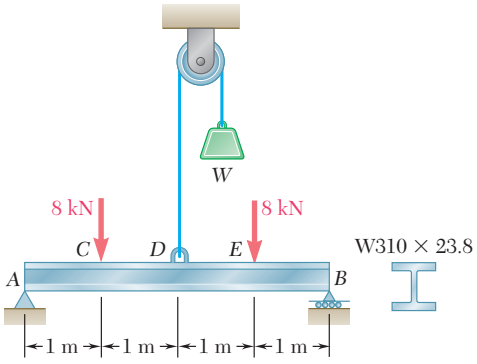
$\max|M| = 4.00$ kN · m occurs at E. ◀

For W310 × 23.8, $S_x = 280 \times 10^3 \text{mm}^3 = 280 \times 10^{-6} \text{m}^3$

Normal stress: $\sigma_{\max} = \dfrac{|M|_{\max}}{S_x} = \dfrac{4 \times 10^3}{280 \times 10^{-6}}$

$$= 14.29 \times 10^6 \text{ Pa} \qquad \sigma_{\max} = 14.29 \text{ MPa} \blacktriangleleft$$

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PROBLEM 5.27

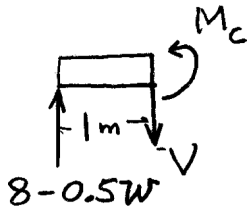
Determine (*a*) the magnitude of the counterweight W for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (*Hint:* Draw the bending-moment diagram and equate the absolute values of the largest positive and negative bending moments obtained.)

SOLUTION

By symmetry, $A = B$

$$+\uparrow \Sigma F_y = 0: \quad A - 8 + W - 8 + B = 0$$

$$A = B = 8 - 0.5W$$

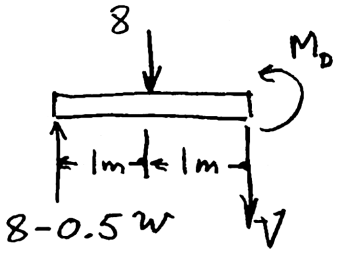


Bending moment at C:

$$+\circlearrowleft \Sigma M_C = 0: \quad -(8 - 0.5W)(1) + M_C = 0$$

$$M_C = (8 - 0.5W)\ \text{kN} \cdot \text{m}$$

Bending moment at D:



$$+\circlearrowleft \Sigma M_D = 0: \quad -(8 - 0.5W)(2) + (8)(1) + M_D = 0$$

$$M_D = (8 - W)\ \text{kN} \cdot \text{m}$$

Equate: $-M_D = M_C \qquad W - 8 = 8 - 0.5W$

$$W = 10.67\ \text{kN} \blacktriangleleft$$

(*a*) $W = 10.6667\ \text{kN}$

$M_C = -2.6667\ \text{kN} \cdot \text{m}$

$M_D = 2.6667\ \text{kN} \cdot \text{m} = 2.6667.10^3\ \text{N} \cdot \text{m}$

$|M|_{\max} = 2.6667\ \text{kN} \cdot \text{m}$

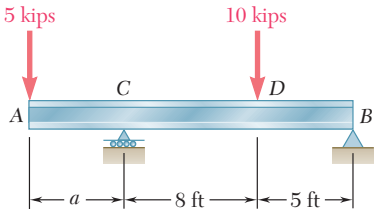
For W310 × 23.8 rolled-steel shape,

$$S_x = 280 \times 10^3\ \text{mm}^3 = 280 \times 10^{-6}\ \text{m}^3$$

(*b*) $$\sigma_{\max} = \frac{|M|_{\max}}{S_x} = \frac{2.6667 \times 10^3}{280 \times 10^{-6}} = 9.52 \times 10^6\ \text{Pa}$$

$$\sigma_{\max} = 9.52\ \text{MPa} \blacktriangleleft$$

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PROBLEM 5.28

Determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

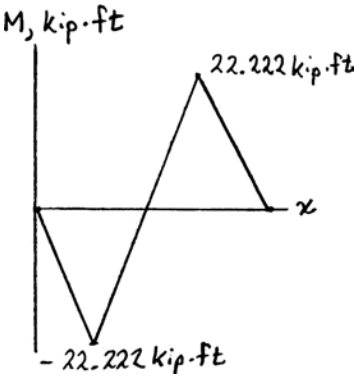
Reaction at *B*: $+\circlearrowleft \Sigma M_C = 0: \quad 5a - (8)(10) + 13R_B = 0$

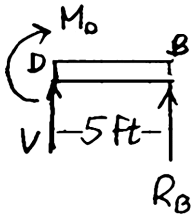
$$R_B = \frac{1}{18}(80 - 5a)$$

Bending moment at *D*:

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D + 5R_B = 0$$

$$M_D = 5R_B = \frac{5}{13}(80 - 5a)$$

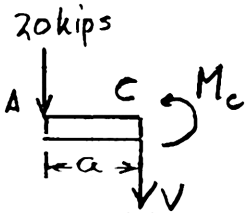




Bending moment at *C*:

$$+\circlearrowleft M_C = 0 \quad 5a + M_C = 0$$

$$M_C = -5a$$



Equate: $-M_C = M_D$

$$5a = \frac{5}{13}(80 - 5a)$$

$$a = 4.4444 \text{ ft}$$

(*a*) $a = 4.44$ ft ◀

Then $-M_C = M_D = (5)(4.4444) = 22.222 \text{ kip} \cdot \text{ft}$

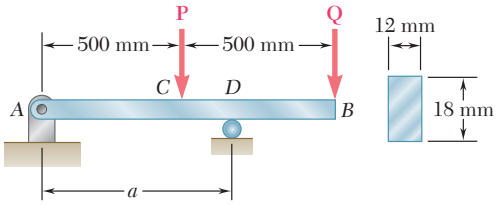
$$|M|_{\max} = 22.222 \text{ kip} \cdot \text{ft} = 266.67 \text{ kip} \cdot \text{in.}$$

For W14 × 22 rolled-steel section, $S = 29.0 \text{ in}^3$

Normal stress: $\sigma = \frac{M}{S} = \frac{266.67}{29.0} = 9.20 \text{ ksi}$

(*b*) 9.20 ksi ◀

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PROBLEM 5.29

Knowing that $P = Q = 480$ N, determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

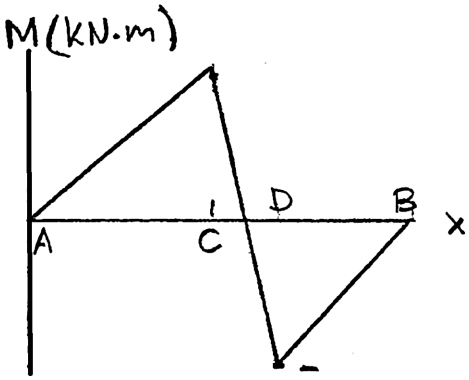
SOLUTION

$$P = 480 \text{ N} \qquad Q = 480 \text{ N}$$

Reaction at *A*: $+\circlearrowleft \Sigma M_D = 0: \quad -Aa + 480(a - 0.5)$

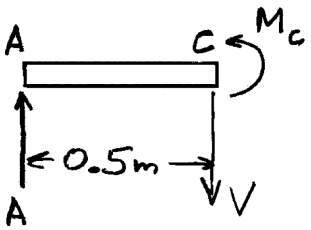
$$-480(1 - a) = 0$$

$$A = \left(960 - \frac{720}{a}\right) \text{N}$$



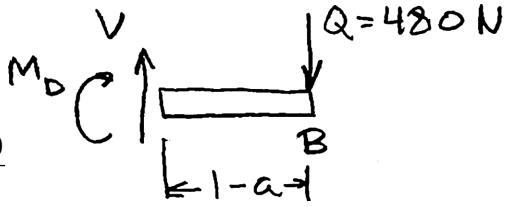
Bending moment at *C*: $+\circlearrowleft \Sigma M_C = 0: \quad -0.5A + M_C = 0$

$$M_C = 0.5A = \left(480 - \frac{360}{a}\right) \text{N} \cdot \text{m}$$



Bending moment at *D*: $+\circlearrowleft \Sigma M_D = 0: \quad -M_D - 480(1 - a) = 0$

$$M_D = -480(1 - a) \text{ N} \cdot \text{m}$$



(*a*) Equate: $-M_D = M_C \qquad 480(1 - a) = 480 - \dfrac{360}{a}$

$$a = 0.86603 \text{ m} \qquad a = 866 \text{ mm} \blacktriangleleft$$

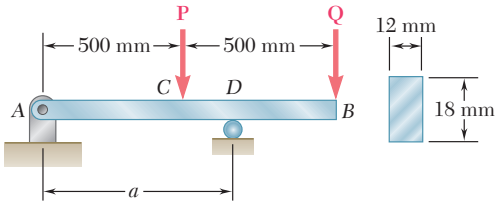
$$A = 128.62 \text{ N} \qquad M_C = 64.31 \text{ N} \cdot \text{m} \qquad M_D = -64.31 \text{ N} \cdot \text{m}$$

(*b*) For rectangular section, $S = \dfrac{1}{6}bh^2$

$$S = \frac{1}{6}(12)(13)^2 = 648 \text{ mm}^3 = 648 \times 10^{-9} \text{m}^3$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{64.31}{6.48 \times 10^{-9}} = 99.2 \times 10^6 \text{ Pa} \qquad \sigma_{\max} = 99.2 \text{ MPa} \blacktriangleleft$$

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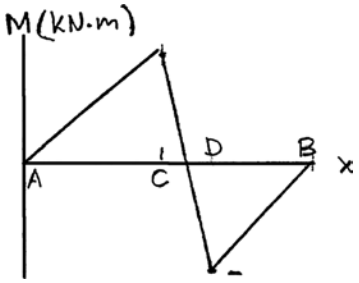
PROBLEM 5.30

Solve Prob. 5.29, assuming that $P = 480$ N and $Q = 320$ N.

PROBLEM 5.29 Knowing that $P = Q = 480$ N, determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

$$P = 480 \text{ N} \qquad Q = 320 \text{ N}$$

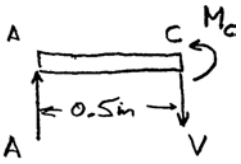


Reaction at *A*: $+\circlearrowleft \Sigma M_D = 0: \quad Aa + 480(a - 0.5) - 320(1 - a) = 0$

$$A = \left(800 - \frac{560}{a}\right) \text{ N}$$

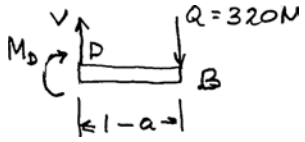
Bending moment at *C*: $+\circlearrowleft \Sigma M_C = 0: \quad -0.5A + M_C = 0$

$$M_C = 0.5A = \left(400 - \frac{280}{a}\right) \text{ N} \cdot \text{m}$$



Bending moment at *D*: $+\circlearrowleft \Sigma M_D = 0: \quad -M_D - 320(1 - a) = 0$

$$M_D = (-320 + 320a) \text{ N} \cdot \text{m}$$



(*a*) Equate: $-M_D = M_C \qquad 320 - 320a = 400 - \dfrac{280}{a}$

$$320a^2 + 80a - 280 = 0 \qquad a = 0.81873 \text{ m}, \ -1.06873 \text{ m}$$

Reject negative root. $a = 819$ mm ◀

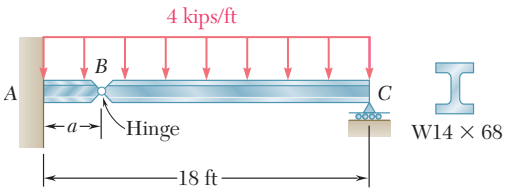
$$A = 116.014 \text{ N} \qquad M_C = 58.007 \text{ N} \cdot \text{m} \qquad M_D = -58.006 \text{ N} \cdot \text{m}$$

(*b*) For rectangular section, $S = \dfrac{1}{6}bh^2$

$$S = \frac{1}{6}(12)(18)^2 = 648 \text{ mm}^3 = 648 \times 10^{-9} \text{ m}^3$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{58.0065}{648 \times 10^{-9}} = 89.5 \times 10^6 \text{ Pa} \qquad \sigma_{\max} = 89.5 \text{ MPa} \blacktriangleleft$$

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PROBLEM 5.31

Determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

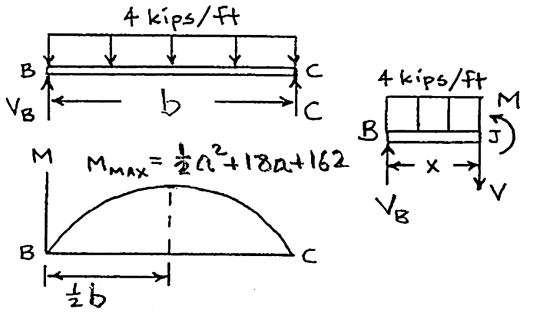
SOLUTION

For W14 × 68, $S_x = 103 \text{ in}^3$

Let $b = (18 - a)$ ft

Segment *BC*:

By symmetry, $V_B = C$



$$+\uparrow \Sigma F_y = 0: \quad V_B + C - 4b = 0$$

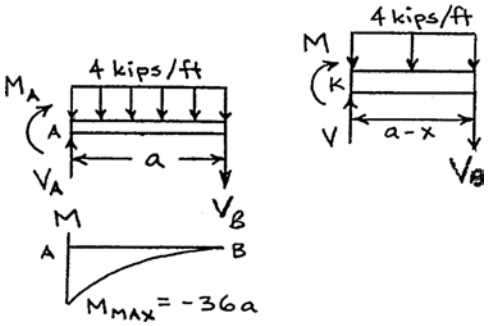
$$V_B = 2b$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -V_B x + (4x)\left(\frac{x}{2}\right) - M = 0$$

$$M = V_B x - 2x^2 = 2bx - 2x^2 \text{ lb} \cdot \text{ft}$$

$$\frac{dM}{dx} = 2b - x_m = 0 \qquad x_m = \frac{1}{2}b$$

$$M_{\max} = b^2 - \frac{1}{2}b^2 = \frac{1}{2}b^2$$



Segment *AB*:

$$+\circlearrowleft \Sigma M_K = 0: \quad -4(a - x)\frac{(a - x)}{2}$$

$$-V_B(a - x) - M = 0$$

$$M = -2(a - x)^2 + 2b(a - x)$$

$|M_{\max}|$ occurs at $x = 0$.

$$|M_{\max}| = -2a^2 - 2ab = -2a^2 - 2a(18 - a) = 36a$$

(*a*) Equate the two values of $|M_{\max}|$: $36a = \frac{1}{2}b^2 = \frac{1}{2}(18 - a)^2 = 162 - 18a + \frac{1}{2}a^2$

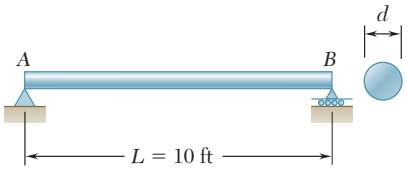
$$\frac{1}{2}a^2 - 54a + 162 = 0 \qquad a = 54 \pm \sqrt{(54)^2 - (4)\left(\tfrac{1}{2}\right)(162)}$$

$$a = 54 \pm 50.9118 = 3.0883 \text{ ft} \qquad a = 3.09 \text{ ft} \blacktriangleleft$$

(*b*) $|M|_{\max} = 36a = 111.179 \text{ kip} \cdot \text{ft} = 1334.15 \text{ kip} \cdot \text{in.}$

$$\sigma = \frac{|M|_{\max}}{S_x} = \frac{1334.15}{103} = 12.95 \text{ kips/in}^2 \qquad \sigma_m = 12.95 \text{ ksi} \blacktriangleleft$$

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PROBLEM 5.32

A solid steel rod of diameter d is supported as shown. Knowing that for steel $\gamma = 490$ lb/ft^3, determine the smallest diameter d that can be used if the normal stress due to bending is not to exceed 4 ksi.

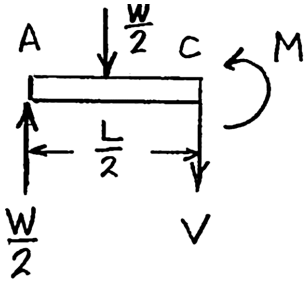
SOLUTION

Let W = total weight.

$$W = AL\gamma = \frac{\pi}{4}d^2L\gamma$$

Reaction at A:

$$A = \frac{1}{2}W$$



Bending moment at center of beam:

$$+\circlearrowleft \Sigma M_C = 0: \quad -\left(\frac{W}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) + M = 0$$

$$M = \frac{WL}{8} = \frac{\pi}{32}d^2L^2\gamma$$

For circular cross section, $\left(c = \frac{1}{2}d\right)$

$$I = \frac{\pi}{4}c^4, \qquad S = \frac{I}{c} = \frac{\pi}{4}c^3 = \frac{\pi}{32}d^3$$

Normal stress:

$$\sigma = \frac{M}{S} = \frac{\frac{\pi}{32}d^2L^2\gamma}{\frac{\pi}{32}d^3} = \frac{L^2\gamma}{d}$$

Solving for d,

$$d = \frac{L^2\gamma}{\sigma}$$

Data:

$$L = 10 \text{ ft} = (12)(10) = 120 \text{ in.}$$

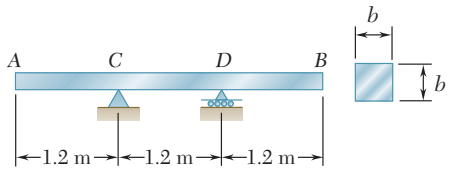
$$\gamma = 490 \text{ lb/ft}^3 = \frac{490}{12^3} = 0.28356 \text{ lb/in}^3$$

$$\sigma = 4 \text{ ksi} = 4000 \text{ lb/in}^2$$

$$d = \frac{(120)^2(0.28356)}{4000}$$

$$d = 1.021 \text{ in.} \blacktriangleleft$$

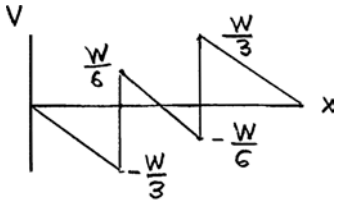
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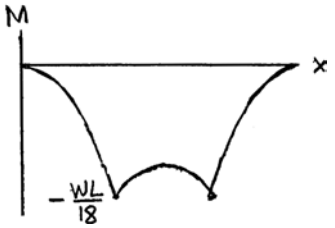


PROBLEM 5.33

A solid steel bar has a square cross section of side b and is supported as shown. Knowing that for steel $\rho = 7860\ \text{kg/m}^3$, determine the dimension b for which the maximum normal stress due to bending is (*a*) 10 MPa, (*b*) 50 MPa.

SOLUTION



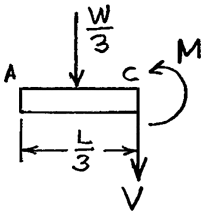


Weight density: $\gamma = \rho g$

Let L = total length of beam.

$$W = AL\rho g = b^2 L\rho g$$

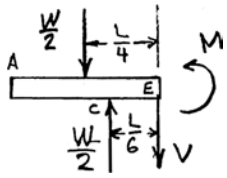
Reactions at C and D: $$C = D = \frac{W}{2}$$



Bending moment at C:

$$+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{L}{6}\right)\left(\frac{W}{3}\right) + M = 0$$

$$M = -\frac{WL}{18}$$



Bending moment at center of beam:

$$+\circlearrowleft \Sigma M_E = 0: \quad \left(\frac{L}{4}\right)\left(\frac{W}{2}\right) - \left(\frac{L}{6}\right)\left(\frac{W}{2}\right) + M = 0 \qquad M = -\frac{WL}{24}$$

$$\max|M| = \frac{WL}{18} = \frac{b^2L^2\rho g}{18}$$

For a square section, $$S = \frac{1}{6}b^3$$

Normal stress: $$\sigma = \frac{|M|}{S} = \frac{b^2L^2\rho g/18}{b^3/6} = \frac{L^2\rho g}{3b}$$

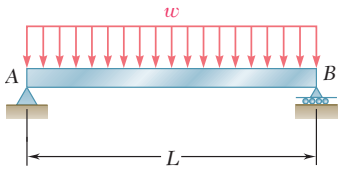
Solve for b: $$b = \frac{L^2\rho g}{3\sigma}$$

Data: $L = 3.6\ \text{m}$ $\rho = 7860\ \text{kg/m}^3$ $g = 9.81\ \text{m/s}^2$ (*a*) $\sigma = 10\times10^6\ \text{Pa}$ (*b*) $\sigma = 50\times10^6\ \text{Pa}$

(*a*) $$b = \frac{(3.6)^2(7860)(9.81)}{(3)(10\times10^6)} = 33.3\times10^{-3}\ \text{m}$$ $b = 33.3\ \text{mm}$ ◀

(*b*) $$b = \frac{(3.6)^2(7860)(9.81)}{(3)(50\times10^6)} = 6.66\times10^{-3}\ \text{m}$$ $b = 6.66\ \text{mm}$ ◀

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PROBLEM 5.34

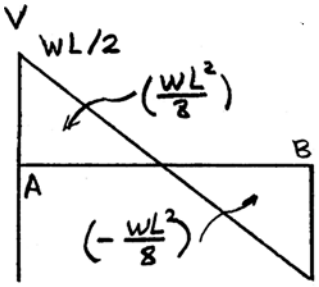
Using the method of Sec. 5.2, solve Prob. 5.1*a*.

PROBLEM 5.1 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -AL + wL \cdot \frac{L}{2} = 0 \qquad A = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad BL - wL \cdot \frac{L}{2} = 0 \qquad B = \frac{wL}{2}$$



$$\frac{dV}{dx} = -w$$

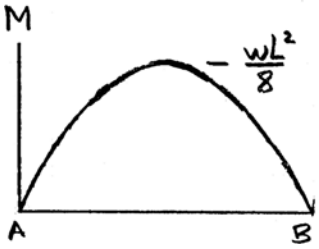
$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = A - wx \qquad V = \frac{wL}{2} - wx \blacktriangleleft$$

$$\frac{dM}{dx} = V$$

$$M - M_A = \int_0^x V\,dx = \int_0^x \left(\frac{wL}{2} - wx\right) dx$$

$$= \frac{wLx}{2} - \frac{wx^2}{2}$$

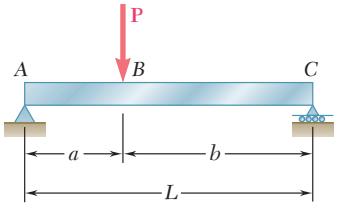


$$M = M_A + \frac{wLx}{2} - \frac{wx^2}{2} \qquad M = \frac{w}{2}(Lx - x^2) \blacktriangleleft$$

Maximum M occurs at $x = \frac{1}{2}$, where

$$V = \frac{dM}{dx} = 0 \qquad |M|_{\max} = \frac{wL^2}{8} \blacktriangleleft$$

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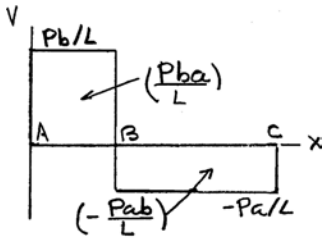


PROBLEM 5.35

Using the method of Sec. 5.2, solve Prob. 5.2*a*.

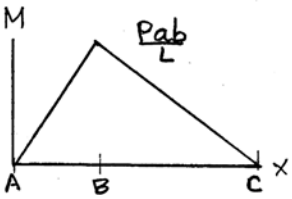
PROBLEM 5.2 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



$$+\circlearrowleft \Sigma M_C = 0: \quad LA - bP = 0 \qquad A = \frac{Pb}{L}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad LC - aP = 0 \qquad C = \frac{Pa}{L}$$



At A^+: $\quad V = A = \dfrac{Pb}{L} \qquad M = 0$

A to B^-: $\quad 0 < x < a$

$$w = 0 \qquad \int_0^x w\,dx = 0$$

$$V - V_A = 0 \qquad V = \frac{Pb}{L} \blacktriangleleft$$

$$M_B - M_A = \int_0^a V\,dx = \int_0^a \frac{Pb}{L}\,dx = \frac{Pba}{L} \qquad M_B = \frac{Pba}{L} \blacktriangleleft$$

At B^+: $\quad V = A - P = \dfrac{Pb}{L} - P = -\dfrac{Pa}{L}$

B^+ to C: $\quad a < x < L$

$$w = 0 \qquad \int_a^x w\,dx = 0$$

$$V_C - V_B = 0 \qquad V = -\frac{Pa}{L} \blacktriangleleft$$

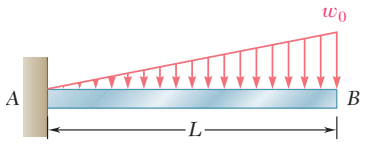
$$M_C - M_B = \int_a^L V\,dx = -\frac{Pa}{L}(L - a) = -\frac{Pab}{L}$$

$$M_C = M_B - \frac{Pab}{L} = \frac{Pba}{L} - \frac{Pab}{L} = 0$$

$$|M|_{\max} = \frac{Pab}{L} \blacktriangleleft$$

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PROBLEM 5.36

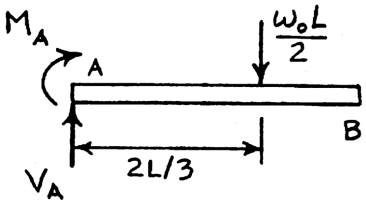


Using the method of Sec. 5.2, solve Prob. 5.3*a*.

PROBLEM 5.3 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Free body diagram for determining reactions.

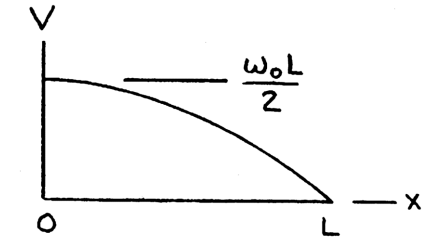


Reactions:

$$+\uparrow \Sigma F_y = 0: \quad V_A - \frac{w_0 L}{2} = 0 \qquad V_A = \frac{w_0 L}{2}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - \left(\frac{w_0 L}{2}\right)\left(\frac{2L}{3}\right) = 0$$

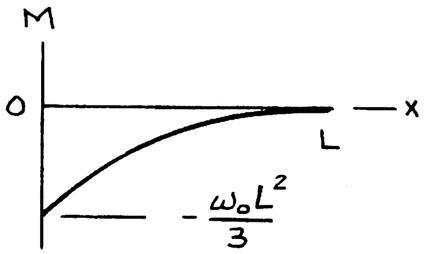
$$M_A = -\frac{w_0 L^2}{3}$$



$$w = w_0 \frac{x}{L}, \quad V_A = \frac{w_0 L}{2}, \quad M_A = -\frac{w_0 L^2}{3}$$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$V - V_A = -\int_0^x \frac{w_0 x}{L} dx = -\frac{w_0 x^2}{2L}$$



$$V = \frac{w_0 L}{2} - \frac{w_0 x^2}{2L} \blacktriangleleft$$

$$\frac{dM}{dx} = V = \frac{w_o L}{2} - \frac{w_o x^2}{2L}$$

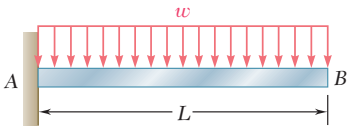
$$M - M_A = \int_0^x V\,dx = \int_0^x \left(\frac{w_0 L}{2} - \frac{w_0 x^2}{2L}\right) dx$$

$$= \frac{w_0 L}{2} x - \frac{w_0 x^3}{6L}$$

$$M = -\frac{w_0 L^2}{3} + \frac{w_0 L}{2} x - \frac{w_0 x^3}{6L} \blacktriangleleft$$

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PROBLEM 5.37



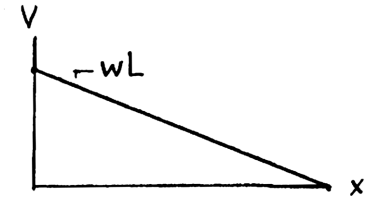
Using the method of Sec. 5.2, solve Prob. 5.4*a*.

PROBLEM 5.4 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

$$+\uparrow \Sigma F_y = 0: \quad V_A - wL = 0 \qquad V_A = wL$$

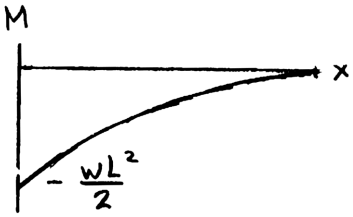
$$+\circlearrowleft \Sigma M_A = 0: \quad -M - (wL)\left(\frac{L}{2}\right) = 0 \qquad M_A = -\frac{wL^2}{2}$$



$$\frac{dV}{dx} = -w$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = wL - wx \blacktriangleleft$$



$$\frac{dM}{dx} = V = wL - wx$$

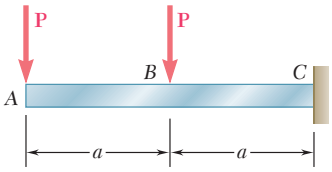
$$M - M_A = -\int_0^x (wL - wx)dx = wLx - \frac{wx^2}{2}$$

$$M = -\frac{wL^2}{2} + wLx - \frac{wx^2}{2} \blacktriangleleft$$

$$|V|_{\max} = wL$$

$$|M|_{\max} = \frac{wL^2}{2}$$

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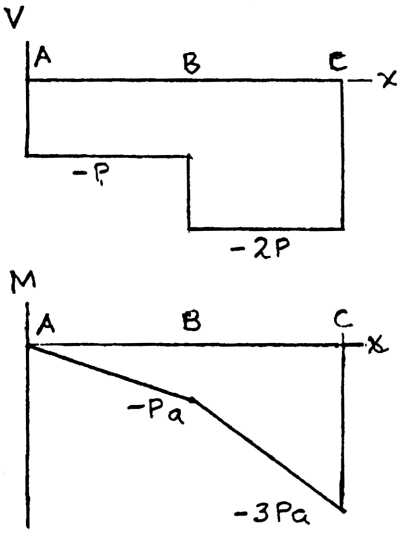


PROBLEM 5.38

Using the method of Sec. 5.2, solve Prob. 5.5*a*.

PROBLEM 5.5 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



At A^+: $V_A = -P$

Over AB: $\dfrac{dV}{dx} = -w = 0$

$$\frac{dM}{dx} = V = V_A = -P \blacktriangleleft$$

$$M = -Px + C$$

$M = 0$ at $x = 0$ $C_1 = 0$

$$M = -Px \blacktriangleleft$$

At point B: $x = a$ $M = -Pa$

At point B^+: $V = -P - P = -2P$

Over BC: $\dfrac{dV}{dx} = -w = 0$

$$\frac{dM}{dx} = V = -2P \blacktriangleleft$$

$$M = -2Px + C_2$$

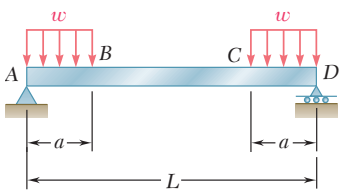
At B: $x = a$ $M = -Pa$

$-Pa = -2Pa + C_2$ $C_2 = Pa$

$$M = -2Px + Pa \blacktriangleleft$$

At C: $x = 2a$ $M = -3Pa$

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PROBLEM 5.39

Using the method of Sec. 5.2, solve Prob. 5.6*a*.

PROBLEM 5.6 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

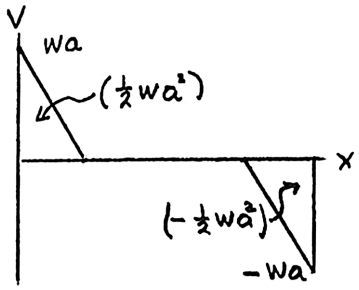
Reactions: $A = D = wa$

A to *B*: $0 < x < a \qquad w = w$

$$V_A = A = wa, \qquad M_A = 0$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

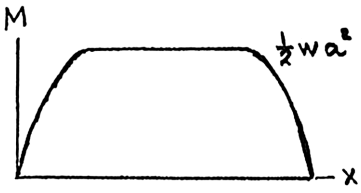
$$V = w(a - x) \blacktriangleleft$$



$$\frac{dM}{dx} = V = wa - wx$$

$$M - M_A = \int_0^x V\,dx = \int_0^x (wa - wx)dx$$

$$M = wax - \frac{1}{2}wx^2 \blacktriangleleft$$



$$V_B = 0 \qquad M_B = \frac{1}{2}wa^2$$

B to *C*: $a < x < L - a$ $\qquad V = 0 \blacktriangleleft$

$$\frac{dM}{dx} = V = 0$$

$$M - M_B = \int_a^x V\,dx = 0$$

$$M = M_B \qquad M = \frac{1}{2}wa^2 \blacktriangleleft$$

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PROBLEM 5.39 ***(Continued)***

$$L - a < x < L$$

C to D: $$V - V_C = -\int_{L-a}^{x} w\,dx = -w[x - (L - a)]$$

$$V = w[L - x - a)] \blacktriangleleft$$

$$M - M_C = \int_{L-a}^{x} V\,dx = \int_{L-a}^{x} -w[x - (L - a)]dx$$

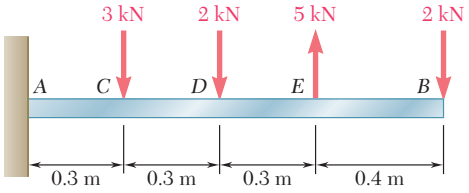
$$= -w\left[\frac{x^2}{2} - (L - a)x\right]\Bigg|_{L-a}^{x}$$

$$= -w\left[\frac{x^2}{2} - (L - a)x - \frac{(L - a)^2}{2} + (L - a)^2\right]$$

$$= -w\left[\frac{x^2}{2} - (L - a)x + \frac{(L - a)^2}{2}\right]$$

$$M = \frac{1}{2}wa^2 - w\left[\frac{x^2}{2} - (L - a)x + \frac{(L - a)^2}{2}\right] \blacktriangleleft$$

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PROBLEM 5.40

Using the method of Sec. 5.2, solve Prob. 5.7.

PROBLEM 5.7 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

Free body diagram for determining reactions.

Reactions:

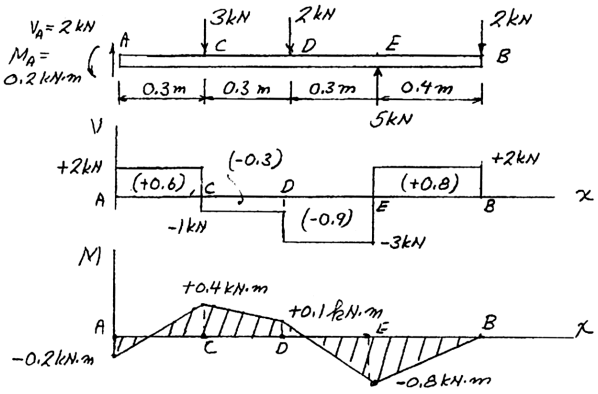
$$+\uparrow \Sigma F_y = 0: \quad V_A - 3\,\text{kN} - 2\,\text{kN} + 5\,\text{kN} + 2\,\text{kN} = 0$$

$$V_A = 2\,\text{kN}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (3\,\text{kN})(0.3\,\text{m}) - (2\,\text{kN})(0.6\,\text{m}) + (5\,\text{kN})(0.9\,\text{m}) - (2\,\text{kN})(1.3\,\text{m}) = 0$$

$$M_A = 0.2\,\text{kN}\cdot\text{m}$$

Between concentrated loads and the vertical reaction, the scope of the shear diagram is ϕ, i.e., the shear is constant. Thus, the area under the shear diagram is equal to the change in bending moment.



A to *C*:

$$V = 2\,\text{kN} \quad M_C - M_A = +0.6 \quad M_C = +0.4\,\text{kN}$$

C to *D*:

$$V = -1\,\text{kN} \quad M_D - M_C = -0.3 \quad M_D = +0.1\,\text{kN}\cdot\text{m}$$

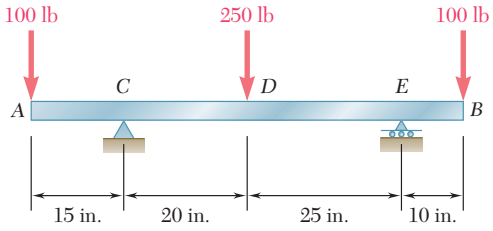
D to *E*:

$$V = -3\,\text{kN} \quad M_E - M_D = -0.9 \quad M_E = -0.8\,\text{kN}\cdot\text{m}$$

E to *B*:

$$V = +2\,\text{kN} \quad M_B - M_E = +0.8 \quad M_B = 0 \text{ (Checks)}$$

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PROBLEM 5.41

Using the method of Sec. 5.2, solve Prob. 5.8.

PROBLEM 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

Free body diagram for determining reactions.

Reactions:

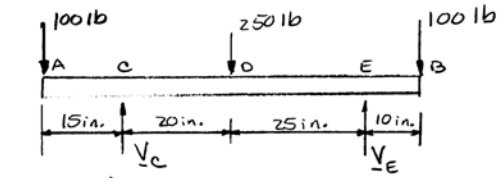
$$+\uparrow \Sigma F_Y = 0: \quad V_C + V_E - 100\text{ lb} - 250\text{ lb} - 100\text{ lb} = 0$$

$$V_C = V_E = 450\text{ lb}$$

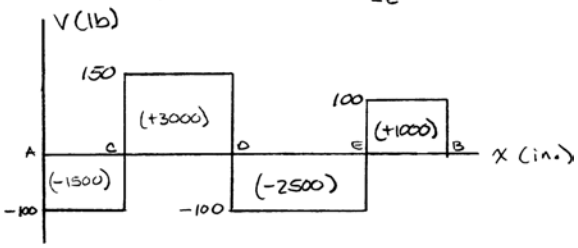
$$+\circlearrowleft \Sigma M_C = 0: \quad V_E(45\text{ in.}) + (100\text{ lb})(15\text{ in.}) - (250\text{ lb})(20\text{ in.}) - (100\text{ lb})(55\text{ in.}) = 0$$

$$V_E = 200\text{ lb}$$

$$\therefore \quad V_C = 250\text{ lb}$$



Between concentrated loads and the vertical reaction, the scope of the shear diagram is ϕ, i.e., the shear is constant. Thus, the area under the shear diagram is equal to he change in bending moment.



A to *C*:

$$V = -100\text{ lb}, \quad M_C - M_A = -1500, \quad M_C = -1500\text{ lb} \cdot \text{in.}$$

C to *D*:

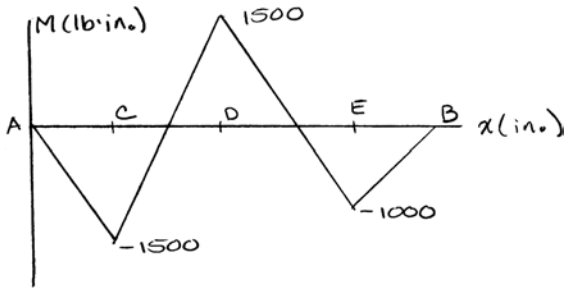
$$V = 150\text{ lb} \quad M_D - M_C = +3000, \quad M_D = +1500\text{ lb} \cdot \text{in.}$$

D to *E*:

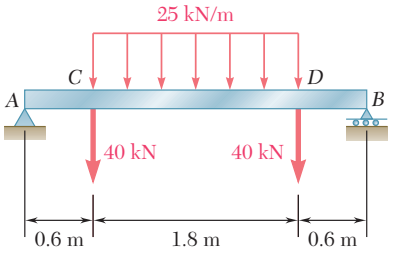
$$V = -100\text{ lb}, \quad M_E - M_D = -2500, \quad M_E = -1000\text{ lb} \cdot \text{in.}$$

E to *B*:

$$V = 100\text{ lb}, \quad M_B - M_E = +1000, \quad M_B = 0 \text{ (Checks)}$$



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PROBLEM 5.42

Using the method of Sec. 5.2, solve Prob. 5.9.

PROBLEM 5.9 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

Free body diagram to determine reactions:

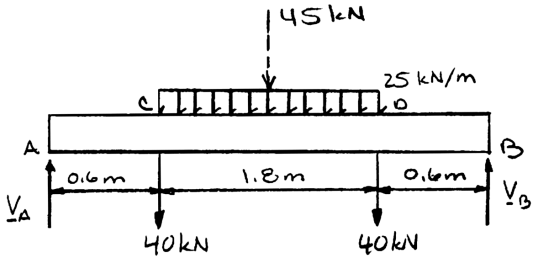
$+\circlearrowleft \Sigma M_A = 0$:

$$V_B(3.0\text{ m}) - 45\text{ kN}(1.5\text{ m}) - (40\text{ kN})(0.6\text{ m}) - (40\text{ kN})(2.4\text{ m}) = 0$$

$$V_B = 62.5\text{ kN}$$

$+\uparrow \Sigma F_y = 0$: $V_A - 40\text{ kN} - 45\text{ kN} - 40\text{ kN} + 62.5\text{ kN} = 0$

$$V_A = 62.5\text{ kN}$$



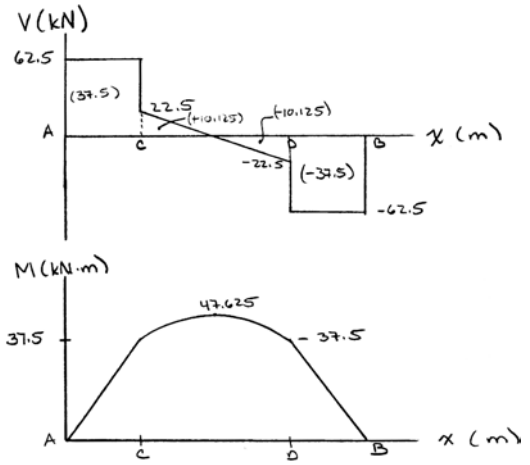
Change in bending moment is equal to area under shear diagram.

A to *C*: $(62.5\text{ kN})(0.6\text{ m}) = 37.5\text{ kN}\cdot\text{m}$

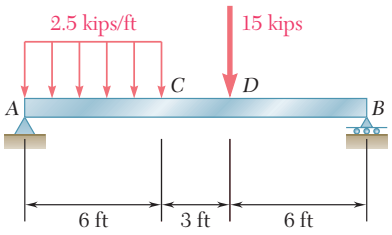
C to *E*: $\frac{1}{2}(0.9\text{ m})(22.5\text{ kN}) = 10.125\text{ kN}\cdot\text{m}$

E to *D*: $\frac{1}{2}(0.9\text{ m})(-22.5\text{ kN}) = -10.125\text{ kN}\cdot\text{m}$

D to *B*: $(-62.5\text{ kN})(0.6\text{ m}) = -37.5\text{ kN}\cdot\text{m}$



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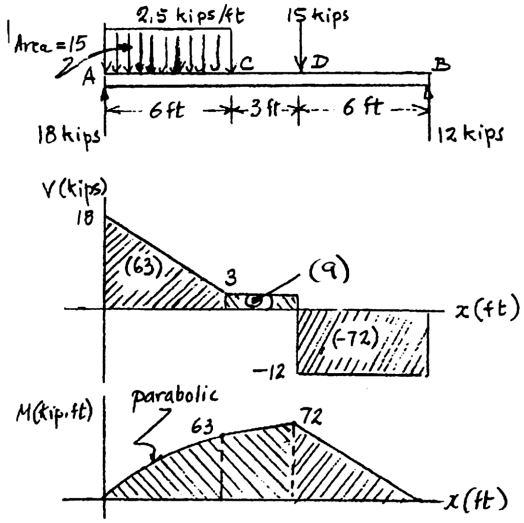


PROBLEM 5.43

Using the method of Sec. 5.2, solve Prob. 5.10.

PROBLEM 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



Reactions at supports *A* and *B*:

$$+\circlearrowleft \Sigma M_B = 0: \quad -15(R_A) + (12)(6)(2.5) + (6)(15) = 0$$

$$R_A = 18 \text{ kips}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 15R_B - (3)(6)(2.5) - (9)(15) = 0$$

$$R_B = 12 \text{ kips}$$

Areas under shear diagram:

A to *C*: $(6)(3) + \frac{1}{2}(6)(15) = 63 \text{ kip} \cdot \text{ft}$

C to *D*: $(3)(3) = 9 \text{ kip} \cdot \text{ft}$

D to *B*: $(6)(-12) = -72 \text{ kip} \cdot \text{ft}$

Bending moments:

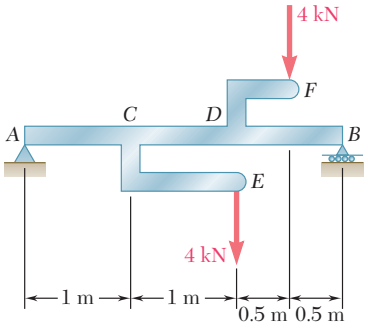
$$M_A = 0$$

$$M_C = 0 + 63 = 63 \text{ kip} \cdot \text{ft}$$

$$M_D = 63 + 9 = 72 \text{ kip} \cdot \text{ft}$$

$$M_B = 72 - 72 = 0$$

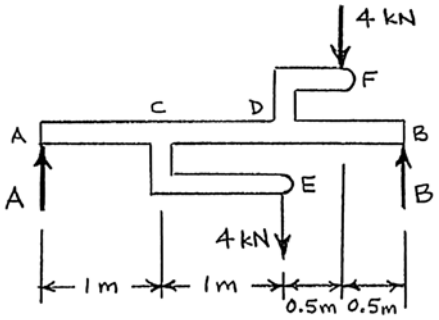
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PROBLEM 5.44

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0:$$

$$-3A + (1)(4) + (0.5)(4) = 0$$

$$\boldsymbol{A} = 2\ \text{kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: 3B - (2)(4) - (2.5)(4) = 0$$

$$\boldsymbol{B} = 6\ \text{kN} \uparrow$$

Shear diagram:

A to C: $V = 2$ kN

C to D: $V = 2 - 4 = -2$ kN

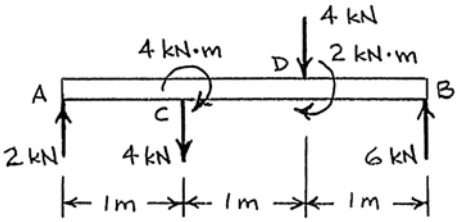
D to B: $V = -2 - 4 = -6$ kN

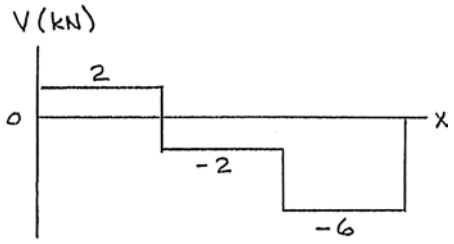
Areas of shear diagram:

A to C: $\int V\,dx = (1)(2) = 2\ \text{kN} \cdot \text{m}$

C to D: $\int V\,dx = (1)(-2) = -2\ \text{kN} \cdot \text{m}$

D to E: $\int V\,dx = (1)(-6) = -6\ \text{kN} \cdot \text{m}$





Bending moments:

$M_A = 0$

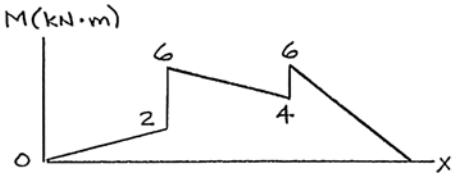
$M_{C^-} = 0 + 2 = 2\ \text{kN} \cdot \text{m}$

$M_{C^+} = 2 + 4 = 6\ \text{kN} \cdot \text{m}$

$M_{D^-} = 6 - 2 = 4\ \text{kN} \cdot \text{m}$

$M_{D^+} = 4 + 2 = 6\ \text{kN} \cdot \text{m}$

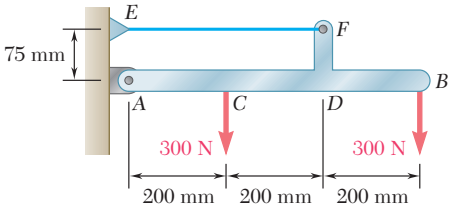
$M_B = 6 - 6 = 0$



(*a*) $|V|_{\max} = 6.00\ \text{kN}$ ◀

(*b*) $|M|_{\max} = 6.00\ \text{kN} \cdot \text{m}$ ◀

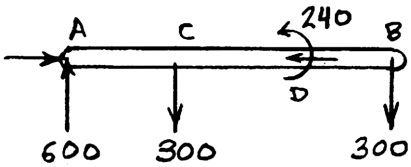
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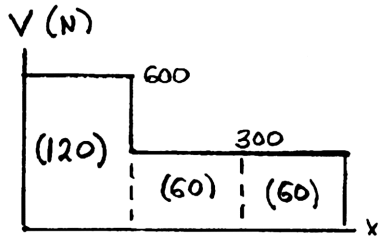


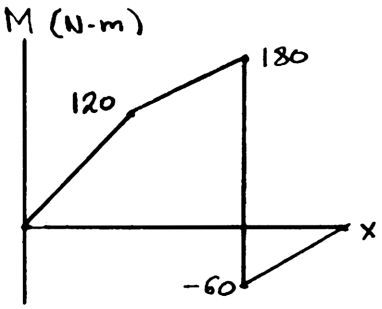
PROBLEM 5.45

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION







$+\circlearrowleft \Sigma M_A = 0$:

$$0.075\,F_{EF} - (0.2)(300) - (0.6)(300) = 0$$

$$F_{EF} = 3.2\times10^3 \text{ N}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0$: $A_x - F_{EF} = 0$ $A_x = 3.2\times10^3$ N

$+\uparrow \Sigma F_y = 0$: $A_y - 300 - 300 = 0$

$$A_y = 600 \text{ N}$$

Couple at D: $M_D = (0.075)(3.2\times10^3)$

$$= 240 \text{ N}\cdot\text{m}$$

Shear:

A to C: $V = 600$ N

C to B: $V = 600 - 300 = 300$ N

Areas under shear diagram:

A to C: $\int V\,dx = (0.2)(600) = 120 \text{ N}\cdot\text{m}$

C to D: $\int V\,dx = (0.2)(300) = 60 \text{ N}\cdot\text{m}$

D to B: $\int V\,dx = (0.2)(300) = 60 \text{ N}\cdot\text{m}$

Bending moments:

$M_A = 0$

$M_C = 0 + 120 = 120 \text{ N}\cdot\text{m}$

$M_{D^-} = 120 + 60 = 180 \text{ N}\cdot\text{m}$

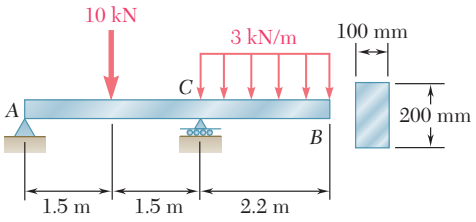
$M_{D^+} = 180 - 240 = -60 \text{ N}\cdot\text{m}$

$M_B = -60 + 60 = 0$

Maximum $|V| = 600$ N ◀

Maximum $|M| = 180.0 \text{ N}\cdot\text{m}$ ◀

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PROBLEM 5.46

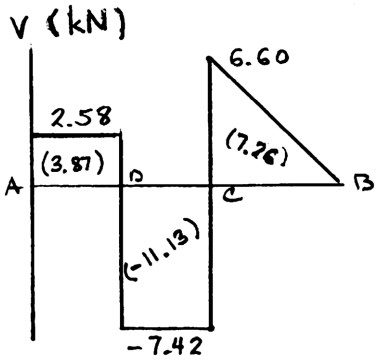
Using the method of Sec. 5.2, solve Prob. 5.15.

PROBLEM 5.15 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0:$$
$$-3A + (1.5)(10) - (1.1)(2.2)(3) = 0$$
$$A = 2.58 \text{ kN}$$

$$+\circlearrowleft \Sigma M_A = 0:$$
$$-(1.5)(10) + 3C - (4.1)(2.2)(3) = 0$$
$$C = 14.02 \text{ kN}$$



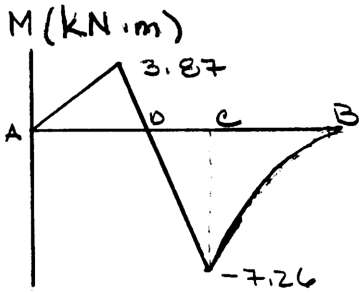
Shear:

A to D^-: $V = 2.58 \text{ kN}$

D^+ to C^-: $V = 2.58 - 10 = -7.42 \text{ kN}$

C^+: $V = -7.42 + 14.02 = 6.60 \text{ kN}$

B: $V = 6.60 - (2.2)(3) = 0$



Areas under shear diagram:

A to D: $\int V\,dx = (1.5)(2.58) = 3.87 \text{ kN} \cdot \text{m}$

D to C: $\int V\,dx = (1.5)(-7.42) = -11.13 \text{ kN} \cdot \text{m}$

C to B: $\int V\,dx = \left(\frac{1}{2}\right)(2.2)(6.60) = 7.26 \text{ kN} \cdot \text{m}$

Bending moments:

$$M_A = 0$$
$$M_D = 0 + 3.87 = 3.87 \text{ kN} \cdot \text{m}$$
$$M_C = 3.87 - 11.13 = -7.26 \text{ kN} \cdot \text{m}$$
$$M_B = 7.26 - 7.26 = 0$$
$$|M_C| = 7.26 \text{ kN} \cdot \text{m} = 7.26 \times 10^3 \text{ N} \cdot \text{m}$$

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PROBLEM 5.46 ***(Continued)***

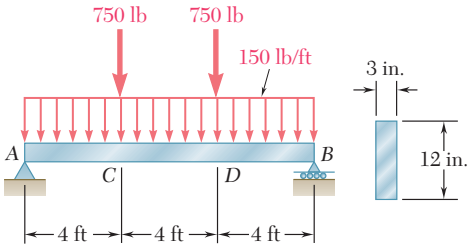
For rectangular cross section, $S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(100)(200)^2$

$= 666.67 \times 10^3 \text{ mm}^3 = 666.67 \times 10^{-6} \text{ m}^2$

Normal stress: $\sigma = \frac{|M_C|}{S} = \frac{7.26 \times 10^3}{666.67 \times 10^{-6}} = 10.89 \times 10^6 \text{ Pa}$

10.89 MPa ◀

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PROBLEM 5.47

Using the method of Sec. 5.2, solve Prob. 5.16.

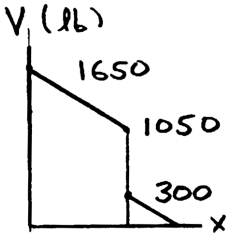
PROBLEM 5.16 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION

Reactions: $C = A$ by symmetry

$$+\uparrow \Sigma F_y = 0: \quad A + C - (2)(750) - (12)(150) = 0$$

$$A = C = 1650 \text{ lb}$$



Shear:

$$V_A = 1650 \text{ lb}$$

$$V_{C^-} = 1650 - (4)(150) = 1050 \text{ lb}$$

$$V_{C^+} = 1050 - 750 = 300 \text{ lb}$$

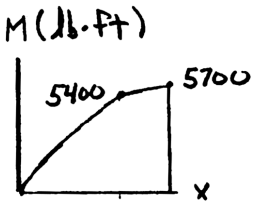
$$V_E = 300 - (2)(150) = 0$$

Areas under shear diagram:

A to C: $\int V\,dx = \left(\frac{1}{2}\right)(1650 + 1050)(4)$

$$= 5400 \text{ lb} \cdot \text{ft}$$

C to E: $\int V\,dx = \left(\frac{1}{2}\right)(300)(2) = 300 \text{ lb} \cdot \text{ft}$



Bending moments:

$$M_A = 0$$

$$M_C = 0 + 5400 = 5400 \text{ lb} \cdot \text{ft}$$

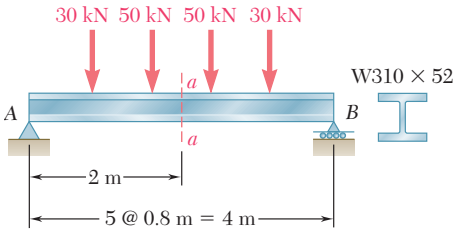
$$M_E = 5400 + 300 = 5700 \text{ lb} \cdot \text{ft}$$

$$M_E = 5700 \text{ lb} \cdot \text{ft} = 68.4 \times 10^3 \text{ lb} \cdot \text{in.}$$

For rectangular cross section, $S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(3)(12)^2 = 72 \text{ in}^3$

Normal stress: $\sigma = \frac{|M|}{S} = \frac{68.4 \times 10^3}{72} = 950 \text{ psi}$ ◀

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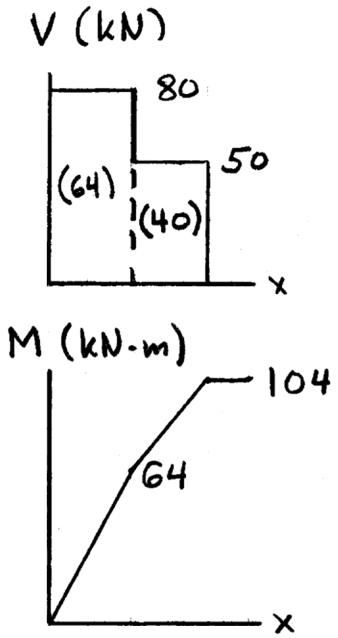


PROBLEM 5.48

Using the method of Sec. 5.2, solve Prob. 5.18.

PROBLEM 5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section *a-a*.

SOLUTION



Reactions: By symmetry, $A = B$.

$+\uparrow \sum F_y = 0$: $\boldsymbol{A} = \boldsymbol{B} = 80\ \text{kN} \uparrow$

Shear diagram:

A to C: $V = 80\ \text{kN}$

C to D: $V = 80 - 30 = 50\ \text{kN}$

D to E: $V = 50 - 50 = 0$

Areas of shear diagram:

A to C: $\int V\,dx = (80)(0.8) = 64\ \text{kN} \cdot \text{m}$

C to D: $\int V\,dx = (50)(0.8) = 40\ \text{kN} \cdot \text{m}$

D to E: $\int V\,dx = 0$

Bending moments:

$M_A = 0$

$M_C = 0 + 64 = 64\ \text{kN} \cdot \text{m}$

$M_D = 64 + 40 = 104\ \text{kN} \cdot \text{m}$

$M_E = 104 + 0 = 104\ \text{kN} \cdot \text{m}$

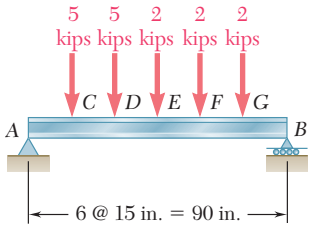
$$|M|_{\max} = 104\ \text{kN} \cdot \text{m} = 104 \times 10^3\ \text{N} \cdot \text{m}$$

For W310 × 52, $S = 747 \times 10^3\ \text{mm}^3 = 747 \times 10^{-6}\ \text{m}^3$

Normal stress: $$\sigma = \frac{|M|}{S} = \frac{104 \times 10^3}{747 \times 10^{-6}} = 139.2 \times 10^6\ \text{Pa}$$

$\sigma = 139.2\ \text{MPa}$ ◀

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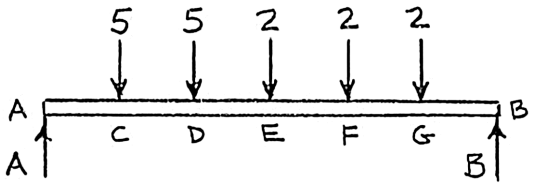


PROBLEM 5.49

Using the method of Sec. 5.2, solve Prob. 5.20.

PROBLEM 5.20 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION



Use entire beam as free body.

$+\circlearrowleft \Sigma M_B = 0$:

$$-90A + (75)(5) + (60)(5) + (45)(2) + (30)(2) + (15)(2) = 0$$

$$\boldsymbol{A} = 9.5 \text{ kips} \uparrow$$

Shear *A* to *C*: $V = 9.5$ kips

Area under shear curve *A* to *C*: $\int V\,dx = (15)(9.5)$

$= 142.5 \text{ kip} \cdot \text{in.}$

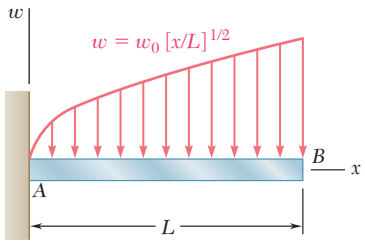
$M_A = 0$

$M_C = 0 + 142.5 = 142.5 \text{ kip} \cdot \text{in.}$

For S8 × 18.4, $S = 14.4 \text{ in}^3$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{142.5}{14.4}$ $\sigma = 9.90$ ksi ◀

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PROBLEM 5.50

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0\left(\frac{x}{L}\right)^{\frac{1}{2}} = -\frac{w_0 x^{1/2}}{L^{1/2}}$$

$$V = -\frac{2}{3}\frac{w_o x^{3/2}}{L^{1/2}} + C_1$$

$$V = 0 \quad \text{at} \quad x = L$$

$$0 = -\frac{2}{3}w_0 L + C_1 \qquad C_1 = \frac{2}{3}w_0 L$$

$$V = \frac{2}{3}w_0 L - \left(\frac{2}{3}\right)\frac{w_0 x^{3/2}}{L^{1/2}} \blacktriangleleft$$

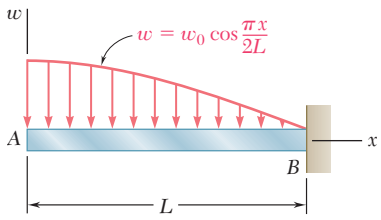
$$\frac{dM}{dx} = V \qquad M = C_2 + \frac{2}{3}w_0 L x - \frac{2}{3}\cdot\frac{2}{5}\frac{w_0 x^{5/2}}{L^{1/2}}$$

$$M = 0 \quad \text{at} \quad x = L \qquad 0 = C + \frac{2}{3}w_0 L^2 - \frac{4}{15}w_0 L^2 \qquad C_2 = -\frac{2}{5}w_0 L^2$$

$$M = \frac{2}{3}w_0 L x - \frac{4}{15}\frac{w_0 x^{5/2}}{L^{1/2}} - \frac{2}{5}w_0 L^2 \blacktriangleleft$$

$$|M|_{\max} \text{ occurs at } x = 0 \qquad |M|_{\max} = \frac{2}{5}w_0 L^2 \blacktriangleleft$$

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PROBLEM 5.51

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \cos\frac{\pi x}{2L}$$

$$V = -\frac{2Lw_0}{\pi}\sin\frac{\pi x}{2L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{4L^2 w_0}{\pi^2}\cos\frac{\pi x}{2L} + C_1 x + C_2$$

$V = 0$ at $x = 0$. Hence, $C_1 = 0$.

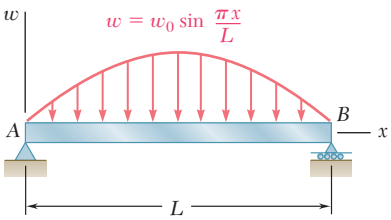
$M = 0$ at $x = 0$. Hence, $C_2 = -\frac{4L^2 w_0}{\pi^2}$.

(*a*) $V = -(2Lw_0/\pi)\sin(\pi x/2L)$ ◀

$M = -(4L^2 w_0/\pi^2)[1 - \cos(\pi x/2L)]$ ◀

(*b*) $|M|_{\max} = 4w_0 L^2/\pi^2$ ◀

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PROBLEM 5.52

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$$M = 0 \text{ at } x = 0 \qquad C_2 = 0$$

$$M = 0 \text{ at } x = L \qquad 0 = 0 + C_1 L + 0$$

$$C_1 = 0$$

(*a*)
$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} \blacktriangleleft$$

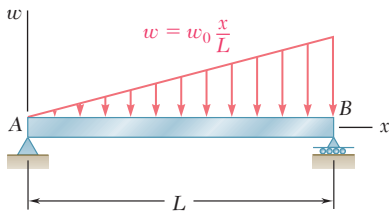
$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} \blacktriangleleft$$

$$\frac{dM}{dx} = V = 0 \quad \text{at} \quad x = \frac{L}{2}$$

(*b*)
$$M_{\max} = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi}{2}$$

$$M_{\max} = \frac{w_0 L^2}{\pi^2} \blacktriangleleft$$

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PROBLEM 5.53

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \frac{x}{L}$$

$$V = -\frac{1}{2} w_0 \frac{x^2}{L} + C_1 = \frac{dM}{dx}$$

$$M = -\frac{1}{6} w_0 \frac{x^3}{L} + C_1 x + C_2$$

$$M = 0 \quad \text{at} \quad x = 0 \qquad C_2 = 0$$

$$M = 0 \quad \text{at} \quad x = L \qquad 0 = -\frac{1}{6} w_0 L^2 + C_1 L \qquad C_1 = \frac{1}{6} w_0 L$$

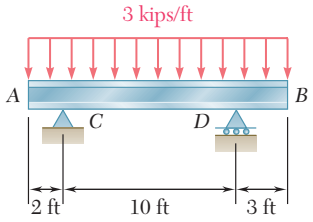
(*a*) $$V = -\frac{1}{2} w_0 \frac{x^2}{L} + \frac{1}{6} w_0 L^2 \qquad V = \frac{1}{6} w_0 (L^2 - 3x^2)/L \blacktriangleleft$$

$$M = -\frac{1}{6} w_0 \frac{x^3}{L} + \frac{1}{6} w_0 L x \qquad M = \frac{1}{6} w_0 (Lx - x^3/L) \blacktriangleleft$$

(*b*) $M_{\max}$ occurs when $\frac{dM}{dx} = V = 0.$ $L^2 - 3x_m^2 = 0$

$$x_m = \frac{L}{\sqrt{3}} \qquad M_{\max} = \frac{1}{6} w_0 \left(\frac{L^2}{\sqrt{3}} - \frac{L^2}{3\sqrt{3}} \right) \qquad M_{\max} = 0.0642 w_0 L^2 \blacktriangleleft$$

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S10 × 25.4

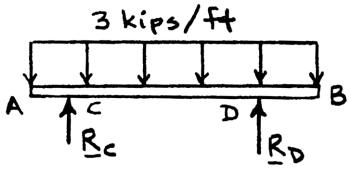
PROBLEM 5.54

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

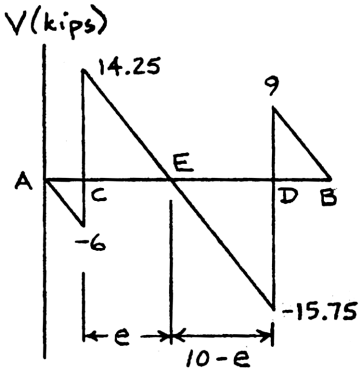
$+\circlearrowleft \Sigma M_C = 0: \quad R_D(10) - (5.5)(15)(3) = 0 \qquad R_D = 24.75 \text{ kips} \uparrow$

$+\circlearrowleft \Sigma M_D = 0: \quad (4.5)(15)(3) - R_C(10) = 0 \qquad R_C = 20.25 \text{ kips} \uparrow$



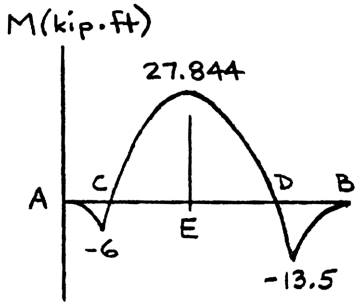
Shear:

$$V_A = 0$$
$$V_C = 0 - (2)(3) = -6 \text{ kips}$$
$$V_{C^+} = -6 + 20.50 = 14.25 \text{ kips}$$
$$V_{D^-} = 14.25 - (10)(3) = -15.75 \text{ kips}$$
$$V_{D^+} = -15.75 + 24.75 = 9 \text{ kips}$$
$$V_B = 9 - (3)(3) = 0$$



Locate point E where $V = 0$:

$$\frac{e}{14.25} = \frac{10 - e}{15.75} \qquad e = 4.75 \text{ ft} \qquad 10 - e = 5.25 \text{ ft}$$



Areas of shear diagram:

A to C: $\int V\,dx = \left(\frac{1}{2}\right)(2)(-6) = -6 \text{ kip} \cdot \text{ft}$

C to E: $\int V\,dx = \left(\frac{1}{2}\right)(4.75)(14.25) = 33.84375 \text{ kip} \cdot \text{ft}$

E to D: $\int V\,dx = \left(\frac{1}{2}\right)(5.25)(-15.75) = -41.34375 \text{ kip} \cdot \text{ft}$

D to B: $\int V\,dx = \left(\frac{1}{2}\right)(3)(9) = 13.5 \text{ kip} \cdot \text{ft}$

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PROBLEM 5.54 *(Continued)*

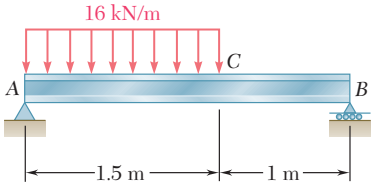
Bending moments:

$$M_A = 0$$
$$M_C = 0 - 6 = -6 \text{ kip} \cdot \text{ft}$$
$$M_E = -6 + 33.84375 = 27.84375 \text{ kip} \cdot \text{ft}$$
$$M_D = 27.84375 - 41.34375 = -13.5 \text{ kip} \cdot \text{ft}$$
$$M_B = -13.5 + 13.5 = 0$$

Maximum $|M| = 27.84375 \text{ kip} \cdot \text{ft} = 334.125 \text{ kip} \cdot \text{in.}$

For $\text{S10} \times 25.4$, $S = 24.6 \text{ in}^3$ Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{334.125}{24.6} = 13.58 \text{ ksi}$ ◀

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S150 × 18.6

PROBLEM 5.55

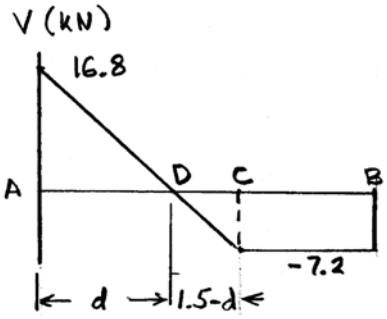
Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -2.5A + (1.75)(1.5)(16) = 0$$
$$A = 16.8 \text{ kN}$$
$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.75) + (1.5)(16) + 2.5B = 0$$
$$B = 7.2 \text{ kN}$$

Shear diagram:

$$V_A = 16.8 \text{ kN}$$
$$V_C = 16.8 - (1.5)(16) = -7.2 \text{ kN}$$
$$V_B = -7.2 \text{ kN}$$



Locate point D where $V = 0$.

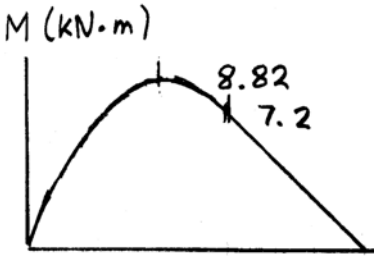
$$\frac{d}{16.8} = \frac{1.5 - d}{7.2} \qquad 24d = 25.2$$
$$d = 1.05 \text{ m} \quad 1.5 - d = 0.45 \text{ m}$$

Areas of the shear diagram:

A to D: $$\int V\,dx = \left(\frac{1}{2}\right)(1.05)(16.8) = 8.82 \text{ kN} \cdot \text{m}$$

D to C: $$\int V\,dx = \left(\frac{1}{2}\right)(0.45)(-7.2) = -1.62 \text{ kN} \cdot \text{m}$$

C to B: $$\int V\,dx = (1)(-7.2) = -7.2 \text{ kN} \cdot \text{m}$$



Bending moments:

$$M_A = 0$$
$$M_D = 0 + 8.82 = 8.82 \text{ kN} \cdot \text{m}$$
$$M_C = 8.82 - 1.62 = 7.2 \text{ kN} \cdot \text{m}$$
$$M_B = 7.2 - 7.2 = 0$$

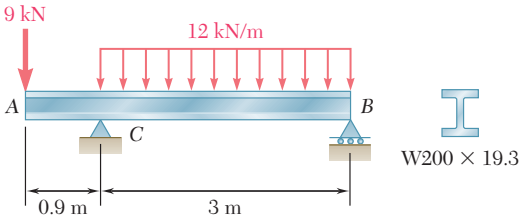
Maximum $|M| = 8.82 \text{ kN} \cdot \text{m} = 8.82 \times 10^3 \text{ N} \cdot \text{m}$

For S150×18.6 rolled-steel section, $S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

Normal stress: $$\sigma = \frac{|M|}{S} = \frac{8.82 \times 10^3}{120 \times 10^{-6}} = 73.5 \times 10^6 \text{ Pa}$$

$$\sigma = 73.5 \text{ MPa} \blacktriangleleft$$

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PROBLEM 5.56

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

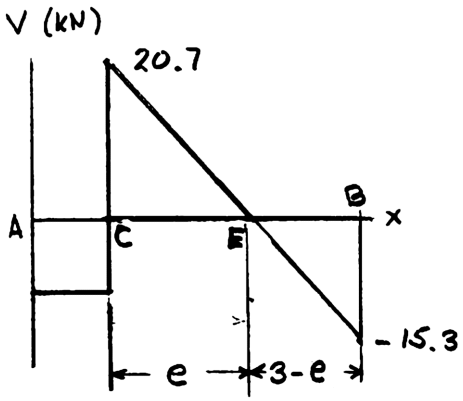
SOLUTION

$+\circlearrowleft \Sigma M_C = 0$: $(0.9)(9) - (1.5)(3)(12) + 3B = 0$

$$B = 15.3 \text{ kN}$$

$+\circlearrowleft \Sigma M_B = 0$: $(3.9)(9) - 3C + (1.5)(3)(12) = 0$

$$C = 29.7 \text{ kN}$$



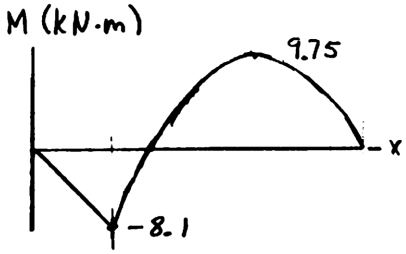
Shear:

A to C: $V = -9$ kN

C^+: $V = -9 + 29.7 = 20.7$ kN

B: $V = 20.7 - (3)(12) = -15.3$ kN

$$|V|_{\max} = 20.7 \text{ kN} \blacktriangleleft$$



Locate point E where $V = 0$.

$$\frac{e}{20.7} = \frac{3 - e}{15.3} \qquad 36e = (20.7)(3)$$

$e = 1.725$ ft $\qquad 3 - e = 1.275$ ft

Areas under shear diagram:

A to C: $\int V\,dx = (0.9)(9) = 8.1 \text{ kN} \cdot \text{m}$

C to E: $\int V\,dx = \left(\frac{1}{2}\right)(1.725)(20.7) = 17.8538 \text{ kN} \cdot \text{m}$

E to B: $\int V\,dx = \left(\frac{1}{2}\right)(-1.275)(15.3) = -9.7538 \text{ kN} \cdot \text{m}$

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PROBLEM 5.56 *(Continued)*

Bending moments:

$$M_A = 0$$
$$M_C = 0 - 8.1 = -8.1\,\text{kN} \cdot \text{m}$$
$$M_E = -8.1 + 17.8538 = 9.7538\,\text{kN} \cdot \text{m}$$
$$M_B = 9.7538 - 9.7538 = 0$$

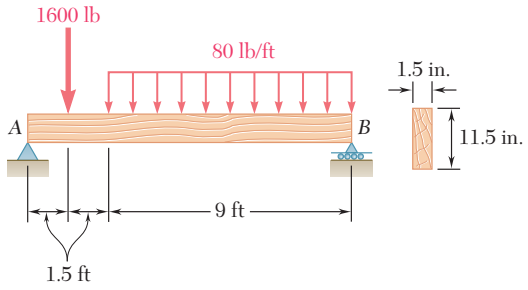
$$|M|_{\max} = 9.7538 \times 10^3\ \text{N} \cdot \text{m at point } E \blacktriangleleft$$

For $\text{W}200 \times 19.3$ rolled-steel section, $S = 162 \times 10^3\ \text{mm}^3 = 162 \times 10^{-6}\ \text{m}^3$

Normal stress: $$\sigma = \frac{|M|}{S} = \frac{9.7538 \times 10^3}{162 \times 10^{-6}} = 60.2 \times 10^6\ \text{Pa} = 60.2\ \text{MPa}$$

$$\sigma = 60.2\ \text{MPa} \blacktriangleleft$$

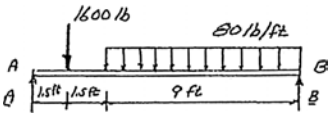
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PROBLEM 5.57

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION

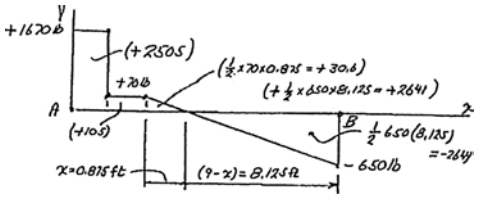


$$+\circlearrowleft \sum M_A = 0: \quad (1600\text{ lb})(1.5\text{ ft}) + [(80\text{ lb/ft})(9\text{ ft})](7.5\text{ ft}) - 12B = 0$$

$$B = +650\text{ lb} \qquad \mathbf{B} = 650\text{ lb} \uparrow$$

$$+\uparrow \sum F_y = 0: \quad A - 1600\text{ lb} - [(80\text{ lb/ft})(9\text{ ft})] + 650\text{ lb} = 0$$

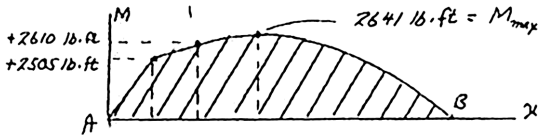
$$A = +1670\text{ lb} \qquad \mathbf{A} = 1670\text{ lb} \uparrow$$



$$\frac{x}{70\text{ lb}} = \frac{9 - x}{650\text{ lb}} \qquad x = 0.875\text{ ft} \qquad 2641\text{ lb} \cdot \text{ft} = M_{\max}$$

$$c = \frac{1}{2}(11.5\text{ in.}) = 5.75\text{ in.}$$

$$I = \frac{1}{12}(1.5\text{ in.})(11.5\text{ in.})^3 = 190.1\text{ in}^4$$



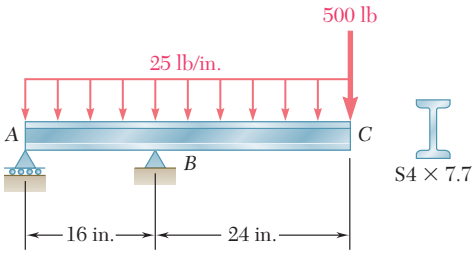
◀



$$M_{\max} = 2641\text{ lb} \cdot \text{ft} = 31{,}690\text{ lb} \cdot \text{in.}$$

$$\sigma_m = \frac{M_{\max} c}{I} = \frac{(31{,}690\text{ lb} \cdot \text{in.})(5.75\text{ in.})}{190.1\text{ in}^4} \qquad \sigma_m = 959\text{ psi}$$ ◀

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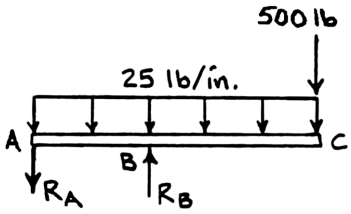
PROBLEM 5.58

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

$+\circlearrowleft \Sigma M_B = 0: \quad R_A(16) - (4)(40)(25) - (24)(500) = 0 \quad R_A = 1000 \text{ lb} \downarrow$

$+\circlearrowleft \Sigma M_A = 0: \quad R_B(16) - (20)(40)(25) - (40)(500) = 0 \quad R_B = 2500 \text{ lb} \uparrow$



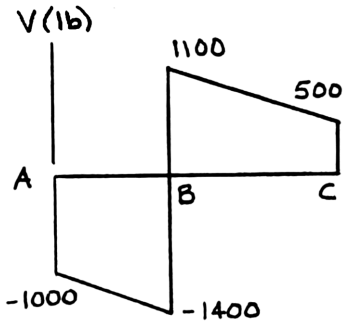
Shear:

$$V_A = -1000 \text{ lb}$$

$$V_B = -1000 - (16)(25) = -1400 \text{ lb}$$

$$V_{B^+} = -1400 + 2500 = 1100 \text{ lb}$$

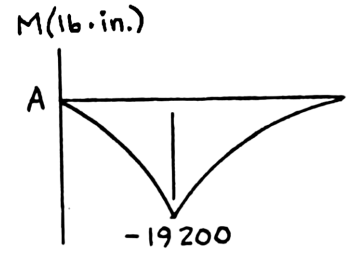
$$V_C = 1100 - (24)(25) = 500 \text{ lb}$$



Areas of shear diagram:

A to B: $\int V\,dx = \frac{1}{2}(-1000 - 1400)(16) = -19,200 \text{ lb} \cdot \text{in.}$

B to C: $\int V\,dx = \left(\frac{1}{2}\right)(1100 + 500)(24) = 19,200 \text{ lb} \cdot \text{in.}$



Bending moments:

$$M_A = 0$$

$$M_B = 0 - 19,200 = -19,200 \text{ lb} \cdot \text{in.}$$

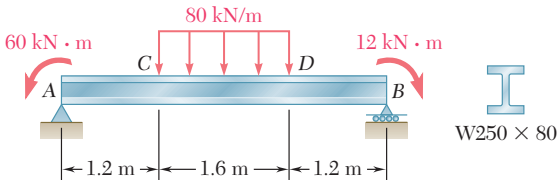
$$M_C = -19,200 + 19,200 = 0$$

Maximum $|M| = 19.2 \text{ kip} \cdot \text{in.}$

For $S4 \times 7.7$ rolled-steel section, $S = 3.03 \text{ in}^3$

Normal stress: $\sigma = \frac{|M|}{S} = \frac{19.2}{3.03} = 6.34 \text{ ksi}$ ◀

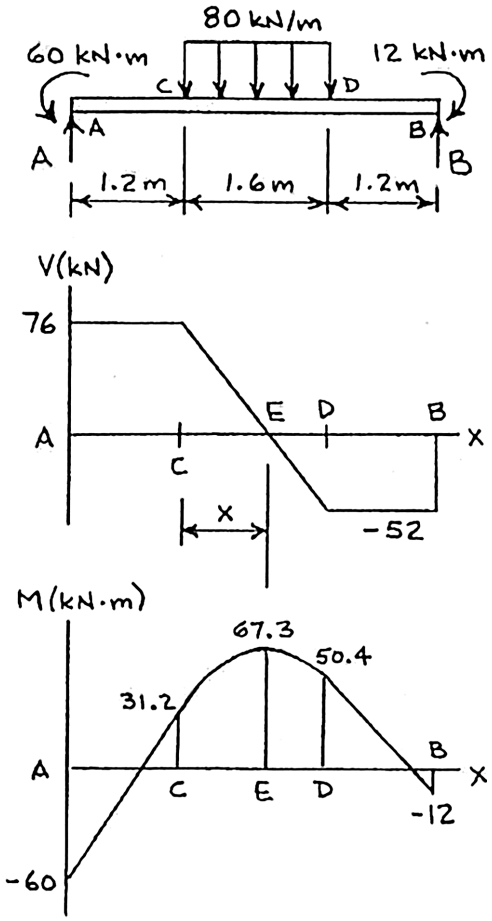
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PROBLEM 5.59

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION



Reaction:

$$+\circlearrowleft \Sigma M_B = 0: \quad -4A + 60 + (80)(1.6)(2) - 12 = 0$$

$$\mathbf{A} = 76 \text{ kN} \uparrow$$

Shear: $V_A = 76$ kN

A to C: $V = 76.0$ kN ◀

$$V_D = 76 - (80)(1.6) = -52 \text{ kN}$$

D to C: $V = -52$ kN

Locate point where $V = 0$.

$V(x) = -80x + 76 = 0 \quad x = 0.95$ m

Areas of shear diagram:

A to C: $\int V\,dx = (1.2)(76) = 91.2$ kN · m

C to E: $\int V\,dx = \frac{1}{2}(0.95)(76) = 36.1$ kN · m

E to D: $\int V\,dx = \frac{1}{2}(0.65)(-52) = -16.9$ kN · m

D to B: $\int V\,dx = (1.2)(-52) = -62.4$ kN · m

Bending moments: $M_A = -60$ kN · m

$M_C = -60 + 91.2 = 31.2$ kN · m

$M_E = 31.2 + 36.1 = 67.3$ kN · m ◀

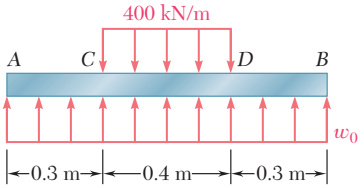
$M_D = 67.3 - 16.9 = 50.4$ kN · m

$M_B = 50.4 - 62.4 = -12$ kN · m

For W250 × 80, $S = 983 \times 10^3 \text{mm}^3$

Normal stress: $\sigma_{\max} = \dfrac{|M|}{S} = \dfrac{67.3 \times 10^3 \text{ N} \cdot \text{m}}{983 \times 10^{-6} \text{ m}^3} = 68.5 \times 10^6 \text{ Pa}$ $\sigma_m = 68.5$ MPa ◀

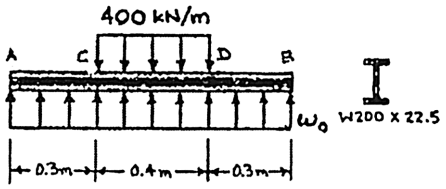
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PROBLEM 5.60

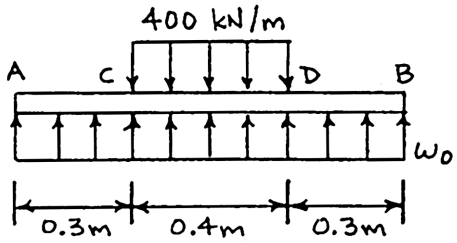
Knowing that beam *AB* is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION



$$+\uparrow \sum F_y = 0: \ (1)(w_0) - (0.4)(400) = 0$$

$$w_0 = 160 \text{ kN/m}$$

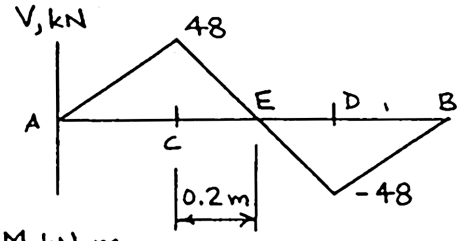


Shear diagram: $V_A = 0$

$$V_C = 0 + (0.3)(160) = 48 \text{ kN}$$

$$V_D = 48 - (0.3)(400) + (0.3)(160) = -48 \text{ kN}$$

$$V_B = -48 + (0.3)(160) = 0$$



Locate point *E* where $V = 0$.

By symmetry, *E* is the midpoint of *CD*.

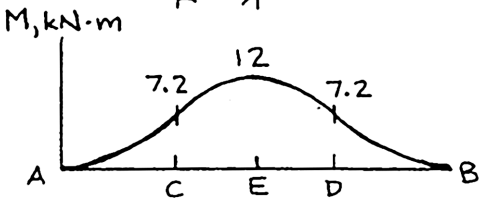
Areas of shear diagram:

$$A \text{ to } C: \quad \frac{1}{2}(0.3)(48) = 7.2 \text{ kN} \cdot \text{m}$$

$$C \text{ to } E: \quad \frac{1}{2}(0.2)(48) = 4.8 \text{ kN} \cdot \text{m}$$

$$E \text{ to } D: \quad \frac{1}{2}(0.2)(-48) = -4.8 \text{ kN} \cdot \text{m}$$

$$D \text{ to } B: \quad \frac{1}{2}(0.3)(-48) = -7.2 \text{ kN} \cdot \text{m}$$



Bending moments: $M_A = 0$

$$M_C = 0 + 7.2 = 7.2 \text{ kN}$$

$$M_E = 7.2 + 4.8 = 12.00 \text{ kN}$$

$$M_D = 12.0 - 4.8 = 7.2 \text{ kN}$$

$$M_B = 7.2 - 7.2 = 0$$

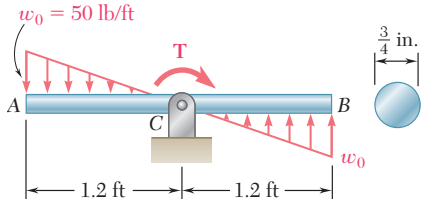
$$|M|_{\max} = 12.00 \text{ kN} \cdot \text{m} = 12.00 \times 10^3 \text{ N} \cdot \text{m}$$

For W200 × 22.5 rolled-steel shape, $S_x = 193 \times 10^3 \text{ mm}^3 = 193 \times 10^{-6} \text{ m}^3$

Normal stress: $$\sigma = \frac{|M|}{S} = \frac{12.00 \times 10^3}{193 \times 10^{-6}} = 62.2 \times 10^6 \text{ Pa}$$

$$\sigma = 62.2 \text{ MPa} \blacktriangleleft$$

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PROBLEM 5.61

Knowing that beam *AB* is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION

A to *C*: $0 < x < 1.2$ ft

$$w = 50\left(1 - \frac{x}{1.2}\right) = 50 - 41.667x$$

$$\frac{dV}{dx} = -w = 41.667x - 50$$

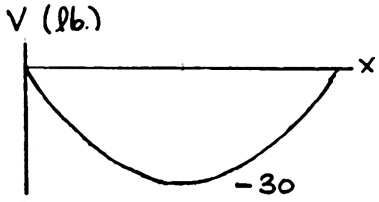
$$V = V_A + \int_0^x (41.667x - 50)dx$$

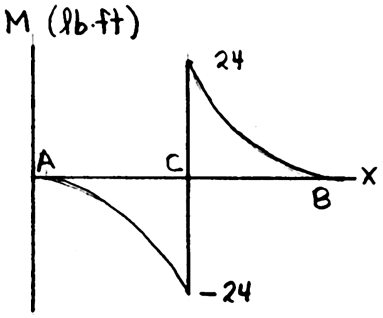
$$= 0 + 20.833x^2 - 50x = \frac{dM}{dx}$$

$$M = M_A + \int_0^x V\,dx$$

$$= 0 + \int_0^x (20.833x^2 - 50x)dx$$

$$= 6.944x^3 - 25x^2$$





At *C* to *B*, use symmetry conditions. $x = 1.2$ ft, $V = -30.0$ lb

$M = -24.0$ lb · in.

Maximum $|M| = 24.0$ lb · ft $= 288$ lb · in.

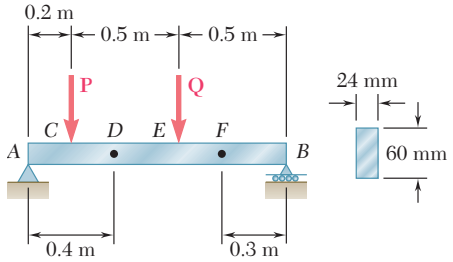
Cross section: $$c = \frac{d}{2} = \left(\frac{1}{2}\right)(0.75) = 0.375 \text{ in.}$$

$$I = \frac{\pi}{4}c^4 = \left(\frac{\pi}{4}\right)(0.375) = 15.532 \times 10^{-3} \text{in}^4$$

Normal stress: $$\sigma = \frac{|M|c}{I} = \frac{(2.88)(0.375)}{15.532 \times 10^{-3}} = 6.95 \times 10^3 \text{ psi}$$

$\sigma = 6.95$ ksi ◀

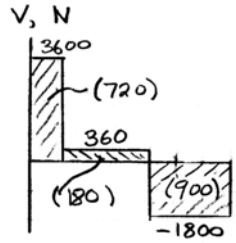
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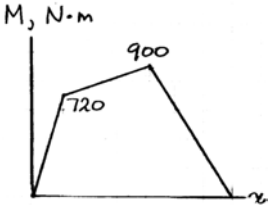


PROBLEM 5.62*

The beam *AB* supports two concentrated loads **P** and **Q**. The normal stress due to bending on the bottom edge of the beam is +55 MPa at *D* and +37.5 MPa at *F*. (*a*) Draw the shear and bending-moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending that occurs in the beam.

SOLUTION





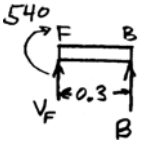
(*a*) $I = \frac{1}{12}(24)(60)^3 = 432\times10^3\ \text{mm}^4 \qquad c = 30\ \text{mm}$

$$S = \frac{I}{c} = 14.4\times10^3\ \text{mm}^3 = 14.4\times10^{-6}\ \text{m}^3 \qquad M = S\sigma$$

At *D*, $M_D = (14.4\times10^{-6})(55\times10^6) = 792\ \text{N}\cdot\text{m}$

At *F*, $M_F = (14.4\times10^{-6})(37.5\times10^6) = 540\ \text{N}\cdot\text{m}$

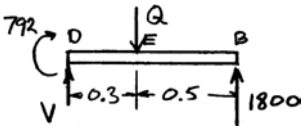
Using free body *FB*,



$$+\circlearrowleft \Sigma M_F = 0: \quad -540 + 0.3B = 0$$

$$B = \frac{540}{0.3} = 1800\ \text{N}$$

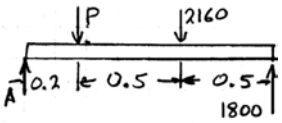
Using free body *DEFB*,



$$+\circlearrowleft \Sigma M_D = 0: \quad -792 - 3Q + (0.8)(1800) = 0$$

$$Q = 2160\ \text{N}$$

Using entire beam,



$$+\circlearrowleft \Sigma M_A = 0: \quad -0.2P - (0.7)(2160) + (1.2)(1800) = 0$$

$$P = 3240\ \text{N}$$

$$+\uparrow \Sigma F_y = 0: \quad A - 3240 - 2160 + 1800 = 0$$

$$A = 3600\ \text{N}$$

Shear diagram and its areas:

A to C^-:	$V = 3600\ \text{N}$	$A_{AC} = (0.2)(3600) = 720\ \text{N}\cdot\text{m}$
C^+ to E^-:	$V = 3600 - 3240 = 360\ \text{N}$	$A_{CE} = (0.5)(360) = 180\ \text{N}\cdot\text{m}$
E^+ to *B*:	$V = 360 - 2160 = -1800\ \text{N}$	$A_{EB} = (0.5)(-1800) = -900\ \text{N}\cdot\text{m}$

Bending moments:

$M_A = 0$

$M_C = 0 + 720 = 720\ \text{N}\cdot\text{m}$

$M_E = 720 + 180 = 900\ \text{N}\cdot\text{m}$

$M_B = 900 - 900 = 0$

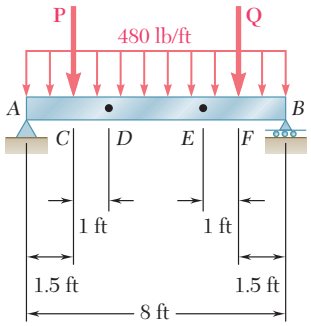
$|M|_{\max} = 900\ \text{N}\cdot\text{m}$

(*b*) Normal stress:

$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{900}{14.4\times10^{-6}} = 62.5\times10^6\ \text{Pa}$$

$\sigma_{\max} = 62.5\ \text{MPa}$ ◀

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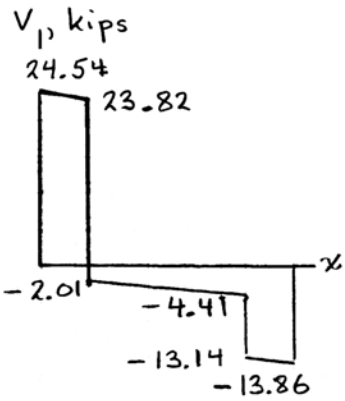


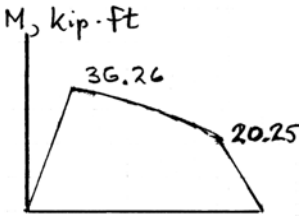


PROBLEM 5.63*

The beam *AB* supports a uniformly distributed load of 480 lb/ft and two concentrated loads **P** and **Q**. The normal stress due to bending on the bottom edge of the lower flange is +14.85 ksi at *D* and +10.65 ksi at *E*. (*a*) Draw the shear and bending-moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending that occurs in the beam.

SOLUTION





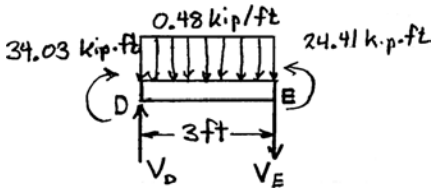
(*a*) For W8 × 31 rolled-steel section, $S = 27.5\ \text{in}^3$

$$M = S\sigma$$

At *D*, $M_D = (27.5)(14.85) = 408.375\ \text{kip} \cdot \text{in.}$

At *E*, $M_E = (27.5)(10.65) = 292.875\ \text{kip} \cdot \text{in.}$

$$M_D = 34.03\ \text{kip} \cdot \text{ft} \quad M_E = 24.41\ \text{kip} \cdot \text{ft}$$



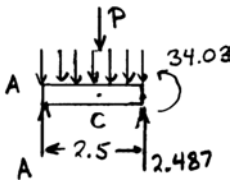
Use free body *DE*.

$$+\circlearrowleft \Sigma M_E = 0: \quad -34.03 + 24.41 + (1.5)(3)(0.48) - 3V_D = 0$$

$$V_D = -2.487\ \text{kips}$$

$$+\circlearrowleft M_D = 0: \quad -34.03 + 24.41 - (1.5)(3)(0.48) - 3V_E = 0$$

$$V_E = -3.927\ \text{kips}$$



Use free body *ACD*.

$$+\circlearrowleft \Sigma M_A = 0: \quad -1.5P - (1.25)(2.5)(0.48) + (2.5)(2.487) + 34.03 = 0$$

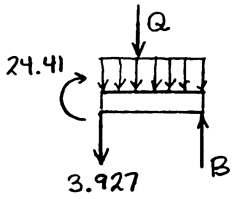
$$\mathbf{P} = 25.83\ \text{kips} \downarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A - (2.5)(0.48) + 2.487 - 25.83 = 0$$

$$\mathbf{A} = 24.54\ \text{kips} \uparrow$$

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PROBLEM 5.63* *(Continued)*



Use free body *EFB*.

$+\circlearrowleft \sum M_B = 0$: $1.5Q + (1.25)(2.5)(0.48) + (2.5)(3.927) - 24.41 = 0$

$Q = 8.728$ kips

$+\uparrow \sum F_y = 0$: $B - 3.927 - (2.5)(0.48) - 8.7 = 0$

$B = 13.855$ kips

Areas of load diagram:

A to *C*: $(1.5)(0.48) = 0.72$ kip · ft

C to *F*: $(5)(0.48) = 2.4$ kip · ft

F to *B:* $(1.5)(0.48) = 0.72$ kip · ft

Shear diagram: $V_A = 24.54$ kips

$V_C^- = 24.54 - 0.72 = 23.82$ kips

$V_C^+ = 23.82 - 25.83 = -2.01$ kips

$V_F^- = -2.01 - 2.4 = 4.41$ kips

$V_F^+ = -4.41 - 8.728 = -13.14$ kips

$V_B = -13.14 - 0.72 = -13.86$ kips

Areas of shear diagram:

A to *C*: $\frac{1}{2}(1.5)(24.52 + 23.82) = 36.23$ kip · ft

C to *F*: $\frac{1}{2}(5)(-2.01 - 4.41) = -16.05$ kip · ft

F to *B*: $\frac{1}{2}(1.5)(-13.14 - 13.86) = 20.25$ kip · ft

Bending moments: $M_A = 0$

$M_C = 0 + 36.26 = 36.26$ kip · ft

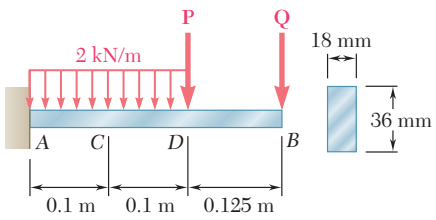
$M_F = 36.26 - 16.05 = 20.21$ kip · ft

$M_B = 20.21 - 20.25 \approx 0$

Maximum $|M|$ occurs at *C*: $|M|_{\max} = 36.26$ kip · ft $= 435.1$ kip · in.

(*b*) Maximum stress: $\sigma = \frac{|M|_{\max}}{S} = \frac{435.1}{27.5}$ $\sigma = 15.82$ ksi ◀

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PROBLEM 5.64*

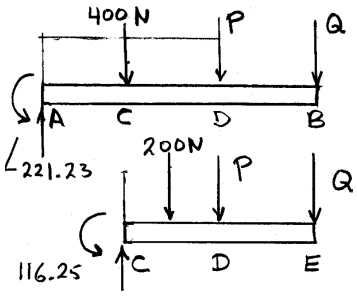
Beam *AB* supports a uniformly distributed load of 2 kN/m and two concentrated loads **P** and **Q**. It has been experimentally determined that the normal stress due to bending in the bottom edge of the beam is −56.9 MPa at *A* and −29.9 MPa at *C*. Draw the shear and bending-moment diagrams for the beam and determine the magnitudes of the loads **P** and **Q**.

SOLUTION

$$I = \frac{1}{12}(18)(36)^3 = 69.984\times10^3\ \text{mm}^4$$

$$c = \frac{1}{2}d = 18\ \text{mm}$$

$$S = \frac{I}{c} = 3.888\times10^3\ \text{mm}^3 = 3.888\times10^{-6}\ \text{m}^3$$



At *A*, $M_A = S\sigma_A = (3.888\times10^{-6})(-56.9) = -221.25\ \text{N}\cdot\text{m}$

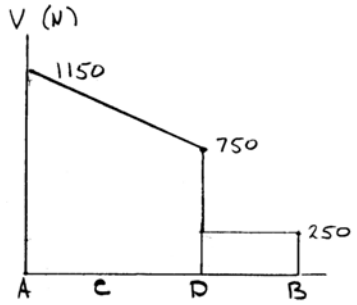
At *C*, $M_C = S\sigma_C = (3.888\times10^{-6})(-29.9) = -116.25\ \text{N}\cdot\text{m}$

$$+\circlearrowleft \Sigma M_A = 0: \quad 221.23 - (0.1)(400) - 0.2P - 0.325Q = 0$$

$$0.2P + 0.325Q = 181.25 \qquad (1)$$

$$+\circlearrowleft \Sigma M_C = 0: \quad 116.25 - (0.05)(200) - 0.1P - 0.225Q = 0$$

$$0.1P + 0.225Q = 106.25 \qquad (2)$$



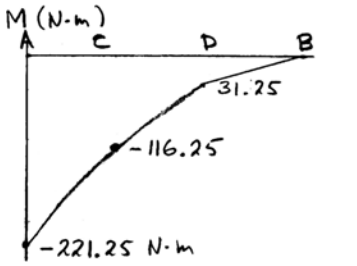
Solving (1) and (2) simultaneously, $P = 500\ \text{N}$ ◀

$Q = 250\ \text{N}$ ◀

Reaction force at *A*: $R_A - 400 - 500 - 250 = 0 \quad R_A = 1150\ \text{N}\cdot\text{m}$

$V_A = 1150\ \text{N} \quad V_D = 250$

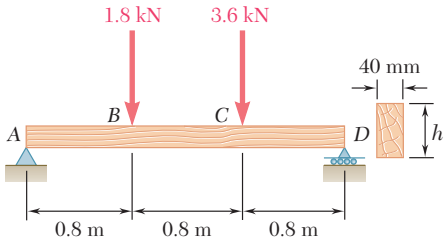
$M_A = -221.25\ \text{N}\cdot\text{m} \quad M_C = -116.25\ \text{N}\cdot\text{m} \quad M_D = -31.25\ \text{N}\cdot\text{m}$



$|V|_{\max} = 1150\ \text{N}$ ◀

$|M|_{\max} = 221\ \text{N}\cdot\text{m}$ ◀

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PROBLEM 5.65

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION

Reactions:

$$+\circlearrowleft \Sigma M_D = 0: \quad -2.4A + (1.6)(1.8) + (0.8)(3.6) = 0 \qquad A = 2.4 \text{ kN}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.8)(1.8) - (1.6)(3.6) + 2.4D = 0 \qquad D = 3 \text{ kN}$$

Construct shear and bending moment diagrams:

$$|M|_{\max} = 2.4 \text{ kN} \cdot \text{m} = 2.4 \times 10^3 \text{ N} \cdot \text{m}$$

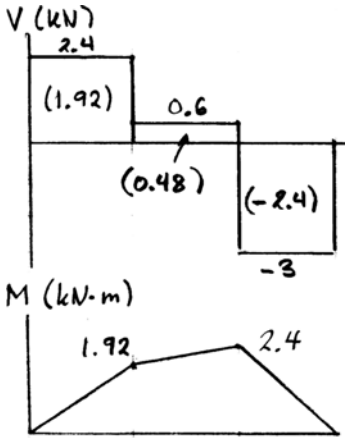
$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{2.4 \times 10^3}{12 \times 10^6} = 200 \times 10^{-6} \text{ m}^3 = 200 \times 10^3 \text{ mm}^3$$

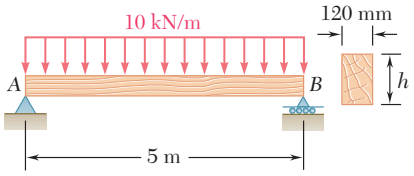
$$S = \frac{1}{6} bh^2 = \frac{1}{6}(40)h^2 = 200 \times 10^3$$

$$h^2 = \frac{(6)(200 \times 10^3)}{40} = 30 \times 10^3 \text{ mm}^2$$

$$h = 173.2 \text{ mm} \blacktriangleleft$$



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PROBLEM 5.66

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION

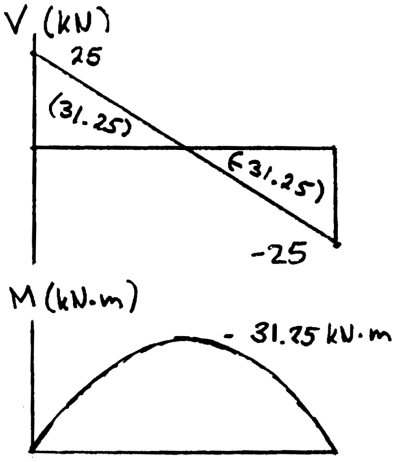
Reactions:

$A = B$ by symmetry

$$+\uparrow \Sigma F_y = 0: \quad A + B - (5)(10) = 0$$

$$A = B = 25 \text{ kN}$$

From bending moment diagram,



$$|M|_{\max} = 31.25 \text{ kN} \cdot \text{m} = 31.25 \times 10^3 \text{ N} \cdot \text{m}$$

$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{31.25 \times 10^3}{12 \times 10^6} = 2.604 \times 10^{-3} \text{ m}^3$$

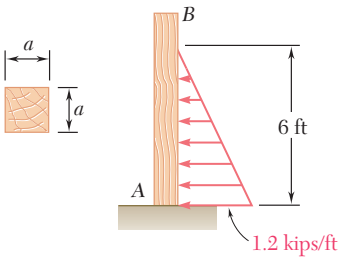
$$= 2.604 \times 10^6 \text{ mm}^3$$

$$S = \frac{1}{6} bh^2 = \left(\frac{1}{6}\right)(120)h^2 = 2.604 \times 10^6$$

$$h^2 = \frac{(6)(2.064 \times 10^6)}{120} = 130.21 \times 10^3 \text{ mm}^2$$

$$h = 361 \text{ mm} \blacktriangleleft$$

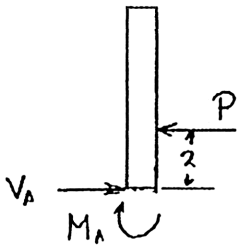
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PROBLEM 5.67

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

SOLUTION



Equivalent concentrated load:

$$P = \left(\frac{1}{2}\right)(6)(1.2) = 3.6 \text{ kips}$$

Bending moment at A:

$$M_A = (2)(3.6) = 7.2 \text{ kip} \cdot \text{ft} = 86.4 \text{ kip} \cdot \text{in.}$$

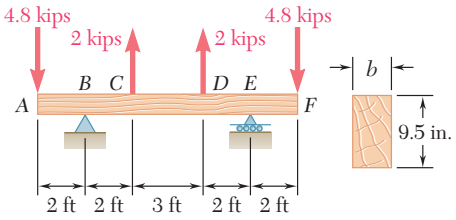
$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{86.4}{1.75} = 49.37 \text{ in}^3$$

For a square section, $S = \frac{1}{6}a^3$

$$a = \sqrt[3]{6S}$$

$$a_{\min} = \sqrt[3]{(6)(49.37)} \qquad a_{\min} = 6.67 \text{ in.} \blacktriangleleft$$

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PROBLEM 5.68

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

SOLUTION

For equilibrium, $B = E = 2.8$ kips

Shear diagram:

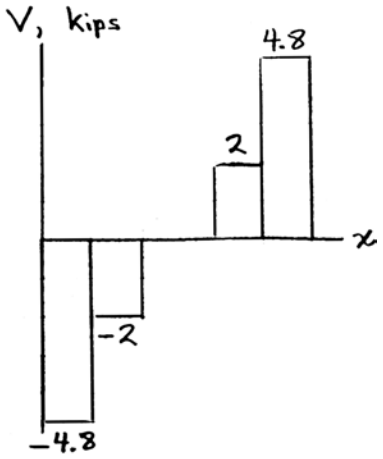
A to B^-: $V = -4.8$ kips

B^+ to C^-: $V = -4.8 + 2.8 = -2$ kips

C^+ to D^-: $V = -2 + 2 = 0$

D^+ to E^-: $V = 0 + 2 = 2$ kips

E^+ to F: $V = 2 + 2.8 = 4.8$ kips



Areas of shear diagram:

A to B: $(2)(-4.8) = -9.6$ kip · ft

B to C: $(2)(-2) = -4$ kip · ft

C to D: $(3)(0) = 0$

D to E: $(2)(2) = 4$ kip · ft

E to F: $(2)(4.8) = 9.6$ kip · ft

Bending moments:

$M_A = 0$

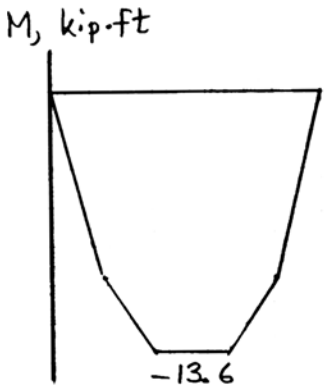
$M_B = 0 - 9.6 = -9.6$ kip · ft

$M_C = -9.6 - 4 = -13.6$ kip · ft

$M_D = -13.6 + 0 = -13.6$ kip · ft

$M_E = -13.6 + 4 = -9.6$ kip · ft

$M_F = -9.6 + 9.6 = 0$



$$|M|_{\max} = 13.6 \text{ kip} \cdot \text{ft} = 162.3 \text{ kip} \cdot \text{in.} = 162.3 \times 10^3 \text{ lb} \cdot \text{in.}$$

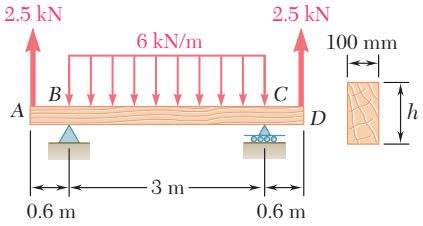
Required value for S:

$$S = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{162.3 \times 10^3}{1750} = 93.257 \text{ in}^3$$

For a rectangular section, $I = \frac{1}{12} bh^3$, $c = \frac{1}{2} h$ $S = \frac{I}{c} = \frac{bh^2}{6} = \frac{(b)(9.5)^2}{6} = 15.0417b$

Equating the two expressions for S, $15.0417b = 93.257$ $b = 6.20$ in. ◀

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PROBLEM 5.69

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION

By symmetry, $B = C$

$$+\uparrow \Sigma F_y = 0: \quad B + C + 2.5 + 2.5 - (3)(6) = 0 \qquad B = C = 6.5 \text{ kN}$$

Shear:

A to B: $V = 2.5$ kN

$V_{B^+} = 2.5 + 6.5 = 9$ kN

$V_{C^-} = 9 - (3)(6) = -9$ kN

C to D: $V = -9 + 6.5 = -2.5$ kN

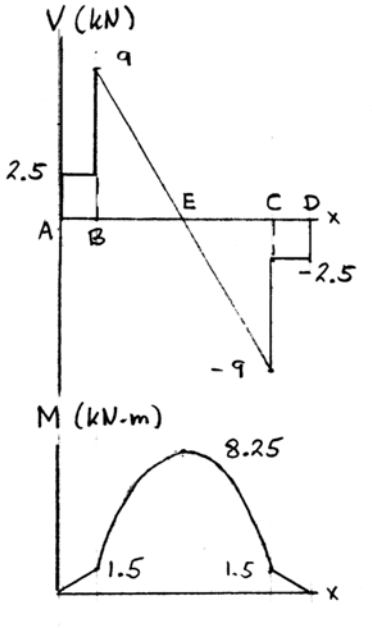
Areas of the shear diagram:

A to B: $\int V\,dx = (0.6)(2.5) = 1.5 \text{ kN}\cdot\text{m}$

B to E: $\int V\,dx = \left(\frac{1}{2}\right)(1.5)(9) = 6.75 \text{ kN}\cdot\text{m}$

E to C: $\int V\,dx = -6.75 \text{ kN}\cdot\text{m}$

C to D: $\int V\,dx = -1.5 \text{ kN}\cdot\text{m}$



Bending moments:

$M_A = 0$

$M_B = 0 + 1.5 = 1.5 \text{ kN}\cdot\text{m}$

$M_E = 1.5 + 6.75 = 8.25 \text{ kN}\cdot\text{m}$

$M_C = 8.25 - 6.75 = 1.5 \text{ kN}\cdot\text{m}$

$M_D = 1.5 - 1.5 = 0$

Maximum $|M| = 8.25 \text{ kN}\cdot\text{m} = 8.25\times10^3 \text{ N}\cdot\text{m}$

$$\sigma_{\text{all}} = 12 \text{ MPa} = 12\times10^6 \text{ Pa}$$

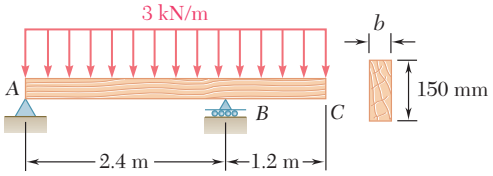
$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{8.25\times10^3}{12\times10^6} = 687.5\times10^{-6} \text{ m}^3 = 687.5\times10^3 \text{ mm}^3$$

For a rectangular section, $S = \frac{1}{6}bh^2$

$$687.5\times10^3 = \left(\frac{1}{6}\right)(100)h^2$$

$$h^2 = \frac{(6)(687.5\times10^3)}{100} = 41.25\times10^3 \text{ mm}^2 \qquad h = 203 \text{ mm} \blacktriangleleft$$

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PROBLEM 5.70

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

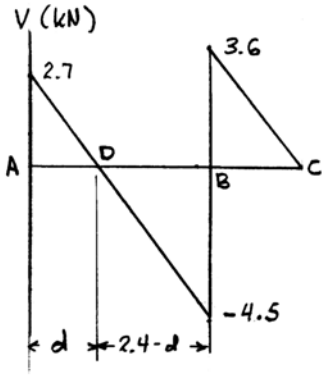
SOLUTION

$$+\circlearrowleft M_B = 0: \quad -2.4A + (0.6)(3.6)(3) = 0 \qquad A = 2.7 \text{ kN}$$

$$+\circlearrowleft M_A = 0: \quad -(1.8)(3.6)(3) + 2.4B = 0 \qquad B = 8.1 \text{ kN}$$

Shear:

$$V_A = 2.7 \text{ kN}$$
$$V_{B^-} = 2.7 - (2.4)(3) = -4.5 \text{ kN}$$
$$V_{B^+} = -4.5 + 8.1 = 3.6 \text{ kN}$$
$$V_C = 3.6 - (1.2)(3) = 0$$



Locate point D where $V = 0$. $\dfrac{d}{2.7} = \dfrac{2.4 - d}{4.5}$ $\quad 7.2d = 6.48$

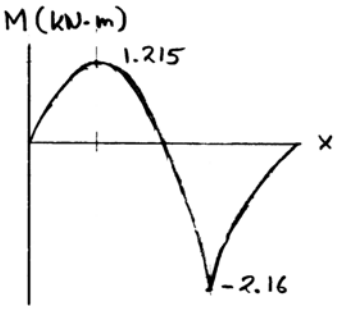
$d = 0.9$ m $\quad 2.4 - d = 1.5$ m

Areas of the shear diagram:

A to D: $\displaystyle\int V\,dx = \left(\frac{1}{2}\right)(0.9)(2.7) = 1.215 \text{ kN}\cdot\text{m}$

D to B: $\displaystyle\int V\,dx = \left(\frac{1}{2}\right)(1.5)(-4.5) = -3.375 \text{ kN}\cdot\text{m}$

B to C: $\displaystyle\int V\,dx = \left(\frac{1}{2}\right)(1.2)(3.6) = 2.16 \text{ kN}\cdot\text{m}$



Bending moments:

$$M_A = 0$$
$$M_D = 0 + 1.215 = 1.215 \text{ kN}\cdot\text{m}$$
$$M_B = 1.215 - 3.375 = -2.16 \text{ kN}\cdot\text{m}$$
$$M_C = -2.16 + 2.16 = 0$$

Maximum $|M| = 2.16 \text{ kN}\cdot\text{m} = 2.16\times10^3 \text{ N}\cdot\text{m}$ $\qquad \sigma_{all} = 12 \text{ MPa} = 12\times10^6 \text{ Pa}$

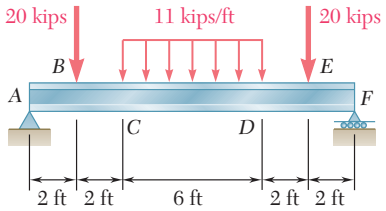
$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{2.16\times10^3}{12\times10^6} = 180\times10^{-6} \text{ m}^3 = 180\times10^3 \text{ mm}^3$$

For rectangular section, $S = \dfrac{1}{6}bh^2 = \dfrac{1}{6}b(150)^2 = 180\times10^3$

$$b = \frac{(6)(180\times10^3)}{150^2}$$

$b = 48.0$ mm ◀

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PROBLEM 5.71

Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

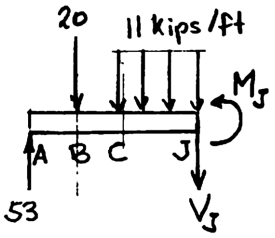
SOLUTION

By symmetry, $R_A = R_F$

$$+\uparrow \Sigma F_y = 0: \quad R_A - 20 - (6)(11) - 20 + R_F = 0$$

$$R_A = R_F = 50 \text{ kips}$$

Maximum bending moment occurs at center of beam.



$$+\circlearrowleft \Sigma M_J = 0: \quad -(7)(53) + (5)(20) + (1.5)(3)(11) + M_J = 0$$

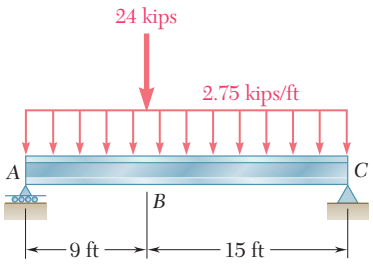
$$M_J = 221.5 \text{ kip.ft} = 2658 \text{ kip} \cdot \text{in.}$$

$$S_{\min} = \frac{M_{\max}}{\sigma_{\text{all}}} = \frac{2658}{24} = 110.75 \text{ in}^3$$

Shape	S (in^2)
W24 × 68	154
W21 × 62	127
W18 × 76	146
W16 × 77	134
W14 × 82	123
W12 × 96	131

Use W21 × 62. ◀

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PROBLEM 5.72

Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

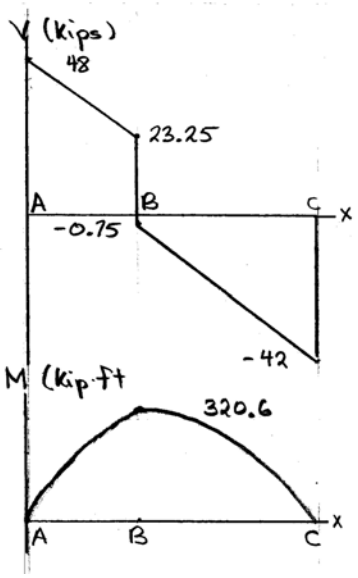
SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -24A + (12)(24)(2.75) + (15)(24) = 0 \qquad A = 48 \text{ kips}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 24C - (12)(24)(2.75) - (9)(24) = 0 \qquad C = 42 \text{ kips}$$

Shear:

$$V_A = 48$$
$$V_{B^-} = 48 - (9)(2.75) = 23.25 \text{ kips}$$
$$V_{B^+} = 23.25 - 24 = -0.75 \text{ kips}$$
$$V_C = -0.75 - (15)(2.75) = -42 \text{ kips}$$



Areas of the shear diagram:

A to *B*: $\int V\,dx = \left(\frac{1}{2}\right)(9)(48 + 23.25) = 320.6 \text{ kip} \cdot \text{ft}$

B to *C*: $\int V\,dx = \left(\frac{1}{2}\right)(15)(-0.75 - 42) = -320.6 \text{ kip} \cdot \text{ft}$

Bending moments:

$$M_A = 0$$
$$M_B = 0 + 320.6 = 320.6 \text{ kip} \cdot \text{ft}$$
$$M_C = 320.6 - 320.6 = 0$$

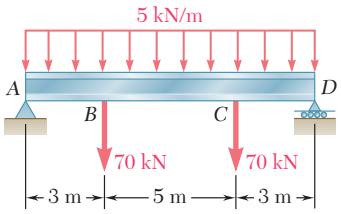
Maximum $|M| = 320.6 \text{ kip} \cdot \text{ft} = 3848 \text{ kip} \cdot \text{in.}$

$$\sigma_{all} = 24 \text{ ksi}$$
$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3848}{24} = 160.3 \text{ in}^3$$

Shape	S, (in^3)
W30×99	269
W27×84	213 ←
W24×104	258
W21×101	227
W18×106	204

Lightest wide flange beam: W27×84 @ 84 lb/ft ◀

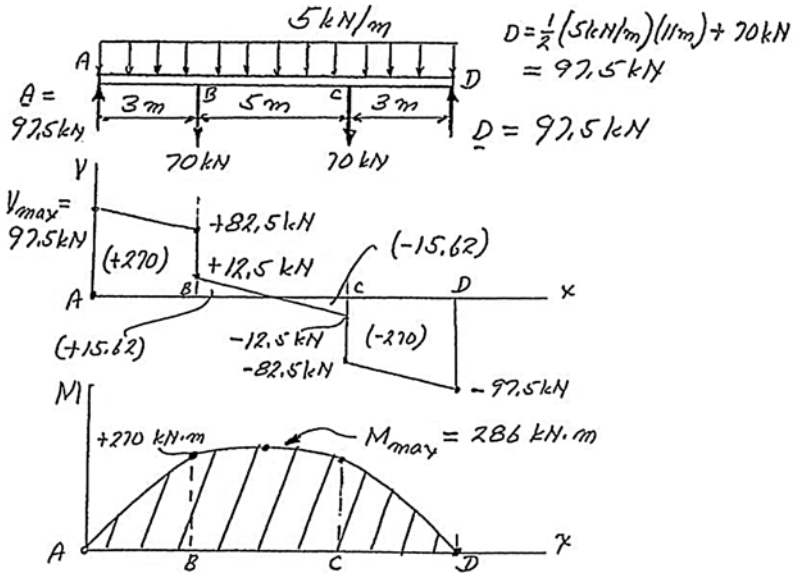
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PROBLEM 5.73

Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION



$$\frac{\text{Section modulus}}{\sigma_{all} = 160 \text{ Mpa}}$$

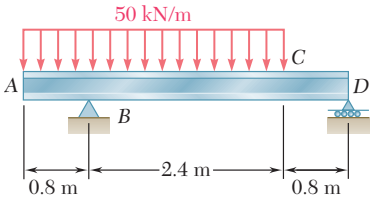
$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{286 \text{ kN} \cdot \text{m}}{160 \text{ MPa}} = 1787 \times 10^{-6} \text{ m}^3$$

$$= 1787 \times 10^{3} \text{ mm}^3$$

Shape	S, (10^3 mm^3)	
W610 × 101	2520	
W530 × 92	2080 ←	◀
W460 × 113	2390	
W410 × 114	2200	
W360 × 122	2020	
W310 × 143	2150	

Use W530 × 92. ◀

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PROBLEM 5.74

Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -3.2B + (24)(3.2)(50) = 0 \qquad B = 120 \text{ kN}$$

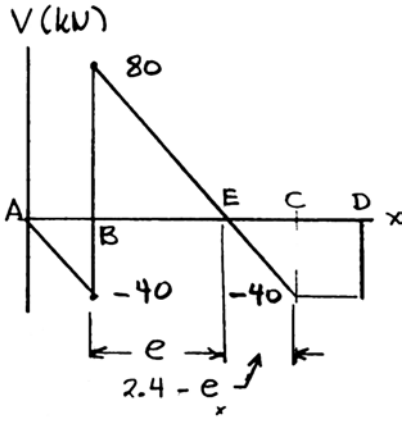
$$+\circlearrowleft \Sigma M_B = 0: \quad 3.2D - (0.8)(3.2)(50) = 0 \qquad D = 40 \text{ kN}$$

Shear:

$$V_A = 0$$
$$V_{B^-} = 0 - (0.8)(50) = -40 \text{ kN}$$
$$V_{B^+} = -40 + 120 = 80 \text{ kN}$$
$$V_C = 80 - (2.4)(50) = -40 \text{ kN}$$
$$V_D = -40 + 0 = -40 \text{ kN}$$

Locate point E where $V = 0$.

$$\frac{e}{80} = \frac{2.4 - e}{40} \qquad 120e = 192$$
$$e = 1.6 \text{ m} \qquad 2.4 - e = 0.8 \text{ m}$$



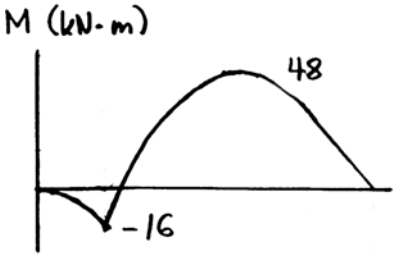
Areas:

$$A \text{ to } B: \quad \int V\,dx = \left(\frac{1}{2}\right)(0.8)(-40) = -16 \text{ kN} \cdot \text{m}$$

$$B \text{ to } E: \quad \int V\,dx = \left(\frac{1}{2}\right)(1.6)(80) = 64 \text{ kN} \cdot \text{m}$$

$$E \text{ to } C: \quad \int V\,dx = \left(\frac{1}{2}\right)(0.8)(-40) = -16 \text{ kN} \cdot \text{m}$$

$$C \text{ to } D: \quad \int V\,dx = (0.8)(-40) = -32 \text{ kN} \cdot \text{m}$$



Bending moments:

$$M_A = 0$$
$$M_B = 0 - 16 = -16 \text{ kN} \cdot \text{m}$$
$$M_E = -16 + 64 = 48 \text{ kN} \cdot \text{m}$$
$$M_C = 48 - 16 = 32 \text{ kN} \cdot \text{m}$$
$$M_D = 32 - 32 = 0$$

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PROBLEM 5.74 ***(Continued)***

Maximum $|M| = 48 \text{ kN} \cdot \text{m} = 48 \times 10^3 \text{ N} \cdot \text{m}$

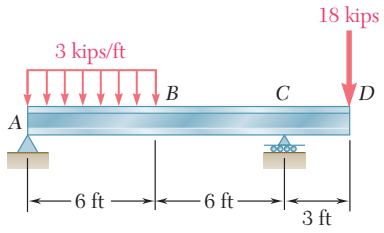
$$\sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{48 \times 10^3}{160 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$$

Shape	$S(10^3 \text{ mm}^3)$
W310×32.7	415
W250×28.4	308 ←
W200×35.9	342

Lightest wide flange beam: W250×28.4 @ 28.4 kg/m ◀

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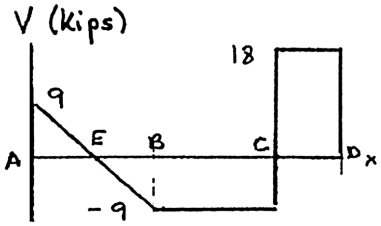
PROBLEM 5.75

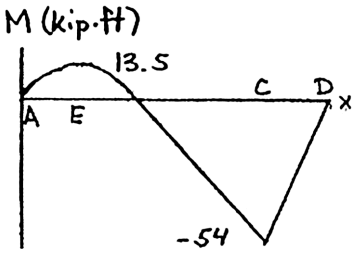
Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.

SOLUTION

$+\circlearrowleft \Sigma M_C = 0$: $-12A + (9)(6)(3) - (3)(18) = 0$ $A = 9$ kips

$+\circlearrowleft \Sigma M_A = 0$: $12C - (3)(6)(3) - (15)(18) = 0$ $C = 27$ kips





Shear: $V_A = 9$ kips

B to C: $V = 9 - (6)(3) = -9$ kips

C to D: $V = -9 + 27 = 18$ kips

Areas:

A to E: $(0.5)(3)(9) = 13.5$ kip · ft

E to B: $(0.5)(3)(-9) = -13.5$ kip · ft

B to C: $(6)(-9) = -54$ kip · ft

C to D: $(3)(18) = 54$ kip · ft

Bending moments: $M_A = 0$

$M_E = 0 + 13.5 = 13.5$ kip · ft

$M_B = 13.5 - 13.5 = 0$

$M_C = 0 + 54 = 54$ kip · ft

$M_D = 54 - 54 = 0$

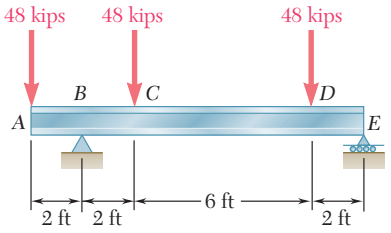
Maximum $|M| = 54$ kip · ft $= 648$ kip · in. $\sigma_{all} = 24$ ksi

$$S_{min} = \frac{648}{24} = 27 \text{ in}^3$$

Shape	$S(\text{in}^3)$
S12 × 31.8	36.2
S10 × 35	29.4

Lightest S-shaped beam: S12 × 31.8 ◀

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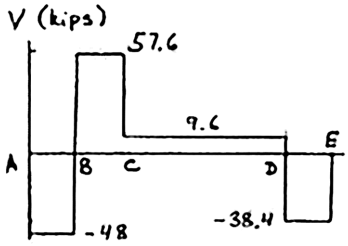
PROBLEM 5.76

Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.

SOLUTION

$+\circlearrowleft \Sigma M_E = 0$: $(12)(48) - 10B + (8)(48) + (2)(48) = 0$ $\quad \boldsymbol{B} = 105.6 \text{ kips} \uparrow$

$+\circlearrowleft \Sigma M_B = 0$: $(2)(48) - (2)(48) - (8)(48) + 10E = 0$ $\quad \boldsymbol{E} = 38.4 \text{ kips} \uparrow$

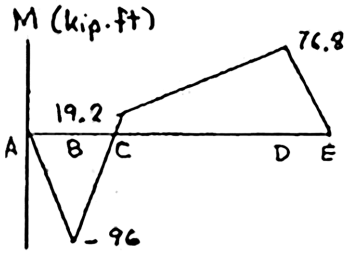


Shear: A to B: $V = -48$ kips

B to C: $V = -48 + 105.6 = 57.6$ kips

C to D: $V = 57.6 - 48 = 9.6$ kips

D to E: $V = 9.6 - 48 = -38.4$ kips



Areas: A to B: $(2)(-48) = -96$ kip · ft

B to C: $(2)(57.6) = 115.2$ kip · ft

C to D: $(6)(9.6) = 57.6$ kip · ft

D to E: $(2)(-38.4) = 76.8$ kip · ft

Bending moments: $M_A = 0$

$M_B = 0 - 96 = -96$ kip · ft

$M_C = -96 + 115.2 = 19.2$ kip · ft

$M_D = 19.2 + 57.2 = 76.8$ kip · ft

$M_E = 76.8 - 76.8 = 0$

Maximum $|M| = 96$ kip · ft $= 1152$ kip · in.

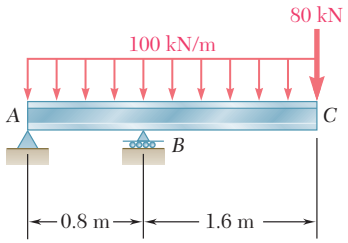
$\sigma_{all} = 24$ ksi

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{1152}{24} = 48 \text{ in}^3$$

Shape	S (in^3)
S15 × 42.9	59.4
S12 × 50	50.6

Lightest S-shaped beam: S15 × 42.9 ◀

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PROBLEM 5.77

Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.

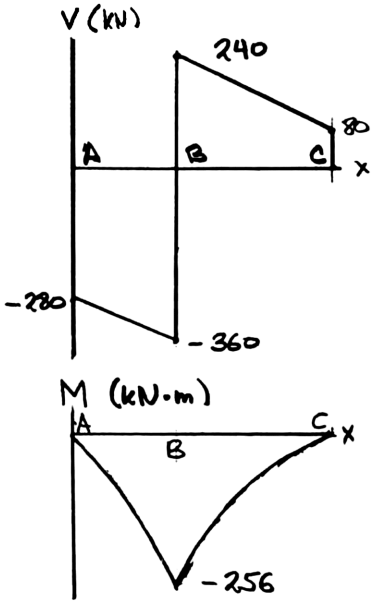
SOLUTION

$$+\circlearrowleft \sum M_B = 0: \quad 0.8A - (0.4)(2.4)(100) - (1.6)(80) = 0$$

$$A = 280 \text{ kN} \downarrow$$

$$+\circlearrowleft \sum M_A = 0: \quad 0.8B - (1.2)(2.4)(100) - (2.4)(80) = 0$$

$$B = 600 \text{ kN} \uparrow$$



Shear: $V_A = -280 \text{ kN}$

$$V_{B^-} = -280 - (0.8)(100) = -360 \text{ kN}$$

$$V_{B^+} = -360 + 600 = 240 \text{ kN}$$

$$V_C = 240 - (1.6)(100) = 80 \text{ kN}$$

Areas under shear diagram:

A to B: $\frac{1}{2}(0.8)(-280 - 360) = -256 \text{ kN} \cdot \text{m}$

B to C: $\frac{1}{2}(1.6)(240 + 80) = 256 \text{ kN} \cdot \text{m}$

Bending moments: $M_A = 0$

$$M_B = 0 - 256 = -256 \text{ kN} \cdot \text{m}$$

$$M_C = -256 + 256 = 0$$

Maximum $|M| = 256 \text{ kN} \cdot \text{m} = 256 \times 10^3 \text{ N} \cdot \text{m}$

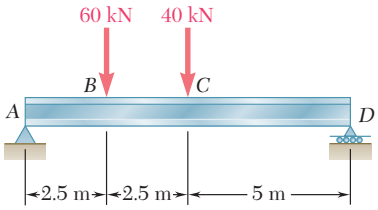
$$\sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{256 \times 10^3}{160 \times 10^6} = 1.6 \times 10^{-3} \text{ m}^3 = 1600 \times 10^3 \text{ mm}^3$$

Shape	S (10^3 mm^3)
S510×98.2	1950
S460×104	1685

Lightest S-section: S510×98.2 ◀

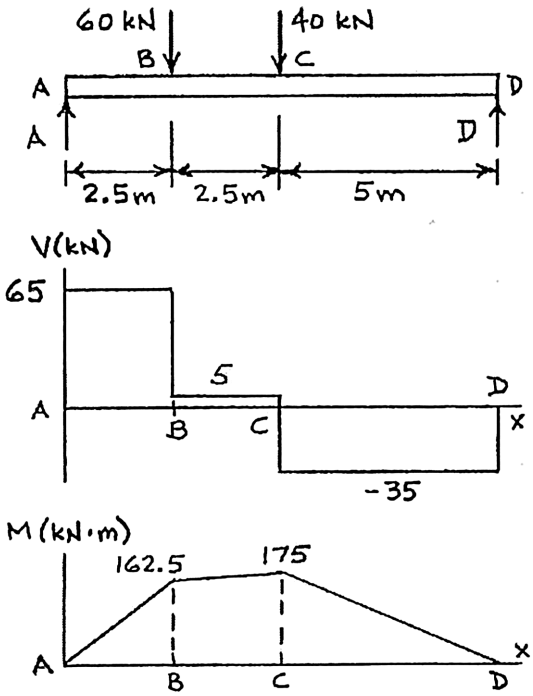
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PROBLEM 5.78

Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.

SOLUTION



Reaction: $+\circlearrowleft \sum M_D = 0$: $-10A + (7.5)(60) + (5)(40) = 0$

$$\boldsymbol{A} = 65\,\text{kN} \uparrow$$

Shear diagram:

A to B: $V = 65$ kN

B to C: $V = 65 - 60 = 5$ kN

C to D: $V = 5 - 40 = -35$ kN

Areas of shear diagram:

A to B: $(2.5)(65) = 162.5\ \text{kN} \cdot \text{m}$

B to C: $(2.5)(5) = 12.5\ \text{kN} \cdot \text{m}$

C to D: $(5)(-35) = -175\ \text{kN} \cdot \text{m}$

Bending moments: $M_A = 0$

$$M_B = 0 + 162.5 = 162.5\ \text{kN} \cdot \text{m}$$

$$M_C = 162.5 + 12.5 = 175\ \text{kN} \cdot \text{m}$$

$$M_D = 175 - 175 = 0$$

$$|M|_{\max} = 175\ \text{kN} \cdot \text{m} = 175 \times 10^3\ \text{N} \cdot \text{m}$$

$$\sigma_{\text{all}} = 160\ \text{MPa} = 160 \times 10^6\ \text{Pa}$$

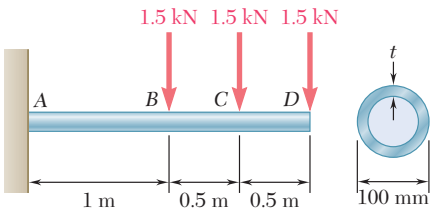
Shape	S_x, $(10^3\ \text{mm}^3)$
S610 × 119	2870
S510 × 98.2	1950
S460 × 81.4	1460 ←

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{175 \times 10^3}{160 \times 10^6} = 1093.75 \times 10^{-6}\ \text{m}^3$$

$$= 1093.75 \times 10^3\ \text{mm}^3$$

Lightest S-section: S460 × 81.4 ◀

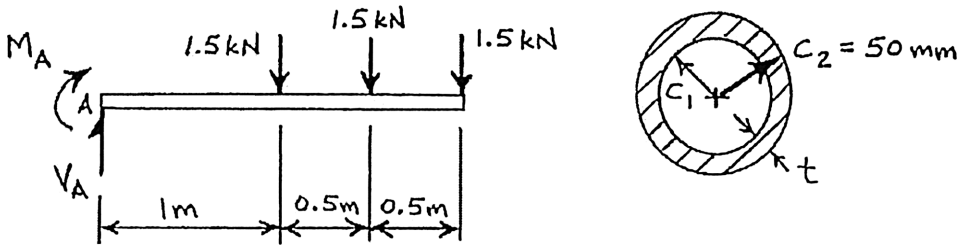
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PROBLEM 5.79

A steel pipe of 100-mm diameter is to support the loading shown. Knowing that the stock of pipes available has thicknesses varying from 6 mm to 24 mm in 3-mm increments, and that the allowable normal stress for the steel used is 150 MPa, determine the minimum wall thickness t that can be used.

SOLUTION



$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (1)(1.5) - (1.5)(1.5) - (2)(1.5) = 0 \qquad M_A = -6.75 \text{ kN} \cdot \text{m}$$

$$|M|_{\max} = |M_A| = 6.75 \text{ kN} \cdot \text{m}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{6.75 \times 10^3 \text{ N} \cdot \text{m}}{150 \times 10^6 \text{ Pa}} = 45 \times 10^{-6} \text{ m}^3 = 45 \times 10^3 \text{ mm}^3$$

$$S_{\min} = \frac{I_{\min}}{c_2} \qquad I_{\min} = c_2 S_{\min} = (50)(45 \times 10^3) = 2.25 \times 10^6 \text{ mm}^4$$

$$I_{\min} = \frac{\pi}{4}\left(c_2^4 - c_{1\max}^4\right)$$

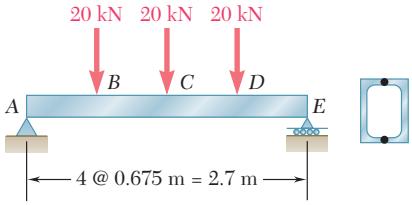
$$c_{1\max}^4 = c_2^4 - \frac{4}{\pi} I_{\min} = (50)^4 - \frac{4}{\pi}(2.25 \times 10^6) = 3.3852 \times 10^6 \text{ mm}^4$$

$$c_{1\max} = 42.894 \text{ mm}$$

$$t_{\min} = c_2 - c_{1\max} = 50 - 42.894 = 7.106 \text{ mm}$$

$$t = 9 \text{ mm} \blacktriangleleft$$

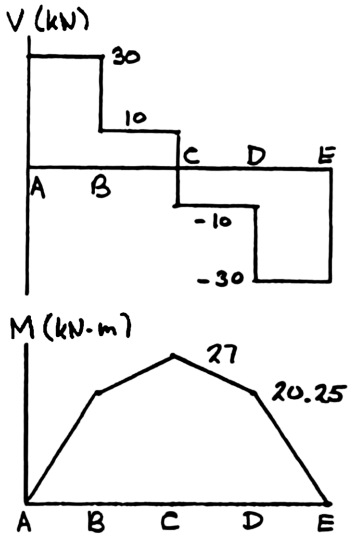
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PROBLEM 5.80

Two metric rolled-steel channels are to be welded along their edges and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 150 MPa, determine the most economical channels that can be used.

SOLUTION



By symmetry, $A = E$

$$+\uparrow \Sigma F_y = 0: \quad A + E - 20 - 20 - 20 = 0$$

$$A = E = 30 \text{ kN}$$

Shear:

A to B: $V = 30$ kN

B to C: $V = 30 - 20 = 10$ kN

C to D: $V = 10 - 20 = -10$ kN

D to E: $V = -10 - 20 = -30$ kN

Areas:

A to B: $(0.675)(30) = 20.25$ kN · m

B to C: $(0.675)(10) = 6.75$ kN · m

C to D: $(0.675)(-10) = -6.75$ kN · m

D to E: $(0.675)(-30) = -20.25$ kN · m

Bending moments: $M_A = 0$

$M_B = 0 + 20.25 = 20.25$ kN · m

$M_C = 20.25 + 6.75 = 27$ kN · m

$M_D = 27 - 6.75 = 20.25$ kN · m

$M_E = 20.25 - 20.25 = 0$

Maximum $|M| = 27$ kN · m $= 27 \times 10^3$ N · m

$\sigma_{all} = 150$ MPa $= 150 \times 10^6$ Pa

For a section consisting of two channels,

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{27 \times 10^3}{150 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$$

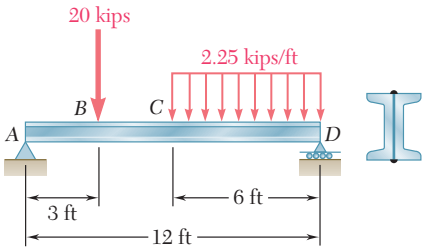
For each channels,

$$S_{min} = \left(\frac{1}{2}\right)(180 \times 10^3) = 90 \times 10^3 \text{ mm}^3$$

Shape	$S(10^3 \text{ mm}^3)$
C180 × 14.6	100
C150 × 19.3	94.7 ←

Lightest channel section: C180 × 14.6 ◀

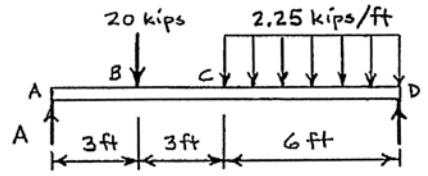
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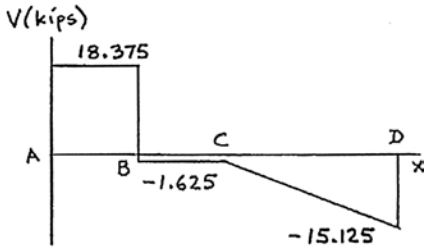


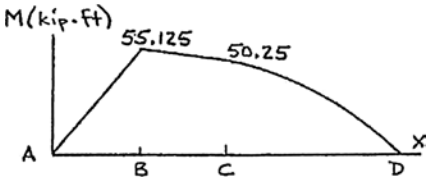
PROBLEM 5.81

Two rolled-steel channels are to be welded back to back and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 30 ksi, determine the most economical channels that can be used.

SOLUTION







Shape	$S(\text{in})^3$
C10 × 15.3	13.5
C9 × 15	11.3 ←
C8 × 18.7	11.0

Reaction: $+\circlearrowleft \Sigma M_D = 0$: $-12A + 9(20) + (6)(2.25)(3) = 0$

$$\boldsymbol{A} = 18.375 \text{ kips} \uparrow$$

Shear diagram:

A to B: $V = 18.375$ kips

B to C: $V = 18.375 - 20 = -1.625$ kips

$$V_D = -1.625 - (6)(2.25) = -15.125 \text{ kips}$$

Areas of shear diagram:

A to B: $(3)(18.375) = 55.125$ kip · ft

B to C: $(3)(-1.625) = -4.875$ kip · ft

C to D: $0.5(6)(-1.625 - 15.125) = -50.25$ kip · ft

Bending moments: $M_A = 0$

$M_B = 0 + 55.125 = 55.125$ kip · ft

$M_C = 55.125 - 4.875 = 50.25$ kip · ft

$M_D = 50.25 - 50.25 = 0$

$|M|_{\max} = 55.125 \text{ kip} \cdot \text{ft} = 661.5 \text{ kip} \cdot \text{in.}$

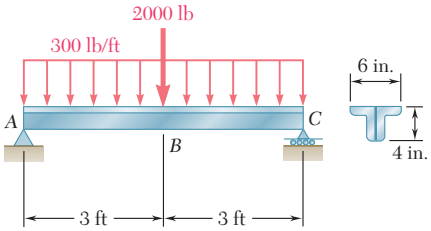
$\sigma_{\text{all}} = 30$ ksi

For double channel, $S_{\min} = \dfrac{|M|}{\sigma_{\text{all}}} = \dfrac{661.5}{30} = 22.05 \text{ in}^3$

For single channel, $S_{\min} = 0.5(22.05) = 11.025 \text{ in}^3$

Lightest channel section: C9 × 15 ◀

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PROBLEM 5.82

Two L4 × 3 rolled-steel angles are bolted together and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 24 ksi, determine the minimum angle thickness that can be used.

SOLUTION

By symmetry, $A = C$

$$+\uparrow \Sigma F_y = 0: \quad A + C - 2000 - (6)(300) = 0$$

$$A = C = 1900 \text{ lb}$$

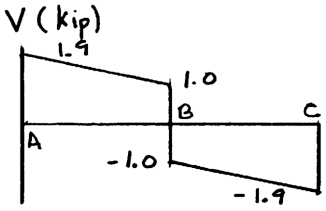
Shear:

$$V_A = 1900 \text{ lb} = 1.9 \text{ kips}$$

$$V_{B^-} = 1900 - (3)(300) = 1000 \text{ lb} = 1 \text{ kip}$$

$$V_{B^+} = 1000 - 2000 = -1000 \text{ lb} = -1 \text{ kip}$$

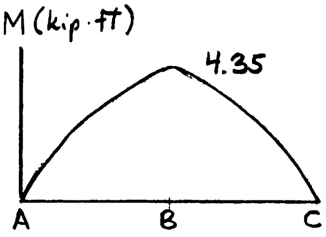
$$V_C = -1000 - (3)(300) = -1900 \text{ lb} = -1.9 \text{ kip}$$



Areas:

A to B: $\left(\frac{1}{2}\right)(3)(1.9 + 1) = 4.35 \text{ kip} \cdot \text{ft}$

B to C: $\left(\frac{1}{2}\right)(3)(-1 - 1.9) = -4.35 \text{ kip} \cdot \text{ft}$



Bending moments: $M_A = 0$

$$M_B = 0 + 4.35 = 4.35 \text{ kip} \cdot \text{ft}$$

$$M_C = 4.35 - 4.35 = 0$$

Maximum $|M| = 4.35 \text{ kip} \cdot \text{ft} = 52.2 \text{ kip} \cdot \text{in.}$

$$\sigma_{all} = 24 \text{ ksi}$$

For section consisting of two angles,

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{52.2}{24} = 2.175 \text{ in}^3$$

For each angle,

$$S_{min} = \left(\frac{1}{2}\right)(2.175) = 1.0875 \text{ in}^3$$

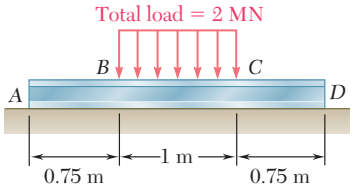
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PROBLEM 5.82 ***(Continued)***

Angle section	$S(\text{in}^3)$
$\text{L}4 \times 3 \times \frac{1}{2}$	1.87
$\text{L}4 \times 3 \times \frac{3}{8}$	1.44
$\text{L}4 \times 3 \times \frac{1}{4}$	0.988

Smallest allowable thickness: $t = \frac{3}{8}$ in. ◀

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PROBLEM 5.83

Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 170 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION

Downward distributed load: $w = \dfrac{2}{1.0} = 2$ MN/m

Upward distributed reaction: $q = \dfrac{2}{2.5} = 0.8$ MN/m

Net distributed load over BC: 1.2 MN/m

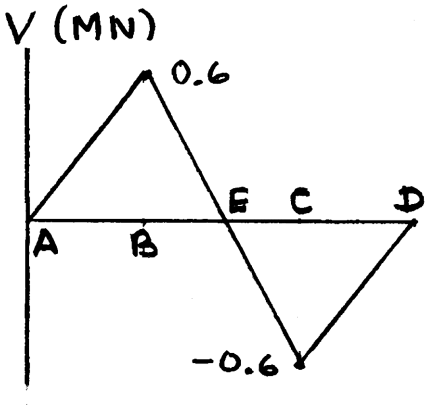
Shear:

$V_A = 0$

$V_B = 0 + (0.75)(0.8) = 0.6$ MN

$V_C = 0.6 - (1.0)(1.2) = -0.6$ MN

$V_D = -0.6 + (0.75)(0.8) = 0$



Areas:

A to B: $\left(\dfrac{1}{2}\right)(0.75)(0.6) = 0.225$ MN · m

B to E: $\left(\dfrac{1}{2}\right)(0.5)(0.6) = 0.150$ MN · m

E to C: $\left(\dfrac{1}{2}\right)(0.5)(-0.6) = -0.150$ MN · m

C to D: $\left(\dfrac{1}{2}\right)(0.75)(-0.6) = -0.225$ MN · m

Bending moments:

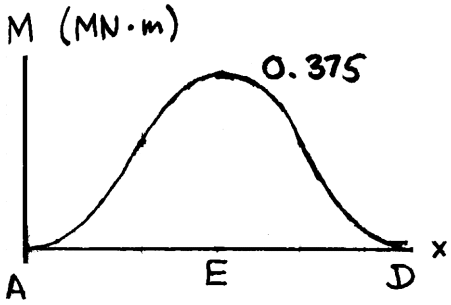
$M_A = 0$

$M_B = 0 + 0.225 = 0.225$ MN · m

$M_E = 0.225 + 0.150 = 0.375$ MN · m

$M_C = 0.375 - 0.150 = 0.225$ MN · m

$M_D = 0.225 - 0.225 = 0$



Maximum $|M| = 0.375$ MN · m $= 375 \times 10^3$ N · m

$$\sigma_{all} = 170 \text{ MPa} = 170 \times 10^6 \text{ Pa}$$

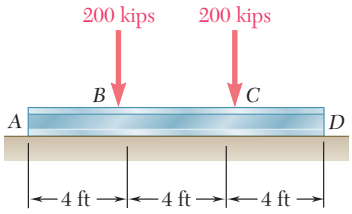
$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{375 \times 10^3}{170 \times 10^6} = 2.206 \times 10^{-3} \text{ m}^3 = 2206 \times 10^3 \text{ mm}^3$$

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PROBLEM 5.83 ***(Continued)***

Shape	$S\,(10^3\,\text{mm}^3)$	
$W690 \times 125$	3490	
$W610 \times 101$	2520 ←	Lightest wide flange section: $W610 \times 101$ ◀
$W530 \times 150$	3720	
$W460 \times 113$	2390	

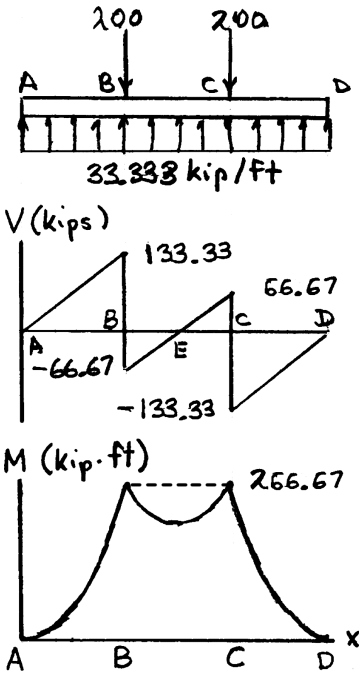
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PROBLEM 5.84

Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

SOLUTION



Distributed reaction: $q = \dfrac{400}{12} = 33.333 \text{ kip/ft}$

Shear:

$V_A = 0$

$V_{B^-} = 0 + (4)(33.333) = 133.33 \text{ kips}$

$V_{B^+} = 133.33 - 200 = -66.67 \text{ kips}$

$V_{C^-} = -66.67 + 4(33.333) = 66.67 \text{ kips}$

$V_{C^+} = 66.67 - 200 = -133.33 \text{ kips}$

$V_D = -133.33 + (4)(33.333) = 0 \text{ kips}$

Areas:

A to B: $\left(\dfrac{1}{2}\right)(4)(133.33) = 266.67 \text{ kip} \cdot \text{ft}$

B to E: $\left(\dfrac{1}{2}\right)(2)(-66.67) = -66.67 \text{ kip} \cdot \text{ft}$

E to C: $\left(\dfrac{1}{2}\right)(2)(66.67) = 66.67 \text{ kip} \cdot \text{ft}$

C to D: $\left(\dfrac{1}{2}\right)(4)(-133.33) = -266.67 \text{ kip} \cdot \text{ft}$

Bending moments:

$M_A = 0$

$M_B = 0 + 266.67 = 266.67 \text{ kip} \cdot \text{ft}$

$M_E = 266.67 - 66.67 = 200 \text{ kip} \cdot \text{ft}$

$M_C = 200 + 66.67 = 266.67 \text{ kip} \cdot \text{ft}$

$M_D = 266.67 - 266.67 = 0$

Maximum $|M| = 266.67 \text{ kip} \cdot \text{ft} = 3200 \text{ kip} \cdot \text{in.}$

$\sigma_{\text{all}} = 24 \text{ ksi}$

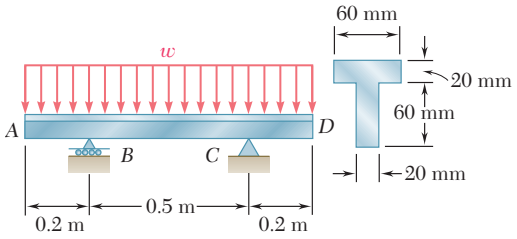
$S_{\text{min}} = \dfrac{|M|}{\sigma_{\text{all}}} = \dfrac{3200}{24} = 133.3 \text{ in}^3$

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PROBLEM 5.84 ***(Continued)***

Shape	$S(\text{in}^3)$		
W27 × 84	213		
W24 × 68	154	←	Lightest W-shaped section: W24 × 68 ◀
W21 × 101	227		
W18 × 76	146		
W16 × 77	134		
W14 × 145	232		

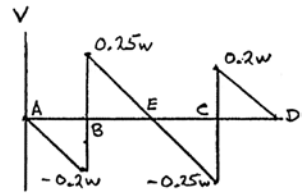
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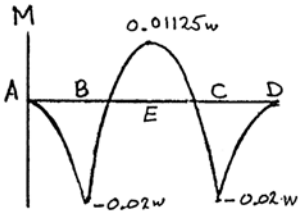


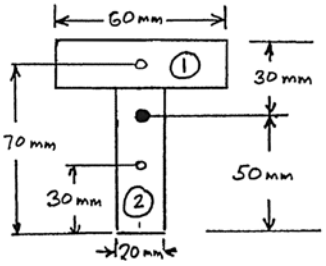
PROBLEM 5.85

Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is +80 MPa in tension and −130 MPa in compression.

SOLUTION







Reactions: By symmetry, $B = C$

$$+\uparrow \sum F_y = 0: \quad B + C - 0.9w = 0$$

$$B = C = 0.45w \uparrow$$

Shear: $V_A = 0$

$$V_{B^-} = 0 - 0.2w = -0.2w$$

$$V_{B^+} = -0.2w + 0.45w = 0.25w$$

$$V_{C^-} = 0.25w - 0.5w = -0.25w$$

$$V_{C^+} = -0.25w + 0.45w = 0.2w$$

$$V_D = 0.2w - 0.2w = 0$$

Areas: A to B: $\frac{1}{2}(0.2)(-0.2w) = -0.02w$

B to E: $\frac{1}{2}(0.25)(0.25w) = 0.03125w$

Bending moments: $M_A = 0$

$$M_B = 0 - 0.02w = -0.02w$$

$$M_E = -0.02w + 0.03125w = 0.01125w$$

Centroid and moment of inertia:

Part	A, mm^2	$\bar{y}$, mm	$A\bar{y}$(10^3 mm^3)	d, mm	Ad^2(10^3 mm^4)	$\bar{I}$(10^3 mm^4)
①	1200	70	84	20	480	40
②	1200	30	36	20	480	360
Σ	2400		120		960	400

$$\bar{Y} = \frac{120 \times 10^3}{2400} = 50 \text{ mm}$$

$$I = \sum Ad^2 + \sum \bar{I} = 1360 \times 10^3 \text{ mm}^4$$

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PROBLEM 5.85 *(Continued)*

Top: $I/y = (1360 \times 10^3)/30 = 45.333 \times 10^3 \text{ mm}^3 = 45.333 \times 10^{-6} \text{ m}^3$

Bottom: $I/y = (1360 \times 10^3)/(-50) = -27.2 \times 10^3 \text{ mm}^3 = -27.2 \times 10^{-6} \text{ m}^3$

Bending moment limits $(M = -\sigma I/y)$ and load limits w.

Tension at B and C: $-0.02w = -(80 \times 10^6)(45.333 \times 10^{-6})$ $w = 181.3 \times 10^3$ N/m

Compression at B and C: $-0.02w = -(-130 \times 10^6)(27.2 \times 10^{-6})$ $w = 176.8 \times 10^3$ N/m

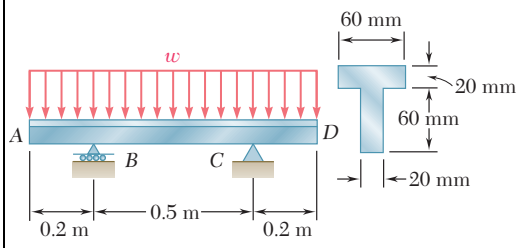
Tension at E: $0.01125w = -(80 \times 10^6)(27.2 \times 10^{-6})$ $w = 193.4 \times 10^3$ N/m

Compression at E: $0.01125w = -(-130 \times 10)(45.333 \times 10^{-6})$ $w = 523.8 \times 10^3$ N/m

The smallest allowable load controls. $w = 176.8 \times 10^3$ N/m

$w = 176.8$ kN/m ◀

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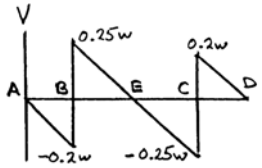


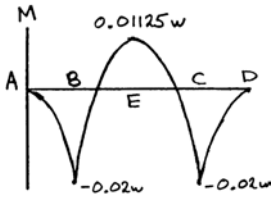
PROBLEM 5.86

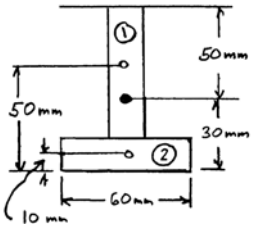
Solve Prob. 5.85, assuming that the cross section of the beam is inverted, with the flange of the beam resting on the supports at *B* and *C*.

PROBLEM 5.85 Determine the largest permissible distributed load *w* for the beam shown, knowing that the allowable normal stress is +80 MPa in tension and −130 MPa in compression.

SOLUTION







Reactions: By symmetry, $B = C$

$$+\uparrow \Sigma F_y = 0: \quad B + C - 0.9w = 0$$

$$\boldsymbol{B = C = 0.45w\uparrow}$$

Shear:

$$V_A = 0$$
$$V_{B^-} = 0 - 0.2w = -0.2w$$
$$V_{B^+} = -0.2w + 0.45w = 0.25w$$
$$V_{C^-} = 0.25w - 0.5w = -0.25w$$
$$V_{C^+} = -0.25w + 0.45w = 0.2w$$
$$V_D = 0.2w - 0.2w = 0$$

Areas:

A to *B*: $\frac{1}{2}(0.2)(-0.2w) = -0.02w$

B to *E*: $\frac{1}{2}(0.25)(0.25w) = 0.03125w$

Bending moments:

$$M_A = 0$$
$$M_B = 0 - 0.02w = -0.02w$$
$$M_E = -0.02w + 0.03125w = 0.01125w$$

Centroid and moment of inertia:

Part	A, mm^2	$\bar{y}$, mm	$A\bar{y}$, (10^3 mm^3)	d, mm	Ad^2(10^3 mm^4)	$\bar{I}$, (10^3 mm^4)
①	1200	50	60	20	480	360
②	1200	10	12	20	480	40
Σ	2400		72		960	400

$$\bar{Y} = \frac{72 \times 10^3}{2400} = 30 \text{ mm} \qquad I = \Sigma Ad^2 + \Sigma \bar{I} = 1360 \times 10^3 \text{ mm}^3$$

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PROBLEM 5.86 *(Continued)*

Top: $I/y = (1360\times10^3)/(50) = 27.2\times10^3\ \text{mm}^3 = 27.2\times10^{-6}\ \text{m}^3$

Bottom: $I/y = (1360\times10^3)/(-30) = -45.333\times10^8\ \text{mm}^3 = -45.333\times10^{-6}\ \text{m}^3$

Bending moment limits $(M = -\sigma I/y)$ and load limits w.

Tension at B and C: $-0.02w = -(80\times10^6)(27.2\times10^{-6})$ $w = 108.8\times10^3$ N/m

Compression at B and C: $-0.02w = -(-130\times10^6)(-45.333\times10^{-6})$ $w = 294.7\times10^3$ N/m

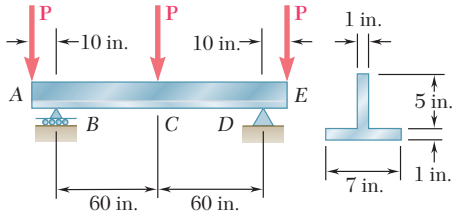
Tension at E: $0.01125w = -(80\times10^6)(-45.333\times10^{-6})$ $w = 322.4\times10^3$ N/m

Compression at E: $0.01125w = -(-130\times10^6)(27.2\times10^{-6})$ $w = 314.3\times10^3$ N/m

The smallest allowable load controls. $w = 108.8\times10^3$ N/m

$w = 108.8$ kN/m ◀

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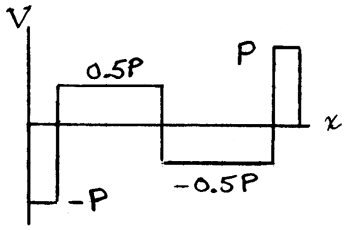
PROBLEM 5.87

Determine the largest permissible value of **P** for the beam and loading shown, knowing that the allowable normal stress is +8 ksi in tension and −18 ksi in compression.

SOLUTION

Reactions: $\boldsymbol{B} = \boldsymbol{D} = 1.5P \uparrow$

Shear diagram:

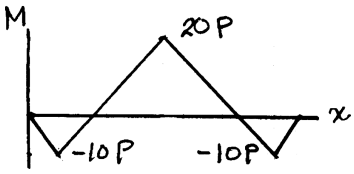


A to B^-: $V = -P$

B^+ to C^-: $V = -P + 1.5P = 0.5P$

C^+ to D^-: $V = 0.5P - P = -0.5P$

D^+ to E: $V = -0.5P + 1.5P = P$



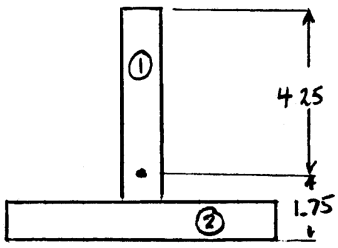
Areas:

A to B: $(10)(-P) = -10P$

B to C: $(60)(0.5P) = 30P$

C to D: $(60)(-0.5P) = -30P$

D to E: $(10)(P) = 10P$



Bending moments:

$M_A = 0$

$M_B = 0 - 10P = -10P$

$M_C = -10P + 30P = 20P$

$M_D = 20P - 30P = -10P$

$M_E = -10P + 10P = 0$

Largest positive bending moment: $20P$

Largest negative bending moment: $-10P$

Centroid and moment of inertia:

Part	A, in^2	$\bar{y}_0$, in.	$A\bar{y}_0$, in^3	d, in.	Ad^2, in^4	$\bar{I}$, in^4
①	5	3.5	17.5	1.75	15.3125	10.417
②	7	0.5	3.5	1.25	10.9375	0.583
Σ	12		21		26.25	11.000

$$\bar{Y} = \frac{21}{12} = 1.75 \text{ in.} \qquad I = \Sigma Ad^2 + \Sigma I = 37.25 \text{ in}^4$$

Top: $y = 4.25$ in. Bottom: $y = -1.75$ in. $\sigma = -\dfrac{My}{I}$

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PROBLEM 5.87 ***(Continued)***

Top, tension: $8 = -\dfrac{(-10P)(4.25)}{37.25}$ $P = 7.01$ kips

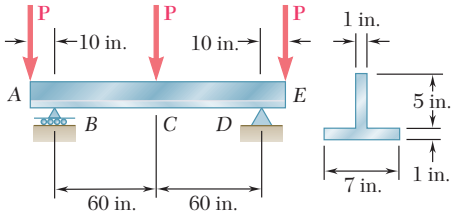
Top, comp.: $-18 = -\dfrac{(20P)(4.25)}{37.25}$ $P = 7.89$ kips

Bottom. tension: $8 = -\dfrac{(20P)(-1.75)}{37.25}$ $P = 8.51$ kips

Bottom. comp.: $-18 = -\dfrac{(-10P)(-1.75)}{37.25}$ $P = 38.3$ kips

Smallest value of P is the allowable value. $P = 7.01$ kips ◀

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PROBLEM 5.88

Solve Prob. 5.87, assuming that the T-shaped beam is inverted.

PROBLEM 5.87 Determine the largest permissible value of **P** for the beam and loading shown, knowing that the allowable normal stress is +8 ksi in tension and −18 ksi in compression.

SOLUTION

Reactions: $B = D = 1.5P \uparrow$

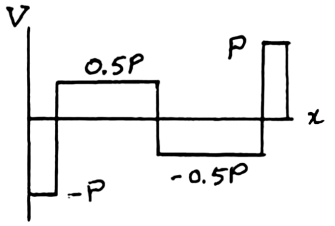
Shear diagram:

A to B^-: $V = -P$

B^+ to C^-: $V = -P + 1.5P = 0.5P$

C^+ to D^-: $V = 0.5P - P = -0.5P$

D^+ to E: $V = -0.5P + 1.5P = P$



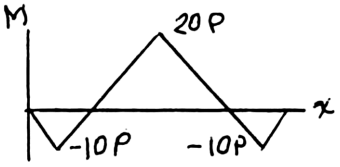
Areas:

A to B: $(10)(-P) = -10P$

B to C: $(60)(0.5P) = 30P$

C to D: $(60)(-0.5P) = -30P$

D to E: $(10)(P) = 10P$



Bending moments:

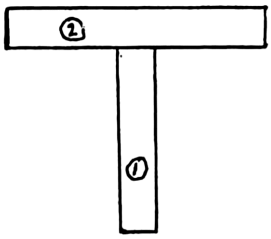
$M_A = 0$

$M_B = 0 - 10P = -10P$

$M_C = -10P + 30P = 20P$

$M_D = 20P - 30P = -10P$

$M_E = -10P + 10P = 0$



Largest positive bending moment = $20P$

Largest negative bending moment = $-10P$

Centroid and moment of inertia:

Part	A, in^2	$\bar{y}_0$, in.	$A\bar{y}_0$, in^3	d, in.	Ad^2, in^4	$\bar{I}$, in^4
①	5	2.5	12.5	1.75	15.3125	10.417
②	7	5.5	38.5	1.25	10.9375	0.583
Σ	12		51		26.25	11.000

Top: $y = 1.75$ in. $\quad \bar{Y} = \dfrac{51}{12} = 4.25$ in.

Bottom: $y = -4.25$ in. $\quad I = \sum Ad^2 + \sum I = 37.25$ in^4

$$\sigma = -\frac{My}{I}$$

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PROBLEM 5.88 ***(Continued)***

Top, tension: $8 = -\dfrac{(-10P)(1.75)}{37.25}$ $P = 17.03$ kips

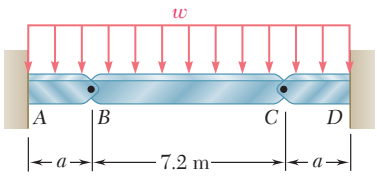
Top, compression: $-18 = -\dfrac{(20P)(1.75)}{37.25}$ $P = 19.16$ kips

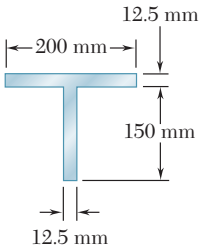
Bottom, tension: $8 = -\dfrac{(20P)(-4.25)}{37.25}$ $P = 3.51$ kips

Bottom, compression: $-18 = -\dfrac{(-10P)(-4.25)}{37.25}$ $P = 15.78$ kips

Smallest value of P is the allowable value. $P = 3.51$ kips ◀

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PROBLEM 5.89

Beams *AB*, *BC*, and *CD* have the cross section shown and are pin-connected at *B* and *C*. Knowing that the allowable normal stress is +110 MPa in tension and –150 MPa in compression, determine (*a*) the largest permissible value of *w* if beam *BC* is not to be overstressed, (*b*) the corresponding maximum distance *a* for which the cantilever beams *AB* and *CD* are not overstressed.

SOLUTION

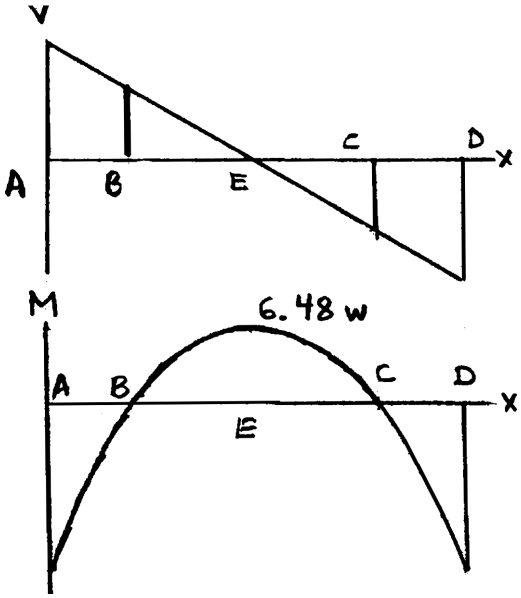
$M_B = M_C = 0$

$$V_B = -V_C = \left(\frac{1}{2}\right)(7.2)w = 3.6w$$

Area *B* to *E* of shear diagram:

$$\left(\frac{1}{2}\right)(3.6)(3.6w) = 6.48w$$

$$M_E = 0 + 6.48w = 6.48w$$

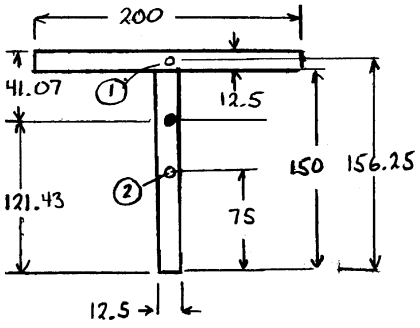


Centroid and moment of inertia:

Part	$A(\text{mm}^2)$	$\bar{y}\,(\text{mm})$	$A\bar{y}\,(\text{mm}^3)$	$d\,(\text{mm})$	$Ad^2\,(\text{mm}^4)$	$\bar{I}\,(\text{mm}^4)$
①	2500	156.25	390,625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140,625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531,250		7.073×10^6	3.548×10^6

$$\bar{Y} = \frac{531,250}{4375} = 121.43 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 10.621\times10^6 \text{ mm}^4$$



Location	$y\,(\text{mm})$	$I/y(10^3\,\text{mm}^3)$ ← also $(10^{-6}\,\text{m}^3)$
Top	41.07	258.6
Bottom	–121.43	–87.47

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PROBLEM 5.89 *(Continued)*

Bending moment limits: $M = -\sigma I/y$

Tension at E: $-(110\times10^{6})(-87.47\times10^{-6}) = 9.622\times10^{3}\ \text{N}\cdot\text{m}$

Compression at E: $-(-150\times10^{-6})(258.6\times10^{-6}) = 38.8\times10^{3}\ \text{N}\cdot\text{m}$

Tension at A and D: $-(110\times10^{6})(258.6\times10^{-6}) = -28.45\times10^{3}\ \text{N}\cdot\text{m}$

Compression at A and D: $-(-150\times10^{6})(-87.47\times10^{-6}) = -13.121\times10^{3}\ \text{N}\cdot\text{m}$

(a) Allowable load w: $6.48w = 9.622\times10^{3}$ $w = 1.485\times10^{3}$ N/m $w = 1.485$ kN/m ◀

Shear at A: $V_A = (a+3.6)\,w$

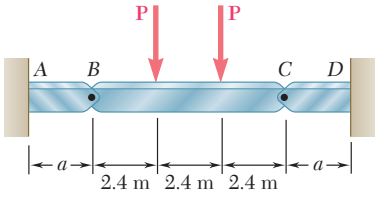
Area A to B of shear diagram: $\frac{1}{2}a(V_A+V_B) = \frac{1}{2}a(a+7.2)\,w$

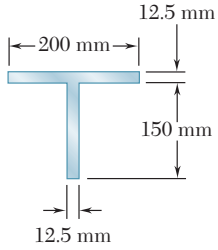
Bending moment at A (also D): $M_A = -\frac{1}{2}a(a+7.2)\,w$

$$-\frac{1}{2}a(a+7.2)(4.485\times10^{3}) = -13.121\times10^{3}$$

(b) Distance a: $\frac{1}{2}a^2 + 3.6a - 8.837 = 0$ $a = 1.935$ m ◀

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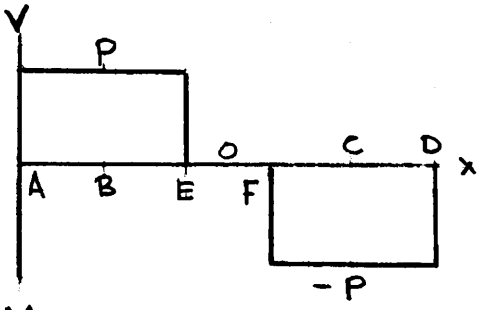


PROBLEM 5.90

Beams *AB*, *BC*, and *CD* have the cross section shown and are pin-connected at *B* and *C*. Knowing that the allowable normal stress is +110 MPa in tension and –150 MPa in compression, determine (*a*) the largest permissible value of **P** if beam *BC* is not to be overstressed, (*b*) the corresponding maximum distance *a* for which the cantilever beams *AB* and *CD* are not overstressed.

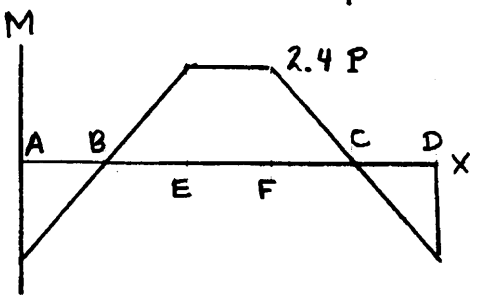
SOLUTION

$$M_B = M_C = 0$$
$$V_B = -V_C = P$$



Area *B* to *E* of shear diagram: $2.4P$

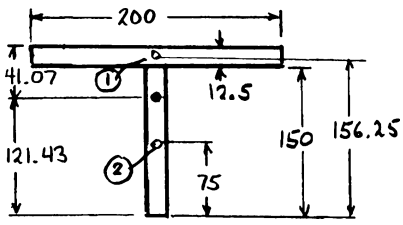
$$M_E = 0 + 2.4P = 2.4P = M_F$$



Centroid and moment of inertia:

Part	$A\,(\text{mm}^2)$	$\bar{y}\,(\text{mm})$	$A\bar{y}\,(\text{mm}^3)$	$d\,(\text{mm})$	$Ad^2\,(\text{mm}^4)$	$\bar{I}\,(\text{mm}^4)$
①	2500	156.25	390,625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140,625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531,250		7.073×10^6	3.548×10^6

$$\bar{Y} = \frac{531,250}{4375} = 121.43 \text{ mm}$$
$$I = \Sigma Ad^2 + \Sigma\bar{I} = 10.621\times10^6 \text{ mm}^4$$



Location	$y\,(\text{mm})$	$I/y\,(10^3\text{ mm}^3)$ ← also (10^{-6} m^3)
Top	41.07	258.6
Bottom	–121.43	–87.47

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PROBLEM 5.90 *(Continued)*

Bending moment limits: $M = -\sigma I/y$

Tension at E and F: $-(110\times10^{6})(-87.47\times10^{-6}) = 9.622\times10^{3}\ \text{N}\cdot\text{m}$

Compression at E and F: $-(-150\times10^{6})(258.6\times10^{-6}) = 38.8\times10^{3}\ \text{N}\cdot\text{m}$

Tension at A and D: $-(110\times10^{6})(258.6\times10^{-6}) = -28.45\times10^{3}\ \text{N}\cdot\text{m}$

Compression at A and D: $-(-150\times10^{6})(-87.47\times10^{-6}) = -13.121\times10^{3}\ \text{N}\cdot\text{m}$

(*a*) Allowable load P: $2.4P = 9.622\times10^{3}$ $P = 4.01\times10^{3}\ \text{N}$ $P = 4.01\ \text{kN}$ ◀

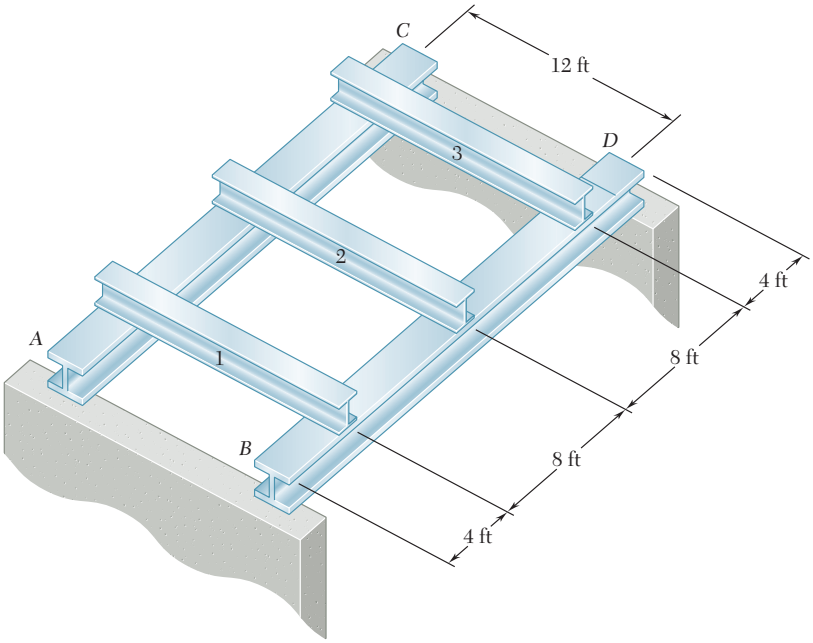
Shear at A: $V_A = P$

Area A to B of shear diagram: $aV_A = aP$

Bending moment at A: $M_A = -aP = -4.01\times10^{3}a$

(*b*) Distance a: $-4.01\times10^{3}a = -13.121\times10^{3}$ $a = 3.27\ \text{m}$ ◀

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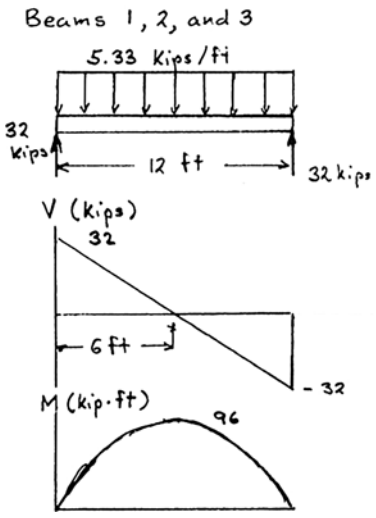
PROBLEM 5.91

Each of the three rolled-steel beams shown (numbered 1, 2, and 3) is to carry a 64-kip load uniformly distributed over the beam. Each of these beams has a 12-ft span and is to be supported by the two 24-ft rolled-steel girders *AC* and *BD*. Knowing that the allowable normal stress for the steel used is 24 ksi, select (*a*) the most economical S shape for the three beams, (*b*) the most economical W shape for the two girders.

SOLUTION

For beams 1, 2, and 3,

$$\text{Maximum } M = \left(\frac{1}{2}\right)(6)(32) = 96 \text{ kip} \cdot \text{ft} = 1152 \text{ kip} \cdot \text{in.}$$



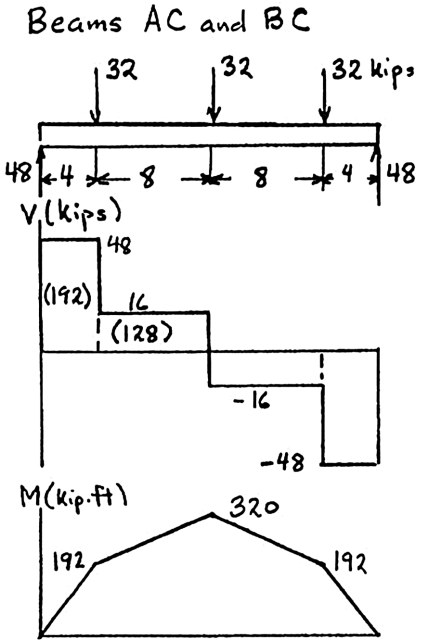
$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{1152}{24} = 48 \text{ in}^3$$

Shape	$S(\text{in}^3)$
S15 × 42.9	59.4
S12 × 50	50.6

(*a*) Use S15 × 42.9. ◀

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PROBLEM 5.91 *(Continued)*



For beams *AC* and *BC*,
areas under shear digram:

$(4)(48) = 192 \text{ kip} \cdot \text{ft}$

$(8)(16) = 128 \text{ kip} \cdot \text{ft}$

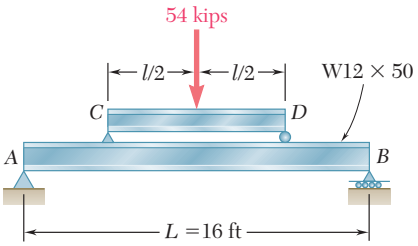
Maximum $M = 192 + 128 = 320 \text{ kip} \cdot \text{ft} = 3840 \text{ kip} \cdot \text{in.}$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{3840}{24} = 160 \text{ in}^3$$

Shape	$S(\text{in}^3)$
$W30 \times 99$	269
$W27 \times 84$	213
$W24 \times 104$	258
$W21 \times 101$	227
$W18 \times 106$	204

(*b*) Use $W27 \times 84$. ◀

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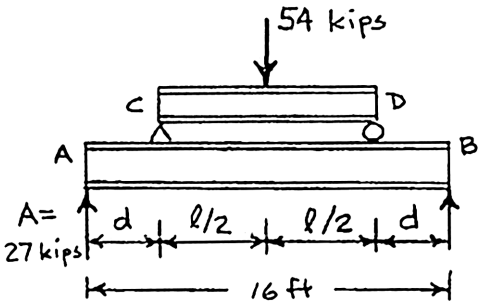


PROBLEM 5.92

A 54-kip is load is to be supported at the center of the 16-ft span shown. Knowing that the allowable normal stress for the steel used is 24 ksi, determine (*a*) the smallest allowable length *l* of beam *CD* if the W12 × 50 beam *AB* is not to be overstressed, (*b*) the most economical W shape that can be used for beam *CD*. Neglect the weight of both beams.

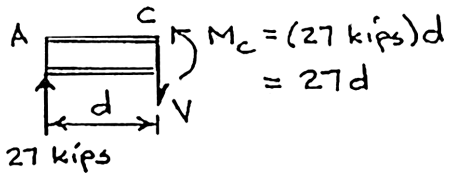
SOLUTION

(*a*)



$$d = 8\,\text{ft} - \frac{l}{2} \qquad l = 16\,\text{ft} - 2d \tag{1}$$

Beam *AB* (Portion *AC*):



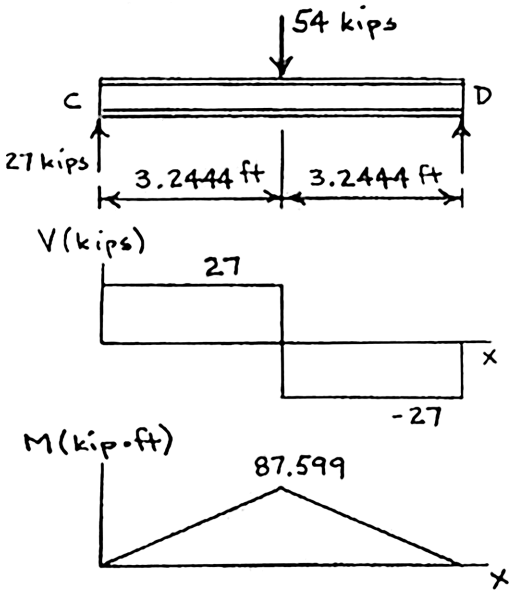
For W12 × 50, $S_x = 64.2\,\text{in}^3$ $\sigma_{\text{all}} = 24$ ksi

$$M_{\text{all}} = \sigma_{\text{all}} S_x = (24)(64.2) = 1540.8\ \text{kip} \cdot \text{in.} = 128.4\ \text{kip} \cdot \text{ft}$$

$$M_C = 27d = 128.4\ \text{kip} \cdot \text{ft} \qquad d = 4.7556\ \text{ft}$$

Using (1), $l = 16 - 2d = 16 - 2(4.7556) = 6.4888\ \text{ft}$ $\quad l = 6.49\ \text{ft}$ ◀

(*b*)



Beam *CD*: $l = 6.4888\ \text{ft}$ $\sigma_{\text{all}} = 24$ ksi

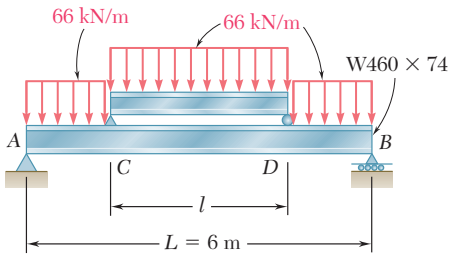
$$S_{\text{min}} = \frac{M_{\text{max}}}{\sigma_{\text{all}}} = \frac{(87.599 \times 12)\ \text{kip} \cdot \text{in.}}{24\ \text{ksi}}$$

$$= 43.800\ \text{in}^3$$

Shape	$S(\text{in}^3)$	
W18 × 35	57.6	
W16 × 31	47.2	←
W14 × 38	54.6	
W12 × 35	45.6	
W10 × 45	49.1	

W16 × 31 ◀

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PROBLEM 5.93

A uniformly distributed load of 66 kN/m is to be supported over the 6-m span shown. Knowing that the allowable normal stress for the steel used is 140 MPa, determine (*a*) the smallest allowable length l of beam *CD* if the W460×74 beam *AB* is not to be overstressed, (*b*) the most economical W shape that can be used for beam *CD*. Neglect the weight of both beams.

SOLUTION

For W460×74,

$$S = 1460\times10^{3}\ \text{mm}^3 = 1460\times10^{-6}\ \text{m}^3$$

$$\sigma_{\text{all}} = 140\ \text{MPa} = 140\times10^{6}\ \text{Pa}$$

$$M_{\text{all}} = S\sigma_{\text{all}} = (1460\times10^{-6})(140\times10^{6})$$

$$= 204.4\times10^{3}\ \text{N}\cdot\text{m} = 204.4\ \text{kN}\cdot\text{m}$$

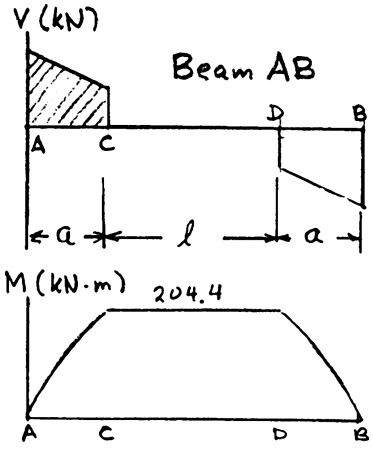
Reactions: By symmetry, $A = B, \quad C = D$

$$+\uparrow \Sigma F_y = 0:\ A + B - (6)(66) = 0$$

$$A = B = 198\ \text{kN} = 198\times10^{3}\ \text{N}$$

$$+\Sigma F_y = 0:\ C + D - 66l = 0$$

$$C = D = (33l)\ \text{kN} \qquad (1)$$



Shear and bending moment in beam *AB*:

$$0 < x < a, \qquad V = 198 - 66x\ \text{kN}$$

$$M = 198x - 33x^2\ \text{kN}\cdot\text{m}$$

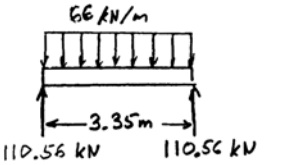
At *C*, $x = a$. $\qquad M = M_{\max}$

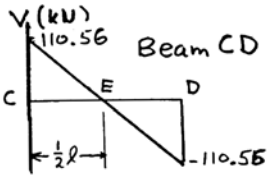
$$M = 198a - 33a^2\ \text{kN}\cdot\text{m}$$

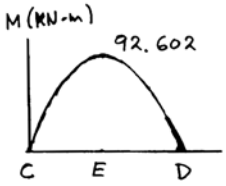
Set $M = M_{\text{all}}$. $\qquad 198a - 33a^2 = 204.4$

$$33a^2 - 198a + 204.4 = 0$$

$$a = \cancel{4.6751\ \text{m}}, \qquad 1.32487\ \text{m}$$







(*a*) By geometry, $l = 6 - 2a = 3.35\ \text{m}$ $\qquad l = 3.35\ \text{m}$ ◀

From (1), $C = D = 110.56\ \text{kN}$

Draw shear and bending moment diagrams for beam *CD*. $V = 0$ at point *E*, the midpoint of *CD*.

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PROBLEM 5.93 ***(Continued)***

Area from A to E:

$$\int V\,dx = \frac{1}{2}(110.560)\left(\frac{1}{2}l\right) = 92.602 \text{ kN} \cdot \text{m}$$

$$M_E = 92.602 \text{ kN} \cdot \text{m} = 92.602 \times 10^3 \text{ N} \cdot \text{m}$$

$$S_{\min} = \frac{M_E}{\sigma_{\text{all}}} = \frac{92.602 \times 10^3}{140 \times 10^6} = 661.44 \times 10^{-6} \text{ m}^3$$

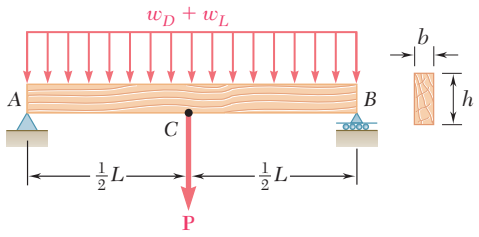
$$= 661.44 \times 10^3 \text{ mm}^3$$

Shape	$S(10^3 \text{ mm}^3)$	
$W410 \times 46.1$	773	
$W360 \times 44$	688	←
$W310 \times 52$	747	
$W250 \times 58$	690	
$W200 \times 71$	708	

(*b*) Use $W360 \times 44$. ◀

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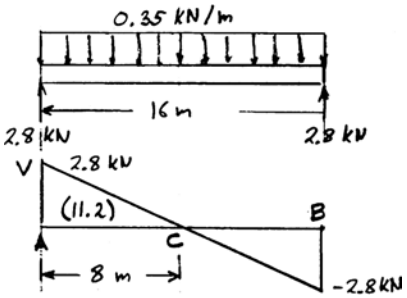
PROBLEM 5.94



A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load **P** applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

SOLUTION

$$L = 16 \text{ m}, \quad w_D = 350 \text{ N/m} = 0.35 \text{ kN/m}$$
$$w_L = 600 \text{ N/m} = 0.6 \text{ kN/m}, \quad P = 6 \text{ kN}$$



Dead load: $R_A = \left(\frac{1}{2}\right)(16)(0.35) = 2.8 \text{ kN}$

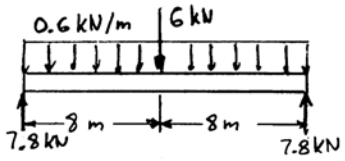
Area A to C of shear diagram: $\left(\frac{1}{2}\right)(8)(2.8) = 11.2 \text{ kN} \cdot \text{m}$

Bending moment at C: $11.2 \text{ kN} \cdot \text{m} = 11.2 \times 10^3 \text{ N} \cdot \text{m}$



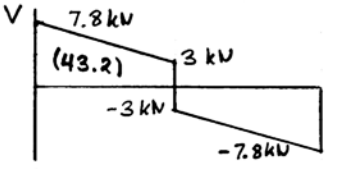
Live load: $R_A = \frac{1}{2}[(16)(0.6) + 6] = 7.8 \text{ kN}$

Shear at C^-: $V = 7.8 - (8)(0.6) = 3 \text{ kN}$



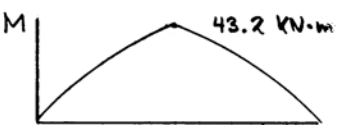
Area A to C of shear diagram: $\left(\frac{1}{2}\right)(8)(7.8 + 3) = 43.2 \text{ kN} \cdot \text{m}$

Bending moment at C: $43.2 \text{ kN} \cdot \text{m} = 43.2 \times 10^3 \text{ N} \cdot \text{m}$



Design: $\gamma_D M_D + \gamma_L M_L = \varphi M_U = \varphi \sigma_U S$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\varphi \sigma_U} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(43.2 \times 10^3)}{(0.9)(50 \times 10^6)}$$
$$= 1.8347 \times 10^{-3} \text{ m}^3 = 1.8347 \times 10^6 \text{ mm}^3$$



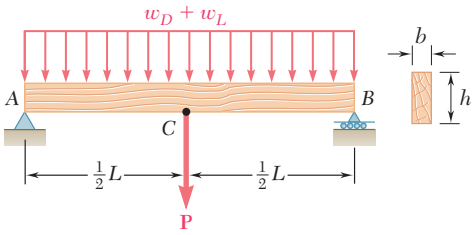
For a rectangular section, $S = \frac{1}{6}bh^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.8347 \times 10^6)}{75}} \qquad h = 383 \text{ mm} \blacktriangleleft$$

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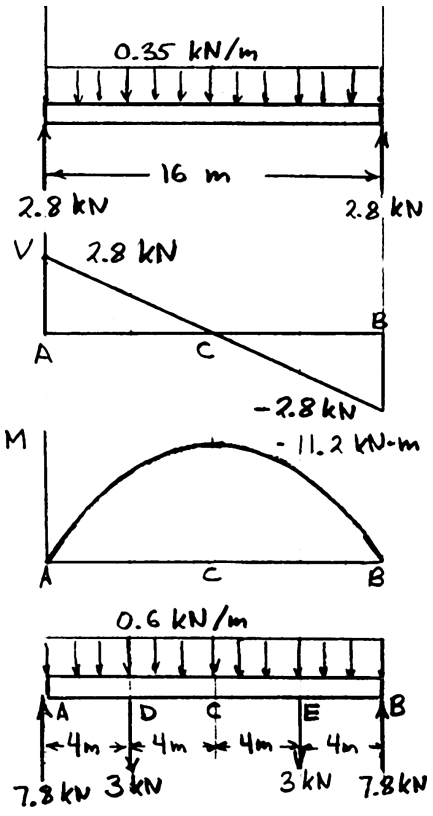
PROBLEM 5.95*

Solve Prob. 5.94, assuming that the 6-kN concentrated load **P** applied to each beam is replaced by 3-kN concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ applied at a distance of 4 m from each end of the beams.



PROBLEM 5.94* A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load **P** applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

SOLUTION



$L = 16$ m, $a = 4$ m, $w_D = 350$ N/m $= 0.35$ kN/m

$w_L = 600$ N/m $= 0.6$ kN/m, $P = 3$ kN

Dead load: $R_A = \left(\frac{1}{2}\right)(16)(0.35) = 2.8$ kN

Area A to C of shear diagram: $\left(\frac{1}{2}\right)(8)(2.8) = 11.2$ kN·m

Bending moment at C: 11.2 kN·m $= 11.2 \times 10^3$ N·m

Live load: $R_A = \frac{1}{2}[(16)(0.6) + 3 + 3] = 7.8$ kN

Shear at D^-: $7.8 - (4)(0.6) = 5.4$ kN

Shear at D^+: $5.4 - 3 = 2.4$ kN

Area A to D: $\left(\frac{1}{2}\right)(4)(7.8 + 5.4) = 26.4$ kN·m

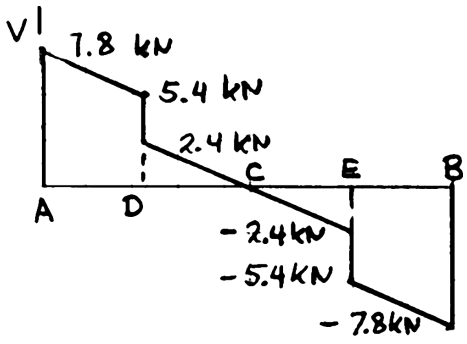
Area D to C: $\left(\frac{1}{2}\right)(4)(2.4) = 4.8$ kN·m

Bending moment at C: $26.4 + 4.8 = 31.2$ kN·m

$= 31.2 \times 10^3$ N·m

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PROBLEM 5.95* ***(Continued)***



Design:
$$\gamma_D M_D + \gamma_L M_L = \varphi M_U = \varphi \sigma_U S$$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\varphi \sigma_U} = \frac{(1.2)(11.2\times10^3) + (1.6)(31.2\times10^3)}{(0.9)(50\times10^6)}$$

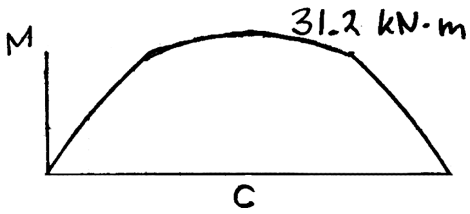
$$= 1.408\times10^{-3}\,\text{m}^3 = 1.408\times10^6\,\text{mm}^3$$

For a rectangular section,

$$S = \frac{1}{6}bh^2$$

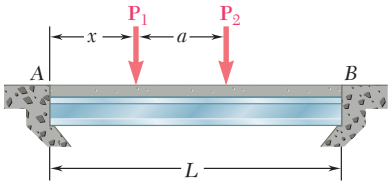
$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.408\times10^6)}{75}}$$

$$h = 336\ \text{mm} \blacktriangleleft$$



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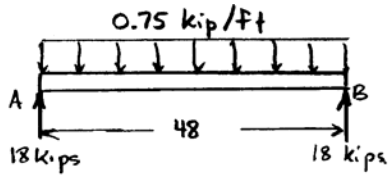
PROBLEM 5.96*



A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength of $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kips/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint:* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/2(P_1 + P_2)$.]

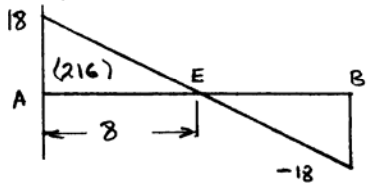
SOLUTION

$$L = 48 \text{ ft} \quad a = 14 \text{ ft} \quad P_1 = 24 \text{ kips}$$
$$P_2 = 6 \text{ kips} \quad W = 0.75 \text{ kip/ft}$$



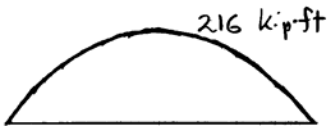
Dead load: $$R_A = R_B = \left(\frac{1}{2}\right)(48)(0.75) = 18 \text{ kips}$$

Area A to E of shear diagram:



$$\left(\frac{1}{2}\right)(8)(18) = 216 \text{ kip} \cdot \text{ft}$$

$$M_{\max} = 216 \text{ kip} \cdot \text{ft} = 2592 \text{ kip} \cdot \text{in. at point } E.$$



Live load: $$u = \frac{aP_2}{2(P_1 + P_2)} = \frac{(14)(6)}{(2)(30)} = 1.4 \text{ ft}$$

$$x = \frac{L}{2} - u = 24 - 1.4 = 22.6 \text{ ft}$$

$$x + a = 22.6 + 14 = 36.6 \text{ ft}$$

$$L - x - a = 48 - 36.6 = 11.4 \text{ ft}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -48R_A + (25.4)(24) + (11.4)(6) = 0$$

$$R_A = 14.125 \text{ kips}$$

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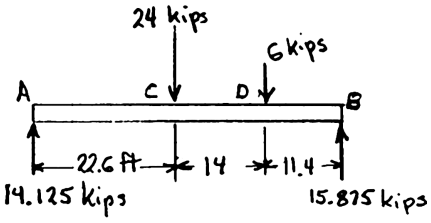
PROBLEM 5.96* *(Continued)*

Shear:

A to *C*: $V = 14.125$ kips

C to *D*: $V = 14.125 - 24 = -9.875$ kips

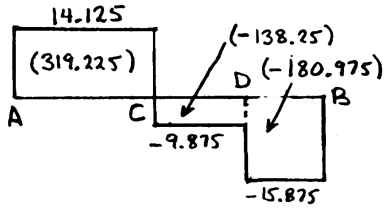
D to *B*: $V = -15.875$ kips



Area:

A to *C*: $(22.6)(14.125) = 319.225$ kip · ft

Bending moment: $M_C = 319.225 \text{ kip} \cdot \text{ft} = 3831 \text{ kip} \cdot \text{in.}$



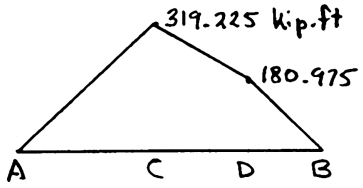
Design:

$$\gamma_D M_D + \gamma_L M_L = \varphi M_U = \varphi \sigma_U S_{\min}$$

$$S_{\min} = \frac{\gamma_D M_D + \gamma_L M_L}{\varphi \sigma_U}$$

$$= \frac{(1.25)(2592) + (1.75)(3831)}{(0.9)(60)}$$

$$= 184.2 \text{ in}^3$$

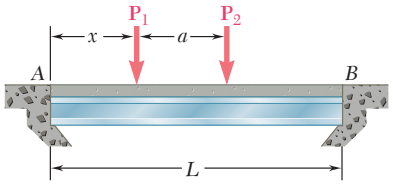


Shape	$S(\text{in}^3)$	
W30×99	269	
W27×84	213	←
W24×104	258	
W21×101	227	
W18×106	204	

Use W27×84. ◀

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PROBLEM 5.97*



Assuming that the front and rear axle loads remain in the same ratio as for the truck of Prob. 5.96, determine how much heavier a truck could safely cross the bridge designed in that problem.

PROBLEM 5.96* A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength of $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kips/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint:* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/2(P_1 + P_2)$.]

SOLUTION

$$L = 48 \text{ ft} \qquad a = 14 \text{ ft} \qquad P_1 = 24 \text{ kips}$$
$$P_2 = 6 \text{ kips} \qquad W = 0.75 \text{ kip/ft}$$

See solution to Prob. 5.96 for calculation of the following:

$$M_D = 2592 \text{ kip} \cdot \text{in.} \qquad M_L = 3831 \text{ kip} \cdot \text{in.}$$

For rolled-steel section W27 × 84, $S = 213 \text{ in}^3$

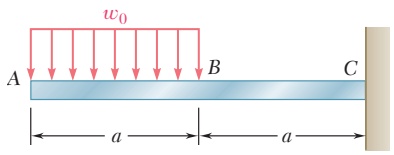
Allowable live load moment M_L^*:

$$\gamma_D M_D + \gamma_L M_L^* = \varphi M_U = \varphi \sigma_U S$$
$$M_L^* = \frac{\varphi \sigma_U S - \gamma_D M_D}{\gamma_L}$$
$$= \frac{(0.9)(60)(213) - (1.25)(2592)}{1.75}$$
$$= 4721 \text{ kip} \cdot \text{in.}$$

Ratio:
$$\frac{M_L^*}{M_L} = \frac{4721}{3831} = 1.232 = 1 + 0.232$$

Increase: 23.2% ◀

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PROBLEM 5.98

(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = w_0 - w_0\langle x-a\rangle^0$$

$$= -\frac{dV}{dx}$$

(*a*) $V = -w_0 x + w_0\langle x-a\rangle^1 = \dfrac{dM}{dx}$ ◀

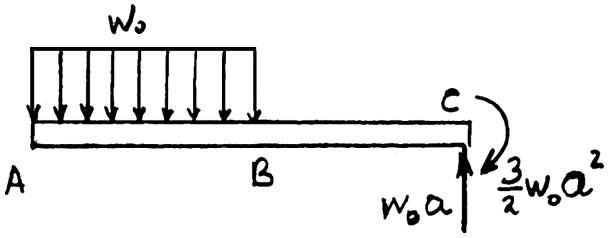
$$M = -\frac{1}{2}w_0 x^2 + \frac{1}{2}w_0\langle x-a\rangle^2 \quad \blacktriangleleft$$

At point *C*, $x = 2a$

(*b*) $M_C = -\dfrac{1}{2}w_0(2a)^2 + \dfrac{1}{2}w_0 a^2$ $\qquad M_C = -\dfrac{3}{2}w_0 a^2$ ◀

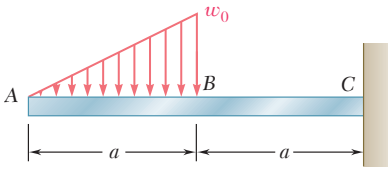
Check: $+\circlearrowleft \Sigma M_C = 0$: $\left(\dfrac{3a}{2}\right)(w_0 a) + M_C = 0$

$$M_C = -\frac{3}{2}w_0 a^2$$



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PROBLEM 5.99



(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = \frac{w_0 x}{a} - w_0\langle x-a\rangle^0 - \frac{w_0}{a}\langle x-a\rangle^1$$

$$= -\frac{dV}{dx}$$

(*a*) $$V = -\frac{w_0 x^2}{2a} + w_0\langle x-a\rangle^1 + \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$ ◀

$$M = -\frac{w_0 x^3}{6a} + \frac{w_0}{2}\langle x-a\rangle^2 + \frac{w_0}{6a}\langle x-a\rangle^3$$ ◀

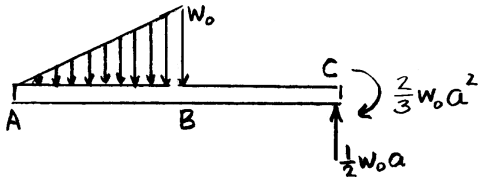
At point C, $x = 2a$

(*b*) $$M_C = -\frac{w_0(2a)^3}{6a} + \frac{w_0 a^2}{2} + \frac{w_0 a^3}{6a}$$

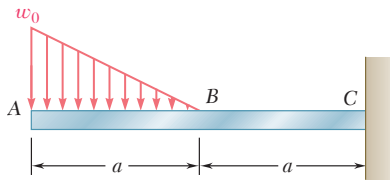
$$M_C = -\frac{2}{3}w_0 a^2$$ ◀

Check: $+\circlearrowleft \Sigma M_C = 0$: $$\left(\frac{4a}{3}\right)\left(\frac{1}{2}w_0 a\right) + M_C = 0$$

$$M_C = -\frac{2}{3}w_0 a^2$$



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PROBLEM 5.100

(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = w_0 - \frac{w_0 x}{a} + \frac{w_0}{a}\langle x-a\rangle^1$$

$$= -\frac{dV}{dx}$$

(*a*) $$V = -w_0 x + \frac{w_0 x^2}{2a} - \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$ ◀

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6a} - \frac{w_0}{6a}\langle x-a\rangle^3$$ ◀

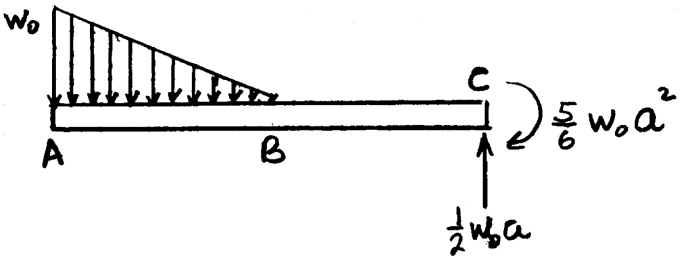
At point *C*, $x = 2a$

(*b*) $$M_C = -\frac{w_0(2a)^2}{2} + \frac{w_0(2a)^3}{6a} - \frac{w_0 a^3}{6a}$$

$$M_C = -\frac{5}{6} w_0 a^2$$ ◀

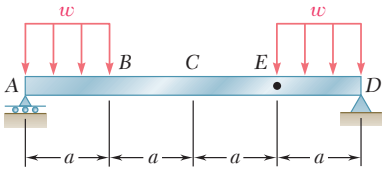
Check: $+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{5}{3}a\right)\left(\frac{1}{2}w_0 a\right) + M_C = 0$

$$M_C = -\frac{5}{6} w_0 a^2$$



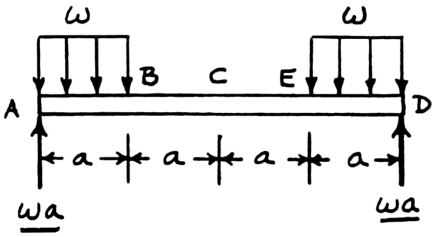
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PROBLEM 5.101



(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E*, and check your answer by drawing the free-body diagram of the portion of the beam to the right of *E*.

SOLUTION



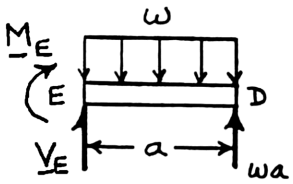
$$w = w - w\langle x-a\rangle^0 + w\langle x-3a\rangle^0$$

(*a*) $$V = wa - \int w\,dx$$

$$= wa - wx + w\langle x-a\rangle^1 - w\langle x-3a\rangle^1 \quad \blacktriangleleft$$

$$M = \int V\,dx$$

$$= wax - wx^2/2 + (w/2)\langle x-a\rangle^2 - (w/2)\langle x-3a\rangle^2 \quad \blacktriangleleft$$



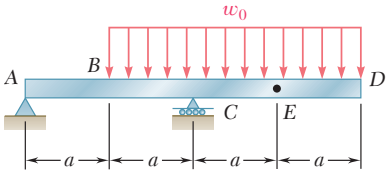
(*b*) At point *E*, $x = 3a$

$$M_E = wa(3a) - w(3a)^2/2 + (w/2)(2a)^2 \qquad = wa^2/2 \quad \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_E = 0: \quad wa(a) - (wa)\left(\frac{a}{2}\right) - M_E = 0 \qquad M_E = wa^2/2 \qquad \text{(Checks)}$$

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PROBLEM 5.102



(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for M to determine the bending moment at point E, and check your answer by drawing the free-body diagram of the portion of the beam to the right of E.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -2aA - \left(\frac{a}{2}\right) + (3aw_0) = 0 \qquad A = -\frac{3}{4}w_0 a$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 2aC - \left(\frac{5a}{2}\right) + (3aw_0) = 0 \qquad C = \frac{15}{4}w_0 a$$

$$w = w_0\langle x-a\rangle^0 = -\frac{dV}{dx}$$

(*a*) $$V = -w_0\langle x-a\rangle^1 - \frac{3}{4}w_0 a + \frac{15}{4}w_0 a\langle x-2a\rangle^0 = \frac{dM}{dx}$$ ◀

$$M = -\frac{1}{2}w_0\langle x-a\rangle^2 - \frac{3}{4}w_0 ax + \frac{15}{4}w_0 a\langle x-2a\rangle^1 + 0$$ ◀

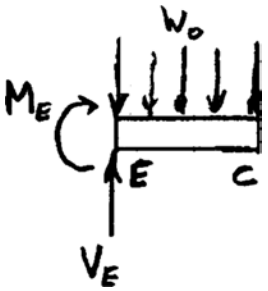
At point E, $x = 3a$

(*b*) $$M_E = -\frac{1}{2}w_0(2a)^2 - \frac{3}{4}w_0 a(3a) + \frac{15}{4}w_0 a(a)$$

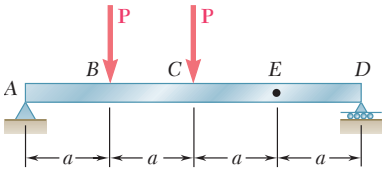
$$M_E = -\frac{1}{2}w_0 a^2$$ ◀

Check: $$+\circlearrowleft \Sigma M_E = 0: \quad -M_E - \frac{a}{2}(w_0 a) = 0$$

$$M_E = -\frac{1}{2}w_0 a^2$$



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PROBLEM 5.103

(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E*, and check your answer by drawing the free-body diagram of the portion of the beam to the right of *E*.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -4aA + 3aP + 2aP = 0 \qquad A = 1.25P$$

(*a*) $V = 1.25P - P\langle x-a\rangle^0 - P\langle x-2a\rangle^0$ ◀

$M = 1.25Px - P\langle x-a\rangle^1 - P\langle x-2a\rangle^1$ ◀

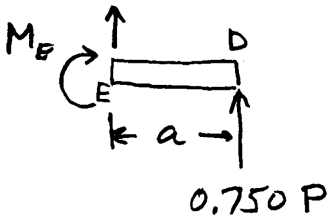
(*b*) At point *E*, $x = 3a$

$M_E = 1.25P(3a) - P(2a) - P(a) = 0.750Pa$ ◀

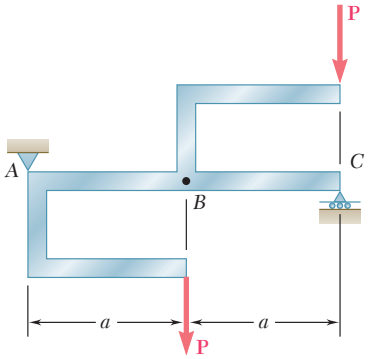
Reaction: $+\uparrow \Sigma F_y = 0: \quad A - P - P + D = 0 \qquad \boldsymbol{D} = 0.750P \uparrow$

$+\circlearrowleft \Sigma M_E = 0: \quad -M_E + 0.750Pa = 0$

$M_E = 0.750Pa$



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PROBLEM 5.104

(*a*) Using singularity functions, write the equations for the shear and bending moment for beam *ABC* under the loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment just to the right of point *B*.

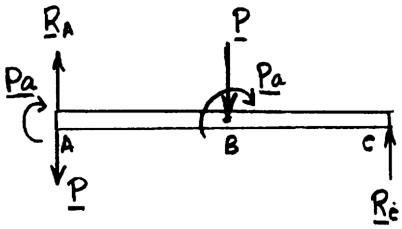
SOLUTION

(*a*) $\sum M_C = 0: \quad (2a)P + aP - 2(Pa) + 2aR_A = 0$

$$R_A = \frac{1}{2}P$$

$$V = (R_A - P) - P\langle x - a\rangle^0$$

$$= -\frac{1}{2}P - P\langle x - a\rangle^0 \quad \blacktriangleleft$$



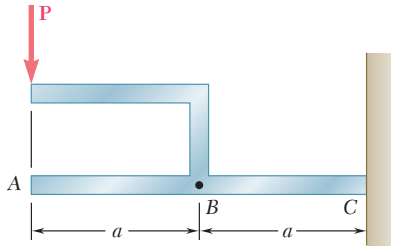
$$\frac{dM}{dx} = -\frac{1}{2}P - P\langle x - a\rangle^0$$

$$M = -\frac{1}{2}Px - P\langle x - a\rangle^1 + Pa + Pa\langle x - a\rangle^0$$

(*b*) Just to the right of point *B*, $x = a^+$

$$M = -\frac{1}{2}Pa - 0 + Pa + Pa = \frac{3}{2}Pa \quad \blacktriangleleft$$

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PROBLEM 5.105

(*a*) Using singularity functions, write the equations for the shear and bending moment for beam *ABC* under the loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment just to the right of point *B*.

SOLUTION

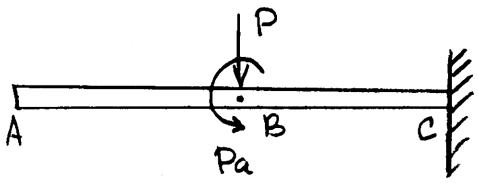
(*a*) $V = -P\langle x-a\rangle^0$ ◀

$$\frac{dM}{dx} = -P\langle x-a\rangle^0$$

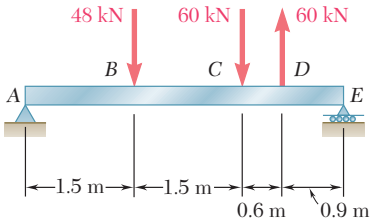
$M = -P\langle x-a\rangle^1 - Pa\langle x-a\rangle^0$ ◀

Just to the right of B, $x = a^1$.

(*b*) $M = -\,0 - Pa$ $M = -Pa$ ◀



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PROBLEM 5.106

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_E = 0: \quad -4.5R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$$

$$R_A = 40 \text{ kN}$$

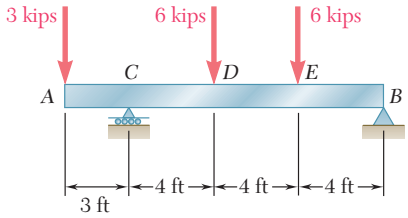
(*a*) $V = 40 - 48\langle x - 1.5\rangle^0 - 60\langle x - 3.0\rangle^0 + 60\langle x - 3.6\rangle^0 \text{ kN}$ ◀

$M = 40x - 48\langle x - 1.5\rangle^1 - 60\langle x - 3.0\rangle^1 + 60\langle x - 3.6\rangle^1 \text{ kN} \cdot \text{m}$ ◀

Pt.	x(m)	M (kN · m)
A	0	0
B	1.5	$(40)(1.5) = 60$ kN · m
C	3.0	$(40)(3.0) - (48)(1.5) = 48$ kN · m
D	3.6	$(40)(3.6) - (48)(2.1) - (60)(0.6) = 7.2$ kN · m
E	4.5	$(40)(4.5) - (48)(3.0) - (60)(1.5) + (60)(0.9) = 0$

(*b*) $M_{\max} = 60 \text{ kN} \cdot \text{m}$ ◀

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PROBLEM 5.107

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \ (15)(3) - 12C + (8)(6)C + (4)(6) = 0$$

$$\boldsymbol{C} = 9.75 \text{ kips} \uparrow$$

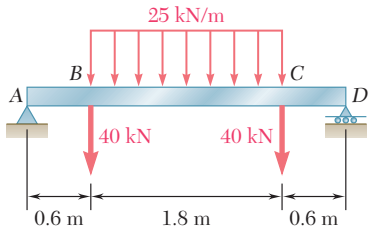
(*a*) $V = -3 + 9.75\langle x-3\rangle^0 - 6\langle x-7\rangle^0 - 6\langle x-11\rangle^0$ kips ◀

$M = -3x + 9.75\langle x-3\rangle^1 - 6\langle x-7\rangle^1 - 6\langle x-11\rangle^1$ kip · ft ◀

Pt.	x(ft)	M(kip · ft)
A	0	0
C	3	$-(3)(3) = -9$
D	7	$-(3)(7) + (9.75)(4) = 18$
E	11	$-(3)(11) + (9.75)(8) - (6)(4) = 21$ ← maximum
B	15	$-(3)(15) + (9.75)(12) - (6)(8) - (6)(4) = 0$

(*b*) $|M|_{\max} = 21.0$ kip · ft ◀

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PROBLEM 5.108

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

(*a*) By symmetry, $R_A = R_D$

$$+\uparrow \sum F_y = 0: \quad R_A + R_D - 40 - (1.8)(25) - 40 = 0$$

$$R_A = R_D = 62.5 \text{ kN}$$

$$w = 25\langle x - 0.6\rangle^0 - 25\langle x - 2.4\rangle^0 = -\frac{dV}{dx}$$

$$V = 62.5 - 25\langle x - 0.6\rangle^1 + 25\langle x - 2.4\rangle^1 - 40\langle x - 0.6\rangle^0 - 40\langle x - 2.4\rangle^0 \text{ kN} \quad \blacktriangleleft$$

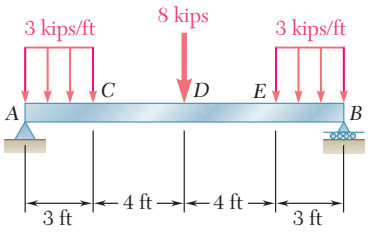
$$M = 62.5x - 12.5\langle x - 0.6\rangle^2 + 12.5\langle x - 2.4\rangle^2 - 40\langle x - 0.6\rangle^1 - 40\langle x - 2.4\rangle^1 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

(*b*) Locate point where $V = 0$. Assume $0.6 < x^* < 1.8$

$$0 = 62.5 - 25(x^* - 0.6) + 0 - 40 - 0 \qquad x^* = 1.5 \text{ m}$$

$$M = (62.5)(1.5) - (12.5)(0.9)^2 + 0 - (40)(0.9) - 0 = 47.6 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

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PROBLEM 5.109

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -14A + (12.5)(3)(3) + (7)(8) + (1.5)(3)(3) = 0$$

$$\boldsymbol{A} = 13 \text{ kips} \uparrow$$

$$w = 3 - 3\langle x-3\rangle^0 + 3\langle x-11\rangle^0 = -\frac{dV}{dx}$$

(*a*) $$V = 13 - 3x + 3\langle x-3\rangle^1 - 8\langle x-7\rangle^0 - 3\langle x-11\rangle^1 \text{ kips}$$ ◀

$$M = 13x - 1.5x^2 + 1.5\langle x-3\rangle^2 - 8\langle x-7\rangle^1 - 1.5\langle x-11\rangle^2 \text{ kip}\cdot\text{ft}$$ ◀

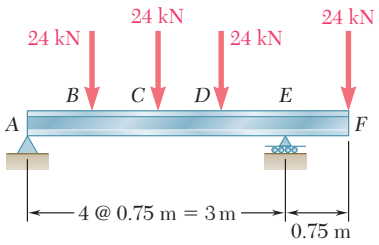
$$V_C = 13 - (3)(3) = 4 \text{ kips}$$
$$V_{D^-} = 13 - (3)(7) + (3)(4) = 4 \text{ kips}$$
$$V_{D^+} = 13 - (3)(7) + (3)(4) - 8 = -4 \text{ kips}$$
$$V_E = 13 - (3)(11) + (3)(8) - 8 = -4 \text{ kips}$$
$$V_B = 13 - (3)(14) + (3)(11) - 8 - (3)(3) = -13 \text{ kips}$$

(*b*) Note that V changes sign at D.

$$|M|_{\max} = M_D = (13)(7) - (1.5)(7)^2 + (1.5)(4)^2 - 0 - 0$$

$$|M|_{\max} = 41.5 \text{ kip}\cdot\text{ft}$$ ◀

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PROBLEM 5.110

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_E = 0: \quad -3R_A + (2.25)(24) - (1.5)(24) - (0.75)(24) + (0.75)(24) = 0$$

$$R_A = 30 \text{ kips}$$

$$+\circlearrowright \Sigma M_A = 0: \quad -(0.75)(24) - (1.5)(24) - (2.25)(24) + 3R_E - (3.75)(24) = 0$$

$$R_E = 66 \text{ kips}$$

(*a*) $V = 30 - 24\langle x - 0.75\rangle^0 - 24\langle x - 1.5\rangle^0 - 24\langle x - 2.25\rangle^0 + 66\langle x - 3\rangle^0 \text{ kN}$ ◀

$M = 30x - 24\langle x - 0.75\rangle^1 - 24\langle x - 1.5\rangle^1 - 24\langle x - 2.25\rangle^1 + 66\langle x - 3\rangle^1 \text{ kN} \cdot \text{m}$ ◀

Point	x(m)	M (kN · m)
B	0.75	$(30)(0.75) = 22.5$ kN · m
C	1.5	$(30)(1.5) - (24)(0.75) = 27$ kN · m
D	2.25	$(30)(2.25) - (24)(1.5) - (24)(0.75) = 13.5$ kN · m
E	3.0	$(30)(3.0) - (24)(2.25) - (24)(1.5) - (24)(0.75) = -18$ kN · m
F	3.75	$(30)(3.75) - (24)(3.0) - (24)(2.25) - (24)(1.5) + (66)(0.75) = 0$ ✓

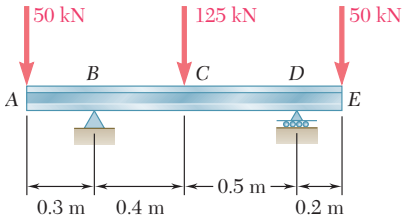
Maximum $|M| = 27 \text{ kN} \cdot \text{m} = 27 \times 30^3 \text{ N} \cdot \text{m}$

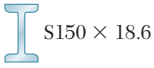
For rolled-steel section W250 × 28.4, $S = 308 \times 10^3 \text{ mm}^3 = 308 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress:

$$\sigma = \frac{|M|}{S} = \frac{27 \times 10^3}{308 \times 10^{-6}} = 87.7 \times 10^6 \text{ Pa} \qquad \sigma = 87.7 \text{ MPa}$$ ◀

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PROBLEM 5.111

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad (1.2)(50) - 0.9B + (0.5)(125) - (0.2)(50) = 0$$

$$\boldsymbol{B} = 125 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (0.3)(50) - (0.4)(125) + 0.9D - (1.1)(50) = 0$$

$$\boldsymbol{D} = 100 \text{ kN} \uparrow$$

(*a*) $V = -50 + 125\langle x - 0.3\rangle^0 - 125\langle x - 0.7\rangle^0 + 100\langle x - 1.2\rangle^0 \text{ kN}$ ◀

$M = -50x + 125\langle x - 0.3\rangle^1 - 125\langle x - 0.7\rangle^1 + 100\langle x - 1.2\rangle^1 \text{ kN} \cdot \text{m}$ ◀

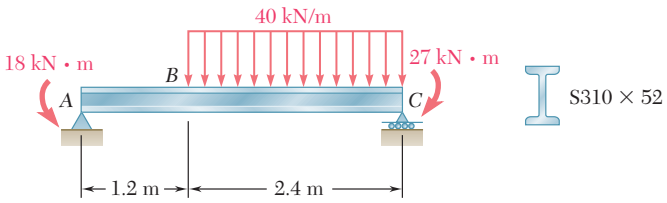
Point	x(m)	M(kN · m)
B	0.3	$-(50)(0.3) + 0 - 0 + 0 = -15$ kN · m
C	0.7	$-(50)(0.7) + (125)(0.4) - 0 + 0 = 15$ kN · m
D	1.2	$-(50)(1.2) + (125)(0.9) - (125)(0.5) + 0 = -10$ kN · m
E	1.4	$-(50)(1.4) + (125)(1.1) - (125)(0.7) + (100)(0.2) = 0$ (checks)

Maximum $|M| = 15 \text{ kN} \cdot \text{m} = 15 \times 10^3 \text{ N} \cdot \text{m}$

For S150×18.6 rolled-steel section, $S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{15 \times 10^3}{120 \times 10^{-6}} = 125 \times 10^6 \text{ Pa}$ $\sigma = 125.0 \text{ MPa}$ ◀

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PROBLEM 5.112

(*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft M_c = 0: \quad 18 - 3.6A + (1.2)(2.4)(40) - 27 = 0$$

$$\boldsymbol{A} = 29.5\ \text{kN} \uparrow$$

$$V = 29.5 - 40\langle x - 1.2\rangle^1\ \text{kN}$$

Point D. $\quad V = 0 \qquad 29.5 - 40(x_D - 1.2) = 0$

$$x_D = 1.9375\ \text{m}$$

$$M = -18 + 29.5x - 20\langle x - 1.2\rangle^2\ \text{kN} \cdot \text{m}$$

$$M_A = -18\ \text{kN} \cdot \text{m}$$

$$M_D = -18 + (29.5)(1.9375) - (20)(0.7375)^2 = 28.278\ \text{kN} \cdot \text{m}$$

$$M_E = -18 + (29.5)(3.6) - (20)(2.4)^2 = -27\ \text{kN} \cdot \text{m}$$

(*a*) Maximum $|M| = 28.3\ \text{kN} \cdot \text{m}$ at $x = 1.938\ \text{m}$ ◀

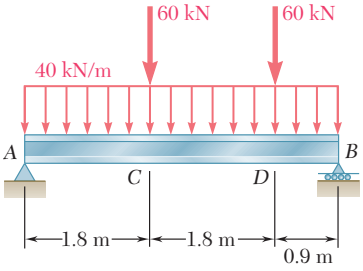
For S310 × 52 rolled-steel section, $\quad S = 624 \times 10^3\ \text{mm}^3$

$$= 624 \times 10^{-6}\ \text{m}^3$$

(*b*) Normal stress: $\quad \sigma = \dfrac{|M|}{S} = \dfrac{28.278 \times 10^3}{624 \times 10^{-6}} = 45.3 \times 10^6\ \text{Pa}$

$\sigma = 45.3\ \text{MPa}$ ◀

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PROBLEM 5.113

(*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -4.5A + (2.25)(4.5)(40) + (2.7)(60) + (0.9)(60) = 0$$

$$\mathbf{A} = 138 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(2.25)(4.5)(40) - (1.8)(60) - (3.6)(60) + 4.5B = 0$$

$$\mathbf{B} = 162 \text{ kN} \uparrow$$

$$w = 40 \text{ kN/m} = \frac{dV}{dx}$$

$$V = -40x + 138 - 60\langle x - 1.8\rangle^0 - 60\langle x - 3.6\rangle^0 = \frac{dM}{dx}$$

$$M = -20x^2 - 138x - 60\langle x - 1.8\rangle^1 - 60\langle x - 3.6\rangle^1$$

$$V_C^+ = -(40)(1.8) + 138 - 60 = 6 \text{ kN}$$

$$V_D^- = -(40)(3.6) + 138 - 60 = -66 \text{ kN}$$

Locate point E where $V = 0$. It lies between C and D.

$$V_E = -40x_E + 138 - 60 + 0 = 0 \qquad x_E = 1.95 \text{ m}$$

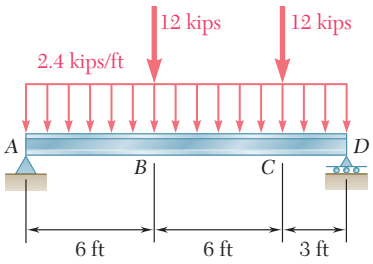
$$M_E = -(20)(1.95)^2 + (138)(1.95) - (60)(1.95 - 1.8) = 184 \text{ kN} \cdot \text{m}$$

(*a*) $|M|_{\max} = 184 \text{ kN} \cdot \text{m} = 184 \times 10^3 \text{ N} \cdot \text{m}$ at $x = 1.950 \text{ m}$ ◀

For W530 × 66 rolled-steel section, $S = 1340 \times 10^3 \text{ mm}^3 = 1340 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress: $\sigma = \dfrac{|M|_{\max}}{S} = \dfrac{184 \times 10^3}{1340 \times 10^{-6}} = 137.3 \times 10^6 \text{ Pa}$ $\sigma = 137.3 \text{ MPa}$ ◀

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PROBLEM 5.114

A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable normal stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that can be used.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -15R_A + (7.5)(15)(2.4) - (9)(12) - (3)(12) = 0$$

$$R_A = 27.6 \text{ kips}$$

$$w = 2.4 \text{ kips/ft} = -\frac{dV}{dx}$$

$$V = 27.6 - 2.4x - 12\langle x - 6\rangle^0 - 12\langle x - 12\rangle^0 \text{ kips}$$

$$V_{B^-} = 27.6 - (2.4)(6) = 13.2 \text{ kips}$$

$$\left.\begin{aligned} V_{B^+} &= 27.6 - (2.4)(6) - 12 = 1.2 \text{ kips} \\ V_{C^-} &= 27.6 - (2.4)(12) - 12 = -13.2 \text{ kips} \end{aligned}\right\} \text{Point where } V = 0 \text{ lies between } B \text{ and } C.$$

Locate point E where $V = 0$.

$$0 = 27.6 - 2.4x_E - 12 - 0 \qquad x_E = 6.50 \text{ ft}$$

$$M = 27.6x - 1.2x^2 - 12\langle x - 6\rangle^1 - 12\langle x - 12\rangle^1 \text{ kip} \cdot \text{ft}$$

At point E, $(x = 6.5 \text{ ft}) \quad M = (27.6)(6.5) - (1.2)(6.5)^2 - (12)(0.5) - 0$

$$= 122.70 \text{ kip} \cdot \text{ft} = 1472.40 \text{ kip} \cdot \text{in.}$$

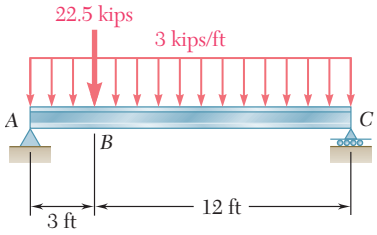
Maximum $|M|$ 122.7 kip · ft at $x = 6.50$ ft ◀

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{1472.40}{24} = 61.35 \text{ in}^3$$

Shape	$S(\text{in}^3)$	
W21×44	81.6	
W18×50	88.9	
W16×40	64.7	←
W14×43	62.6	
W12×50	64.2	
W10×68	75.7	

Answer: W16×40 ◀

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PROBLEM 5.115

A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable normal stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that can be used.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -15R_A + (7.5)(15)(3) + (12)(22.5) = 0$$

$$\mathbf{R}_A = 40.5 \text{ kips} \uparrow$$

$$w = 3 \text{ kips/ft} = -\frac{dV}{dx}$$

$$V = 40.5 - 3x - 22.5\langle x - 3\rangle^0 \text{ kips}$$

$$M = 40.5x - 1.5x^2 - 22.5\langle x - 3\rangle^1 \text{ kip} \cdot \text{ft}$$

(*a*) Location of point D where $V = 0$. Assume $3 \text{ ft} < x_D < 12 \text{ ft}$.

$$0 = 40.5 - 3x_D - 22.5 \qquad x_D = 6 \text{ ft}$$

At point D, $(x = 6 \text{ ft})$. $M = (40.5)(6) - (1.5)(6)^2 - (22.5)(3)$

$= 121.5 \text{ kip} \cdot \text{ft} = 1458 \text{ kip} \cdot \text{in.}$

Maximum $|M|$: $|M|_{\max} = 121.5 \text{ kip} \cdot \text{ft}$ at $x = 6.00 \text{ ft}$ ◀

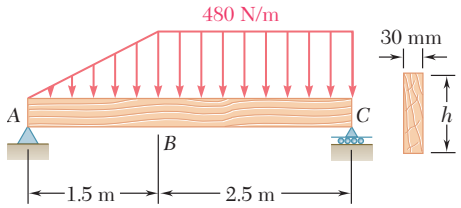
$$S_{\min} = \frac{M}{\sigma_{\text{all}}} = \frac{1458}{24} = 60.75 \text{ in}^3$$

(*b*)

Shape	S (in^3)	
W21×44	81.6	
W18×50	88.9	
W16×40	64.7	←
W14×43	62.6	
W12×50	64.2	
W10×68	75.7	

Wide-flange shape: W16×40 ◀

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PROBLEM 5.116

A timber beam is being designed with supports and loads as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with an allowable normal stress of 12 MPa and a rectangular cross section of 30-mm width and depth h varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.

SOLUTION

$$480 \text{ N/m} = 0.48 \text{ kN/m}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -4R_A + (3)\left(\frac{1}{2}\right)(1.5)(0.48) + (1.25)(2.5)(0.48) = 0$$

$$\mathbf{R}_A = 0.645 \text{ kN} \uparrow$$

$$w = \frac{0.48}{1.5}x - \frac{0.48}{1.5}\langle x-1.5\rangle^1 = 0.32x - 0.32\langle x-1.5\rangle^1 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.645 - 0.16x^2 + 0.16\langle x-1.5\rangle^2 \text{ kN}$$

$$M = 0.645x - 0.053333x^3 + 0.053333\langle x-1.5\rangle^3 \text{ kN}\cdot\text{m}$$

(*a*) Locate point D where $V = 0$.

Assume $1.5 \text{ m} < x_D < 4 \text{ m}$.

$$0 = 0.645 - 0.16x_D^2 + 0.16(x_D - 1.5)^2$$

$$= 0.645 - \cancel{0.16x_D^2} + \cancel{0.16x_D^2} - 0.48x_D + 0.36$$

$$x_D = 2.0938 \text{ m} \qquad x_D = 2.09 \text{ m} \blacktriangleleft$$

At point D,

$$M_D = (0.645)(2.0938) - (0.053333)(2.0938)^3 + (0.053333)(0.59375)^3$$

$$M_D = 0.87211$$

$$M_D = 0.872 \text{ kN}\cdot\text{m} \blacktriangleleft$$

$$S_{\min} = \frac{M_D}{\sigma_{\text{all}}} = \frac{0.87211\times10^3}{12\times10^6} = 72.676\times10^{-6} \text{ m}^3 = 72.676\times10^3 \text{ mm}^3$$

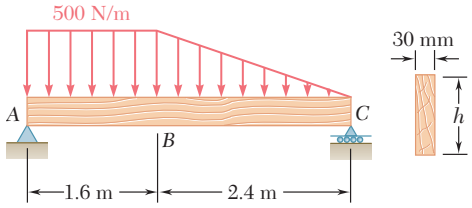
For a rectangular cross section, $S = \frac{1}{6}bh^2 \qquad h = \sqrt{\frac{6S}{b}}$

$$h_{\min} = \sqrt{\frac{(6)(72.676\times10^3)}{30}} = 120.562 \text{ mm}$$

(*b*) At next larger 10-mm increment, $h = 130 \text{ mm} \blacktriangleleft$

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PROBLEM 5.117



A timber beam is being designed with supports and loads as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of 30-mm width and depth h varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.

SOLUTION

$$500 \text{ N/m} = 0.5 \text{ kN/m}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -4R_A + (3.2)(1.6)(0.5) + (1.6)\left(\frac{1}{2}\right)(2.4)(0.5) = 0 \qquad \mathbf{R}_A = 0.880 \text{ kN} \uparrow$$

$$w = 0.5 - \frac{0.5}{2.4}\langle x - 1.6\rangle^1 = 0.5 - 0.20833\langle x - 1.6\rangle^1 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.880 - 0.5x + 0.104167\langle x - 1.6\rangle^2 \text{ kN}$$

$$V_A = 0.880 \text{ kN}$$

$$\left.\begin{aligned} V_B &= 0.880 - (0.5)(1.6) = 0.080 \text{ kN} \\ V_C &= 0.880 - (0.5)(4) + (0.104167)(2.4)^2 = -0.520 \text{ kN} \end{aligned}\right\} \text{ Sign change}$$

Locate point D (between B and C) where $V = 0$. $\qquad 0 = 0.880 - 0.5x_D + 0.104167\,(x_D - 1.6)^2$

$$0.104167x_D^2 - 0.83333x_D + 1.14667 = 0$$

$$x_D = \frac{0.83333 \pm \sqrt{(0.83333)^2 - (4)(0.104167)(1.14667)}}{(2)(0.104167)} = \cancel{6.2342}, \quad 1.7658 \text{ m} \qquad \blacktriangleleft$$

$$M = 0.880x - 0.25x^2 + 0.347222\langle x - 1.6\rangle^3 \text{ kN} \cdot \text{m}$$

$$M_D = (0.880)(1.7658) - (0.25)(1.7658)^2 + (0.34722)(0.1658)^3 = 0.77597 \text{ kN} \cdot \text{m}$$

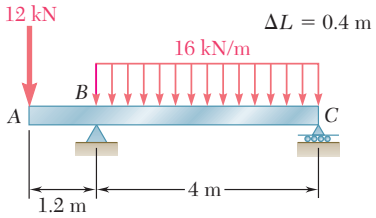
(*a*) $\qquad M_{\max} = 0.776 \text{ kN} \cdot \text{m} \quad \text{at} \quad x = 1.766 \text{ m} \qquad \blacktriangleleft$

$$S_{\min} = \frac{M_{\max}}{\sigma_{\text{all}}} = \frac{0.77597 \times 10^3}{12 \times 10^6} = 64.664 \times 10^{-6} \text{ m}^3 = 64.664 \times 10^3 \text{ mm}^3$$

For a rectangular cross section, $\quad S = \frac{1}{6}bh^2 \qquad h = \sqrt{\frac{6S}{b}} \qquad h_{\min} = \sqrt{\frac{(6)(64.664 \times 10^3)}{30}} = 113.7 \text{ mm}$

(*b*) At next higher 10-mm increment, $\qquad h = 120 \text{ mm} \qquad \blacktriangleleft$

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PROBLEM 5.118

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad (5.2)(12) - 4B + (2)(4)(16) = 0$$

$$\mathbf{B} = 47.6 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (1.2)(12) - (2)(4)(16) + 4C = 0$$

$$\mathbf{C} = 28.4 \text{ kN} \uparrow$$

$$w = 16\langle x - 1.2\rangle^0 = -\frac{dV}{dx}$$

$$V = -16\langle x - 1.2\rangle^1 - 12 + 47.6\langle x - 1.2\rangle^0 \blacktriangleleft$$

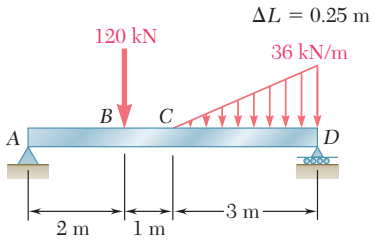
$$M = -8\langle x - 1.2\rangle^2 - 12x + 47.6\langle x - 1.2\rangle^1 \blacktriangleleft$$

x	V	M
m	kN	kN · m
0.0	–12.0	0.00
0.4	–12.0	–4.80
0.8	–12.0	–9.60
1.2	35.6	–14.40
1.6	29.2	–1.44
2.0	22.8	8.96
2.4	16.4	16.80
2.8	10.0	22.08
3.2	3.6	24.80
3.6	–2.8	24.96
4.0	–9.2	22.56
4.4	–15.6	17.60
4.8	–22.0	10.08
5.2	–28.4	–0.00

$$|V|_{\max} = 35.6 \text{ kN} \blacktriangleleft$$

$$|M|_{\max} = 25.0 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 5.119

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -6R_A + (4)(120) + (1)\left(\frac{1}{2}\right)(3)(36) = 0$$

$$R_A = 89 \text{ kN}$$

$$w = \frac{36}{3}\langle x-3\rangle^1 = 12\langle x-3\rangle^1$$

$$V = 89 - 120\langle x-2\rangle^0 - 6\langle x-3\rangle^2 \text{ kN} \quad \blacktriangleleft$$

$$M = 89x - 120\langle x-2\rangle^1 - 2\langle x-3\rangle^3 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

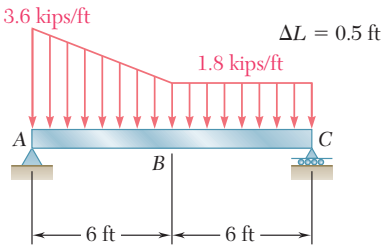
x	V	M
m	kN	kN · m
0.0	89.0	0.0
0.3	89.0	22.3
0.5	89.0	44.5
0.8	89.0	66.8
1.0	89.0	89.0
1.3	89.0	111.3
1.5	89.0	133.5
1.8	89.0	155.8
2.0	–31.0	178.0
2.3	–31.0	170.3
2.5	–31.0	162.5
2.8	–31.0	154.8
3.0	–31.0	147.0
3.3	–31.4	139.2
3.5	–32.5	131.3
3.8	–34.4	122.9
4.0	–37.0	114.0
4.3	–40.4	104.3
4.5	–44.5	93.8
4.8	–49.4	82.0

x	V	M
m	kN	kN · m
5.0	–55.0	69.0
5.3	–61.4	54.5
5.5	–68.5	38.3
5.8	–76.4	20.2
6.0	–85.0	–0.0

$$|V|_{\max} = 89.0 \text{ kN} \quad \blacktriangleleft$$

$$|M|_{\max} = 178.0 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

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PROBLEM 5.120

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -12R_A + (6)(12)(1.8) + (10)\left(\frac{1}{2}\right)(6)(1.8) = 0$$

$$R_A = 15.3 \text{ kips}$$

$$w = 3.6 - \frac{1.8}{6}x + \frac{1.8}{6}\langle x-6\rangle^1$$

$$= 3.6 - 0.3x + 0.3\langle x-6\rangle^1$$

$$V = 15.3 - 3.6x + 0.15x^2 - 0.15\langle x-6\rangle^2 \text{ kips} \blacktriangleleft$$

$$M = 15.3x - 1.8x^2 + 0.05x^3 - 0.05\langle x-6\rangle^3 \text{ kip}\cdot\text{ft} \blacktriangleleft$$

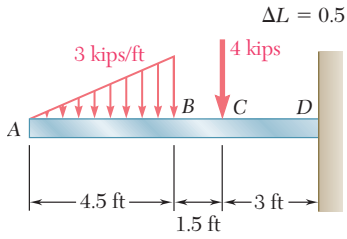
x	V	M
ft	kips	kip · ft
0.0	15.30	0.0
0.5	13.54	7.2
1.0	11.85	13.6
1.5	10.24	19.1
2.0	8.70	23.8
2.5	7.24	27.8
3.0	5.85	31.1
3.5	4.54	33.6
4.0	3.30	35.6
4.5	2.14	37.0
5.0	1.05	37.8
5.5	0.04	38.0
6.0	–0.90	37.8
6.5	–1.80	37.1
7.0	–2.70	36.0
7.5	–3.60	34.4
8.0	–4.50	32.4
8.5	–5.40	29.9
9.0	–6.30	27.0

x	V	M
ft	kips	kip · ft
9.5	–7.20	23.6
10.0	–8.10	19.8
10.5	–9.00	15.5
11.0	–9.90	10.8
11.5	–10.80	5.6
12.0	–11.70	0.0

$$|V|_{\max} = 15.30 \text{ kips} \blacktriangleleft$$

$$|M|_{\max} = 38.0 \text{ kip}\cdot\text{ft} \blacktriangleleft$$

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PROBLEM 5.121

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$w = \frac{3}{4.5}x - 3\langle x-4.5\rangle^0 - \frac{3}{4.5}\langle x-4.5\rangle^1$$

$$= \frac{2}{3}x - 3\langle x-4.5\rangle^0 - \frac{2}{3}\langle x-4.5\rangle^1 = -\frac{dV}{dx}$$

$$V = -\frac{1}{3}x^2 + 3\langle x-4.5\rangle^1 + \frac{1}{3}\langle x-4.5\rangle^2 - 4\langle x-6\rangle^0$$

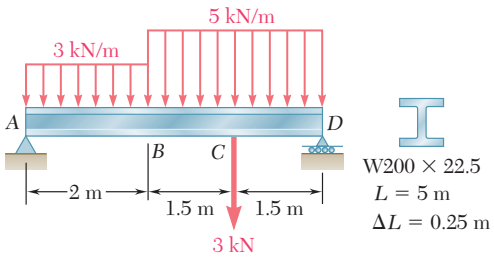
$$M = -\frac{1}{9}x^3 + \frac{3}{2}\langle x-4.5\rangle^2 + \frac{1}{9}\langle x-4.5\rangle^3 - 4\langle x-6\rangle^1$$

x	V	M
ft	kips	kip · ft
0.0	0.00	0.00
0.5	–0.08	–0.01
1.0	–0.33	–0.11
1.5	–0.75	–0.38
2.0	–1.33	–0.89
2.5	–2.08	–1.74
3.0	–3.00	–3.00
3.5	–4.08	–4.76
4.0	–5.33	–7.11
4.5	–6.75	–10.13
5.0	–6.75	–13.50
5.5	–6.75	–16.88
6.0	–10.75	–20.25
6.5	–10.75	–25.63
7.0	–10.75	–31.00
7.5	–10.75	–36.38
8.0	–10.75	–41.75
8.5	–10.75	–47.13
9.0	–10.75	–52.50

$$|V|_{\max} = 10.75 \text{ kips} \blacktriangleleft$$

$$|M|_{\max} = 52.5 \text{ kip} \cdot \text{ft} \blacktriangleleft$$

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PROBLEM 5.122

For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0:$$

$$-5R_A + (4.0)(2.0)(3) + (1.5)(3)(5) + (1.5)(3) = 0$$

$$R_A = 10.2 \text{ kN}$$

$$w = 3 + 2\langle x - 2\rangle^0 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 10.2 - 3x - 2\langle x - 2\rangle^1 - 3\langle x - 3.5\rangle^0 \text{ kN} \quad \blacktriangleleft$$

$$M = 10.2x - 1.5x^2 - \langle x - 2\rangle^2 - 3\langle x - 3.5\rangle^1 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

(*b*) For rolled-steel section W200 × 22.5,

$$S = 193 \times 10^3 \text{ mm}^3 = 193 \times 10^{-6} \text{ m}^3$$

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{16.164 \times 10^3}{193 \times 10^{-6}} = 83.8 \times 10^6 \text{ Pa}$$

$$\sigma = 83.8 \text{ MPa} \quad \blacktriangleleft$$

x	V	M	σ
m	kN	kN · m	MPa
0.00	10.20	0.00	0.0
0.25	9.45	2.46	12.7
0.50	8.70	4.73	24.5
0.75	7.95	6.81	35.3
1.00	7.20	8.70	45.1
1.25	6.45	10.41	53.9
1.50	5.70	11.93	61.8
1.75	4.95	13.26	68.7
2.00	4.20	14.40	74.6
2.25	2.95	15.29	79.2
2.50	1.70	15.88	82.3
2.75	0.45	16.14	83.6

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PROBLEM 5.122 *(Continued)*

x	V	M	σ	
m	kN	kN · m	MPa	
3.00	–0.80	16.10	83.4	
3.25	–2.05	15.74	81.6	
3.50	–6.30	15.08	78.1	
3.75	–7.55	13.34	69.1	
4.00	–8.80	11.30	58.5	
4.25	–10.05	8.94	46.3	
4.50	–11.30	6.28	32.5	
4.75	–12.55	3.29	17.1	
5.00	–13.80	0.00	0.0	
2.83	0.05	16.164	83.8	
2.84	0.00	16.164	83.8	←
2.85	–0.05	16.164	83.8	

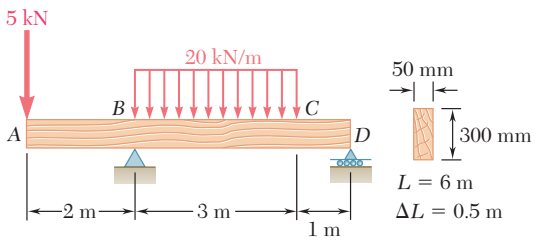
(*a*)

$$|V|_{\max} = 13.80 \text{ kN} \blacktriangleleft$$

$$|M|_{\max} = 16.16 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 5.123



For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the *x* axis at end *A* of the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -4R_B + (6)(5) + (2.5)(3)(20) = 0 \qquad R_B = 45 \text{ kN}$$

$$w = 20\langle x-2\rangle^0 - 20\langle x-5\rangle^0 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = -5 + 45\langle x-2\rangle^0 - 20\langle x-2\rangle^1 + 20\langle x-5\rangle^1 \text{ kN} \quad \blacktriangleleft$$

$$M = -5x + 45\langle x-2\rangle^1 - 10\langle x-2\rangle^2 + 10\langle x-5\rangle^2 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

(*a*)

x	*V*	*M*	stress	
m	kN	kN · m	MPa	
0.00	–5	0.00	0.0	
0.50	–5	–2.50	–3.3	
1.00	–5	–5.00	–6.7	
1.50	–5	–7.50	–10.0	
2.00	40	–10.00	–13.3	
2.50	30	7.50	10.0	
3.00	20	20.00	26.7	
3.50	10	27.50	36.7	
4.00	0	30.00	40.0	←
4.50	–10	27.50	36.7	
5.00	–20	20.00	26.7	
5.50	–20	10.00	13.3	
6.00	–20	0.00	0.0	

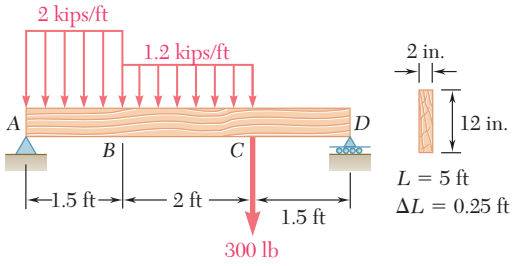
(*b*) Maximum $|M| = 30.0 \text{ kN}\cdot\text{m}$ at $x = 4.0 \text{ m}$ ◀

Maximum $|V| = 40.0 \text{ kN}$ ◀

For rectangular cross section, $S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(50)(300)^2 = 750\times10^3 \text{ mm}^3 = 750\times10^{-6} \text{ m}^3$

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{30\times10^3}{750\times10^{-6}} = 40\times10^6 \text{ Pa} \qquad \sigma_{\max} = 40.0 \text{ MPa} \quad \blacktriangleleft$$

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PROBLEM 5.124

For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

SOLUTION

$$300 \text{ lb} = 0.3 \text{ kips}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -5R_A + (4.25)(1.5)(2) + (2.5)(2)(1.2) + (1.5)(0.3) = 0$$

$$R_A = 3.84 \text{ kips}$$

$$w = 2 - 0.8\langle x - 1.5\rangle^0 - 1.2\langle x - 3.5\rangle^0 \text{ kip/ft}$$

$$V = 3.84 - 2x + 0.8\langle x - 1.5\rangle^1 + 1.2\langle x - 3.5\rangle^1 - 0.3\langle x - 3.5\rangle^0 \text{ kips} \quad \blacktriangleleft$$

$$M = 3.84x - x^2 + 0.4\langle x - 1.5\rangle^2 + 0.6\langle x - 3.5\rangle^2 - 0.3\langle x - 3.5\rangle^1 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$

x	V	M	stress
ft	kips	kip · ft	ksi
0.00	3.84	0.00	0.000
0.25	3.34	0.90	0.224
0.50	2.84	1.67	0.417
0.75	2.34	2.32	0.579
1.00	1.84	2.84	0.710
1.25	1.34	3.24	0.809
1.50	0.84	3.51	0.877
1.75	0.54	3.68	0.921
2.00	0.24	3.78	0.945
2.25	−0.06	3.80	0.951
2.50	−0.36	3.75	0.937
2.75	−0.66	3.62	0.906
3.00	−0.96	3.42	0.855
3.25	−1.26	3.14	0.786
3.50	−1.86	2.79	0.697
3.75	−1.86	2.32	0.581

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PROBLEM 5.124 *(Continued)*

x	V	M	stress	
ft	kips	kip · ft	ksi	
4.00	–1.86	1.86	0.465	
4.25	–1.86	1.39	0.349	
4.50	–1.86	0.93	0.232	
4.75	–1.86	0.46	0.116	
5.00	–1.86	–0.00	–0.000	
2.10	0.12	3.80	0.949	
2.20	0.00	3.80	0.951	←
2.30	–0.12	3.80	0.949	

Maximum $|M| = 3.804 \text{ kip} \cdot \text{ft} = 45.648 \text{ kip} \cdot \text{in.}$ at $x = 2.20 \text{ ft}$ ◀

Maximum $|V| = 3.84 \text{ kip}$ ◀

Rectangular section:

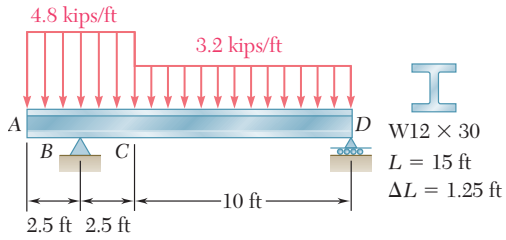
$$2 \text{ in.} \times 12 \text{ in.}$$

$$S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(2)(12)^2$$

$$= 48 \text{ in}^3$$

$$\sigma = \frac{M}{S} = \frac{45.648}{48} \qquad \sigma = 0.951 \text{ ksi} \blacktriangleleft$$

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PROBLEM 5.125

For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -12.5R_B + (12.5)(5.0)(4.8) + (5)(10)(3.2) = 0$$

$$R_B = 36.8 \text{ kips}$$

$$w = 4.8 - 1.6\langle x - 5\rangle^0 \text{ kips/ft}$$

$$V = -4.8x + 36.8\langle x - 2.5\rangle^0 + 1.6\langle x - 5\rangle^1 \text{ kips} \quad \blacktriangleleft$$

$$M = -2.4x^2 + 36.8\langle x - 2.5\rangle^1 + 0.8\langle x - 5\rangle^2 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$

x	V	M	stress	
ft	kips	kip · ft	ksi	
0.00	0.00	0.00	0.00	
1.25	–6.0	–3.75	–1.17	
2.50	24.8	–15.00	–4.66	
3.75	18.8	12.25	3.81	
5.00	12.8	32.00	9.95	
6.25	8.8	45.50	14.15	
7.50	4.8	54.00	16.79	
8.75	0.8	57.50	17.88	
10.00	–3.2	56.00	17.41	
11.25	–7.2	49.50	15.39	
12.50	–11.2	38.00	11.81	
13.75	–15.2	21.50	6.68	
15.00	–19.2	0.00	0.00	
8.90	0.32	57.58	17.90	
9.00	–0.00	57.60	17.91	←
9.10	–0.32	57.58	17.90	

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PROBLEM 5.125 ***(Continued)***

Maximum $|V| = 24.8$ kips ◀

Maximum $|M| = 57.6 \text{ kip} \cdot \text{ft} = 691.2 \text{ kip} \cdot \text{in.}$ at $x = 9.0$ ft ◀

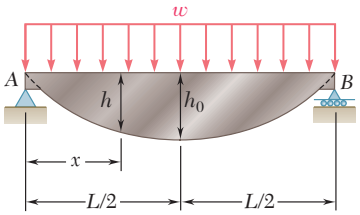
For rolled-steel section W12 × 30,

$$S = 38.6 \text{ in}^3$$

Maximum normal stress:

$$\sigma = \frac{M}{S} = \frac{691.2}{38.6} \qquad \sigma = 17.91 \text{ ksi} \blacktriangleleft$$

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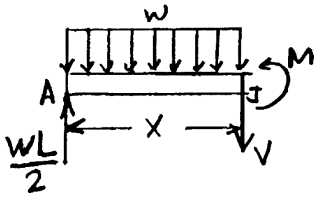
PROBLEM 5.126

The beam *AB*, consisting of a cast-iron plate of uniform thickness *b* and length *L*, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0. (*b*) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{all} = 24$ ksi.

SOLUTION

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wL = 0 \qquad R_A = R_B = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{wL}{2}x - wx\frac{x}{2} + M = 0$$



$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}} \qquad M = \frac{w}{2}x(L-x)$$

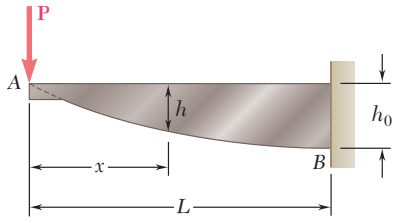
For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating, $\frac{1}{6}bh^2 = \frac{wx(L-x)}{2\sigma_{all}}$ $\qquad h = \left\{\frac{3wx(L-x)}{\sigma_{all}b}\right\}^{1/2}$

(*a*) At $x = \frac{L}{2}$, $\qquad h = h_0 = \left\{\frac{3wL^2}{4\sigma_{all}b}\right\}^{1/2}$ $\qquad h = h_0\left[\frac{x}{L}\left(1-\frac{x}{L}\right)\right]^{1/2}$ ◀

(*b*) Solving for *w*, $\qquad w = \frac{4\sigma_{all}bh_0^2}{3L^2} = \frac{(4)(24)(1.25)(12)^2}{(3)(36)^2}$ $\qquad w = 4.44$ kip/in. ◀

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PROBLEM 5.127

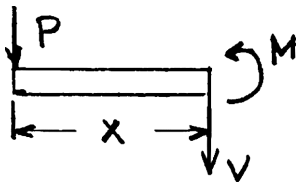
The beam *AB*, consisting of a cast-iron plate of uniform thickness *b* and length *L*, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0. (*b*) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{all} = 24$ ksi.

SOLUTION

$$V = -P$$

$$M = -Px \qquad |M| = Px$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{P}{\sigma_{all}}x$$



For a rectangular cross section, $S = \frac{1}{6}bh^2$

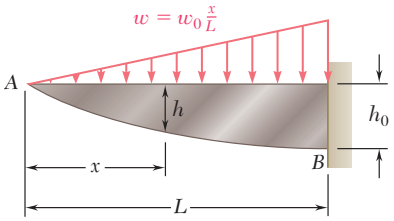
Equating, $$\frac{1}{6}bh^2 = \frac{Px}{\sigma_{all}} \qquad h = \left(\frac{6Px}{\sigma_{all}b}\right)^{1/2} \tag{1}$$

At $x = L$, $$h = h_0 = \left\{\frac{6PL}{\sigma_{all}b}\right\}^{1/2} \tag{2}$$

(*a*) Divide Eq. (1) by Eq. (2) and solve for *h*. $h = h_0(x/L)^{1/2}$ ◀

(*b*) Solving for *P*, $$P = \frac{\sigma_{all}bh_0^2}{6L} = \frac{(24)(1.25)(12)^2}{(6)(36)}$$ $P = 20.0$ kips ◀

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PROBLEM 5.128

The beam *AB*, consisting of a cast-iron plate of uniform thickness *b* and length *L*, is to support the distributed load *w*(*x*) shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0. (*b*) Determine the smallest value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 200$ MPa.

SOLUTION

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$V = -\frac{w_0 x^2}{2L} = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^3}{6L} \qquad |M| = \frac{w_0 x^3}{6L}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w_0 x^3}{6L\sigma_{all}}$$

For a rectangular cross section, $S = \frac{1}{6} bh^2$

Equating, $\frac{1}{6} bh^2 = \frac{w_0 x^3}{6L\sigma_{all}} \qquad h = \sqrt{\frac{w_0 x^3}{\sigma_{all} bL}}$

At $x = L$, $h = h_0 = \sqrt{\frac{w_0 L^2}{\sigma_{all} b}}$

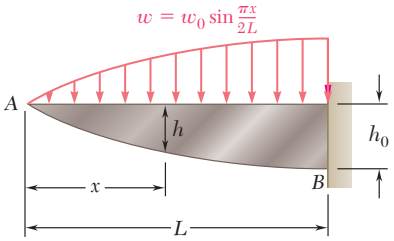
(*a*) $h = h_0 \left(\frac{x}{L}\right)^{3/2}$ ◀

Data: $L = 750 \text{ mm} = 0.75 \text{ m}, \quad b = 30 \text{ mm} = 0.030 \text{ m}$

$w_0 = 300 \text{ kN/m} = 300 \times 10^3 \text{ N/m}, \quad \sigma_{all} = 200 \text{ MPa} = 200 \times 10^6 \text{ Pa}$

(*b*) $h_0 = \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(200 \times 10^6)(0.030)}} = 167.7 \times 10^{-3} \text{ m}$ $\qquad h_0 = 167.7 \text{ mm}$ ◀

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PROBLEM 5.129

The beam *AB*, consisting of a cast-iron plate of uniform thickness *b* and length *L*, is to support the distributed load $w(x)$ shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0. (*b*) Determine the smallest value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 200$ MPa.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin\frac{\pi x}{2L}$$

$$V = \frac{2w_0 L}{\pi}\cos\frac{\pi x}{2L} + C_1$$

$$V = 0 \quad \text{at} \quad x = 0 \rightarrow C_1 = \frac{2w_0 L}{\pi}$$

$$\frac{dM}{dx} = V = -\frac{2w_0 L}{\pi}\left(1 - \cos\frac{\pi x}{2L}\right)$$

$$M = -\frac{2w_0 L}{\pi}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right) \qquad |M| = \frac{2w_0 L}{\pi}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{L}\right)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{2w_0 L}{\pi\sigma_{all}}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right)$$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating, $$\frac{1}{6}bh^2 = \frac{2w_0 L}{\pi\sigma_{all}}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right)$$

$$h = \left\{\frac{12w_0 L}{\pi\sigma_{all} b}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right)\right\}^{1/2}$$

At $x = L$, $$h = h_0 = \left\{\frac{12w_0 L^2}{\pi\sigma_{all} b}\left(1 - \frac{2}{\pi}\right)\right\}^{1/2} = 1.178\sqrt{\frac{w_0 L^2}{\sigma_{all} b}}$$

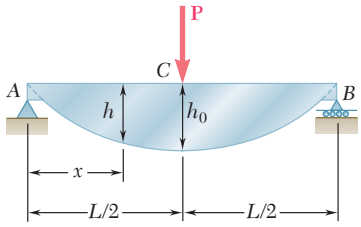
(*a*) $$h = h_0\left[\left(\frac{x}{L} - \frac{2}{\pi}\sin\frac{\pi x}{2L}\right)\Big/\left(1 - \frac{2}{\pi}\right)\right]^{1/2} \qquad h = 1.659\, h_0\left[\frac{x}{L} - \frac{2}{\pi}\sin\frac{\pi x}{2L}\right]^{1/2}$$ ◀

Data: $L = 750 \text{ mm} = 0.75 \text{ m}, \quad b = 30 \text{ mm} = 0.030 \text{ m}$

$w_0 = 300 \text{ kN/m} = 300\times10^3 \text{ N/m}, \quad \sigma_{all} = 200 \text{ MPa} = 200\times10^6 \text{ Pa}$

(*b*) $$h_0 = 1.178\sqrt{\frac{(300\times10^3)(0.75)^2}{(200\times10^6)(0.030)}} = 197.6\times10^{-3} \text{ m} \qquad h_0 = 197.6 \text{ mm}$$ ◀

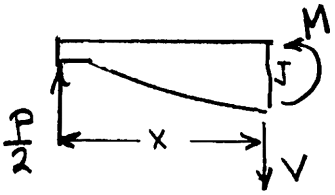
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PROBLEM 5.130

The beam *AB*, consisting of an aluminum plate of uniform thickness *b* and length *L*, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0 for portion *AC* of the beam. (*b*) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.

SOLUTION



$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{P}{2} \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{P}{2}x + M = 0$$

$$M = \frac{Px}{2} \qquad \left(0 < x < \frac{L}{2}\right)$$

$$S = \frac{M}{\sigma_{all}} = \frac{Px}{2\sigma_{all}}$$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

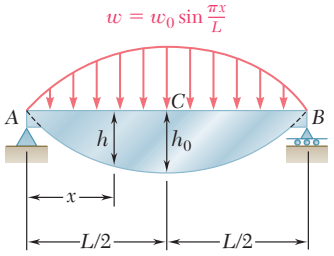
Equating, $\frac{1}{6}bh^2 = \frac{Px}{2\sigma_{all}} \qquad h = \sqrt{\frac{3Px}{\sigma_{all}b}}$

(*a*) At $x = \frac{L}{2}$, $h = h_0 = \sqrt{\frac{3PL}{2\sigma_{all}b}}$ $\qquad h = h_0\sqrt{\frac{2x}{L}}, \quad 0 < x < \frac{L}{2}$ ◀

For $x > \frac{L}{2}$, replace *x* by $L - x$.

(*b*) Solving for *P*, $P = \frac{2\sigma_{all}bh_0^2}{3L} = \frac{(2)(72\times10^6)(0.025)(0.200)^2}{(3)(0.8)} = 60.0\times10^3 \text{ N}$ $\qquad P = 60.0 \text{ kN}$ ◀

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PROBLEM 5.131

The beam *AB*, consisting of an aluminum plate of uniform thickness *b* and length *L*, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0 for portion *AC* of the beam. (*b*) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin\frac{\pi x}{L}$$

$$\frac{dM}{dx} = V = \frac{w_0 L}{\pi}\cos\frac{\pi x}{L} + C_1$$

$$M = \frac{w_0 L^2}{\pi^2}\sin\frac{\pi x}{L} + C_1 x + C_2$$

At *A*, $x = 0$ $M = 0$ $\quad 0 = 0 + 0 + C_2 \quad C_2 = 0$

At *B*, $x = L$ $M = 0$ $\quad 0 = \frac{w_0 L^2}{\pi^2}\sin\pi + C_1 L \quad C_1 = 0$

$$M = \frac{w_0 L^2}{\pi^2}\sin\frac{\pi x}{L}$$

For constant strength, $S = \frac{|M|}{\sigma_{all}} = \frac{w_0 L^2}{\pi^2 \sigma_{all}}\sin\frac{\pi x}{L}$

For a rectangular section, $I = \frac{1}{12}bh^3, \quad c = \frac{h}{2}, \quad S = \frac{I}{C} = \frac{1}{6}bh^2$

Equating the two expressions for *S*, $\frac{1}{6}bh^2 = \frac{w_0 L^2}{\pi^2 \sigma_{all}}\sin\frac{\pi x}{L}$ (1)

At $x = \frac{L}{2}$, $h = h_0$ $\quad \frac{1}{6}bh_0^2 = \frac{w_0 L^2}{\pi^2 \sigma_{all}}$ (2)

(*a*) Dividing Eq. (1) by Eq. 2, $\frac{h^2}{h_0^2} = \sin\frac{\pi x}{L}$

$$h = h_0\left(\sin\frac{\pi x}{L}\right)^{\frac{1}{2}} \blacktriangleleft$$

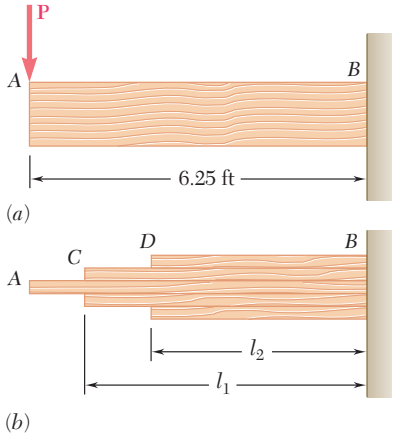
(*b*) Solving Eq. (1) for w_0, $w_0 = \frac{\pi^2 \sigma_{all} b h_0^2}{6L^2}$

Data: $\sigma_{all} = 72 \times 10^6$ Pa, $L = 800$ mm $= 0.800$ m, $h_0 = 200$ mm $= 0.200$ m,
$b = 25$ mm $= 0.025$ m

$$w_0 = \frac{\pi^2 (72\times10^6)(0.025)(0.200)^2}{(6)(0.800)^2} = 185.1\times10^3 \text{ N/m}$$

185.1 kN/m ◀

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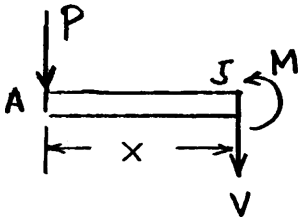
PROBLEM 5.132

A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2×2-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$+\circlearrowleft \Sigma M_J = 0: \quad Px + M = 0 \qquad M = -Px$$

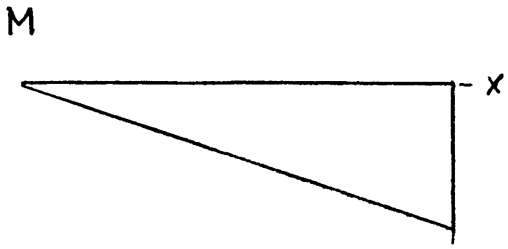
$$|M| = Px$$



At B, $\quad |M|_B = M_{\max}$

At C, $\quad |M|_C = M_{\max} x_C / 6.25$

At D, $\quad |M|_D = M_{\max} x_D / 6.25$



$$S_B = \frac{1}{6} bh^2 = \frac{1}{6} \cdot b(5b)^2 = \frac{25}{6} b^3$$

A to C: $\quad S_C = \frac{1}{6} \cdot b(b)^2 = \frac{1}{6} b^3$

C to D: $\quad S_D = \frac{1}{6} b(3b)^2 = \frac{9}{6} b^3$

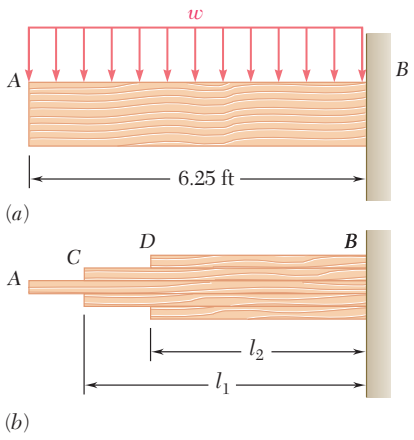
$$\frac{|M|_C}{|M|_B} = \frac{x_C}{6.25} = \frac{S_C}{S_B} = \frac{1}{25} \qquad x_C = \frac{(1)(6.25)}{25} = 0.25 \text{ ft}$$

$$l_1 = 6.25 - 0.25 \qquad l_1 = 6.00 \text{ ft} \blacktriangleleft$$

$$\frac{|M|_D}{|M|_B} = \frac{x_D}{6.25} = \frac{S_D}{S_B} = \frac{9}{25} \qquad x_D = \frac{(9)(6.25)}{25} = 2.25 \text{ ft}$$

$$l_2 = 6.25 - 2.25 \qquad l_2 = 4.00 \text{ ft} \blacktriangleleft$$

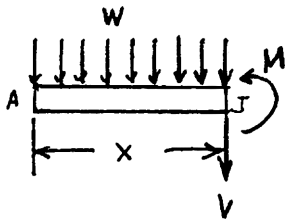
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PROBLEM 5.133

A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2×2-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION



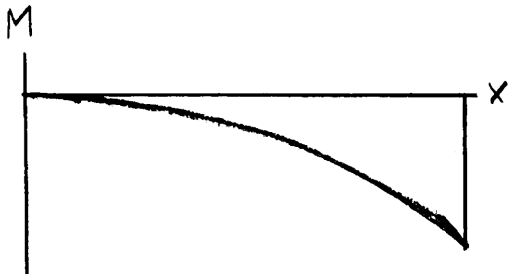
$$+\circlearrowleft \Sigma M_J = 0: \quad wx\frac{x}{2} + M = 0$$

$$M = -\frac{wx^2}{2} \qquad |M| = \frac{wx^2}{2}$$

At *B*, $|M|_B = |M|_{\max}$

At *C*, $|M|_C = |M|_{\max}\ (x_C/6.25)^2$

At *D*, $|M|_D = |M|_{\max}\ (x_D/6.25)^2$



At *B*, $S_B = \frac{1}{6}bh^2 = \frac{1}{6}b(5b)^2 = \frac{25}{6}b^3$

A to *C*: $S_C = \frac{1}{6}bh^2 = \frac{1}{6}b(b)^2 = \frac{1}{6}b^3$

C to *D*: $S_D = \frac{1}{6}bh^2 = \frac{1}{6}b(3b)^2 = \frac{9}{6}b^3$

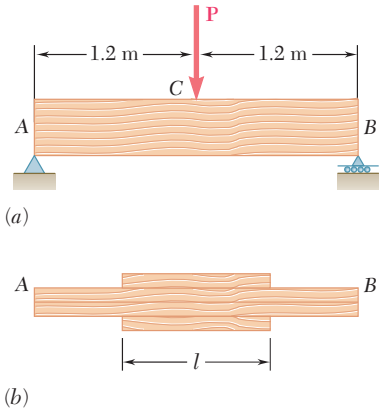
$$\frac{|M|_C}{|M|_B} = \left(\frac{x_C}{6.25}\right)^2 = \frac{S_C}{S_B} = \frac{1}{25} \qquad x_C = \frac{6.25}{\sqrt{25}} = 1.25 \text{ ft}$$

$$l_1 = 6.25 - 1.25 \text{ ft} \qquad l_1 = 5.00 \text{ ft} \blacktriangleleft$$

$$\frac{|M|_D}{|M|_B} = \left(\frac{x_D}{6.25}\right)^2 = \frac{S_D}{S_B} = \frac{9}{25} \qquad x_D = \frac{6.25\sqrt{9}}{\sqrt{25}} = 3.75 \text{ ft}$$

$$l_2 = 6.25 - 3.75 \text{ ft} \qquad l_2 = 2.50 \text{ ft} \blacktriangleleft$$

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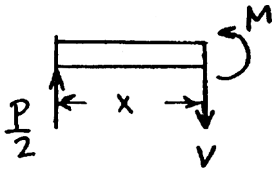
PROBLEM 5.134

A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50×50-mm cross section. Determine the length l of the two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$R_A = R_B = \frac{P}{2}$$

$$0 < x < \frac{1}{2}$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{P}{2}x + M = 0$$

$$M = \frac{Px}{2} \quad \text{or} \quad M = \frac{M_{\max} x}{1.2}$$

Bending moment diagram is two straight lines.



At *C*, $S_C = \frac{1}{6} b h_C^2 \qquad M_C = M_{\max}$

Let *D* be the point where the thickness changes.

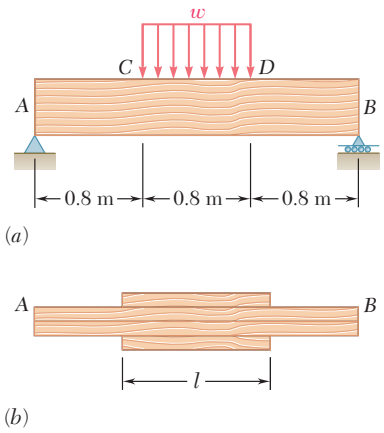
At *D*, $S_D = \frac{1}{6} b h_D^2 \qquad M_D = \frac{M_{\max} x_D}{1.2}$

$$\frac{S_D}{S_C} = \frac{h_D^2}{h_C^2} = \left(\frac{100 \text{ mm}}{200 \text{ mm}} \right)^2 = \frac{1}{4} = \frac{M_D}{M_C} = \frac{x_D}{1.2} \qquad x_D = 0.3 \text{ m}$$

$$\frac{l}{2} = 1.2 - x_D = 0.9$$

$$l = 1.800 \text{ m} \blacktriangleleft$$

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PROBLEM 5.135

A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50×50-mm cross section. Determine the length l of the two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$R_A = R_B = \frac{0.8 \text{ N}}{2} = 0.4w$$

Shear:

A to C: $V = 0.4w$

D to B: $V = -0.4w$

Areas:

A to C: $(0.8)(0.4)w = 0.32w$

C to E: $\left(\frac{1}{2}\right)(0.4)(0.4)w = 0.08w$

Bending moments:

At C, $M_C = 0.40w$

A to C: $M = 0.40wx$

At C, $S_C = \frac{1}{6}bh_C^2 \qquad M_C = M_{\max} = 0.40w$

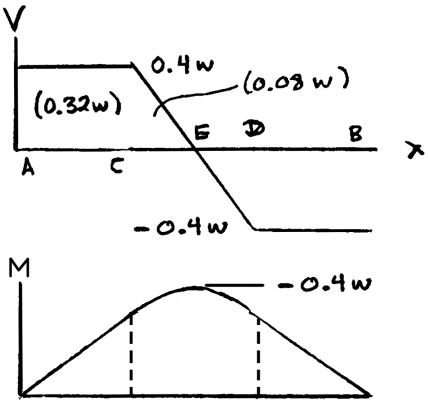
Let F be the point where the thickness changes.

At F, $S_F = \frac{1}{6}bh_F^2 \qquad M_F = 0.40wx_F$

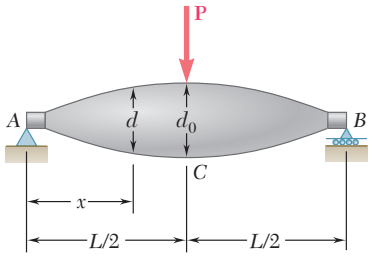
$$\frac{S_F}{S_C} = \frac{h_F^2}{h_C^2} = \left(\frac{100 \text{ mm}}{200 \text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_F}{M_C} = \frac{0.40wx_F}{0.40w}$$

$$x_F = 0.25 \text{ m} \qquad \frac{l}{2} = 1.2 - x_F = 0.95 \text{ m}$$

$$l = 1.900 \text{ m} \blacktriangleleft$$



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PROBLEM 5.136

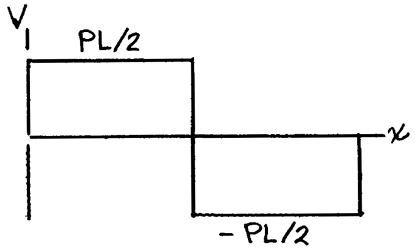
A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x, L, and d_0.

SOLUTION

Draw shear and bending moment diagrams.

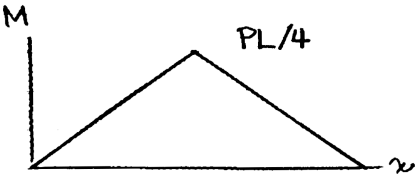
$$0 \le x \le \frac{L}{2}, \qquad M = \frac{Px}{2}$$

$$\frac{L}{2} \le x \le L, \qquad M = \frac{P(L-x)}{2}$$



For a solid circular section, $c = \frac{1}{2}d$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^4 \qquad S = \frac{I}{c} = \frac{\pi}{32}d^3$$



For constant strength design, $\sigma = \text{constant}$. $\qquad S = \frac{M}{\sigma}$

For $0 \le x \le \frac{L}{2}$, $\qquad \frac{\pi}{32}d^3 = \frac{Px}{2}$ (1a)

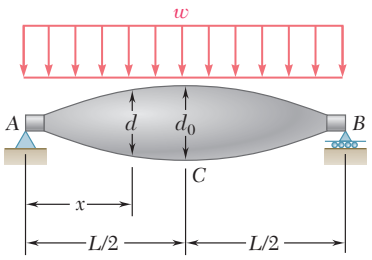
For $\frac{L}{2} \le x \le L$, $\qquad \frac{\pi}{32}d^3 = \frac{P(L-x)}{2}$ (1b)

At point C, $\qquad \frac{\pi}{32}d_0^3 = \frac{PL}{4}$ (2)

Dividing Eq. (1a) by Eq. (2), $\quad 0 \le x \le \frac{L}{2}, \quad \frac{d^3}{d_0^3} = \frac{2x}{L}$ $\qquad d = d_0(2x/L)^{1/3}$ ◀

Dividing Eq. (1b) by Eq. (2), $\quad \frac{L}{2} \le x \le L, \quad \frac{d^3}{d_0^3} = \frac{2(L-x)}{L}$ $\qquad d = d_0[2(L-x)/L]^{1/3}$ ◀

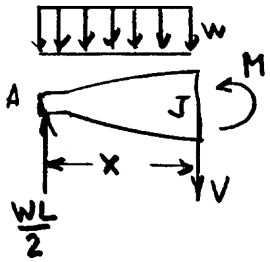
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PROBLEM 5.137

A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x, L, and d_0.

SOLUTION



$$R_A = R_B = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}x(L-x)$$

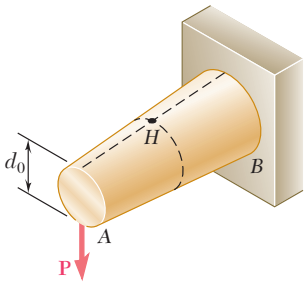
$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{wx(L-x)}{2\sigma_{\text{all}}}$$

For a solid circular cross section, $c = \frac{d}{2}$ $I = \frac{\pi}{4}c^3$ $S = \frac{I}{c} = \frac{\pi d^3}{32}$

Equating, $\frac{\pi d^3}{32} = \frac{wx(L-x)}{2\sigma_{\text{all}}}$ $d = \left\{\frac{16\,wx(L-x)}{\pi\sigma_{\text{all}}}\right\}^{1/3}$

At $x = \frac{L}{2}$, $d = d_0 = \left\{\frac{4wL^2}{\pi\sigma_{\text{all}}}\right\}^{1/3}$ $d = d_0\left\{4\frac{x}{L}\left(1-\frac{x}{L}\right)\right\}^{1/3}$ ◀

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PROBLEM 5.138

A transverse force **P** is applied as shown at end *A* of the conical taper *AB*. Denoting by d_0 the diameter of the taper at *A*, show that the maximum normal stress occurs at point *H*, which is contained in a transverse section of diameter $d = 1.5d_0$.

SOLUTION

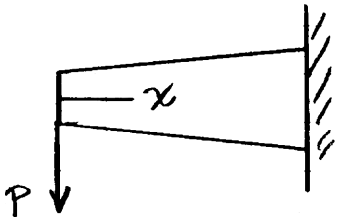
$$V = -P = \frac{dM}{dx} \qquad M = -Px$$

Let $d = d_0 + kx$

For a solid circular section, $I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^3$

$$c = \frac{d}{2} \qquad S = \frac{I}{c} = \frac{\pi}{32}d^3 = \frac{\pi}{32}(d_0 + kx)^3$$

$$\frac{dS}{dx} = \frac{3\pi}{32}(d_0 + kx)^2 k = \frac{3\pi}{32}d^2 k$$



Stress: $\sigma = \frac{|M|}{S} = \frac{Px}{S}$

At *H*, $\frac{d\sigma}{dx} = \frac{1}{S^2}\left(PS - Px_H \frac{dS}{dx}\right) = 0$

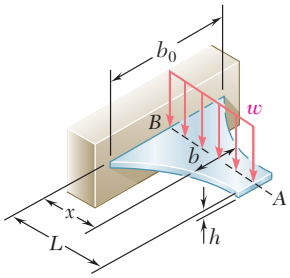
$$S - x_H \frac{dS}{dx} = \frac{\pi}{32}d^3 - x_H \frac{3\pi}{32}d^2 k$$

$$kx_H = \frac{1}{3}d = \frac{1}{3}(d_0 + k_H x_H) \qquad kx_H = \frac{1}{2}d_0$$

$$d = d_0 + \frac{1}{2}d_0 = \frac{3}{2}d_0$$

$d = 1.5d_0$ ◀

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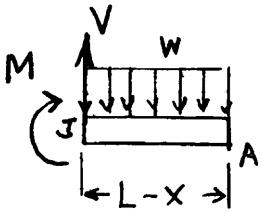


PROBLEM 5.139

A cantilever beam *AB* consisting of a steel plate of uniform depth *h* and variable width *b* is to support the distributed load *w* along its centerline *AB*. (*a*) Knowing that the beam is to be of constant strength, express *b* in terms of *x*, *L*, and b_0. (*b*) Determine the maximum allowable value of *w* if $L = 15$ in., $b_0 = 8$ in., $h = 0.75$ in., and $\sigma_{all} = 24$ ksi.

SOLUTION

$$+\circlearrowleft \Sigma M_J = 0: \quad -M - w(L-x)\frac{L-x}{2} = 0$$



$$M = -\frac{w(L-x)^2}{2} \qquad |M| = \frac{w(L-x)^2}{2}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w(L-x)^2}{2\sigma_{all}}$$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

$$\frac{1}{6}bh^2 = \frac{w(L-x)^2}{2\sigma_{all}} \qquad b = \frac{3w(L-x)^2}{\sigma_{all}h^2}$$

(*a*) At $x = 0$, $b = b_0 = \dfrac{3wL^2}{\sigma_{all}h^2}$ $\qquad b = b_0\left(1 - \dfrac{x}{L}\right)^2$ ◀

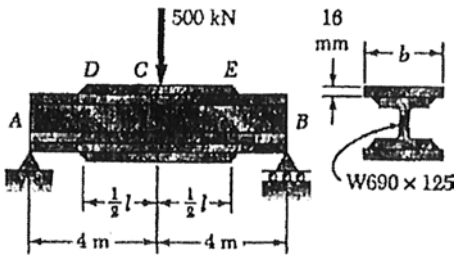
(*b*) Solving for *w*, $w = \dfrac{\sigma_{all}b_0h^2}{3L^2} = \dfrac{(24)(8)(0.75)^2}{(3)(15)^2} = 0.160$ kip/in. $\qquad w = 160.0$ lb/in. ◀

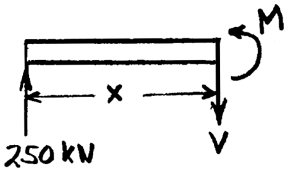
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PROBLEM 5.140

Assuming that the length and width of the cover plates used with the beam of Sample Prob. 5.12 are, respectively, $l = 4$ m and $b = 285$ mm, and recalling that the thickness of each plate is 16 mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION





$A = B = 250$ kN ↑

$+\circlearrowleft \Sigma M_J = 0: \quad -250x + M = 0$

$M = 250x$ kN · m

At center of beam, $x = 4$ m $\quad M_C = (250)(4) = 1000$ kN · m

At *D*, $x = \frac{1}{2}(8 - l) = \frac{1}{2}(8 - 4) = 2$ m $\qquad M_0 = 500$ kN · m

At center of beam, $I = I_{\text{beam}} + 2I_{\text{plate}}$

$$= 1190 \times 10^6 + 2\left\{(285)(16)\left(\frac{678}{2} + \frac{16}{2}\right)^2 + \frac{1}{12}(285)(16)^3\right\}$$

$$= 2288 \times 10^6 \text{ mm}^4$$

$$c = \frac{678}{2} + 16 = 355 \text{ mm} \qquad S = \frac{I}{c} = 6445 \times 10^3 \text{mm}^3$$

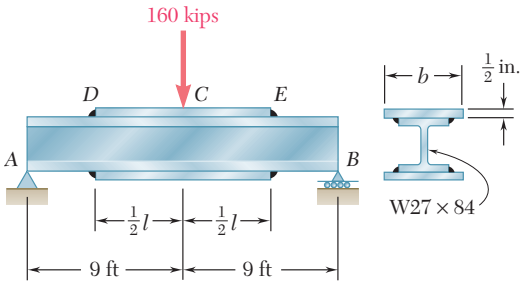
$$= 6445 \times 10^{-6} \text{m}^3$$

(*a*) Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{1000 \times 10^3}{6445 \times 10^{-6}} = 155.2 \times 10^6$ Pa $\qquad \sigma = 155.2$ MPa ◀

At *D*, $S = 3490 \times 10^3 \text{mm}^3 = 3510 \times 10^{-6} \text{m}^3$

(*b*) Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{500 \times 10^3}{3490 \times 10^{-6}} = 143.3 \times 10^6$ Pa $\qquad \sigma = 143.3$ MPa ◀

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PROBLEM 5.141

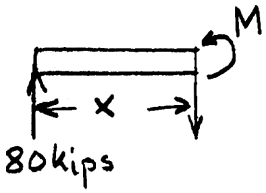
Two cover plates, each $\frac{1}{2}$ in. thick, are welded to a W27×84 beam as shown. Knowing that $l = 10$ ft and $b = 10.5$ in., determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION

$$\mathbf{R}_A = \mathbf{R}_B = 80 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -80x + M = 0$$

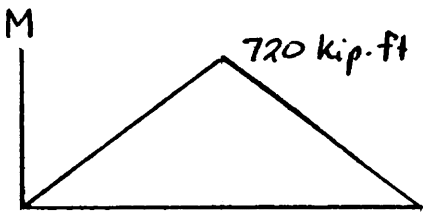
$$M = 80x \text{ kip} \cdot \text{ft}$$



At *C*, $x = 9$ ft $\quad M_C = 720 \text{ kip} \cdot \text{ft} = 8640 \text{ kip} \cdot \text{in.}$

At *D*, $x = 9 - 5 = 4$ ft

$$M_D = (80)(4) = 320 \text{ kip} \cdot \text{ft} = 3840 \text{ kip} \cdot \text{in.}$$



At center of beam,

$$I = I_{\text{beam}} + 2I_{\text{plate}}$$

$$I = 2850 + 2\left\{(10.5)(0.500)\left(\frac{26.71}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(10.5)(0.500)^3\right\}$$

$$= 4794 \text{ in}^4$$

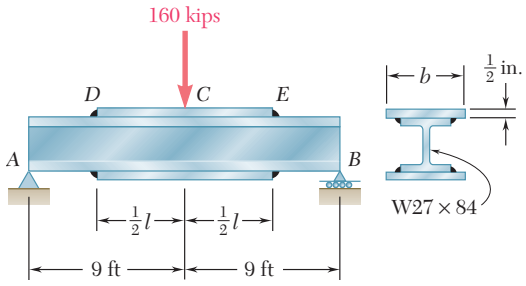
$$c = \frac{26.71}{2} + 0.500 = 13.855 \text{ in.}$$

(*a*) Normal stress: $\sigma = \dfrac{Mc}{I} = \dfrac{(8640)(13.855)}{4794}$ $\qquad \sigma = 25.0 \text{ ksi}$ ◀

At point *D*, $S = 213 \text{ in}^3$

(*b*) Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{3840}{213}$ $\qquad \sigma = 18.03 \text{ ksi}$ ◀

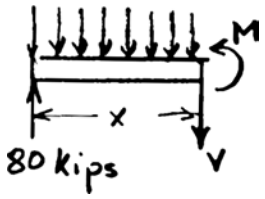
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PROBLEM 5.142

Two cover plates, each $\frac{1}{2}$ in. thick, are welded to a W27 × 84 beam as shown. Knowing that $\sigma_{all} = 24$ ksi for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.

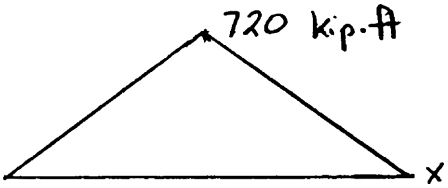
SOLUTION



$$\boldsymbol{R}_A = \boldsymbol{R}_B = 80 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -80x + M = 0$$

$$M = 80x \text{ kip} \cdot \text{ft}$$



At *D*, $S = 213 \text{ in}^3$

Allowable bending moment:

$$M_{all} = \sigma_{all} S = (24)(213) = 5112 \text{ kip} \cdot \text{in.}$$

$$= 426 \text{ kip} \cdot \text{ft}$$

Set $M_D = M_{all}$. $\quad 80x_D = 426 \quad x_D = 5.325$ ft

(*a*) $\quad l = 18 - 2x_D \quad\quad l = 7.35$ ft ◀

At center of beam, $\quad M = (80)(9) = 720 \text{ kip} \cdot \text{ft} = 8640 \text{ kip} \cdot \text{in.}$

$$S = \frac{M}{\sigma_{all}} = \frac{8640}{24} = 360 \text{ in}^3$$

$$c = \frac{26.7}{2} + 0.500 = 13.85 \text{ in.}$$

Required moment of inertia: $\quad I = Sc = 4986 \text{ in}^4$

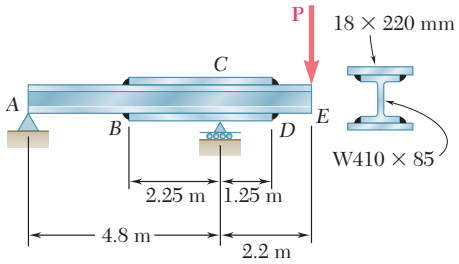
But $\quad I = I_{beam} + 2I_{plate}$

$$4986 = 2850 + 2\left\{(b)(0.500)\left(\frac{26.7}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(b)(0.500)^3\right\}$$

$$= 2850 + 184.981b$$

(*b*) $\quad b = 11.55$ in. ◀

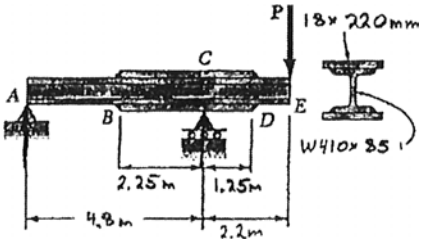
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PROBLEM 5.143

Knowing that $\sigma_{all} = 150$ MPa, determine the largest concentrated load **P** that can be applied at end *E* of the beam shown.

SOLUTION



$$+\circlearrowleft \Sigma M_C = 0: \quad -4.8A - 2.2P = 0$$

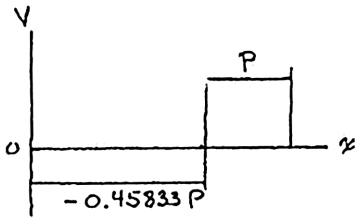
$$A = -0.45833P \qquad \boldsymbol{A} = 0.45833P \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 4.8D - 7.0P = 0$$

$$\boldsymbol{D} = 1.45833P \uparrow$$

Shear: A to C: $V = -0.45833P$

C to E: $V = P$



Bending moments: $M_A = 0$

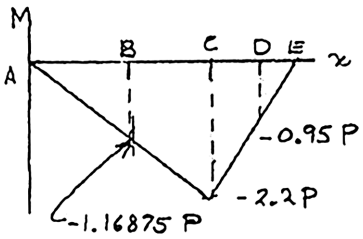
$$M_C = 0 + (4.8)(-0.45833P) = -2.2P$$

$$M_E = -2.2P + 2.2P = 0$$

$$M_B = \left(\frac{4.8 - 2.25}{48}\right)(-2.2P) = -1.16875P$$

$$M_D = \left(\frac{2.2 - 1.25}{2.2}\right)(-2.2P) = -0.95P$$

$$|M_D| < |M_B|$$



For W410 × 85, $S = 1510 \times 10^3 \text{ mm}^3 = 1510 \times 10^{-6} \text{ m}^3$

Allowable value of *P* based on strength at *B*. $\sigma = \dfrac{|M_B|}{S}$

$$150 \times 10^6 = \frac{1.16875P}{1510 \times 10^{-6}} \qquad P = 193.8 \times 10^3 \text{ N}$$

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PROBLEM 5.143 *(Continued)*

Section properties over portion *BCD*:

$$\text{W}410 \times 85: \quad d = 417 \text{ mm}, \quad \frac{1}{2}d = 208.5 \text{ mm}, \quad I_x = 316 \times 10^6 \text{mm}^4$$

Plate: $A = (18)(220) = 3960 \text{ mm}^2 \qquad d = 208.5 + \left(\frac{1}{2}\right)(18) = 217.5 \text{ mm}$

$$\bar{I} = \frac{1}{12}(220)(18)^3 = 106.92 \times 10^3 \text{ mm}^4 \quad Ad^2 = 187.333 \times 10^6 \text{ mm}^4$$

$$I_x = \bar{I} + Ad^2 = 187.440 \times 10^6 \text{ mm}^4$$

For section, $I = 316 \times 10^6 + (2)(187.440 \times 10^6) = 690.88 \times 10^6 \text{ mm}^4$

$$c = 208.5 + 18 = 226.5 \text{ mm}$$

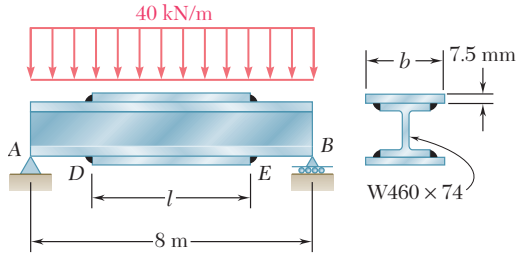
$$S = \frac{I}{c} = \frac{690.88 \times 10^6}{226.5} = 3050.2 \times 10^3 \text{mm}^3 = 3050.2 \times 10^{-6} \text{ m}^3$$

Allowable load based on strength at *C*: $\sigma = \dfrac{|M_C|}{S}$

$$150 \times 10^6 = \frac{2.2P}{3050.2 \times 10^{-6}} \qquad P = 208.0 \times 10^3 \text{ N}$$

The smaller allowable load controls. $P = 193.8 \times 10^3 \text{N}$ $\qquad P = 193.8 \text{ kN}$ ◀

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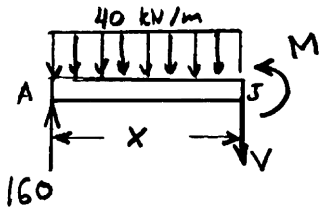


PROBLEM 5.144

Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $l = 5$ m and $b = 200$ mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION

$$\boldsymbol{R}_A = \boldsymbol{R}_B = 160 \text{ kN} \uparrow$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -160x + (40x)\frac{x}{2} + M = 0$$

$$M = 160x - 20x^2 \text{ kN} \cdot \text{m}$$

At center of beam, $x = 4$ m $\quad M_C = 320$ kN · m

At *D*, $x = \frac{1}{2}(8 - l) = 1.5$ m $\quad M_D = 195$ kN · m

At center of beam, $I = I_{\text{beam}} + 2I_{\text{plate}}$

$$= 333 \times 10^6 + 2\left\{(200)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(200)(7.5)^3\right\}$$

$$= 494.8 \times 10^6 \text{ mm}^4$$

$$c = \frac{457}{2} + 7.5 = 236 \text{ mm}$$

$$S = \frac{I}{c} = 2097 \times 10^3 \text{ mm}^3 = 2097 \times 10^{-6} \text{ m}^3$$

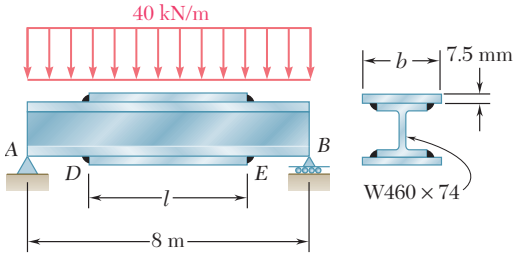
(*a*) Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{320 \times 10^3}{2097 \times 10^{-6}} = 152.6 \times 10^6 \text{ Pa}$

$\sigma = 152.6$ MPa ◀

At *D*, $S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{195 \times 10^3}{1460 \times 10^{-6}} = 133.6 \times 10^6 \text{ Pa}$ $\quad \sigma = 133.6$ MPa ◀

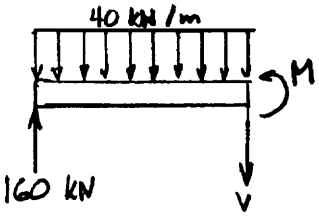
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PROBLEM 5.145

Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $\sigma_{all} = 150$ MPa for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.

SOLUTION

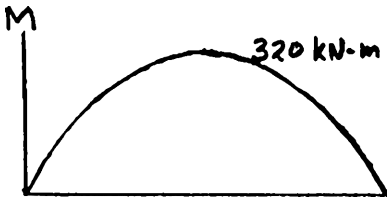


$$\boldsymbol{R}_A = \boldsymbol{R}_B = 160 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -160x + (40x)\left(\frac{x}{2}\right) + M = 0$$

$$M = 160x - 20x^2 \text{ kN} \cdot \text{m}$$

For W460 × 74 rolled-steel beam,



$$S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$$

Allowable bending moment:

$$M_{all} = \sigma_{all} S = (150 \times 10^6)(1460 \times 10^{-6})$$

$$= 219 \times 10^3 \text{ N} \cdot \text{m} = 219 \text{ kN} \cdot \text{m}$$

To locate points *D* and *E*, set $M = M_{all}$

$$160x - 20x^2 = 219 \qquad 20x^2 - 160x + 219 = 0$$

$$x = \frac{160 \pm \sqrt{160^2 - (4)(20)(219)}}{(2)(20)} \qquad x = 1.753 \text{ m and } x = 6.247 \text{ m}$$

(*a*) $x_D = 1.753$ ft $x_E = 6.247$ ft $\qquad l = x_E - x_D = 4.49$ m ◀

At center of beam, $M = 320 \text{ kN} \cdot \text{m} = 320 \times 10^3 \text{ N} \cdot \text{m}$ $\qquad c = \frac{457}{2} + 7.5 = 236 \text{ mm}^4$

$$S = \frac{M}{\sigma_{all}} = \frac{320 \times 10^3}{150 \times 10^6} = 2133 \times 10^{-6} \text{ m}^3 = 2133 \times 10^3 \text{ mm}^3$$

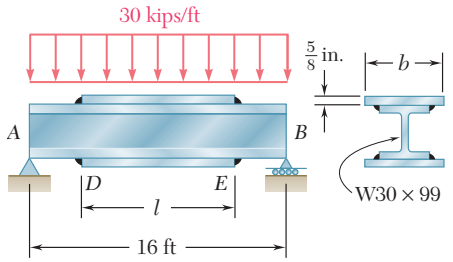
Required moment of inertia: $I = Sc = 503.4 \times 10^6 \text{ mm}^4$

But $I = I_{beam} + 2I_{plate}$

$$503.4 \times 10^6 = 333 \times 10^6 + 2\left\{(b)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(b)(7.5)^3\right\}$$

(*b*) $$= 333 \times 10^6 + 809.2 \times 10^3 b$$ $\qquad b = 211$ mm ◀

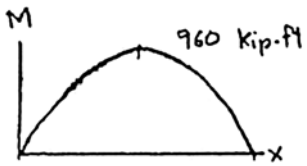
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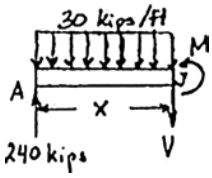
PROBLEM 5.146

Two cover plates, each $\frac{5}{8}$ in. thick, are welded to a W30 × 99 beam as shown. Knowing that $l = 9$ ft and $b = 12$ in., determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION



$$\boldsymbol{A} = \boldsymbol{B} = 240 \text{ kips} \uparrow$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -240x + 30x\frac{x}{2} + M = 0$$

$$M = 240x - 15x^2 \text{ kip} \cdot \text{ft}$$

At center of beam, $x = 8$ ft

$$M_C = 960 \text{ kip} \cdot \text{ft} = 11{,}520 \text{ kip} \cdot \text{in.}$$

At point *D*, $x = \frac{1}{2}(16 - 9) = 3.5$ ft

$$M_D = 656.25 \text{ kip} \cdot \text{ft} = 7875 \text{ kip} \cdot \text{in.}$$

At center of beam, $I = I_{\text{beam}} + 2I_{\text{plate}}$

$$I = 3990 + 2\left\{(12)(0.625)\left(\frac{29.7}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(12)(0.625)^3\right\} = 7439 \text{ in}^4$$

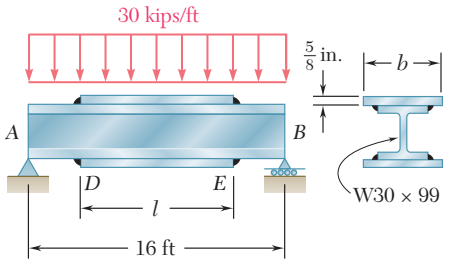
$$c = \frac{29.7}{2} + 0.625 = 15.475 \text{ in.}$$

(*a*) Normal stress: $\sigma = \dfrac{Mc}{I} = \dfrac{(11{,}520)(15.475)}{7439}$ $\sigma = 24.0$ ksi ◀

At point *D*, $S = 269 \text{ in}^3$

(*b*) Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{7875}{269}$ $\sigma = 29.3$ ksi ◀

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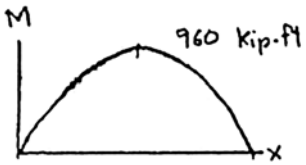


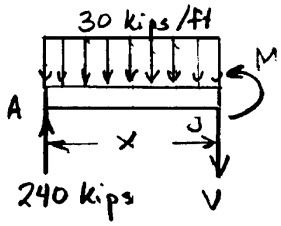
PROBLEM 5.147

Two cover plates, each $\frac{5}{8}$ in. thick, are welded to a W30 × 99 beam as shown. Knowing that $\sigma_{all} = 22$ ksi for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.

SOLUTION

$$\boldsymbol{R}_A = \boldsymbol{R}_B = 240 \text{ kips} \uparrow$$





$$+\circlearrowleft \Sigma M_J = 0: \quad -240x + 30x\frac{x}{2} + M = 0$$

$$M = 240x - 15x^2 \text{ kip} \cdot \text{ft}$$

For W30 × 99 rolled-steel section, $S = 269 \text{ in}^3$

Allowable bending moment:

$$M_{all} = \sigma_{all} S = (22)(269) = 5918 \text{ kip} \cdot \text{in.} = 493.167 \text{ kip} \cdot \text{ft}$$

To locate points *D* and *E*, set $M = M_{all}$.

$$240x - 15x^2 = 493.167 \qquad 15x^2 - 240x + 493.167 = 0$$

$$x = \frac{240 \pm \sqrt{(240)^2 - (4)(15)(493.167)}}{(2)(15)} = 2.42 \text{ ft}, \quad 13.58 \text{ ft}$$

(*a*) $l = x_E - x_D = 13.58 - 2.42$ $\qquad l = 11.16 \text{ ft}$ ◀

Center of beam: $M = 960 \text{ kip} \cdot \text{ft} = 11{,}520 \text{ kip} \cdot \text{in.}$

$$S = \frac{M}{\sigma_{all}} = \frac{11{,}520}{22} = 523.64 \text{ in}^3 \qquad c = \frac{29.7}{2} + 0.625 = 15.475 \text{ in.}$$

Required moment of inertia: $I = Sc = 8103.3 \text{ in}^4$

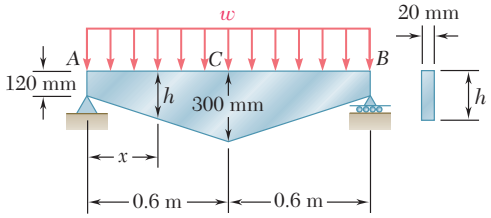
But $I = I_{beam} + 2I_{plate}$

$$8103.3 = 3990 + 2\left\{(b)(0.625)\left(\frac{29.7}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(b)(0.625)^3\right\}$$

$$= 3990 + 287.42b$$

(*b*) $\qquad b = 14.31 \text{ in.}$ ◀

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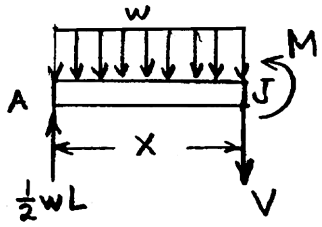


PROBLEM 5.148

For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest distributed load *w* that can be applied, knowing that $\sigma_{all} = 140$ MPa.

SOLUTION

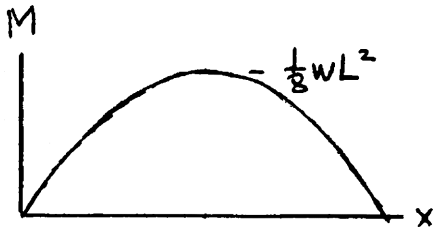
$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{1}{2}wL \uparrow \qquad L = 1.2 \text{ m}$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{2}wL + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L - x)$$



For the tapered beam, $h = a + kx$

$$a = 120 \text{ mm}$$

$$k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For rectangular cross section, $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a + kx)^2$

Bending stress: $\sigma = \frac{M}{S} = \frac{3w}{b}\frac{Lx - x^2}{(a + kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a + kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a + kx)^2(L - 2x) - (Lx - x^2)2(a + kx)k}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - \cancel{2kx^2} - 2kLx + \cancel{2kx^2}}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a + kL)x}{(a + kx)^3}\right\} = 0$$

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PROBLEM 5.148 *(Continued)*

(*a*) $x_m = \dfrac{aL}{2a + kL} = \dfrac{(120)(1.2)}{(2)(120) + (300)(1.2)} = 0.240 \text{ m}$ $\quad x_m = 240 \text{ mm}$ ◀

$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$

$S_m = \dfrac{1}{6} b h_m^2 = \dfrac{1}{6}(20)(192)^2 = 122.88 \times 10^3 \text{ mm}^3 = 122.88 \times 10^{-6} \text{ m}^3$

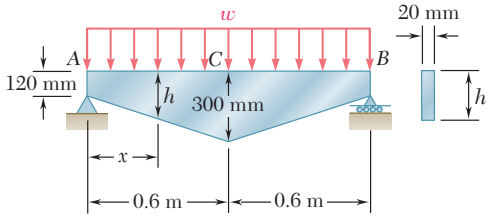
Allowable value of M_m: $M_m = S_m \sigma_{\text{all}} = (122.88 \times 10^{-6})(140 \times 10^6)$

$= 17.2032 \times 10^3 \text{ N} \cdot \text{m}$

(*b*) Allowable value of *w*: $w = \dfrac{2M_m}{x_m(L - x_m)} = \dfrac{(2)(17.2032 \times 10^3)}{(0.24)(0.96)}$

$= 149.3 \times 10^3 \text{ N/m}$ $\quad w = 149.3 \text{ kN/m}$ ◀

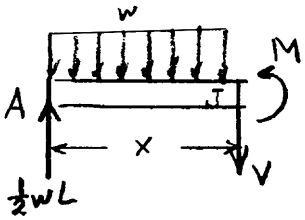
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PROBLEM 5.149

For the tapered beam shown, knowing that $w = 160$ kN/m, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the corresponding value of the normal stress.

SOLUTION

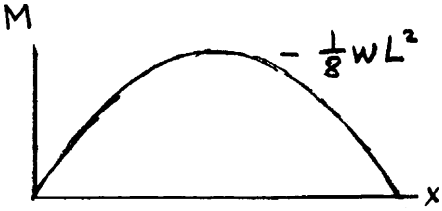


$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{1}{2}wL \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{2}wLx + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L - x)$$



where $w = 160$ kN/m and $L = 1.2$ m.

For the tapered beam, $h = a + kx$

$a = 120$ mm

$$k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For a rectangular cross section, $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a + kx)^2$

Bending stress: $\sigma = \frac{M}{S} = \frac{3w}{b}\frac{Lx - x^2}{(a + kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a + kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a + kx)^2(L - 2x) - (Lx - x^2)2(a + kx)k}{(a + kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - \cancel{2kx^2} - 2kLx + \cancel{2kx^2}}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - 2ax + kLx}{(a + kx)^3}\right\} = 0$$

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PROBLEM 5.149 *(Continued)*

(*a*) $x_m = \dfrac{aL}{2a + kL} = \dfrac{(120)(1.2)}{(2)(120) + (300)(1.2)} = 0.240 \text{ m}$ $\quad x_m = 240 \text{ mm}$ ◀

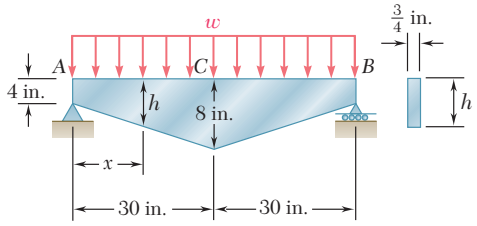
$h_m = a + k x_m = 120 + (300)(0.24) = 192 \text{ mm}$

$S_m = \dfrac{1}{6} b h_m^2 = \dfrac{1}{6}(20)(192)^2 = 122.88 \times 10^3 \text{ mm}^3 = 122.88 \times 10^{-6} \text{ m}^3$

$M_m = \dfrac{w}{2} x_m (L - x_m) = \dfrac{160 \times 10^3}{2}(0.24)(0.96) = 18.432 \times 10^3 \text{ N} \cdot \text{m}$

(*b*) Maximum bending stress: $\sigma_m = \dfrac{M_m}{S_m} = \dfrac{18.432 \times 10^3}{122.88 \times 10^{-6}} = 150 \times 10^6 \text{ Pa}$ $\quad \sigma_m = 150.0 \text{ MPa}$ ◀

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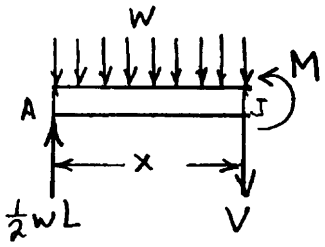


PROBLEM 5.150

For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest distributed load *w* that can be applied, knowing that $\sigma_{all} = 24$ ksi.

SOLUTION

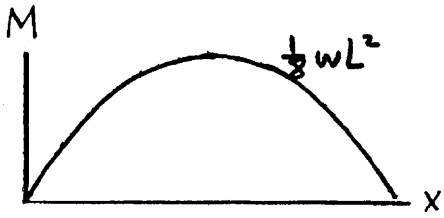
$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{1}{2}wL \uparrow \quad L = 60 \text{ in.}$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{2}wLx + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L - x)$$



For the tapered beam, $h = a + kx$

$$a = 4 \text{ in.} \quad k = \frac{8-4}{30} = \frac{2}{15} \text{ in./in.}$$

For a rectangular cross section, $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress: $\sigma = \frac{M}{S} = \frac{3w}{b} \cdot \frac{Lx - x^2}{(a+kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a+kx)^2}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)^2(L-2x) - (Lx-x^2)2(a+kx)k}{(a+kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)(L-2x) - 2k(Lx-x^2)}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a+kL)x}{(a+kx)^3}\right\} = 0$$

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PROBLEM 5.150 *(Continued)*

(*a*)
$$x_m = \frac{aL}{2a + kL} = \frac{(4)(60)}{(2)(4) + \left(\frac{2}{15}\right)(6.0)}$$
$x_m = 15.00$ in. ◀

$$h_m = a + kx_m = 4 + \left(\frac{2}{15}\right)(15) = 6.00 \text{ in.}$$

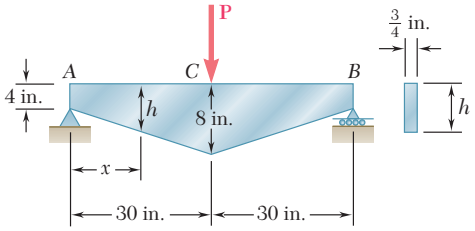
$$S_m = \frac{1}{6} b h_m^3 = \left(\frac{1}{6}\right)\left(\frac{3}{4}\right)(6.00)^2 = 4.50 \text{ in}^3$$

Allowable value of M_m: $M_m = S_m \sigma_{\text{all}} = (4.50)(24) = 180.0 \text{ kip} \cdot \text{in.}$

(*b*) Allowable value of *w*:
$$w = \frac{2M_m}{x_m(L - x_m)} = \frac{(2)(108.0)}{(15)(45)} = 0.320 \text{ kip/in.}$$

$w = 320$ lb/in. ◀

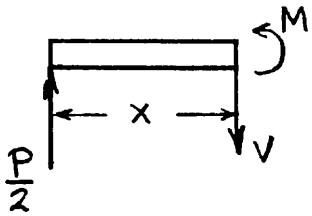
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PROBLEM 5.151

For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest concentrated load **P** that can be applied, knowing that $\sigma_{all} = 24$ ksi.

SOLUTION



$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{P}{2} \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \qquad \left(0 < x < \frac{L}{2}\right)$$

For a tapered beam, $h = a + kx$

For a rectangular cross section, $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress: $\sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{3P}{b}\frac{d}{dx}\left\{\frac{x}{(a+kx)^2}\right\} = \frac{3P}{b}\frac{(a+kx)^2 - x - 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b}\frac{a-kx}{(a+kx)^3} = 0 \qquad x_m = \frac{a}{k}$$

Data: $a = 4$ in., $\quad k = \frac{8-4}{30} = 0.13333$ in./in.

(*a*) $x_m = \frac{4}{0.13333} = 30$ in. $\qquad x_m = 30.0$ in. ◀

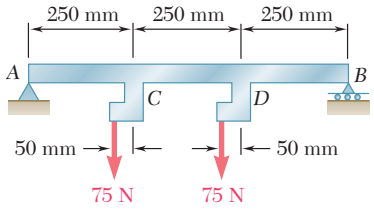
$h_m = a + kx_m = 8$ in.

$S_m = \frac{1}{6}bh_m^2 = \left(\frac{1}{6}\right)\left(\frac{3}{4}\right)(8)^2 = 8 \text{ in}^3$

$M_m = \sigma_{all} S_m = (24)(8) = 192 \text{ kip} \cdot \text{in.}$

(*b*) $P = \frac{2M_m}{x_m} = \frac{(2)(192)}{30} = 12.8$ kips $\qquad P = 12.80$ kips ◀

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PROBLEM 5.152

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

Reaction at *A*: $+\circlearrowleft M_B = 0: \; -0.750R_A + (0.550)(75) + (0.300)(75) = 0$

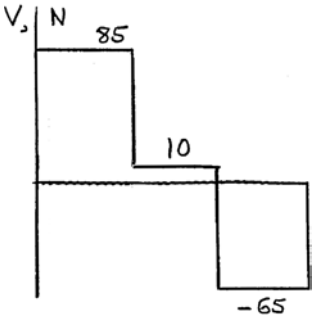
$$\boldsymbol{R}_A = 85\,\text{N}\uparrow$$

Also, $$\boldsymbol{R}_B = 65\,\text{N}\uparrow$$

A to *C*: $V = 85$ N

C to *D*: $V = 10$ N

D to *B*: $V = -65$ N

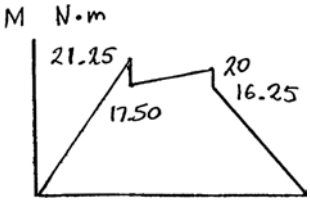


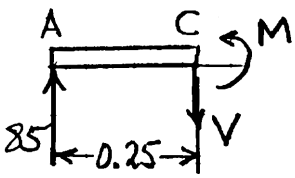
At *A* and *B*, $M = 0$

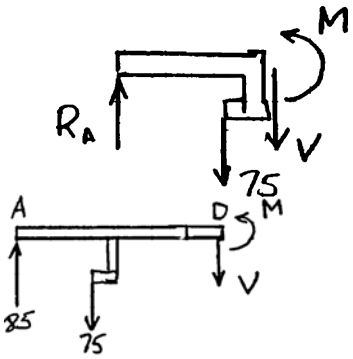
Just to the left of *C*,

$$+\circlearrowleft \Sigma M_C = 0: \; -(0.25)(85) + M = 0$$

$$M = 21.25\ \text{N} \cdot \text{m}$$







Just to the right of *C*,

$$+\circlearrowleft \Sigma M_C = 0: \; -(0.25)(85) + (0.050)(75) + M = 0$$

$$M = 17.50\ \text{N} \cdot \text{m}$$

Just to the left of *D*,

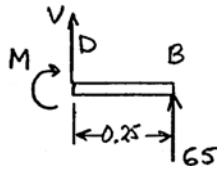
$$+\circlearrowleft \Sigma M_D = 0: \; -(0.50)(85) + (0.300)(75) + M = 0$$

$$M = 20\ \text{N} \cdot \text{m}$$

Just to the right of *D*,

$$+\circlearrowleft \Sigma M_D = 0: \; -M + (0.25)(65) = 0$$

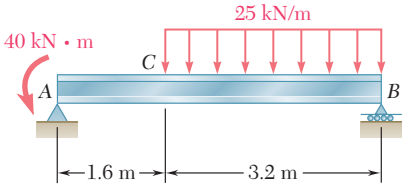
$$M = 16.25\ \text{kN}$$



(*a*) $|V|_{\max} = 85.0$ N ◀

(*b*) $|M|_{\max} = 21.3\ \text{N} \cdot \text{m}$ ◀

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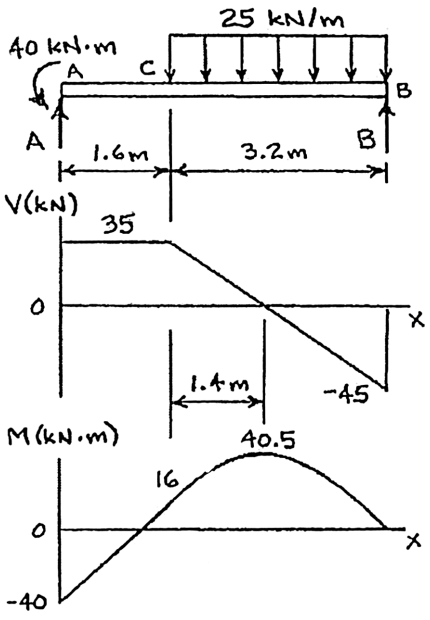




PROBLEM 5.153

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

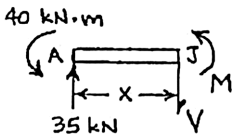


Reaction at A:

$$+\circlearrowleft \sum M_B = 0: \quad -4.8A + 40 + (25)(3.2)(1.6) = 0$$

$$\mathbf{A} = 35\ \text{kN} \uparrow$$

A to *C*: $0 < x < 1.6$ m



$$+\uparrow \sum F_y = 0: \quad 35 - V = 0 \quad V = 35\ \text{kN}$$

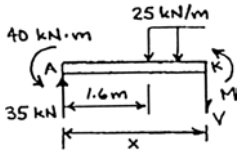
$$+\circlearrowleft \sum M_J = 0: \quad M + 40 - 35x = 0$$

$$M = (30x - 40)\ \text{kN} \cdot \text{m}$$

C to *B*: $1.6\ \text{m} < x < 4.8\ \text{m}$

$$+\uparrow \sum F_y = 0: \quad 35 - 25(x - 1.6) - V = 0$$

$$V = (-25x + 75)\ \text{kN}$$



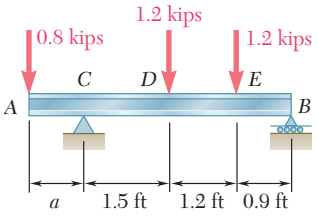
$$+\circlearrowleft \sum M_K = 0: \quad M + 40 - 35x + (25)(x - 1.6)\left(\frac{x - 1.6}{2}\right) = 0$$

$$M = (-12.5x^2 + 75x - 72)\ \text{kN} \cdot \text{m}$$

Normal stress: For W200 × 31.3, $S = 298 \times 10^3\ \text{mm}^3$

$$\sigma = \frac{|M|}{S} = \frac{40.5 \times 10^3\ \text{N} \cdot \text{m}}{298 \times 10^{-6}\ \text{m}^3} = 135.9 \times 10^6\ \text{Pa} \qquad \sigma = 135.9\ \text{MPa} \blacktriangleleft$$

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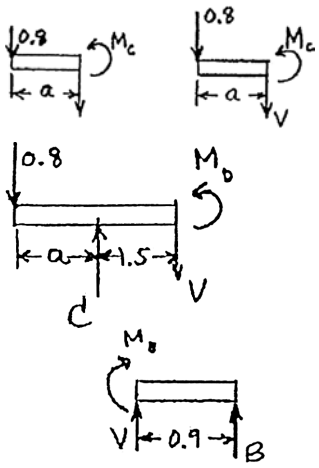
PROBLEM 5.154

Determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

$+\circlearrowleft \Sigma M_C = 0$: $0.8a - (1.5)(1.2) - (2.7)(1.2) + (3.6)B = 0$ $\quad \mathbf{B} = 1.4 - 0.22222a \uparrow$

$+\circlearrowleft \Sigma M_B = 0$: $(0.8)(3.6 + a) - 3.6C + (2.1)(1.2) + (0.9)(1.2) = 0$ $\quad \mathbf{C} = 1.8 + 0.22222a \uparrow$



Bending moment at *C*: $+\circlearrowleft \Sigma M_C = 0$: $M_C + (0.8)(a) = 0$

$M_C = -0.8a$

Bending moment at *D*: $\Sigma M_D = 0$:

$M_D + (0.8)(a + 1.5) - 1.5C = 0$

$M_D = 1.5 - 0.46667a$

Bending moment at *E*: $+\circlearrowleft \Sigma M_E = 0$: $-M_E + 0.9B = 0$

$M_E = 1.26 - 0.2a$

Assume $-M_C = M_E$: $\quad 0.8a = 1.26 - 0.2a$ $\quad a = 1.260 \text{ ft}$ ◀

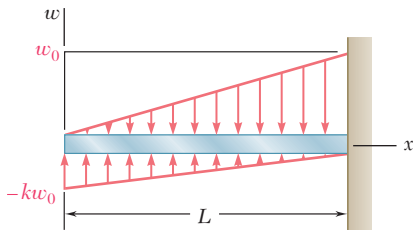
$M_C = -1.008 \text{ kip} \cdot \text{ft}$ $\quad M_E = 1.008 \text{ kip} \cdot \text{ft}$ $\quad M_D = 0.912 \text{ kip} \cdot \text{ft}$

Note that $|M_D| < 1.008 \text{ kip} \cdot \text{ft}$ $\quad \max|M| = 1.008 \text{ kip} \cdot \text{ft} = 12.096 \text{ kip} \cdot \text{in.}$

For rolled-steel section S3 × 5.7: $\quad S = 1.67 \text{ in}^3$

Normal stress: $\quad \sigma = \dfrac{|M|}{S} = \dfrac{12.096}{1.67}$ $\quad \sigma = 7.24 \text{ ksi}$ ◀

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PROBLEM 5.155

For the beam and loading shown, determine the equations of the shear and bending-moment curves, and the maximum absolute value of the bending moment in the beam, knowing that (*a*) $k = 1$, (*b*) $k = 0.5$.

SOLUTION

$$w = \frac{w_0 x}{L} - \frac{k w_0 (L - x)}{L} = (1 + k)\frac{w_0 x}{L} - kw$$

$$\frac{dV}{dx} = -w = k w_0 - (1 + k)\frac{w_0 x}{L}$$

$$V = k w_0 x - (1 + k)\frac{w_0 x^2}{2L} + C_1$$

$$V = 0 \quad \text{at} \quad x = 0 \quad C_1 = 0$$

$$\frac{dM}{dx} = V = k w_0 x - (1 + k)\frac{w_0 x^2}{2L}$$

$$M = \frac{k w_0 x^2}{2} - (1 + k)\frac{w_0 x^3}{6L} + C_2$$

$$M = 0 \quad \text{at} \quad x = 0 \quad C_2 = 0$$

$$M = \frac{k w_0 x^2}{2} - \frac{(1 + k) w_0 x^3}{6L}$$

(*a*) $k = 1$.

$$V = w_0 x - \frac{w_0 x^2}{L} \blacktriangleleft$$

$$M = \frac{w_0 x^2}{2} - \frac{w_0 x^3}{3L} \blacktriangleleft$$

Maximum M occurs at $x = L$.

$$|M|_{\max} = \frac{w_0 L^2}{6} \blacktriangleleft$$

(*b*) $k = \frac{1}{2}$.

$$V = \frac{w_0 x}{2} - \frac{3 w_0 x^2}{4L} \blacktriangleleft$$

$$M = \frac{w_0 x^2}{4} - \frac{w_0 x^3}{4L} \blacktriangleleft$$

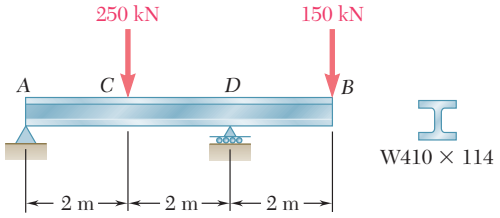
$$V = 0 \quad \text{at} \quad x = \frac{2}{3}L$$

At $x = \frac{2}{3}L$, $M = \frac{w_0 \left(\frac{2}{3}L\right)^2}{4} - \frac{w_0 \left(\frac{2}{3}L\right)^3}{4L} = \frac{w_0 L^2}{27} = 0.03704 w_0 L^2$

At $x = L$, $M = 0$

$$|M|_{\max} = \frac{w_0 L^2}{27} \blacktriangleleft$$

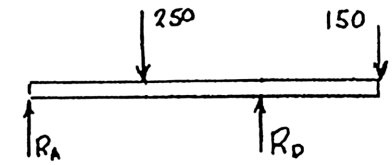
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PROBLEM 5.156

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION



$$w = 0$$

$$+\circlearrowleft \Sigma M_D = 0:$$

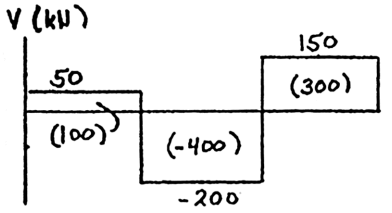
$$-4R_A + (2)(250) - (2)(150) = 0$$

$$\mathbf{R}_A = 50 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0:$$

$$4R_D - (2)(250) - (6)(150) = 0$$

$$\mathbf{R}_D = 350 \text{ kN} \uparrow$$

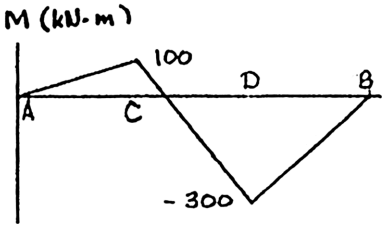


Shear: $V_A = 50$ kN

A to C: $V = 50$ kN

C to D: $V = 50 - 250 = -200$ kN

D to B: $V = -200 + 350 = 150$ kN



Areas of shear diagram:

A to C: $\int V\,dx = (50)(2) = 100$ kN · m

C to D: $\int V\,dx = (-200)(2) = -400$ kN · m

D to B: $\int V\,dx = (150)(2) = 300$ kN · m

Bending moments: $M_A = 0$

$$M_C = M_A + \int V\,dx = 0 + 100 = 100 \text{ kN} \cdot \text{m}$$

$$M_D = M_C + \int V\,dx = 100 - 400 = -300 \text{ kN} \cdot \text{m}$$

$$M_B = M_D + \int V\,dx = -300 + 300 = 0$$

Maximum $|V| = 200$ kN

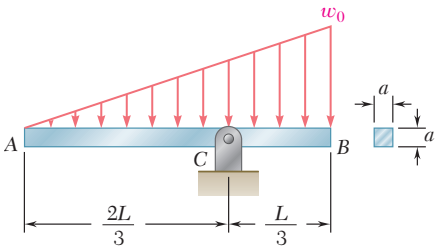
Maximum $|M| = 300 \text{ kN} \cdot \text{m} = 300 \times 10^3 \text{ N} \cdot \text{m}$

For W410 × 114 rolled-steel section, $S_x = 2200 \times 10^3 \text{ mm}^3 = 2200 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{\max}}{S_x} = \frac{300 \times 10^3}{2200 \times 10^{-6}} = 136.4 \times 10^6 \text{ Pa}$$

$$\sigma_m = 136.4 \text{ MPa} \blacktriangleleft$$

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PROBLEM 5.157

Beam *AB*, of length *L* and square cross section of side *a*, is supported by a pivot at *C* and loaded as shown. (*a*) Check that the beam is in equilibrium. (*b*) Show that the maximum normal stress due to bending occurs at *C* and is equal to $w_0L^2/(1.5a)^3$.

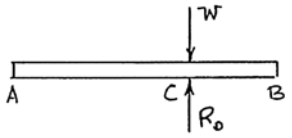
SOLUTION

(*a*) Replace distributed load by equivalent concentrated load at the centroid of the area of the load diagram.

For the triangular distribution, the centroid lies at $x = \frac{2L}{3}$. $W = \frac{1}{2}w_0L$

(*a*) $+\uparrow \Sigma F_y = 0$: $R_D - W = 0$ $R_D = \frac{1}{2}w_0L$ $+\circlearrowleft \Sigma M_C = 0$: $0 = 0$ equilibrium ◀

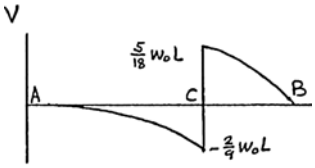
$V = 0,\ M = 0,$ at $x = 0$



$0 < x < \frac{2L}{3}$, $\frac{dV}{dx} = -w = -\frac{w_0x}{L}$

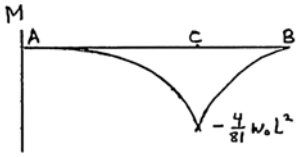
$$\frac{dM}{dx} = V = -\frac{w_0x^2}{2L} + C_1 = -\frac{w_0x^2}{2L}$$

$$M = -\frac{w_0x^3}{6L} + C_2 = -\frac{w_0x^3}{6L}$$



Just to the left of *C*, $V = -\frac{w_0(2L/3)^2}{2L} = -\frac{2}{9}w_0L$

Just to the right of *C*, $V = -\frac{2}{9}w_0L + R_D = \frac{5}{18}w_0L$



Note sign change. Maximum $|M|$ occurs at *C*. $M_C = -\frac{w_0(2L/3)^3}{6L} = -\frac{4}{81}w_0L^2$

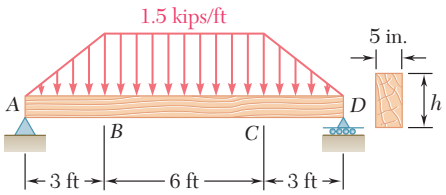
Maximum $|M| = \frac{4}{81}w_0L^2$

For square cross section, $I = \frac{1}{12}a^4$ $c = \frac{1}{2}a$

(*b*) $\sigma_m = \frac{|M|_{\max}c}{I} = \frac{4}{81}\frac{w_0L^2 6}{a^3} = \frac{8}{27}\frac{w_0L^2}{a^3} = \left(\frac{2}{3}\right)^3\frac{w_0L^2}{a^3}$

$$\sigma_m = \frac{w_0L^2}{(1.5a)^3}$$ ◀

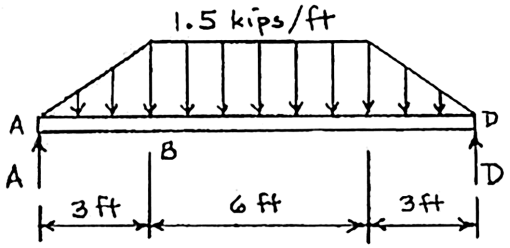
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PROBLEM 5.158

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

SOLUTION



Reactions: By symmetry, $A = D$.

$$+\uparrow \sum F_y = 0: \quad A - \frac{1}{2}(3)(1.5) - (6)(1.5) - \frac{1}{2}(3)(1.5) - D = 0$$

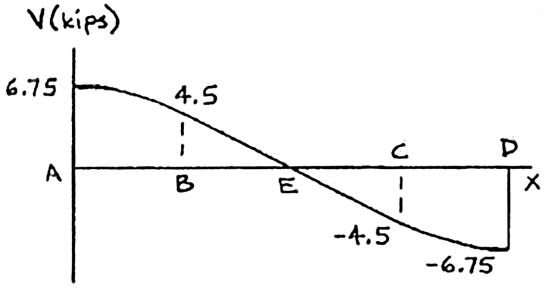
$$A = D = 6.75 \text{ kips} \uparrow$$

Shear diagram: $V_A = 6.75$ kips

$$V_B = 6.75 - \frac{1}{2}(3)(1.5) = 4.5 \text{ kips}$$

$$V_C = 4.5 - (6)(1.5) = -4.5 \text{ kips}$$

$$V_D = -4.5 - \frac{1}{2}(3)(1.5) = -6.75 \text{ kips}$$



Locate point E where $V = 0$.

By symmetry, E is the midpoint of BC.

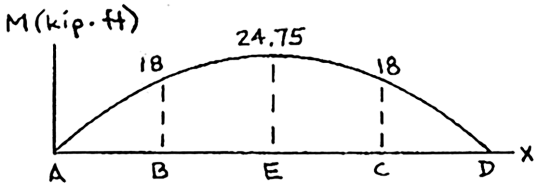
Areas of the shear diagram:

$$A \text{ to } B: \quad (3)(4.5) + \frac{2}{3}(3)(2.25) = 18 \text{ kip} \cdot \text{ft}$$

$$B \text{ to } E: \quad \frac{1}{2}(3)(4.5) = 6.75 \text{ kip} \cdot \text{ft}$$

$$E \text{ to } C: \quad \frac{1}{2}(3)(-4.5) = -6.75 \text{ kip} \cdot \text{ft}$$

C to D: By antisymmetry, -18 kip · ft



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PROBLEM 5.158 ***(Continued)***

Bending moments: $M_A = 0$

$$M_B = 0 + 18 = 18 \text{ kip} \cdot \text{ft}$$

$$M_E = 18 + 6.75 = 24.75 \text{ kip} \cdot \text{ft}$$

$$M_C = 24.75 - 6.75 = 18 \text{ kip} \cdot \text{ft}$$

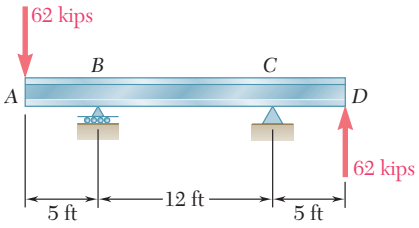
$$M_D = 18 - 18 = 0$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} \qquad S = \frac{|M|_{\max}}{\sigma_{\max}} = \frac{(24.75 \text{ kip} \cdot \text{ft})(12 \text{ in./ft})}{1.750 \text{ ksi}} = 169.714 \text{ in}^3$$

For a rectangular section, $S = \frac{1}{6}bh^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{6(169.714)}{5}} = 14.27 \text{ in.} \qquad h = 14.27 \text{ in.} \blacktriangleleft$$

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PROBLEM 5.159

Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

SOLUTION

$+\circlearrowleft \Sigma M_C = 0$: $(17)(62) - 12B + (5)(62) = 0$ $\qquad B = 113.667 \text{ kips} \uparrow$

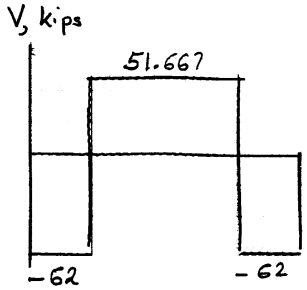
$+\circlearrowleft \Sigma M_B = 0$: $(5)(62) + 12C + (17)(62) = 0$ $\qquad C = -113.667 \text{ kips}$ or $C = 113.667 \text{ kips} \downarrow$

Shear diagram:

A to B^-: $V = -62$ kips

B^+ to C^-: $V = -62 + 113.667 = 51.667$ kips

C^+ to D: $V = 51.667 - 113.667 = -62$ kips

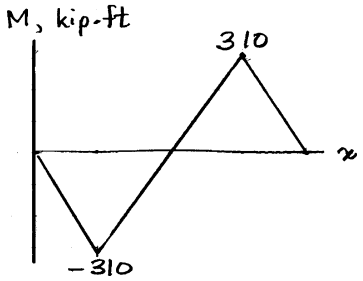


Areas of shear diagram:

A to B: $(5)(-62) = -310 \text{ kip} \cdot \text{ft}$

B to C: $(12)(51.667) = 620 \text{ kip} \cdot \text{ft}$

C to D: $(5)(-62) = -310 \text{ kip} \cdot \text{ft}$



Bending moments: $M_A = 0$

$M_B = 0 - 310 = -310 \text{ kip} \cdot \text{ft}$

$M_C = -310 + 620 = 310 \text{ kip} \cdot \text{ft}$

$M_D = 310 - 310 = 0$

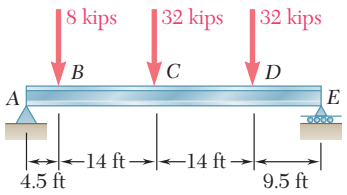
$|M|_{\max} = 310 \text{ kip} \cdot \text{ft} = 3.72 \times 10^3 \text{ kip} \cdot \text{in.}$

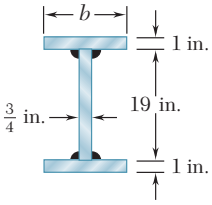
Required $S_{\min}$: $S_{\min} = \dfrac{|M|_{\max}}{\sigma_{\text{all}}} = \dfrac{3.72 \times 10^3}{24} = 155 \text{ in}^3$

Shape	$S(\text{in}^3)$
W27 × 84	213
W21 × 101	227
W18 × 106	204
W14 × 145	232

Use W27 × 84. ◀

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PROBLEM 5.160

Three steel plates are welded together to form the beam shown. Knowing that the allowable normal stress for the steel used is 22 ksi, determine the minimum flange width b that can be used.

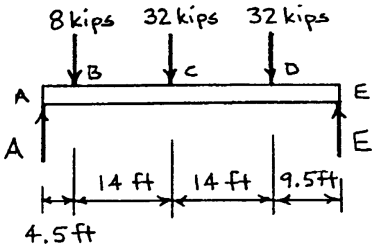
SOLUTION

Reactions: $+\circlearrowleft \Sigma M_E = 0: \quad -42A + (37.5)(8) + (23.5)(32) - (9.5)(32) = 0$

$$\boldsymbol{A} = 32.2857 \text{ kips} \uparrow$$

$+\circlearrowleft \Sigma M_A = 0: \quad 42E - (4.5)(8) - (18.5)(32) - (32.5)(32) = 0$

$$\boldsymbol{E} = 39.7143 \text{ kips} \uparrow$$



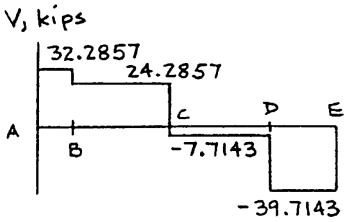
Shear:

A to B: 32.2857 kips

B to C: $32.2857 - 8 = 24.2857$ kips

C to D: $24.2857 - 32 = -7.7143$ kips

D to E: $-7.7143 - 32 = -39.7143$ kips



Areas:

A to B: $(4.5)(32.2857) = 145.286$ kip · ft

B to C: $(14)(24.2857) = 340$ kip · ft

C to D: $(14)(-7.7143) = -108$ kip · ft

D to E: $(9.5)(-39.7143) = -377.286$ kip · ft

Bending moments:

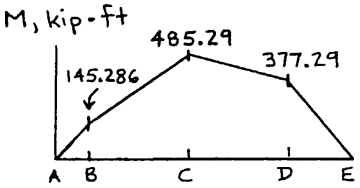
$M_A = 0$

$M_B = 0 + 145.286 = 145.286$ kip · ft

$M_C = 145.286 + 340 = 485.29$ kip · ft

$M_D = 485.29 - 108 = 377.29$ kip · ft

$M_E = 377.29 - 377.286 = 0$



Maximum $|M| = 485.29 \text{ kip} \cdot \text{ft} = 5.2834 \times 10^3 \text{ kip} \cdot \text{in.} \quad \sigma_{all} = 22 \text{ ksi}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{5.2834 \times 10^3}{22} = 264.70 \text{ in}^3$$

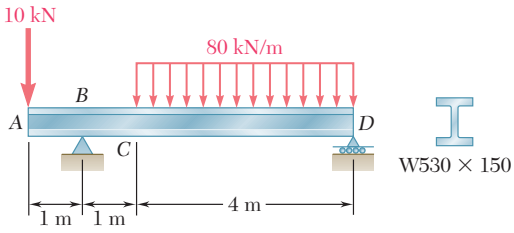
$$I = \frac{1}{12}\left(\frac{3}{4}\right)(19)^3 + 2\left[\frac{1}{12}(b)(1)^3 + (b)(1)(10)^2\right] = 428.69 + 200.17b$$

$$c = 9.5 + 1 = 10.5 \text{ in.}$$

$$S_{min} = \frac{I}{c} = 40.828 + 19.063b = 264.70$$

$$b = 11.74 \text{ in.} \blacktriangleleft$$

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PROBLEM 5.161

(*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft M_D = 0: \quad (6)(10) - 5R_B + (2)(4)(80) = 0$$

$$R_B = 140 \text{ kN}$$

$$w = 80\langle x-2\rangle^0 \text{ kN/m} = -dV/dx$$

$$V = -10 + 140\langle x-1\rangle^0 - 80\langle x-2\rangle^1 \text{ kN}$$

A to *B*: $V = -10 \text{ kN}$

B to *C*: $V = -10 + 140 = 130 \text{ kN}$

D: $(x = 6)$ $V = -10 + 140 - 80(4) = -190 \text{ kN}$

V changes sign at *B* and at point *E* $(x = x_E)$ between *C* and *D*.

$$V = 0 = -10 + 140\langle x_E - 1\rangle^0 - 80\langle x_E - 2\rangle^1$$

$$= -10 + 140 - 80(x_E - 2) \qquad x_E = 3.625 \text{ m}$$

$$M = -10x + 140\langle x-1\rangle^1 - 40\langle x-2\rangle^2 \text{ kN} \cdot \text{m}$$

At pt. *B*, $x = 1$ $M_B = -(10)(1) = -10 \text{ kN} \cdot \text{m}$

At pt. *E*, $x = 3.625$

$$M_E = -(10)(3.625) + (140)(2.625) - (40)(1.625)^2 = 225.6 \text{ kN} \cdot \text{m}$$

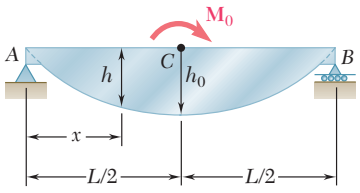
(*a*) $|M|_{\max} = 225.6 \text{ kN} \cdot \text{m}$ at $x = 3.63 \text{ m}$ ◀

For W530 × 150, $S = 3720 \times 10^3 \text{ mm}^3 = 3720 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{225.6 \times 10^3}{3720 \times 10^{-6}} = 60.6 \times 10^6 \text{ Pa}$

$\sigma = 60.6 \text{ MPa}$ ◀

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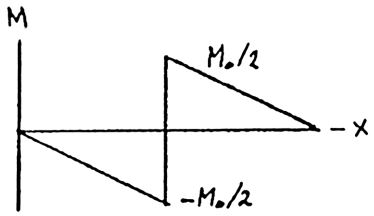


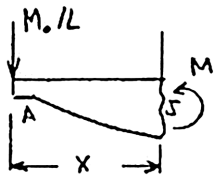
PROBLEM 5.162

The beam *AB*, consisting of an aluminum plate of uniform thickness *b* and length *L*, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0 for portion *AC* of the beam. (*b*) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.

SOLUTION

$$\mathbf{A} = M_0/L \downarrow \qquad \mathbf{B} = M_0/L \uparrow$$





$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{M_0}{L}x + M = 0$$

$$M = -\frac{M_0 x}{L} \quad \left(0 < x < \frac{L}{2}\right)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{M_0 x}{\sigma_{all} L} \qquad \text{for} \qquad \left(0 < x \frac{L}{2}\right)$$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

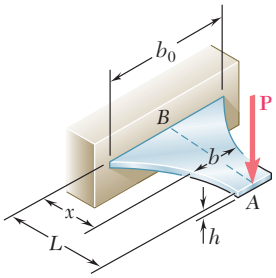
Equating, $\frac{1}{6}bh^2 = \frac{M_0 x}{\sigma_{all} L} \qquad h = \sqrt{\frac{6M_0 x}{\sigma_{all} bL}}$

(*a*) At $x = \frac{L}{2}$, $\qquad h = h_0 = \sqrt{\frac{3M_0}{\sigma_{all} b}} \qquad h = h_0\sqrt{2x/L}$ ◀

(*b*) Solving for M_0, $\quad M_0 = \frac{\sigma_{all} bh_0^2}{3} = \frac{(72 \times 10^6)(0.025)(0.200)^2}{3} = 24 \times 10^3 \text{ N} \cdot \text{m}$

$$M_0 = 24.0 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 5.163

A cantilever beam *AB* consisting of a steel plate of uniform depth *h* and variable width *b* is to support the concentrated load **P** at point *A*. (*a*) Knowing that the beam is to be of constant strength, express *b* in terms of *x*, *L*, and b_0. (*b*) Determine the smallest allowable value of *h* if $L = 300$ mm, $b_0 = 375$ mm, $P = 14.4$ kN, and $\sigma_{all} = 160$ MPa.

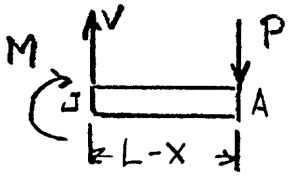
SOLUTION

$$+\circlearrowleft \Sigma M_J = 0: \quad -M - P(L-x) = 0$$

$$M = -P(L-x)$$

$$|M| = P(L-x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{P(L-x)}{\sigma_{all}}$$



For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating, $\frac{1}{6}bh^2 = \frac{P(L-x)}{\sigma_{all}}$ $\qquad b = \frac{6P(L-x)}{\sigma_{all}h^2}$

(*a*) At $x = 0$, $\qquad b = b_0 = \frac{6PL}{\sigma_{all}h^2}$ $\qquad b = b_0\left(1 - \frac{x}{L}\right)$ ◀

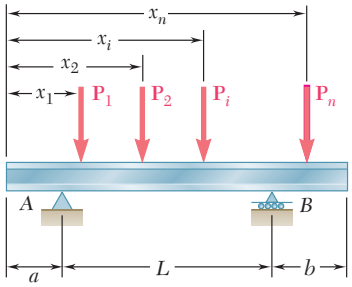
Solving for *h*, $\qquad h = \sqrt{\frac{6PL}{\sigma_{all}b_0}}$

Data: $L = 300 \text{ mm} = 0.300 \text{ m}, \quad b_0 = 375 \text{ mm} = 0.375 \text{ m}$

$P = 14.4 \text{ kN} = 14.4 \times 10^3 \text{ N} \cdot \text{m}, \quad \sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

(*b*) $$h = \sqrt{\frac{(6)(14.4 \times 10^3)(0.300)}{(160 \times 10^6)(0.375)}} = 20.8 \times 10^{-3} \text{ m}$$ $\qquad h = 20.8 \text{ mm}$ ◀

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PROBLEM 5.C1

Several concentrated loads $P_i (i = 1, 2, \ldots, n)$ can be applied to a beam as shown. Write a computer program that can be used to calculate the shear, bending moment, and normal stress at any point of the beam for a given loading of the beam and a given value of its section modulus. Use this program to solve Probs. 5.18, 5.21, and 5.25. (*Hint:* Maximum values will occur at a support or under a load.)

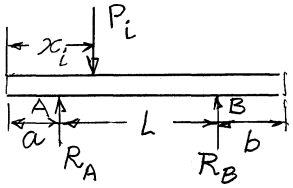
SOLUTION

Reactions at A and B

$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - \sum_i P_i (x_i - a)$$

$$R_B = \left(\frac{1}{L}\right) \sum_i P_i (x_i - a)$$

$$R_A = \sum_i P_i - R_B$$



We use step functions (See bottom of Page 348 of text.)

We define:

If $x \geq a$	Then STP $A = 1$	Else STP $A = 0$
If $x \geq a + L$	Then STP $B = 1$	Else STP $B = 0$
If $x \geq x_i$	Then STP $(I) = 1$	Else STP (I) $= 0$

$$V = R_A \text{ STP } A + R_B \text{ STP } B - \sum_i P_i \text{ STP (I)}$$

$$M = R_A (x - a) \text{ STP } A + R_B (x - a - L) \text{ STP } B - \sum_i P_i (x - x_i) \text{ STP (I)}$$

$\sigma = M/S$, where S is obtained from Appendix C.

Program Outputs

Problem 5.18

$R_A = 80.0$ kN $\quad R_B = 80.0$ kN

X m	V kN	M kN · m	σ MPa
2.00	0.00	104.00	139.0

◀

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PROBLEM 5.C1 *(Continued)*

Program Outputs *(Continued)*

Problem 5.21

$R_1 = 52.5$ kips $R_2 = 22.5$ kips

X ft	V kips	M kip · ft	σ ksi
0.00	–25.00	0.00	0.00
1.00	27.50	–25.00	–7.85
3.00	2.50	30.00	9.42
9.00	–22.50	45.00	14.14
11.00	0.00	0.00	0.00

◀

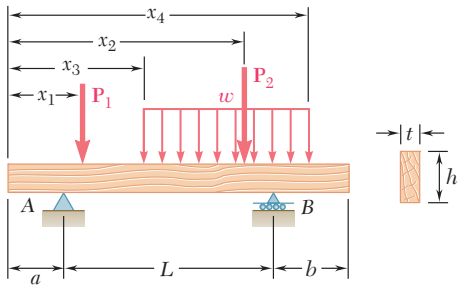
Problem 5.25

$R_1 = 10.77$ kips $R_2 = 4.23$ kips

X ft	V kips	M kip · in.	σ ksi
0	–5.00	0	0
5	5.69	–300.00	–10.34
13	–4.23	253.8	8.75
18	–4.23	0	0

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PROBLEM 5.C2



A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress in the beam will not exceed a given allowable value σ_{all}. Write a computer program that can be used to calculate at given intervals ΔL the shear, the bending moment, and the smallest acceptable value of the unknown dimension. Apply this program to solve the following problems, using the intervals ΔL indicated: (*a*) Prob. 5.65 ($\Delta L = 0.1$ m), (*b*) Prob. 5.69 ($\Delta L = 0.3$ m), (*c*) Prob. 5.70 ($\Delta L = 0.2$ m).

SOLUTION

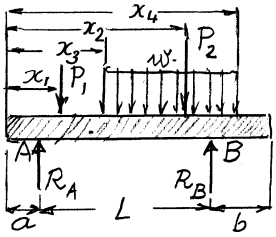
Reactions at *A* and *B*:

$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - P_1(x_1 - a) - P_2(x_2 - a) - w(x_4 - x_3)\left(\frac{x_4 + x_3}{2} - a\right) = 0$$

$$R_B = \frac{1}{L}\left[P_1(x_1 - a) + P_2(x_2 - a) + \frac{1}{2}w(x_4 - x_3)(x_4 + x_3 - 2a)\right]$$

$$R_A = P_1 + P_2 + w(x_4 - x_3) - R_B$$

We use step functions. (See bottom of Page 348 of text.)



Set $n = (a + b + L)/\Delta L$

For $i = 0$ to n: $x = (\Delta L)i$

We define:

If $x \geq a$	Then STP $A = 1$	Else STP $A = 0$
If $x \geq a + L$	Then STP $B = 1$	Else STP $B = 0$
If $x \geq x_1$	Then STP $1 = 1$	Else STP $1 = 0$
If $x \geq x_2$	Then STP $2 = 1$	Else STP $2 = 0$
If $x \geq x_3$	Then STP $3 = 1$	Else STP $3 = 0$
If $x \geq x_4$	Then STP $4 = 1$	Else STP $4 = 0$

$$V = R_A \text{ Step } A + R_B \text{ STP } B - P_1 \text{ STP} 1 - P_2 \text{ STP} 2 - w(x - x_3)\text{STP} 3 + w(x - x_4)\text{STP} 4$$

$$M = R_A(x - a)\text{STP } A + R_B(x - a - L)\text{STP } B - P_1(x - x_1)\text{STP} 1$$
$$-P_2(x - x_2)\text{STP} 2 - \frac{1}{2}w(x - x_3)^2 \text{ STP} 3 + \frac{1}{2}w(x - x_4)^4 \text{ STP} 4$$

$$S_{\min} = \frac{|M|}{\sigma_{all}}$$

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PROBLEM 5.C2 *(Continued)*

If unknown dimension is *h*:

From $S = \frac{1}{6}th^2$, we have $h = \sqrt{6S/t}$

If unknown dimension is *t*:

From $S = \frac{1}{6}th^2$, we have $t = 6S/h^2$

Program Outputs

Problem 5.65 $R_A = 2.40$ kN $R_B = 3.00$ kN

X m	*V* kN	*M* kN · m	*H* mm
0.00	2.40	0.000	0.00
0.10	2.40	0.240	54.77
0.20	2.40	0.480	77.46
0.30	2.40	0.720	94.87
0.40	2.40	0.960	109.54
0.50	2.40	1.200	122.47
0.60	2.40	1.440	134.16
0.70	2.40	1.680	144.91
0.80	0.60	1.920	154.92
0.90	0.60	1.980	157.32
1.00	0.60	2.040	159.69
1.10	0.60	2.100	162.02
1.20	0.60	2.160	164.32
1.30	0.60	2.220	166.58
1.40	0.60	2.280	168.82
1.50	0.60	2.340	171.03
1.60	–3.00	2.400	173.21
1.70	–3.00	2.100	162.02
1.80	–3.00	1.800	150.00
1.90	–3.00	1.500	136.93
2.00	–3.00	1.200	122.47
2.10	–3.00	0.900	106.07
2.20	–3.00	0.600	86.60
2.30	–3.00	0.300	61.24
2.40	0.00	0.000	0.05

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PROBLEM 5.C2 *(Continued)*

Program Outputs *(Continued)*

Problem 5.69

$R_A = 6.50$ kN $R_B = 6.50$ kN

X m	*V* kN	*M* kN · m	*H* mm
0.00	2.50	0.000	0.00
0.30	2.50	0.750	61.24
0.60	9.00	1.500	86.60
0.90	7.20	3.930	140.18
1.20	5.40	5.820	170.59
1.50	3.60	7.170	189.34
1.80	1.80	7.980	199.75
2.10	–0.00	8.250	203.10
2.40	–1.80	7.980	199.75
2.70	–3.60	7.170	189.34
3.00	–5.40	5.820	170.59
3.30	–7.20	3.930	140.18
3.60	–2.50	1.500	86.60
3.90	–2.50	0.750	61.24
4.20	0.00	0.000	0.06

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PROBLEM 5.C2 *(Continued)*

Program Outputs ***(Continued)***

Problem 5.70

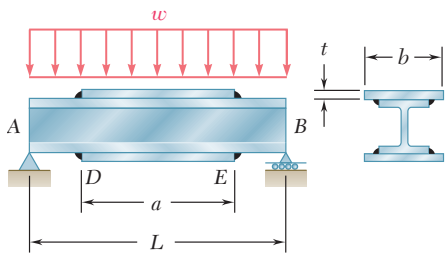
$R_A = 2.70$ kN $\quad R_B = 8.10$ kN

X m	V kN	M kN · m	T mm
0.00	2.70	0.000	0.00
0.20	2.10	0.480	10.67
0.40	1.50	0.840	18.67
0.60	0.90	1.080	24.00
0.80	0.30	1.200	26.67
1.00	–0.30	1.200	26.67
1.20	–0.90	1.080	24.00
1.40	–1.50	0.840	18.67
1.60	–2.10	0.480	10.67
1.80	–2.70	0.000	0.00
2.00	–3.30	–0.600	13.33
2.20	–3.90	–1.320	29.33
2.40	3.60	–2.160	48.00
2.60	3.00	–1.500	33.33
2.80	2.40	–0.960	21.33
3.00	1.80	–0.540	12.00
3.20	1.20	–0.240	5.33
3.40	0.60	–0.060	1.33
3.60	0.00	–0.000	0.00

◀

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PROBLEM 5.C3



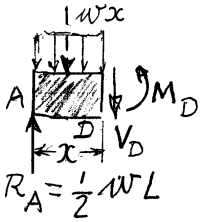
Two cover plates, each of thickness t, are to be welded to a wide-flange beam of length L that is to support a uniformly distributed load w. Denoting by σ_{all} the allowable normal stress in the beam and in the plates, by d the depth of the beam, and by I_b and S_b, respectively, the moment of inertia and the section modulus of the cross section of the unreinforced beam about a horizontal centroidal axis, write a computer program that can be used to calculate the required value of (*a*) the length a of the plates, (*b*) the width b of the plates. Use this program to solve Prob. 5.145.

SOLUTION

(*a*) Required length of plates:

$FB = AD$:

$$+\circlearrowleft \Sigma M_D = 0: \quad M_D + wx\left(\frac{x}{2}\right) - R_A x = 0$$



But

$$R_A = \frac{1}{2}wL \quad \text{and} \quad M_D = S\sigma_{all}.$$

Divide by $\frac{1}{2}w$:

$$x^2 - Lx + (2S\sigma_{all}/w) = 0$$

Set $k = \dfrac{2S\sigma_{all}}{w}$:

$$x^2 - Lx + k = 0$$

Solving the quadratic, $$x = \frac{L - \sqrt{L^2 - 4k}}{2}$$

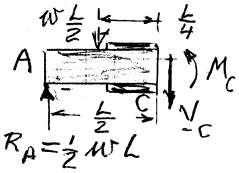
Compute x and $$a = L - 2x$$ ◀

(*b*) Required width of plates:

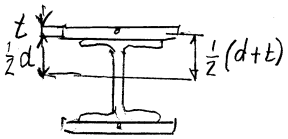
At midpoint C of beam:

$FB = AC$:

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + \frac{wL}{2}\frac{L}{4} - \frac{wL}{2}\frac{L}{2} = 0$$



Compute $$M_C = \frac{1}{8}wL^2$$



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PROBLEM 5.C3 *(Continued)*

Compute $C = t + \frac{1}{2}d$

From $\sigma_{\text{all}} = \frac{M_C C}{I}$ compute $I = \frac{M_C C}{\sigma_{\text{all}}}$

But $I = I_{\text{beam}} + I_{\text{plates}} = I_b + 2\left[\frac{1}{12}bt^3 + bt\left(\frac{d+t}{2}\right)^2\right]$

Solving for b, $b = \frac{6(I - I_b)}{t[t^2 + 3(d+t)^2]}$ ◀

Program Outputs

Problems 5.155:

$\text{W}460 \times 74$, $\sigma_{\text{all}} = 150$ MPa

$w = 40$ kN/m, $L = 8$ m, $t = 7.5$ mm

$d = 457$ mm, $I_b = 333 \times 10^6\,\text{mm}^4$, $S = 416 \times 10^3\,\text{mm}^3$

Problem 5.145

$a = 4.49$ m

$b = 211$ mm

Problem 5.157:

$\text{W}30 \times 99$, $\sigma_{\text{all}} = 22$ ksi

$w = 30$ kips/ft, $L = 16$ ft, $t = 5/8$ in.

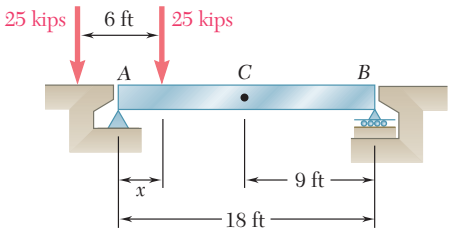
$d = 29.65$ in., $I_b = 3990$ in.^{4}, $S = 269$ in.^{3}

Problem 5.143

$a = 11.16$ ft

$b = 14.31$ in.

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PROBLEM 5.C4

Two 25-kip loads are maintained 6 ft apart as they are moved slowly across the 18-ft beam *AB*. Write a computer program and use it to calculate the bending moment under each load and at the midpoint *C* of the beam for values of *x* from 0 to 24 ft at intervals $\Delta x = 1.5$ ft.

SOLUTION

Notation: Length of beam $= L = 18$ ft

Loads: $P_1 = P_2 = P = 25$ kips

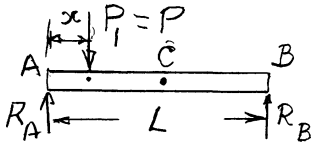
Distance between loads $= d = 6$ ft

We note that $d < L/2$.

(1) From $x = 0$ to $x = d$:

$$+\circlearrowleft \Sigma M_B = 0: \quad P(L-x) - R_A L = 0$$

$$R_A = P(L-x)/L$$



Under P_1: $M_1 = R_A x$

At *C*: $M_C = R_A\left(\frac{L}{2}\right) - P\left(\frac{L}{2} - x\right)$

(2) From $x = d$ to $x = L$:

$$+\circlearrowleft \Sigma M_B = 0: \quad P(L-x) + P(L-x+d) - R_A L = 0$$

$$R_A = P(2L - 2x + d)/L$$

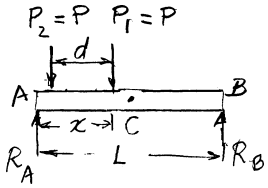
Under P_1: $M_1 = R_A x - Pd$

Under P_2: $M_2 = R_A(x-d)$

(2A) From $x = d$ to $x = L/2$:

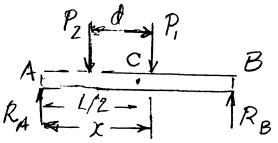
$$M_C = R_A\left(\frac{L}{2}\right) - P\left(\frac{L}{2} - x\right) - P\left(\frac{L}{2} - x + d\right)$$

$$= R_A(L/2) - P(L - 2x + d)$$



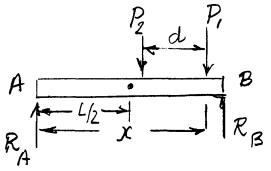
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PROBLEM 5.C4 *(Continued)*



(2B) From $x = L/2$ to $x = L/2 + d$:

$$M_C = R_A(L/2) - P\left(\frac{L}{2} - x + d\right)$$



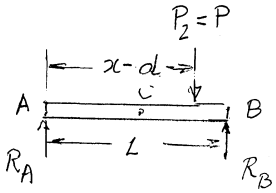
(2C) From $x = L/2 + d$ to $x = L$:

$$M_C = R_A L/2$$

(3) From $x = L$ to $x = L + d$:

$$+\circlearrowleft \Sigma M_B = 0: \quad P(L - x + d) - R_A L = 0$$

$$R_A = P(L - x + d)/L$$



Under P_2: $M_z = R_A(x - d)$

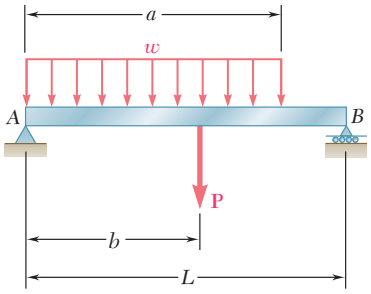
At C: $M_C = R_A(L/2)$

Program Output

$P = 25$ kips, $L = 18$ ft, $D = 6$ ft

X ft	M_C kip · ft	M_1 kip · ft	M_2 kip · ft
0.0	0.00	0.00	0.00
1.5	18.75	34.38	0.00
3.0	37.50	62.50	0.00
4.5	56.25	84.38	0.00
6.0	75.00	100.00	0.00
7.5	112.50	131.25	56.25
9.0	150.00	150.00	100.00
10.5	150.00	156.25	131.25
12.0	150.0	150.00	150.00
13.5	150.00	131.25	156.25
15.0	150.00	100.00	150.00
16.5	112.50	56.25	131.25
18.0	75.00	0.00	100.00
19.5	56.25	0.00	84.38
21.0	37.50	0.00	62.50
22.5	18.75	0.00	34.38
24.0	0.00	0.00	0.00

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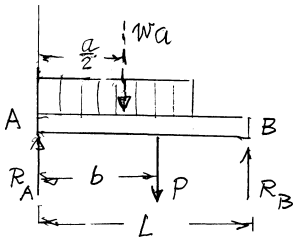
PROBLEM 5.C5

Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.2$ ft to the beam and loading of (*a*) Prob. 5.72, (*b*) Prob. 5.115.

SOLUTION

Reactions at *A* and *B*:

Using *FB* diagram of beam,



$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - Pb - wa(a/2) = 0$$

$$R_B = (1/L)\left(Pb + \frac{1}{2}wa^2\right)$$

$$R_A = P + wa - R_B$$

We use step functions (see bottom of Page 348 of text).

$$\text{set } n = L/\Delta L. \quad \text{For} \quad i = 0 \quad \text{to} \quad n: \quad x = (\Delta L)i$$

We define:

If $x \geqslant a$ Then STP $A = 1$ Else STP $A = 0$

If $x \geqslant b$ Then STP $B = 1$ Else STP $B = 0$

$$V = R_A - wx + w(x-a)\,\text{STP}\,A - P\,\text{STP}\,B$$

$$M = R_A x - \frac{1}{2}wx^2 + \frac{1}{2}w(x-a)^2\,\text{STP}\,A - P(x-b)\,\text{STP}\,B$$

Locate and Print $(x,\ v)$ and $(x,\ M)$

See next pages for program outputs

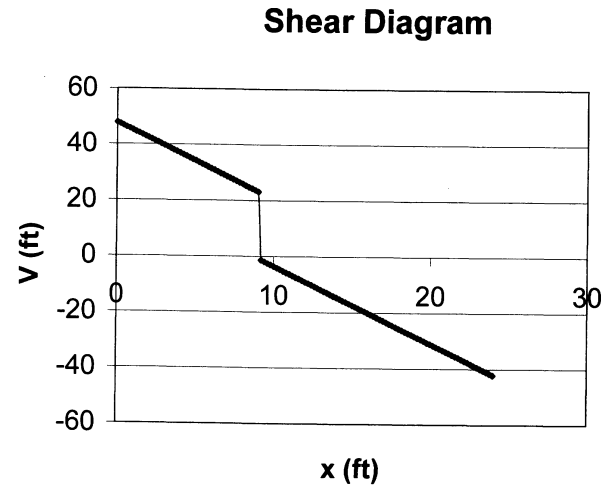
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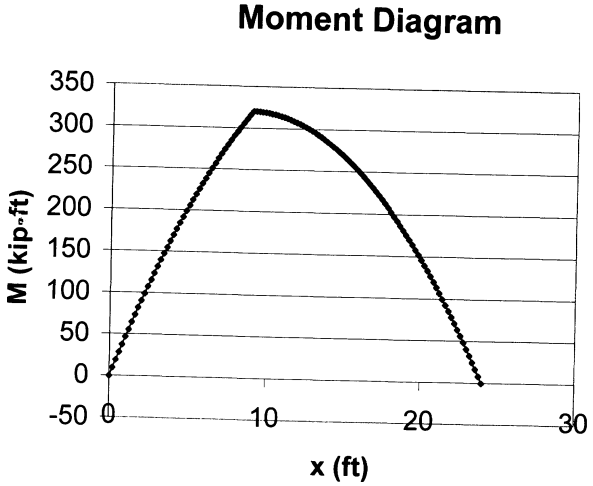
PROBLEM 5.C5 *(Continued)*

Program Outputs

Problem 5.72

$R_A = 48.00$ kips $R_B = 42.00$ kips





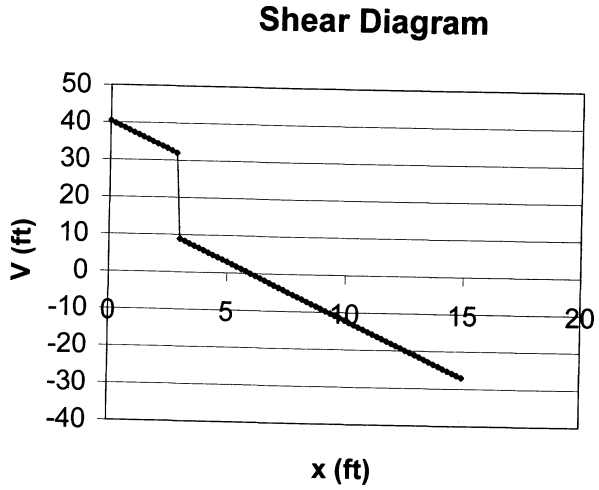
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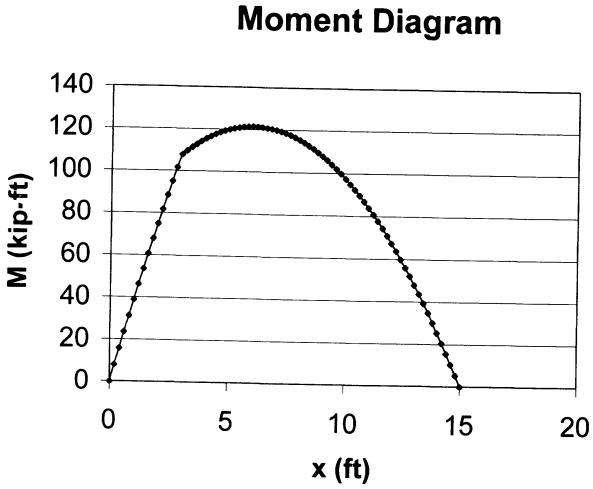
PROBLEM 5.C5 *(Continued)*

Program Outputs ***(Continued)***

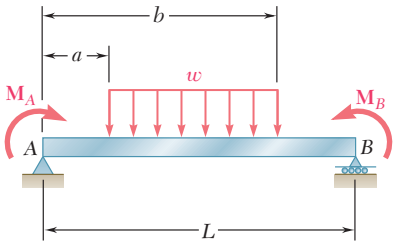
Problem 5.115

$R_A = 40.50$ kips $R_B = 27.00$ kips





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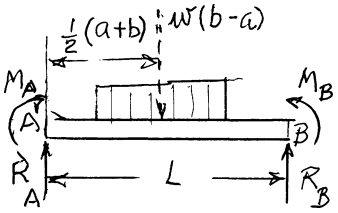


PROBLEM 5.C6

Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.025$ m to the beam and loading of Prob. 5.112.

SOLUTION

Reactions at *A* and *B*:



$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L + M_B - M_A = w(b-a)\frac{1}{2}(a+b) = 0$$

$$R_B = (1/L)\left[M_A - M_B + \frac{1}{2}w(b^2 - a^2)\right]$$

$$R_A = w(b-a) - R_B$$

We use step functions (see bottom of Page 348 of text).

Set $$n = \frac{L}{\Delta L}$$

For $$i = 0 \text{ to } n: \quad x = (\Delta L)i$$

We define:

$$\text{If } x \geqslant a \quad \text{Then STP}A = 1 \quad \text{Else STP}A = 0$$
$$\text{If } x \geqslant b \quad \text{Then STP}B = 1 \quad \text{Else STP}B = 0$$

$$V = R_A - w(x-a)\,\text{STP}A + w(x-b)\,\text{STP}B$$

$$M = M_A + R_A x - \frac{1}{2}w(x-a)^2\,\text{STP}A + \frac{1}{2}w(x-b)^2\,\text{STP}B$$

Locate and print (x, V) and (x, M).

Program output on next page

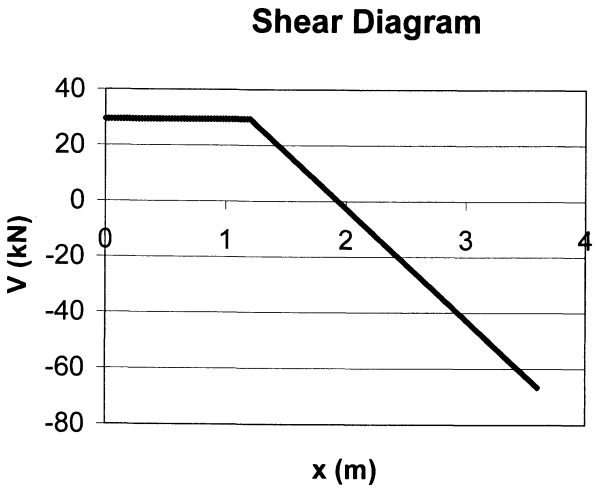
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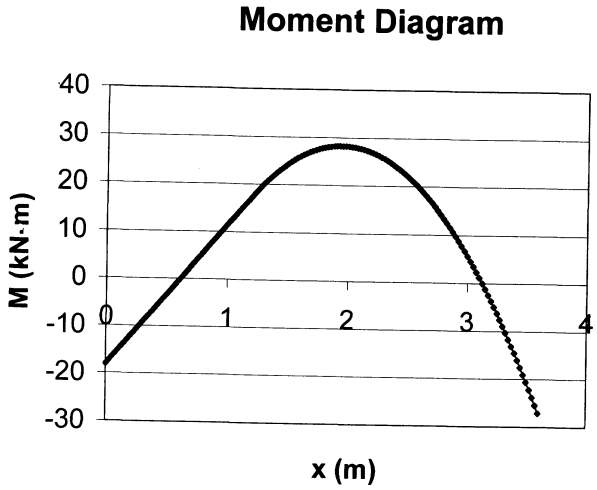
PROBLEM 5.C6 ***(Continued)***

Program Output

Problem 5.112

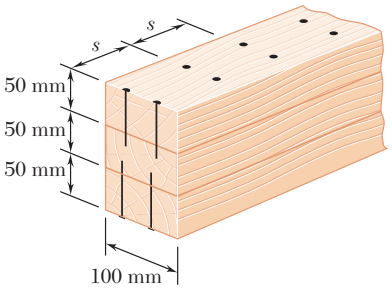
$R_A = 29.50$ kips $R_B = 66.50$ kips





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CHAPTER 6



PROBLEM 6.1

Three full-size 50 × 100-mm boards are nailed together to form a beam that is subjected to a vertical shear of 1500 N. Knowing that the allowable shearing force in each nail is 400 N, determine the largest longitudinal spacing s that can be used between each pair of nails.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4$$

$$= 28.125 \times 10^{-6} \text{ m}^4$$

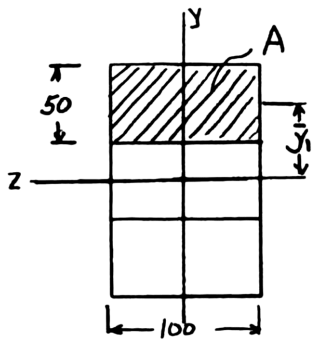
$$A = (100)(50) = 5000 \text{ mm}^2$$

$$\bar{y}_1 = 50 \text{ mm}$$

$$Q = A\bar{y}_1 = 250 \times 10^3 \text{ mm}^3 = 250 \times 10^{-6} \text{ m}^3$$

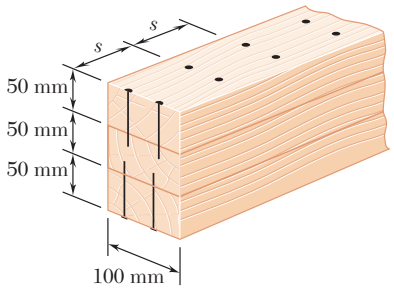
$$q = \frac{VQ}{I} = \frac{(1500)(250 \times 10^{-6})}{28.125 \times 10^{-6}} = 13.3333 \times 10^3 \text{ N/m}$$

$$qs = 2F_{\text{nail}} \qquad s = \frac{2F_{\text{nail}}}{q} = \frac{(2)(400)}{13.3333 \times 10^3} = 60.0 \times 10^{-3} \text{ m}$$



$$s = 60.0 \text{ mm} \blacktriangleleft$$

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PROBLEM 6.2

For the built-up beam of Prob. 6.1, determine the allowable shear if the spacing between each pair of nails is $s = 45$ mm.

PROBLEM 6.1 Three full-size 50 × 100-mm boards are nailed together to form a beam that is subjected to a vertical shear of 1500 N. Knowing that the allowable shearing force in each nail is 400 N, determine the largest longitudinal spacing s that can be used between each pair of nails.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4$$

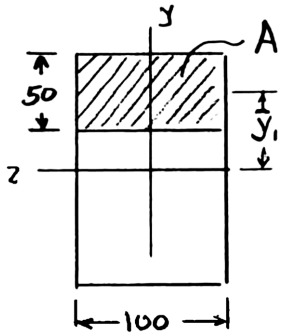
$$= 28.125 \times 10^{-6} \text{ m}^4$$

$$A = (100)(50) = 5000 \text{ mm}^2$$

$$\bar{y}_1 = 50 \text{ mm}$$

$$Q = A\bar{y}_1 = 250 \times 10^3 \text{ mm}^3 = 250 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} \qquad qs = 2F_{\text{nail}}$$

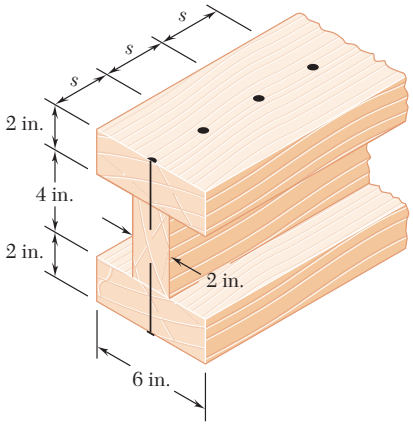


Eliminating q, $$\frac{VQ}{I} = \frac{2F_{\text{nail}}}{s}$$

Solving for V, $$V = \frac{2IF_{\text{nail}}}{Qs} = \frac{(2)(28.125 \times 10^{-6})(400)}{(250 \times 10^{-6})(45 \times 10^{-3})} = 2.00 \times 10^3 \text{ N}$$

$$V = 2.00 \text{ kN} \blacktriangleleft$$

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PROBLEM 6.3

Three boards, each 2 in. thick, are nailed together to form a beam that is subjected to a vertical shear. Knowing that the allowable shearing force in each nail is 150 lb, determine the allowable shear if the spacing s between the nails is 3 in.

SOLUTION

$$I_1 = \frac{1}{12}bh^3 + Ad^2$$

$$= \frac{1}{12}(6)(2)^3 + (6)(2)(3)^2 = 112 \text{ in}^4$$

$$I_2 = \frac{1}{12}bh^3 = \frac{1}{12}(2)(4)^3 = 10.6667 \text{ in}^4$$

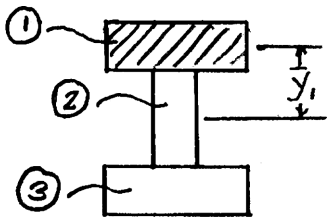
$$I_3 = I_1 = 112 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 234.67 \text{ in}^4$$

$$Q = A_1\bar{y}_1 = (6)(2)(3) = 36 \text{ in}^3$$

$$qs = F_{\text{nail}} \quad (1)$$

$$q = \frac{VQ}{I} \quad (2)$$



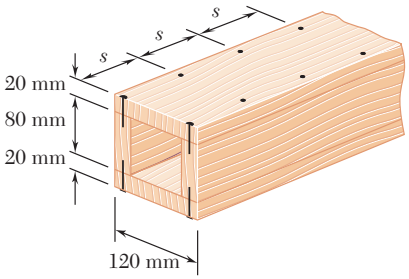
Dividing Eq. (2) by Eq. (1),

$$\frac{1}{s} = \frac{VQ}{F_{\text{nail}}I}$$

$$V = \frac{F_{\text{nail}}I}{Qs} = \frac{(150)(234.67)}{(36)(3)}$$

$$V = 326 \text{ lb} \blacktriangleleft$$

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PROBLEM 6.4

A square box beam is made of two 20×80-mm planks and two 20×120-mm planks nailed together as shown. Knowing that the spacing between the nails is $s=30$ mm and that the vertical shear in the beam is $V=1200$ N, determine (*a*) the shearing force in each nail, (*b*) the maximum shearing stress in the beam.

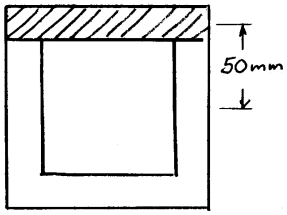
SOLUTION

$$I=\frac{1}{12}b_2h_2^3-\frac{1}{12}b_1h_1^3$$
$$=\frac{1}{12}(120)(120)^3-\frac{1}{12}(80)(80)^3=13.8667\times10^6\,\text{mm}^4$$
$$=13.8667\times10^{-6}\,\text{m}^4$$

(*a*) $A_1=(120)(20)=2400\ \text{mm}^2$

$\bar{y}_1=50$ mm

$Q_1=A_1\bar{y}_1=120\times10^3\ \text{mm}^3=120\times10^{-6}\,\text{m}^3$



$$q=\frac{VQ}{I}=\frac{(1200)(120\times10^{-6})}{13.8667\times10^{-6}}=10.3846\times10^3\ \text{N/m}$$

$qs=2F_{\text{nail}}$

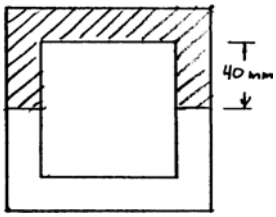
$$F_{\text{nail}}=\frac{qs}{2}=\frac{(10.3846\times10^3)(30\times10^{-3})}{2}$$

$F_{\text{nail}}=155.8$ N ◀

(*b*) $Q=Q_1+(2)(20)(40)(20)$

$=120\times10^3+32\times10^3=152\times10^3\ \text{mm}^3$

$=152\times10^{-6}\ \text{m}^3$

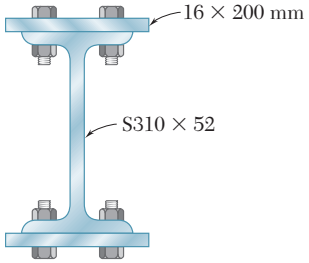


$$\tau_{\text{max}}=\frac{VQ}{It}=\frac{(1200)(152\times10^{-6})}{(13.8667\times10^{-6})(2\times20\times10^{-3})}$$

$=329\times10^3\,\text{Pa}$

$\tau_{\text{max}}=329$ kPa ◀

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PROBLEM 6.5

The American Standard rolled-steel beam shown has been reinforced by attaching to it two 16×200-mm plates, using 18-mm-diameter bolts spaced longitudinally every 120 mm. Knowing that the average allowable shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shearing force.

SOLUTION

Calculate moment of inertia:

Part	$A\,(\text{mm}^2)$	$d\,(\text{mm})$	$Ad^2\,(10^6\,\text{mm}^4)$	$\bar{I}\,(10^6\,\text{mm}^4)$
Top plate	3200	$^*160.5$	82.43	0.07
S310 × 52	6650	0		95.3
Bot. plate	3200	$^*160.5$	82.43	0.07
Σ			164.86	95.44

$$^*d = \frac{305}{2} + \frac{16}{2} = 160.5 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 260.3 \times 10^6\,\text{mm}^4 = 260.3 \times 10^{-6}\,\text{m}^4$$

$$Q = A_{\text{plate}}d_{\text{plate}} = (3200)(160.5) = 513.6 \times 10^3\,\text{mm}^3 = 513.6 \times 10^{-6}\,\text{m}^3$$

$$A_{\text{bolt}} = \frac{\pi}{4}d_{\text{bolt}}^2 = \frac{\pi}{4}(18 \times 10^{-3})^2 = 254.47 \times 10^{-6}\,\text{m}^2$$

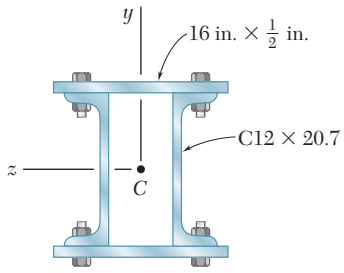
$$F_{\text{bolt}} = \tau_{\text{all}}A_{\text{bolt}} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.90 \times 10^3\,\text{N}$$

$$qs = 2F_{\text{bolt}} \qquad q = \frac{2F_{\text{bolt}}}{s} = \frac{(2)(22.90 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3\,\text{N/m}$$

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(260.3 \times 10^{-6})(381.7 \times 10^3)}{513.6 \times 10^{-6}} = 193.5 \times 10^3\,\text{N}$$

$$V = 193.5\text{ kN} \blacktriangleleft$$

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PROBLEM 6.6

The beam shown is fabricated by connecting two channel shapes and two plates, using bolts of $\frac{3}{4}$-in. diameter spaced longitudinally every 7.5 in. Determine the average shearing stress in the bolts caused by a shearing force of 25 kips parallel to the *y* axis.

SOLUTION

C12 × 20.7: $d = 12.00$ in., $I_x = 129$ in^4

For top plate, $\bar{y} = \dfrac{12.00}{2} + \left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right) = 6.25$ in.

$$I_t = \frac{1}{12}(16)\left(\frac{1}{2}\right)^3 + (16)\left(\frac{1}{2}\right)(6.25)^2 = 312.667 \text{ in}^4$$

For bottom plate, $I_b = 312.667$ in^4

Moment of inertia of fabricated beam:

$$I = (2)(129) + 312.667 + 312.667$$
$$= 883.33 \text{ in}^4$$

$$Q = A_{\text{plate}}\bar{y}_{\text{plate}} = (16)\left(\frac{1}{2}\right)(6.25) = 50 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(25)(50)}{883.33} = 1.41510 \text{ kips/in.}$$

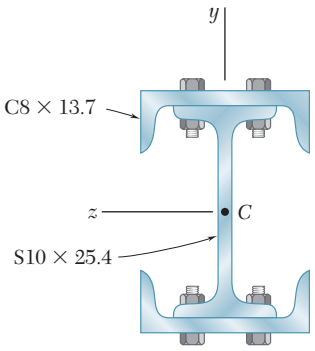
$$F_{\text{bolt}} = \frac{1}{2}qs = \left(\frac{1}{2}\right)(1.41510)(7.5) = 5.3066 \text{ kips}$$

$$A_{\text{bolt}} = \frac{\pi}{4}(d_{\text{bolt}})^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.44179 \text{ in}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{5.3066}{0.44179} = 12.01 \text{ ksi}$$

$\tau_{\text{bolt}} = 12.01$ ksi ◀

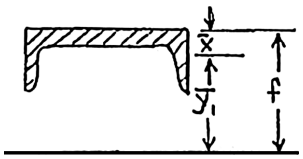
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PROBLEM 6.7

A columm is fabricated by connecting the rolled-steel members shown by bolts of $\frac{3}{4}$-in. diameter spaced longitudinally every 5 in. Determine the average shearing stress in the bolts caused by a shearing force of 30 kips parallel to the *y* axis.

SOLUTION



Geometry:

$$f = \left(\frac{d}{2}\right)_s + (t_w)_C$$

$$= \frac{10.0}{2} + 0.303 = 5.303 \text{ in.}$$

$$\bar{x} = 0.534 \text{ in.}$$

$$\bar{y}_1 = f - \bar{x} = 5.303 - 0.534 = 4.769 \text{ in.}$$

Determine moment of inertia.

Part	$A(\text{in}^2)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}\ (\text{in}^4)$
C8 × 13.7	4.04	4.769	91.88	1.52
S10 × 25.4	7.45	0	0	123
C8 × 13.7	4.04	4.769	91.88	1.52
Σ			183.76	126.04

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 183.76 + 126.04 = 309.8 \text{ in}^4$$

$$Q = A\bar{y}_1 = (4.04)(4.769) = 19.2668 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(30)(19.2668)}{309.8} = 1.86573 \text{ kip/in.}$$

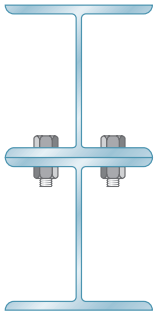
$$F_{\text{bolt}} = \frac{1}{2}qs = \left(\frac{1}{2}\right)(1.86573)(5) = 4.6643 \text{ kips}$$

$$A_{\text{bolt}} = \frac{\pi}{4}d_{\text{bolt}}^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.44179 \text{ in}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{4.6643}{0.44179} = 10.56 \text{ ksi}$$

$\tau_{\text{bolt}} = 10.56 \text{ ksi}$ ◀

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PROBLEM 6.8

The composite beam shown is fabricated by connecting two $W6 \times 20$ rolled-steel members, using bolts of $\frac{5}{8}$-in. diameter spaced longitudinally every 6 in. Knowing that the average allowable shearing stress in the bolts is 10.5 ksi, determine the largest allowable vertical shear in the beam.

SOLUTION

$W6 \times 20$: $A = 5.87 \text{ in}^2$, $d = 6.20 \text{ in.}$, $I_x = 41.4 \text{ in}^4$

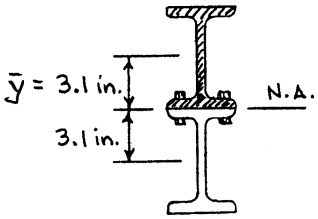
$$\bar{y} = \frac{1}{2}d = 3.1 \text{ in.}$$

Composite:

$$I = 2[41.4 + (5.87)(3.1)^2]$$

$$= 195.621 \text{ in}^4$$

$$Q = A\bar{y} = (5.87)(3.1) = 18.197 \text{ in}^3$$



Bolts:

$$d = \frac{5}{8} \text{ in.}, \quad \tau_{all} = 10.5 \text{ ksi}, \quad s = 6 \text{ in.}$$

$$A_{bolt} = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.30680 \text{ in}^2$$

$$F_{bolt} = \tau_{all} A_{bolt} = (10.5)(0.30680) = 3.2214 \text{ kips}$$

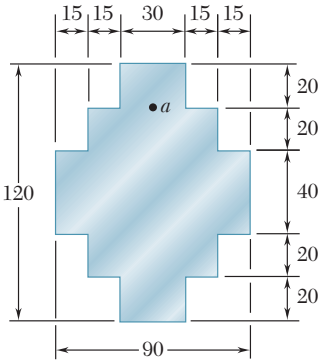
$$q = \frac{2F_{bolt}}{s} = \frac{(2)(3.2214)}{6} = 1.07380 \text{ kips/in.}$$

Shear:

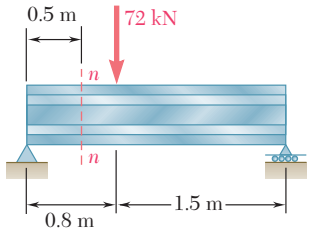
$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(195.621)(1.0780)}{18.197}$$

$V = 11.54 \text{ kips}$ ◀

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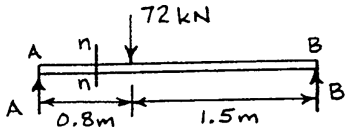
Dimensions in mm



PROBLEM 6.9

For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad -2.3A + (1.5)(72) = 0$$

$$\boldsymbol{A} = 46.957 \text{ kN} \uparrow$$

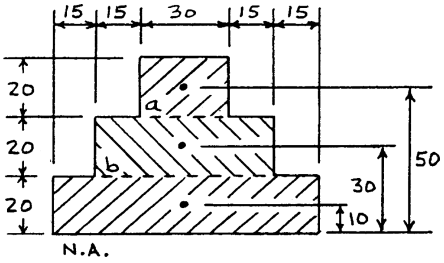
At section *n-n*, $V = A = 46.957$ kN

Calculate moment of inertia:

$$I = 2\left[\frac{1}{12}(15)(40)^3\right] + 2\left[\frac{1}{12}(15)(80)^3\right] + \frac{1}{12}(30)(120^3)$$

$$= 5.76\times10^{6}\,\text{mm}^4 = 5.76\times10^{-6}\,\text{m}^4$$

At *a*, $t_a = 30 \text{ mm} = 0.030 \text{ m}$



$$Q_a = (30\times20)(50) = 30\times10^{3}\,\text{mm}^3$$

$$= 30\times10^{-6}\,\text{m}^3$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(46.957\times10^{3})(30\times10^{-6})}{(5.76\times10^{-6})(0.030)}$$

$$= 8.15\times10^{6} \text{ Pa} = 8.15 \text{ MPa}$$

At *b*, $t_b = 60 \text{ mm} = 0.060 \text{ m}$

$$Q_b = Q_a + (60\times20)(30) = 30\times10^{3} + 36\times10^{3} = 66\times10^{3}\,\text{mm}^3 = 66\times10^{-6}\,\text{m}^4$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(46.957\times10^{3})(66\times10^{-6})}{(5.76\times10^{-6})(0.060)} = 8.97\times10^{6}\,\text{Pa} = 8.97 \text{ MPa}$$

At *NA*, $t_{NA} = 90 \text{ mm} = 0.090 \text{ m}$

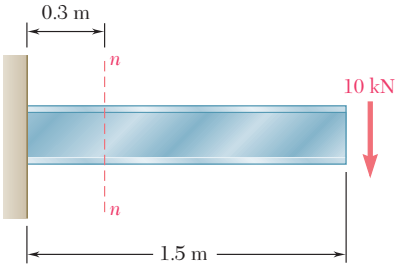
$$Q_{NA} = Q_b + (90\times20)(10) = 66\times10^{3} + 18\times10^{3} = 84\times10^{3}\,\text{mm}^3 = 84\times10^{-6}\,\text{m}^3$$

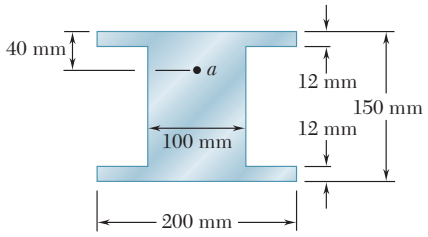
$$\tau_{NA} = \frac{VQ_{NA}}{It_{NA}} = \frac{(46.957\times10^{3})(84\times10^{-6})}{(5.76\times10^{-6})(0.090)} = 7.61\times10^{6}\,\text{Pa} = 7.61 \text{ MPa}$$

(*a*) τ_{max} occurs at *b*. $\tau_{max} = 8.97$ MPa ◀

(*b*) $\tau_a = 8.15$ MPa ◀

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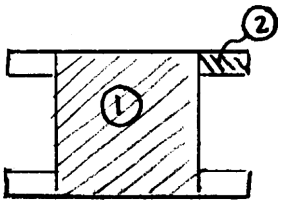


PROBLEM 6.10

For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION

At section *n-n*, $V = 10$ kN.



$$I = I_1 + 4I_2$$

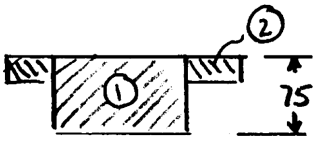
$$= \frac{1}{12}b_1h_1^3 + 4\left(\frac{1}{12}b_2h_2^3 + A_2d_2^2\right)$$

$$= \frac{1}{12}(100)(150)^3 + 4\left[\left(\frac{1}{12}\right)(50)(12)^3 + (50)(12)(69)^2\right]$$

$$= 28.125\times10^6 + 4\left[0.0072\times10^6 + 2.8566\times10^6\right]$$

$$= 39.58\times10^6\ \text{mm}^4 = 39.58\times10^{-6}\ \text{m}^4$$

(*a*)



$$Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$$

$$= (100)(75)(37.5) + (2)(50)(12)(69)$$

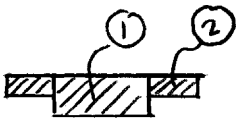
$$= 364.05\times10^3\ \text{mm}^3 = 364.05\times10^{-6}\ \text{m}^3$$

$$t = 100\ \text{mm} = 0.100\ \text{m}$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(10\times10^3)(364.05\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 920\times10^3\ \text{Pa}$$

$\tau_{\max} = 920$ kPa ◀

(*b*)



$$Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$$

$$= (100)(40)(55) + (2)(50)(12)(69)$$

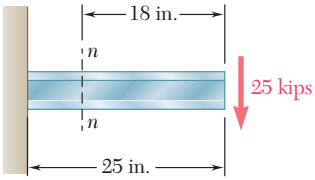
$$= 302.8\times10^3\ \text{mm}^3 = 302.8\times10^{-6}\ \text{m}^3$$

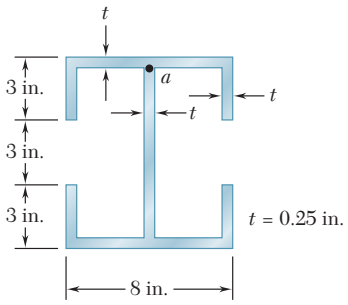
$$t = 100\ \text{mm} = 0.100\ \text{m}$$

$$\tau_a = \frac{VQ}{It} = \frac{(10\times10^3)(302.8\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 765\times10^3\ \text{Pa}$$

$\tau_a = 765$ kPa ◀

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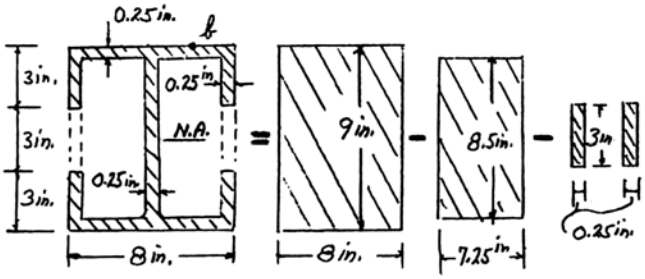




PROBLEM 6.11

For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION



$$I = \frac{1}{12}(8\text{ in.})(9\text{ in.})^3 - \frac{1}{12}(7.25\text{ in.})(8.5\text{ in.})^3 - \frac{2}{12}(0.25\text{ in.})(3\text{ in.})^3$$

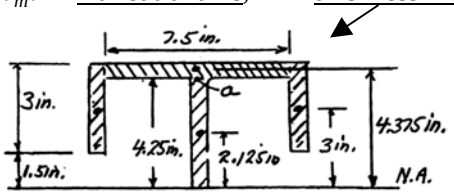
$$I = 113.8\text{ in}^4$$



$$V = 25\text{ kips}$$

$$M = (25\text{ kips})(18\text{ in.}) = 450\text{ kip}\cdot\text{in.}$$

(*a*) τ_m: At neutral axis, thickness = 0.25 in.



$$Q = 2(3\text{ in.} \times 0.25\text{ in.})(3\text{ in.}) + (7.5\text{ in.})(0.25\text{ in.})(4.375\text{ in.}) + (4.25\text{ in.})(0.25\text{ in.})(2.125\text{ in.})$$

$$Q = 4.5\text{ in}^3 + 8.203\text{ in}^3 + 2.258\text{ in}^3 = 14.96\text{ in}^3$$

$$t = 0.25\text{ in.}$$

$$\tau_m = \frac{V_Q}{It} = \frac{(25\text{ kips})(14.96\text{ in}^3)}{(113.8\text{ in}^4)(0.25\text{ in.})}$$

$$\tau_m = 13.15\text{ ksi} \blacktriangleleft$$

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PROBLEM 6.11 *(Continued)*

(*b*) τ_a: At point *a*, $t = 0.25$ in.

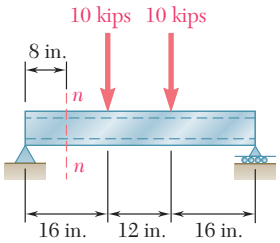
See sketch above.

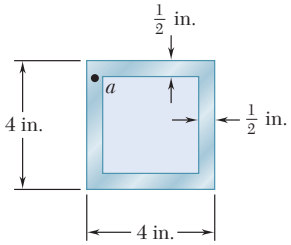
$Q_a = 4.5 \text{ in}^3 + 8.203 \text{ in}^3 = 12.70 \text{ in}^3$

$$\tau_a = \frac{VQ_a}{It} = \frac{(25 \text{ kips})(12.70 \text{ in}^3)}{(113.8 \text{ in}^4)(0.25 \text{ in.})}$$

$\tau_a = 11.16$ ksi ◄

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PROBLEM 6.12

For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

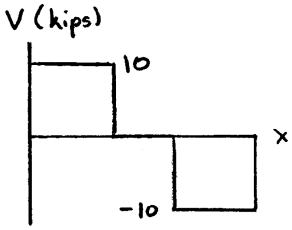
SOLUTION

By symmetry, $R_A = R_B$.

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - 10 - 10 = 0$$

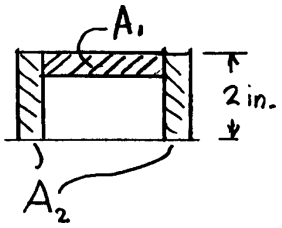
$$R_A = R_B = 10 \text{ kips}$$

From the shear diagram, $V = 10$ kips at *n-n*.



$$I = \frac{1}{12}b_2h_2^3 - \frac{1}{12}b_1h_1^3$$

$$= \frac{1}{12}(4)(4)^3 - \frac{1}{12}(3)(3)^3 = 14.5833 \text{ in}^4$$

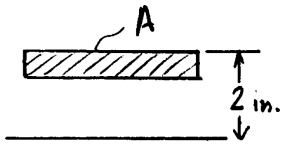


(*a*) $$Q = A_1\bar{y}_1 + A_2\bar{y}_2 = (3)\left(\frac{1}{2}\right)(1.75) + (2)\left(\frac{1}{2}\right)(2)(1) = 4.625 \text{ in}^3$$

$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(10)(4.625)}{(14.5833)(1)}$$

$\tau_{\max} = 3.17$ ksi ◀



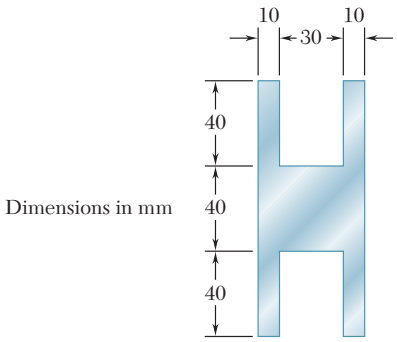
(*b*) $$Q = A\bar{y} = (4)\left(\frac{1}{2}\right)(1.75) = 3.5 \text{ in}^3$$

$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(10)(3.5)}{(14.5833)(1)}$$

$\tau_a = 2.40$ ksi ◀

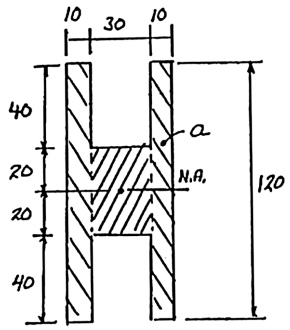
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PROBLEM 6.13

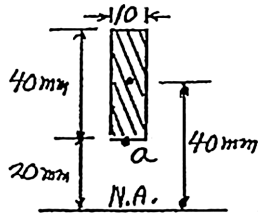
For a beam having the cross section shown, determine the largest allowable vertical shear if the shearing stress is not to exceed 60 MPa.

SOLUTION



Calculate moment of inertia.

$$I = 2\left[\frac{1}{12}(10\text{ mm})(120\text{ mm})^3\right] + \frac{1}{12}(30\text{ mm})(40\text{ mm})^3$$
$$= 2[1.440\times10^6\text{ mm}^4] + 0.160\times10^6\text{ mm}^4$$
$$= 3.04\times10^6\text{ mm}^4$$
$$I = 3.04\times10^{-6}\text{ m}^4$$

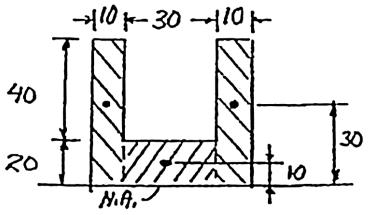


Assume that τ_m occurs at point *a*.

$$t = 10\text{ mm} = 0.01\text{ m}$$
$$Q = (10\text{ mm}\times40\text{ mm})(40\text{ mm})$$
$$= 16\times10^3\text{ mm}^3 \quad Q = 16\times10^{-6}\text{ m}^3$$

For $\tau_{\text{all}} = 60\text{ MPa}$, $\tau_m = \tau_{\text{all}} = \dfrac{VQ}{It}$

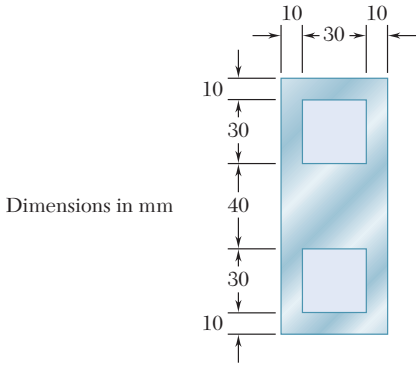
$$60\times10^6\text{ Pa} = \frac{V(16\times10^{-6}\text{ m}^3)}{(3.04\times10^{-6}\text{ m}^4)(0.01\text{m})} \qquad V = 114.0\text{ kN} \blacktriangleleft$$



Check τ at neutral axis:

$$t = 50\text{ mm} = 0.05\text{ m}$$
$$Q = 2[(10\times60)(30)] + (30\times20)(10) = 42\times10^3\text{ m}^3 = 42\times10^{-6}\text{ m}^3$$
$$\tau_{NA} = \frac{VQ}{It} = \frac{(114.0\text{ kN})(42\times10^{-6}\text{ m}^3)}{(3.04\times10^{-6}\text{ m}^4)(0.05\text{ m})} = 31.5\text{ MPa} < 60\text{ MPa} \quad \underline{\text{OK}}$$

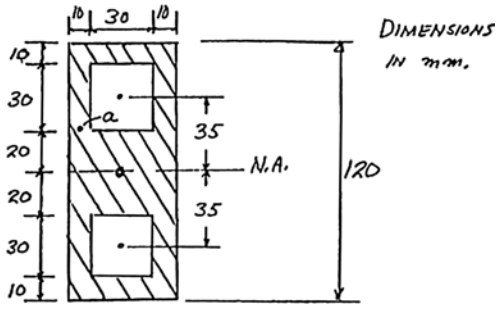
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PROBLEM 6.14

For a beam having the cross section shown, determine the largest allowable vertical shear if the shearing stress is not to exceed 60 MPa.

SOLUTION



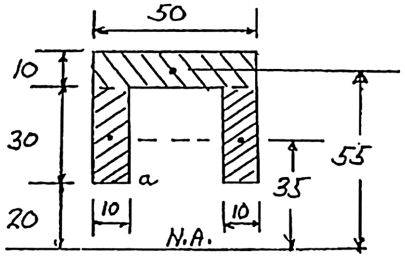
Calculate moment of inertia.

$$I = \frac{1}{12}(50\text{ mm})(120\text{ mm})^3 - 2\left[\frac{1}{12}(30\text{ mm})^4 + (30\text{ mm} \times 30\text{ mm})(35\text{ mm})^2\right]$$

$$I = 7.2 \times 10^6\,\text{mm}^4 - 2[1.170 \times 10^6\,\text{mm}^4] = 4.86 \times 10^6\,\text{mm}^4$$

$$= 4.86 \times 10^{-6}\,\text{m}^4$$

Assume that τ_m occurs at point *a*.



$$t = 2(10\text{ mm}) = 0.02\text{ m}$$

$$Q = (10\text{ mm} \times 50\text{ mm})(55\text{ mm}) + 2[(10\text{ mm} \times 30\text{ mm})(35\text{ mm})]$$

$$= 48.5 \times 10^3\,\text{mm}^3 = 48.5 \times 10^{-6}\,\text{m}^3$$

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PROBLEM 6.14 ***(Continued)***

For $\tau_{\text{all}} = 60$ MPa, $\tau_m = \tau_{\text{all}} = \dfrac{VQ}{It}$

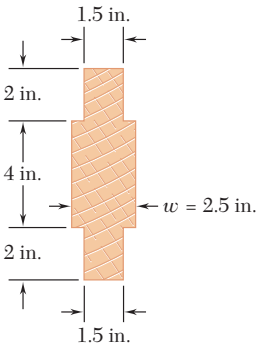
$$60 \times 10^6 \text{ Pa} = \frac{V(48.5 \times 10^{-6} \text{ m}^3)}{(4.86 \times 10^{-6} \text{ m}^4)(0.02 \text{ m})} \qquad V = 120.3 \text{ kN} \blacktriangleleft$$

Check τ at neutral axis: $t = 50 \text{ mm} = 0.05 \text{ m}$

$$Q = (50 \times 60)(30) - (30 \times 30)(35) = 58.5 \times 10^3 \text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{(120.3 \text{ kN})(58.5 \times 10^{-6} \text{ m}^3)}{(4.86 \times 10^{-6} \text{ m}^4)(0.05 \text{ m})} = 29.0 \text{ MPa} < 60 \text{ MPa} \quad \underline{\text{OK}}$$

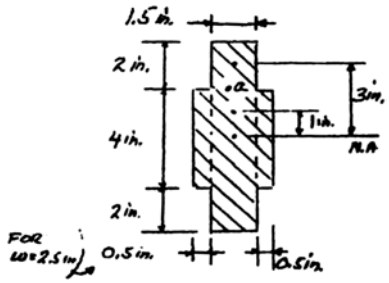
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PROBLEM 6.15

For a timber beam having the cross section shown, determine the largest allowable vertical shear if the shearing stress is not to exceed 150 psi.

SOLUTION



$$I = \frac{1}{12}(1.5 \times 8^3 + 2(0.5)(4)^3)$$

$$I = 69.333 \text{ in}^4$$

$$\tau_{all} = 150 \text{ psi}$$

At point *a*: $Q = (1.5 \text{ in.})(2 \text{ in.})(3 \text{ in.}) = 9 \text{ in}^3; \quad t = 1.5 \text{ in.}$

$$\tau_m = \frac{VQ}{It}; \quad 150 \text{ psi} = \frac{V(9 \text{ in}^3)}{(69.333 \text{ in}^4)(1.5 \text{ in.})}; \qquad V = 1733 \text{ lb} \triangleleft$$

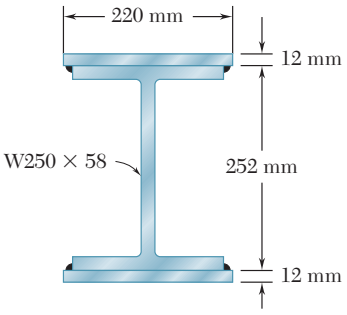
At neutral axis:

$$Q = (1.5 \text{ in.})(2 \text{ in.})(3 \text{ in.}) + (2.5 \text{ in.})(2 \text{ in.})(1 \text{ in.}) = 14 \text{ in}^3, t = 2.6 \text{ in.}$$

$$\tau_m = \frac{VQ}{It}; \quad 150 \text{ psi} = \frac{V(14 \text{ in}^3)}{(69.333 \text{ in}^4)(2.5 \text{ in.})}; \qquad V = 1857 \text{ lb} \triangleleft$$

We choose smaller shear. $V = 1733 \text{ lb} \blacktriangleleft$

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PROBLEM 6.16

Two steel plates of 12 × 220-mm rectangular cross section are welded to the W250 × 58 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

SOLUTION

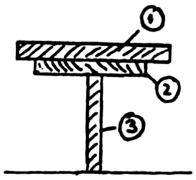
Calculate moment of inertia.

Part	$A(\text{mm}^2)$	$d(\text{mm})$	$Ad^2(10^6\,\text{mm}^4)$	$\bar{I}(10^6\,\text{mm}^4)$
Top plate	2640	*132	45.999	0.032
W250 × 58	7420	0	0	87.3
Bot. plate	2640	*132	45.999	0.032
Σ			91.999	87.364

$$*d = \frac{252}{2} + \frac{12}{2}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 179.363 \times 10^6\,\text{mm}^4 = 179.363 \times 10^{-6}\,\text{m}^4$$

$\tau_{\max}$ occurs at neutral axis. $t = 8.0\,\text{mm} = 8.0 \times 10^{-3}\,\text{m}$



Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3\,\text{mm}^3)$
① Top plate	2640	132	348.48
② Top flange	2740.5	119.25	326.805
③ Half web	900	56.25	50.625
Σ			725.91

Dimensions in mm: ① 12×220; ② 13.5×203; ③ 8.0×112.5

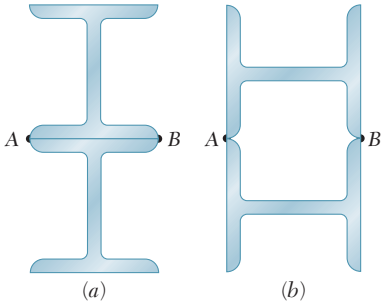
$$Q = \Sigma A\bar{y} = 725.91 \times 10^3\,\text{mm}^3 = 725.91 \times 10^{-6}\,\text{m}^3$$

$$\tau = \frac{VQ}{It} \qquad V = \frac{It\tau}{Q} = \frac{(179.363 \times 10^{-6})(8.0 \times 10^{-3})(90 \times 10^6)}{725.91 \times 10^{-6}}$$

$$= 177.9 \times 10^3\,\text{N}$$

$$V = 177.9\,\text{kN} \blacktriangleleft$$

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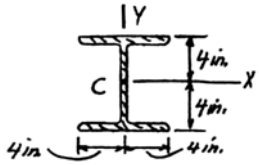


PROBLEM 6.17

Two W8 × 31 rolled-steel sections may be welded at *A* and *B* in either of the two ways shown in order to form a composite beam. Knowing that for each weld the allowable shearing force is 3000 lb per inch of weld, determine for each arrangement the maximum allowable vertical shear in the composite beam.

SOLUTION

W8 × 31

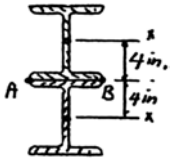


$A = 9.12 \text{ in}^2$

$I_x = 110 \text{ in}^4$

$I_y = 37.1 \text{ in}^4$

(*a*)



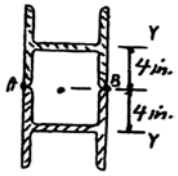
$$I = 2[I_x + Ad^2]$$
$$= 2[110 \text{ in}^4 + (9.12 \text{ in}^2)(4 \text{ in.})^2]$$
$$I = 511.84 \text{ in}^4$$
$$Q = A\bar{y} = (9.12 \text{ in}^2)(4 \text{ in.}) = 36.4 \text{ in}^3$$

For *two* welds each with allowable shearing force of 3 kips/in.,

$$q = 2(3 \text{ kips/in.}) = 6 \text{ kips/in.}$$

$$q = \frac{VQ}{I}; \quad 6 \text{ kips/in.} = \frac{V(36.4 \text{ in}^3)}{(511.84 \text{ in}^4)} \qquad V = 84.2 \text{ kips} \blacktriangleleft$$

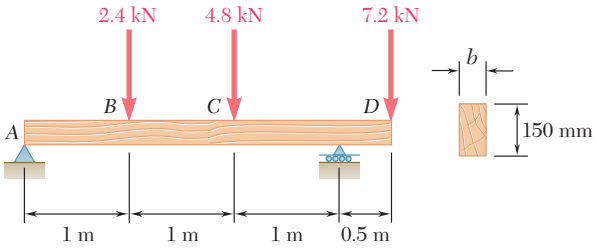
(*b*)



$$I = 2[I_y + Ad^2]$$
$$= 2[37.1 \text{ in}^4 + (9.12 \text{ in}^2)(4 \text{ in.})^2]$$
$$I = 366.04 \text{ in}^4$$
$$Q = A\bar{y} = (9.12 \text{ in}^2)(4 \text{ in.}) = 36.4 \text{ in}^3$$
$$q = 6 \text{ kips/in.} \quad \text{(same as in part } a\text{)}$$

$$q = \frac{VQ}{I}; \quad 6 \text{ kips/in.} = \frac{V(36.4 \text{ in}^3)}{(366.04 \text{ in}^4)} \qquad V = 60.2 \text{ kips} \blacktriangleleft$$

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PROBLEM 6.18

For the beam and loading shown, determine the minimum required width b, knowing that for the grade of timber used, $\sigma_{all} = 12$ MPa and $\tau_{all} = 825$ kPa.

SOLUTION

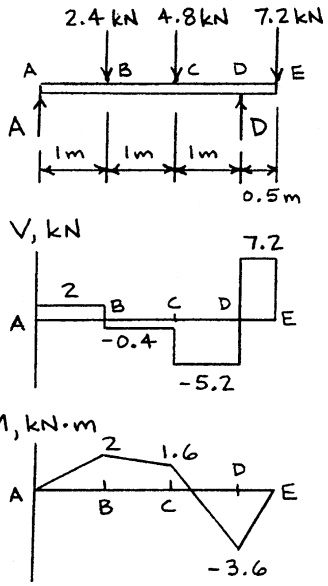
$$+\circlearrowleft \Sigma M_D = 0: \; -3A + (2)(2.4) + (1)(4.8) - (0.5)(7.2) = 0$$

$$\boldsymbol{A} = 2 \text{ kN} \uparrow$$

Draw the shear and bending moment diagrams.

$$|V|_{\max} = 7.2 \text{ kN} = 7.2 \times 10^3 \text{ N}$$

$$|M|_{\max} = 3.6 \text{ kN} \cdot \text{m} = 3.6 \times 10^3 \text{ N} \cdot \text{m}$$



Bending:

$$\sigma = \frac{M}{S}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma} = \frac{3.6 \times 10^3}{12 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$$

For a rectangular section,

$$S = \frac{1}{6} bh^2$$

$$b = \frac{6S}{h^2} = \frac{(6)(300 \times 10^3)}{(150)^2} = 80 \text{ mm}$$

Shear: Maximum shearing stress occurs at the neutral axis of bending for a rectangular section.

$$A = \frac{1}{2} bh, \quad \bar{y} = \frac{1}{4} h, \quad Q = A\bar{y} = \frac{1}{8} bh^2$$

$$I = \frac{1}{12} bh^3 \quad t = b$$

$$\tau = \frac{VQ}{It} = \frac{V(\frac{1}{8} bh^2)}{(\frac{1}{12} bh^3)(b)} = \frac{3}{2} \frac{V}{bh}$$

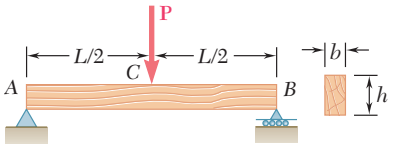
$$b = \frac{3V}{2h\tau} = \frac{(3)(7.2 \times 10^3)}{(2)(150 \times 10^{-3})(825 \times 10^3)} = 87.3 \times 10^{-3} \text{ m}$$

The required value of b is the larger one.

$$b = 87.3 \text{ mm} \blacktriangleleft$$

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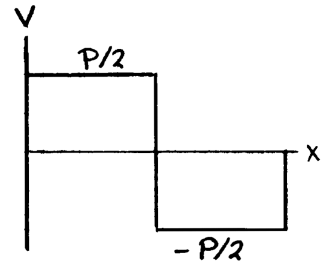
PROBLEM 6.19

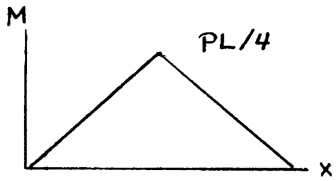


A timber beam *AB* of length *L* and rectangular cross section carries a single concentrated load **P** at its midpoint *C*. (*a*) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where *h* and *L* are, respectively, the depth and the length of the beam. (*b*) Determine the depth *h* and the width *b* of the beam, knowing that $L = 2$ m, $P = 40$ kN, $\tau_m = 960$ kPa, and $\sigma_m = 12$ MPa.

SOLUTION

Reactions: $R_A = R_B = P/2 \uparrow$





(1) $V_{\max} = R_A = \dfrac{P}{2}$

(2) $A = bh$ for rectangular section.

(3) $\tau_m = \dfrac{3}{2}\dfrac{V_{\max}}{A} = \dfrac{3P}{4bh}$ for rectangular section.

(4) $M_{\max} = \dfrac{PL}{4}$

(5) $S = \dfrac{1}{6}bh^2$ for rectangular section.

(6) $\sigma_m = \dfrac{M_{\max}}{S} = \dfrac{3PL}{2bh^2}$

(*a*) $\dfrac{\tau_m}{\sigma_m} = \dfrac{h}{2L}$ ◀

(*b*) Solving for *h*,

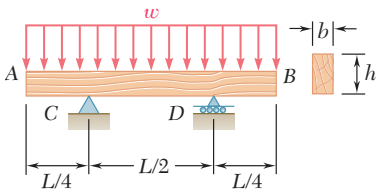
$$h = \frac{2L\tau_m}{\sigma_m} = \frac{(2)(2)(960 \times 10^3)}{12 \times 10^6} = 320 \times 10^{-3}\ \text{m} \qquad h = 320\ \text{mm} \blacktriangleleft$$

Solving Eq. (3) for *b*,

$$b = \frac{3P}{4h\tau_m} = \frac{(3)(40 \times 10^3)}{(4)(320 \times 10^{-3})(960 \times 10^3)} = 97.7 \times 10^{-3}\ \text{m} \qquad b = 97.7\ \text{mm} \blacktriangleleft$$

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PROBLEM 6.20

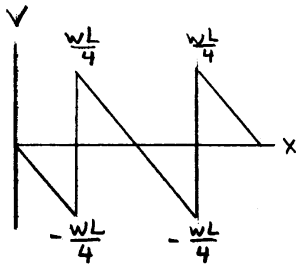


A timber beam AB of length L and rectangular cross section carries a uniformly distributed load w and is supported as shown. (*a*) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where h and L are, respectively, the depth and the length of the beam. (*b*) Determine the depth h and the width b of the beam, knowing that $L = 5$ m, $w = 8$ kN/m, $\tau_m = 1.08$ MPa, and $\sigma_m = 12$ MPa.

SOLUTION

$$R_A = R_B = \frac{wL}{2}$$

From shear diagram, $$|V|_m = \frac{wL}{4} \tag{1}$$

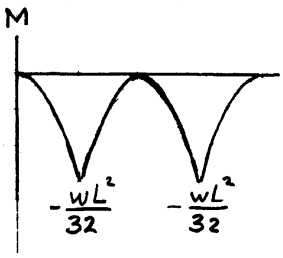


For rectangular section, $$A = bh \tag{2}$$

$$\tau_m = \frac{3}{2}\frac{V_m}{A} = \frac{3wL}{8bh} \tag{3}$$

From bending moment diagram, $$|M|_m = \frac{wL^2}{32} \tag{4}$$

For a rectangular cross section,



$$S = \frac{1}{6}bh^2 \tag{5}$$

$$\sigma_m = \frac{|M|_m}{S} = \frac{3wL^2}{16bh^2} \tag{6}$$

(*a*) Dividing Eq. (3) by Eq. (6), $$\frac{\tau_m}{\sigma_m} = \frac{2h}{L}$$ ◀

(*b*) Solving for h,

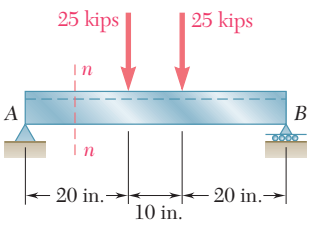
$$h = \frac{L\tau_m}{2\sigma_m} = \frac{(5)(1.08\times10^6)}{(2)(12\times10^6)} = 225\times10^{-3}\text{m} \qquad h = 225\text{ mm}$$ ◀

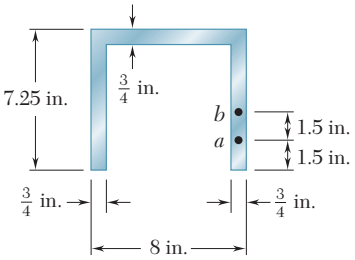
Solving Eq. (3) for b,

$$b = \frac{3wL}{8h\tau_m} = \frac{(3)(8\times10^3)(5)}{(8)(225\times10^{-3})(1.08\times10^6)}$$

$$= 61.7\times10^{-3}\text{ m} \qquad b = 61.7\text{ mm}$$ ◀

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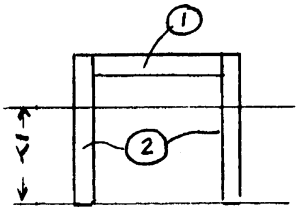




PROBLEM 6.21

For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION



$$R_A = R_B = 25 \text{ kips}$$

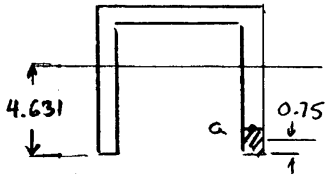
At section *n-n*, $V = 25$ kips

Locate centroid and calculate moment of inertia.

Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.91

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in.}$$

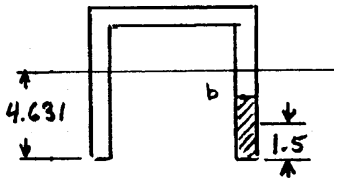
$$I = \Sigma Ad^2 + \Sigma\bar{I} = 35.56 + 47.91 = 83.47 \text{ in}^4$$



(*a*) $$Q_a = A\bar{y} = \left(\frac{3}{4}\right)(1.5)(4.631 - 0.75) = 4.366 \text{ in}^3$$

$$t = \frac{3}{4} = 0.75 \text{ in.}$$

$$\tau_a = \frac{VQ}{It} = \frac{(25)(4.366)}{(83.47)(0.75)} \qquad \tau_a = 1.744 \text{ ksi} \blacktriangleleft$$

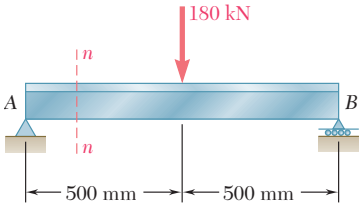


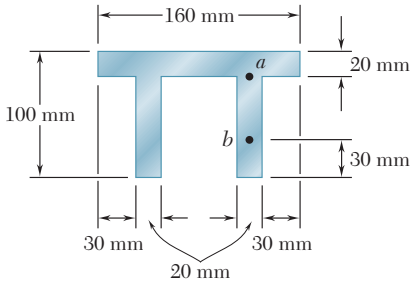
(*b*) $$Q_b = A\bar{y} = \left(\frac{3}{4}\right)(3)(4.631 - 1.5) = 7.045 \text{ in}^3$$

$$t = 0.75 \text{ in.}$$

$$\tau_b = \frac{VQ}{It} = \frac{(25)(7.045)}{(83.47)(0.75)} \qquad \tau_b = 2.81 \text{ ksi} \blacktriangleleft$$

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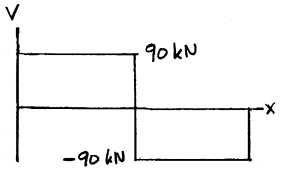
PROBLEM 6.22

For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

Draw the shear diagram. $|V|_{\max} = 90$ kN

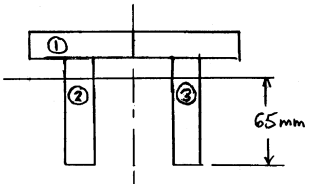
Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3\,\text{mm}^3)$	$d(\text{mm})$	$Ad^2(10^6\,\text{mm}^4)$	$\bar{I}(10^6\,\text{mm}^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	–25	1.000	0.8533
③	1600	40	64	–25	1.000	0.8533
Σ	6400		416		4.000	1.8133



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416 \times 10^3}{6400} = 65 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = (4.000 + 1.8133) \times 10^6\,\text{mm}^4$$

$$= 5.8133 \times 10^6\,\text{mm}^4 = 5.8133 \times 10^{-6}\,\text{m}^4$$



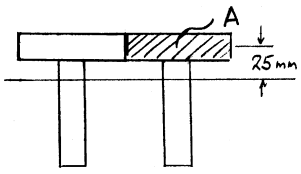
(*a*) $A = (80)(20) = 1600\ \text{mm}^2$

$\bar{y} = 25$ mm

$Q_a = A\bar{y} = 40 \times 10^3\text{mm}^3 = 40 \times 10^{-6}\,\text{m}^3$

$$\tau_a = \frac{VQ_a}{It} = \frac{(90 \times 10^3)(40 \times 10^{-6})}{(5.8133 \times 10^{-6})(20 \times 10^{-3})} = 31.0 \times 10^6\ \text{Pa}$$

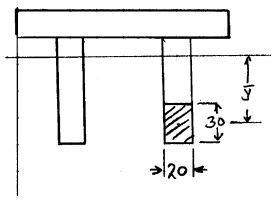
$\tau_a = 31.0$ MPa ◀



(*b*) $A = (30)(20) = 600\ \text{mm}^2 \quad \bar{y} = 65 - 15 = 50$ mm

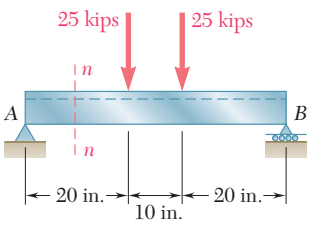
$Q_b = A\bar{y} = 30 \times 10^3\text{mm}^3 = 30 \times 10^{-6}\text{m}^3$

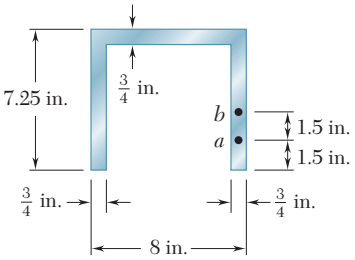
$$\tau_b = \frac{VQ_b}{It} = \frac{(90 \times 10^3)(30 \times 10^{-6})}{(5.8133 \times 10^{-6})(20 \times 10^{-3})} = 23.2 \times 10^6\ \text{Pa}$$



$\tau_b = 23.2$ MPa ◀

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PROBLEM 6.23

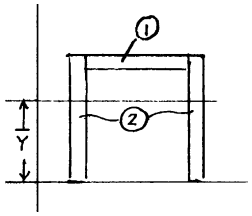
For the beam and loading shown, determine the largest shearing stress in section *n-n*.

SOLUTION

$$R_A = R_B = 25 \text{ kips}$$

At section *n-n*, $V = 25$ kips

Locate centroid and calculate moment of inertia.

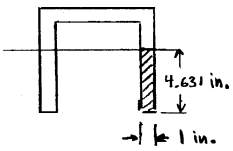


Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.91

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 35.56 + 47.91 = 83.47 \text{ in}^4$$

Largest shearing stress occurs on section through centroid of entire cross section.



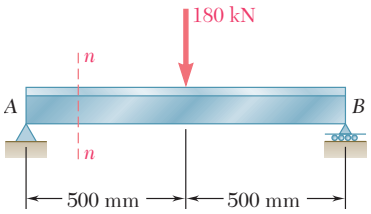
$$Q = Ay = \left(\frac{3}{4}\right)(4.631)\left(\frac{4.631}{2}\right) = 8.042 \text{ in}^3$$

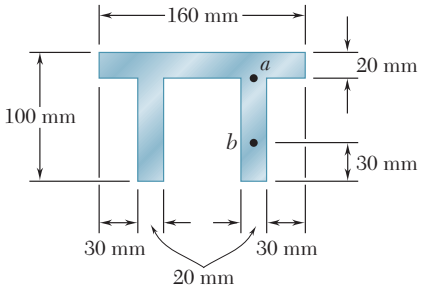
$$t = \frac{3}{4} = 0.75 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(25)(8.042)}{(83.47)(0.75)}$$

$$\tau_m = 3.21 \text{ ksi} \blacktriangleleft$$

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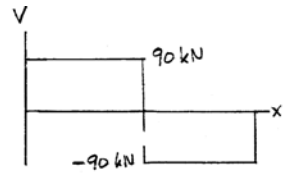


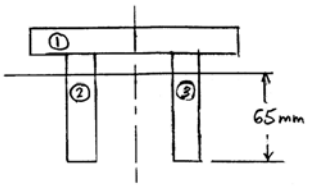


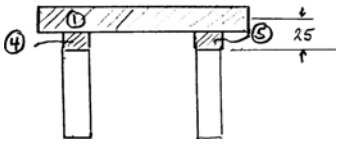
PROBLEM 6.24

For the beam and loading shown, determine the largest shearing stress in section *n-n*.

SOLUTION







Draw the shear diagram. $|V|_{\max} = 90$ kN

Part	$A(\text{mm}^2)$	x'	$A\bar{y}(10^3\,\text{mm}^3)$	$d(\text{mm})$	$Ad^2(10^6\,\text{mm}^4)$	$\bar{I}(10^6\,\text{mm}^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	−25	1.000	0.8533
③	1600	40	64	−25	1.000	0.8533
Σ	6400		416		4.000	1.8133

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416 \times 10^3}{6400} = 65 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = (4.000 + 1.8133) \times 10^6\,\text{mm}^4$$

$$= 5.8133 \times 10^6\,\text{mm}^4 = 5.8133 \times 10^{-6}\,\text{m}^4$$

Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3\,\text{mm}^3)$
①	3200	25	80
④	300	7.5	2.25
⑤	300	7.5	2.25
Σ			84.5

$$Q = \Sigma A\bar{y} = 84.5 \times 10^3\text{mm}^3 = 84.5 \times 10^{-6}\text{m}^3$$

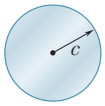
$$t = (2)(20) = 40 \text{ mm} = 40 \times 10^{-3}\text{m}$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(90 \times 10^3)(84.5 \times 10^{-6})}{(5.8133 \times 10^{-6})(40 \times 10^{-3})}$$

$$= 32.7 \times 10^6 \text{ Pa} \qquad \tau_m = 32.7 \text{ MPa} \blacktriangleleft$$

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PROBLEM 6.25



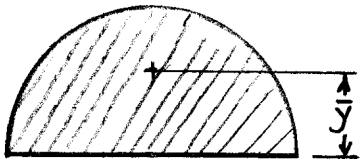
A beam having the cross section shown is subjected to a vertical shear **V.** Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

$$\tau_{\max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

SOLUTION

$$I = \frac{\pi}{4}c^4 \quad \text{and} \quad A = \pi c^2$$



For semicircle, $A_s = \frac{\pi}{2}c^2 \qquad \bar{y} = \frac{4c}{3\pi}$

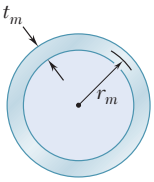
$$Q = A_s\bar{y} = \frac{\pi}{2}c^2 \cdot \frac{4c}{3\pi} = \frac{2}{3}c^3$$

(*a*) $\tau_{\max}$ occurs at center where $t = 2c$ ◀

(*b*) $\tau_{\max} = \frac{VQ}{It} = \frac{V \cdot \frac{2}{3}c^3}{\frac{\pi}{4}c^4 \cdot 2c} = \frac{4V}{3\pi c^2} = \frac{4}{3}\frac{V}{A}$ $\qquad k = \frac{4}{3} = 1.333$ ◀

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PROBLEM 6.26



A beam having the cross section shown is subjected to a vertical shear **V.** Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

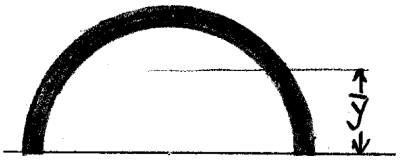
$$\tau_{\max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

SOLUTION

For a thin-walled circular section, $A = 2\pi r_m t_m$

$$J = Ar_m^2 = 2\pi r_m^3 t_m, \qquad I = \frac{1}{2}J = \pi r_m^3 t_m$$



For a semicircular arc, $\bar{y} = \dfrac{2r_m}{\pi}$

$$A_s = \pi r_m t_m$$

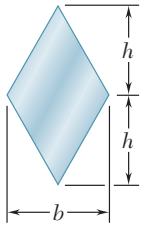
$$Q = A_s\bar{y} = \pi r_m t_m \frac{2r_m}{\pi} = 2r_m^2 t_m$$

(*a*) $t = 2t_m$ at neutral axis where maximum occurs. ◀

(*b*) $\tau_{\max} = \dfrac{VQ}{It} = \dfrac{V(2r_m^2 t_m)}{(\pi r_m^3 t_m)(2t_m)} = \dfrac{V}{\pi r_m t_m} = \dfrac{2V}{A}$ $k = 2.00$ ◀

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PROBLEM 6.27



A beam having the cross section shown is subjected to a vertical shear **V.** Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

$$\tau_{\max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

SOLUTION

$$A = 2\left(\frac{1}{2}bh\right) = bh \qquad I = 2\left(\frac{1}{12}bh^3\right) = \frac{1}{6}bh^3$$

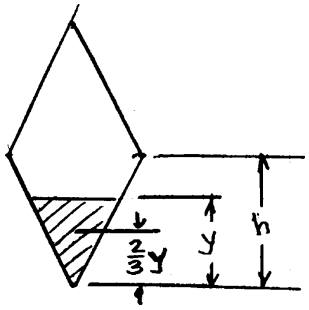
For a cut at location *y*, where $y \le h$,

$$A(y) = \frac{1}{2}\left(\frac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = h - \frac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{2} - \frac{by^3}{3h}$$

$$t(y) = \frac{by}{h}$$



$$\tau(y) = \frac{VQ}{It} = V\frac{6}{bh^3}\cdot\frac{h}{by}\cdot\frac{by^2}{2} - \frac{by^3}{3h} = \frac{V}{bh}\left[3\left(\frac{y}{h}\right) - 2\left(\frac{y}{h}\right)^2\right]$$

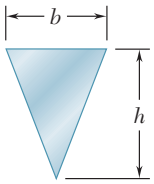
(*a*) To find location of maximum of τ, set $\frac{d\tau}{dy} = 0$.

$$\frac{d\tau}{dy} = \frac{V}{bh^2}[3 - 4\frac{y_m}{h}] = 0 \qquad \frac{y_m}{h} = \frac{3}{4}, \text{ i.e., } \pm\frac{1}{4}h \text{ from neutral axis.} \blacktriangleleft$$

(*b*) $$\tau(y_m) = \frac{V}{bh}\left[3\left(\frac{3}{4}\right) - 2\left(\frac{3}{4}\right)^2\right] = \frac{9}{8}\frac{V}{bh} = 1.125\frac{V}{A} \qquad k = 1.125 \blacktriangleleft$$

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PROBLEM 6.28



A beam having the cross section shown is subjected to a vertical shear **V.** Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

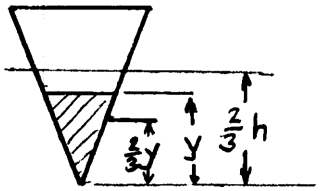
$$\tau_{\max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

SOLUTION

$$A = \frac{1}{2}bh \qquad I = \frac{1}{36}bh^3$$

For a cut at location *y*,



$$A(y) = \frac{1}{2}\left(\frac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = \frac{2}{3}h - \frac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{3}(h - y)$$

$$t(y) = \frac{by}{h}$$

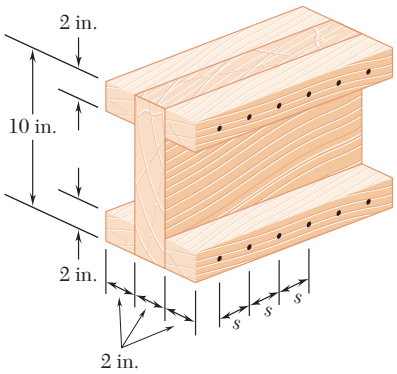
$$\tau(y) = \frac{VQ}{It} = \frac{V\frac{by^2}{3}(h-y)}{(\frac{1}{36}bh^3)\frac{by}{h}} = \frac{12Vy(h-y)}{bh^3} = \frac{12V}{bh^3}(hy - y^2)$$

(*a*) To find location of maximum of τ, set $\frac{d\tau}{dy} = 0$.

$$\frac{d\tau}{dy} = \frac{12V}{bh^3}(h - 2y_m) = 0 \qquad y_m = \frac{1}{2}h, \text{ i.e., at mid-height} \blacktriangleleft$$

(*b*) $$\tau_m = \frac{12V}{bh^3}(hy_m - y_m^2) = \frac{12V}{bh^3}\left[\frac{1}{2}h^2 - \left(\frac{1}{2}h\right)^2\right] = \frac{3V}{bh} = \frac{3}{2}\frac{V}{A} \qquad k = \frac{3}{2} = 1.500 \blacktriangleleft$$

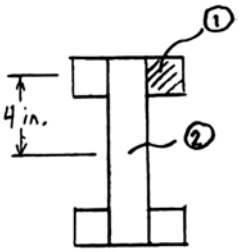
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PROBLEM 6.29

The built-up timber beam shown is subjected to a vertical shear of 1200 lb. Knowing that the allowable shearing force in the nails is 75 lb, determine the largest permissible spacing s of the nails.

SOLUTION



$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2$$

$$= \frac{1}{12}(2)(2)^3 + (2)(2)(4)^2 = 65.333\ \text{in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(2)(10)^3 = 166.67\ \text{in}^4$$

$$I = 4I_1 + I_2 = 428\ \text{in}^4$$

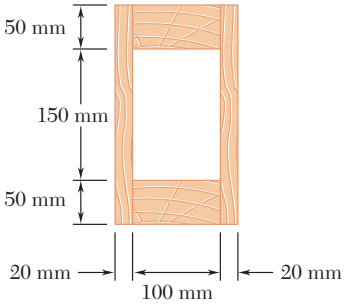
$$Q = Q_1 = A_1\bar{y}_1 = (2)(2)(4) = 16\ \text{in}^3$$

$$q = \frac{VQ}{I} = \frac{(1200)(16)}{428} = 44.86\ \text{lb/in.}$$

$$F_{\text{nail}} = qs$$

$$s = \frac{F_{\text{nail}}}{q} = \frac{75}{44.86} = 1.672\ \text{in.}$$ ◀

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PROBLEM 6.30

The built-up beam shown is made by gluing together two 20 × 250-mm plywood strips and two 50 × 100-mm planks. Knowing that the allowable average shearing stress in the glued joints is 350 kPa, determine the largest permissible vertical shear in the beam.

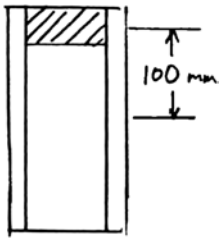
SOLUTION

$$I = \frac{1}{12}(140)(250)^3 - \frac{1}{12}(100)(150)^3 = 154.167 \times 10^6\ \text{mm}^4$$

$$= 154.167 \times 10^{-6}\ \text{m}^4$$

$$Q = A\bar{y} = (100)(50)(100) = 500 \times 10^3\ \text{mm}^3$$

$$= 500 \times 10^{-6}\ \text{m}^3$$



$$t = 50\ \text{mm} + 50\ \text{mm} = 100\ \text{mm} = 100 \times 10^{-3}\ \text{m}$$

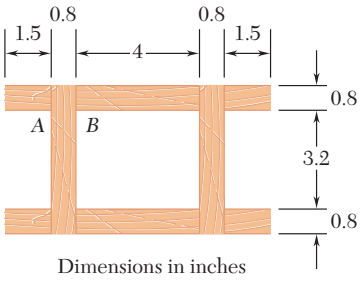
$$\tau = \frac{VQ}{It}$$

$$V = \frac{It\tau}{Q} = \frac{(154.167 \times 10^{-6})(100 \times 10^{-3})(350 \times 10^3)}{500 \times 10^{-6}}$$

$$= 10.79 \times 10^3\ \text{N}$$

$$V = 10.79\ \text{kN} \blacktriangleleft$$

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PROBLEM 6.31

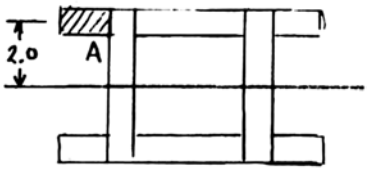
The built-up beam was made by gluing together several wooden planks. Knowing that the beam is subjected to a 1200-lb vertical shear, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.

SOLUTION

$$I = 2\left[\frac{1}{12}(0.8)(4.8)^3 + \frac{1}{12}(7)(0.8)^3 + (7)(0.8)(2.0)^2\right]$$

$$= 60.143\text{ in}^4$$

(*a*) $A_a = (1.5)(0.8) = 1.2\text{ in}^2 \qquad \bar{y}_a = 2.0\text{ in.}$



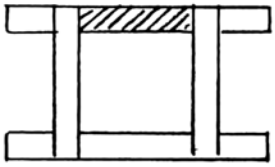
$$Q_a = A_a\bar{y}_a = 2.4\text{ in}^3$$

$$t_a = 0.8\text{ in.}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(1200)(2.4)}{(60.143)(0.8)}$$

$$\tau_a = 59.9\text{ psi} \blacktriangleleft$$

(*b*) $A_b = (4)(0.8) = 3.2\text{ in}^2 \qquad \bar{y}_b = 2.0\text{ in.}$



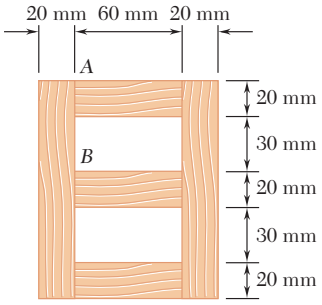
$$Q_b = A_b\bar{y}_b = (3.2)(2.0) = 6.4\text{ in}^3$$

$$t_b = (2)(0.8) = 1.6\text{ in.}$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(1200)(6.4)}{(60.143)(1.6)}$$

$$\tau_b = 79.8\text{ psi} \blacktriangleleft$$

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PROBLEM 6.32

Several wooden planks are glued together to form the box beam shown. Knowing that the beam is subjected to a vertical shear of 3 kN, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.

SOLUTION

$$I_A = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(60)(20)^3 + (60)(20)(50)^2$$

$$= 3.04 \times 10^6 \text{ mm}^4$$

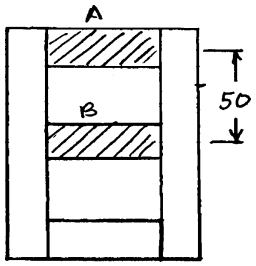
$$I_B = \frac{1}{12}bh^3 = \frac{1}{12}(60)(20)^3 = 0.04 \times 10^6 \text{ mm}^4$$

$$I_C = \frac{1}{12}bh^3 = \frac{1}{12}(20)(120)^3 = 2.88 \times 10^6 \text{ mm}^4$$

$$I = 2I_A + I_B + 2I_C = 11.88 \times 10^6 \text{ mm}^4 = 11.88 \times 10^{-6} \text{ m}^4$$

$$Q_A = A\bar{y} = (60)(20)(50) = 60 \times 10^3 \text{ mm}^3 = 60 \times 10^{-6} \text{ m}^3$$

$$t = 20 \text{ mm} + 20 \text{ mm} = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$



(*a*) $$\tau_A = \frac{VQ_A}{It} = \frac{(3 \times 10^3)(60 \times 10^{-6})}{(11.88 \times 10^{-6})(40 \times 10^{-3})} = 379 \times 10^3 \text{ Pa}$$

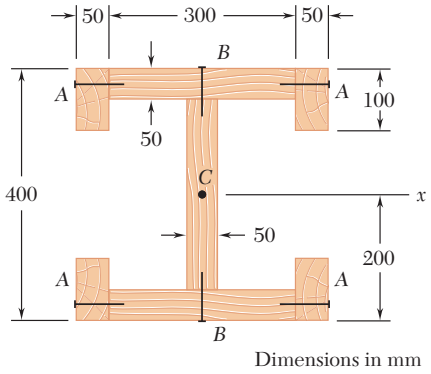
$$Q_B = 0$$

$$\tau_A = 379 \text{ kPa} \blacktriangleleft$$

(*b*) $$\tau_B = \frac{VQ_B}{It} = 0$$

$$\tau_B = 0 \blacktriangleleft$$

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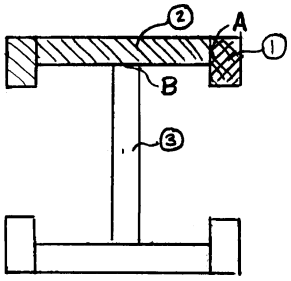


PROBLEM 6.33

The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at *A* and every 25 mm at *B*, determine the shearing force in the nails (*a*) at *A*, (*b*) at *B*. (Given: $I_x = 1.504 \times 10^9\ \text{mm}^4$.)

SOLUTION

$$I_x = 1.504 \times 10^9 \text{mm}^4 = 1504 \times 10^{-6} \text{m}^4$$
$$s_A = 60 \text{ mm} = 0.060 \text{ m}$$
$$s_B = 25 \text{ mm} = 0.025 \text{ m}$$



(*a*)
$$Q_A = Q_1 = A_1\bar{y}_1 = (50)(100)(150) = 750 \times 10^3 \text{mm}^3 = 750 \times 10^{-6} \text{m}^3$$

$$F_A = q_A s_A = \frac{VQ_1 s_A}{I} = \frac{(8 \times 10^3)(750 \times 10^{-6})(0.060)}{1504 \times 10^{-6}}$$

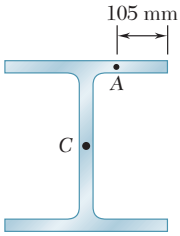
$$F_A = 239 \text{ N} \blacktriangleleft$$

(*b*)
$$Q_2 = A_2\bar{y}_2 = (300)(50)(175) = 2625 \times 10^3 \text{ mm}^3$$
$$Q_B = 2Q_1 + Q_2 = 4125 \times 10^3 \text{ mm}^3 = 4125 \times 10^{-6} \text{ m}^3$$

$$F_B = q_B s_B = \frac{VQ_B s_B}{I} = \frac{(8 \times 10^3)(4125 \times 10^{-6})(0.025)}{1504 \times 10^{-6}}$$

$$F_B = 549 \text{ N} \blacktriangleleft$$

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PROBLEM 6.34

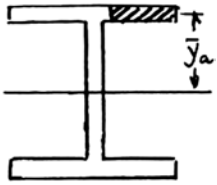
Knowing that a W360 × 122 rolled-steel beam is subjected to a 250-kN vertical shear, determine the shearing stress (*a*) at point *A*, (*b*) at the centroid *C* of the section.

SOLUTION

For W360 × 122, $d = 363\text{ mm}$, $b_F = 257\text{ mm}$, $t_F = 21.70\text{ mm}$, $t_w = 13.0\text{ mm}$

$$I = 367 \times 10^6\text{ mm}^4 = 367 \times 10^{-6}\text{ m}^4$$

(*a*) $A_a = (105)(21.70) = 2278.5\text{ mm}^2$



$$\bar{y}_a = \frac{d}{2} - \frac{t_F}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65\text{ mm}$$

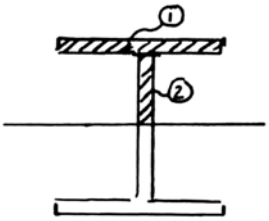
$$Q_a = A_a\bar{y}_a = 388.8 \times 10^3\text{ mm}^3 = 388.8 \times 10^{-6}\text{ m}^3$$

$$t_a = t_F = 21.70\text{ mm} = 21.7 \times 10^{-3}\text{ m}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(250 \times 10^3)(388.8 \times 10^{-6})}{(367 \times 10^{-6})(21.7 \times 10^{-3})} = 12.21 \times 10^6\text{ Pa}$$

$\tau_a = 12.21\text{ MPa}$ ◀

(b) $A_1 = b_F t_F = (257)(21.70) = 5577\text{ mm}^2$



$$\bar{y}_1 = \frac{d}{2} - \frac{t_F}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65\text{ mm}$$

$$A_2 = t_w\left(\frac{d}{2} - t_F\right) = (13.0)(159.8) = 2077\text{ mm}^2$$

$$\bar{y}_2 = \frac{1}{2}\left(\frac{d}{2} - t_F\right) = 79.9\text{ mm}$$

$$Q_c = \Sigma A\bar{y} = (5577)(170.65) + (2077)(79.9) = 1117.7 \times 10^3\text{ mm}^3$$

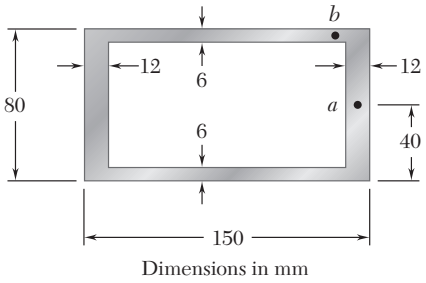
$$= 1117.7 \times 10^{-6}\text{ m}^3$$

$$t_c = t_w = 13.0\text{ mm} = 13 \times 10^{-3}\text{ m}$$

$$\tau_c = \frac{VQ_c}{It_c} = \frac{(250 \times 10^3)(1117.7 \times 10^{-6})}{(367 \times 10^{-6})(13 \times 10^{-3})} = 58.6 \times 10^6\text{ Pa}$$

$\tau_c = 58.6\text{ MPa}$ ◀

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PROBLEM 6.35

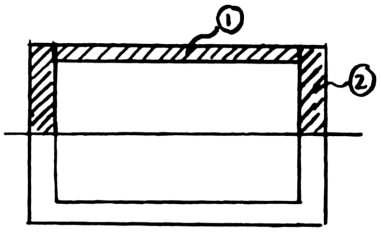
An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 150 kN, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

$$I = \frac{1}{12}(150)(80)^3 - \frac{1}{12}(126)(68)^3$$

$$= 3.098 \times 10^6 \text{ mm}^4 = 3.0985 \times 10^{-6} \text{ m}^4$$

(*a*) $Q_a = A_1\bar{y}_1 + 2A_2\bar{y}_2$



$$= (126)(6)(37) + (2)(12)(40)(20)$$

$$= 47.172 \times 10^3 \text{ mm}^3 = 47.172 \times 10^{-6} \text{ m}^3$$

$$t_a = (2)(12) = 24 \text{ mm} = 0.024 \text{ m}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(150 \times 10^3)(47.172 \times 10^{-6})}{(3.0985 \times 10^{-6})(0.024)} = 95.2 \times 10^6 \text{ Pa}$$

$$\tau_a = 95.2 \text{ MPa} \blacktriangleleft$$

(*b*) $Q_b = A_1\bar{y}_1 = (126)(6)(37) = 27.972 \times 10^3 \text{ mm}^3$

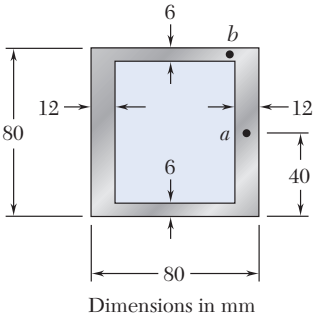
$$= 27.972 \times 10^{-6} \text{ m}^3$$

$$t_b = (2)(6) = 12 \text{ mm} = 0.012 \text{ m}$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(150 \times 10^3)(27.972 \times 10^{-6})}{(3.0985 \times 10^{-6})(0.012)} = 112.8 \times 10^6 \text{ Pa}$$

$$\tau_b = 112.8 \text{ MPa} \blacktriangleleft$$

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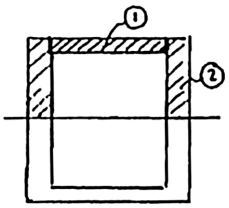


PROBLEM 6.36

An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 150 kN, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

$$I = \frac{1}{12}(80)(80)^3 - \frac{1}{12}(56)(68)^3 = 1.9460 \times 10^6 \text{ mm}^4$$
$$= 1.946 \times 10^{-6} \text{ m}^4$$



(*a*) $Q_a = A_1\bar{y}_1 + 2A_2\bar{y}_2$

$$= (56)(6)(37) + (2)(12)(40)(20) = 31.632 \times 10^3 \text{ mm}^3$$
$$= 31.632 \times 10^{-6} \text{ m}^3$$
$$t_a = (2)(12) = 24 \text{ mm} = 0.024 \text{ m}$$
$$\tau_a = \frac{VQ_a}{It_a} = \frac{(150 \times 10^3)(31.632 \times 10^{-6})}{(1.946 \times 10^{-6})(0.024)} = 101.6 \times 10^6 \text{ Pa}$$

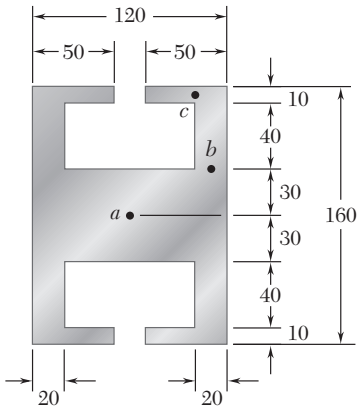
$$\tau_a = 101.6 \text{ MPa} \blacktriangleleft$$

(*b*) $Q_b = A_1\bar{y}_1 = (56)(6)(37) = 12.432 \times 10^3 \text{ mm}^3$

$$= 12.432 \times 10^{-6} \text{ m}^3$$
$$t_b = (2)(6) = 12 \text{ mm} = 0.012 \text{ m}$$
$$\tau_b = \frac{VQ_b}{It_b} = \frac{(150 \times 10^3)(12.432 \times 10^{-6})}{(1.946 \times 10^{-6})(0.012)} = 79.9 \times 10^6 \text{ Pa}$$

$$\tau_b = 79.9 \text{ MPa} \blacktriangleleft$$

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Dimensions in mm

PROBLEM 6.37

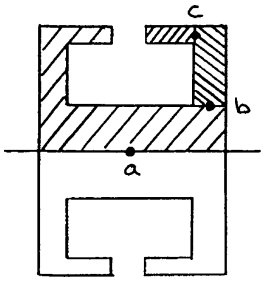
Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa in an extruded beam having the cross section shown, determine the shearing stress at the three points indicated.

SOLUTION

$$\tau = \frac{VQ}{It} \qquad \tau \text{ is proportional to } Q/t.$$

Point *c*:

$$Q_c = (30)(10)(75)$$
$$= 22.5 \times 10^3 \text{mm}^3$$
$$t_c = 10 \text{ mm}$$
$$Q_c/t_c = 2250 \text{ mm}^2$$



Point *b*:

$$Q_b = Q_c + (20)(50)(55)$$
$$= 77.5 \times 10^3 \text{mm}^3$$
$$t_b = 20 \text{ mm}$$
$$Q_b/t_b = 3875 \text{ mm}^2$$

Point *a*:

$$Q_a = 2Q_b + (120)(30)(15)$$
$$= 209 \times 10^3 \text{ mm}^3$$
$$t_a = 120 \text{ mm}$$
$$Q_a/t_a = 1741.67 \text{ mm}^2$$

$(Q/t)_m$ occurs at *b*.

$$\tau_m = \tau_b = 75 \text{ MPa}$$

$$\frac{\tau_a}{Q_a/t_a} = \frac{\tau_b}{Q_b/t_b} = \frac{\tau_c}{Q_c/t_c}$$

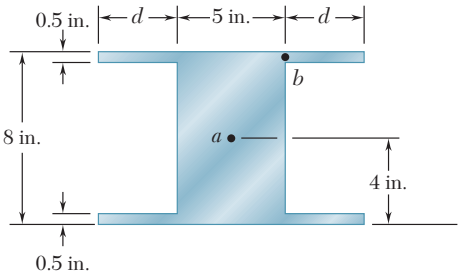
$$\frac{\tau_a}{1741.67 \text{ mm}^2} = \frac{75 \text{ MPa}}{3875 \text{ mm}^2} = \frac{\tau_a}{2250 \text{ mm}^2}$$

$$\tau_a = 33.7 \text{ MPa} \blacktriangleleft$$

$$\tau_b = 75.0 \text{ MPa} \blacktriangleleft$$

$$\tau_c = 43.5 \text{ MPa} \blacktriangleleft$$

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PROBLEM 6.38

The vertical shear is 1200 lb in a beam having the cross section shown. Knowing that $d = 4$ in., determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

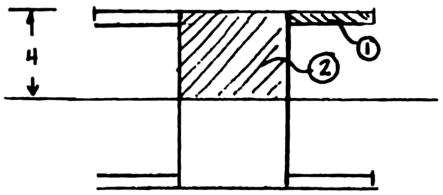
SOLUTION

$$I_1 = \frac{1}{12}(4)(0.5)^3 + (4)(0.5)(3.75)^2 = 28.167 \text{ in}^4$$

$$I_2 = \frac{1}{3}(5)(4)^3 = 106.67 \text{ in}^4$$

$$I = 4I_1 + 2I_2 = 326 \text{ in}^4$$

(*a*)



$$Q_a = 2A_1\bar{y}_1 + A_2\bar{y}_2$$

$$= (2)(4)(0.5)(3.75) + (5)(4)(2) = 55 \text{ in}^3$$

$$t_a = 5 \text{ in.}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(1200)(55)}{(326)(5)} = 40.5 \text{ psi} \quad \blacktriangleleft$$

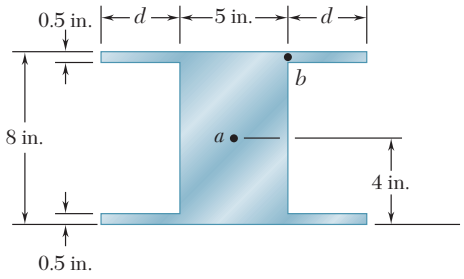
(*b*)

$$Q_b = A_1\bar{y}_1 = (4)(0.5)(3.75) = 7.5 \text{ in}^4$$

$$t_b = 0.5 \text{ in.}$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(1200)(7.5)}{(326)(0.5)} = 55.2 \text{ psi} \quad \blacktriangleleft$$

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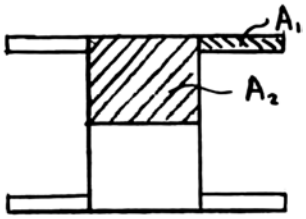
PROBLEM 6.39

The vertical shear is 1200 lb in a beam having the cross section shown. Determine (*a*) the distance *d* for which $\tau_a = \tau_b$, (*b*) the corresponding shearing stress at points *a* and *b*.

SOLUTION

$A_1 = 0.5d \text{ in}^2, \quad \bar{y}_1 = 3.75 \text{ in.}, \quad t_b = 0.5 \text{ in.}$

$A_2 = (5)(4) = 20 \text{ in}^2, \quad \bar{y}_2 = 2 \text{ in.}, \quad t_a = 5 \text{ in.}$



$$Q_b = A_1 y_1 = 1.875d \text{ in}^3$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{V}{I}\frac{1.875d}{0.5} = 3.75\frac{Vd}{I}$$

$$Q_a = A_2\bar{y}_2 + 2Q_b = (20)(2) + (2)(1.875d)$$

$$= 40 + 3.75d$$

$$t_a = 5 \text{ in.}$$

(*a*) $$\tau_a = \frac{VQ_a}{It_a} = \frac{V(40 + 3.75d)}{I(5)} = 8\frac{V}{I} + 0.75\frac{Vd}{I} = \tau_b = 3.75\frac{Vd}{I}$$

$$8 + 0.75d = 3.75d \qquad d = \frac{8}{3} = 2.6667 \text{ in.}$$

$d = 2.67 \text{ in.}$ ◀

(*b*) $$I_1 = \frac{1}{12}(2.6667)(0.5)^3 + (2.6667)(0.5)(3.75)^2 = 18.780 \text{ in}^4$$

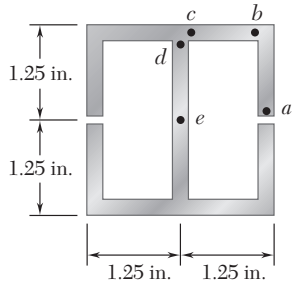
$$I_2 = \frac{1}{3}(0.5)(4)^3 = 106.667 \text{ in}^4$$

$$I = 4I_1 + 2I_2 = 288.46 \text{ in}^4$$

$$\tau_a = \tau_b = 3.75\frac{Vd}{I} = \frac{(3.75)(1200)(2.6667)}{288.46}$$

$\tau_a = 41.6 \text{ psi}$ ◀

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PROBLEM 6.40

The extruded aluminum beam has a uniform wall thickness of $\frac{1}{8}$ in. Knowing that the vertical shear in the beam is 2 kips, determine the corresponding shearing stress at each of the five points indicated.

SOLUTION

$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.23812 \text{ in}^4$$

$$t = 0.125 \text{ in.} \quad \text{at all sections.}$$

$$V = 2 \text{ kips}$$

$Q_a = 0$ $\qquad \tau_a = \dfrac{VQ_a}{It}$ $\qquad \tau_a = 0$ ◀

$$Q_b = (0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.097656 \text{ in}^3$$

$$\tau_b = \frac{VQ_b}{It} = \frac{(2)(0.097656)}{(1.23812)(0.125)} \qquad \tau_b = 1.262 \text{ ksi} \blacktriangleleft$$

$$Q_c = Q_b + (1.0625)(0.125)(1.1875) = 0.25537 \text{ in.}^2$$

$$\tau_c = \frac{VQ_c}{It} = \frac{(2)(0.25537)}{(1.23812)(0.125)} \qquad \tau_c = 3.30 \text{ ksi} \blacktriangleleft$$

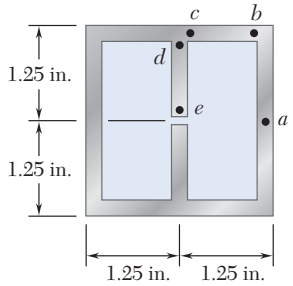
$$Q_d = 2Q_c + (0.125)^2(1.1875) = 0.52929$$

$$\tau_d = \frac{VQ_d}{It} = \frac{(2)(0.52929)}{(1.23812)(0.125)} \qquad \tau_d = 6.84 \text{ ksi} \blacktriangleleft$$

$$Q_e = Q_d + (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.60839$$

$$\tau_e = \frac{VQ}{It} = \frac{(2)(0.60839)}{(1.23812)(0.125)} \qquad \tau_e = 7.86 \text{ ksi} \blacktriangleleft$$

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PROBLEM 6.41

The extruded aluminum beam has a uniform wall thickness of $\frac{1}{8}$ in. Knowing that the vertical shear in the beam is 2 kips, determine the corresponding shearing stress at each of the five points indicated.

SOLUTION

$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.23812 \text{ in}^4$$

Add symmetric points *c*', *b*', and *a*'.

$Q_e = 0$

$Q_d = (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.079102 \text{ in}^3$ $\qquad t_d = 0.125 \text{ in.}$

$Q_c = Q_d + (0.125)^2(1.1875) = 0.097657 \text{ in}^4$ $\qquad t_c = 0.25 \text{ in.}$

$Q_b = Q_c + (2)(1.0625)(0.125)(1.1875) = 0.41309 \text{ in}^3$ $\qquad t_b = 0.25 \text{ in.}$

$Q_a = Q_b + (2)(0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.60840 \text{ in}^3$ $\qquad t_a = 0.25 \text{ in.}$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(2)(0.60840)}{(1.23812)(0.25)} \qquad \tau_a = 3.93 \text{ ksi} \blacktriangleleft$$

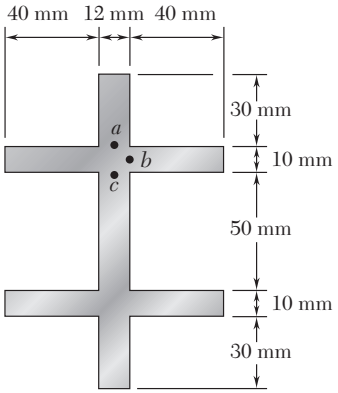
$$\tau_b = \frac{VQ_b}{It_b} = \frac{(2)(0.41309)}{(1.23812)(0.25)} \qquad \tau_b = 2.67 \text{ ksi} \blacktriangleleft$$

$$\tau_c = \frac{VQ_c}{It_c} = \frac{(2)(0.097657)}{(1.23812)(0.25)} \qquad \tau_c = 0.631 \text{ ksi} \blacktriangleleft$$

$$\tau_d = \frac{VQ_d}{It_d} = \frac{(2)(0.079102)}{(1.23812)(0.125)} \qquad \tau_d = 1.02 \text{ ksi} \blacktriangleleft$$

$$\tau_e = \frac{VQ_e}{It_e} \qquad \tau_e = 0 \blacktriangleleft$$

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PROBLEM 6.42

Knowing that a given vertical shear **V** causes a maximum shearing stress of 50 MPa in a thin-walled member having the cross section shown, determine the corresponding shearing stress (*a*) at point *a*, (*b*) at point *b*, (*c*) at point *c*.

SOLUTION

$$Q_a = (12)(30)(25 + 10 + 15) = 18 \times 10^3 \text{ mm}^3$$

$$Q_b = (40)(10)(25 + 5) = 12 \times 10^3 \text{ mm}^3$$

$$Q_c = Q_a + 2Q_b + (12)(10)(25 + 5) = 45.6 \times 10^3 \text{ mm}^3$$

$$Q_m = Q_c + (12)(25)\left(\frac{25}{2}\right) = 49.35 \times 10^3 \text{ mm}^3$$

$$t_a = t_c = t_m = 12 \text{ mm}$$

$$t_b = 10 \text{ mm}$$

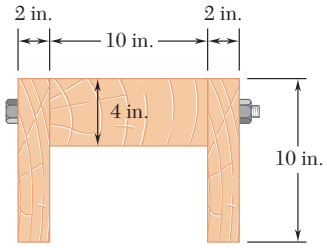
$$\tau_m = 50 \text{ MPa}$$

(*a*) $$\frac{\tau_a}{\tau_m} = \frac{Q_a}{Q_m} \cdot \frac{t_m}{t_a} = \frac{18}{49.35} \cdot \frac{12}{12} = 0.36474 \qquad \tau_a = 18.23 \text{ MPa} \blacktriangleleft$$

(*b*) $$\frac{\tau_b}{\tau_m} = \frac{Q_b}{Q_m} \cdot \frac{t_m}{t_b} = \frac{12}{49.35} \cdot \frac{12}{10} = 0.29179 \qquad \tau_b = 14.59 \text{ MPa} \blacktriangleleft$$

(*c*) $$\frac{\tau_c}{\tau_m} = \frac{Q_c}{Q_m} \cdot \frac{t_m}{t_c} = \frac{45.6}{49.35} \cdot \frac{12}{12} = 0.92401 \qquad \tau_c = 46.2 \text{ MPa} \blacktriangleleft$$

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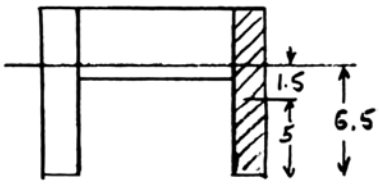


PROBLEM 6.43

Three planks are connected as shown by bolts of $\frac{3}{8}$-in. diameter spaced every 6 in. along the longitudinal axis of the beam. For a vertical shear of 2.5 kips, determine the average shearing stress in the bolts.

SOLUTION

Locate neutral axis.



$$\Sigma A = (2)(2)(10) + (10)(4) = 80\ \text{in}^2$$

$$\Sigma A\bar{y} = (2)(2)(10)(5) + (10)(4)(8) = 520\ \text{in}^3$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{520}{80} = 6.5\ \text{in.}$$

$$I = 2\left[\frac{1}{12}(2)(10)^3 + (2)(10)(1.5)^2\right] + \frac{1}{12}(10)(4)^3 + (10)(4)(1.5)^2 = 566.67\ \text{in}^4$$

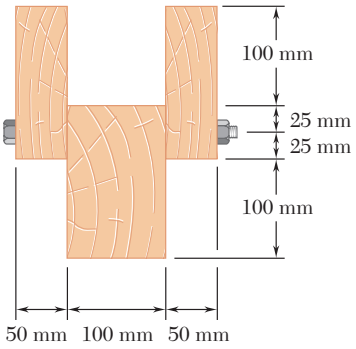
$$Q = (2)(10)(1.5) = 30\ \text{in}^3$$

$$F = qs = \frac{VQs}{I} = \frac{(2.5)(30)(6)}{566.67} = 0.79411\ \text{kips}$$

$$A_{\text{bolt}} = \frac{\pi}{4}d_{\text{bolt}}^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 0.110447\ \text{in}^2$$

$$\tau_{\text{bolt}} = \frac{F}{A_{\text{bolt}}} = \frac{0.79411}{0.110447} = 7.19\ \text{ksi} \blacktriangleleft$$

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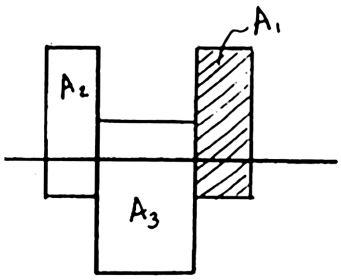


PROBLEM 6.44

A beam consists of three planks connected as shown by steel bolts with a longitudinal spacing of 225 mm. Knowing that the shear in the beam is vertical and equal to 6 kN and that the allowable average shearing stress in each bolt is 60 MPa, determine the smallest permissible bolt diameter that can be used.

SOLUTION

Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}^2(10^6\,\text{mm}^4)$	$\bar{I}\,(10^6\,\text{mm}^4)$
①	7500	50	18.75	14.06
②	7500	50	18.75	14.06
③	15,000	–50	37.50	28.12
Σ			75.00	56.24



$$I = \sum A\bar{y}^2 + \sum \bar{I} = 131.25 \times 10^6\,\text{mm}^4 = 131.25 \times 10^{-6}\,\text{m}^4$$

$$Q = A_1\bar{y}_1 = (7500)(50) = 375 \times 10^3\,\text{mm}^3 = 375 \times 10^{-6}\,\text{m}^3$$

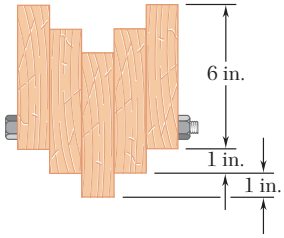
$$F_{\text{bolt}} = \tau_{\text{bolt}} A_{\text{bolt}} = qs = \frac{VQs}{I}$$

$$A_{\text{bolt}} = \frac{VQs}{\tau_{\text{bolt}} I} = \frac{(6 \times 10^3)(375 \times 10^{-6})(0.225)}{(6 \times 10^6)(131.25 \times 10^6)} = 64.286 \times 10^{-6}\,\text{m}^2 = 64.286\,\text{mm}^2$$

$$d_{\text{bolt}} = \sqrt{\frac{4A_{\text{bolt}}}{\pi}} = \sqrt{\frac{(4)(64.286)}{\pi}}$$

$$d_{\text{bolt}} = 9.05\,\text{mm} \blacktriangleleft$$

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PROBLEM 6.45

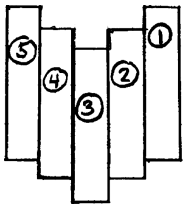
A beam consists of five planks of 1.5×6-in. cross section connected by steel bolts with a longitudinal spacing of 9 in. Knowing that the shear in the beam is vertical and equal to 2000 lb and that the allowable average shearing stress in each bolt is 7500 psi, determine the smallest permissible bolt diameter that can be used.

SOLUTION

Part	$A(\text{in}^2)$	$\bar{y}_0(\text{in.})$	$A\bar{y}_0(\text{in}^3)$	$\bar{y}(\text{in.})$	$A\bar{y}^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	9	5	45	0.8	5.76	27
②	9	4	36	–0.2	0.36	27
③	9	3	27	–1.2	12.96	27
④	9	4	36	–0.2	0.36	27
⑤	9	5	45	0.8	5.76	27
Σ	45		189		25.20	135

$$\bar{Y}_0 = \frac{\Sigma Ay}{\Sigma A} = \frac{189}{45} = 4.2 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 160.2 \text{ in}^4$$



Between ① and ②: $Q_{12} = Q_1 = A\bar{y}_1 = (9)(0.8) = 7.2 \text{ in}^3$

Between ② and ③: $Q_{23} = Q_1 + A\bar{y}_2 = 7.2 + (9)(-0.2) = 5.4 \text{ in}^3$

$$q = \frac{VQ}{I}$$

Maximum q is based on $Q_{12} = 7.2 \text{ in}^3$.

$$q = \frac{(2000)(7.2)}{160.2} = 89.888 \text{ lb/in.}$$

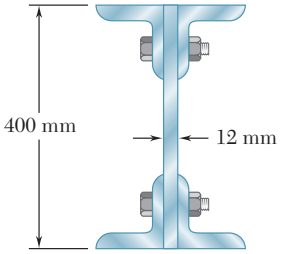
$$F_{\text{bolt}} = qs = (89.888)(9) = 809 \text{ lb}$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} \qquad A_{\text{bolt}} = \frac{F_{\text{bolt}}}{\tau_{\text{bolt}}} = \frac{809}{7500} = 0.1079 \text{ in}^2$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 \qquad d_{\text{bolt}} = \sqrt{\frac{4A_{\text{bolt}}}{\pi}} = \sqrt{\frac{(4)(0.1079)}{\pi}}$$

$$d_{\text{bolt}} = 0.371 \text{ in.} \blacktriangleleft$$

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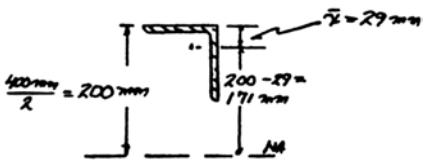


PROBLEM 6.46

Four L102 × 102 × 9.5 steel angle shapes and a 12 × 400-mm plate are bolted together to form a beam with the cross section shown. The bolts are of 22-mm diameter and are spaced longitudinally every 120 mm. Knowing that the beam is subjected to a vertical shear of 240 kN, determine the average shearing stress in each bolt.

SOLUTION

For one L102 × 102 × 9.5,



$$A = 1845\ \text{mm}^2$$

$$I = 1.815 \times 10^6\ \text{mm}^2$$

$$Q = (1845\ \text{mm}^2)(171\ \text{mm})$$

$$= 315.5 \times 10^3\ \text{mm}^3$$

$$= 315.5 \times 10^{-6}\ \text{m}^4$$

For 12-mm × 400-mm plate and four angle,

$$I = \frac{1}{12}(12\ \text{mm})(400\ \text{mm})^3 + 4[1.815 \times 10^6\ \text{mm}^4 + (1845\ \text{mm}^2)(171\ \text{mm})^2]$$

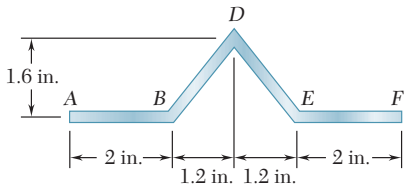
$$= 287.06 \times 10^6\ \text{mm}^4 = 287.06 \times 10^{-6}\ \text{m}^4$$

One angle: $$q = \frac{VQ}{I} = \frac{(240\ \text{kN})(315.5 \times 10^{-6}\ \text{m}^3)}{287.06 \times 10^{-6}\ \text{m}^4} = 263.78\ \text{kN/m}$$

$$F = qs = (263.78\ \text{kN/m})(0.120\ \text{m}) = 31.65\ \text{kN}$$

Diam. bolt = 22 mm $$\tau = \frac{F}{A} = \frac{31.65\ \text{kN}}{\frac{\pi}{4}(0.022\ \text{m})^2};$$ $\tau = 83.3\ \text{MPa}$ ◄

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PROBLEM 6.47

A plate of $\frac{1}{4}$-in. thickness is corrugated as shown and then used as a beam. For a vertical shear of 1.2 kips, determine (*a*) the maximum shearing stress in the section, (*b*) the shearing stress at point *B*. Also, sketch the shear flow in the cross section.

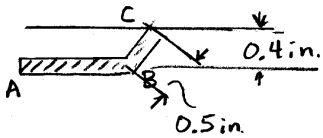
SOLUTION

$$L_{BD} = \sqrt{(1.2)^2 + (1.6)^2} = 2.0 \text{ in.} \quad A_{BD} = (0.25)(2.0) = 0.5 \text{ in}^2$$

Locate neutral axis and compute moment of inertia.

Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}\ (\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
AB	0.5	0	0	0.4	0.080	neglect
BD	0.5	0.8	0.4	0.4	0.080	*0.1067
DE	0.5	0.8	0.4	0.4	0.080	*0.1067
EF	0.5	0	0	0.4	0.080	neglect
Σ	2.0		0.8		0.320	0.2134

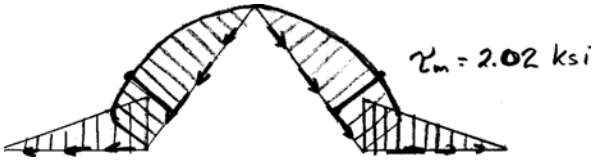
$$^*\frac{1}{12}A_{BD}h^2 = \frac{1}{12}(0.5)(1.6)^2 = 0.1067 \text{ in}^4 \qquad \bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{0.8}{2.0} = 0.4 \text{ in.}$$



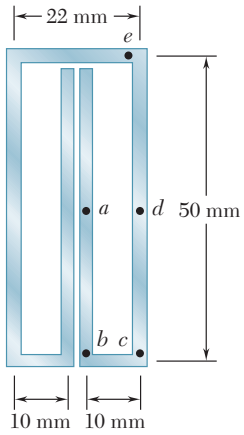
$$I = \Sigma Ad^2 + \Sigma\bar{I} = 0.5334 \text{ in}^4$$

(*a*)
$$Q_m = Q_{AB} + Q_{BC}$$
$$Q_{AB} = (2)(0.25)(0.4) = 0.2 \text{ in}^3$$
$$Q_{BC} = (0.5)(0.25)(0.2) = 0.025 \text{ in}^3$$
$$Q_m = 0.225 \text{ in}^3$$
$$\tau_m = \frac{VQ_m}{It} = \frac{(1.2)(0.225)}{(0.5334)(0.25)} \qquad \tau_m = 2.02 \text{ ksi} \blacktriangleleft$$

(*b*)
$$Q_B = Q_{AB} = 0.2 \text{ in}^3$$
$$\tau_B = \frac{VQ_B}{It} = \frac{(1.2)(0.2)}{(0.5334)(0.25)} \qquad \tau_B = 1.800 \text{ ksi} \blacktriangleleft$$
$$\tau_D = 0$$



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PROBLEM 6.48

A plate of 2-mm thickness is bent as shown and then used as a beam. For a vertical shear of 5 kN, determine the shearing stress at the five points indicated and sketch the shear flow in the cross section.

SOLUTION

$$I = 2\left[\frac{1}{12}(2)(48)^3 + \frac{1}{12}(2)(52)^3 + \frac{1}{12}(20)(2)^3 + (20)(2)(25)^2\right]$$

$$= 133.75 \times 10^3 \text{ mm}^4 = 133.75 \times 10^{-9} \text{ mm}^4$$

$$Q_a = (2)(24)(12) = 576 \text{ mm}^3 = 576 \times 10^{-9} \text{ mm}^3$$

$$Q_a = 0$$

$$Q_c = Q_b - (12)(2)(25) = -600 \text{ mm}^3 = -600 \times 10^{-9} \text{ m}^3$$

$$Q_d = Q_c - (2)(24)(12) = -1.176 \times 10^3 \text{ mm}^3 = -1.176 \times 10^{-6} \text{ m}^3$$

$$Q_e = Q_d + (2)(26)(13) = -600 \text{ mm}^3 = -500 \times 10^{-9} \text{ m}^3$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(5 \times 10^3)(576 \times 10^{-9})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 10.77 \times 10^6 \text{ Pa} \qquad \tau_a = 10.76 \text{ MPa} \blacktriangleleft$$

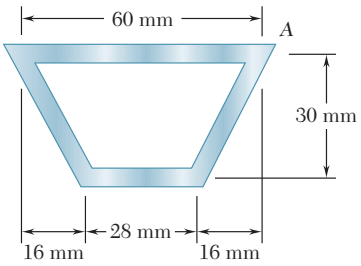
$$\tau_b = \frac{VQ_b}{It} \qquad \tau_b = 0 \blacktriangleleft$$

$$\tau_c = \frac{VQ_c}{It} = \frac{(5 \times 10^3)(600 \times 10^{-9})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 11.21 \times 10^6 \text{ Pa} \qquad \tau_c = 11.21 \text{ MPa} \blacktriangleleft$$

$$\tau_d = \frac{VQ_d}{It} = \frac{(5 \times 10^3)(1.176 \times 10^{-6})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 22.0 \times 10^6 \text{ Pa} \qquad \tau_d = 22.0 \text{ MPa} \blacktriangleleft$$

$$\tau_e = \frac{VQ_e}{It} = \frac{(5 \times 10^3)(500 \times 10^{-9})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 9.35 \times 10^6 \text{ Pa} \qquad \tau_e = 9.35 \text{ MPa} \blacktriangleleft$$

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PROBLEM 6.49

An extruded beam has the cross section shown and a uniform wall thickness of 3 mm. For a vertical shear of 10 kN, determine (*a*) the shearing stress at point *A*, (*b*) the maximum shearing stress in the beam. Also, sketch the shear flow in the cross section.

SOLUTION

$$\tan\alpha = \frac{16}{30} \qquad \alpha = 28.07°$$

Side:
$$A = (3\sec\alpha)(30) = 102 \text{ mm}^2$$
$$\bar{I} = \frac{1}{12}(3\sec\alpha)(30)^3 = 7.6498\times10^3 \text{ mm}^4$$

Part	$A(\text{mm}^2)$	$\bar{y}_0(\text{mm})$	$A\bar{y}(10^3\text{ mm}^3)$	$d(\text{mm})$	$Ad^2(10^3\text{ mm}^4)$	$\bar{I}(10^3\text{ mm}^4)$
Top	180	30	5.4	11.932	25.627	neglect
Side	102	15	1.53	3.077	0.966	7.6498
Side	102	15	1.53	3.077	0.966	7.6498
Bot.	84	0	0	18.077	27.449	neglect
Σ	468		8.46		55.008	15.2996

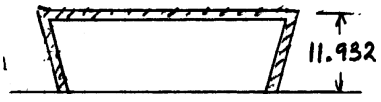
$$\bar{Y}_0 = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8.46\times10^3}{468} = 18.077 \text{ mm}$$
$$I = \Sigma Ad^2 + \Sigma\bar{I} = 70.31\times10^3 \text{ mm}^4 = 70.31\times10^{-9} \text{ m}^4$$

(*a*) $$Q_A = (180)(11.932) = 2.14776\times10^3 \text{ mm}^3 = 2.14776\times10^{-6} \text{ m}^3$$

$$t = (2)(3\times10^{-3}) = 6\times10^{-3} \text{ m}$$
$$\tau_A = \frac{VQ}{It} = \frac{(10\times10^3)(2.14776\times10^{-6})}{(70.31\times10^{-9})(6\times10^{-6})} = 50.9\times10^6 \text{ Pa}$$

$\tau_A = 50.9$ MPa ◀

(*b*) $$Q_m = Q_A + (2)(3\sec\alpha)(11.932)\left(\frac{1}{2}\times11.932\right)$$
$$= 2.14776\times10^3 + 484.06 = 2.6318\times10^3 \text{ mm}^3$$
$$= 2.6318\times10^{-6} \text{ m}^3$$



$$t = 6\times10^{-3} \text{ m}$$
$$\tau_m = \frac{VQ_m}{It} = \frac{(10\times10^3)(2.6318\times10^{-6})}{(70.31\times10^{-9})(6\times10^{-3})} = 62.4\times10^6 \text{ Pa}$$

$\tau_m = 62.4$ MPa ◀

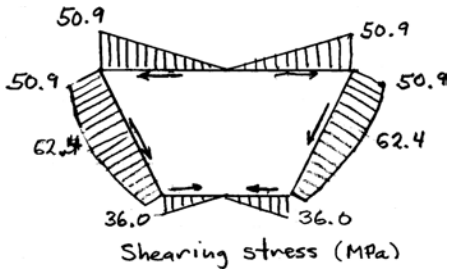
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PROBLEM 6.49 ***(Continued)***

$$Q_B = (28)(3)(18.077) = 1.51847 \times 10^3 \text{ mm}^3$$

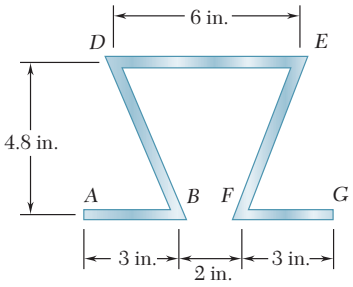
$$\tau_B = \frac{Q_B}{Q_A}\tau_A = \frac{1.51847 \times 10^3}{2.14776 \times 10^3}(50.9)$$

$$= 36.0 \text{ MPa}$$



Multiply shearing stresses by $t(3 \text{ mm} = 0.003 \text{ m})$ to get shear flow.

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PROBLEM 6.50

A plate of thickness t is bent as shown and then used as a beam. For a vertical shear of 600 lb, determine (*a*) the thickness t for which the maximum shearing stress is 300 psi, (*b*) the corresponding shearing stress at point *E*. Also, sketch the shear flow in the cross section.

SOLUTION

$$L_{BD} = L_{EF} = \sqrt{4.8^2 + 2^2} = 5.2 \text{ in.}$$

Neutral axis lies at 2.4 in. above *AB*.

Calculate *I*.

$$I_{AB} = (3t)(2.4)^2 = 17.28t$$

$$I_{BD} = \frac{1}{12}(5.2t)(4.8)^2 = 9.984t$$

$$I_{DE} = (6t)(2.4)^2 = 34.56t$$

$$I_{EF} = I_{DB} = 9.984t$$

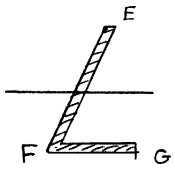
$$I_{FG} = I_{AB} = 17.28t$$

$$I = \Sigma I = 89.09t$$

(*a*) At point *C*, $Q_C = Q_{AB} + Q_{BC} = (3t)(2.4) + (2.6t)(1.2) = 10.32t$

$$\tau = \frac{VQ_C}{It} \quad \therefore \quad t = \frac{VQ}{\tau I} = \frac{(600)(10.32t)}{(300)(89.09t)} = 0.23168 \text{ in.}$$

$t = 0.232$ in. ◀



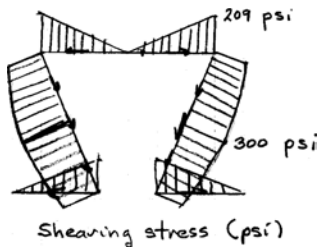
(*b*) $I = (89.09)(0.23168) = 20.64 \text{ in}^4$

$$Q_E = Q_{EF} + Q_{FG}$$

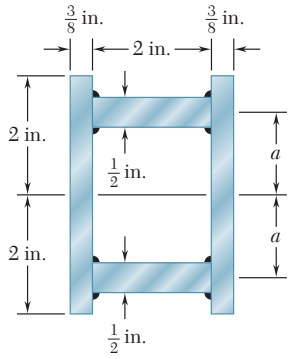
$$= 0 + (3)(0.23168)(2.4) = 1.668 \text{ in}^3$$

$$\tau_E = \frac{VQ_E}{It} = \frac{(600)(1.668)}{(20.64)(0.23168)}$$

$\tau_E = 209$ psi ◀



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PROBLEM 6.51

The design of a beam calls for connecting two vertical rectangular $\frac{3}{8} \times 4$-in. plates by welding them to horizontal $\frac{1}{2} \times 2$-in. plates as shown. For a vertical shear **V**, determine the dimension *a* for which the shear flow through the welded surface is maximum.

SOLUTION

$$I = (2)\left(\frac{1}{12}\right)\left(\frac{3}{8}\right)(4)^3 + (2)\left(\frac{1}{12}\right)(2)\left(\frac{1}{2}\right)^3 + (2)(2)\left(\frac{1}{2}\right)a^2$$

$$= 4.041667 + 2a^2 \text{ in}^4$$

$$Q = (2)\left(\frac{1}{2}\right)a = a \text{ in}^3$$

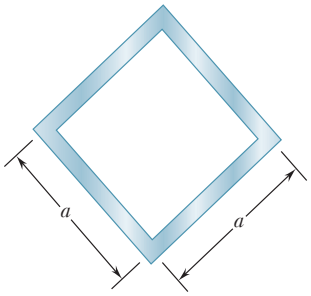
$$q = \frac{VQ}{I} = \frac{Va}{4.041667 + 2a^2} \qquad \text{Set } \frac{dq}{da} = 0.$$

$$\frac{dq}{da} = \left[\frac{(4.041667 + 2a^2) - (a)(4a)}{(4.041667 + 2a^2)^2}\right]V = 0$$

$$2a^2 = 4.041667$$

$$a = 1.422 \text{ in.} \blacktriangleleft$$

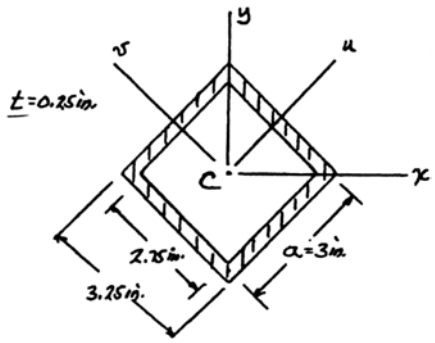
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PROBLEM 6.52

The cross section of an extruded beam is a hollow square of side $a = 3$ in. and thickness $t = 0.25$ in. For a vertical shear of 15 kips, determine the maximum shearing stress in the beam and sketch the shear flow in the cross section.

SOLUTION



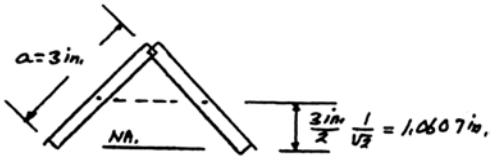
$$I_u = I_v = \frac{1}{12}(3.25^4 - 2.75^4)$$

$$= 4.53125 \text{ in}^4$$

Since products of inertia = 0,

$$I_x = I_y = I_u = I_v$$

$$I_x = 4.53125 \text{ in}^4$$



$$V = 15 \text{ kips}$$

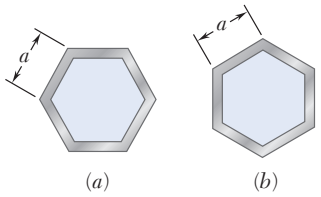
$$Q_{NA} = 2[(3 \text{ in.} \times 0.25 \text{ in.})(1.0607 \text{ in.})] = 1.59105 \text{ in}^3$$

$$\tau_m = \tau_{NA} = \frac{VQ_{NA}}{I(2t)} = \frac{(15 \text{ kips})(1.59105 \text{ in}^3)}{(4.53125 \text{ in}^4)(2)(0.25 \text{ in.})}$$



$$\tau_m = 10.53 \text{ ksi} \blacktriangleleft$$

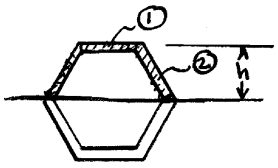
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PROBLEM 6.53

An extruded beam has a uniform wall thickness *t*. Denoting by **V** the vertical shear and by *A* the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{\max} = k(V/A)$ and determine the constant *k* for each of the two orientations shown.

SOLUTION



(*a*) $h = \dfrac{\sqrt{3}}{2}a$

$A_1 = A_2 = at$

$I_1 = A_1h^2 = ath^2 = \dfrac{3}{4}a^3t$

$I_2 = \dfrac{1}{3}A_2h^2 = \dfrac{1}{3}at\dfrac{3}{4}a^2 = \dfrac{1}{4}a^3t$

$I = 2I_1 + 4I_2 = \dfrac{5}{2}a^3t$

$Q_1 = A_1h = \dfrac{\sqrt{3}}{2}a^2t$

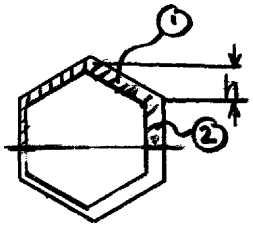
$Q_2 = A_2\dfrac{h}{2} = \dfrac{\sqrt{3}}{4}a^2t$

$Q_m = Q_1 + 2Q_2 = \sqrt{3}a^2t$

$\tau_m = \dfrac{VQ}{I(2t)} = \dfrac{V\sqrt{3}a^2t}{\left(\frac{5}{2}a^3t\right)2t} = \dfrac{\sqrt{3}}{5}\dfrac{V}{at}$

$= \dfrac{6\sqrt{3}}{5}\dfrac{V}{6at} = \dfrac{6\sqrt{3}}{5}\dfrac{V}{A} = k\dfrac{V}{A}$

$k = \dfrac{6\sqrt{3}}{5}$ $\qquad k = 2.08$ ◀



(*b*) $h = \dfrac{a}{2}$

$A_1 = at$

$A_2 = \dfrac{1}{2}at$

$I_1 = \bar{I}_1 + A_1d^2$

$= \dfrac{1}{12}ath^2 + at\left(\dfrac{a}{2} + \dfrac{h}{2}\right)^2$

$= \dfrac{1}{48}a^3t + \dfrac{9}{16}a^3t = \dfrac{7}{12}a^3t$

$I_2 = \dfrac{1}{3}t\left(\dfrac{a}{2}\right)^3 = \dfrac{1}{24}a^3t$

$I = 4I_1 + 4I_2 = \dfrac{5}{2}a^3t$

$Q_1 = at\left(\dfrac{a}{2} + \dfrac{h}{2}\right) = \dfrac{3}{4}a^2t$

$Q_2 = \left(\dfrac{1}{2}at\right)\left(\dfrac{a}{4}\right) = \dfrac{1}{8}a^2t$

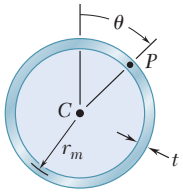
$Q = 2Q_1 + 2Q_2 = \dfrac{7}{4}a^3t$

$\tau_m = \dfrac{VQ}{I(2t)} = \dfrac{V\cdot\frac{7}{4}a^3t}{\left(\frac{5}{2}a^3t\right)(2t)}$

$= \dfrac{7}{20}\dfrac{V}{at} = \dfrac{42}{20}\dfrac{V}{6at} = \dfrac{21}{10}\dfrac{V}{A}$

$= k\dfrac{V}{A}$ $\qquad k = \dfrac{21}{10} = 2.10$ ◀

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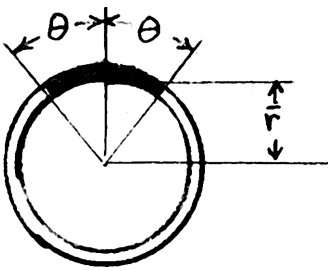
PROBLEM 6.54

(*a*) Determine the shearing stress at point *P* of a thin-walled pipe of the cross section shown caused by a vertical shear **V.** (*b*) Show that the maximum shearing stress occurs for $\theta = 90°$ and is equal to $2V/A$, where *A* is the cross-sectional area of the pipe.

SOLUTION

$$A = 2\pi r_m t \quad J = Ar_m^2 = 2\pi r_m^3 t \quad I = \frac{1}{2}J = \pi r_m^3 t$$

$$\bar{r} = \frac{\sin\theta}{\theta} \text{ for a circular arc.}$$



$$A_P = 2r\theta t$$

$$Q_P = A_P\bar{r} = 2rt\sin\theta$$

(*a*) $$\tau_P = \frac{VQ_P}{I(2t)} = \frac{(V)(2rt\sin\theta)}{\left(\pi r_m^3 t\right)(2t)}$$

$$\tau_P = \frac{V\sin\theta}{\pi r_m t} \blacktriangleleft$$

(*b*) $$\tau_m = \frac{2V\sin\frac{\pi}{2}}{2\pi r_m t}$$

$$\tau_m = \frac{2V}{A} \blacktriangleleft$$

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PROBLEM 6.55

For a beam made of two or more materials with different moduli of elasticity, show that Eq. (6.6)

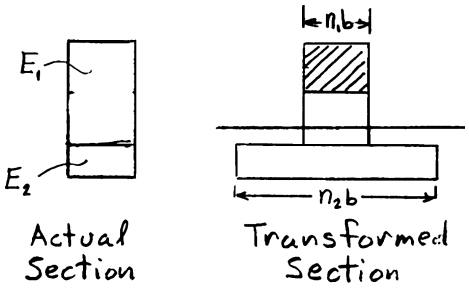
$$\tau_{\text{ave}} = \frac{VQ}{It}$$

remains valid provided that both Q and I are computed by using the transformed section of the beam (see Sec. 4.4), and provided further that t is the actual width of the beam where τ_{ave} is computed.

SOLUTION

Let E_{ref} be a reference modulus of elasticity.

$$n_1 = \frac{E_1}{E_{\text{ref}}}, \quad n_2 = \frac{E_2}{E_{\text{ref}}}, \text{ etc.}$$



Widths b of actual section are multiplied by n's to obtain the transformed section. The bending stress distribution in the cross section is given by

$$\sigma_x = -\frac{nMy}{I}$$

where I is the moment of inertia of the transformed cross section and y is measured from the centroid of the transformed section.

The horizontal shearing force over length Δx is

$$\Delta H = -\int (\Delta\sigma_x)\, dA = \int \frac{n(\Delta M)y}{I}\, dA = \frac{(\Delta M)}{I}\int ny\, dA = \frac{Q(\Delta M)}{I}$$

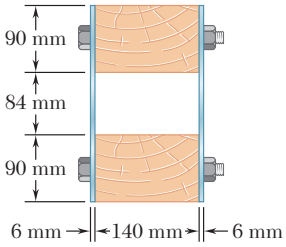
$$Q = \int ny\, dA = \text{ first moment of transformed section.}$$

Shear flow: $$q = \frac{\Delta H}{\Delta x} = \frac{\Delta M}{\Delta x}\frac{Q}{I} = \frac{VQ}{I}$$

q is distributed over actual width t, thus $$\tau = \frac{q}{t}.$$

$$\tau = \frac{VQ}{It}$$

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PROBLEM 6.56

A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. For a vertical shear of 4 kN, determine (*a*) the average shearing stress in the bolts, (*b*) the shearing stress at the center of the cross section. (*Hint:* Use the method indicated in Prob. 6.55.)

SOLUTION

Let steel be the reference material.

$$n_s = 1.0 \qquad n_w = \frac{E_w}{E_s} = \frac{10\text{ GPa}}{200\text{ GPa}} = 0.05$$

Depth of section: $d = 90 + 84 + 90 = 264\text{ mm}$

For steel portion, $I_s = 2\dfrac{1}{12}bd^3 = (2)\left(\dfrac{1}{12}\right)(6)(264)^3 = 18.400\times10^6\text{ mm}^4$

For the wooden portion, $I_w = \dfrac{1}{12}b\left(d_1^3 - d_2^3\right) = \dfrac{1}{12}(140)(264^3 - 84^3) = 207.75\times10^6\text{ mm}^4$

For the transformed section, $I = n_sI_s + n_wI_w$

$$I = (1.0)(18.400\times10^6) + (0.05)(207.75\times10^6) = 28.787\times10^6\text{ mm}^4 = 28.787\times10^{-6}\text{ m}^4$$

(*a*) Shearing stress in the bolts.

For the upper wooden portion, $Q_w = (90)(140)(42 + 45) = 1.0962\times10^6\text{ mm}^3$

For the transformed wooden portion,

$$Q = n_wQ_w = (0.05)(1.0962\times10^6) = 54.81\times10^3\text{ mm}^3 = 54.81\times10^{-6}\text{ m}^3$$

Shear flow on upper wooden portion:

$$q = \frac{VQ}{I} = \frac{(4000)(54.81\times10^{-6})}{28.787\times10^{-6}} = 7616\text{ N/m}$$

$$F_{\text{bolt}} = qs = (7616)(0.200) = 1523.2\text{ N}$$

$$A_{\text{bolt}} = \frac{\pi}{4}d_{\text{bolt}}^2 = \frac{\pi}{4}(12)^2 = 113.1\,\text{mm}^2 = 113.1\times10^{-6}\text{ m}^2$$

Double shear: $\tau_{\text{bolt}} = \dfrac{F_{\text{bolt}}}{2A_{\text{bolt}}} = \dfrac{1523.2}{(2)(113.1\times10^{-6})}$

$$= 6.73\times10^6\text{ Pa} \qquad \tau_{\text{bolt}} = 6.73\text{ MPa} \blacktriangleleft$$

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PROBLEM 6.56 *(Continued)*

(*b*) Shearing stress at the center of the cross section.

For two steel plates, $Q_s = (2)(6)(90 + 42)(90 - 42) = 76.032 \times 10^3 \text{ mm}^3 = 76.032 \times 10^{-6} \text{ m}^3$

For the neutral axis, $Q = 54.81 \times 10^{-6} + 76.032 \times 10^{-6} = 130.842 \times 10^{-6} \text{ m}^3$

Shear flow across the neutral axis:

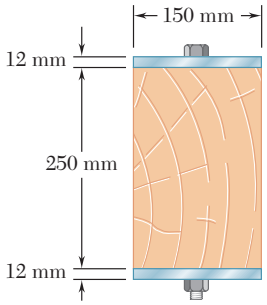
$$q = \frac{VQ}{I} = \frac{(4000)(130.842 \times 10^{-6})}{28.787 \times 10^{-6}} = 18.181 \times 10^3 \text{ N/m}$$

Double thickness: $2t = 12 \text{ mm} = 0.012 \text{ m}$

Shearing stress: $\tau = \frac{q}{2t} = \frac{18.181 \times 10^3}{0.012} = 1.515 \times 10^6 \text{ Pa}$

$\tau = 1.515 \text{ MPa}$ ◀

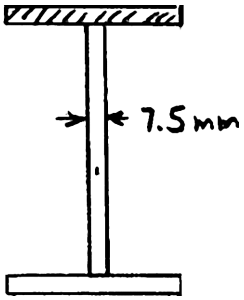
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PROBLEM 6.57

A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. For a vertical shear of 4 kN, determine (*a*) the average shearing stress in the bolts, (*b*) the shearing stress at the center of the cross section. (*Hint:* Use the method indicated in Prob. 6.55.)

SOLUTION



Let $E_{\text{ref}} = E_s = 200 \text{ GPa}$

$$n_s = 1 \qquad n_w = \frac{E_w}{E_s} = \frac{10 \text{ GPa}}{200 \text{ GPa}} = \frac{1}{20}$$

Widths of transformed section:

$$b_s = 150 \text{ mm} \qquad b_w = \left(\frac{1}{20}\right)(150) = 7.5 \text{ mm}$$

$$I = 2\left[\frac{1}{12}(150)(12)^3 + (150)(12)(125+6)^2\right] + \frac{1}{12}(7.5)(250)^3$$

$$= 2[0.0216\times10^6 + 30.890\times10^6] + 9.766\times10^6$$

$$= 71.589\times10^6 \text{ mm}^4 = 71.589\times10^{-6} \text{ m}^4$$

(*a*) $$Q_1 = (150)(12)(125+6) = 235.8\times10^3 \text{ mm}^3 = 235.8\times10^{-6} \text{ m}^3$$

$$q = \frac{VQ_1}{I} = \frac{(4\times10^3)(235.8\times10^{-6})}{71.589\times10^{-6}} = 13.175\times10^3 \text{ N/m}$$

$$F_{\text{bolt}} = qs = (23.187\times10^3)(200\times10^{-3}) = 2.635\times10^3 \text{ N}$$

$$A_{\text{bolt}} = \frac{\pi}{4}d_{\text{bolt}}^2 = \left(\frac{\pi}{4}\right)(12)^2 = 113.1 \text{ mm}^2 = 113.1\times10^{-6} \text{ m}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{2.635\times10^3}{113.1\times10^{-6}} = 23.3\times10^6 \text{ Pa}$$

$\tau_{\text{bolt}} = 23.3 \text{ MPa}$ ◀

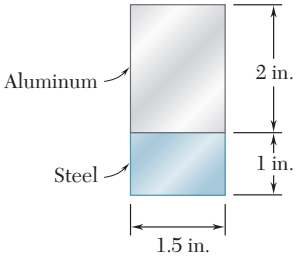
(*b*) $$Q_2 = Q_1 + (7.5)(125)(62.5) = 235.8\times10^3 + 58.594\times10^3 = 294.4\times10^3 \text{ mm}^3 = 294.4\times10^{-6} \text{ m}^3$$

$$t = 150 \text{ mm} = 150\times10^{-3} \text{ m}$$

$$\tau_c = \frac{VQ_2}{It} = \frac{(4\times10^3)(294.4\times10^{-6})}{(71.589\times10^{-6})(150\times10^{-3})} = 109.7\times10^3 \text{ Pa}$$

$\tau_c = 109.7 \text{ kPa}$ ◀

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PROBLEM 6.58

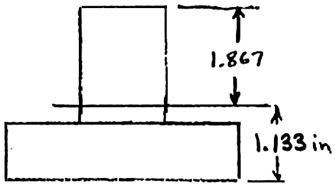
A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum shearing stress in the beam. (*Hint:* Use the method indicated in Prob. 6.55.)

SOLUTION

$n = 1$ in aluminum.

$$n = \frac{29\times10^6 \text{ psi}}{10.6\times10^6 \text{ psi}} = 2.7358 \text{ in steel.}$$

Part	$nA(\text{in}^2)$	$\bar{y}(\text{in.})$	$nA\bar{y}(\text{in}^3)$	$d(\text{in.})$	$nAd^2(\text{in}^2)$	$n\bar{I}(\text{in}^4)$
Alum.	3.0	2.0	6.0	0.8665	2.2525	1.0
Steel	4.1038	0.5	2.0519	0.6335	1.6469	0.3420
Σ	7.1038		8.0519		3.8994	1.3420



$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma nA} = \frac{8.0519}{7.1038} = 1.1335 \text{ in.}$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.2414 \text{ in}^4$$

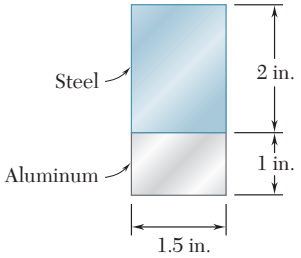
(*a*) At the bonded surface, $Q = (1.5)(2)(0.8665) = 2.5995 \text{ in}^3$

$$\tau = \frac{VQ}{It} = \frac{(4)(2.5995)}{(5.2414)(1.5)} \qquad \tau = 1.323 \text{ ksi} \blacktriangleleft$$

(*b*) At the neutral axis, $Q = (1.5)(1.8665)\left(\frac{1.8665}{2}\right) = 2.6129 \text{ in}^3$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(4)(2.6129)}{(5.2814)(1.5)} \qquad \tau_{\max} = 1.329 \text{ ksi} \blacktriangleleft$$

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PROBLEM 6.59

A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum shearing stress in the beam. (*Hint:* Use the method indicated in Prob. 6.55.)

SOLUTION

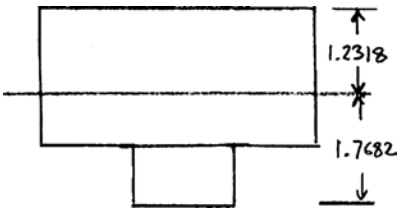
$n = 1$ in aluminum.

$$n = \frac{29\times10^6 \text{ psi}}{10.6\times10^6 \text{ psi}} = 2.7358 \text{ in steel.}$$

Part	$nA(\text{in}^2)$	$\bar{y}(\text{in.})$	$nA\bar{y}(\text{in}^3)$	$d(\text{in.})$	$nAd^2(\text{in}^2)$	$n\bar{I}(\text{in}^4)$
Steel	8.2074	2.0	16.4148	0.2318	0.4410	2.7358
Alum.	1.5	0.5	0.75	1.2682	2.4125	0.1250
Σ	9.7074		17.1648		2.8535	2.8608

$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma A} = \frac{17.1648}{9.7074} = 1.7682 \text{ in.}$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.7143 \text{ in}^4$$



(*a*) At the bonded surface, $Q = (1.5)(1.2682) = 1.9023 \text{ in}^3$

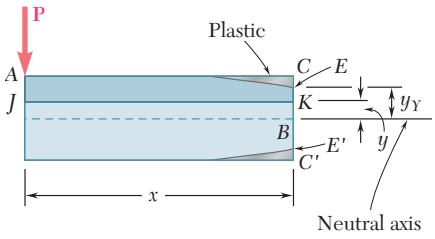
$$\tau = \frac{VQ}{It} = \frac{(4)(1.9023)}{(5.7143)(1.5)} \qquad \tau = 0.888 \text{ ksi} \blacktriangleleft$$

(*b*) At the neutral axis, $Q = (2.7358)(1.5)(1.2318)\left(\frac{1.2318}{2}\right) = 3.1133 \text{ in}^3$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(4)(3.1133)}{(5.7143)(1.5)} \qquad \tau_{\max} = 1.453 \text{ ksi} \blacktriangleleft$$

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PROBLEM 6.60



Consider the cantilever beam *AB* discussed in Sec. 6.5 and the portion *ACKJ* of the beam that is located to the left of the transverse section *CC'* and above the horizontal plane *JK*, where *K* is a point at a distance $y < y_Y$ above the neutral axis. (See figure.) (*a*) Recalling that $\sigma_x = \sigma_Y$ between *C* and *E* and $\sigma_x = (\sigma_Y/y_Y)y$ between *E* and *K*, show that the magnitude of the horizontal shearing force **H** exerted on the lower face of the portion of beam *ACKJ* is

$$H = \frac{1}{2} b\sigma_Y \left(2c - y_Y - \frac{y^2}{y_Y} \right)$$

(*b*) Observing that the shearing stress at *K* is

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta H}{\Delta A} = \lim_{\Delta x \to 0} \frac{1}{b} \frac{\Delta H}{\Delta x} = \frac{1}{b} \frac{\partial H}{\partial x}$$

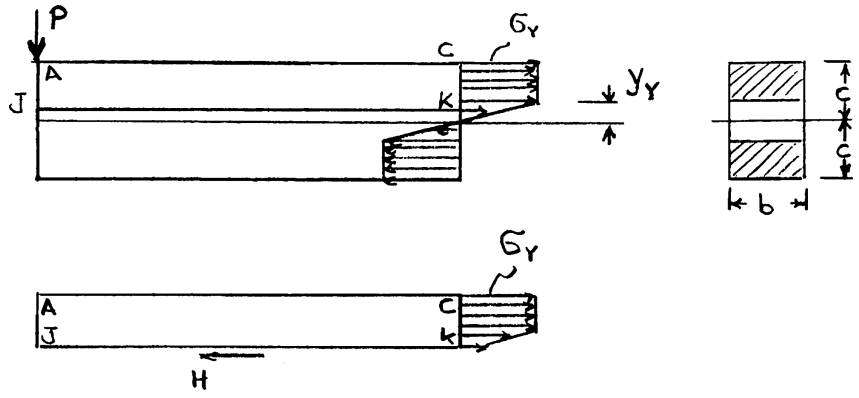
and recalling that y_Y is a function of *x* defined by Eq. (6.14), derive Eq. (6.15).

SOLUTION

Point *K* is located a distance *y* above the neutral axis.

The stress distribution is given by

$$\sigma = \sigma_Y \frac{y}{y_Y} \quad \text{for} \quad 0 \le y < y_Y \quad \text{and} \quad \sigma = \sigma_Y \quad \text{for} \quad y_Y \le y \le c.$$



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PROBLEM 6.60 ***(Continued)***

For equilibrium of horizontal forces acting on *ACKJ*,

$$H = \int \sigma \, dA = \int_{y}^{y_Y} \frac{\sigma_Y y b}{y_Y} dy + \int_{y_Y}^{c} \sigma_Y b \, dy$$

$$= \frac{\sigma_Y b}{y_Y}\left(\frac{y_Y^2 - y^2}{2}\right) + \sigma_y b(c - y_Y)$$

$$H = \frac{1}{2} b \sigma_Y \left(2c - y_Y - \frac{y^2}{y_Y}\right) \quad \blacktriangleleft \quad (a)$$

Note that y_Y is a function of *x*.

$$\tau_{xy} = \frac{1}{b}\frac{\partial H}{\partial x} = \frac{1}{2}\sigma_Y \left(-\frac{\partial y_Y}{\partial x} + \frac{y^2}{y_{Y^2}}\frac{dy_Y}{dx}\right)$$

$$= -\frac{1}{2}\sigma_Y \left(1 - \frac{y^2}{y_{Y^2}}\right)\frac{dy_Y}{dx}$$

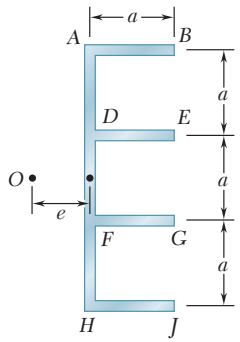
But $$M = Px = \frac{3}{2} M_y \left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right)$$

Differentiating, $$\frac{dM}{dx} = P = \frac{3}{2} M_Y \left(-\frac{2}{3}\frac{y_Y}{c^2}\frac{dy_Y}{dx}\right)$$

$$\frac{dy_Y}{dx} = -\frac{Pc^2}{y_Y M_Y} = -\frac{Pc^2}{y_Y \frac{2}{3}\sigma_Y b c^2} = -\frac{3}{2}\frac{P}{\sigma_Y b y_Y}$$

Then $$\tau_{xy} = \frac{1}{2}\sigma_Y \left(1 - \frac{y^2}{y_{Y^2}}\right)\frac{3}{2}\frac{P}{\sigma_Y b \sigma_Y} = \frac{3P}{4by_Y}\left(1 - \frac{y^2}{y_{Y^2}}\right) \quad \blacktriangleleft \quad (b)$$

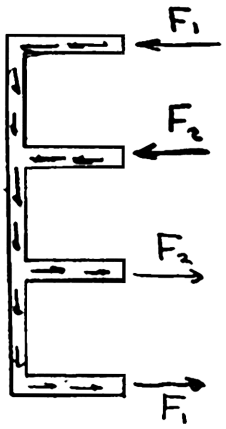
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PROBLEM 6.61

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION



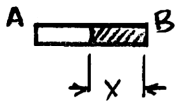
$$I_{AB} = I_{HJ} = at\left(\frac{3a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{9}{4}ta^3$$

$$I_{DE} = I_{FG} = at\left(\frac{a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{1}{4}ta^2$$

$$I_{AH} = \frac{1}{12}t(3a)^3 = \frac{9}{4}ta^3$$

$$I = \Sigma I = \frac{29}{4}ta^3$$

Part AB:

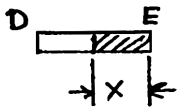


$$A = tx \qquad \bar{y} = \frac{3a}{2} \qquad Q = \frac{3}{2}atx$$

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{3}{2}atx}{\frac{29}{4}ta^3t} = \frac{6Vx}{29a^2t}$$

$$F_1 = \int \tau\, dA = \int_0^a \frac{6Vx}{29a^2t}\, t\, dx = \frac{6V}{29a^2}\int_0^a x\, dx = \frac{3}{29}V$$

Part DE:



$$A = tx \qquad \bar{y} = \frac{a}{2} \qquad Q = \frac{1}{2}atx$$

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{2}atx}{\frac{29}{4}ta^3t} = \frac{2Vx}{29a^2t}$$

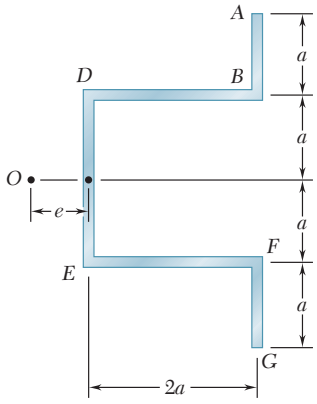
$$F_2 = \int \tau\, dA = \int_0^a \frac{2Vx}{29a^2t}\, t\, dx = \frac{2V}{29a^2}\int_0^a x\, dx = \frac{1}{29}V$$

$$+\circlearrowright \Sigma M_K = +\circlearrowright \Sigma M_K: \quad Ve = F_1(3a) + F_2(a)$$

$$= \frac{9}{29}Va + \frac{1}{29}Va = \frac{10}{29}Va$$

$$e = \frac{10}{29}a \qquad e = 0.345a \blacktriangleleft$$

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PROBLEM 6.62

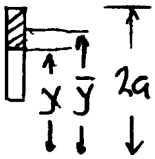
Determine the location of the shear center *O* of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

$$I_{AB} = I_{FG} = \frac{1}{12}ta^3 + (ta)\left(\frac{3a}{2}\right)^2 = \frac{7}{3}ta^3$$

$$I_{DB} = I_{EF} = (2at)a^2 + \frac{1}{12}(2a)t^3 \approx 2a^3t$$

$$I_{DE} = \frac{1}{12}t(2a)^3 = \frac{2}{3}ta^3 \qquad I = \Sigma I = \frac{28}{3}ta^3$$



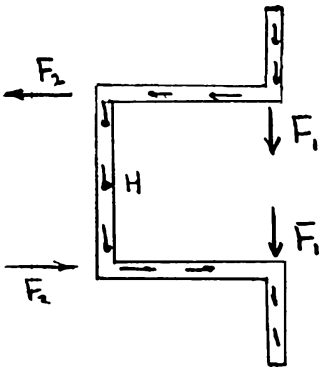
Part *AB*:

$$A = t(2a - y); \qquad \bar{y} = \frac{2a + y}{3}$$

$$Q = A\bar{y} = \frac{1}{2}t(2a - y)(2a + y)$$

$$= \frac{1}{2}t(4a^2 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I}(4a^2 - y^2)$$



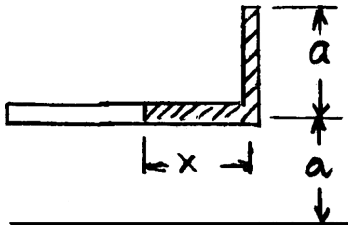
$$F_1 = \int \tau dA = \int_a^{2a} \frac{V}{2I}(4a^2 - y^2)t\,dy$$

$$= \frac{Vt}{2I}\left(4a^2y - \frac{y^3}{3}\right)\Bigg|_a^{2a} = \frac{Vta^3}{2I}\left[(4)(2) - \frac{(2)^3}{3} - (4)(1) + \left(\frac{1}{3}\right)\right]$$

$$= \frac{5}{6}\frac{Vta^3}{I} = \frac{5}{56}V$$

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PROBLEM 6.62 *(Continued)*



Part *DB*:

$$Q = (ta)\frac{3a}{2} + txa$$

$$= ta\left(\frac{3a}{2} + x\right)$$

$$\tau = \frac{VQ}{It} = \frac{Va}{I}\left(\frac{3a}{2} + x\right)$$

$$F_2 = \int \tau\, dA = \int_0^{2a} \frac{Va}{I}\left(\frac{3a}{2} + x\right) t\, dx = \frac{Vta}{I}\int_0^{2a}\left(\frac{3a}{2} + x\right) dx$$

$$= \frac{Vta}{I}\left(\frac{3ax}{2} + \frac{x^2}{2}\right)\Bigg|_0^{2a} = \frac{Vta^3}{I}\left[\frac{(3)(2)}{2} + \frac{(2)^2}{2}\right]$$

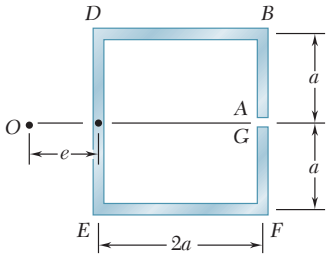
$$= 5\frac{Vta^3}{I} = \frac{15}{28}V$$

$$+\circlearrowright \Sigma M_H = +\circlearrowright \Sigma M_H: \quad Ve = F_2(2a) - 2F_1(2a)$$

$$= \frac{30}{28}Va - \frac{20}{56}Va = \frac{5}{7}Va$$

$$e = \frac{5}{7}a = 0.714a \blacktriangleleft$$

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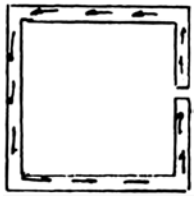
PROBLEM 6.63

Determine the location of the shear center *O* of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

$$I_{AB} = I_{FG} = \frac{1}{3}ta^3 \quad I_{DB} = I_{CP} = 2ata^3 + \frac{1}{12}2att^3 \approx 2ta^3$$

$$I_{DE} = \frac{1}{12}t(2a)^3 = \frac{2}{3}ta^3 \quad I = \Sigma I = \frac{16}{3}ta^3$$



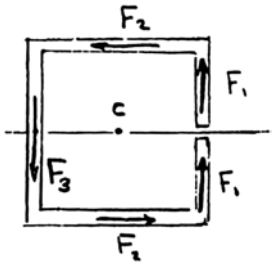
Part *AB*:

$$A = ty \quad \bar{y} = \frac{y}{2} \quad Q = \frac{1}{2}ty^2$$



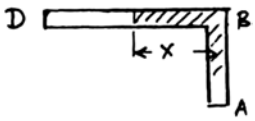
$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{2}ty^2}{\frac{16}{3}ta^3t} = \frac{3Vy^2}{32a^3t}$$

$$F_1 = \int \tau dA = \int_0^a \tau t\, dy = \frac{3V}{32a^3}\int_0^a y^2 dt = \frac{1}{32}V$$



Part *BD*:

$$Q = Q_B + txa = \frac{1}{2}ta^2 + tax$$



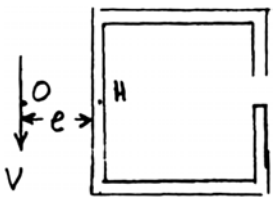
$$\tau = \frac{VQ}{It} = \frac{Vt}{\frac{16}{3}a^3t}\left(\frac{1}{2}a^2 + ax\right)$$

$$= \frac{3}{32}\frac{V}{a^2}(a + 2x)$$

$$F_2 = \int \tau dA = \int_0^{2a} \frac{3V}{32a^2}(a + 2x)dx$$

$$= \frac{3V}{32a^2}(ax + x^2)\Big|_0^{2a}$$

$$= \frac{3V}{32a^2}(2a^2 + 4a^2) = \frac{9}{16}V$$



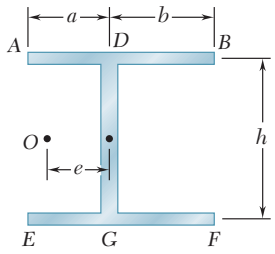
$$+\circlearrowright \Sigma M_H = +\circlearrowright \Sigma M_H:$$

$$Ve = (2a)(2F_1) + (2a)(F_2)$$

$$= \frac{1}{8}Va + \frac{9}{8}Va = \frac{5}{4}Va$$

$$e = \frac{5}{4}a = 1.250a \blacktriangleleft$$

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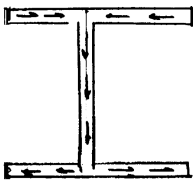
PROBLEM 6.64

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

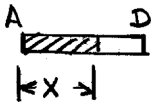
$$I_{AB} = I_{EF} = (a+b)t\left(\frac{h}{2}\right)^2 + \frac{1}{12}(a+b)t^3 \approx \frac{1}{4}t(a+b)h^2$$

$$I_{DG} = \frac{1}{12}th^3 \qquad I = \Sigma I = \frac{1}{12}t(6a+6b+h)h^2$$



Part AD:

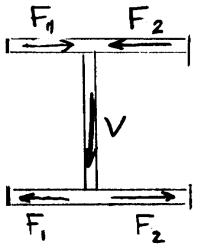
$$Q = tx\frac{h}{2} = \frac{1}{2}thx$$



$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

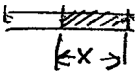
$$F_1 = \int \tau\, dA = \int_0^a \frac{Vhx}{2I} t\, dx = \frac{Vht}{2I}\int_0^a x\, dx$$

$$= \frac{Vht}{2I}\frac{x^2}{2}\bigg|_0^a = \frac{Vhta^2}{4I}$$



Part BD:

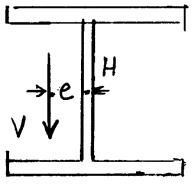
$$Q = tx\frac{h}{2} = \frac{1}{2}thx$$



$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_2 = \int \tau\, dA = \int_0^b \frac{Vhx}{2I} t\, dx = \frac{Vht}{2I}\int_0^b x\, dx$$

$$= \frac{Vht}{2I}\frac{x^2}{2}\bigg|_0^b = \frac{Vhtb^2}{4I}$$



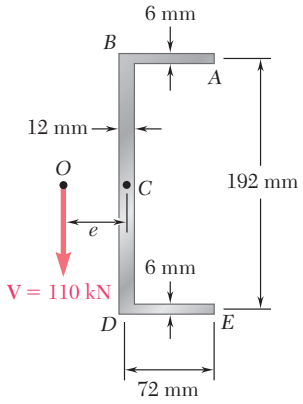
$$+\circlearrowleft \Sigma M_H = +\circlearrowleft \Sigma M_H:$$

$$Ve = F_2 h - F_1 h = \frac{Vh^2 t(b^2 - a^2)}{4I}$$

$$= \frac{Vh^2 t(b^2 - a^2)}{4 \cdot \frac{1}{12}t(6a+6b+h)h^2} = \frac{3V(b^2 - a^2)}{6a+6b+h}$$

$$e = \frac{3(b^2 - a^2)}{6(a+b)+h} \blacktriangleleft$$

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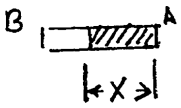
PROBLEM 6.65

An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.

SOLUTION

$$I = 2\left[\left(\frac{1}{12}\right)(72)(6)^3 + (72)(6)\left(\frac{192}{2}\right)^2\right] + \frac{1}{12}(12)(192)^3$$

$$= 15.0431\times10^6\,\text{mm}^4 = 15.0431\times10^{-6}\,\text{m}^4$$



Part AB: $A = 6x \qquad Q = A\bar{y} = (6x)\left(\frac{192}{2}\right) = 576x$

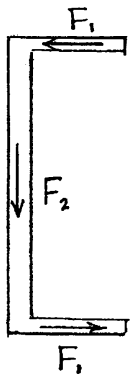
$$q = \frac{VQ}{I} = \frac{576Vx}{I}$$

$x = 0$ at point *A*. $x = l_{AB} = 72$ mm at point *B*.

$$F_1 = \int_{x_A}^{x_B} q\,dx = \int_0^{72} \frac{576Vx}{I}\,dx = \frac{576V}{I}\frac{(72)^2}{2}$$

$$= \frac{(288)(72)^2}{15.0431\times10^6}V = 0.099247V$$

$$+\circlearrowleft M_C = +\circlearrowleft M_C: \quad Ve = (0.099247)V(192)$$



(*a*) $e = 19.0555$ mm $\qquad e = 19.06$ mm ◀

(*b*) Point *A*: $x = 0, \quad Q = 0, \quad q = 0$ $\qquad \tau_A = 0$ ◀

Point *B* in part *AB*: $x = 72$ mm

$$Q = (576)(72) = 41.472\times10^3\,\text{mm}^3 = 41.472\times10^{-6}\,\text{m}^3$$

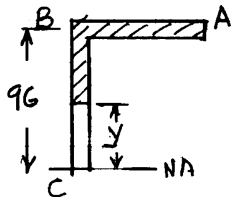
$$t = 6\text{ mm} = 0.006\text{ m}$$

$$\tau_B = \frac{VQ}{It} = \frac{(110\times10^3)(41.472\times10^{-6})}{(15.0431\times10^{-6})(0.006)}$$

$$= 50.5\times10^6\,\text{Pa} \qquad \tau_B = 50.5\text{ MPa}$$ ◀

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PROBLEM 6.65 *(Continued)*



Part *BD*:

Point *B*: $y = 96$ mm $\quad Q = 41.472\times10^{3}\,\text{mm}^3 = 41.472\times10^{-6}\,\text{m}^3$

$t = 12$ mm $= 0.012$ m

$$\tau_B = \frac{VQ}{It} = \frac{(110\times10^{3})(41.472\times10^{-6})}{(15.0431\times10^{-6})(0.012)}$$

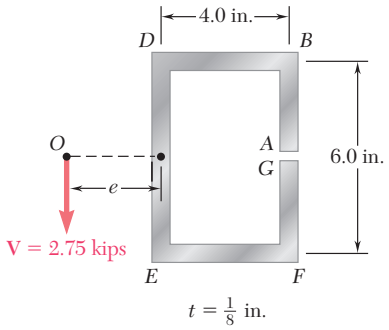
$$= 25.271\times10^{6}\,\text{Pa} \qquad \tau_B = 25.3\ \text{MPa} \blacktriangleleft$$

Point *C*: $y = 0, \quad t = 0.012$ m

$$Q = 41.472\times10^{3} + (12)(96)\left(\frac{96}{2}\right) = 96.768\times10^{3}\,\text{mm}^3 = 96.768\times10^{-6}\,\text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(110\times10^{3})(96.768\times10^{-6})}{(15.0431\times10^{-6})(0.012)} = 58.967\times10^{6}\,\text{Pa} \qquad \tau_C = 59.0\ \text{MPa} \blacktriangleleft$$

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PROBLEM 6.66

An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.

SOLUTION

$$I_{AB} = \frac{1}{3}(0.125)(3)^3 = 1.125 \text{ in}^4$$

$$I_{BD} = \frac{1}{12}(4)(0.125)^3 + (4)(0.125)(3)^2 = 4.50065 \text{ in}^4$$

$$I_{DE} = \frac{1}{12}(0.125)(6)^3 = 2.25 \text{ in}^4$$

$$I_{EF} = I_{BD} = 4.50065 \text{ in}^4$$

$$I_{FG} = I_{AB} = 1.125 \text{ in}^4$$

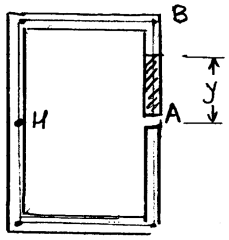
$$I = \Sigma I = 13.50 \text{ in}^4$$

(*a*) Part *AB*:

$$Q(y) = ty\frac{y}{2} = 0.5ty^2$$

$$q(y) = \frac{VQ(y)}{I} = \frac{0.5Vt}{I}y^2$$

$$F_{AB} = \int_0^3 q(y)\,dy = \frac{0.5Vt}{I}\int_0^3 y^2\,dy = 4.5\frac{Vt}{I} \uparrow$$



Its moment about *H* is $4F_{AB} = 18\frac{Vt}{I}$ ↻

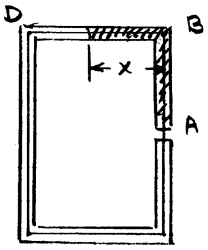
$$Q_B = (0.5)(t)(3)^2 = 4.5t$$

Part *BD*:

$$Q(x) = Q_B + xt(3) = (4.5 + 3x)t$$

$$q(x) = \frac{Vq(x)}{I} = \frac{Vt}{I}(4.5 + 3x)$$

$$F_{BD} = \int_0^4 q(x)\,dx = \frac{\nabla t}{I}\int_0^4 (4.5 + 3x)\,dx = 42\frac{Vt}{I} \leftarrow$$



Its moment about *H*: $3F_{BD} = 126\frac{Vt}{I}$ ↻

$$Q_D = [4.5 + (3)(4)]t = 16.5t$$

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PROBLEM 6.66 *(Continued)*

Part *EF*: By symmetry with part *BD*, $F_{EF} = 42\ \dfrac{Vt}{I} \rightarrow$

Its moment about *H* is $3F_{EF} = 126\ \dfrac{Vt}{I}$ ↻

Part *FG*: By symmetry with part *AB*, $F_{FG} = 4.5\dfrac{VT}{I} \uparrow$

Its moment about *H* is $4\ F_{FG} = 18\ \dfrac{Vt}{I}$ ↻

Moment about *H* of force in part *DE* is zero.

$$Ve = \Sigma M_H = \frac{Vt}{I}(18 + 126 + 0 + 126 + 18) = \frac{144Vt}{I}$$

$$e = \frac{144t}{I} = \frac{(2.88)(0.125)}{13.50} \qquad e = 2.67 \text{ in.} \blacktriangleleft$$

(*b*) $Q_A = Q_G = 0$ $\qquad \tau_A = \tau_G = 0 \blacktriangleleft$

$Q_B = Q_F = 4.5t$

$$\tau_B = \tau_F = \frac{VQ_B}{It} = \frac{(2.75)(4.5t)}{13.50t} \qquad \tau_B = \tau_F = 0.917 \text{ ksi} \blacktriangleleft$$

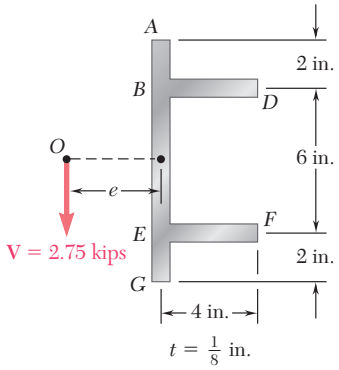
$Q_D = Q_E = 16.5t$

$$\tau_D = \tau_E = \frac{VQ_0}{It} = \frac{(2.75)(16.5t)}{13.50t} \qquad \tau_D = \tau_E = 3.36 \text{ ksi} \blacktriangleleft$$

At *H* (neutral axis), $Q_H = Q_D + t(3)(1.5) = 21t$

$$\tau_H = \frac{VQ_H}{It} = \frac{(2.75)(21t)}{13.50t} \qquad \tau_H = 4.28 \text{ ksi} \blacktriangleleft$$

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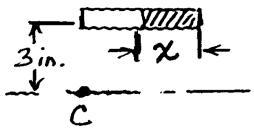
PROBLEM 6.67

An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.

SOLUTION

Part	A (in^2)	d (in.)	Ad^2 (in^4)	$\bar{I}$ (in^4)
BD	0.50	3	4.50	≈ 0
ABEG	1.25	0	0	10.417
EF	0.50	3	4.50	≈ 0
Σ	2.25		9.00	10.417

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 19.417 \text{ in}^4$$

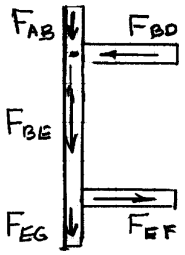


(*a*) Part *BD*: $Q(x) = 3tx$

$$q(x) = \frac{VQ(x)}{I} = \frac{V}{I}(3tx)$$

$$F_{BD} = \frac{3Vt}{I}\int_0^4 x\,dx = \frac{3Vt}{I}(8) = \frac{24Vt}{I}$$

Its moment about *H*: $(M_{BD})_H = 3F_{BD} = \frac{72Vt}{I}$



Part *EF*: By same method, $F_{EF} = \frac{24Vt}{I}$ $(M_{EF})_H = \frac{72Vt}{I}$

Moments of F_{AB}, F_{BE}, and F_{EG} about *H* are zero.

$$Ve = \Sigma M_H = \frac{72Vt}{I} + \frac{72Vt}{I} = \frac{144Vt}{I}$$

$$e = \frac{144t}{I} = \frac{(144)(0.125)}{19.417}$$

$e = 0.927$ in. ◀

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PROBLEM 6.67 *(Continued)*

(*b*) At *A*, *D*, *F*, and *G*, $Q = 0$ $\tau_A = \tau_D = \tau_F = \tau_G = 0$ ◀

Just above *B*: $Q_1 = Q_{AB} = (2t)(4) = 8t$

$$\tau_1 = \frac{VQ_1}{It} = \frac{(2.75)(8t)}{(19.417)t} \qquad \tau_1 = 1.133 \text{ ksi} \blacktriangleleft$$

Just to the right of *B*: $Q_2 = Q_{BD} = (3)t(4) = 12t$

$$\tau_2 = \frac{VQ_2}{It} = \frac{(2.75)(12t)}{(19.417)t} \qquad \tau_2 = 1.700 \text{ ksi} \blacktriangleleft$$

Just below *B*: $Q_3 = Q_1 + Q_2 = 20t$

$$\tau_3 = \frac{VQ_3}{It} = \frac{(2.75)(20t)}{(19.417)t} \qquad \tau_3 = 2.83 \text{ ksi} \blacktriangleleft$$

At *H* (neutral axis), $Q_H = Q_3 + Q_{BH} = 20t + t(3)(1.5) = 24.5t$

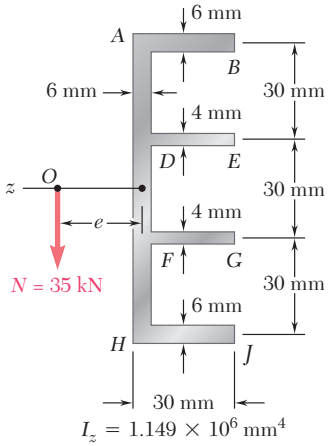
$$\tau_H = \frac{VQ_H}{It} = \frac{(2.75)(24.5t)}{(19.417)t} \qquad \tau_H = 3.47 \text{ ksi} \blacktriangleleft$$

By symmetry, $\tau_4 = \tau_3 = 2.83 \text{ ksi}$ ◀

$\tau_5 = \tau_2 = 1.700 \text{ ksi}$ ◀

$\tau_6 = \tau_1 = 1.133 \text{ ksi}$ ◀

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PROBLEM 6.68

An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.

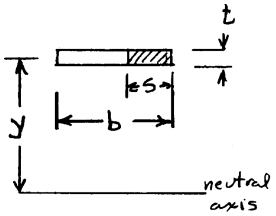
SOLUTION

$$I_{AB} = I_{HJ} = \frac{1}{12}(30)(6)^3 + (30)(6)(45)^2 = 0.365 \times 10^6 \text{ mm}^4$$

$$I_{DE} = I_{FG} = \frac{1}{12}(30)(4)^3 + (30)(4)(15)^2 = 0.02716 \times 10^6 \text{ mm}^4$$

$$I_{AH} = \frac{1}{12}(6)(90)^3 = 0.3645 \times 10^6 \text{ mm}^4$$

$$I = \Sigma I = 1.14882 \times 10^6 \text{ mm}^4$$

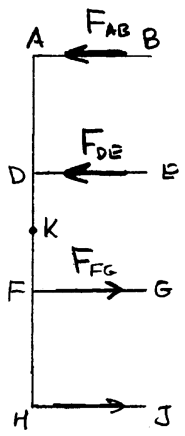


(*a*) For a typical flange, $A(s) = ts$

$$Q(s) = yts$$

$$q(s) = \frac{VQ(s)}{I} = \frac{Vyts}{I}$$

$$F = \int_0^b q(s)ds = \frac{Vytb^2}{2I}$$



Flange *AB*: $$F_{AB} = \frac{V(45)(6)(30^2)}{(2)(1.14882 \times 10^{-6})} = 0.10576V \leftarrow$$

Flange *DE*: $$F_{DE} = \frac{V(15)(4)(30)^2}{(2)(1.14882 \times 10^6)} = 0.023502V \leftarrow$$

Flange *FG*: $F_{FG} = 0.023502V \rightarrow$

Flange *HJ*: $F_{HJ} = 0.10576V \rightarrow$

$$+\circlearrowleft \Sigma M_K = +\circlearrowleft \Sigma M_K: \quad Ve = 45F_{AB} + 15F_{DE} + 15F_{FG} + 45F_{HJ} = 10.223V$$

Dividing by *V*, $e = 10.22$ mm ◀

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PROBLEM 6.68 *(Continued)*

(*b*) Calculation of shearing stresses.

$$V = 35\times10^{3}\,\text{N} \qquad I = 1.14882\times10^{-6}\,\text{m}^4$$

At *B*, *E*, *G*, and *J*, $\tau = 0$ ◀

At *A* and *H*,

$$Q = (30)(6)(45) = 8.1\times10^{3}\,\text{mm}^3 = 8.1\times10^{-6}\,\text{m}^3$$

$$t = 6\times10^{-3}\,\text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(8.1\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 41.1\times10^{6}\,\text{Pa} \qquad \tau = 41.1\ \text{MPa}$$ ◀

Just above *D* and just below *F*:

$$Q = 8.1\times10^{3} + (6)(30)(30) = 13.5\times10^{3}\,\text{mm}^3 = 13.5\times10^{-6}\,\text{m}^3$$

$$t = 6\times10^{-3}\,\text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(13.5\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 68.5\times10^{6}\,\text{Pa} \qquad \tau = 68.5\ \text{MPa}$$ ◀

Just to right of *D* and just to the right of *F*:

$$Q = (30)(4)(15) = 1.8\times10^{3}\,\text{mm}^3 = 1.8\times10^{-6}\,\text{m}^3 \quad t = 4\times10^{-3}\,\text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(1.8\times10^{-6})}{(1.14882\times10^{-6})(4\times10^{-3})} = 13.71\times10^{6}\,\text{Pa} \qquad \tau = 13.71\ \text{MPa}$$ ◀

Just below *D* and just above *F*:

$$Q = 13.5\times10^{3} + 1.8\times10^{3} = 15.3\times10^{3}\,\text{mm}^3 = 15.3\times10^{-6}\,\text{m}^3$$

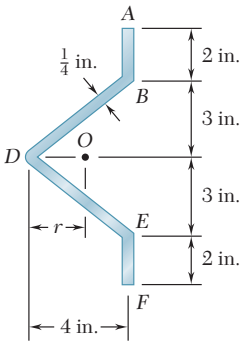
$$t = 6\times10^{-3}\,\text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(15.3\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 77.7\times10^{6}\,\text{Pa} \qquad \tau = 77.7\ \text{MPa}$$ ◀

At *K*, $Q = 15.3\times10^{3} + (6)(15)(7.5) = 15.975\times10^{3}\,\text{mm}^3 = 15.975\times10^{-6}\,\text{m}^3$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(15.975\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 81.1\times10^{6}\,\text{Pa} \qquad \tau = 81.1\ \text{MPa}$$ ◀

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PROBLEM 6.69

Determine the location of the shear center *O* of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

$$L_{DB} = \sqrt{4^2 + 3^2} = 5 \text{ in.} \quad A_{DB} = L_{DB}t = (5)\left(\frac{1}{4}\right) = 1.25 \text{ in}^2$$

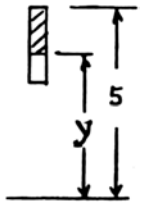
$$I_{DB} = \frac{1}{3}A_{AB}h^2 = \left(\frac{1}{3}\right)(1.25)(3)^2 = 3.75 \text{ in}^4$$

$$I_{AB} = \frac{1}{12}\left(\frac{1}{4}\right)(2)^3 + \left(\frac{1}{4}\right)(2)(4)^2 = 8.1667 \text{ in}^4$$

$$I = (2)(3.75) + (2)(8.1667) = 23.833 \text{ in}^4$$

Part *AB*:

$$A = \frac{1}{4}(5 - y) \text{ in}^2$$



$$\bar{y} = \frac{1}{2}(5 + y) \text{ in.}$$

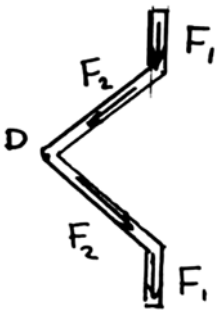
$$Q = A\bar{y} = \frac{1}{8}(5 - y)(5 + y) = \frac{1}{8}(25 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V(25 - y^2)}{(8)(23.833)(0.25)} = \frac{V(25 - y^2)}{47.667}$$

$$F_1 = \int \tau dA = \int_3^5 \frac{V(25 - y^2)}{(47.667)} \cdot \frac{1}{4} dy$$

$$= \frac{V}{190.667}\left[25y - \frac{1}{3}y^3\right]_3^5$$

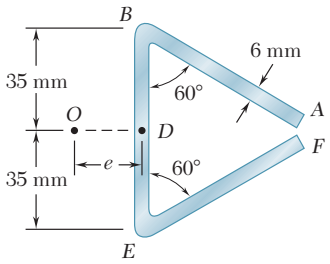
$$= \frac{V}{190.667}\left[(25)(5) - \frac{1}{3}(5)^3 - (25)(3) + \frac{1}{3}(3)^3\right] = 0.09091V$$



$$+\circlearrowright M_D = +\circlearrowright M_D: \quad -Ve = -2F_1(4) = -0.7273V$$

$$e = 0.727 \text{ in.} \blacktriangleleft$$

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PROBLEM 6.70

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

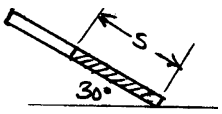
SOLUTION

$$I_{DB} = \frac{1}{3}(6)(35)^3 = 85.75 \times 10^3 \text{ mm}^4$$

$$L_{AB} = 70 \text{ mm} \qquad A_{AB} = (70)(6) = 420 \text{ mm}^2$$

$$I_{AB} = \frac{1}{3}A_{AB}h^2 = \left(\frac{1}{3}\right)(420)(35)^2 = 171.5 \times 10^3 \text{ mm}^4$$

$$I = (2)(85.75 \times 10^3) + (2)(171.5 \times 10^3) = 514.5 \times 10^3 \text{ mm}^4$$



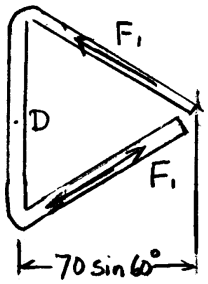
Part AB:

$$A = ts = 6s$$

$$\bar{y} = \frac{1}{2}s \sin 30° = \frac{1}{4}s$$

$$Q = A\bar{y} = \frac{3}{2}s^2$$

$$\tau = \frac{VQ}{It} = \frac{3Vs^2}{It}$$



$$F_1 = \int \tau \, dA = \int_0^{70} \frac{3Vs^2}{2It} t \, ds = \frac{3V}{I}\int_0^{70} s^2 ds$$

$$= \frac{(3)(70)^3}{(2)(3)I}V = \frac{1}{3}V$$

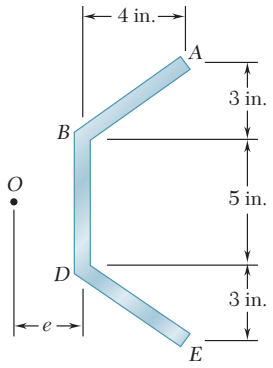
$$+\circlearrowright \Sigma M_D = +\circlearrowright \Sigma M_D: \quad Ve = 2[(F_1 \cos 60°)(70 \sin 60°)]$$

$$= 20.2V$$

Dividing by V,

$$e = 20.2 \text{ mm} \blacktriangleleft$$

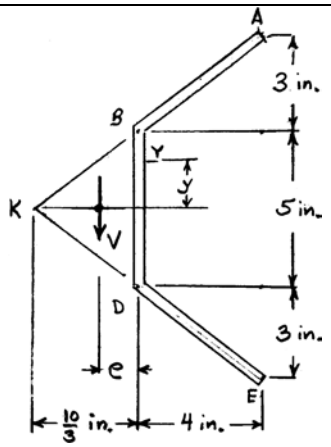
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PROBLEM 6.71

Determine the location of the shear center *O* of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION



$$L_{AB} = \sqrt{4^2 + 3^2} = 5 \text{ in.} \qquad A_{AB} = 5t$$

$$I_{AB} = \frac{1}{12} A_{AB} h^2 + A_{AB} d^2 = \frac{1}{12}(5t)(3)^2 + (5t)(4)^2 = 83.75t \text{ in}^4$$

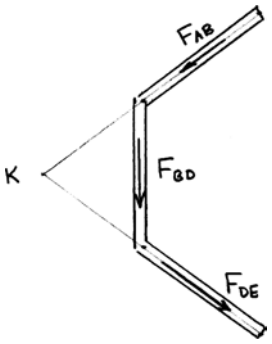
$$I_{BD} = \frac{1}{12}(t)(5)^3 = 10.417t \text{ in}^4$$

$$I = 2I_{AB} + I_{BD} = 177.917t \text{ in}^4$$

In part *BD*, $\quad Q = Q_{AB} + Q_{BY}$

$$Q = (5t)(4) + (2.5 - y)t\left(\frac{1}{2}\right)(2.5 + y)$$

$$= 20t + 3.125t - \frac{1}{2}ty^2 = \left(23.125 - \frac{1}{2}y^2\right)t$$



$$\tau = \frac{VQ}{It} \qquad F_{BD} = \int \tau \, dA = \int_{-2.5}^{2.5} \frac{V(23.125 - \frac{1}{2}y^2)t}{It} \cdot t \, dy$$

$$= \frac{Vt}{I}\int_{-2.5}^{2.5}\left(23.125 - \frac{1}{2}y^2\right)dy = \frac{Vt}{I}\left[23.125y - \frac{1}{6}y^3\right]_{-2.5}^{2.5}$$

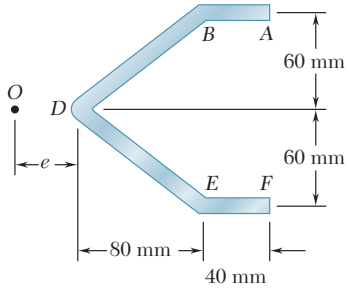
$$= \frac{Vt}{I} \cdot 2\left[(23.125)(2.5) - \frac{(2.5)^3}{6}\right] = \frac{Vt(110.417)}{177.917t} = 0.62061V$$

$$+\circlearrowright \Sigma M_K = +\circlearrowright \Sigma M_K: \quad -V\left(\frac{10}{3} - e\right) = -\frac{10}{3}(0.62061V)$$

$$e = \frac{10}{3}[1 - 0.62061] \qquad e = 1.265 \text{ in.} \blacktriangleleft$$

Note that the lines of action of F_{AB} and F_{DE} pass through point *K*. Thus, these forces have zero moment about point *K*.

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PROBLEM 6.72

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

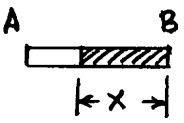
SOLUTION

$$I_{AB} = (40t)(60)^2 = 144\times10^3 t$$

$$L_{DB} = \sqrt{80^2 + 60^2} = 100 \text{ mm} \qquad A_{DB} = 100t$$

$$I_{DB} = \frac{1}{3}A_{DB}h^2 = \frac{1}{3}(100t)(60)^2 = 120\times10^3 t$$

$$I = 2I_{AB} + 2I_{DB} = 528\times10^3 t$$



Part AB:

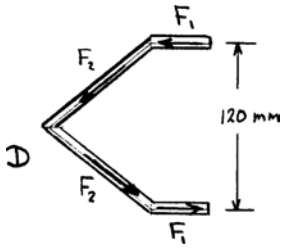
$$A = tx \quad \bar{y} = 60 \text{ mm}$$

$$Q = A\bar{y} = 60tx \text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{V(60tx)}{It} = \frac{60Vx}{I}$$

$$F_1 = \int \tau dA = \int_0^{40} \frac{60Vx}{I} t\,dx = \frac{60Vt}{I}\int_0^{40} x\,dx$$

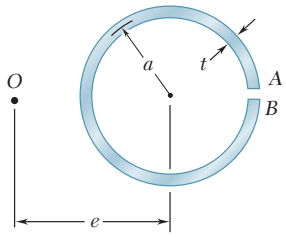
$$= \frac{60Vt}{I}\frac{x^2}{2}\bigg|_0^{30} = \frac{(60)(30)^2 Vt}{(2)(528\times10^3)t} = 0.051136V$$



$$+\circlearrowleft \Sigma M_D = +\circlearrowleft \Sigma M_D: \quad Ve = (0.051136V)(120)$$

$$e = 6.14 \text{ mm} \blacktriangleleft$$

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PROBLEM 6.73

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

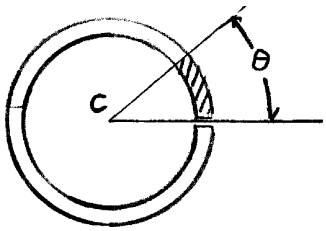
For whole cross section, $A = 2\pi at$

$$J = Aa^2 = 2\pi a^3 t \qquad I = \frac{1}{2}J = \pi a^3 t$$

Use polar coordinate θ for partial cross section.

$A = st = a\theta t$ $\quad$ s = arc length

$$\bar{r} = a\frac{\sin\alpha}{\alpha} \quad \text{where} \quad \alpha = \frac{1}{2}\theta$$



$$\bar{y} = \bar{r}\,\sin\alpha = a\frac{\sin^2\alpha}{\alpha}$$

$$Q = A\bar{y} = a\theta t\; a\frac{\sin^2\alpha}{\alpha} = a^2 t\;\; 2\sin^2\alpha$$

$$= a^2 t\;\; 2\sin^2\frac{\theta}{2} = a^2 t(1-\cos\theta)$$

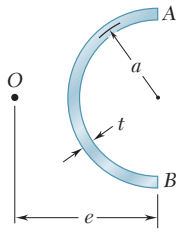
$$\tau = \frac{VQ}{It} = \frac{Va^2}{I}(1-\cos\theta)$$

$$M_C = \int a\tau\, dA = \int_0^{2\pi}\frac{Va^3}{I}(1-\cos\theta)\,tad\theta = \frac{Va^4 t}{I}(\theta-\sin\theta)\Big|_0^{2\pi}$$

$$= \frac{2\pi Va^4 t}{\pi a^3 t} = 2aV$$

But $M_C = Ve$, hence $e = 2a$ ◀

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PROBLEM 6.74

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

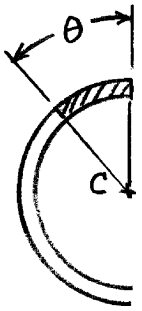
SOLUTION

For a thin-walled hollow circular cross section, $A = 2\pi at$

$$J = a^2A = 2\pi a^3 t \qquad I = \frac{1}{2}J = \pi a^3 t$$

For the half-pipe section, $I = \frac{\pi}{2}a^3 t$

Use polar coordinate θ for partial cross section.



$A = st = a\theta t$ $\qquad s = \text{arc length}$

$\bar{r} = a\dfrac{\sin\alpha}{\alpha}$ where $\alpha = \dfrac{\theta}{2}$

$\bar{y} = \bar{r}\cos\alpha = a\dfrac{\sin\alpha\cos\alpha}{\alpha}$

$Q = A\bar{y} = a\theta t\; a\dfrac{\sin\alpha\cos\alpha}{\alpha} = a^2 t\;(2\sin\alpha\cos\alpha)$

$= a^2 t\;\sin 2\alpha = a^2 t\;\sin\theta$

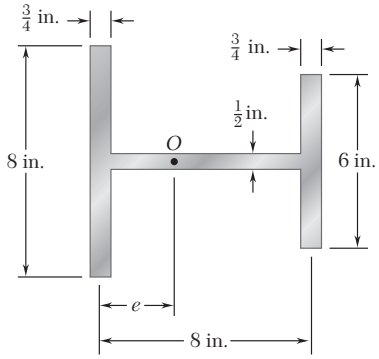
$\tau = \dfrac{VQ}{It} = \dfrac{Va^2}{I}\sin\theta$

$M_H = \int a\,\tau\,dA = \int_0^{\pi} a\,\dfrac{Va^2}{I}\sin\theta\; ta\,d\theta = \dfrac{Va^4 t}{I} - \cos\theta\Big|_0^{\pi}$

$= 2\dfrac{Va^4 t}{I} = \dfrac{4}{\pi}Va$

But $M_H = Ve$, hence $\qquad e = \dfrac{4}{\pi}a = 1.273a$ ◀

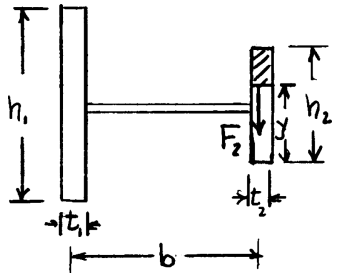
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PROBLEM 6.75

A thin-walled beam has the cross section shown. Determine the location of the shear center *O* of the cross section.

SOLUTION



$$I = \frac{1}{12}t_1h_1^3 + \frac{1}{12}t_2h_2^3$$

Right flange:

$$A = \left(\frac{1}{2}h_2 - y\right)t_2$$

$$\bar{y} = \frac{1}{2}\left(\frac{1}{2}h_2 + y\right)t_2$$

$$Q = A\bar{y}$$

$$= \frac{1}{2}\left(\frac{1}{2}h_2 - y\right)\left(\frac{1}{2}h_2 + y\right)t_2$$

$$= \frac{1}{2}\left(\frac{1}{4}h_2^2 - y^2\right)t_2$$

$$\tau = \frac{VQ}{It_2} = \frac{V}{2It_2}\left(\frac{1}{4}h_2^2 - y^2\right)t_2$$

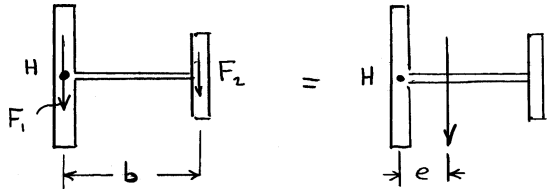
$$F_2 = \int \tau\, dA = \int_{-h_2/2}^{h_2/2} \frac{Vt_2}{2I\cancel{t_2}}\left(\frac{1}{4}h_2^2 - y^2\right)\cancel{t_2}\, dy = \frac{Vt_2}{2I}\left(\frac{1}{4}h_2^2 y - \frac{y^3}{3}\right)\Bigg|_{-h_2/2}^{h_2/2}$$

$$= \frac{Vt_2}{2I}\left\{\frac{1}{4}h_2^2\frac{h_2}{2} - \frac{1}{3}\left(\frac{h_2}{2}\right)^3 + \frac{1}{4}h_2^2\frac{h_2}{2} - \frac{1}{3}\left(\frac{h_2}{2}\right)^3\right\} = \frac{Vt_2h_2^3}{12I} = \frac{Vt_2h_2^3}{t_1h_1^3 + t_2h_2^3}$$

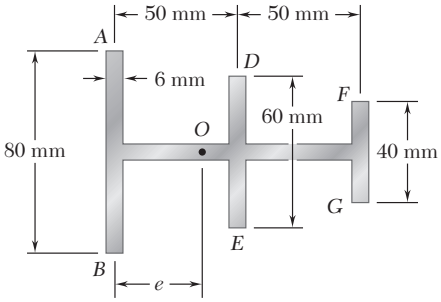
$$+\circlearrowright M_H = +\circlearrowright M_H: \quad -Ve = -F_2b = -V\frac{t_2h_2^3b}{t_1h_1^3 + t_2h_2^3}$$

$$e = \frac{t_2h_2^3b}{t_1h_1^3 + t_2h_2^3} = \frac{(0.75)(6)^3(8)}{(0.75)(8)^3 + (0.75)(6)^3}$$

$$e = 2.37 \text{ in.} \blacktriangleleft$$



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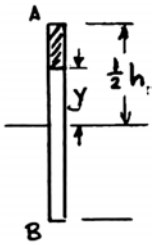
PROBLEM 6.76

A thin-walled beam has the cross section shown. Determine the location of the shear center *O* of the cross section.

SOLUTION

Let $h_1 = \overline{AB} = h, \quad h_2 = \overline{DE}, \quad h_3 = \overline{FG}$

$$I = \frac{1}{12}t\left(h_1^3 + h_2^3 + h_3^3\right)$$



Part *AB*:

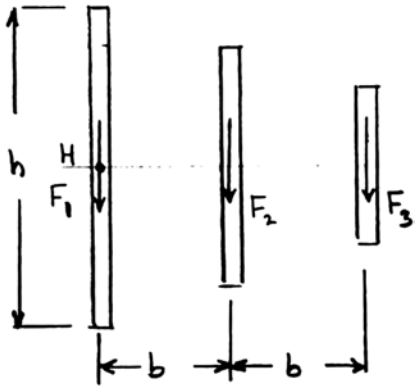
$$A = \left(\frac{1}{2}h_1 - y\right)t$$

$$\bar{y} = \frac{1}{2}\left(\frac{1}{2}h_1 + y\right)$$

$$Q = A\bar{y} = \frac{1}{2}t\left(\frac{1}{2}h_1 - y\right)\left(\frac{1}{2}h_1 + y\right)$$

$$= \frac{1}{2}t\left(\frac{1}{4}h_1^2 - y^2\right)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I}\left(\frac{1}{4}h_1^2 - y^2\right)$$



$$F_1 = \int \tau\, dA = \int_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1} \frac{V}{2I}\left(\frac{1}{4}h_1^2 - y^2\right)t\, dy$$

$$= \frac{Vt}{2I}\left(\frac{1}{4}h_1^2 y - \frac{y^3}{3}\right)\Bigg|_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1}$$

$$= \frac{Vt}{I}\left(\frac{1}{4}h_1^2\frac{1}{2}h_1 - \frac{1}{3}\left(\frac{h_1}{2}\right)^3\right) = \frac{Vth_1^3}{12I}$$

$$= \frac{h_1^3 V}{h_1^3 + h_2^3 + h_3^3}$$

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PROBLEM 6.76 *(Continued)*

Likewise, for Part *DE*,
$$F_2 = \frac{h_2^3 V}{h_1^3 + h_2^3 + h_3^3}$$

and for Part *FG*,
$$F_3 = \frac{h_3^3 V}{h_1^3 + h_2^3 + h_3^3}$$

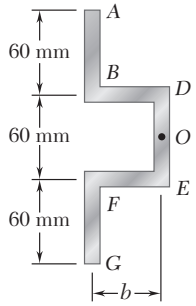
$$+\circlearrowleft \Sigma M_H = +\circlearrowleft \Sigma M_H: \quad Ve = bF_2 + 2bF_3 = \frac{bh_2^3 + 2bh_3^3}{h_1^3 + h_2^3 + h_3^3} V$$

$$e = \frac{h_2^3 + 2h_3^3}{h_1^3 + h_2^3 + h_3^3} b = \frac{(60)^3 + (2)(40)^3}{(80)^3 + (60)^3 + (40)^3} (50)$$

$$= 21.7 \text{ mm}$$

$e = 21.7$ mm ◀

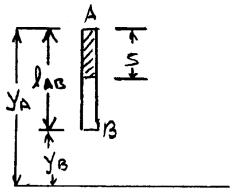
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PROBLEM 6.77

A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.

SOLUTION



Part AB:

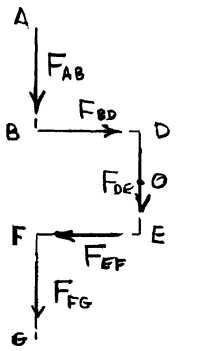
$$A(s) = ts \qquad \bar{y}(s) = y_A - \frac{1}{2}s$$

$$Q(s) = A(s)\bar{y}(s) = ty_A s - \frac{1}{2}ts^2$$

$$q(s) = \frac{VQ(s)}{I} = \frac{Vt}{I}\left(y_A s - \frac{1}{2}s^2\right)$$

$$F_{AB} = \int_0^{l_{AB}} q(s)\,ds$$

$$= \frac{Vt}{I}\left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6}\right)\downarrow$$



At B,

$$Q_B = ty_A l_{AB} - \frac{1}{2}tl_{AB}^2$$

By symmetry,

$$F_{FG} = F_{AB}$$

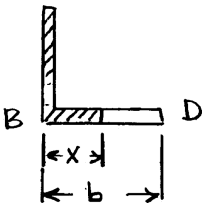
Part BD:

$$A(x) = tx$$

$$Q(x) = Q_B + y_B A(x) = Q_B + ty_B x$$

$$q(x) = \frac{VQ(x)}{I} = \frac{V}{I}(Q_B + ty_B x)$$

$$F_{BD} = \int_0^b q(x)\,dx = \frac{V}{I}\left(Q_B b + \frac{1}{2}ty_B b^2\right)\rightarrow$$



By symmetry,

$$F_{EF} = F_{BD}$$

F_{DE} is not required, since its moment about O is zero.

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PROBLEM 6.77 *(Continued)*

$$\Sigma M_O = 0: \quad b(F_{AB} + F_{FG}) - y_B F_{BD} + y_F F_{EF} = 0$$

$$2b\,F_{AB} - 2y_B F_{BD} = 0$$

$$2b \cdot \frac{Vt}{I}\left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6}\right) - 2y_B \frac{V}{I}\left(Q_B b + \frac{1}{2} t y_B b^2\right) = 0$$

$$\frac{2Vt}{I}\left\{\frac{1}{2} y_A l_{AB}^2 - \frac{1}{6} l_{AB}^3\right\} b - \frac{2Vt}{I}\left\{\left(y_A l_{AB} - \frac{1}{2} l_{AB}^2\right) y_B b - \frac{1}{2} y_B^2 b^2 = 0\right.$$

Dividing by $\frac{2Vt}{I}$ and substituting numerical data,

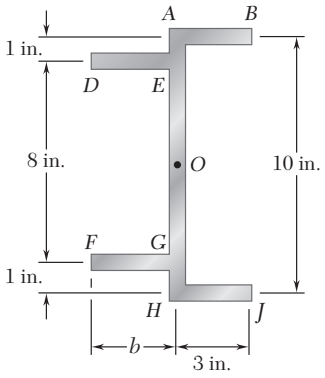
$$\left\{\frac{1}{2}(90)(60)^2 - \frac{1}{6}(60)^3\right\} b - \left\{(90)(60) - \frac{1}{2}(60)^2\right\}(30)b + \frac{1}{2}(30)^2 b^2 = 0$$

$$126 \times 10^3 b - 108 \times 10^3 b + 450 b^2 = 0$$

$$18 \times 10^3 b - 450 b^2 = 0$$

$b = 0$ and $b = 40.0$ mm ◀

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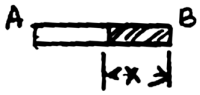


PROBLEM 6.78

A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.

SOLUTION

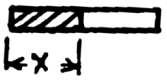
Part AB: $A = tx \qquad \bar{y} = 5 \text{ in.} \qquad Q = A\bar{y} = 5tx$



$$\tau = \frac{VQ}{It} = \frac{V.5\cancel{t}x}{I\cancel{t}} = \frac{5Vx}{I}$$

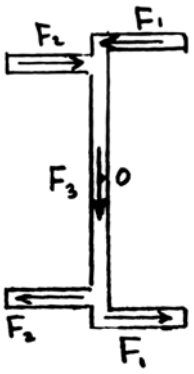
$$F_1 = \int \tau dA = \int_0^3 \frac{5Vx}{I} t\,dx = \frac{5Vt}{I}\int_0^3 x\,dx$$

$$= \frac{(5)(3)^2}{2}\frac{Vt}{I} = 22.5\frac{Vt}{I}$$



Part DE: $A = tx \qquad \bar{y} = 4 \text{ in.} \qquad Q = A\bar{y} = 4tx$

$$\tau = \frac{VQ}{It} = \frac{V4tx}{It} = \frac{4Vx}{I}$$



$$F_2 = \int \tau dA = \int_0^a \frac{4Vx}{I} t\,dx$$

$$= \frac{4Vt}{I}\int_0^a x\,dx$$

$$= \frac{2Vta^2}{I}$$

$$+\circlearrowright \Sigma M_O = +\circlearrowright \Sigma M_O: \quad O = (10)\left(22.5\frac{Vt}{I}\right) - (8)\frac{2Vta^2}{I}$$

$$a^2 = \frac{(10)(22.5)}{(8)(2)} = 14.0625 \text{ in}^2$$

$$a = 3.75 \text{ in.} \blacktriangleleft$$

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PROBLEM 6.79

For the angle shape and loading of Sample Prob. 6.6, check that $\int q\,dz = 0$ along the horizontal leg of the angle and $\int q\,dy = P$ along its vertical leg.

SOLUTION

Refer to Sample Prob. 6.6.

Along horizontal leg:

$$\tau_f = \frac{3P(a-z)(a-3z)}{4ta^3} = \frac{3P}{4ta^3}(a^2 - 4az + 3z^2)$$

$$\int q\,dz = \int_0^a \tau_f\, t\,dz = \frac{3P}{4a^3}\int_0^a (a^2 - 4az + 3z^2)dz$$

$$= \frac{3P}{4a^3}\left(a^2 z - 4a\frac{z^2}{2} + 3\frac{z^2}{3}\right)\Bigg|_0^a$$

$$= \frac{3P}{4a^3}(a^3 - 2a^3 + a^3) = 0 \quad \blacktriangleleft$$

Along vertical leg:

$$\tau_e = \frac{3P(a-y)(a+5y)}{4ta^3} = \frac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$$

$$\int q\,dy = \int_0^a \tau_e t\,dy = \frac{3P}{4a^3}\int_0^a (a^2 + 4ay - 5y^2)dy$$

$$= \frac{3P}{4a^3}\left(a^2 y + 4a\frac{y^2}{2} - 5\frac{y^3}{3}\right)\Bigg|_0^a$$

$$= \frac{3P}{4a^3}\left(a^3 + 2a^3 - \frac{5}{3}a^3\right) = \frac{3P}{4a^3}\cdot\frac{4}{3}a^3 = P \quad \blacktriangleleft$$

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PROBLEM 6.80

For the angle shape and loading of Sample Prob. 6.6, (*a*) determine the points where the shearing stress is maximum and the corresponding values of the stress, (*b*) verify that the points obtained are located on the neutral axis corresponding to the given loading.

SOLUTION

Refer to Sample Prob. 6.6.

(*a*) Along vertical leg: $\tau_e = \dfrac{3P(a-y)(a+5y)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2+4ay-5y^2)$

$$\frac{d\tau_e}{dy} = \frac{3P}{4ta^3}(4a-10y) = 0 \qquad y = \frac{2}{5}a \blacktriangleleft$$

$$\tau_m = \frac{3P}{4ta^3}\left[a^2+(4a)\left(\frac{2}{5}a\right)-(5)\left(\frac{2}{5}a\right)^2\right] = \frac{3P}{4ta^3}\left(\frac{9}{5}a^2\right) \qquad \tau_m = \frac{27}{20}\frac{P}{ta} \blacktriangleleft$$

Along horizontal leg: $\tau_f = \dfrac{3P(a-z)(a-3z)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2-4az+3z^2)$

$$\frac{d\tau_f}{dz} = \frac{3P}{4ta^3}(-4a+6z) = 0 \qquad z = \frac{2}{3}a \blacktriangleleft$$

$$\tau_{\text{m}} = \frac{3P}{4ta^3}\left[a^2-(4a)\left(\frac{2}{3}a\right)+(3)\left(\frac{2}{3}a\right)^2\right] = \frac{3P}{4ta^3}\left(-\frac{5}{3}a^2\right) \qquad \tau_{\text{m}} = -\frac{1}{4}\frac{P}{ta} \blacktriangleleft$$

At the corner: $y = 0, \quad z = 0,$ $\qquad \tau = \dfrac{3}{4}\dfrac{P}{ta} \blacktriangleleft$

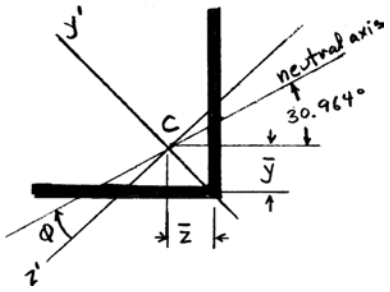
(*b*) $I_{y'} = \dfrac{1}{3}ta^3 \quad I_{z'} = \dfrac{1}{12}ta^3 \quad \theta = 45°$

$$\tan\varphi = \frac{I_{z'}}{I_{y'}}\tan\theta = \frac{1}{4} \quad \varphi = 14.036°$$

$$\theta - \varphi = 45 - 14.036 = 30.964°$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{at\,(a/2)}{2at} = \frac{1}{4}a$$

$$\bar{z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{at\,(a/2)}{2at} = \frac{1}{4}a$$



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PROBLEM 6.80 ***(Continued)***

Neutral axis intersects vertical leg at

$$y = \bar{y} + \bar{z} \tan 30.964°$$

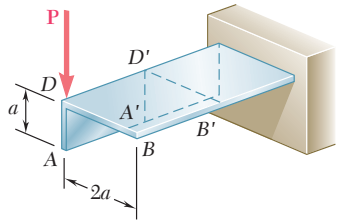
$$= \left(\frac{1}{4} + \frac{1}{4} \tan 30.964° \right) a = 0.400a \qquad y = \frac{2}{5} a \blacktriangleleft$$

Neutral axis intersects horizontal leg at

$$z = \bar{z} + \bar{y} \tan (45° + \varphi)$$

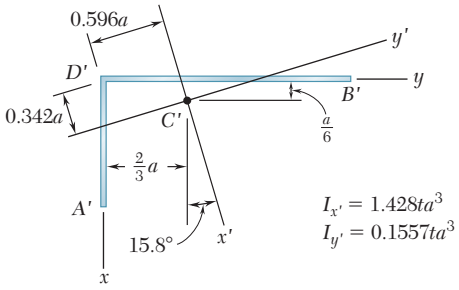
$$= \left(\frac{1}{4} + \frac{1}{4} \tan 59.036° \right) a = 0.667a \qquad z = \frac{2}{3} a \blacktriangleleft$$

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PROBLEM 6.81*

Determine the distribution of the shearing stresses along line $D'B'$ in the horizontal leg of the angle shape for the loading shown. The x' and y' axes are the principal centroidal axes of the cross section.



SOLUTION

$$\beta = 15.8^\circ \qquad V_x' = P\cos\beta \qquad V_y' = -P\sin\beta$$

$$A(y) = (2a - y)t \qquad \bar{y} = \frac{1}{2}(2a + y), \qquad \bar{x} = 0$$

Coordinate transformation.

$$y' = \left(y - \frac{2}{3}a\right)\cos\beta - \left(x - \frac{1}{6}a\right)\sin\beta$$

$$x' = \left(x - \frac{1}{6}a\right)\cos\beta + \left(y - \frac{2}{3}a\right)\sin\beta$$

In particular,

$$\bar{y}' = \left(\bar{y} - \frac{2}{3}a\right)\cos\beta - \left(\bar{x} - \frac{1}{6}a\right)\sin\beta$$

$$= \left(\frac{1}{2}y + \frac{1}{3}a\right)\cos\beta - \left(-\frac{1}{6}a\right)\sin\beta$$

$$= 0.48111y + 0.36612a$$

$$\bar{x}' = \left(\bar{x} - \frac{1}{6}a\right)\cos\beta + \left(\bar{y} - \frac{2}{3}a\right)\sin\beta$$

$$= \left(-\frac{1}{6}a\right)\cos\beta + \left(\frac{1}{2}y + \frac{1}{3}a\right)\sin\beta$$

$$= 0.13614y - 0.06961a$$

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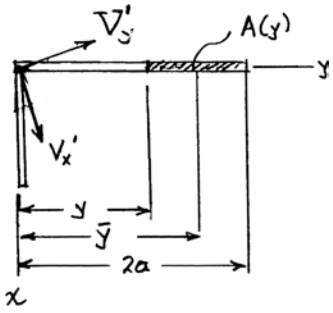
PROBLEM 6.81* *(Continued)*

$$+\rightarrow \quad \tau = \frac{V_{x'}A\bar{x}'}{I_{y'}t} + \frac{V_{y'}A\bar{y}'}{I_{x'}t}$$

$$= \frac{(P\cos\beta)(2a-y)(t)(0.13614y - 0.06961a)}{(0.1557ta^3)(t)}$$

$$+ \frac{(-P\sin\beta)(2a-y)(0.48111y + 0.36612a)}{(1.428a^3t)(t)}$$

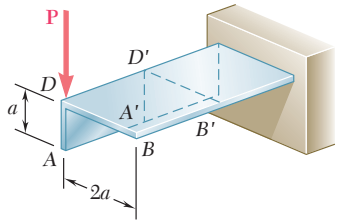
$$= \frac{P(2a-y)(0.750y - 0.500a)}{ta^3}$$



$y(a)$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
$-\tau\left(\frac{P}{at}\right)$	–1.000	–0.417	0	0.250	0.333	0.250	0

◀

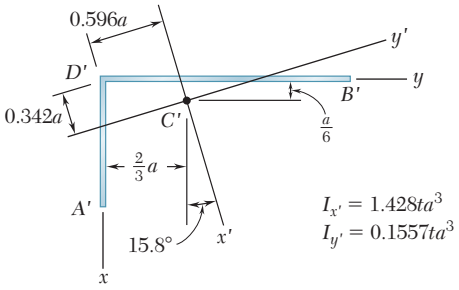
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PROBLEM 6.82*

For the angle shape and loading of Prob. 6.81, determine the distribution of the shearing stresses along line $D'A'$ in the vertical leg.

PROBLEM 6.81* Determine the distribution of the shearing stresses along line $D'B'$ in the horizontal leg of the angle shape for the loading shown. The x' and y' axes are the principal centroidal axes of the cross section.



SOLUTION

$$\beta = 15.8^\circ \qquad V_{x'} = P\cos\beta$$
$$V_{x'} = -P\sin\beta \qquad A(x) = (a - x)t$$
$$\bar{x} = \frac{1}{2}(a + x), \quad \bar{y} = 0$$

Coordinate transformation.

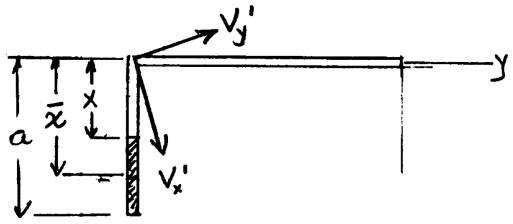
$$y' = \left(y - \frac{2}{3}a\right)\cos\beta - \left(x - \frac{1}{6}a\right)\sin\beta$$
$$x' = \left(x - \frac{1}{6}a\right)\cos\beta + \left(y - \frac{2}{3}a\right)\sin\beta$$

In particular,

$$\bar{y}' = \left(\bar{y} - \frac{2}{3}a\right)\cos\beta - \left(\bar{x} - \frac{1}{6}a\right)\sin\beta$$
$$= \left(-\frac{2}{3}a\right)\cos\beta - \left(\frac{1}{2}x + \frac{1}{3}a\right)\sin\beta$$
$$= -0.13614x - 0.73224a$$
$$\bar{x}' = \left(\bar{x} - \frac{1}{6}a\right)\cos\beta + \left(\bar{y} - \frac{2}{3}a\right)\sin\beta$$
$$= \left(\frac{1}{2}x + \frac{1}{3}a\right)\cos\beta + \left(-\frac{2}{3}a\right)\sin\beta$$
$$= 0.48111x + 0.13922a$$

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PROBLEM 6.82* (*Continued*)



$$+\downarrow \tau = \frac{V_{x'}A(x)\bar{x}'}{I'_y t} - \frac{V_{y'}A(x)\bar{y}'}{I_{x'}t}$$

$$= \frac{(P\cos\beta)(a-x)(t)(0.48111x + 0.13922a)}{(0.1557ta^3)(t)}$$

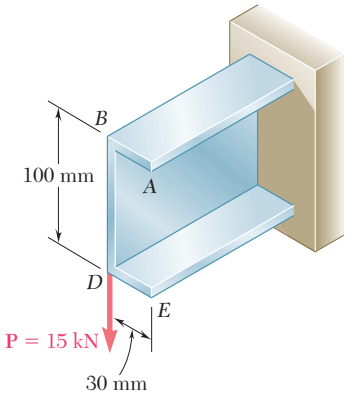
$$+ \frac{(-P\sin\beta)(a-x)(t)(-0.13614x - 0.73224a)}{(1.428ta^3)(t)}$$

$$= \frac{P(a-x)(3.00x + 1.000a)}{ta^3}$$

$x(a)$	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
$\tau \downarrow \left(\frac{P}{at}\right)$	1.000	1.250	1.333	1.250	1.000	0.583	0

◀

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PROBLEM 6.83*

A steel plate, 160 mm wide and 8 mm thick, is bent to form the channel shown. Knowing that the vertical load **P** acts at a point in the midplane of the web of the channel, determine (*a*) the torque **T** that would cause the channel to twist in the same way that it does under the load **P,** (*b*) the maximum shearing stress in the channel caused by the load **P.**

SOLUTION

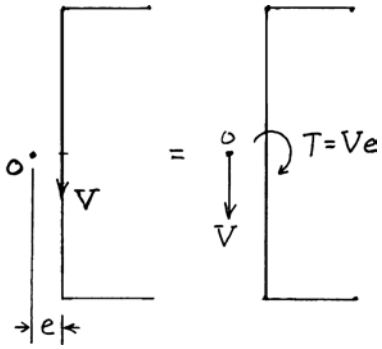
Use results of Example 6.06 with $b = 30$ mm, $h = 100$ mm, and $t = 8$ mm.

$$e = \frac{b}{2 + \frac{h}{3b}} = \frac{30}{2 + \frac{100}{(3)(30)}} = 9.6429 \text{ mm} = 9.6429 \times 10^{-3} \text{m}$$

$$I = \frac{1}{12} th^2(6b + h) = \frac{1}{12}(8)(100)^2[(16)(30) + 100] = 1.86667 \times 10^6 \text{mm}^4 = 1.86667 \times 10^{-6} \text{m}^4$$

$$V = 15 \times 10^3 \text{N}$$

(*a*) $T = Ve = (15 \times 10^3)(9.6429 \times 10^{-3})$ $\qquad T = 144.6 \text{ N} \cdot \text{m}$ ◀



Stress at neutral axis due to *V*:

$$Q = bt\frac{h}{2} + t\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) = \frac{1}{8} th(h + 4b)$$

$$= \frac{1}{8}(8)(100)\left[100 + (4)(30)\right] = 22 \times 10^3 \text{mm}^3 = 22 \times 10^{-6} \text{m}^3$$

$$t = 8 \times 10^{-3} \text{m}$$

$$\tau_V = \frac{VQ}{It} = \frac{(15 \times 10^3)(22 \times 10^{-6})}{(1.86667 \times 10^{-6})(8 \times 10^{-3})} = 22.10 \times 10^6 \text{Pa} = 22.10 \text{ MPa}$$

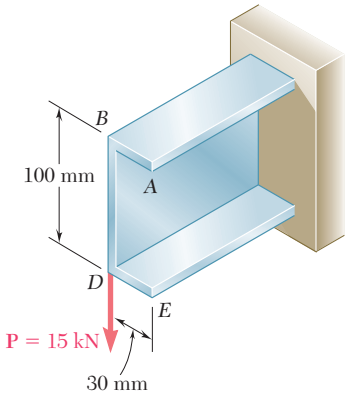
Stress due to *T*: $a = 2b + h = 160 \text{ mm} = 0.160 \text{ m}$

$$c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{a}\right) = \frac{1}{3}\left[1 - (0.630)\frac{8}{160}\right] = 0.3228$$

$$\tau_V = \frac{T}{c_1 a t^2} = \frac{144.64}{(0.3228)(0.160)(8 \times 10^{-3})^2} = 43.76 \times 10^6 \text{ Pa} = 43.76 \text{ MPa}$$

(*b*) By superposition, $\tau_{\max} = \tau_V + \tau_T$ $\qquad \tau_{\max} = 65.9 \text{ MPa}$ ◀

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PROBLEM 6.84*

Solve Prob. 6.83, assuming that a 6-mm-thick plate is bent to form the channel shown.

PROBLEM 6.83* A steel plate, 160 mm wide and 8 mm thick, is bent to form the channel shown. Knowing that the vertical load **P** acts at a point in the midplane of the web of the channel, determine (*a*) the torque **T** that would cause the channel to twist in the same way that it does under the load **P,** (*b*) the maximum shearing stress in the channel caused by the load **P.**

SOLUTION

Use results of Example 6.06 with $b = 30$ mm, $h = 100$ mm, and $t = 6$ mm.

$$e = \frac{b}{2 + \frac{h}{3b}} = \frac{30}{2 + \frac{100}{(3)(30)}} = 9.6429 \text{ mm} = 9.6429 \times 10^{-3} \text{m}$$

$$I = \frac{1}{12} th^2(6b + h) = \frac{1}{12}(6)(100)^2[(6)(30) + 100] = 1.400 \times 10^6 \text{mm}^4 = 1.400 \times 10^{-6} \text{m}^4$$

$$V = 15 \times 10^3 \text{N}$$

(*a*) $T = Ve = (15 \times 10^3)(9.6429 \times 10^{-3})$ $\qquad T = 144.6 \text{ N} \cdot \text{m}$ ◀

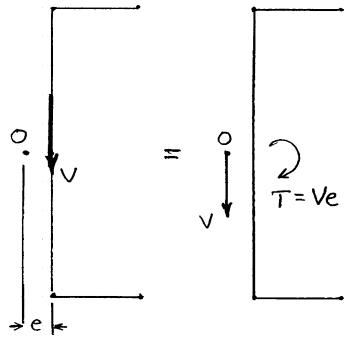
Stress at neutral axis due to *V*:

$$Q = bt\frac{h}{2} + t\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) = \frac{1}{8} th(h + 4b)$$

$$= \frac{1}{8}(6)(100)\left[100 + (4)(30)\right] = 16.5 \times 10^3 \text{mm}^3 = 16.5 \times 10^{-6} \text{m}^3$$

$$t = 6 \times 10^{-3} \text{m}$$

$$\tau_V = \frac{VQ}{It} = \frac{(15 \times 10^3)(16.5 \times 10^{-6})}{(1.400 \times 10^{-6})(6 \times 10^{-6})} = 29.46 \times 10^6 \text{Pa} = 29.46 \text{ MPa}$$



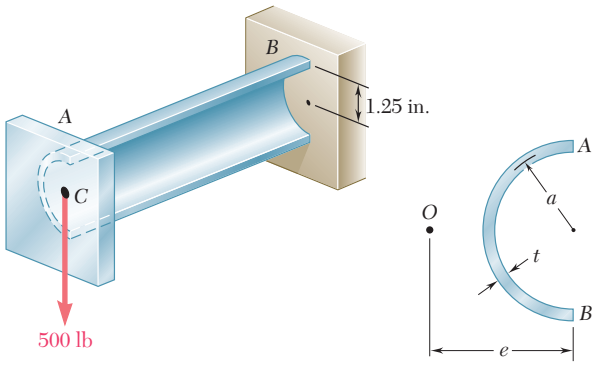
Stress due to *T*: $a = 2b + h = 160 \text{ mm} = 0.160 \text{ m}$

$$c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{a}\right) = \frac{1}{3}\left[1 - (0.630)\left(\frac{6}{160}\right)\right] = 0.32546$$

$$\tau_V = \frac{T}{c_1 a\, t^2} = \frac{144.64}{(0.32546)(0.160)(6 \times 10^{-3})^2} = 77.16 \times 10^6 \text{Pa} = 77.16 \text{ MPa}$$

(*b*) By superposition, $\qquad \tau_{\max} = \tau_V + \tau_T$ $\qquad \tau_{\max} = 106.6 \text{ MPa}$ ◀

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PROBLEM 6.85*

The cantilever beam *AB*, consisting of half of a thin-walled pipe of 1.25-in. mean radius and $\frac{3}{8}$-in. wall thickness, is subjected to a 500-lb vertical load. Knowing that the line of action of the load passes through the centroid *C* of the cross section of the beam, determine (*a*) the equivalent force-couple system at the shear center of the cross section, (*b*) the maximum shearing stress in the beam. (*Hint:* The shear center *O* of this cross section was shown in Prob. 6.74 to be located twice as far from its vertical diameter as its centroid *C*.)

SOLUTION

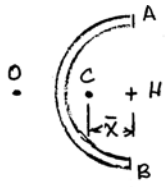
From the solution to Prob. 6.74,

$$I = \frac{\pi}{2}a^3 t \qquad Q = a^2 t \sin\theta$$

$$e = \frac{4}{\pi}a \qquad Q_{\max} = a^2 t$$

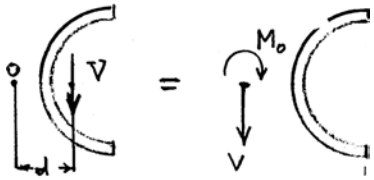
For a half-pipe section, the distance from the center of the semi-circle to the centroid is

$$\bar{x} = \frac{2}{\pi}a$$



At each section of the beam, the shearing force *V* is equal to P. Its line of action passes through the centroid *C*. The moment arm of its moment about the shear center *O* is

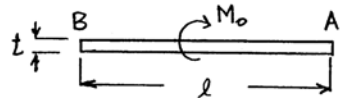
$$d = e - \bar{x} = \frac{4}{\pi}a - \frac{2}{\pi}a = \frac{2}{\pi}a$$



(*a*) Equivalent force-couple system at *O*.

$$V = P \qquad M_O = Vd = \frac{2}{\pi}\text{Pa}$$

Data: $P = 500$ lb $\quad a = 1.25$ in.



$$V = 500 \text{ lb} \blacktriangleleft$$

$$M_O = \left(\frac{2}{\pi}\right)(500)(1.25) \qquad M_O = 398 \text{ lb}\cdot\text{in.} \blacktriangleleft$$

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PROBLEM 6.85* *(Continued)*

(*b*) Shearing stresses.

(1) Due to V: $\tau_V = \dfrac{VQ_{\max}}{It}$

$$\tau_V = \frac{(P)(a^2t)}{\left(\frac{\pi}{2}a^3t\right)(t)} = \frac{2P}{\pi at} = \frac{(2)(500)}{\pi(1.25)(0.375)} = 679 \text{ psi}$$

(2) Due to the torque M_O:

For a long rectangular section of length l and width t, the shearing stress due to torque M_O is

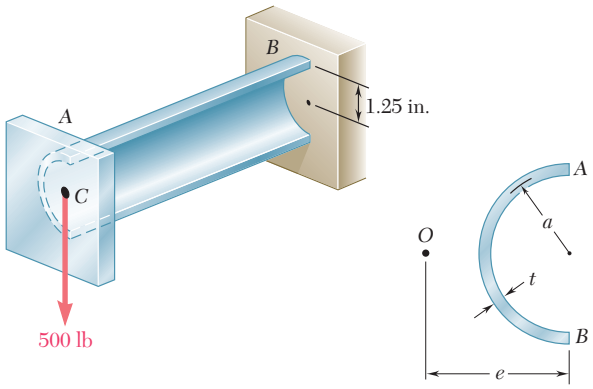
$$\tau_M = \frac{M_O}{c_1 l t^2} \quad \text{where} \quad c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{l}\right)$$

Data: $l = \pi a = \pi(1.25) = 3.927$ in. $t = 0.375$ in. $c_1 = 0.31328$

$$\tau_M = \frac{397.9}{(0.31328)(3.927)(0.375)^2} = 2300 \text{ psi}$$

By superposition, $\tau = \tau_V + \tau_M = 679 \text{ psi} + 2300 \text{ psi}$ $\tau = 2980$ psi ◀

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PROBLEM 6.86*

Solve Prob. 6.85, assuming that the thickness of the beam is reduced to $\frac{1}{4}$ in.

PROBLEM 6.85* The cantilever beam *AB*, consisting of half of a thin-walled pipe of 1.25-in. mean radius and $\frac{3}{8}$-in. wall thickness, is subjected to a 500-lb vertical load. Knowing that the line of action of the load passes through the centroid *C* of the cross section of the beam, determine (*a*) the equivalent force-couple system at the shear center of the cross section, (*b*) the maximum shearing stress in the beam. (*Hint:* The shear center *O* of this cross section was shown in Prob. 6.74 to be located twice as far from its vertical diameter as its centroid *C*.)

SOLUTION

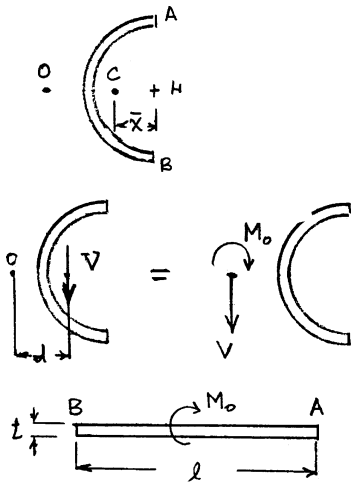
From the solution to Prob. 6.74,

$$I = \pi a^3 t \qquad Q = a^2 t \sin\theta$$

$$e = \frac{4}{\pi} a \qquad Q_{\max} = a^2 t$$

For a half-pipe section, the distance from the center of the semi-circle to the centroid is

$$\bar{x} = \frac{2}{\pi} a$$



At each section of the beam, the shearing force *V* is equal to *P*. Its line of action passes through the centroid *C*. The moment arm of its moment about the shear center *O* is

$$d = e - \bar{x} = \frac{4}{\pi} a - \frac{2}{\pi} a = \frac{2}{\pi} a$$

(*a*) Equivalent force-couple system at *O*.

$$V = P \qquad M_O = Vd = \frac{2}{\pi} Pa$$

Data: $P = 500$ lb $\quad a = 1.25$ in.

$$V = 500 \text{ lb} \blacktriangleleft$$

$$M_O = 398 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

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PROBLEM 6.86* *(Continued)*

(*b*) Shearing stresses.

(1) Due to V, $\tau_V = \dfrac{VQ_{\max}}{It}$

$$\tau_V = \frac{(P)(a^2 t)}{\left(\frac{\pi}{2}a^3 t(t)\right)} = \frac{2P}{\pi a t} = \frac{(2)(500)}{\pi(1.25)(0.250)} = 1019 \text{ psi}$$

(2) Due to the torque M_O:

For a long rectangular section of length l and width t, the shearing stress due to torque M_O is

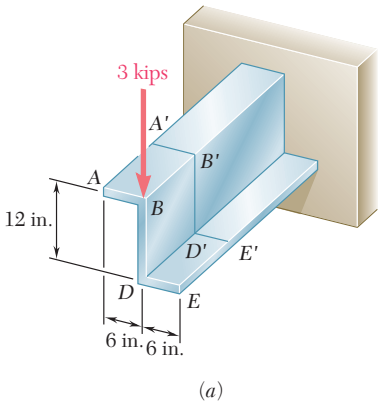
$$\tau_M = \frac{M_O}{c_1 l t^2} \quad \text{where} \quad c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{l}\right)$$

Data: $l = \pi a = \pi(1.25) = 3.927$ in. $t = 0.250$ in. $c_1 = 0.31996$

$$\tau_M = \frac{397.9}{(0.31996)(3.927)(0.250)^2} = 5067 \text{ psi}$$

By superposition, $\tau = \tau_V + \tau_M = 1019 \text{ psi} + 5067 \text{ psi}$ $\tau = 6090$ psi ◀

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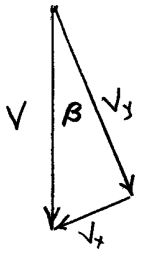
PROBLEM 6.87*

The cantilever beam shown consists of a Z shape of $\frac{1}{4}$-in. thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3 \text{ in}^4$ and $I_{y'} = 13.61 \text{ in}^4$.

SOLUTION

$$V = 3 \text{ kips} \quad \beta = 22.5°$$

$$V_{x'} = V \sin\beta \quad V_{y'} = V \cos\beta$$



In upper horizontal leg, use coordinate x: $(-6 \text{ in} \leq x \leq 0)$

$$A = \frac{1}{4}(6 + x) \text{ in.}$$

$$\bar{x} = \frac{1}{2}(-6 + x) \text{ in.}$$

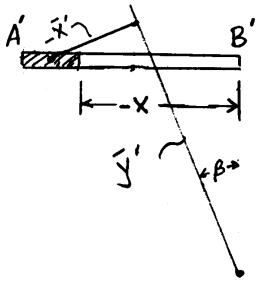
$$\bar{y} = 6 \text{ in.}$$

$$\bar{x}' = \bar{x}\cos\beta + \bar{y}\sin\beta$$

$$\bar{y}' = \bar{y}\cos\beta - \bar{x}\sin\beta$$

Due to $V_{x'}$:

$$\tau_1 = \frac{V_{x'} A \bar{x}'}{I_y t}$$



$$\tau_1 = \frac{(V \sin\beta)\left(\frac{1}{4}\right)(6 + x)\left[\frac{1}{2}(-6 + x)\cos\beta + 6\sin\beta\right]}{(13.61)\left(\frac{1}{4}\right)}$$

$$= 0.084353(6 + x)(-0.47554 + 0.46194x)$$

Due to $V_{y'}$:

$$\tau_2 = \frac{V_{y'} A \bar{y}'}{I_x t} = \frac{(V \cos\beta)\left(\frac{1}{4}\right)(6 + x)\left[6\cos\beta - \frac{1}{2}(-6 + x)\sin\beta\right]}{(166.3)\left(\frac{1}{4}\right)}$$

$$= 0.0166665(6 + x)[6.69132 - 0.19134x]$$

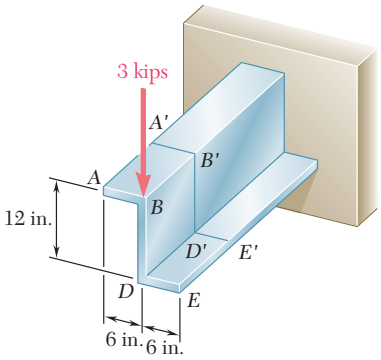
Total:

$$\tau_1 + \tau_2 = (6 + x)[-0.07141 + 0.035396x]$$

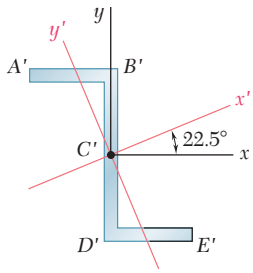
x (in.)	–6	–5	–4	–3	–2	–1	0
τ (ksi)	0	–0.105	–0.140	–0.104	0.003	0.180	0.428

◀

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(a)



(b)

PROBLEM 6.88*

For the cantilever beam and loading of Prob. 6.87, determine the distribution of the shearing stress along line $B'D'$ in the vertical web of the Z shape.

PROBLEM 6.87* The cantilever beam shown consists of a Z shape of $\frac{1}{4}$-in. thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3 \text{ in}^4$ and $I_{y'} = 13.61 \text{ in}^4$.

SOLUTION

$$V = 3 \text{ kips} \quad \beta = 22.5°$$

$$V_{x'} = V \sin\beta \quad V_{y'} = V \cos\beta$$



For part AB',

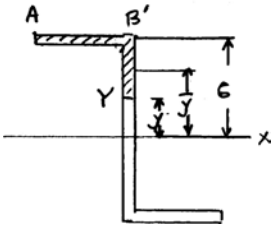
$$A = \left(\frac{1}{4}\right)(6) = 1.5 \text{ in}^2$$

$$\bar{x} = -3 \text{ in.}, \quad y = 6 \text{ in.}$$

For part $B'Y$,

$$A = \frac{1}{4}(6 - y)$$

$$\bar{x} = 0 \quad \bar{y} = \frac{1}{2}(6 + y)$$



$$x' = x\cos\beta + y\sin\beta$$

$$y' = y\cos\beta - x\sin\beta$$

Due to $V_{x'}$:

$$\tau_1 = \frac{V_{x'}(A_{AB}\bar{x}_{AB} + A_{BY}\bar{x}'_{BY})}{I_{y'}t}$$

$$\tau_1 = \frac{(V\sin\beta)[(1.5)(-3\cos\beta + 6\sin\beta) + \frac{1}{4}(6 - y)\frac{1}{2}(6 + y)\sin\beta]}{(13.61)\left(\frac{1}{4}\right)}$$

$$= \frac{(V\sin\beta)[-0.7133 + 1.7221 - 0.047835y^2]}{3.4025} = 0.3404 - 0.01614y^2$$

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PROBLEM 6.88* *(Continued)*

Due to $V_{y'}$:

$$\tau_2 = \frac{V_{y'}(A_{AB}\bar{y}'_{AB} + A_{BY}\bar{y}')}{I_{x'}t}$$

$$\tau_2 = \frac{(V\cos\beta)[(1.5)(6\cos\beta + 3\sin\beta) + \frac{1}{4}(6-y)\frac{1}{2}(6+y)\cos\beta]}{(166.3)\left(\frac{1}{4}\right)}$$

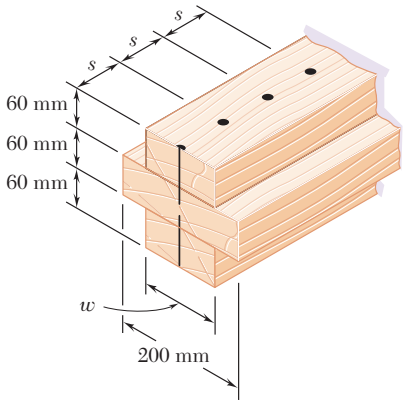
$$= \frac{(V\cos\beta)[10.037 + 4.1575 - 0.11548y^2]}{(166.3)\left(\frac{1}{4}\right)} = 0.9463 - 0.00770y^2$$

Total: $\tau_1 + \tau_2 = 1.2867 - 0.02384y^2$

y (in.)	0	± 2	±4	±6
τ (ksi)	1.287	1.191	0.905	0.428

◀

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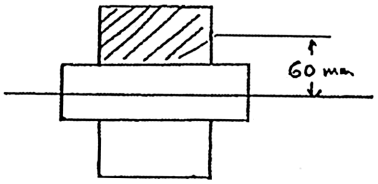


PROBLEM 6.89

Three boards are nailed together to form a beam shown, which is subjected to a vertical shear. Knowing that the spacing between the nails is $s = 75\,\text{mm}$ and that the allowable shearing force in each nail is 400 N, determine the allowable shear when $w = 120$ mm.

SOLUTION

Part	$A(\text{mm}^2)$	d (mm)	$Ad^2(10^6\,\text{mm}^4)$	$\bar{I}(10^6\,\text{mm}^4)$
Top Plank	7200	60	25.92	2.16
Middle Plank	12,000	0	0	3.60
Bottom Plank	7200	60	25.92	2.16
Σ			51.84	7.92



$$I = \sum Ad^2 + \sum \bar{I} = 59.76 \times 10^6\,\text{mm}^4 = 59.76 \times 10^{-6}\,\text{m}^4$$

$$Q = (7200)(60) = 432 \times 10^3\,\text{mm}^3 = 432 \times 10^{-6}\,\text{m}^3$$

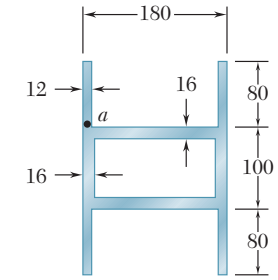
$$q = \frac{VQ}{I} \qquad F_{\text{nail}} = qs$$

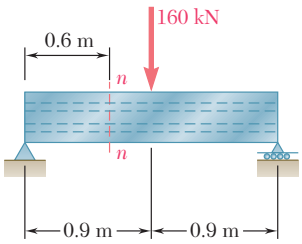
$$q = \frac{F_{\text{nail}}}{s} \qquad V = \frac{Iq}{Q} = \frac{IF_{\text{nail}}}{Qs}$$

$$V = \frac{(59.76 \times 10^{-6})(400)}{(432 \times 10^{-6})(75 \times 10^{-3})}$$

$$V = 738\ \text{N} \blacktriangleleft$$

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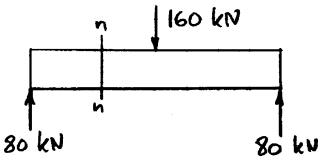




PROBLEM 6.90

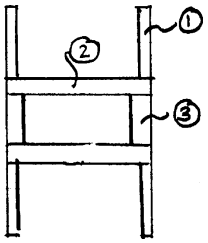
For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION



At section *n-n*, $V = 80$ kN

Consider cross section as composed of rectangles of types ①, ②, and ③.



$$I_1 = \frac{1}{12}(12)(80)^3 + (12)(80)(90)^2 = 8.288 \times 10^6\ \text{mm}^4$$

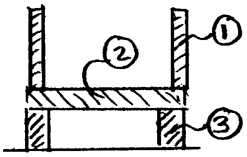
$$I_2 = \frac{1}{12}(180)(16)^3 + (180)(16)(42)^2 = 5.14176 \times 10^6\ \text{mm}^4$$

$$I_3 = \frac{1}{12}(16)(68)^3 = 419.24 \times 10^3\ \text{mm}^4$$

$$I = 4I_1 + 2I_2 + 2I_3 = 44.274 \times 10^6\ \text{mm}^4$$

$$= 44.274 \times 10^{-6}\ \text{m}^4$$

(*a*) Calculate Q at neutral axis.



$$Q_1 = (12)(80)(90) = 86.4 \times 10^3\ \text{mm}^4$$

$$Q_2 = (180)(16)(42) = 120.96 \times 10^3\ \text{mm}^4$$

$$Q_3 = (16)(34)(17) = 9.248 \times 10^3\ \text{mm}^4$$

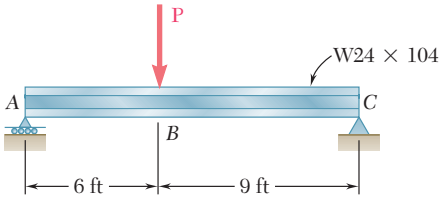
$$Q = 2Q_1 + Q_2 + 2Q_3 = 312.256 \times 10^3\ \text{mm}^3 = 312.256 \times 10^{-6}\ \text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(312.256 \times 10^{-6})}{(44.274 \times 10^{-6})(2 \times 16 \times 10^{-3})} = 17.63 \times 10^6\ \text{Pa} \qquad \tau = 17.63\ \text{MPa} \blacktriangleleft$$

(*b*) At point *a*, $Q = Q_1 = 86.4 \times 10^3\ \text{mm}^4 = 86.4 \times 10^{-6}\ \text{m}^4$

$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(86.4 \times 10^{-6})}{(44.274 \times 10^{-6})(12 \times 10^{-3})} = 13.01 \times 10^6\ \text{Pa} \qquad \tau = 13.01\ \text{MPa} \blacktriangleleft$$

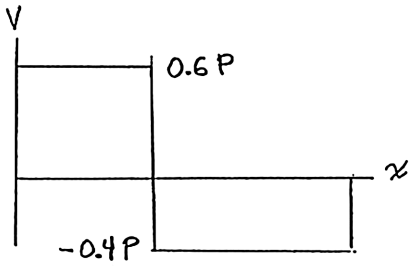
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PROBLEM 6.91

For the wide-flange beam with the loading shown, determine the largest load **P** that can be applied, knowing that the maximum normal stress is 24 ksi and the largest shearing stress using the approximation $\tau_m = V/A_{\text{web}}$ is 14.5 ksi.

SOLUTION



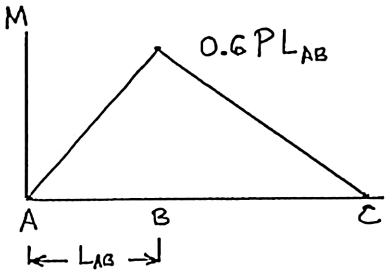
$$+\circlearrowleft \sum M_C = 0: \; -15R_A + qP = 0$$

$$R_A = 0.6P$$

Draw shear and bending moment diagrams.

$$|V|_{\max} = 0.6P \qquad |M|_{\max} = 0.6PL_{AB}$$

$$L_{AB} = 6\,\text{ft} = 72\,\text{in.}$$



Bending. For W24 × 104, $S = 258\,\text{in}^3$

$$S = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{0.6PL_{AB}}{\sigma_{\text{all}}}$$

$$P = \frac{\sigma_{\text{all}} S}{0.6L_{AB}} = \frac{(24)(258)}{(0.6)(72)} = 143.3\,\text{kips}$$

Shear.

$$A_{\text{web}} = dt_w$$

$$= (24.1)(0.500)$$

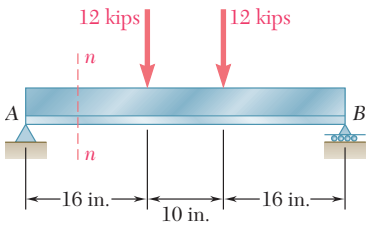
$$= 12.05\,\text{in}^2$$

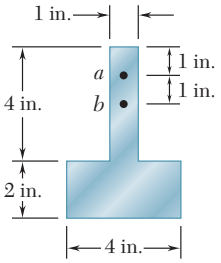
$$\tau = \frac{|V|_{\max}}{A_{\text{web}}} = \frac{0.6P}{A_{\text{web}}}$$

$$P = \frac{\tau A_{\text{web}}}{0.6} = \frac{(14.5)(12.05)}{0.6} = 291\,\text{kips}$$

The smaller value of P is the allowable value. $P = 143.3$ kips ◀

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PROBLEM 6.92

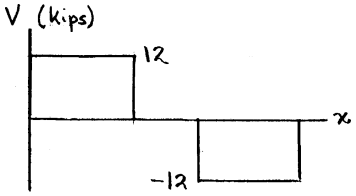
For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

$$R_A = R_B = 12 \text{ kips}$$

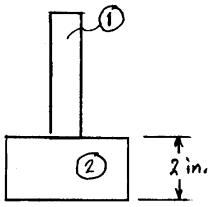
Draw shear diagram.

$$V = 12 \text{ kips}$$



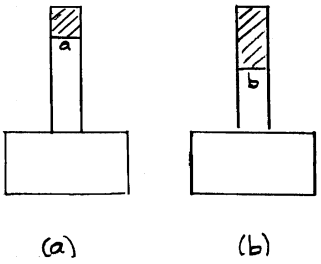
Determine section properties.

Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	4	4	16	2	16	5.333
②	8	1	8	−1	8	2.667
Σ	12		24		24	8.000



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{24}{12} = 2 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 32 \text{ in}^4$$



(*a*) $A = 1 \text{ in}^2 \quad \bar{y} = 3.5 \text{ in.} \quad Q_a = A\bar{y} = 3.5 \text{ in}^3$

$t = 1 \text{ in.}$

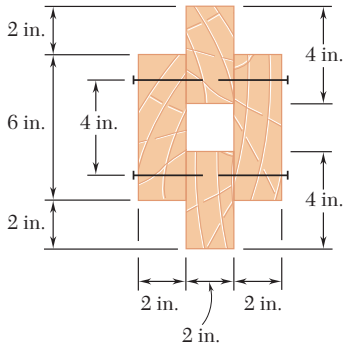
$$\tau_a = \frac{VQ_a}{It} = \frac{(12)(3.5)}{(32)(1)} \qquad \tau_a = 1.313 \text{ ksi} \blacktriangleleft$$

(*b*) $A = 2 \text{ in}^2 \quad \bar{y} = 3 \text{ in.} \quad Q_b = A\bar{y} = 6 \text{ in}^3$

$t = 1 \text{ in.}$

$$\tau_b = \frac{VQ_b}{It} = \frac{(12)(6)}{(32)(1)} \qquad \tau_b = 2.25 \text{ ksi} \blacktriangleleft$$

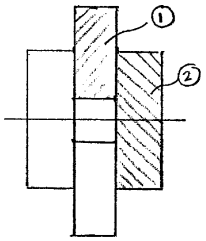
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PROBLEM 6.93

The built-up timber beam is subjected to a 1500-lb vertical shear. Knowing that the longitudinal spacing of the nails is $s = 2.5$ in. and that each nail is 3.5 in. long, determine the shearing force in each nail.

SOLUTION



$$I_1 = \frac{1}{12}(2)(4)^3 + (2)(4)(3)^2$$

$$= 82.6667 \text{ in}^4$$

$$I_2 = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$

$$I = 2I_1 + 2I_2$$

$$= 237.333 \text{ in}^4$$

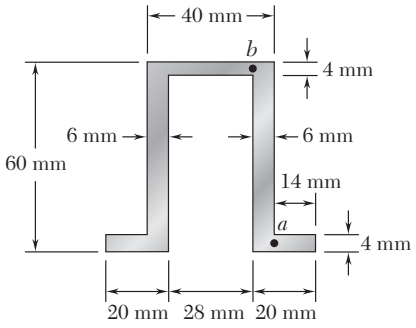
$$Q = A_1\bar{y}_1 = (2)(4)(3) = 24 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(1500)(24)}{237.333} = 151.685 \text{ lb/in.}$$

$$2F_{\text{nail}} = qs \qquad F_{\text{nail}} = \frac{1}{2}qs = \left(\frac{1}{2}\right)(151.685)(2.5)$$

$$F_{\text{nail}} = 189.6 \text{ lb} \blacktriangleleft$$

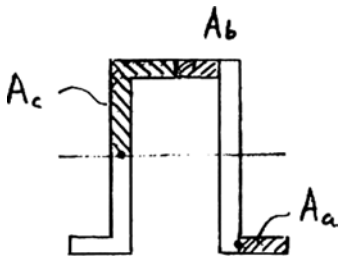
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PROBLEM 6.94

Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa in the hat-shaped extrusion shown, determine the corresponding shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION



Neutral axis lies 30 mm above bottom.

$$\tau_c = \frac{VQ_c}{It} \qquad \tau_a = \frac{VQ_a}{It_a} \qquad \tau_b = \frac{VQ_b}{It_b}$$

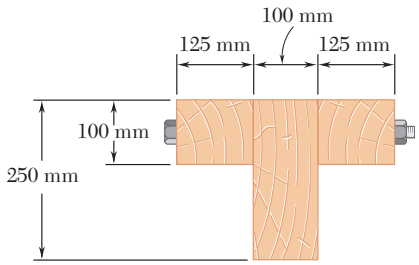
$$\frac{\tau_a}{\tau_c} = \frac{Q_a t_c}{Q_c t_a} \qquad \frac{\tau_b}{\tau_c} = \frac{Q_b t_c}{Q_c t_b}$$

$$Q_c = (6)(30)(15) + (14)(4)(28) = 4260 \text{ mm}^3$$
$$t_c = 6 \text{ mm}$$
$$Q_a = (14)(4)(28) = 1568 \text{ mm}^3$$
$$t_a = 4 \text{ mm}$$
$$Q_b = (14)(4)(28) = 1568 \text{ mm}^3$$
$$t_b = 4 \text{ mm}$$
$$\tau_c = 75 \text{ MPa}$$

(*a*) $\tau_a = \dfrac{Q_a}{Q_c} \cdot \dfrac{t_c}{t_a} \tau_c = \dfrac{1568}{4260} \cdot \dfrac{6}{4} \cdot 75$ $\qquad \tau_a = 41.4 \text{ MPa}$ ◀

(*b*) $\tau_b = \dfrac{Q_b}{Q_c} \cdot \dfrac{t_c}{t_b} \tau_c = \dfrac{1568}{4260} \cdot \dfrac{6}{4} \cdot 75$ $\qquad \tau_b = 41.4 \text{ MPa}$ ◀

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PROBLEM 6.95

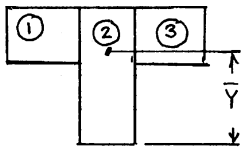
Three planks are connected as shown by bolts of 14-mm diameter spaced every 150 mm along the longitudinal axis of the beam. For a vertical shear of 10 kN, determine the average shearing stress in the bolts.

SOLUTION

Locate neutral axis and compute moment of inertia.

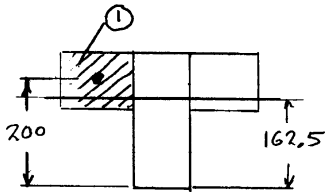
Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
①	12,500	200	2.5×10^6	37.5	17.5781×10^6	10.4167×10^6
②	25,000	125	3.125×10^6	37.5	35.156×10^6	130.208×10^6
③	12,500	200	2.5×10^6	37.5	17.5781×10^6	10.4167×10^6
Σ	50,000		8.125×10^6		70.312×10^6	151.041×10^6

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8.125\times10^6}{50\times10^3} = 162.5 \text{ mm}$$



$$I = \Sigma Ad^2 + \Sigma\bar{I} = 221.35\times10^6 \text{mm}^4$$
$$= 221.35\times10^{-6} \text{m}^4$$

$$Q = A_1\bar{y}_1 = (12{,}500)(37.5) = 468.75\times10^3 \text{ mm}^3$$
$$= 468.75\times10^{-6} \text{ m}^3$$



$$q = \frac{VQ}{I} = \frac{(10\times10^3)(468.75\times10^{-6})}{221.35\times10^{-6}}$$
$$= 21.177\times10^3 \text{ N/m}$$

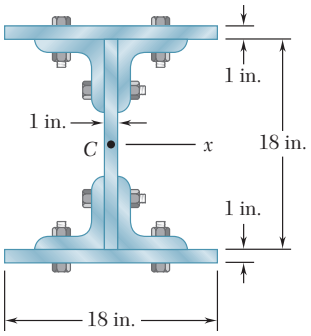
$$F_{\text{bolt}} = qs = (21.177\times10^3)(150\times10^{-3}) = 3.1765\times10^3 \text{ N}$$

$$A_{\text{bolt}} = \frac{\pi}{4}(14)^2 = 153.938 \text{ mm}^2 = 153.938\times10^{-6} \text{ m}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{3.1765\times10^3}{153.938\times10^{-6}} = 20.6\times10^6 \text{ Pa}$$

$$\tau_{\text{bolt}} = 20.6 \text{ MPa} \blacktriangleleft$$

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PROBLEM 6.96

Three 1×18-in. steel plates are bolted to four $L6 \times 6 \times 1$ angles to form a beam with the cross section shown. The bolts have a $\frac{7}{8}$-in. diameter and are spaced longitudinally every 5 in. Knowing that the allowable average shearing stress in the bolts is 12 ksi, determine the largest permissible vertical shear in the beam. (*Given:* $I_x = 6123 \text{ in}^4$.)

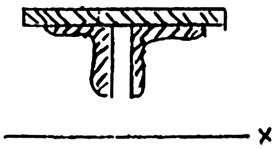
SOLUTION

Flange: $$I_f = \frac{1}{12}(18)(1)^3 + (18)(1)(9.5)^2 = 1626 \text{ in}^4$$

Web: $$I_w = \frac{1}{12}(1)(18)^3 = 486 \text{ in}^4$$

Angle: $$\bar{I} = 35.5 \text{ in}^4, \quad A = 11.0 \text{ in}^2$$
$$\bar{y} = 1.86 \text{ in.} \quad d = 9 - 1.86 = 7.14 \text{ in.}$$
$$I_a = \bar{I} + Ad^2 = 596.18 \text{ in}^4$$

$I = 2I_f + I_w + 4I_a = 6123 \text{ in}^4$, which agrees with the given value.



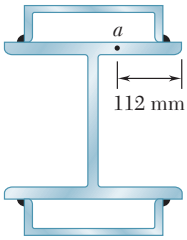
Flange: $$Q_f = (18)(1)(9.5) = 171 \text{ in}^3$$

Angle: $$Q_a = Ad = (11.0)(7.14) = 78.54 \text{ in}^3$$
$$Q = Q_f + 2Q_a = 328.08 \text{ in}^3$$

$$A_{\text{bolt}} = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$$
$$F_{\text{bolt}} = 2\tau_{\text{bolt}} A_{\text{bolt}} = (2)(12)(0.60132) = 14.4317 \text{ kips}$$
$$q_{\text{all}} = \frac{F_{\text{bolt}}}{s} = \frac{14.4317}{5} = 2.8863 \text{ kip/s}$$
$$q = \frac{VQ}{I} \qquad V_{\text{all}} = \frac{q_{\text{all}} I}{Q} = \frac{(2.8863)(6123)}{328.08}$$

$$V_{\text{all}} = 53.9 \text{ kips} \blacktriangleleft$$

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PROBLEM 6.97

The composite beam shown is made by welding C200×17.1 rolled-steel channels to the flanges of a W250×80 wide-flange rolled-steel shape. Knowing that the beam is subjected to a vertical shear of 200 kN, determine (*a*) the horizontal shearing force per meter at each weld, (*b*) the shearing stress at point *a* of the flange of the wide-flange shape.

SOLUTION

For W250 × 80, $d = 257$ mm, $t_f = 15.6$ mm, $I_x = 126 \times 10^6$ mm^4

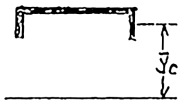
For C200 × 17.1, $A = 2170$ mm^2, $b_f = 57.4$ mm, $t_f = 9.91$ mm

$$I_y = 0.545 \times 10^6 \text{ mm}^4, \quad \bar{x} = 14.5 \text{ mm}$$

For the channel in the composite beam,

$$\bar{y}_c = \frac{257}{2} + 57.4 - 14.5 = 171.4 \text{ mm}$$

For the composite beam,



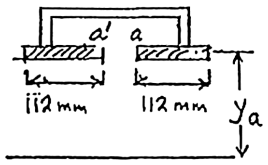
$$I = 126 \times 10^6 + 2\left[0.545 \times 10^6 + (2170)(171.4)^2\right]$$

$$= 254.59 \times 10^6 \text{ mm}^4 = 254.59 \times 10^{-6} \text{ m}^4$$

(*a*) For the two welds,

$$Q_w = A\bar{y}_c = (2170)(171.4) = 371.94 \times 10^3 \text{ mm}^3 = 371.94 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(200 \times 10^3)(371.94 \times 10^{-6})}{254.59 \times 10^{-6}} = 292.2 \times 10^3 \text{ N/m}$$



For one weld, $\frac{q}{2} = 146.1 \times 10^3$ N/m

Shearing force per meter of weld: 146.1 kN/m ◀

(*b*) For cuts at *a* and *a*' together,

$$A_a = 2(112)(15.6) = 3494.4 \text{ mm}^2 \qquad \bar{y}_a = \frac{257}{2} - \frac{15.6}{2} = 120.7 \text{ mm}$$

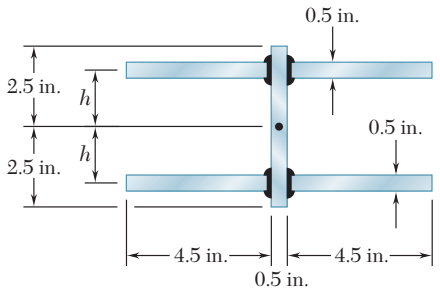
$$Q_a = 371.94 \times 10^3 + (3494.4)(120.7) = 793.71 \times 10^3 \text{ mm}^3 = 793.71 \times 10^{-6} \text{ m}^3$$

Since there are cuts at *a* and *a*', $t = 2t_f = (2)(15.6) = 31.2 \text{ mm} = 0.0312$ m.

$$\tau_a = \frac{VQ_a}{It} = \frac{(200 \times 10^3)(793.71 \times 10^{-6})}{(254.59 \times 10^6)(0.0312)} = 19.99 \times 10^6 \text{ Pa}$$

$\tau_a = 19.99$ MPa ◀

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PROBLEM 6.98

The design of a beam requires welding four horizontal plates to a vertical 0.5×5-in. plate as shown. For a vertical shear **V,** determine the dimension h for which the shear flow through the welded surface is maximum.

SOLUTION

Horizontal plate:

$$I_h = \frac{1}{12}(4.5)(0.5)^3 + (4.5)(0.5)h^2$$
$$= 0.046875 + 2.25h^2$$

Vertical plate: $I_v = \frac{1}{12}(0.5)(5)^3 = 5.2083 \text{ in}^4$

Whole section: $I = 4I_h + I_v = 9h^2 + 5.39583 \text{ in}^4$

For one horizontal plate, $Q = (4.5)(0.5)h = 2.25h \text{ in}^3$

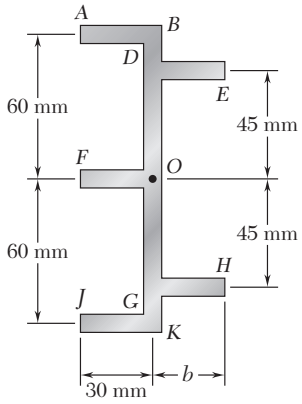
$$q = \frac{VQ}{I} = \frac{2.25Vh}{9h^2 + 5.39583}$$

To maximize q, set $\frac{dq}{dh} = 0.$

$$2.25V\frac{(9h^2 + 5.39583) - 18h^2}{(9h^2 + 5.39583)^2} = 0$$

$h = 0.774$ in. ◀

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PROBLEM 6.99

A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension *b* for which the shear center *O* of the cross section is located at the point indicated.

SOLUTION

Part *AB*:

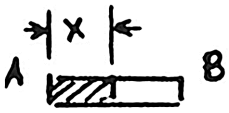
$$A = tx \qquad \bar{y} = 60 \text{ mm}$$

$$Q = A\bar{y} = 60tx \text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{60Vx}{I}$$

$$F_1 = \int \tau \, dA = \int_0^{30} \frac{60\,Vx}{I} t \, dx = \frac{60Vt}{I} \int_0^{30} x \, dx$$

$$= \frac{60Vt}{I} \frac{x^2}{2}\bigg|_0^{30} = \frac{(60)(30)^2}{2} \frac{Vt}{I} = 27 \times 10^3 \frac{Vt}{I}$$



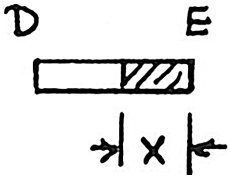
Part *DE*:

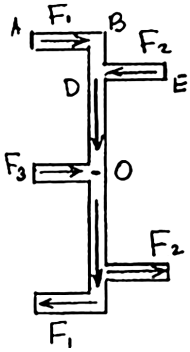
$$A = tx \qquad \bar{y} = 45 \text{ mm}$$

$$Q = A\bar{y} = 45tx$$

$$\tau = \frac{VQ}{It} = \frac{45Vx}{I}$$

$$F_2 = \int \tau \, dA = \int_O^b \frac{45Vx}{I} t \, dx = \frac{45Vt}{I} \int_O^b x \, dx = \frac{45b^2Vt}{2I}$$





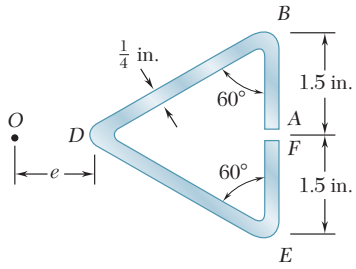
$$+\circlearrowleft \Sigma M_O = +\circlearrowleft \Sigma M_O: \quad 0 = (2)(45)F_2 - (2)(60)F_1$$

$$\left[(45)^2 b^2 - (2)(60)(27 \times 10^3)\right] \frac{Vt}{I} = 0$$

$$b^2 = \frac{(2)(60)(27 \times 10^3)}{45^2} = 1600 \text{ mm}^2 \qquad b = 40 \text{ mm} \blacktriangleleft$$

Note that the pair of F_1 forces form a couple. Likewise, the pair of F_2 forces form a couple. The lines of action of the forces in *BDOGK* pass through point *O*.

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PROBLEM 6.100

Determine the location of the shear center *O* of a thin-walled beam of uniform thickness having the cross section shown.

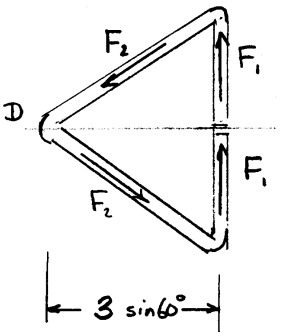
SOLUTION

$$I_{AB} = \frac{1}{3}\left(\frac{1}{4}\right)(1.5)^3 = 0.28125 \text{ in}^4$$

$$L_{BD} = 3 \text{ in.} \quad A_{BD} = (3)\left(\frac{1}{4}\right) = 0.75 \text{ in}^2$$

$$I_{BD} = \frac{1}{3}A_{BD}h^2 = \frac{1}{3}(0.75)(1.5)^2 = 0.5625 \text{ in}^4$$

$$I = (2)(0.28125) + (2)(0.5625) = 1.6875 \text{ in}^4$$



Part *AB*:

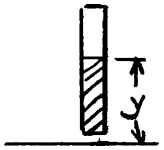
$$A = \frac{1}{4}y \quad \bar{y} = \frac{1}{2}y \quad Q = A\bar{y} = \frac{1}{8}y^2$$

$$\tau = \frac{VQ}{It} = \frac{Vy^2}{(8)(1.6875)(0.25)} = \frac{Vy^2}{3.375}$$

$$F_1 = \int \tau dA = \int_0^{1.5} \frac{Vy^2}{3.375}(0.25dy)$$

$$= \frac{(0.25)V}{3.375}\left.\frac{y^3}{3}\right|_0^{1.5} = \frac{(0.25)(1.5)^3}{(3.375)(3)}$$

$$= 0.083333V$$



$$+\circlearrowright M_D = +\circlearrowright M_D: \quad Ve = 2F_1(3 \sin 60°)$$

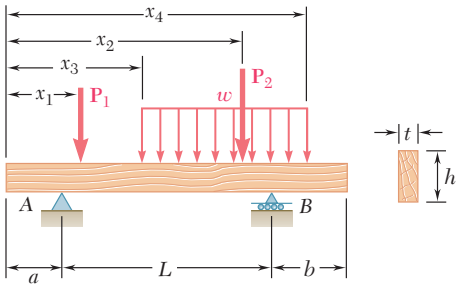
$$Ve = (2)(0.083333)V(3 \sin 60°)$$

$$e = (2)(0.083333)(3 \sin 60°)$$

$$e = 0.433 \text{ in.} \blacktriangleleft$$

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PROBLEM 6.C1



A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress and the maximum shearing stress in the beam will not exceed given allowable values σ_{all} and τ_{all}. Measuring x from end A and using either SI or U.S. customary units, write a computer program to calculate for successive cross sections, from $x = 0$ to $x = L$ and using given increments Δx, the shear, the bending moment, and the smallest value of the unknown dimension that satisfies in that section (1) the allowable normal stress requirement and (2) the allowable shearing stress requirement. Use this program to solve Prob. 5.65, assuming $\sigma_{all} = 12$ MPa and $\tau_{all} = 825$ kPa and using $\Delta x = 0.1$ m.

SOLUTION

See solution of Prob. 5.C2 for the determination of R_A, R_B, $V(x)$, and $M(x)$

We recall that

$$V(x) = R_A\,\text{STP}A + R_B\,\text{STP}B - P_1\,\text{STP}1 - P_2\,\text{STP}2$$
$$-w(x - x_3)\,\text{STP}3 + w(x - x_4)\,\text{STP}4$$
$$M(x) = R_A(x - a)\,\text{STP}A + R_B(x - a - L)\,\text{STP}B - P_1(x - x_1)\,\text{STP}1$$
$$-P_2(x - x_2)\,\text{STP}2 - \frac{1}{2}w(x - x_3)^2\,\text{STP}3 + \frac{1}{2}w(x - x_4)^2\,\text{STP}4$$

where STPA, STPB, STP1, STP2, STP3, and STP4 are step functions defined in Problem 5.C2.

(1) To satisfy the allowable normal stress requirement: If unknown dimension is h:

$$S_{\min} = |M|/\sigma_{all}.$$

From $S = \frac{1}{6}th^2$, we have $h_\sigma = h = \sqrt{6S/t}$

If unknown dimension is t:

$$S_{\min} = |M|/\sigma_{all}.$$

From $S = \frac{1}{6}th^2$, we have $t_\sigma = t = 6S/h^2$

(2) To satisfy the allowable shearing stress requirement:

We use Equation (6.10), Page 378: $\tau_{\max} = \frac{3}{2}\frac{|V|}{A} = \frac{3}{2}\frac{|V|}{th}$

If unknown dimension is h: $h_\tau = h = \frac{3|V|}{2t\tau_{all}}$

If unknown dimension is t: $t_\tau = t = \frac{3M}{2h\tau_{all}}$

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PROBLEM 6.C1 *(Continued)*

Program Outputs

Problem 5.65

$$R_A = 2.40 \text{ kN}$$
$$R_B = 3.00 \text{ kN}$$

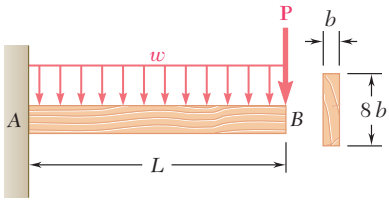
X m	V kN	M kN · m	HSIG mm	HTAU mm	
0.00	2.40	0.000	0.00	109.09	
0.10	2.40	0.240	54.77	109.09	
0.20	2.40	0.480	77.46	109.09	
0.30	2.40	0.720	94.87	109.09	
0.40	2.40	0.960	109.54	109.09	
0.50	2.40	1.200	122.47	109.09	
0.60	2.40	1.440	134.16	109.09	
0.70	2.40	1.680	144.91	109.09	
0.80	0.60	1.920	154.92	27.27	
0.90	0.60	1.980	157.32	27.27	
1.00	0.60	2.040	159.69	27.27	
1.10	0.60	2.100	162.02	27.27	
1.20	0.60	2.160	164.32	27.27	
1.30	0.60	2.220	166.58	27.27	
1.40	0.60	2.280	168.82	27.27	
1.50	0.60	2.340	171.03	27.27	
1.60	–3.00	2.400	173.21	136.36	◀
1.70	–3.00	2.100	162.02	136.36	
1.80	–3.00	1.800	150.00	136.36	
1.90	–3.00	1.500	136.93	136.36	
2.00	–3.00	1.200	122.47	136.36	
2.10	–3.00	0.900	106.07	136.36	
2.20	–3.00	0.600	86.60	136.36	
2.30	–3.00	0.300	61.24	136.36	
2.40	0.00	0.000	0.05	0.00	

The smallest allowable value of h is the largest of the values shown in the last two columns.

For Problem 5.65, $h = h_\sigma = 173.2$ mm ◀

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PROBLEM 6.C2



A cantilever timber beam *AB* of length *L* and of uniform rectangular section shown supports a concentrated load **P** at its free end and a uniformly distributed load *w* along its entire length. Write a computer program to determine the length *L* and the width *b* of the beam for which both the maximum normal stress and the maximum shearing stress in the beam reach their largest allowable values. Assuming $\sigma_{\text{all}} = 1.8$ ksi and $\tau_{\text{all}} = 120$ psi, use this program to determine the dimensions *L* and *b* when (*a*) $P = 1000$ lb and $w = 0$, (*b*) $P = 0$ and $w = 12.5$ lb/in., (*c*) $P = 500$ lb and $w = 12.5$ lb/in.

SOLUTION

Both the maximum shear and the maximum bending moment occur at *A*. We have

$$V_A = P + wL$$

$$M_A = PL + \frac{1}{2}wL^2$$

To satisfy the allowable normal stress requirement:

$$\sigma_{\text{all}} = \frac{M_A}{S} = \frac{M_A}{\frac{1}{6}b(8b)^2} = \frac{3M_A}{32b^3}$$

$$b_\sigma = b = \left[\frac{3}{32}\frac{M_A}{\sigma_{\text{all}}}\right]^{1/3}$$

To satisfy the allowable shearing stress requirement:

We use Equation (6.10), Page 378.

$$\tau_{\text{all}} = \frac{3V}{2A} = \frac{3}{2}\frac{V_A}{b(8b)} = \frac{3V_A}{16b^2}$$

$$b_\tau = b = \left[\frac{3}{16}\frac{V_A}{\tau_{\text{all}}}\right]^{1/2}$$

Program

For $L = 0$, $V_A = P$ and $b_\tau > 0$, while $M_A = 0$ and $b_\sigma = 0$.

Starting with $L = 0$ and using increments $\Delta L = 0.001$ in., we increase *L* until b_σ and b_τ become equal. We then Print *L* and *b*.

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PROBLEM 6.C2 *(Continued)*

Program Outputs

For $P = 1000$ lb, $w = 0.0$ lb/in.

Increment = 0.0010 in.

$L = 37.5$ in., $b = 1.250$ in.

For $P = 500$ lb, $w = 12.5$ lb/in.

Increment = 0.0010 in.

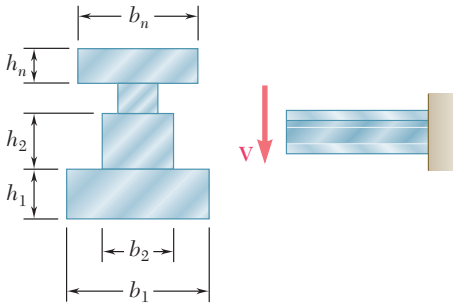
$L = 59.8$ in., $b = 1.396$ in.

For $P = 0$ lb, $w = 12.5$ lb/in.

Increment = 0.0010 in.

$L = 70.3$ in., $b = 1.172$ in.

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PROBLEM 6.C3

A beam having the cross section shown is subjected to a vertical shear **V**. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to calculate the shearing stress along the line between any two adjacent rectangular areas forming the cross section. Use this program to solve (*a*) Prob. 6.10, (*b*) Prob. 6.12, (*c*) Prob. 6.22.

SOLUTION

1. Enter V and the number n of rectangles.
2. For $i = 1$ to n, enter the dimensions b_i and h_i.
3. Determine the area $A_i = b_i h_i$ of each rectangle.
4. Determine the elevation of the centroid of each rectangle:

$$\bar{y}_i = \sum_{k=1}^{i} h_k - 0.5h_i$$

and the elevation $\bar{y}$ of the centroid of the entire section:

$$\bar{y} = \left(\sum_i A_i \bar{y}_i\right) \Big/ \left(\sum_i A_i\right)$$

5. Determine the centroidal moment of inertia of the entire section:

$$I = \sum_i \left[\frac{1}{12} b_i h_i^3 + A_i(\bar{y}_i - \bar{y})^2\right]$$

6. For each surface separating two rectangles i and $i+1$, determine Q_i of the area below that surface:

$$Q_i = \sum_{k=1}^{i} = A_k(\bar{y}_k - \bar{y})$$

7. Select for t_i the smaller of b_i and b_{i+1}.

The shearing stress on the surface between the rectangles i and $i+1$ is

$$\tau_i = \frac{VQ_i}{It_i}$$

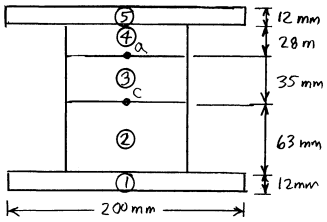
◀

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PROBLEM 6.C3 *(Continued)*

Program Outputs

Problem 6.10



Shearing force $= 10$ kN

$\bar{y} = 75.000$ mm above base

$I = 39.580 \times 10^{-6}\ \text{mm}^4$

Between Elements 1 and 2:

$\tau = 418.39$ kPA

Between Elements 2 and 3:

$\tau = 919.78$ kPA ◀ (*a*)

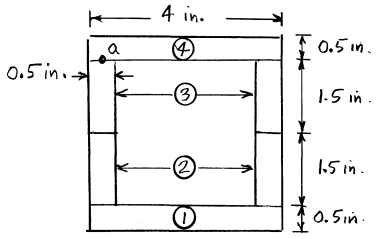
Between Elements 3 and 4:

$\tau = 765.03$ kPA ◀ (*b*)

Between Elements 4 and 5:

$\tau = 418.39$ kPA

Problem 6.12



Shearing force $= 10$ kips

$\bar{y} = 2.000$ in.

$I = 14.58\ \text{in}^4$

Between Elements 1 and 2:

$\tau = 2.400$ ksi

Between Elements 2 and 3:

$\tau = 3.171$ ksi ◀ (*a*)

Between Elements 3 and 4:

$\tau = 2.400$ ksi ◀ (*b*)

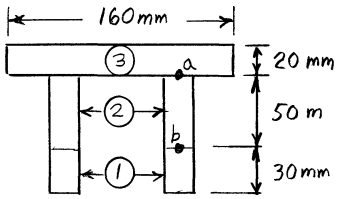
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PROBLEM 6.C3 *(Continued)*

Program Outputs *(Continued)*

Problem 6.22

Shearing force $= 90$ kN



$\bar{y} = 65.000$ mm

$I = 58.133 \times 10^{-6}\ \text{mm}^4$

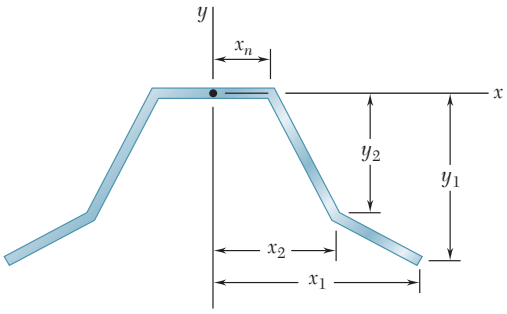
Between Elements 1 and 2:

$\tau = 23.222$ MPA ◀ (*b*)

Between Elements 2 and 3:

$\tau = 30.963$ MPA ◀ (*a*)

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PROBLEM 6.C4

A plate of uniform thickness t is bent as shown into a shape with a vertical plane of symmetry and is then used as a beam. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine the distribution of shearing stresses caused by a vertical shear **V.** Use this program (*a*) to solve Prob. 6.47, (*b*) to find the shearing stress at a Point *E* for the shape and load of Prob. 6.50, assuming a thickness $t = \frac{1}{4}$ in.

SOLUTION

For each element on the right-hand side, we compute (for $i = 1$ to n):

Length of element $= L_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$

Area of element $= A_i = tL_i$

where $$t = \frac{1}{4} \text{ in.}$$

Distance from x axis to centroid of element $= \bar{y}_i = \frac{1}{2}(y_i + y_{i+1})$

Distance from x axis to centroid of section:

$$\bar{y} = \left(\Sigma A_i \bar{y}_i\right) / \Sigma A_i$$

Note that $y_n = 0$ and that $x_{n+1} = y_{n+1} = 0$.

Moment of inertia of section about centroidal axis:

$$\bar{I} = 2\Sigma A_i \left[\frac{1}{12}(y_i - y_{i+1})^2 + (\bar{y}_i - \bar{y})^2\right]$$

Computation of Q at Point P where stress is desired:

$Q = \Sigma A_i(\bar{y}_i - \bar{y})$ where sum extends to the areas located between one end of section and Point P.

Shearing stress at P:

$$\tau = \frac{VQ}{It}$$

Note: $\tau_{\max}$ occurs on neutral axis, i.e., for $y_P = \bar{y}$.

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PROBLEM 6.C4 *(Continued)*

Program Outputs

Part (*a*):

$$I = 0.5333 \text{ in}^4$$

$$\tau_{\max} = 2.02 \text{ ksi} \quad \blacktriangleleft$$

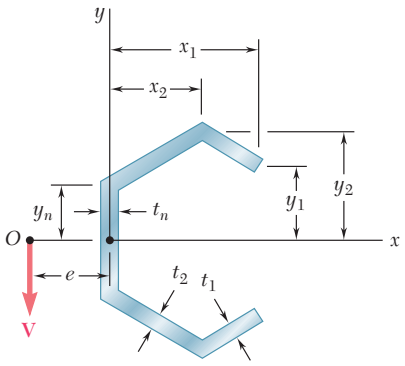
$$\tau_B = 1.800 \text{ ksi} \quad \blacktriangleleft$$

Part (*b*):

$$I = 22.27 \text{ in}^4$$

$$\tau_E = 194.0 \text{ psi} \quad \blacktriangleleft$$

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PROBLEM 6.C5

The cross section of an extruded beam is symmetric with respect to the *x* axis and consists of several straight segments as shown. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine (*a*) the location of the shear center *O*, (*b*) the distribution of shearing stresses caused by a vertical force applied at *O*. Use this program to solve Prob. 6.70.

SOLUTION

Since section is symmetric with *x* axis, computations will be done for top half.

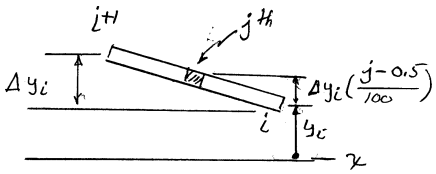
For $i = 1$ to $n+1$: (*Note:* $n+1$ is the origin)

Enter $t_i, \quad x_i, \quad y_i$

Compute length of each segment.

For $i = 1$ to n:

$$\Delta x_i = x_{i+1} - x_i$$
$$\Delta y_i = y_{i+1} - y_i$$
$$L = (\Delta x_i^2 + \Delta y_i^2)^{1/2}$$



Calculate moment of inertia I_x.

Consider each segment as made of 100 equal parts.

For $i = 1$ to n:

$$\Delta\text{Area} = L_i t_i / 100$$

For $j = 1$ to 100:

$$y = y_i + \Delta y_i (j - 0.5)/100$$
$$\Delta I = (\Delta\text{Area}) y^2$$
$$I_x = I_x + \Delta I$$

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PROBLEM 6.C5 *(Continued)*

Since only top half was used,

$$I_x = 2I_x$$

Calculate shearing stress at ends of segments and shear forces in segments.

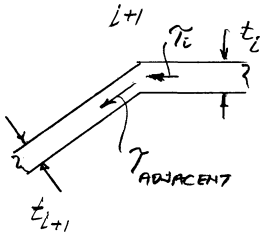
For $i = 1$ to n:

$$\Delta\text{Area} = L_i t_i/100, \qquad \tau_{\text{new}} = \tau_{\text{next}}$$

For $j = 1$ to 100:

$$y = y_i + \Delta y_i\,(j - 0.5)/100$$
$$\Delta Q = (\Delta\text{Area})\,y$$
$$\tau_{\text{old}} = \tau_{\text{new}}, \qquad Q = Q + \Delta Q$$
$$\tau_{\text{new}} = VQ/I_x t_i$$
$$\tau_{\text{ave}} = 0.5(\tau_{\text{old}} + \tau_{\text{new}})$$
$$\tau = \tau + \tau_{\text{ave}}$$

Force $i = \tau\ (\Delta\text{Area})$



$$\tau_i = VQ/I_x t_i$$
$$(\tau_{\text{adjacent}})_i = VQ/I_x t_{i+1}$$
$$Q_i = Q$$
$$\tau_{\text{next}} = (\tau_{\text{adjacent}})_i$$

Location of shear center.

Calculate moment of shear forces about origin.

For $L = 1$ to n,

$$(F_x)_i = \text{Force}_i\,(\Delta x_i)/L_i$$
$$(F_y)_i = \text{Force}_i\,(\Delta y_i)/L_i$$
$$\text{Moment}_i = -(F_x)_i\,y_L + (F_y)_i\,x_i$$
$$\text{Moment} = \text{Moment} + \text{Moment}_i$$

For whole section, moment = 2(moment),

Shear center is at $e = \text{Moment}/V$

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PROBLEM 6.C5 *(Continued)*

Program Output

Problem 6.70

	T(K) mm	X(K) mm	Y(K) mm	L(K) mm
1	6.00	60.62	0.00	70.00
2	6.00	0.00	35.00	35.00
3	6.00	0.00	0.00	

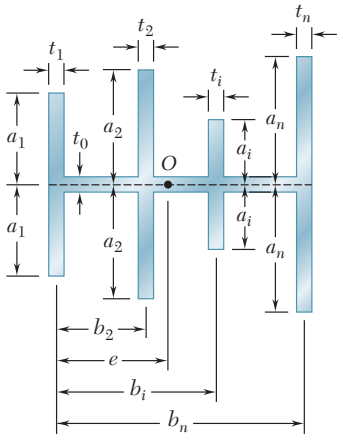
Moment of inertia: $I_x = 514487 \text{ mm}^4$ Shear $= 1000.000$ N

Junction of Segments	Q mm^3	Tau Before MPa	Tau After MPa	Force in Segment kN
1 and 2	7350.00	2.38	2.38	335.01
2 and 3	11,025.00	3.57	3.57	666.27

Moment of shear forces about origin: $M = 20.309 \text{ N} \cdot \text{m} +$ counterclockwise

Distance from origin to shear center: $e = 20.309 \text{ mm}$

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PROBLEM 6.C6

A thin-walled beam has the cross section shown. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine the location of the shear center O of the cross section. Use the program to solve Prob. 6.75.

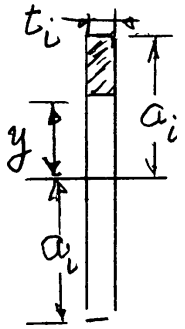
SOLUTION

Distribution of shearing stresses in element i.

Let $V =$ Shear in cross section

$\bar{I} =$ Centroidal moment of inertia of section

We have for shaded area

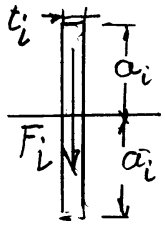


$$Q = A\bar{y} = t_i(a_i - y)\frac{a_i + y}{2}$$

$$= \frac{1}{2}t_i(a_i^2 - y^2)$$

$$\tau = \frac{QV}{It_i} = \frac{V}{2I}(a_i^2 - y^2)$$

Force exerted on element i.



$$F_i = \int_{-a_i}^{a_i} \tau(t_i dy)$$

$$= \frac{Vt_i}{2I}\int_{-a_i}^{a_i}\left(a_i^2 - y^2\right)dy$$

$$= \frac{Vt_i}{I}\int_{0}^{a_i}\left(a_i^2 - y^2\right)dy$$

$$= \frac{Vt_i}{I}\left(a_i^3 - \frac{1}{3}a_i^3\right) = \frac{2}{3}\frac{V}{I}t_i a_i^3$$

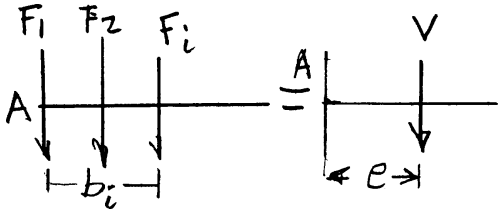
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PROBLEM 6.C6 *(Continued)*

The system of the forces F_i must be equivalent to V at shear center.

$$\Sigma F = \Sigma F: \qquad \frac{2}{3}\frac{V}{I}\Sigma t_i a_i^3 = V \tag{1}$$

$$\Sigma M_A = \Sigma M_A: \qquad \frac{2}{3}\frac{V}{I}\Sigma t_i a_i^3 b_i = eV \tag{2}$$



Divide (2) by (1): $$e = \frac{\Sigma t_i a_i^3 b_i}{\Sigma t_i a_i^3}$$ ◀

Program Output

Problem 6.75

For Element 1:

$t = 0.75$ in., $a = 4$ in., $b = 0$

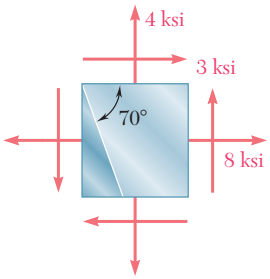
For Element 2:

$t = 0.75$ in., $a = 3$ in., $b = 8$ in.

Answer: $e = 2.37$ in. ◀

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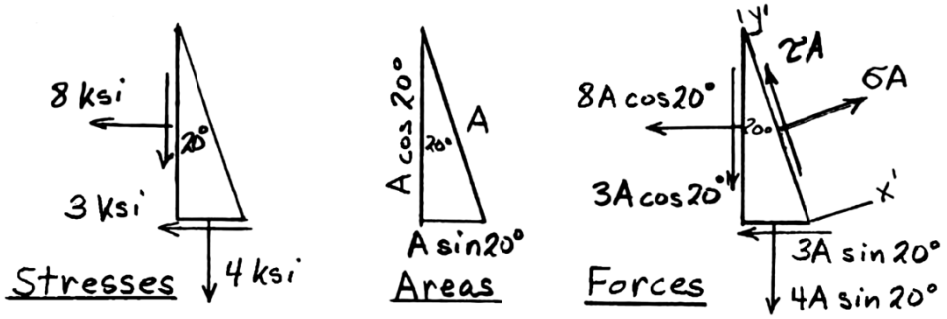
CHAPTER 7



PROBLEM 7.1

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.1A.

SOLUTION



$$+/ \quad \Sigma F = 0: \quad \sigma A - 8A\cos 20^\circ \cos 20^\circ - 3A\cos 20^\circ \sin 20^\circ - 3A\sin 20^\circ \cos 20^\circ - 4A\sin 20^\circ \sin 20^\circ = 0$$

$$\sigma = 8\cos^2 20^\circ + 3\cos 20^\circ \sin 20^\circ + 3\sin 20^\circ \cos 20^\circ + 4\sin^2 20^\circ = 0$$

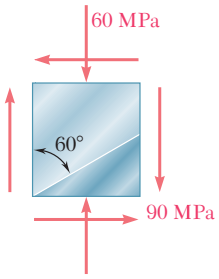
$$\sigma = 9.46 \text{ ksi} \blacktriangleleft$$

$$+\nwarrow \quad \Sigma F = 0: \quad \tau A + 8A\cos 20^\circ \sin 20^\circ - 3A\cos 20^\circ \cos 20^\circ + 3A\sin 20^\circ \sin 20^\circ - 4A\sin 20^\circ \cos 20^\circ = 0$$

$$\tau = -8\cos 20^\circ \sin 20^\circ + 3(\cos^2 20^\circ - \sin^2 20^\circ) + 4\sin 20^\circ \cos 20^\circ$$

$$\tau = 1.013 \text{ ksi} \blacktriangleleft$$

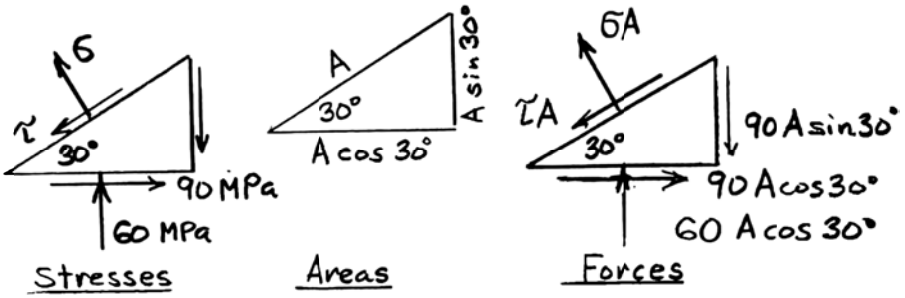
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PROBLEM 7.2

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.1A.

SOLUTION



$$+\nwarrow \Sigma F = 0: \quad \sigma A - 90A\sin 30^\circ\cos 30^\circ - 90A\cos 30^\circ\sin 30^\circ + 60A\cos 30^\circ\cos 30^\circ = 0$$

$$\sigma = 180\sin 30^\circ\cos 30^\circ - 60\cos^2 30^\circ$$

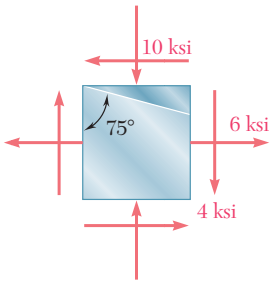
$$\sigma = 32.9 \text{ MPa} \blacktriangleleft$$

$$+\swarrow \Sigma F = 0: \quad \tau A + 90A\sin 30^\circ\sin 30^\circ - 90A\cos 30^\circ\cos 30^\circ - 60A\cos 30^\circ\sin 30^\circ = 0$$

$$\tau = 90(\cos^2 30^\circ - \sin^2 30^\circ) + 60\cos 30^\circ\sin 30^\circ$$

$$\tau = 71.0 \text{ MPa} \blacktriangleleft$$

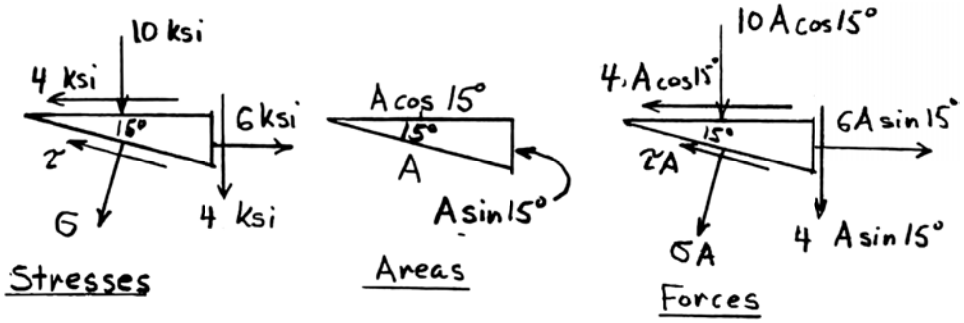
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PROBLEM 7.3

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.1A.

SOLUTION



$$+\nearrow \Sigma F = 0: \quad \sigma A + 4A\cos 15^\circ \sin 15^\circ + 10A\cos 15^\circ \cos 15^\circ - 6A\sin 15^\circ \sin 15^\circ + 4A\sin 15^\circ \cos 15^\circ = 0$$

$$\sigma = -4\cos 15^\circ \sin 15^\circ - 10\cos^2 15^\circ + 6\sin^2 15^\circ - 4\sin 15^\circ \cos 15^\circ$$

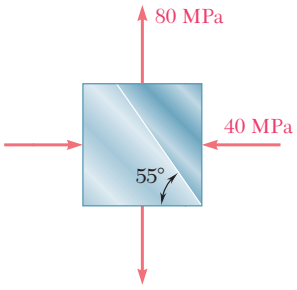
$$\sigma = 10.93 \text{ ksi} \blacktriangleleft$$

$$+\nwarrow \Sigma F = 0: \quad \tau A + 4A\cos 15^\circ \cos 15^\circ - 10A\cos 15^\circ \sin 15^\circ - 6A\sin 15^\circ \cos 15^\circ - 4A\sin 15^\circ \sin 15^\circ = 0$$

$$\tau = -4(\cos^2 15^\circ - \sin^2 15^\circ) + (10 + 6)\cos 15^\circ \sin 15^\circ$$

$$\tau = 0.536 \text{ ksi} \blacktriangleleft$$

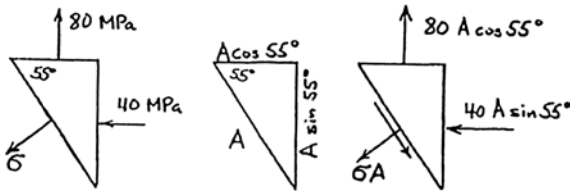
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PROBLEM 7.4

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.1A.

SOLUTION



Stresses Areas Forces

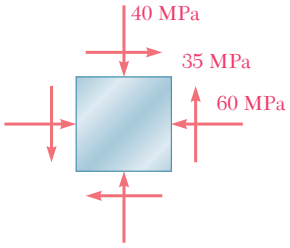
$+\nearrow \Sigma F = 0$: $\sigma A - 80A \cos 55° \cos 55° + 40A \sin 55° \sin 55° = 0$

$\sigma = 80 \cos^2 55° - 40 \sin^2 55°$ $\sigma = -0.521$ MPa ◀

$+\searrow \Sigma F = 0$: $\tau A - 80A \cos 55° \sin 55° - 40A \sin 55° \cos 55°$

$\tau = 120 \cos 55° \sin 55°$ $\tau = 56.4$ MPa ◀

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PROBLEM 7.5

For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(35)}{-60+40} = -3.50$$

$$2\theta_p = -74.05°$$

$$\theta_p = -37.0°, \quad 53.0° \blacktriangleleft$$

(*b*) $$\sigma_{\max,\,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

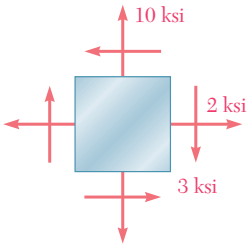
$$= \frac{-60-40}{2} \pm \sqrt{\left(\frac{-60+40}{2}\right)^2 + (35)^2}$$

$$= -50 \pm 36.4 \text{ MPa}$$

$$\sigma_{\max} = -13.60 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = -86.4 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.6

For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$$\sigma_x = 2 \text{ ksi} \qquad \sigma_y = 10 \text{ ksi} \qquad \tau_{xy} = -3 \text{ ksi}$$

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-3)}{2 - 10} = 0.750$$

$$2\theta_p = 36.87°$$

$$\theta_p = 18.4°, 108.4° \blacktriangleleft$$

(*b*) $$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

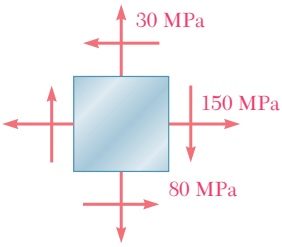
$$= \frac{2 + 10}{2} \pm \sqrt{\left(\frac{2 - 10}{2}\right)^2 + (-3)^2}$$

$$= 6 \pm 5 \text{ ksi}$$

$$\sigma_{\max} = 11.00 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\min} = 1.000 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.7

For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$$\sigma_x = +150 \text{ MPa}, \qquad \sigma_y = +30 \text{ MPa}, \qquad \tau_{xy} = -80 \text{ MPa}$$

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(-80 \text{ MPa})}{(+150 \text{ MPa} - 30 \text{ MPa})} = -1.33333 \text{ MPa}$$

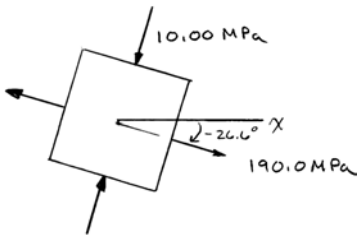
$$2\theta_p = -53.130° \quad \text{and} \quad 126.870°$$

$$\theta_p = -26.6° \text{ and } +63.4° \blacktriangleleft$$

(*b*) $$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right) + \tau_{xy}^2}$$

$$= \frac{150 \text{ MPa} + 30 \text{ MPa}}{2} \pm \sqrt{\left(\frac{150 \text{ MPa} - 30 \text{ MPa}}{2}\right)^2 + (-80 \text{ MPa})^2}$$

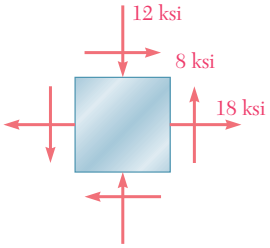
$$= 90 \text{ MPa} \pm 100 \text{ MPa}$$



$$\sigma_{\max} = 190.0 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = -10.00 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.8

For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$$\sigma_x = 18 \text{ ksi} \qquad \sigma_y = -12 \text{ ksi} \qquad \tau_{xy} = 8 \text{ ksi}$$

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(8)}{18 + 12} = 0.5333$$

$$2\theta_p = 28.07°$$

$$\theta_p = 14.0°, 104.0° \blacktriangleleft$$

(*b*) $$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

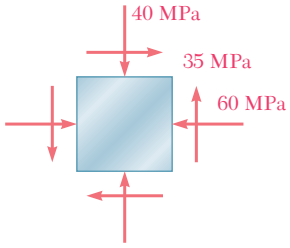
$$= \frac{18 - 12}{2} \pm \sqrt{\left(\frac{18 + 12}{2}\right)^2 + (8)^2}$$

$$= 3 \pm 17 \text{ ksi}$$

$$\sigma_{\max} = 20.0 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\min} = -14.00 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.9

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

(*a*) $$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{-60+40}{(2)(35)} = 0.2857$$

$$2\theta_s = 15.95°$$

$$\theta_s = 8.0°, \quad 98.0° \blacktriangleleft$$

(*b*) $$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

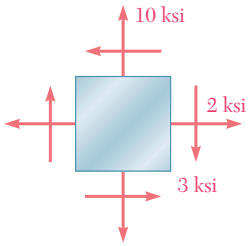
$$= \sqrt{\left(\frac{-60+40}{2}\right)^2 + (35)^2}$$

$$\tau_{\max} = 36.4 \text{ MPa} \blacktriangleleft$$

(*c*) $$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-60-40}{2}$$

$$\sigma' = -50.0 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.10

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 2 \text{ ksi} \qquad \sigma_y = 10 \text{ ksi} \qquad \tau_{xy} = -3 \text{ ksi}$$

(*a*) $$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{2 - 10}{(2)(-3)} = -1.33333$$

$$2\theta_s = -53.13°$$

$$\theta_s = -26.6°, 63.4° \blacktriangleleft$$

(*b*) $$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

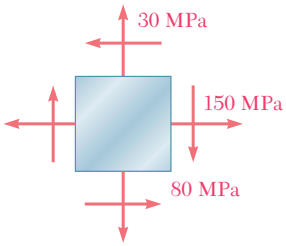
$$= \sqrt{\left(\frac{2 - 10}{2}\right)^2 + (-3)^2}$$

$$\tau_{\max} = 5.00 \text{ ksi} \blacktriangleleft$$

(*c*) $$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{2 + 10}{2}$$

$$\sigma' = 6.00 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.11

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = +150 \text{ MPa}, \qquad \sigma_y = +30 \text{ MPa}, \qquad \tau_{xy} = -80 \text{ MPa}$$

(*a*) $$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{150 - 30}{2(-80)} = +0.750$$

$$2\theta_s = 36.87° \text{ and } 216.87°$$

$$\theta_s = 18.4° \text{ and } 108.4° \blacktriangleleft$$

(*b*) $$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{150 - 30}{2}\right)^2 + (-80)^2}$$

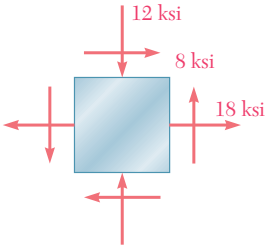
$$\tau_{\max} = 100.0 \text{ MPa} \blacktriangleleft$$

(*c*) $$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2}$$

$$= \left(\frac{150 + 30}{2}\right)$$

$$\sigma' = 90.0 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.12

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 18 \text{ ksi} \qquad \sigma_y = -12 \text{ ksi} \qquad \tau_{xy} = 8 \text{ ksi}$$

(*a*) $$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{18 + 12}{(2)(8)} = -1.875$$

$$2\theta_s = -61.93°$$

$$\theta_s = -31.0°, 59.0° \blacktriangleleft$$

(*b*) $$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

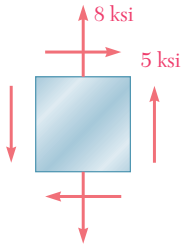
$$= \sqrt{\left(\frac{18 + 12}{2}\right)^2 + (8)^2}$$

$$\tau_{\max} = 17.00 \text{ ksi} \blacktriangleleft$$

(*c*) $$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{18 - 12}{2}$$

$$\sigma' = 3.00 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.13

For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = 0 \quad \sigma_y = 8 \text{ ksi} \quad \tau_{xy} = 5 \text{ ksi}$$

$$\frac{\sigma_x + \sigma_y}{2} = 4 \text{ ksi} \quad \frac{\sigma_x - \sigma_y}{2} = -4 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(*a*) $\underline{\theta = -25°}$ $2\theta = -50°$

$$\sigma_{x'} = 4 - 4 \cos(-50°) + 5 \sin(-50°) \qquad \sigma_{x'} = -2.40 \text{ ksi} \blacktriangleleft$$

$$\tau_{x'y'} = 4 \sin(-50°) + 5 \cos(-50°) \qquad \tau_{x'y'} = 0.1498 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = 4 + 4 \cos(-50°) - 5 \sin(-50) \qquad \sigma_{y'} = 10.40 \text{ ksi} \blacktriangleleft$$

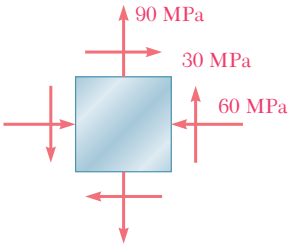
(*b*) $\underline{\theta = 10°}$ $2\theta = 20°$

$$\sigma_{x'} = 4 - 4 \cos(20°) + 5 \sin(20°) \qquad \sigma_{x'} = 1.951 \text{ ksi} \blacktriangleleft$$

$$\tau_{x'y'} = 4 \sin(20°) + 5 \cos(20°) \qquad \tau_{x'y'} = 6.07 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = 4 + 4 \cos(20°) - 5 \cos(20°) \qquad \sigma_{y'} = 6.05 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.14

For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = 90 \text{ MPa} \quad \tau_{xy} = 30 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa} \quad \frac{\sigma_x - \sigma_y}{2} = -75 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{xy'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(*a*) $\underline{\theta = -25°}$ $2\theta = -50°$

$$\sigma_{x'} = 15 - 75 \cos(-50°) + 30 \sin(-50°) \qquad \sigma_{x'} = -56.2 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = +75 \sin(-50°) + 30 \cos(-50°) \qquad \tau_{x'y'} = -38.2 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = 15 + 75 \cos(-50°) - 30 \sin(-50°) \qquad \sigma_{y'} = 86.2 \text{ MPa} \blacktriangleleft$$

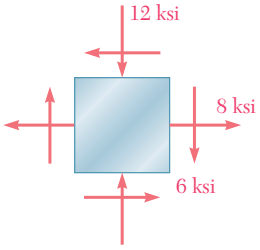
(*b*) $\underline{\theta = 10°}$ $2\theta = 20°$

$$\sigma_{x'} = 15 - 75 \cos(20°) + 30 \sin(20°) \qquad \sigma_{x'} = -45.2 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = +75 \sin(20°) + 30 \cos(20°) \qquad \tau_{x'y'} = 53.8 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = 15 + 75 \cos(20°) - 30 \sin(20°) \qquad \sigma_{y'} = 75.2 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.15

For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = 8 \text{ ksi} \quad \sigma_y = -12 \text{ ksi} \quad \tau_{xy} = -6 \text{ ksi}$$

$$\frac{\sigma_x + \sigma_y}{2} = -2 \text{ ksi} \quad \frac{\sigma_x - \sigma_y}{2} = 10 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(*a*) $\underline{\theta = -25°}$ $2\theta = -50°$

$$\sigma_{x'} = -2 + 10 \cos(-50°) - 6 \sin(-50°) \qquad \sigma_{x'} = 9.02 \text{ ksi} \blacktriangleleft$$

$$\tau_{x'y'} = -10 \sin(-50°) - 6 \cos(-50°) \qquad \tau_{x'y'} = 3.80 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = -2 - 10 \cos(-50°) + 6 \sin(-50°) \qquad \sigma_{y'} = -13.02 \text{ ksi} \blacktriangleleft$$

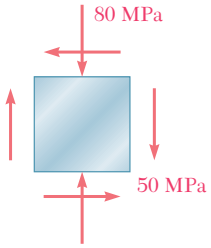
(*b*) $\underline{\theta = 10°}$ $2\theta = 20°$

$$\sigma_{x'} = -2 + 10 \cos(20°) - 6 \sin(20°) \qquad \sigma_{x'} = 5.34 \text{ ksi} \blacktriangleleft$$

$$\tau_{x'y'} = -10 \sin(20°) - 6 \cos(20°) \qquad \tau_{x'y'} = -9.06 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = -2 - 10 \cos(20°) + 6 \sin(20°) \qquad \sigma_{y'} = -9.34 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.16

For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = 0 \quad \sigma_y = -80 \text{ MPa} \quad \tau_{xy} = -50 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa} \quad \frac{\sigma_x - \sigma_y}{2} = 40 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy} \sin 2\theta$$

(*a*) $\underline{\theta = -25°}$ $2\theta = -50°$

$$\sigma_{x'} = -40 + 40\cos(-50°) - 50\sin(-50°) \qquad \sigma_{x'} = 24.0 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = -40\sin(-50°) - 50\cos(-50°) \qquad \tau_{x'y'} = -1.498 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = -40 - 40\cos(-50°) + 50\sin(-50°) \qquad \sigma_{y'} = -104.0 \text{ MPa} \blacktriangleleft$$

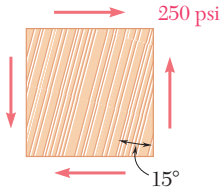
(*b*) $\underline{\theta = 10°}$ $2\theta = 20°$

$$\sigma_{x'} = -40 + 40\cos(20°) - 50\sin(20°) \qquad \sigma_{x'} = -19.51 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = -40\sin(20°) - 50\cos(20°) \qquad \tau_{x'y'} = -60.7 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = -40 - 40\cos(20°) + 50\sin(20°) \qquad \sigma_{y'} = -60.5 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.17

The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = \sigma_y = 0 \qquad \tau_{xy} = +250 \text{ psi} \qquad \theta = -15°$$

(*a*) $$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= 0 + 250\cos(-30°)$$

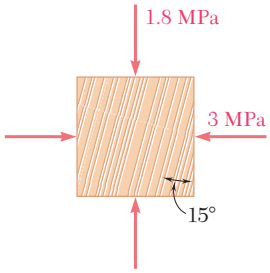
$$\tau_{x'y'} = 217 \text{ psi} \blacktriangleleft$$

(*b*) $$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 0 + 0 + 250\sin(-30°)$$

$$\sigma'_x = -125.0 \text{ psi} \blacktriangleleft$$

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PROBLEM 7.18

The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = -3 \text{ MPa} \qquad \sigma_y = -1.8 \text{ MPa} \qquad \tau_{xy} = 0$$

$$\theta = -15° \qquad 2\theta = -30°$$

(*a*) $$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\sin 2\theta$$

$$= -\frac{-3 + 1.8}{2}\sin(-30°) + 0$$

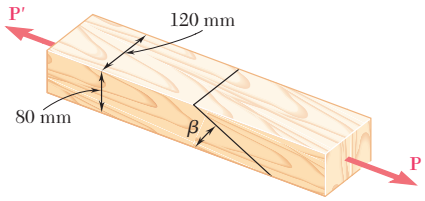
$$\tau_{x'y'} = -0.300 \text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-3 - 1.8}{2} + \frac{-3 + 1.8}{2}\cos(-30°) + 0$$

$$\sigma_{x'} = -2.92 \text{ MPa} \blacktriangleleft$$

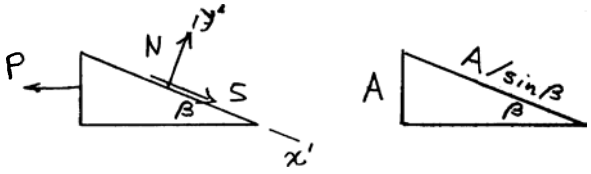
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PROBLEM 7.19

Two wooden members of 80 × 120-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 22°$ and that the maximum allowable stresses in the joint are, respectively, 400 kPa in tension (perpendicular to the splice) and 600 kPa in shear (parallel to the splice), determine the largest centric load **P** that can be applied.

SOLUTION



Forces Areas

$$A = (80)(120) = 9.6\times10^{3}\,\text{mm}^2 = 9.6\times10^{-3}\,\text{m}^2$$

$$N_{\text{all}} = \sigma_{\text{all}}\ A/\sin\beta = \frac{(400\times10^{3})(9.6\times10^{-3})}{\sin 22°} = 10.251\times10^{3}\,\text{N}$$

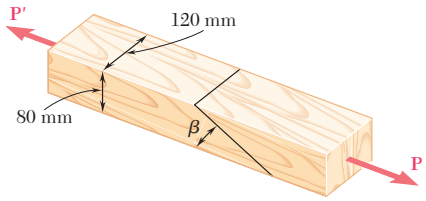
$$+\nearrow \Sigma F_{y'} = 0: \quad N - P\sin\beta = 0 \qquad P = \frac{N}{\sin\beta} = \frac{10.251\times10^{3}}{\sin 22°} = 27.4\times10^{3}\,\text{N}$$

$$S_{\text{all}} = \tau_{\text{all}}\ A/\sin\beta = \frac{(600\times10^{3})(9.6\times10^{-3})}{\sin 22°} = 15.376\times10^{3}\,\text{N}$$

$$+\searrow \Sigma F_{x'} = 0: \quad S - P\cos\beta = 0 \qquad P = \frac{S}{\cos\beta} = \frac{15.376\times10^{3}}{\cos 22°} = 16.58\times10^{3}\,\text{N}$$

The smaller value for P governs. $P = 16.58$ kN ◀

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PROBLEM 7.20

Two wooden members of 80 × 120-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 25°$ and that centric loads of magnitude $P = 10$ kN are applied to the members as shown, determine (*a*) the in-plane shearing stress parallel to the splice, (*b*) the normal stress perpendicular to the splice.

SOLUTION



Forces Areas

$$A = (80)(120) = 9.6\times10^{3}\,\text{mm}^2 = 9.6\times10^{-3}\,\text{m}^2$$

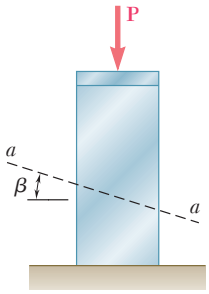
(*a*) $+\searrow \Sigma F_{x'} = 0: \quad S - P\cos\beta = 0 \qquad S = P\cos\beta = (10\times10^{3})\cos 25° = 9.063\times10^{3}\,\text{N}$

$$\tau = \frac{N}{A/\sin\beta} = \frac{(9.063\times10^{3})\sin 25°}{9.6\times10^{-3}} = 399\times10^{3}\,\text{Pa} \qquad \tau = 399\ \text{kPa} \blacktriangleleft$$

(*b*) $+\nearrow \Sigma F_{y'} = 0: \quad N - P\sin\beta = 0 \qquad N = P\sin\beta = (10\times10^{3})\sin 25° = 4.226\times10^{3}\,\text{N}$

$$\sigma = \frac{N}{A/\sin\beta} = \frac{(4.226\times10^{3})\sin 25°}{9.6\times10^{3}} = 186.0\times10^{3}\,\text{Pa} \qquad \sigma = 186.0\ \text{kPa} \blacktriangleleft$$

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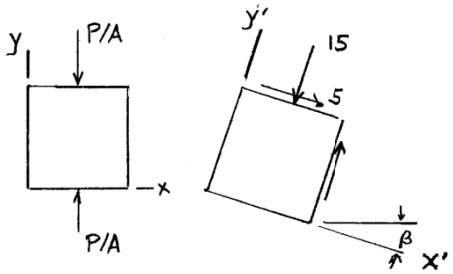


PROBLEM 7.21

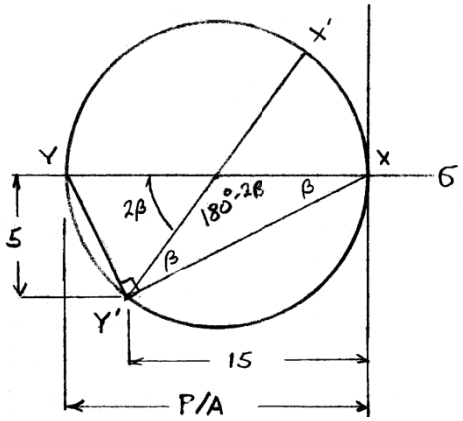
The centric force **P** is applied to a short post as shown. Knowing that the stresses on plane *a*-*a* are $\sigma = -15$ ksi and $\tau = 5$ ksi, determine (*a*) the angle β that plane *a*-*a* forms with the horizontal, (*b*) the maximum compressive stress in the post.

SOLUTION

$$\sigma_x = 0$$
$$\tau_{xy} = 0$$
$$\sigma_y = -P/A$$



(*a*) From the Mohr's circle,

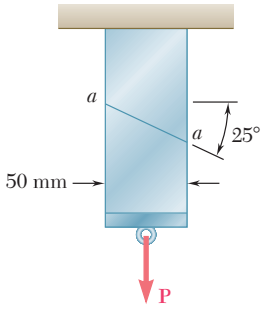


$$\tan\beta = \frac{5}{15} = 0.3333 \qquad \beta = 18.4° \blacktriangleleft$$

$$-\sigma = \frac{P}{2A} + \frac{P}{2A}\cos 2\beta$$

(*b*) $$\frac{P}{A} = \frac{2(-\sigma)}{1+\cos 2\beta} = \frac{(2)(15)}{1+\cos 2\beta} \qquad \frac{P}{A} = 16.67 \text{ ksi} \blacktriangleleft$$

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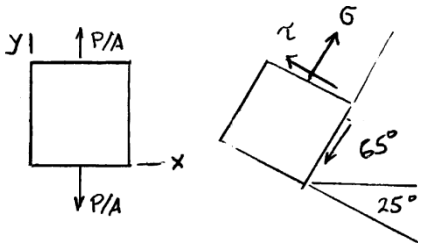


PROBLEM 7.22

Two members of uniform cross section 50×80 mm are glued together along plane *a-a* that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load **P** that can be applied.

SOLUTION

For plane *a-a*, $\theta = 65°$.



$$\sigma_x = 0, \quad \tau_{xy} = 0, \quad \sigma_y = \frac{P}{A}$$

$$\sigma = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta = 0 + \frac{P}{A}\sin^2 65° + 0$$

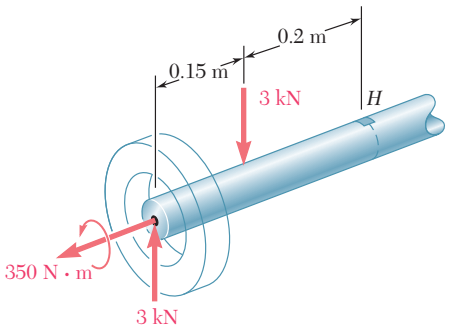
$$P = \frac{A\sigma}{\sin^2 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(800\times10^{3})}{\sin^2 65°} = 3.90\times10^{3}\ \text{N}$$

$$\tau = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) = \frac{P}{A}\sin 65°\cos 65° + 0$$

$$P = \frac{A\tau}{\sin 65°\cos 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(600\times10^{3})}{\sin 65°\cos 65°} = 6.27\times10^{3}\ \text{N}$$

Allowable value of *P* is the smaller one. $P = 3.90$ kN ◀

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PROBLEM 7.23

The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 32 mm, determine (*a*) the principal planes and principal stresses at point *H* located on top of the axle, (*b*) the maximum shearing stress at the same point.

SOLUTION

$$c = \frac{1}{2}d = \frac{1}{2}(32) = 16\text{ mm} = 16 \times 10^{-3}\text{ m}$$

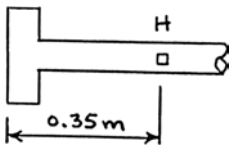
Torsion:
$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(350\text{ N}\cdot\text{m})}{\pi(16 \times 10^{-3}\text{ m})^3} = 54.399 \times 10^6\text{ Pa} = 54.399\text{ MPa}$$

Bending:
$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(16 \times 10^{-3})^4 = 51.472 \times 10^{-9}\text{ m}^4$$

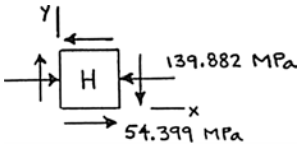
$$M = (0.15\text{ m})(3 \times 10^3\text{ N}) = 450\text{ N}\cdot\text{m}$$

$$\sigma = -\frac{My}{I} = -\frac{(450)(16 \times 10^{-3})}{51.472 \times 10^{-9}} = -139.882 \times 10^6\text{ Pa} = -139.882\text{ MPa}$$

Top view:



Stresses:



$$\sigma_x = -139.882\text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -54.399\text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(-139.882 + 0) = -69.941\text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(-69.941)^2 + (-54.399)^2} = 88.606\text{ MPa}$$

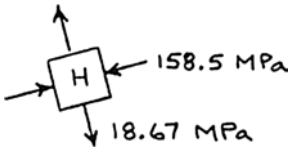
(*a*)
$$\sigma_{max} = \sigma_{ave} + R = -69.941 + 88.606 \qquad \sigma_{max} = 18.67\text{ MPa} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R = -69.941 - 88.606 \qquad \sigma_{min} = -158.5\text{ MPa} \blacktriangleleft$$

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PROBLEM 7.23 *(Continued)*

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-54.399)}{-139.882} = 0.77778 \qquad 2\theta_p = 37.88°$$

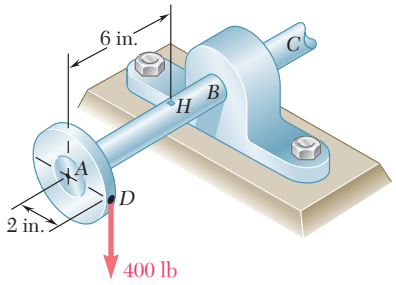


$\theta_p = 18.9°$ and $108.9°$ ◀

(*b*) $\tau_{\max} = R = 88.6$ MPa

$\tau_{\max} = 88.6$ MPa ◀

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PROBLEM 7.24

A 400-lb vertical force is applied at D to a gear attached to the solid l-in. diameter shaft AB. Determine the principal stresses and the maximum shearing stress at point H located as shown on top of the shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point H:

$$V = 400 \text{ lb} \qquad M = (400)(6) = 2400 \text{ lb} \cdot \text{in.}$$

$$T = (400)(2) = 800 \text{ lb} \cdot \text{in.}$$

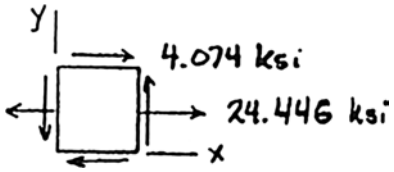
Shaft cross section: $d = 1 \text{ in.} \quad c = \frac{1}{2}d = 0.5 \text{ in.}$

$$J = \frac{\pi}{2}c^4 = 0.098175 \text{ in}^4 \qquad I = \frac{1}{2}J = 0.049087 \text{ in}^4$$

Torsion: $$\tau = \frac{Tc}{J} = \frac{(800)(0.5)}{0.098175} = 4.074 \times 10^3 \text{ psi} = 4.074 \text{ ksi}$$

Bending: $$\sigma = \frac{Mc}{I} = \frac{(2400)(0.5)}{0.049087} = 24.446 \times 10^3 \text{ psi} = 24.446 \text{ ksi}$$

Transverse shear: Stress at point H is zero.



$$\sigma_x = 24.446 \text{ ksi}, \quad \sigma_y = 0, \quad \tau_{xy} = 4.074 \text{ ksi}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 12.223 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(12.223)^2 + (4.074)^2}$$

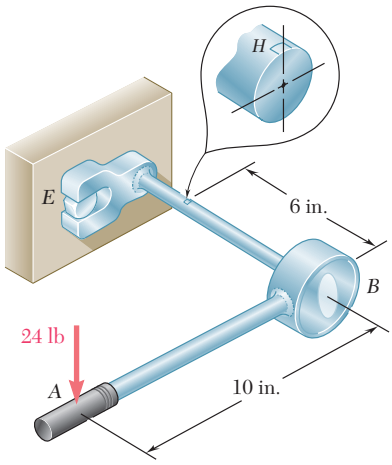
$$= 12.884 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 25.1 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -0.661 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 12.88 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.25

A mechanic uses a crowfoot wrench to loosen a bolt at E. Knowing that the mechanic applies a vertical 24-lb force at A, determine the principal stresses and the maximum shearing stress at point H located as shown as on top of the $\frac{3}{4}$-in. diameter shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point H:

$$V = 24 \text{ lb} \quad M = (24)(6) = 144 \text{ lb} \cdot \text{in.}$$
$$T = (24)(10) = 240 \text{ lb} \cdot \text{in.}$$

Shaft cross section: $d = 0.75$ in., $c = \frac{1}{2}d = 0.375$ in.

$$J = \frac{\pi}{2}c^4 = 0.031063 \text{ in}^4 \quad I = \frac{1}{2}J = 0.015532 \text{ in}^4$$

Torsion:
$$\tau = \frac{Tc}{J} = \frac{(240)(0.375)}{0.031063} = 2.897 \times 10^3 \text{ psi} = 2.897 \text{ ksi}$$

Bending:
$$\sigma = \frac{Mc}{I} = \frac{(144)(0.375)}{0.015532} = 3.477 \times 10^3 \text{ psi} = 3.477 \text{ ksi}$$

Transverse shear: At point H, the stress due to transverse shear is zero.

Resultant stresses: $\sigma_x = 3.477$ ksi, $\sigma_y = 0$, $\tau_{xy} = 2.897$ ksi

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 1.738 \text{ ksi}$$

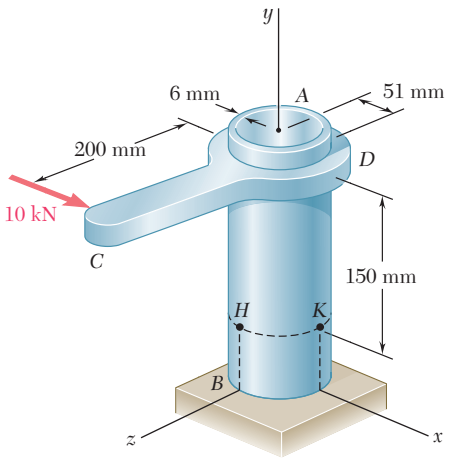
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{1.738^2 + 2.897^2} = 3.378 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 5.12 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -1.640 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 3.38 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.26

The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *K*.

SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = 4.1855 \times 10^6 \text{ mm}^4$$

$$= 4.1855 \times 10^{-6} \text{ m}^4$$

$$I = \frac{1}{2}J = 2.0927 \times 10^{-6} \text{m}^4$$

Force-couple system at center of tube in the plane containing points *H* and *K*:

$$F_x = 10 \text{ kN}$$

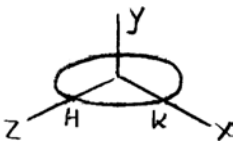
$$= 10 \times 10^3 \text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3})$$

$$= 2000 \text{ N} \cdot \text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3})$$

$$= -1500 \text{ N} \cdot \text{m}$$



Torsion: At point *K*, place local *x*-axis in negative global *z*-direction.

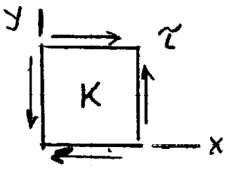
$$T = M_y = 2000 \text{ N} \cdot \text{m}$$

$$c = r_o = 51 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^6}$$

$$= 24.37 \times 10^6 \text{ Pa}$$

$$= 24.37 \text{ MPa}$$



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PROBLEM 7.26 *(Continued)*

Transverse shear: Stress due to transverse shear $V = F_x$ is zero at point *K*.

Bending:

$$|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51\times10^{-3})}{2.0927\times10^{-6}} = 36.56\times10^{6}\text{ Pa} = 36.56\text{ MPa}$$

Point *K* lies on compression side of neutral axis.

$$\sigma_y = -36.56\text{ MPa}$$

Total stresses at point *K*:

$$\sigma_x = 0, \quad \sigma_y = -36.56\text{ MPa}, \quad \tau_{xy} = 24.37\text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28\text{ MPa}$$

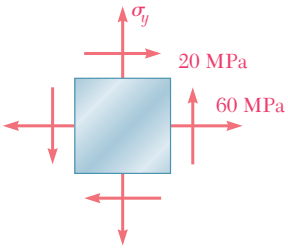
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46\text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = -18.28 + 30.46 \qquad \sigma_{\max} = 12.18\text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R = -18.28 - 30.46 \qquad \sigma_{\min} = -48.7\text{ MPa} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 30.5\text{ MPa} \blacktriangleleft$$

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PROBLEM 7.27

For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 75 MPa.

SOLUTION

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_y = ?, \quad \tau_{xy} = 20 \text{ MPa}$$

Let

$$u = \frac{\sigma_x - \sigma_y}{2}.$$

Then

$$\sigma_y = \sigma_x - 2u$$

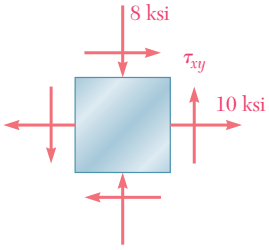
$$R = \sqrt{u^2 + \tau_{xy}^2} = 75 \text{ MPa}$$

$$u = \pm\sqrt{R^2 - \tau_{xy}^2} = \pm\sqrt{75^2 - 20^2} = 72.284 \text{ MPa}$$

$$\sigma_y = \sigma_x - 2u = 60 \mp (2)(72.284) = -84.6 \text{ MPa} \quad \text{or} \quad 205 \text{ MPa}$$

Largest value of σ_y is required. $\sigma_y = 205 \text{ MPa}$ ◀

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PROBLEM 7.28

For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (*b*) the corresponding principal stresses.

SOLUTION

$$\sigma_x = 10 \text{ ksi}, \quad \sigma_y = -8 \text{ ksi}, \quad \tau_{xy} = ?$$

$$\tau_{\max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{z}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{10-(-8)}{z}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{9^2 + \tau_{xy}^2} = 12 \text{ ksi}$$

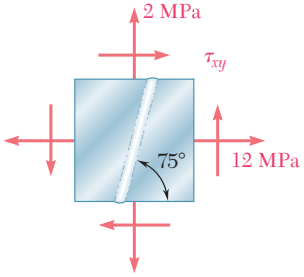
(*a*) $\tau_{xy} = \sqrt{12^2 - 9^2}$ $\qquad \tau_{xy} = 7.94 \text{ ksi}$ ◀

(*b*) $\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 1 \text{ ksi}$

$\sigma_a = \sigma_{\text{ave}} + R = 1 + 12 = 13 \text{ ksi}$ $\qquad \sigma_a = 13.00 \text{ ksi}$ ◀

$\sigma_b = \sigma_{\text{ave}} - R = 1 - 12 = -11 \text{ ksi}$ $\qquad \sigma_b = -11.00 \text{ ksi}$ ◀

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PROBLEM 7.29

For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.

SOLUTION

$$\sigma_x = 12 \text{ MPa}, \quad \sigma_y = 2 \text{ MPa}, \quad \tau_{xy} = ?$$

Since $\tau_{x'y'} = 0$, x'-direction is a principal direction.



$$\theta_p = -15°$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

(*a*) $$\tau_{xy} = \frac{1}{2}(\sigma_x - \sigma_y) \tan 2\theta_p = \frac{1}{2}(12-2)\tan(-30°)$$

$\tau_{xy} = -2.89$ MPa ◀

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5^2 + 2.89^2} = 5.7735 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 7 \text{ MPa}$$

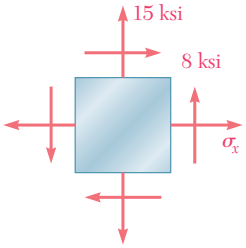
(*b*) $\sigma_a = \sigma_{\text{ave}} + R = 7 + 5.7735$

$\sigma_a = 12.77$ MPa ◀

$\sigma_b = \sigma_{\text{ave}} - R = 7 - 5.7735$

$\sigma_b = 1.226$ MPa ◀

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PROBLEM 7.30

Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 10 ksi.

SOLUTION

$$\sigma_x = ?, \quad \sigma_y = 15 \text{ ksi}, \quad \tau_{xy} = 8 \text{ ksi}$$

Let $u = \dfrac{\sigma_x - \sigma_y}{2}$ $\qquad \sigma_x = \sigma_y + 2u$

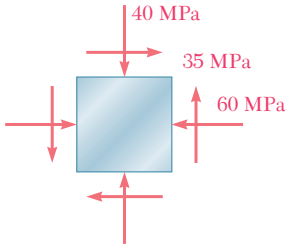
$$R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{\max} = 10 \text{ ksi}$$

$$u = \pm\sqrt{R^2 - \tau_{xy}^2} = \pm\sqrt{10^2 - 8^{2z}} = \pm 6 \text{ ksi}$$

$$\sigma_x = \sigma_y + 2u = 15 \pm (2)(6) = 27 \text{ ksi} \quad \text{or} \quad 3 \text{ ksi}$$

Allowable range: $3 \text{ ksi} \le \sigma_x \le 27 \text{ ksi}$ ◀

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PROBLEM 7.31

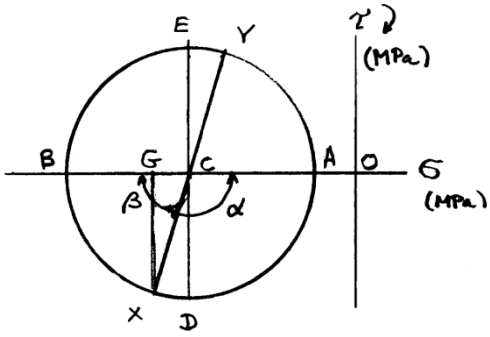
Solve Probs. 7.5 and 7.9, using Mohr's circle.

PROBLEM 7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = -60 \text{ MPa},$$
$$\sigma_y = -40 \text{ MPa},$$
$$\tau_{xy} = 35 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -50 \text{ MPa}$$



Plotted points for Mohr's circle:

$$X: (\sigma_x, -\tau_{xy}) = (-60 \text{ MPa}, -35 \text{ MPa})$$
$$Y: (\sigma_y, \tau_{xy}) = (-40 \text{ MPa}, 35 \text{ MPa})$$
$$C: (\sigma_{\text{ave}}, 0) = (-50 \text{ MPa}, 0)$$

(*a*) $\tan\beta = \dfrac{GX}{CG} = \dfrac{35}{10} = 3.500$

$\beta = 74.05°$

$\theta_b = -\dfrac{1}{2}\beta = -37.03°$ $\qquad \theta_b = -37.0° \blacktriangleleft$

$\alpha = 180° - \beta = 105.95°$

$\theta_a = \dfrac{1}{2}\alpha = 52.97°$ $\qquad \theta_a = 53.0° \blacktriangleleft$

$R = \sqrt{\overline{CG}^2 + \overline{GX}^2} = \sqrt{10^2 + 35^2} = 36.4 \text{ MPa}$

(*b*) $\sigma_{\min} = \sigma_{\text{ave}} - R = -50 - 36.4$ $\qquad \sigma_{\min} = -86.4 \text{ MPa} \blacktriangleleft$

$\sigma_{\max} = \sigma_{\text{ave}} + R = -50 + 36.4$ $\qquad \sigma_{\max} = -13.60 \text{ MPa} \blacktriangleleft$

(*a′*) $\theta_d = \theta_B + 45° = 7.97°$ $\qquad \theta_d = 8.0° \blacktriangleleft$

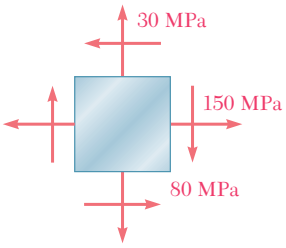
$\theta_e = \theta_A + 45° = 97.97°$ $\qquad \theta_e = 98.0° \blacktriangleleft$

(*b′*) $\tau_{\max} = R = 36.4 \text{ MPa}$ $\qquad \tau_{\max} = 36.4 \text{ MPa} \blacktriangleleft$

(*c′*) $\sigma' = \sigma_{\text{ave}} = -50 \text{ MPa}$ $\qquad \sigma' = -50.0 \text{ MPa} \blacktriangleleft$

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PROBLEM 7.32



Solve Probs. 7.7 and 7.11, using Mohr's circle.

PROBLEM 7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 150 \text{ MPa}$$

$$\sigma_y = 30 \text{ MPa}$$

$$\tau_{xy} = -80 \text{ MPa}$$

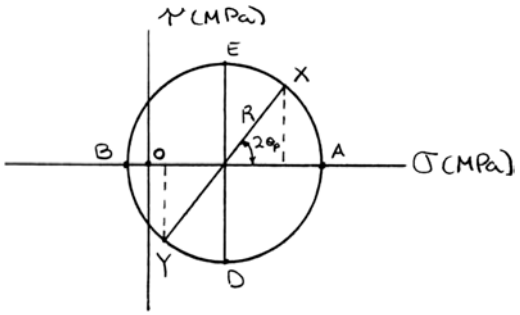
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = 90 \text{ MPa}$$

Plotted points for Mohr's circle:

$$X: (\sigma_x, -\tau_{xy}) = (150 \text{ MPa}, 80 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (30 \text{ MPa}, -80 \text{ MPa})$$

$$C: (\sigma_{\text{ave}}, 0) = (90 \text{ MPa}, 0)$$



$$\frac{\sigma_x - \sigma_y}{2} = \frac{(150 - 30)}{2} = 60$$

$$R = \sqrt{(60)^2 + (80)^2} = 100$$

(*a*) $\tan 2\theta_p = \dfrac{80}{60}$

$2\theta_p = 53.130°$ $\qquad \theta_p = -26.6° \text{ and } 63.4°$ ◀

(*b*) $\sigma_{\max} = \sigma_{\text{ave}} + R = 90 + 100$ $\qquad \sigma_{\max} = 190.0 \text{ MPa}$ ◀

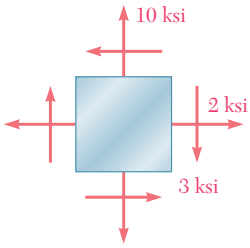
$\sigma_{\min} = \sigma_{\text{ave}} - R = 90 - 100$ $\qquad \sigma_{\min} = -10.00 \text{ MPa}$ ◀

(*a*′) $\theta_s = \theta_p + 45°$ $\qquad \theta_s = 18.4° \text{ and } 108.4°$ ◀

(*b*′) $\tau_{\max} = R$ $\qquad \tau_{\max} = 100.0 \text{ MPa}$ ◀

(c′) $\sigma' = \sigma_{\text{ave}}$ $\qquad \sigma' = 90.0 \text{ MPa}$ ◀

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PROBLEM 7.33

Solve Prob. 7.10, using Mohr's circle.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 2 \text{ ksi} \qquad \sigma_y = 10 \text{ ksi} \qquad \tau_{xy} = -3 \text{ ksi}$$

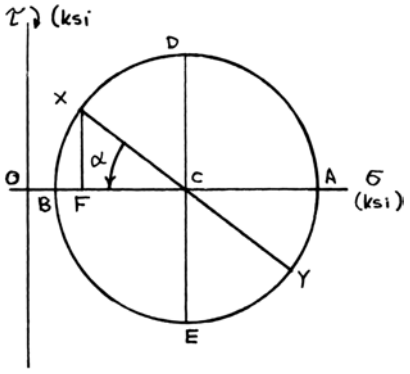
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{2 + 10}{2} = 6 \text{ ksi}$$

Plotted points for Mohr's circle:

$$X: \ (\sigma_x, -\tau_{xy}) = (2 \text{ ksi}, 3 \text{ ksi})$$

$$Y: \ (\sigma_y, \tau_{xy}) = (10 \text{ ksi}, -3 \text{ ksi})$$

$$C: \ (\sigma_{\text{ave}}, 0) = (6 \text{ ksi}, 0)$$



$$\tan \alpha = \frac{FX}{FC} = \frac{3}{4} = 0.75$$

$$\alpha = 36.87°$$

$$\theta_B = \frac{1}{2}\alpha = 18.43°$$

(*a*) $\theta_D = \theta_B - 45° = -26.6°$ $\qquad \theta_D = -26.6° \blacktriangleleft$

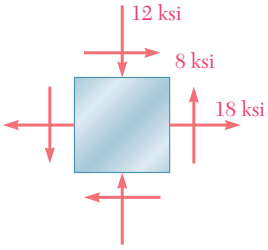
$\theta_E = \theta_B + 45° = 63.4°$ $\qquad \theta_E = 63.4° \blacktriangleleft$

$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{4^2 + 3^2} = 5 \text{ ksi}$$

(*b*) $\tau_{\max} = R = 5.00 \text{ ksi}$ $\qquad \tau_{\max} = 5.00 \text{ ksi} \blacktriangleleft$

(*c*) $\sigma' = \sigma_{\text{ave}} = 6.00 \text{ ksi}$ $\qquad \sigma' = 6.00 \text{ ksi} \blacktriangleleft$

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PROBLEM 7.34

Solve Prob. 7.12, using Mohr's circle.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 18 \text{ ksi} \qquad \sigma_y = -12 \text{ ksi} \qquad \tau_{xy} = 8 \text{ ksi}$$

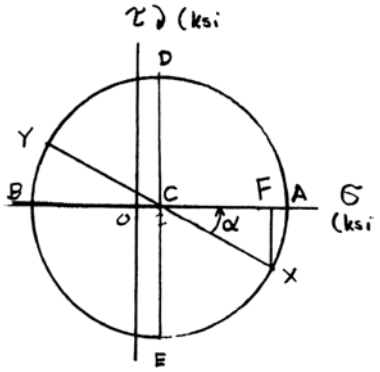
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = 3 \text{ ksi}$$

Plotted points for Mohr's circle:

$$X\text{: } (\sigma_x, -\tau_{xy}) = (18 \text{ ksi}, -8 \text{ ksi})$$

$$Y\text{: } (\sigma_y, \tau_{xy}) = (-12 \text{ ksi}, 8 \text{ ksi})$$

$$C\text{: } (\sigma_{\text{ave}}, 0) = (3 \text{ ksi}, 0)$$



$$\tan\alpha = \frac{FX}{CF} = \frac{8}{15} = 0.5333$$

$$\alpha = 28.07°$$

$$\theta_A = \frac{1}{2}\alpha = 14.04°$$

(*a*) $\theta_D = \theta_A + 45° = 59.0°$ $\qquad \theta_D = 59.0°$ ◀

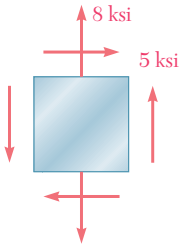
$\theta_E = \theta_A - 45° = -30.1°$ $\qquad \theta_E = -30.1°$ ◀

$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{15^2 + 8^2} = 17 \text{ ksi}$$

(*b*) $\tau_{\max} = R = 17.00 \text{ ksi}$ $\qquad \tau_{\max} = 17.00 \text{ ksi}$ ◀

(*c*) $\sigma' = \sigma_{\text{ave}} = 3.00 \text{ ksi}$ $\qquad \sigma' = 3.00 \text{ ksi}$ ◀

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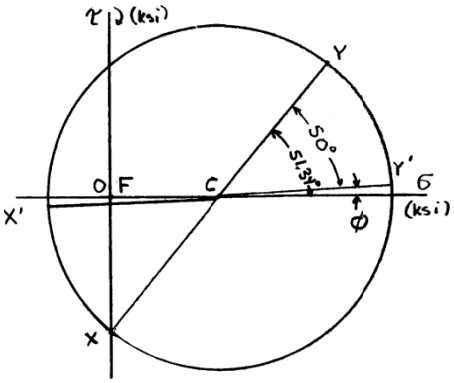
PROBLEM 7.35

Solve Prob. 7.13, using Mohr's circle.

PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = 0,$$
$$\sigma_y = 8 \text{ ksi},$$
$$\tau_{xy} = 5 \text{ ksi}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = 4 \text{ ksi}$$

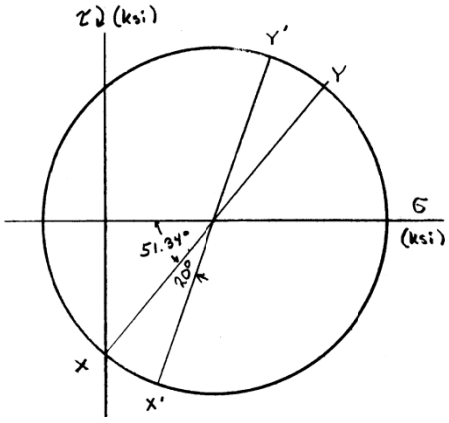


Plotted points for Mohr's circle:

$$X: (0, -5 \text{ ksi})$$
$$Y: (8 \text{ ksi}, 5 \text{ ksi})$$
$$C: (4 \text{ ksi}, 0)$$

$$\tan 2\theta_p = \frac{FX}{FC} = \frac{5}{4} = 1.25$$
$$2\theta_p = 51.34°$$
$$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{4^2 + 5^2} = 6.4031 \text{ ksi}$$

(*a*) $\theta = 25°$ ↻. $2\theta = 50°$ ↻



$$\varphi = 51.34° - 50° = 1.34°$$

$$\sigma_{x'} = \sigma_{\text{ave}} - R\cos\varphi \qquad \sigma_{x'} = -2.40 \text{ ksi} \blacktriangleleft$$

$$\tau_{x'y'} = R\sin\varphi \qquad \tau_{x'y'} = 0.1497 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{\text{ave}} + R\cos\varphi \qquad \sigma_{y'} = 10.40 \text{ ksi} \blacktriangleleft$$

(*b*) $\theta = 10°$ ↺. $2\theta = 20°$ ↺

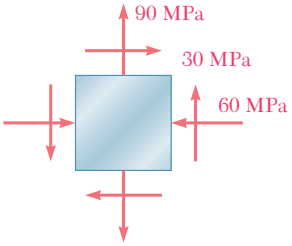
$$\varphi = 51.34° + 20° = 71.34°$$

$$\sigma_{x'} = \sigma_{\text{ave}} - R\cos\varphi \qquad \sigma_{x'} = 1.951 \text{ ksi} \blacktriangleleft$$

$$\tau_{x'y'} = R\sin\varphi \qquad \tau_{x'y'} = 6.07 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{\text{ave}} + R\cos\varphi \qquad \sigma_{y'} = 6.05 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.36

Solve Prob. 7.14, using Mohr's circle.

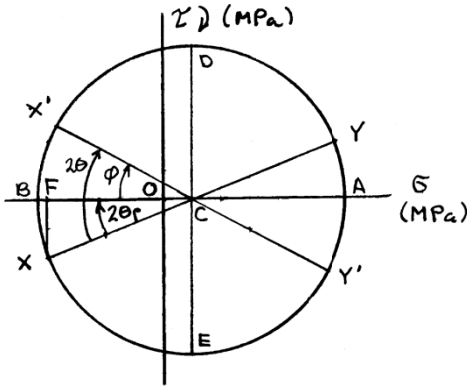
PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = -60 \text{ MPa},$$
$$\sigma_y = 90 \text{ MPa},$$
$$\tau_{xy} = 30 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa}$$

Plotted points for Mohr's circle:

X: (–60 MPa, – 30 MPa)
Y: (90 MPa, 30 MPa)
C: (15 MPa, 0)



$$\tan 2\theta_p = \frac{FX}{FC} = \frac{30}{75} = 0.4$$
$$2\theta_p = 21.80° \quad \theta_P = 10.90° \circlearrowleft$$
$$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{75^2 + 30^2} = 80.78 \text{ MPa}$$

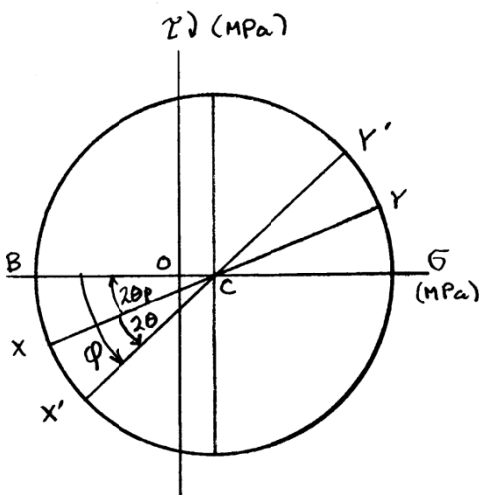
(*a*) $\theta = 25° \circlearrowleft$. $2\theta = 50° \circlearrowleft$

$$\varphi = 2\theta - 2\theta_P = 50° - 21.80° = 28.20° \circlearrowleft$$

$$\sigma_{x'} = \sigma_{\text{ave}} - R\cos\varphi \qquad \sigma_{x'} = -56.2 \text{ MPa} \blacktriangleleft$$
$$\tau_{x'y'} = -R\sin\varphi \qquad \tau_{x'y'} = -38.2 \text{ MPa} \blacktriangleleft$$
$$\sigma_{y'} = \sigma_{\text{ave}} + R\cos\varphi \qquad \sigma_{y'} = 86.2 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.36 *(Continued)*



(*b*) $\underline{\theta = 10^\circ}$ ↻. $2\theta = 20^\circ$ ↻

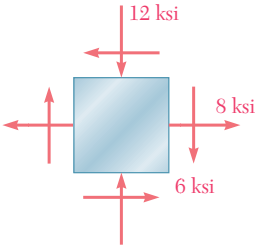
$\varphi = 2\theta_p + 2\theta = 21.80^\circ + 20^\circ = 41.80^\circ$ ↻

$\sigma_{x'} = \sigma_{\text{ave}} - R\cos\varphi$ $\sigma_{x'} = -45.2$ MPa ◀

$\tau_{x'y'} = R\sin\varphi$ $\tau_{x'y'} = 53.8$ MPa ◀

$\sigma_{y'} = \sigma_{\text{ave}} + R\cos\varphi$ $\sigma_{y'} = 75.2$ MPa ◀

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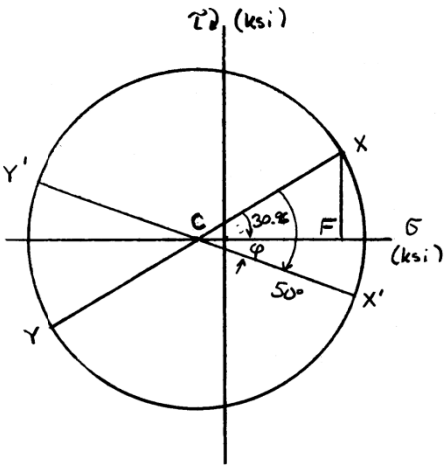


PROBLEM 7.37

Solve Prob. 7.15, using Mohr's circle.

PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

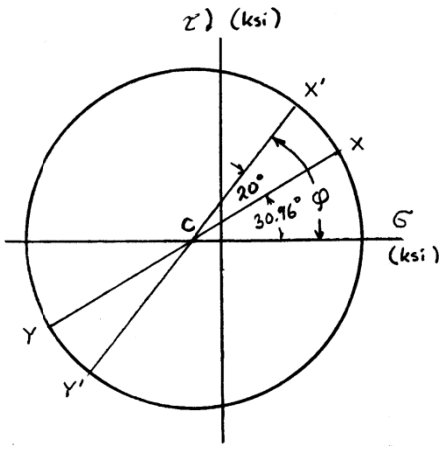


$$\sigma_x = 8 \text{ ksi},$$
$$\sigma_y = -12 \text{ ksi},$$
$$\tau_{xy} = -6 \text{ ksi}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -2 \text{ ksi}$$

Plotted points for Mohr's circle:

X: (8 ksi, 6 ksi)
Y: (−12 ksi, −6 ksi)
C: (−2 ksi, 0)

$$\tan 2\theta_p = \frac{FX}{CF} = \frac{6}{10} = 0.6$$
$$2\theta_p = 30.96°$$
$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{10^2 + 6^2} = 11.66 \text{ ksi}$$



(*a*) $\theta = 25°$ ↻. $2\theta = 50°$ ↻

$$\varphi = 50° - 30.96° = 19.04°$$

$\sigma_{x'} = \sigma_{\text{ave}} + R\cos\varphi$ $\quad \sigma_{x'} = 9.02$ ksi ◀

$\tau_{x'y'} = R\sin\varphi$ $\quad \tau_{x'y'} = 3.80$ ksi ◀

$\sigma_{y'} = \sigma_{\text{ave}} - R\cos\varphi$ $\quad \sigma_{y'} = -13.02$ ksi ◀

(*b*) $\theta = 10°$ ↺. $2\theta = 20°$ ↺

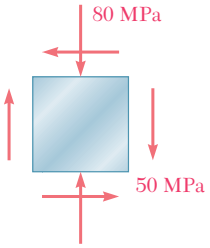
$$\varphi = 30.96° + 20° = 50.96°$$

$\sigma_{x'} = \sigma_{\text{ave}} + R\cos\varphi$ $\quad \sigma_{x'} = 5.34$ ksi ◀

$\tau_{x'y'} = -R\sin\varphi$ $\quad \tau_{x'y'} = -9.06$ ksi ◀

$\sigma_{y'} = \sigma_{\text{ave}} - R\cos\varphi$ $\quad \sigma_{y'} = -9.34$ ksi ◀

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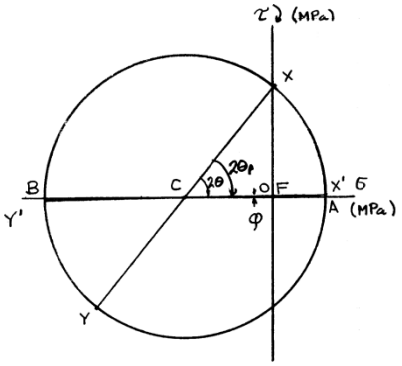


PROBLEM 7.38

Solve Prob. 7.16, using Mohr's circle.

PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

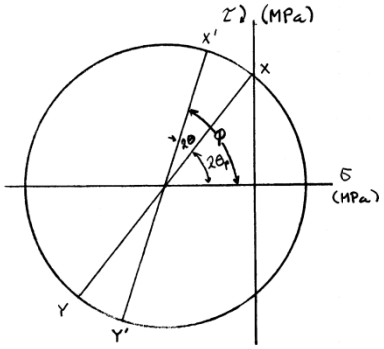


$$\sigma_x = 0,$$
$$\sigma_y = -80 \text{ MPa},$$
$$\tau_{xy} = -50 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa}$$

Plotted points for Mohr's circle:

$$X: (0, 50 \text{ MPa})$$
$$Y: (-80 \text{ MPa}, -50 \text{ MPa})$$
$$C: (-40 \text{ MPa}, 0)$$

$$\tan 2\theta_p = \frac{FX}{CF} = \frac{50}{40} = 1.25$$
$$2\theta_p = 51.34°$$
$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{40^2 + 50^2}$$
$$= 64.031 \text{ MPa}$$



(*a*) $\theta = 25°$ ↻. $2\theta = 50°$ ↻

$$\varphi = 51.34° - 50° = 1.34° \text{ ↺}$$

$$\sigma_{x'} = \sigma_{\text{ave}} + R\cos\varphi \qquad \sigma_{x'} = 24.0 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = -R\sin\varphi \qquad \tau_{x'y'} = -1.497 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{\text{ave}} - R\cos\varphi \qquad \sigma_{y'} = -104.0 \text{ MPa} \blacktriangleleft$$

(*b*) $\theta = 10°$ ↺. $2\theta = 20°$ ↺

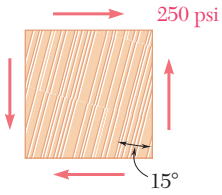
$$\varphi = 51.34° + 20° = 71.34°$$

$$\sigma_{x'} = \sigma_{\text{ave}} + R\cos\varphi \qquad \sigma_{x'} = -19.51 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = -R\sin\varphi \qquad \tau_{x'y'} = -60.7 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{\text{ave}} - R\cos\varphi \qquad \sigma_{y'} = -60.5 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.39

Solve Prob. 7.17, using Mohr's circle.

PROBLEM 7.17 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = \sigma_y = 0$$

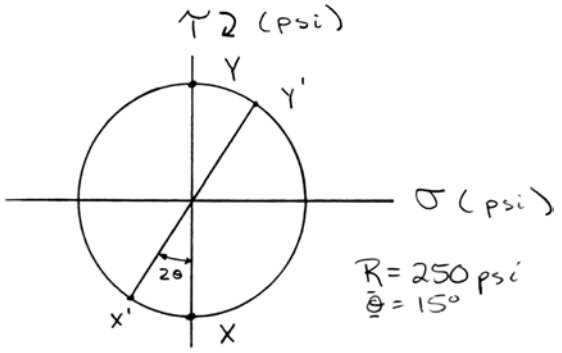
$$\tau_{xy} = 250 \text{ psi}$$

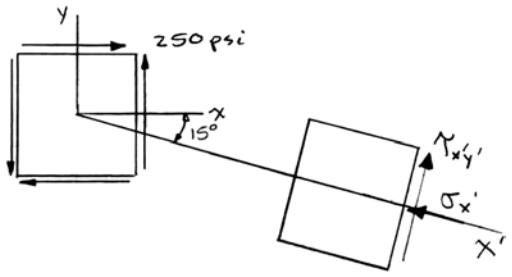
Plotted points for Mohr's circle:

$$X = (0, -250 \text{ psi})$$

$$Y = (0, 250 \text{ psi})$$

$$C = (0, 0)$$





(*a*) $\tau_{x'y'} = R\cos 2\theta$

$= (250 \text{ psi})\cos 30°$

$= 217 \text{ psi}$

$\tau_{x'y'} = 217 \text{ psi}$ ◀

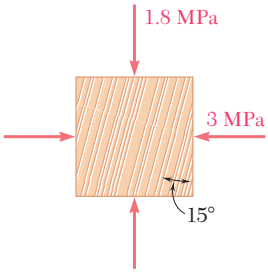
(*b*) $\sigma_{x'} = -R\sin 2\theta$

$= -(250 \text{ psi})\sin 30°$

$= -125.0 \text{ psi}$

$\sigma_{x'} = -125.0 \text{ psi}$ ◀

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PROBLEM 7.40

Solve Prob. 7.18, using Mohr's circle.

PROBLEM 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = -3 \text{ MPa} \qquad \sigma_y = -1.8 \text{ MPa} \qquad \tau_{xy} = 0$$

$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -2.4 \text{ MPa}$$

Points.

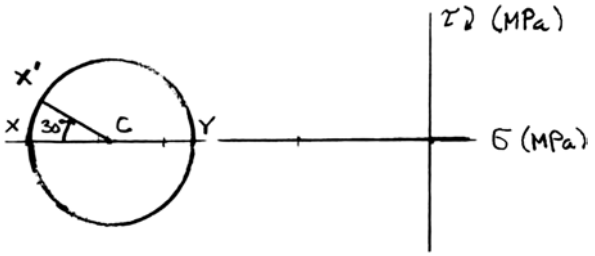
$$X: (\sigma_x, -\tau_{xy}) = (-3 \text{ MPa}, 0)$$

$$Y: (\sigma_y, -\tau_{xy}) = (-1.8 \text{ MPa}, 0)$$

$$C: (\sigma_{\text{ave}}, 0) = (-2.4 \text{ MPa}, 0)$$

$$\theta = -15° \qquad 2\theta = -30°$$

$$\overline{CX} = 0.6 \text{ MPa} \qquad R = 0.6 \text{ MPa}$$

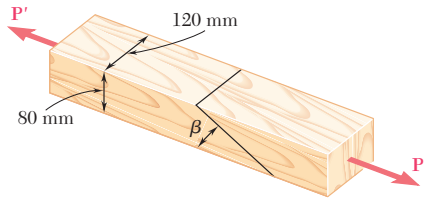


(*a*) $\tau_{x'y'} = -\overline{CX'} \sin 30° = -R \sin 30° = -0.6 \sin 30° = -0.300 \text{ MPa}$ $\qquad \tau_{x'y'} = -0.300 \text{ MPa}$ ◀

(*b*) $\sigma_{x'} = \sigma_{\text{ave}} - \overline{CX'} \cos 30° = -2.4 - 0.6 \cos 30° = -2.92 \text{ MPa}$ $\qquad \sigma_{x'} = -2.92 \text{ MPa}$ ◀

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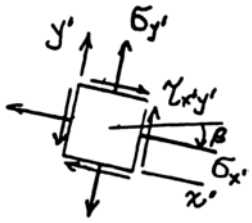
PROBLEM 7.41



Solve Prob. 7.19, using Mohr's circle.

PROBLEM 7.19 Two wooden members of 80 × 120-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 22°$ and that the maximum allowable stresses in the joint are, respectively, 400 kPa in tension (perpendicular to the splice) and 600 kPa in shear (parallel to the splice), determine the largest centric load **P** that can be applied.

SOLUTION



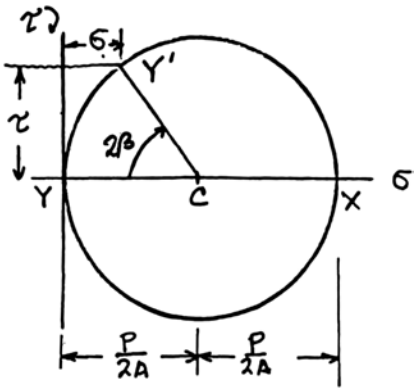
$$\sigma_x = \frac{P}{A}, \qquad \sigma_y = 0 \qquad \tau_{xy} = 0$$

Plotted points for Mohr's circle:

$$X: \left(\frac{P}{A}, 0\right), \qquad Y: (0, 0)$$

$$C: \left(\frac{P}{2}, 0\right)$$

$$R = \overline{CX} = \frac{P}{2A}$$



Coordinates of point Y':

$$\sigma = \frac{P}{2A}(1 - \cos 2\beta)$$

$$\tau = \frac{P}{2A}\sin 2\beta$$

Data: $A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$

If $\sigma = 400 \text{ kPa} = 400 \times 10^3 \text{ Pa}$,

$$P = \frac{2A\sigma}{1 - \cos 2\beta} = \frac{(2)(9.6 \times 10^{-3})(400 \times 10^3)}{(1 - \cos 44°)}$$

$$= 27.4 \times 10^3 \text{ N} = 27.4 \text{ kN}$$

If $\tau = 600 \text{ kPa} = 600 \times 10^3 \text{ Pa}$,

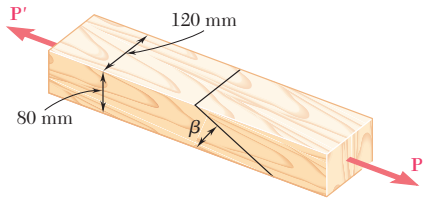
$$P = \frac{2A\tau}{\sin 2\beta} = \frac{(2)(9.6 \times 10^{-3})(600 \times 10^3)}{(\sin 44°)}$$

$$= 16.58 \times 10^3 \text{ N} = 16.58 \text{ kN}$$

The smaller value of P governs. $P = 16.58 \text{ kN}$ ◀

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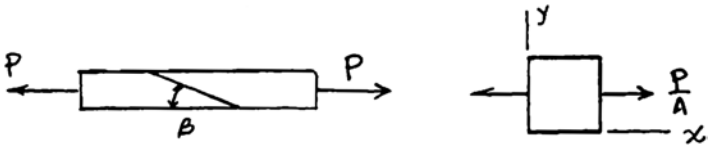
PROBLEM 7.42

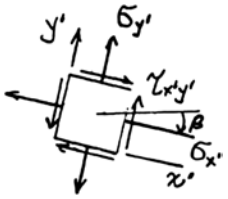


Solve Prob. 7.20, using Mohr's circle.

PROBLEM 7.20 Two wooden members of 80 × 120-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 25°$ and that centric loads of magnitude $P = 10$ kN are applied to the members as shown, determine (*a*) the in-plane shearing stress parallel to the splice, (*b*) the normal stress perpendicular to the splice.

SOLUTION



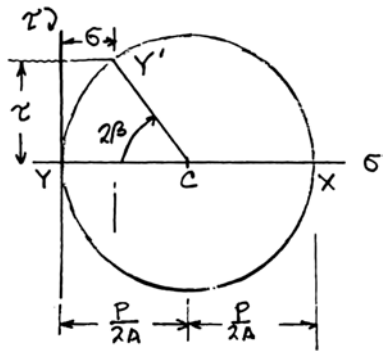


$$\sigma_x = \frac{P}{A} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0$$

Plotted points for Mohr's circle:

$$X: \left(\frac{P}{A}, 0\right) \qquad Y: (0, 0)$$

$$C: \left(\frac{P}{2A}, 0\right) \qquad R = \overline{CX} = \frac{P}{2A}$$



Coordinates of point Y:

$$\sigma = \frac{P}{2A}(1 - \cos 2\beta)$$

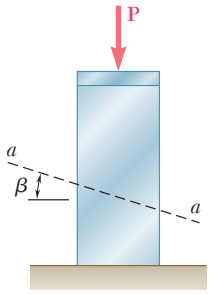
$$\tau = \frac{P}{2A}\sin 2\beta$$

Data: $A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$

(*a*) $$\tau = \frac{(10 \times 10^3)\sin 50°}{(2)(9.6 \times 10^{-3})} = 399 \times 10^3 \text{ Pa} = 399 \text{ kPa}$$ ◀

(*b*) $$\sigma = \frac{(10 \times 10^3)(1 - \cos 50°)}{(2)(9.6 \times 10^{-3})} = 186.0 \times 10^3 \text{ Pa} = 186.0 \text{ kPa}$$ ◀

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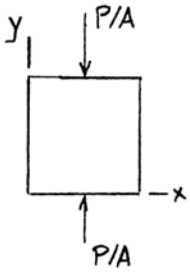
PROBLEM 7.43

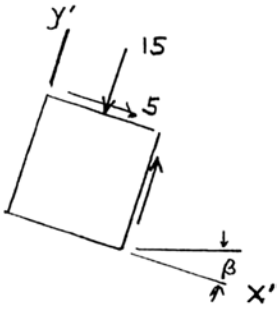
Solve Prob. 7.21, using Mohr's circle.

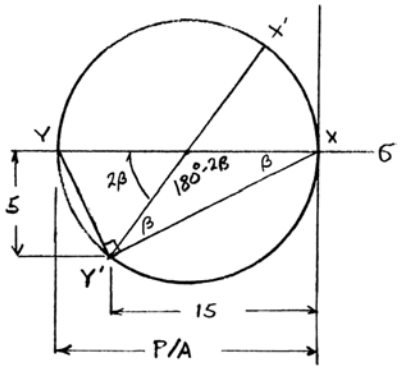
PROBLEM 7.21 The centric force **P** is applied to a short post as shown. Knowing that the stresses on plane *a*-*a* are $\sigma = -15$ ksi and $\tau = 5$ ksi, determine (*a*) the angle β that plane *a*-*a* forms with the horizontal, (*b*) the maximum compressive stress in the post.

SOLUTION

$$\sigma_x = 0$$
$$\tau_{xy} = 0$$
$$\sigma_y = -\frac{P}{A}$$







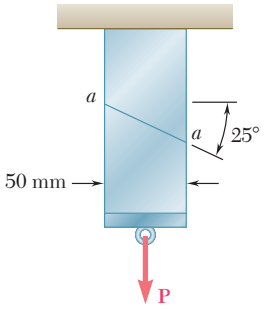
(*a*) From the Mohr's circle,

$$\tan\beta = \frac{5}{15} = 0.3333 \qquad \beta = 18.4° \blacktriangleleft$$

(*b*)
$$-\sigma = \frac{P}{2A} + \frac{P}{2A}\cos 2\beta$$
$$\frac{P}{A} = \frac{2(-\sigma)}{1+\cos 2\beta} = \frac{(2)(15)}{1+\cos 2\beta}$$
$$= 16.67 \text{ ksi}$$

16.67 ksi ◀

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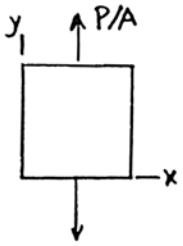


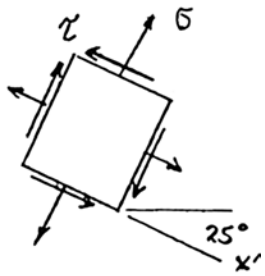
PROBLEM 7.44

Solve Prob. 7.22, using Mohr's circle.

PROBLEM 7.22 Two members of uniform cross section 50×80 mm are glued together along plane *a-a* that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load **P** that can be applied.

SOLUTION





$$\sigma_x = 0$$

$$\tau_{xy} = 0$$

$$\sigma_y = P/A$$

$$A = (50 \times 10^{-3})(80 \times 10^{-3})$$

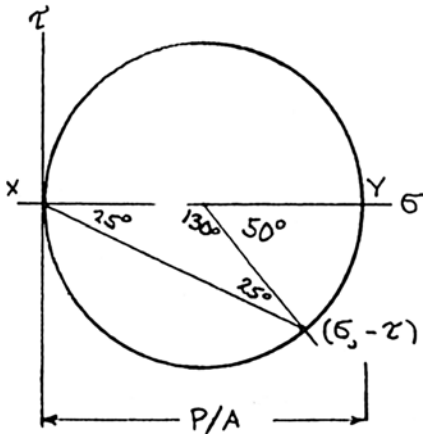
$$= 4 \times 10^{-3}\ \text{m}^2$$

$$\sigma = \frac{P}{2A}(1 + \cos 50°)$$

$$P = \frac{2A\sigma}{1 + \cos 50°}$$

$$P \le \frac{(2)(4 \times 10^{-3})(800 \times 10^{3})}{1 + \cos 50°}$$

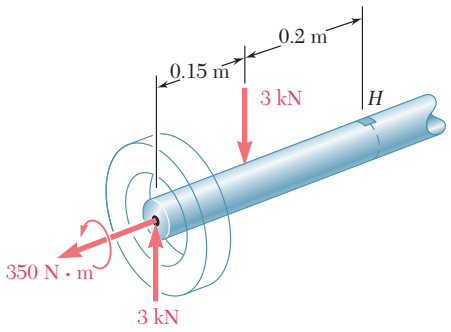
$$P \le 3.90 \times 10^{3}\ \text{N}$$



$$\tau = \frac{P}{2A}\sin 50° \qquad P = \frac{2A\tau}{\sin 50°} \le \frac{(2)(4 \times 10^{-3})(600 \times 10^{3})}{\sin 50°} = 6.27 \times 10^{3}\ \text{N}$$

Choosing the smaller value, $P = 3.90$ kN ◀

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PROBLEM 7.45

Solve Prob. 7.23, using Mohr's circle.

PROBLEM 7.23 The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 32 mm, determine (*a*) the principal planes and principal stresses at point *H* located on top of the axle, (*b*) the maximum shearing stress at the same point.

SOLUTION

$$c = \frac{1}{2}d = \frac{1}{2}(32) = 16 \text{ mm} = 16 \times 10^{-3} \text{ m}$$

Torsion: $$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

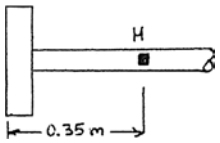
$$\tau = \frac{2(350 \text{ N} \cdot \text{m})}{\pi(16 \times 10^{-3}\text{m})^3} = 54.399 \times 10^6 \text{ Pa} = 54.399 \text{ MPa}$$

Bending: $$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(16 \times 10^{-3})^4 = 51.472 \times 10^{-9}\text{m}^4$$

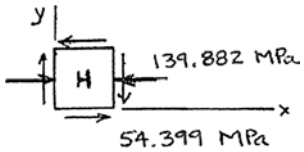
$$M = (0.15\text{m})(3 \times 10^3 \text{ N}) = 450 \text{ N} \cdot \text{m}$$

$$\sigma = -\frac{My}{I} = -\frac{(450)(16 \times 10^{-3})}{51.472 \times 10^{-9}} = -139.882 \times 10^6 \text{ Pa} = -139.882 \text{ MPa}$$

Top view

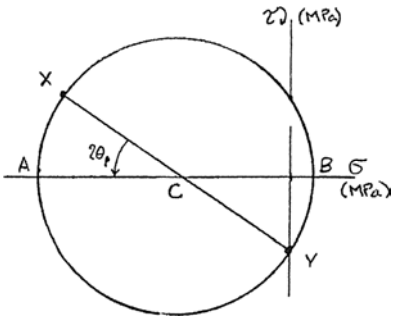


Stresses



$$\sigma_x = -139.882 \text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = -54.399 \text{ MPa}$$

Plotted points: $X: (-139.882, 54.399)$; $Y: (0, -54.399)$; $C: (-69.941, 0)$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -69.941 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-139.882}{2}\right)^2 + (54.399)^2} = 88.606 \text{ MPa}$$

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PROBLEM 7.45 ***(Continued)***

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-54.399)}{-139.882}$$

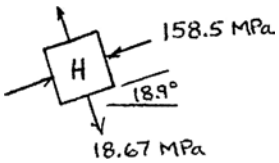
$$= 0.77778$$

(*a*) $\theta_a = 18.9°$ ↻, $\theta_b = 108.9°$ ↻ ◀

$\sigma_a = \sigma_{\text{ave}} - R = -69.941 - 88.606$ $\sigma_a = -158.5$ MPa ◀

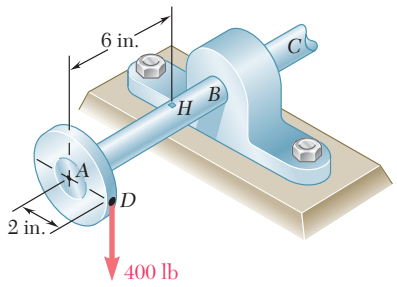
$\sigma_b = \sigma_{\text{ave}} + R = -69.941 + 88.606$ $\sigma_b = 18.67$ MPa ◀

(*b*)



$\tau_{\max} = R$ $\tau_{\max} = 88.6$ MPa ◀

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PROBLEM 7.46

Solve Prob. 7.24, using Mohr's circle.

PROBLEM 7.24 A 400-lb vertical force is applied at D to a gear attached to the solid 1-in.-diameter shaft AB. Determine the principal stresses and the maximum shearing stress at point H located as shown on top of the shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point H:

$$V = 400 \text{ lb} \qquad M = (400)(6) = 2400 \text{ lb} \cdot \text{in.}$$

$$T = (400)(2) = 800 \text{ lb} \cdot \text{in.}$$

Shaft cross section:

$$d = 1 \text{ in.} \qquad c = \frac{1}{2}d = 0.5 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = 0.098175 \text{ in}^4 \qquad I = \frac{1}{2}J = 0.049087 \text{ in}^4$$

Torsion:

$$\tau = \frac{Tc}{J} = \frac{(800)(0.5)}{0.098175} = 4.074 \times 10^3 \text{ psi} = 4.074 \text{ ksi}$$

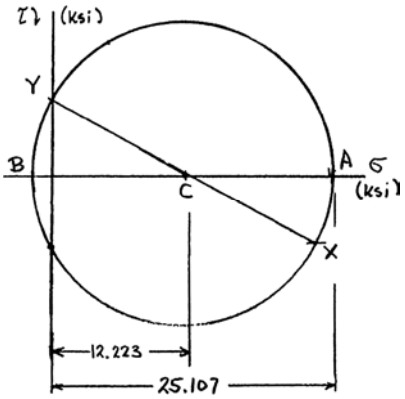
Bending:

$$\sigma = \frac{Mc}{I} = \frac{(2400)(0.5)}{0.049087} = 24.446 \times 10^3 \text{ psi} = 24.446 \text{ ksi}$$

Transverse shear: Stress at point H is zero.

Resultant stresses:

$$\sigma_x = 24.446 \text{ ksi}, \qquad \sigma_y = 0, \qquad \tau_{xy} = 4.074 \text{ ksi}$$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 12.223 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

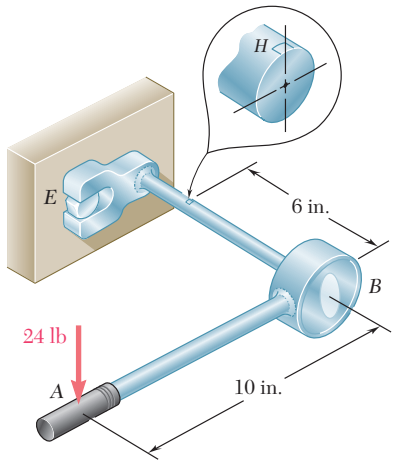
$$= \sqrt{(12.223)^2 + (4.074)^2} = 12.884 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 25.1 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -0.661 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 12.88 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.47

Solve Prob. 7.25, using Mohr's circle.

PROBLEM 7.25 A mechanic uses a crowfoot wrench to loosen a bolt at *E*. Knowing that the mechanic applies a vertical 24-lb force at *A*, determine the principal stresses and the maximum shearing stress at point *H* located as shown as on top of the $\frac{3}{4}$-in.-diameter shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point *H*:

$$V = 24 \text{ lb} \qquad M = (24)(6) = 144 \text{ lb} \cdot \text{in.}$$

$$T = (24)(10) = 240 \text{ lb} \cdot \text{in.}$$

Shaft cross section:

$$d = 0.75 \text{ in.} \qquad c = \frac{1}{2}d = 0.375 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = 0.031063 \text{ in}^4 \qquad I = \frac{1}{2}J = 0.015532 \text{ in}^4$$

Torsion:

$$\tau = \frac{Tc}{J} = \frac{(240)(0.375)}{0.031063} = 2.897 \times 10^3 \text{ psi} = 2.897 \text{ ksi}$$

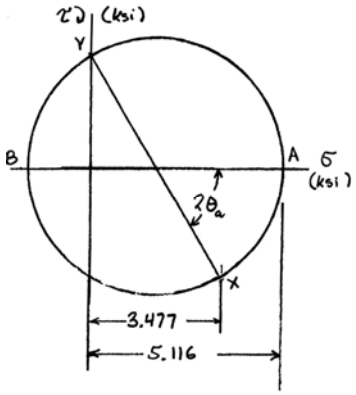
Bending:

$$\sigma = \frac{Mc}{I} = \frac{(144)(0.375)}{0.015532} = 3.477 \times 10^3 \text{ psi} = 3.477 \text{ ksi}$$

Transverse shear: At point *H*, stress due to transverse shear is zero.

Resultant stresses:

$$\sigma_x = 3.477 \text{ ksi}, \qquad \sigma_y = 0, \qquad \tau_{xy} = 2.897 \text{ ksi}$$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 1.738 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

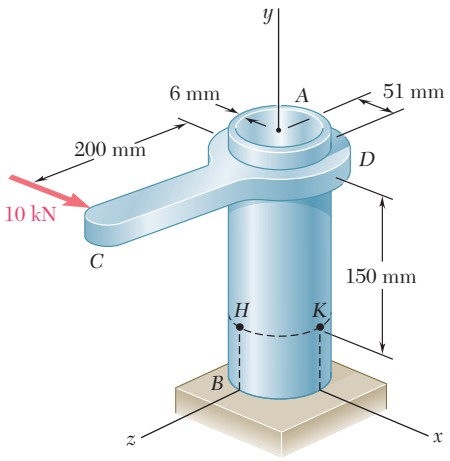
$$= \sqrt{1.738^2 + 2.897^2} = 3.378 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 5.12 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -1.640 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 3.38 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.48

Solve Prob. 7.26, using Mohr's circle.

PROBLEM 7.26 The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *K*.

SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51\text{ mm} \qquad r_i = r_o - t = 45\text{ mm}$$

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = 4.1855 \times 10^6\text{ mm}^4 = 4.1855 \times 10^{-6}\text{ m}^4$$

$$I = \frac{1}{2}J = 2.0927 \times 10^{-6}\text{ m}^4$$

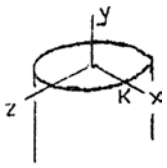
Force-couple system at center of tube in the plane containing points *H* and *K*:

$$F_x = 10 \times 10^3\text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000\text{ N} \cdot \text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500\text{ N} \cdot \text{m}$$

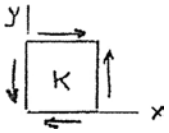
Torsion:



$$T = M_y = 2000\text{ N} \cdot \text{m}$$

$$c = r_o = 51 \times 10^{-3}\text{ m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37\text{ MPa}$$



Note that the local *x*-axis is taken along a negative global *z* direction.

Transverse shear: Stress due to $V = F_x$ is zero at point *K*.

Bending:
$$\left|\sigma_y\right| = \frac{|M_z|c}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56\text{ MPa}$$

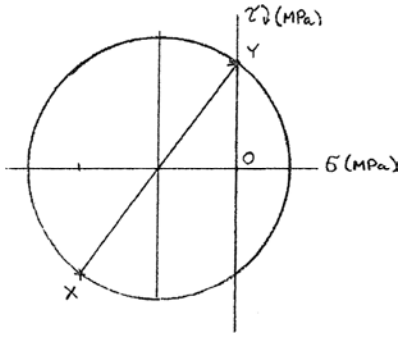
Point *K* lies on compression side of neutral axis. $\sigma_y = -36.56\text{ MPa}$

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PROBLEM 7.48 *(Continued)*

Total stresses at point K: $\sigma_x = 0, \quad \sigma_y = -36.56 \text{ MPa}, \quad \tau_{xy} = 24.37 \text{ MPa}$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = -18.28 + 30.46$$

$$\sigma_{\max} = 12.18 \text{ MPa} \blacktriangleleft$$

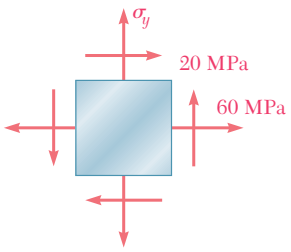
$$\sigma_{\min} = \sigma_{\text{ave}} - R = -18.28 - 30.46$$

$$\sigma_{\min} = -48.7 \text{ MPa} \blacktriangleleft$$

$$\tau_{\max} = R$$

$$\tau_{\max} = 30.5 \text{ MPa} \blacktriangleleft$$

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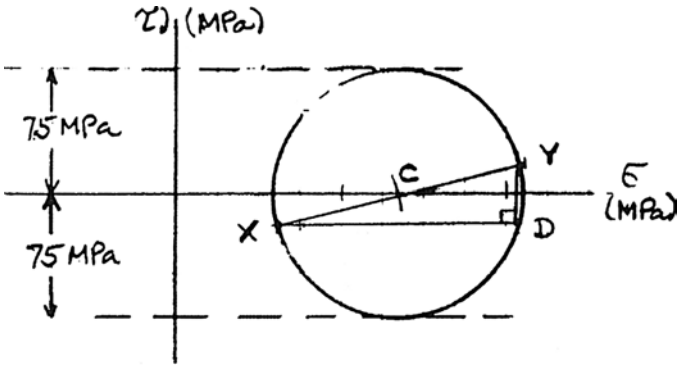
PROBLEM 7.49

Solve Prob. 7.27, using Mohr's circle.

PROBLEM 7.27 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 75 MPa.

SOLUTION

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_y = ?, \quad \tau_{xy} = 20 \text{ MPa}$$



Given:

$$\tau_{\max} = R = 75 \text{ MPa}$$

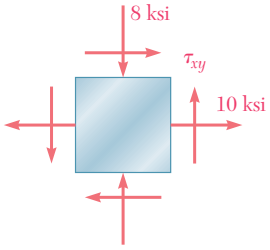
$$\overline{XY} = 2R = 150 \text{ MPa}$$

$$\overline{DY} = (2)(\tau_{xy}) = 40 \text{ MPa}$$

$$\overline{XD} = \sqrt{\overline{XY}^2 - \overline{DY}^2} = \sqrt{150^2 - 40^2} = 144.6 \text{ MPa}$$

$$\sigma_y = \sigma_x + \overline{XD} = 60 + 144.6 \qquad \sigma_y = 205 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.50

Solve Prob. 7.28, using Mohr's circle.

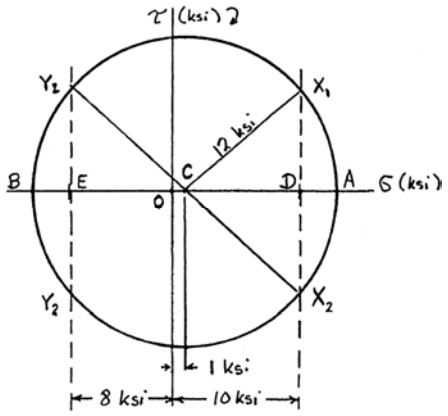
PROBLEM 7.28 For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (*b*) the corresponding principal stresses.

SOLUTION

The center of the Mohr's circle lies at point *C* with coordinates

$$\left(\frac{\sigma_x+\sigma_y}{2}, 0\right)=\left(\frac{10-8}{2}, 0\right)=(1\text{ ksi}, 0).$$

The radius of the circle is $\tau_{\max(\text{in-plane})}=12$ ksi.



The stress point $(\sigma_x, -\tau_{xy})$ lies along the line X_1X_2 of the Mohr circle diagram. The extreme points with $R \le 12$ ksi are X_1 and X_2.

(*a*) The largest allowable value of τ_{xy} is obtained from triangle *CDX*.

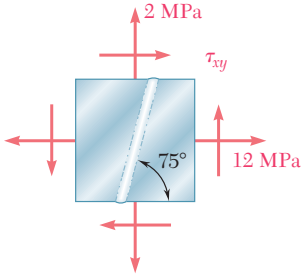
$$\overline{DX_1}^2=\overline{DX_2}^2=\sqrt{\overline{CX_1}^2-\overline{CD}^2}$$

$$\tau_{xy}=\sqrt{12^2-9^2} \qquad \tau_{xy}=7.94\text{ ksi} \blacktriangleleft$$

(*b*) The principal stresses are $\sigma_a=1+12$ $\qquad \sigma_a=13.00\text{ ksi} \blacktriangleleft$

$$\sigma_b=1-12 \qquad \sigma_b=-11.00\text{ ksi} \blacktriangleleft$$

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PROBLEM 7.51

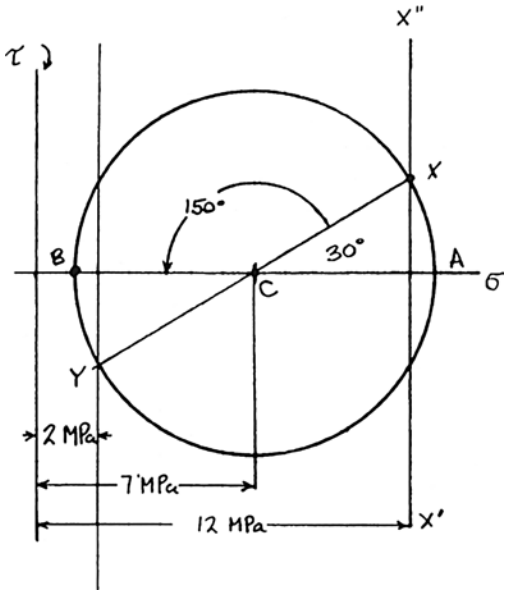
Solve Prob. 7.29, using Mohr's circle.

PROBLEM 7.29 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.

SOLUTION

Point X of Mohr's circle must lie on $X'X''$ so that $\sigma_x = 12$ MPa. Likewise, point Y lies on line $Y'Y''$ so that $\sigma_y = 2$ MPa. The coordinates of C are

$$\frac{2+12}{2}, \quad 0 = (7 \text{ MPa}, 0).$$



Counterclockwise rotation through 150° brings line CX to CB, where $\tau = 0$.

$$R = \frac{\sigma_x - \sigma_y}{2} \sec 30° = \frac{12-2}{2} \sec 30° = 5.7735 \text{ MPa}$$

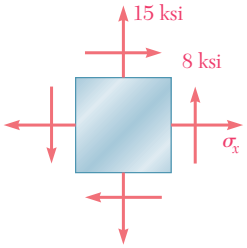
(*a*) $$\tau_{xy} = -\frac{\sigma_x - \sigma_y}{2} \tan 30°$$

$$= -\frac{12-2}{2} \tan 30° \qquad \tau_{xy} = -2.89 \text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma_a = \sigma_{\text{ave}} + R = 7 + 5.7735 \qquad \sigma_a = 12.77 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R = 7 - 5.7735 \qquad \sigma_b = 1.226 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.52

Solve Prob. 7.30, using Mohr's circle.

PROBLEM 7.30 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 10 ksi.

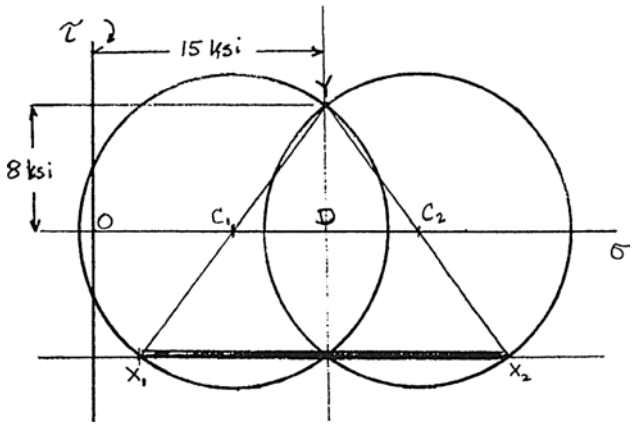
SOLUTION

For the Mohr's circle, point Y lies at (15 ksi, 8 ksi). The radius of limiting circles is $R = 10$ ksi.

Let C_1 be the location of the leftmost limiting circle and C_2 be that of the rightmost one.

$$\overline{C_1Y} = 10 \text{ ksi}$$

$$\overline{C_2Y} = 10 \text{ ksi}$$



Noting right triangles C_1DY and C_2DY,

$$\overline{C_1D}^2 + \overline{DY}^2 = \overline{C_1Y}^2 \qquad \overline{C_1D}^2 + 8^2 = 10^2 \qquad \overline{C_1D} = 6 \text{ ksi}$$

Coordinates of point C_1 are $(0, 15-6) = (0, 9 \text{ ksi})$.

Likewise, coordinates of point C_2 are $(0, 15+6) = (0, 21 \text{ ksi})$.

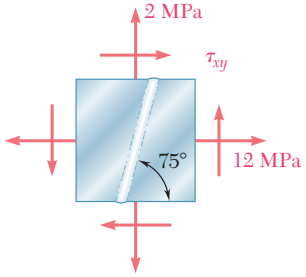
Coordinates of point X_1: $(9-6, -8) = (3 \text{ ksi}, -8 \text{ ksi})$

Coordinates of point X_2: $(21+6, -8) = (27 \text{ ksi}, -8 \text{ ksi})$

The point $(\sigma_x, -\tau_{xy})$ must lie on the line X_1X_2.

Thus, $3 \text{ ksi} \le \sigma_x \le 27 \text{ ksi}$ ◀

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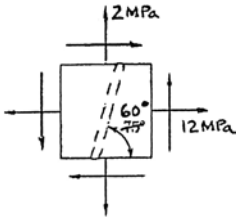


PROBLEM 7.53

Solve Problem 7.29, using Mohr's circle and assuming that the weld forms an angle of 60° with the horizontal.

PROBLEM 7.29 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.

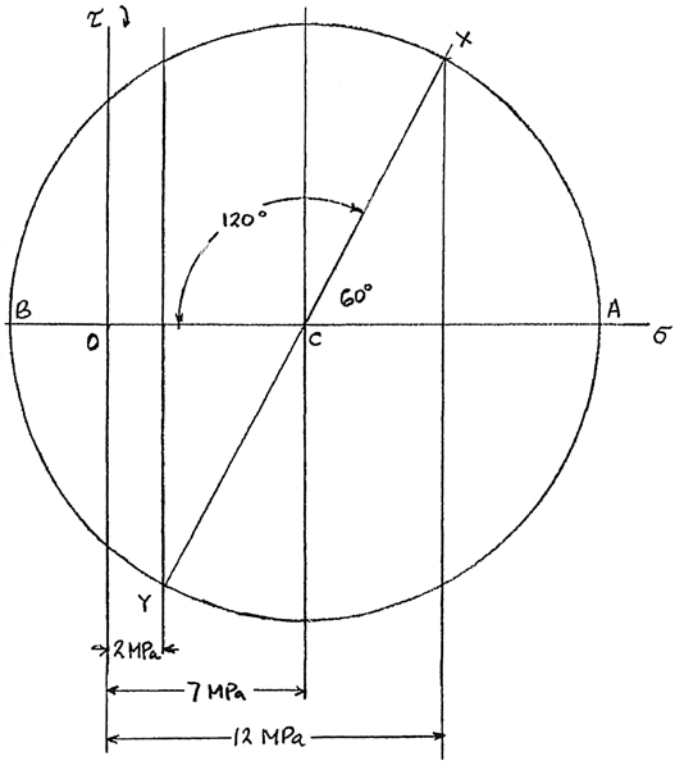
SOLUTION



Locate point C at $\sigma = \dfrac{12+2}{2} = 7$ MPa with $\tau = 0$.

Angle $XCB = 120°$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{12-2}{2}$$
$$= 5 \text{ MPa}$$



$$R = 5\sec 60°$$
$$= 10 \text{ MPa}$$

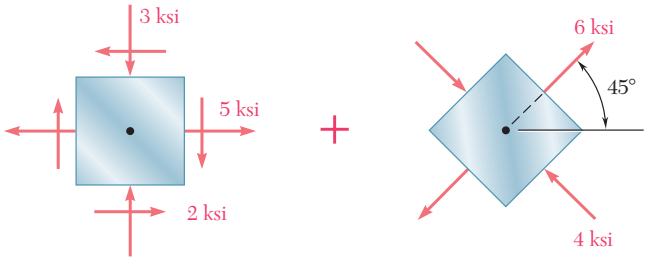
$$\tau_{xy} = -5\tan 60°$$

$$\tau_{xy} = -8.66 \text{ MPa} \blacktriangleleft$$

$$\sigma_a = \sigma_{\text{ave}} + R$$
$$= 7 + 10 \qquad \sigma_a = 17.00 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R$$
$$= 7 - 10 \qquad \sigma_b = -3.00 \text{ MPa} \blacktriangleleft$$

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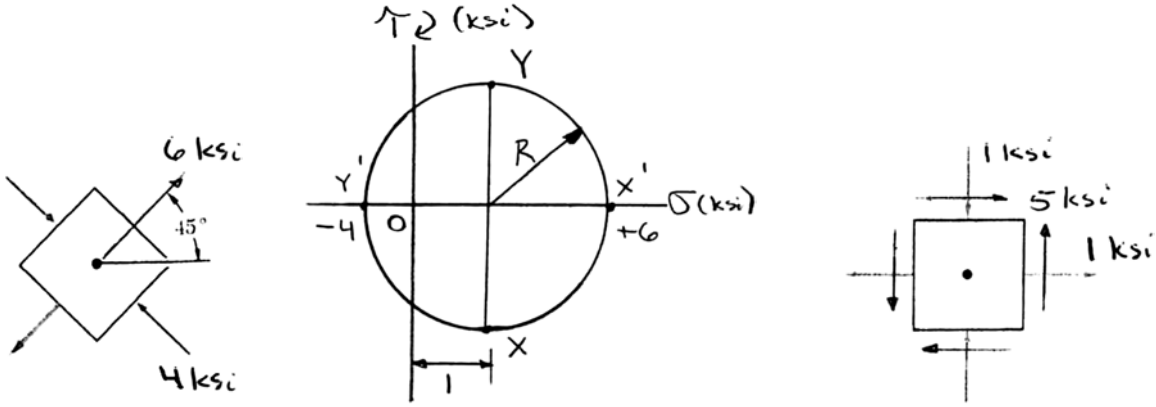


PROBLEM 7.54

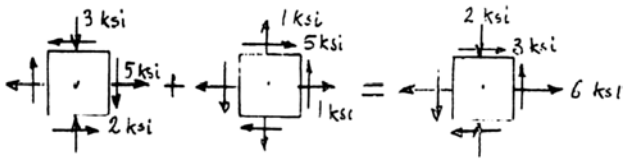
Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

SOLUTION

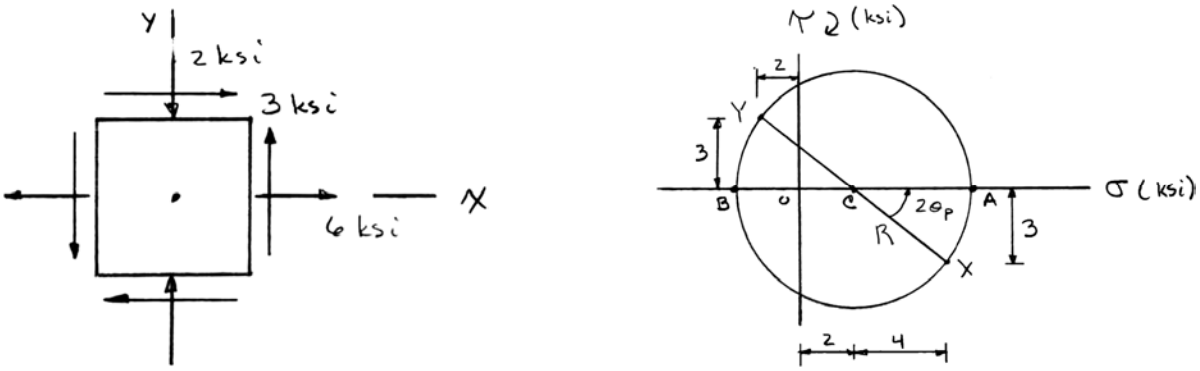
Consider state of stress on the right. We shall express it in terms of horizontal and vertical components.



We now can add the two stress elements by superposition.



Principal planes and principal stresses:



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PROBLEM 7.54 ***(Continued)***

$$\sigma_{\text{ave}} = \frac{1}{2}(6 - 2) = +2$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{1}{2}(6 + 2) = 4$$

$$R = \sqrt{(4)^2 + (3)^2} = 5$$

$$\tan 2\theta_p = \frac{3}{4}$$

$$2\theta_p = 36.87°$$

$$\theta_p = 18.4°, 108.4° \blacktriangleleft$$

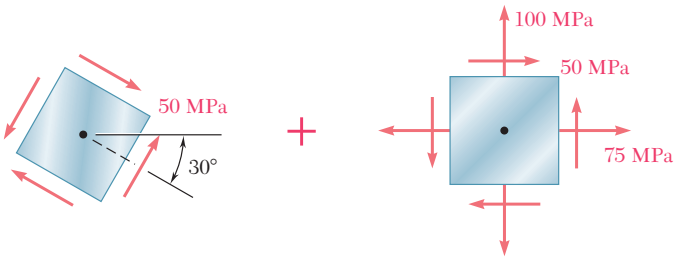
$$\sigma_{\max} = \sigma_{\text{ave}} + R = 2 + 5$$

$$\sigma_{\max} = 7.00 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R = 2 - 5$$

$$\sigma_{\min} = -3.00 \text{ ksi} \blacktriangleleft$$

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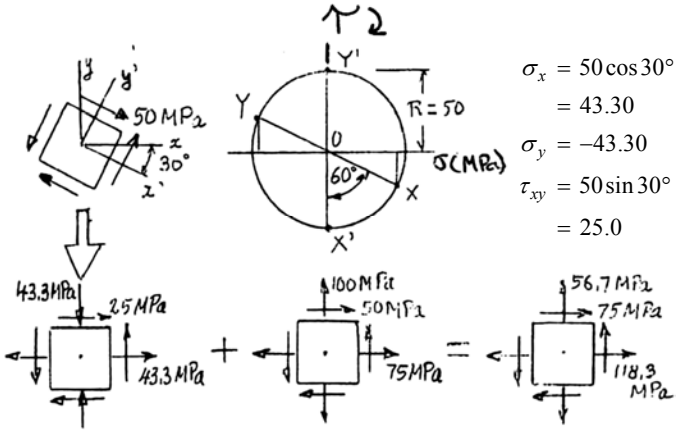


PROBLEM 7.55

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

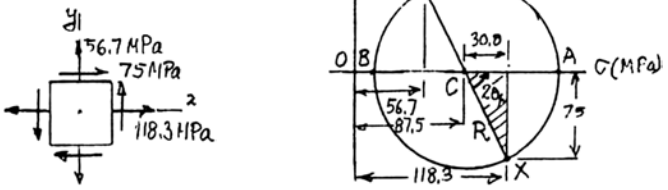
SOLUTION

Consider the state of stress on the left. We shall express it in terms of horizontal and vertical components.



$$\sigma_x = 50\cos 30° = 43.30$$
$$\sigma_y = -43.30$$
$$\tau_{xy} = 50\sin 30° = 25.0$$

Principal axes and principal stress:



$$\sigma_{ave} = \frac{1}{2}(118.3 + 56.7) = +87.5$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{1}{2}(118.3 - 56.7) = 30.8$$

$$R = \sqrt{(30.8)^2 + (75)^2} = 81.08$$

$$\tan 2\theta_p = \frac{75}{30.8} \qquad 2\theta_p = 67.67°$$

$$\theta_p = 33.8° \text{ and } 123.8° \blacktriangleleft$$

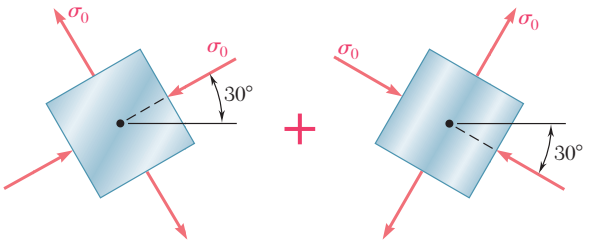
$$\sigma_{max} = \sigma_{ave} + R = 87.5 + 81.08$$

$$\sigma_{max} = +168.6 \text{ MPa} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R = 87.5 - 81.08$$

$$\sigma_{min} = +6.42 \text{ MPa} \blacktriangleleft$$

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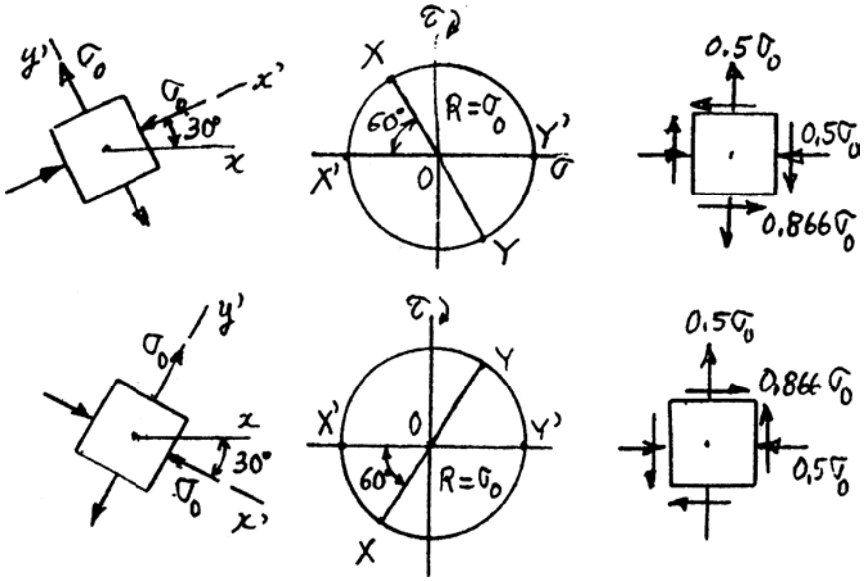


PROBLEM 7.56

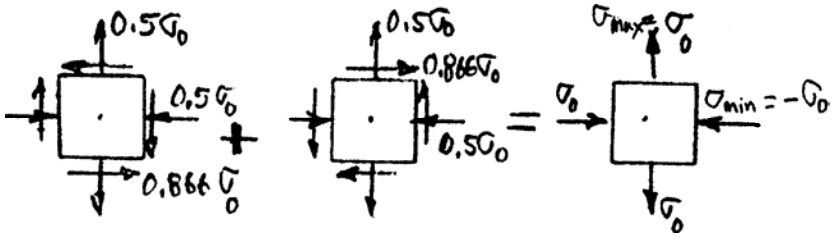
Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

SOLUTION

Express each state of stress in terms of horizontal and vertical components.



Adding the two states of stress,



$\theta_p = 0$ and $90°$ ◀

$\sigma_{\max} = \sigma_0$ ◀

$\sigma_{\min} = -\sigma_0$ ◀

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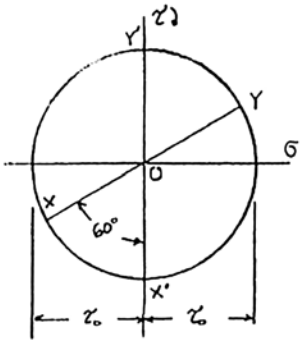
PROBLEM 7.57

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

SOLUTION

Mohr's circle for 2nd state of stress:

$$\sigma_{x'} = 0$$
$$\sigma_{y'} = 0$$
$$\tau_{x'y'} = \tau_0$$



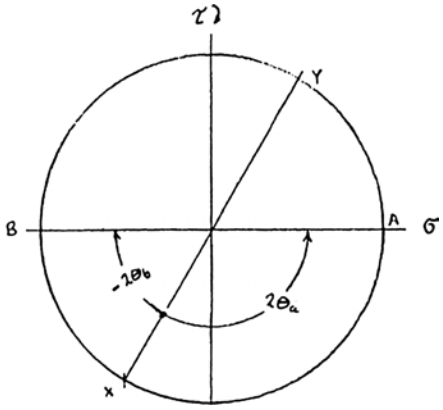
$$\sigma_x = -\tau_0 \sin 60° = -\frac{\sqrt{3}}{2}\tau_0 \qquad \sigma_y = \tau_0 \sin 60° = \frac{\sqrt{3}}{2}\tau_0$$

$$\tau_{xy} = \tau_0 \cos 60° = \frac{1}{2}\tau_0$$

Resultant stresses:

$$\sigma_x = 0 - \frac{\sqrt{3}}{2}\tau_0 = -\frac{\sqrt{3}}{2}\tau_0 \qquad \sigma_y = 0 + \frac{\sqrt{3}}{2}\tau_0 = \frac{\sqrt{3}}{2}\tau_0$$

$$\tau_{xy} = \tau_0 + \frac{1}{2}\tau_0 = \frac{3}{2}\tau_0$$

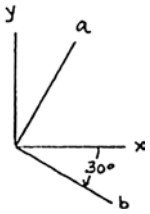


$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sqrt{3}}{2}\tau_0\right)^2 + \left(\frac{3}{2}\tau_0\right)^2} = \sqrt{3}\,\tau_0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)\left(\frac{3}{2}\right)}{-\sqrt{3}} = -\sqrt{3}$$

$$2\theta_p = -60°$$



$$\theta_b = -30° \qquad \theta_a = 60° \blacktriangleleft$$

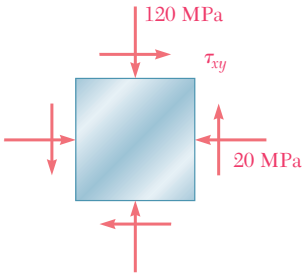
$$\sigma_a = \sigma_{\text{ave}} + R$$

$$\sigma_a = \sqrt{3}\,\tau_0 \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R$$

$$\sigma_b = -\sqrt{3}\,\tau_0 \blacktriangleleft$$

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PROBLEM 7.58

For the element shown, determine the range of values of τ_{xy} for which the maximum tensile stress is equal to or less than 60 MPa.

SOLUTION

$$\sigma_x = -20 \text{ MPa} \quad \sigma_y = -120 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -70 \text{ MPa}$$

Set

$$\sigma_{\max} = 60 \text{ MPa} = \sigma_{\text{ave}} + R$$

$$R = \sigma_{\max} - \sigma_{\text{ave}} = 130 \text{ MPa}$$

But

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_x}{2}\right)^2}$$

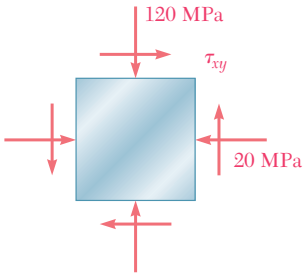
$$= \sqrt{130^2 - 50^2}$$

$$= 120.0 \text{ MPa}$$

Range of τ_{xy}:

$$-120.0 \text{ MPa} \le \tau_{xy} \le 120.0 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.59

For the element shown, determine the range of values of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 150 MPa.

SOLUTION

$$\sigma_x = -20\,\text{MPa} \quad \sigma_y = -120\ \text{MPa}$$

$$\frac{1}{2}(\sigma_x - \sigma_y) = 50\ \text{MPa}$$

Set $$\tau_{\max\,(\text{in-plane})} = R = 150\ \text{MPa}$$

But $$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

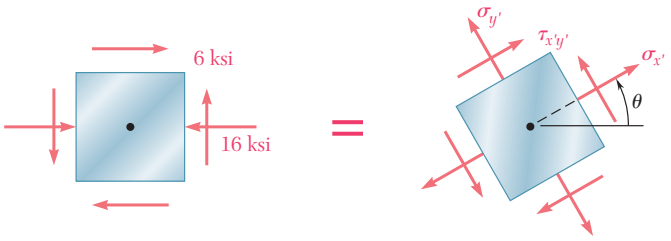
$$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$

$$= \sqrt{150^2 - 50^2}$$

$$= 141.4\ \text{MPa}$$

Range of τ_{xy}: $$-141.4\ \text{MPa} \le \tau_{xy} \le 141.4\ \text{MPa}$$ ◀

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PROBLEM 7.60

For the state of stress shown, determine the range of values of θ for which the magnitude of the shearing stress $\tau_{x'y'}$ is equal to or less than 8 ksi.

SOLUTION

$$\sigma_x = -16 \text{ ksi}, \quad \sigma_y = 0$$

$$\tau_{xy} = 6 \text{ ksi}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -8 \text{ ksi}$$

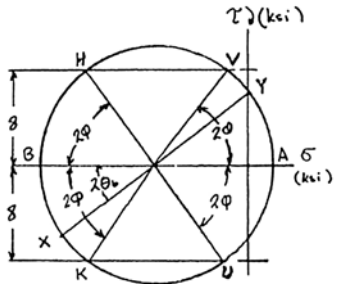
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-8)^2 + (6)^2} = 10 \text{ ksi}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(6)}{-16} = -0.75$$

$$2\theta_p = -36.870°$$

$$\theta_b = -18.435°$$



$|\tau_{x'y'}| \le 8$ ksi for states of stress corresponding to arcs *HBK* and *UAV* of Mohr's circle. The angle φ is calculated from

$$R \sin 2\varphi = 8 \qquad \sin 2\varphi = \frac{8}{10} = 0.8$$

$$2\varphi = 53.130° \qquad \varphi = 26.565°$$

$$\theta_k = \theta_b - \varphi = -18.435° - 26.565° = -45°$$

$$\theta_k = \theta_b + \varphi = -18.435° + 26.565° = 8.13°$$

$$\theta_u = \theta_h + 90° = 45°$$

$$\theta_v = \theta_k + 90° = 98.13°$$

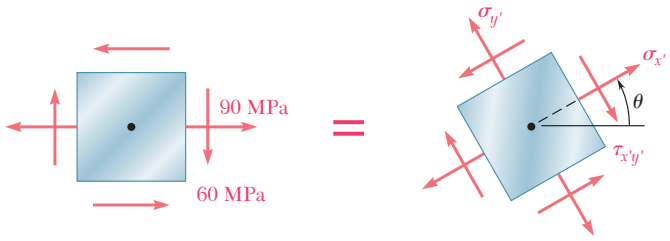
Permissible range of θ:

$$\theta_h \le \theta \le \theta_k \qquad -45° \le \theta \le 8.13° \blacktriangleleft$$

$$\theta_u \le \theta \le \theta_v \qquad 45° \le \theta \le 98.13° \blacktriangleleft$$

Also, $135° \le \theta \le 188.13°$ and $225° \le \theta \le 278.13°$

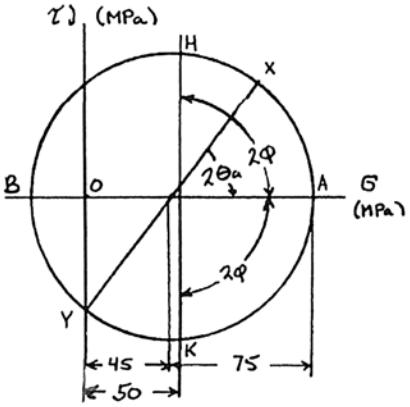
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PROBLEM 7.61

For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 50 MPa.

SOLUTION



$$\sigma_x = 90 \text{ MPa}, \quad \sigma_y = 0$$

$$\tau_{xy} = -60 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-60)}{90} = -\frac{4}{3}$$

$$2\theta_p = -53.13°$$

$$\theta_a = -26.565°$$

$\sigma_{x'} \le 50$ MPa for states of stress corresponding to the arc *HBK* of Mohr's circle. From the circle,

$$R\cos 2\varphi = 50 - 45 = 5 \text{ MPa}$$

$$\cos 2\varphi = \frac{5}{75} = 0.066667$$

$$2\varphi = 86.177° \qquad \varphi = 43.089°$$

$$\theta_h = \theta_a + \varphi = -26.565° + 43.089° = 16.524°$$

$$2\theta_k = 2\theta_h + 360° - 4\varphi = 32.524° + 360° - 172.355° = 220.169°$$

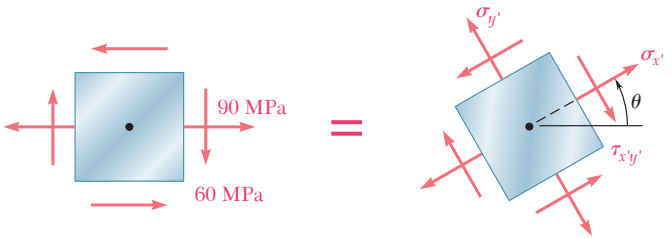
$$\theta_k = 110.085°$$

Permissible range of θ: $\theta_h \le \theta \le \theta_k$

$$16.5° \le \theta \le 110.1° \blacktriangleleft$$

Also, $196.5° \le \theta \le 290.1°$

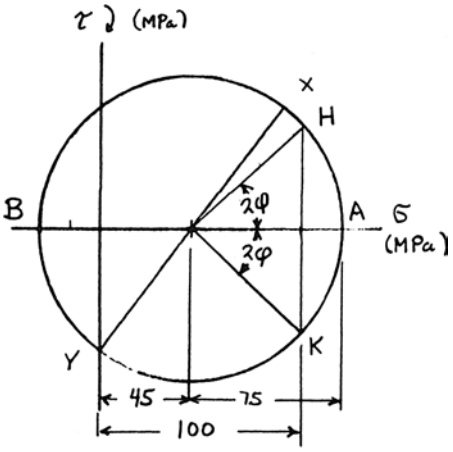
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PROBLEM 7.62

For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 100 MPa.

SOLUTION



$$\sigma_x = 90 \text{ MPa}, \quad \sigma_y = 0$$

$$\tau_{xy} = -60 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-60)}{90} = -\frac{4}{3}$$

$$2\theta_p = -53.13°$$

$$\theta_a = -26.565°$$

$\sigma_{x'} \le 100$ MPa for states of stress corresponding to arc *HBK* of Mohr's circle. From the circle,

$$R\cos 2\varphi = 100 - 45 = 55 \text{ MPa}$$

$$\cos 2\varphi = \frac{55}{75} = 0.73333$$

$$2\varphi = 42.833° \qquad \varphi = 21.417°$$

$$\theta_h = \theta_a + \varphi = -26.565° + 21.417° = -5.15°$$

$$2\theta_k = 2\theta_h + 360° - 4\varphi = -10.297° + 360° - 85.666° = 264.037°$$

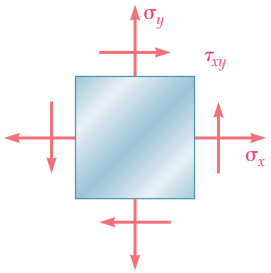
$$\theta_k = 132.02°$$

Permissible range of θ is $\theta_h \le \theta \le \theta_k$

$$-5.1° \le \theta \le 132.0°$$ ◀

Also, $174.8° \le \theta \le 312.0°$

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PROBLEM 7.63

For the state of stress shown, it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 14$ ksi, $\sigma_y = 9$ ksi, and $\sigma_{\min} = 5$ ksi. Determine (*a*) the orientation of the principal planes, (*b*) the principal stress $\sigma_{\max}$, (*c*) the maximum in-plane shearing stress.

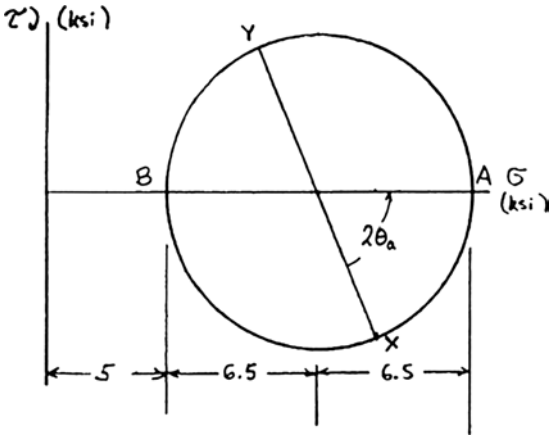
SOLUTION

$$\sigma_x = 14 \text{ ksi}, \quad \sigma_y = 9 \text{ ksi}, \quad \sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 11.5 \text{ ksi}$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R \quad \therefore \quad R = \sigma_{\text{ave}} - \sigma_{\min}$$
$$= 11.5 - 5 = 6.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm\sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \pm\sqrt{6.5^2 - 2.5^2} = \pm 6 \text{ ksi}$$



But it is given that τ_{xy} is positive, thus $\tau_{xy} = +6$ ksi.

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$
$$= \frac{(2)(6)}{5} = 2.4$$
$$2\theta_p = 67.38°$$

$$\theta_a = 33.7° \blacktriangleleft$$

$$\theta_b = 123.7° \blacktriangleleft$$

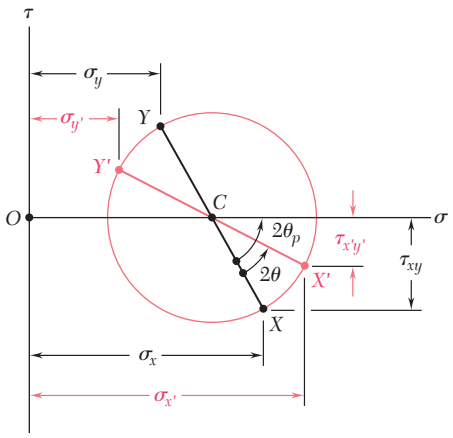
(*b*) $$\sigma_{\max} = \sigma_{\text{ave}} + R$$

$$\sigma_{\max} = 18.00 \text{ ksi} \blacktriangleleft$$

(*c*) $$\tau_{\max(\text{in-plane})} = R$$

$$\tau_{\max(\text{in-plane})} = 6.50 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.64

The Mohr's circle shown corresponds to the state of stress given in Fig. 7.5*a* and *b*. Noting that $\sigma_{x'} = OC + (CX')\cos(2\theta_p - 2\theta)$ and that $\tau_{x'y'} = (CX')\sin(2\theta_p - 2\theta)$, derive the expressions for $\sigma_{x'}$ and $\tau_{x'y'}$ given in Eqs. (7.5) and (7.6), respectively. [*Hint:* Use $\sin(A + B) = \sin A\cos B + \cos A\sin B$ and $\cos(A + B) = \cos A\cos B - \sin A\sin B$.]

SOLUTION

$$\overline{OC} = \frac{1}{2}(\sigma_x + \sigma_y) \qquad \overline{CX'} = \overline{CX}$$

$$\overline{CX'}\cos 2\theta_p = \overline{CX}\cos 2\theta_p = \frac{\sigma_x - \sigma_y}{2}$$

$$\overline{CX'}\sin 2\theta_p = \overline{CX}\sin 2\theta_p = \tau_{xy}$$

$$\begin{aligned}\sigma_{x'} &= \overline{OC} + \overline{CX'}\cos(2\theta_p - 2\theta)\\ &= \overline{OC} + \overline{CX'}(\cos 2\theta_p \cos 2\theta + \sin 2\theta_p \sin 2\theta)\\ &= \overline{OC} + \overline{CX'}\cos 2\theta_p \cos 2\theta + \overline{CX'}\sin 2\theta_p \sin 2\theta\\ &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad \blacktriangleleft\end{aligned}$$

$$\begin{aligned}\tau_{x'y'} &= \overline{CX'}\sin(2\theta_p - 2\theta) = \overline{CX'}(\sin 2\theta_p \cos 2\theta - \cos 2\theta_p \sin 2\theta)\\ &= \overline{CX'}\sin 2\theta_p \cos 2\theta - \overline{CX'}\cos 2\theta_p \sin 2\theta\\ &= \tau_{xy}\cos 2\theta - \frac{\sigma_x - \sigma_y}{2}\sin 2\theta \quad \blacktriangleleft\end{aligned}$$

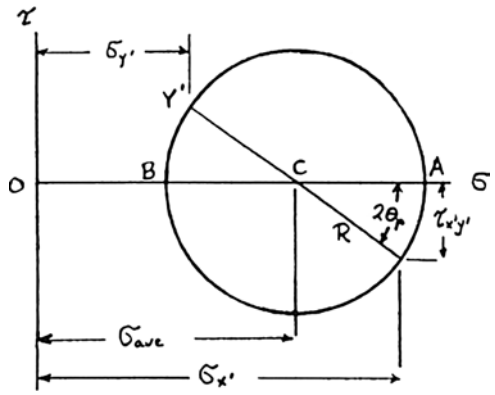
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PROBLEM 7.65

(*a*) Prove that the expression $\sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$, where $\sigma_{x'}, \sigma_{y'}$, and $\tau_{x'y'}$ are components of the stress along the rectangular axes x' and y', is independent of the orientation of these axes. Also, show that the given expression represents the square of the tangent drawn from the origin of the coordinates to Mohr's circle. (*b*) Using the invariance property established in part *a*, express the shearing stress τ_{xy} in terms of σ_x, σ_y, and the principal stresses $\sigma_{\max}$ and $\sigma_{\min}$.

SOLUTION

(*a*) From Mohr's circle,



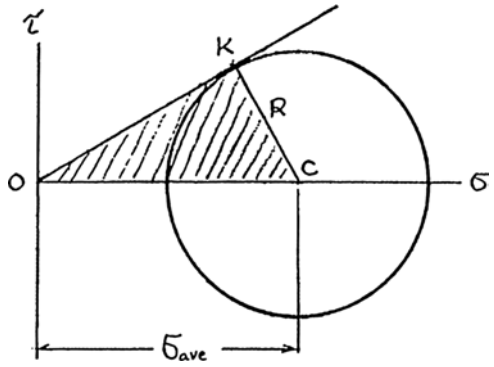
$$\tau_{x'y'} = R\sin 2\theta_p$$
$$\sigma_{x'} = \sigma_{\text{ave}} + R\cos 2\theta_p$$
$$\sigma_{y'} = \sigma_{\text{ave}} - R\cos 2\theta_p$$

$$\sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$$
$$= \sigma_{\text{ave}}^2 - R^2\cos^2 2\theta_p - R^2\sin^2 2\theta_p$$
$$= \sigma_{\text{ave}}^2 - R^2;\ \text{independent of } \theta_p.$$

Draw line $\overline{OK}$ from origin tangent to the circle at *K*. Triangle *OCK* is a right triangle.

$$\overline{OC}^2 = \overline{OK}^2 + \overline{CK}^2$$
$$\overline{OK}^2 = \overline{OC}^2 - \overline{CK}^2$$
$$= \sigma_{\text{ave}}^2 - R^2$$
$$= \sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$$

(*b*) Applying above to σ_x, σ_y, and τ_{xy}, and to σ_a, σ_b,



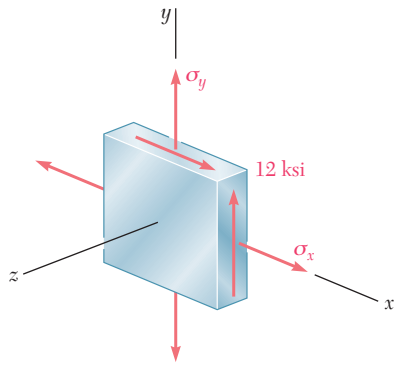
$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_a\sigma_b - \tau_{ab}^2 = \sigma_{\text{ave}}^2 - R^2$$

But $\tau_{ab} = 0, \quad \sigma_a = \sigma_{\max}, \quad \sigma_b = \sigma_{\min}$

$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_{\max}\sigma_{\min}$$
$$\tau_{xy}^2 = \sigma_x\sigma_y - \sigma_{\max}\sigma_{\min}$$
$$\left|\tau_{xy}\right| = \pm\sqrt{\sigma_x\sigma_y - \sigma_{\max}\sigma_{\min}}$$

The sign cannot be determined from above equation. ◀

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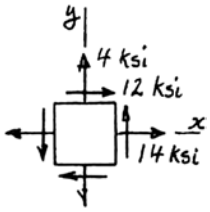


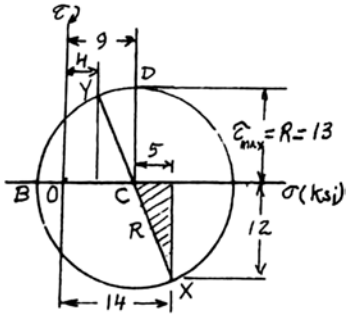
PROBLEM 7.66

For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 14$ ksi and $\sigma_y = 4$ ksi, (*b*) $\sigma_x = 21$ ksi and $\sigma_y = 14$ ksi. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(*a*)





$$\sigma_{ave} = \frac{1}{2}(14 + 4) = 9$$

$$\frac{1}{2}(\sigma_x - \sigma_y) = \frac{1}{2}(14 - 4) = 5$$

$$R = \sqrt{(5)^2 + (12)^2} = 13$$

$$\sigma_{max} = \sigma_{ave} + R = 9 + 13 = 22$$

$$\sigma_{min} = \sigma_{ave} - R = 9 - 13 = -4$$

Since σ_{max} and σ_{min} have opposite signs, the maximum shearing stress is equal to the maximum in-plane shearing stress.

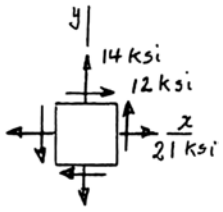
$$\tau_{max} = R = 13.00 \text{ ksi}$$

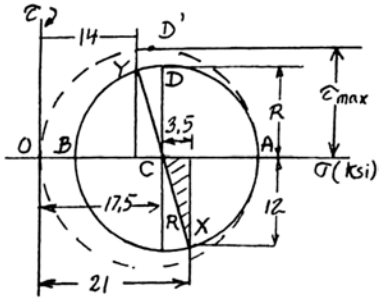
$\tau_{max} = 13.00$ ksi ◀

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PROBLEM 7.66 *(Continued)*

(*b*)





$$\sigma_{\text{ave}} = \frac{1}{2}(21 + 14) = 17.5$$

$$\frac{1}{2}(\sigma_x - \sigma_y) = \frac{1}{2}(21 - 14) = 3.5$$

$$R = \sqrt{(3.5)^2 + (12)^2} = 12.5$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = 17.5 + 12.5 = 30$$

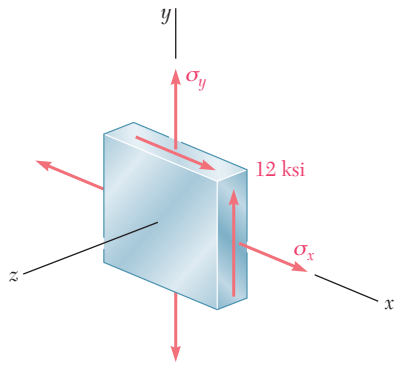
$$\sigma_{\min} = \sigma_{\text{ave}} - R = 17.5 - 12.5 = 5$$

Since $\sigma_{\max}$ and $\sigma_{\min}$ have the same sign, $\tau_{\max}$ is out of the plane of stress. Using Mohr's circle through *O* and *A,* we have

$$\sigma_{\max} = \frac{1}{2}\sigma_{\max} = \frac{1}{2}(30 \text{ ksi})$$

$$\tau_{\max} = 15.00 \text{ ksi} \blacktriangleleft$$

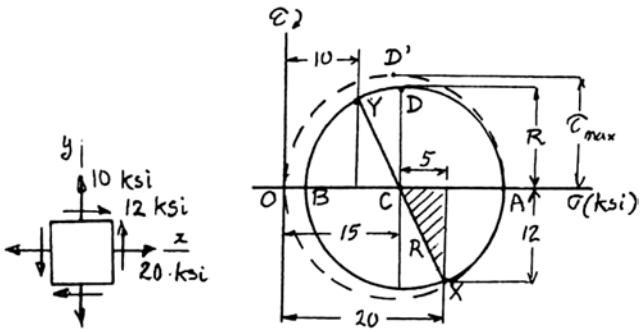
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PROBLEM 7.67

For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 20$ ksi and $\sigma_y = 10$ ksi, (*b*) $\sigma_x = 12$ ksi and $\sigma_y = 5$ ksi. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION



(*a*) $\sigma_{ave} = \frac{1}{2}(20 + 10) = 15$

$$\frac{1}{2}(\sigma_x - \sigma_y) = \frac{1}{2}(20 - 10) = 5$$

$$R = \sqrt{(5)^2 + (12)^2} = 13$$

$$\sigma_{max} = \sigma_{ave} + R = 15 + 13 = 28$$

$$\sigma_{min} = \sigma_{ave} - R = 15 - 13 = 2$$

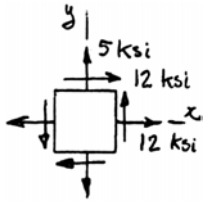
Since σ_{max} and σ_{min} have the same sign, σ_{max} is out of the plane of stress. Using Mohr's circle through *O* and *A*, we have

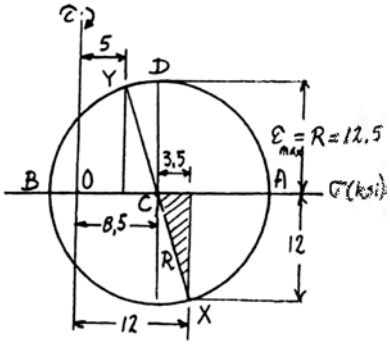
$$\tau_{max} = \frac{1}{2}\sigma_{max} = \frac{1}{2}(28 \text{ ksi}) \qquad \tau_{max} = 14.00 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.67 ***(Continued)***

(*b*)





$$\sigma_{\text{ave}} = \frac{1}{2}(12 + 5) = 8.5$$

$$\frac{1}{2}(\sigma_x - \sigma_y) = \frac{1}{2}(12 - 5) = 3.5$$

$$R = \sqrt{(3.5)^2 + (12)^2} = 12.5$$

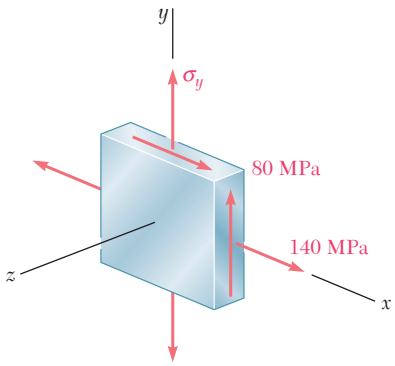
$$\sigma_{\max} = \sigma_{\text{ave}} + R = 8.5 + 12.5 = 21$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R = 8.5 - 12.5 = -4$$

Since $\sigma_{\max}$ and $\sigma_{\min}$ have opposite signs, the maximum shearing stress is equal to the maximum in-plane shearing stress.

$$\tau_{\max} = R = 12.50 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.68

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 40$ MPa, (*b*) $\sigma_y = 120$ MPa. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(*a*) $\sigma_x = 140$ MPa, $\sigma_y = 40$ MPa, $\tau_{xy} = 80$ MPa

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 90 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{50^2 + 80^2} = 94.34 \text{ MPa}$$

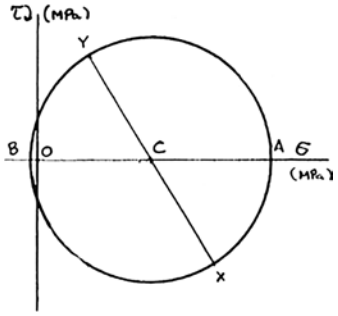
$$\sigma_a = \sigma_{\text{ave}} + R = 184.34 \text{ MPa} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -4.34 \text{ MPa} \quad \text{(min)}$$

$$\sigma_c = 0$$

$$\tau_{\max(\text{in-plane})} = \frac{1}{2}(\sigma_a - \sigma_b) = R = 94.34 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = \frac{1}{2}(\sigma_a - \sigma_b) = 94.3 \text{ MPa}$$



$\tau_{\max} = 94.3$ MPa ◀

(*b*) $\sigma_x = 140$ MPa, $\sigma_y = 120$ MPa, $\tau_{xy} = 80$ MPa

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 130 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{10^2 + 80^2} = 80.62 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 210.62 \text{ MPa} \quad \text{(max)}$$

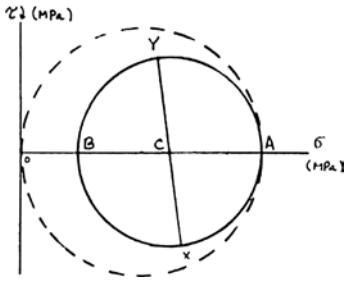
$$\sigma_b = \sigma_{\text{ave}} - R = 49.38 \text{ MPa}$$

$$\sigma_c = 0 \quad \text{(min)}$$

$$\sigma_{\max} = \sigma_a = 210.62 \text{ MPa} \quad \sigma_{\min} = \sigma_c = 0$$

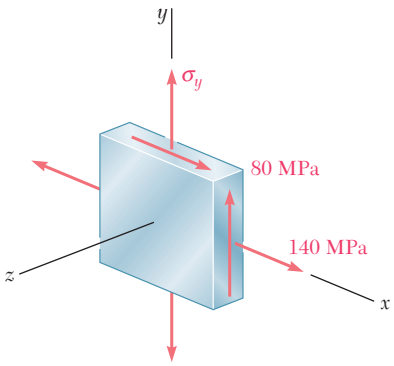
$$\tau_{\max(\text{in-plane})} = R = 86.62 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 105.3 \text{ MPa}$$



$\tau_{\max} = 105.3$ MPa ◀

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PROBLEM 7.69

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 20$ MPa, (*b*) $\sigma_y = 140$ MPa. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(*a*) $\sigma_x = 140$ MPa, $\sigma_y = 20$ MPa, $\tau_{xy} = 80$ MPa

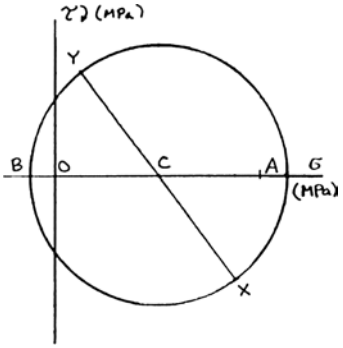
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 80 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{60^2 + 80^2} = 100 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 80 + 100 = 180 \text{ MPa} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 80 - 100 = -20 \text{ MPa} \quad \text{(min)}$$

$$\sigma_c = 0$$



$$\tau_{\max\,(\text{in-plane})} = \frac{1}{2}(\sigma_a - \sigma_b) = 100 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 100 \text{ MPa}$$

$\tau_{\max} = 100.0$ MPa ◀

(*b*) $\sigma_x = 140$ MPa, $\sigma_y = 140$ MPa, $\tau_{xy} = 80$ MPa

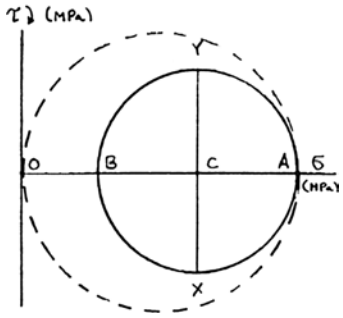
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 140 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + 80^2} = 80 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 220 \text{ MPa} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 60 \text{ MPa}$$

$$\sigma_c = 0 \quad \text{(min)}$$

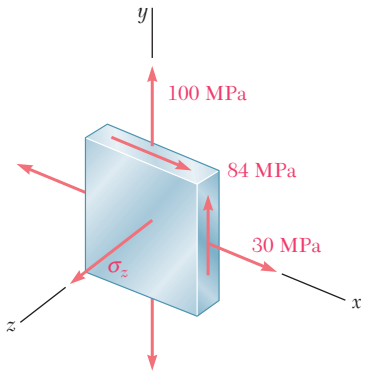


$$\tau_{\max\,(\text{in-plane})} = \frac{1}{2}(\sigma_a - \sigma_b) = 80 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 110 \text{ MPa}$$

$\tau_{\max} = 110.0$ MPa ◀

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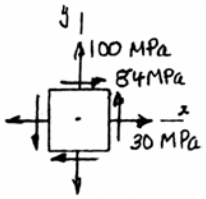


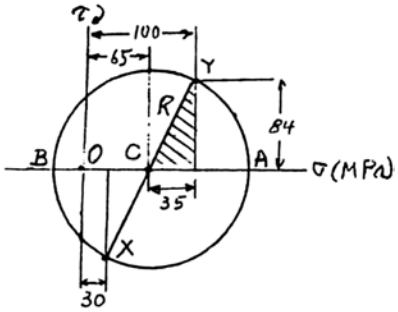
PROBLEM 7.70

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = 0$, (*b*) $\sigma_z = +60$ MPa, (*c*) $\sigma_z = -60$ MPa.

SOLUTION

The *z* axis is a principal axis. We determine the other two principal axes by drawing Mohr's circle for a rotation in the *xy* plane.





$$\sigma_{\text{ave}} = \frac{1}{2}(30 + 100) = 65$$

$$\frac{1}{2}(\sigma_x - \sigma_y) = \frac{1}{2}(30 - 100) = -35$$

$$R = \sqrt{(35)^2 + (84)^2} = 91$$

$$\sigma_A = \sigma_{\text{ave}} + R = 65 + 91 = 156 \text{ MPa}$$

$$\sigma_B = \sigma_{\text{ave}} - R = 65 - 91 = -26 \text{ MPa}$$

(*a*) $\sigma_z = 0$. Point *Z* corresponding to the *z* axis is located at *O* between *A* and *B*. Therefore, the largest of the 3 Mohr's circles is the circle we drew through *A* and *B*. We have

$$\tau_{\max} = R = 91.0 \text{ MPa} \blacktriangleleft$$

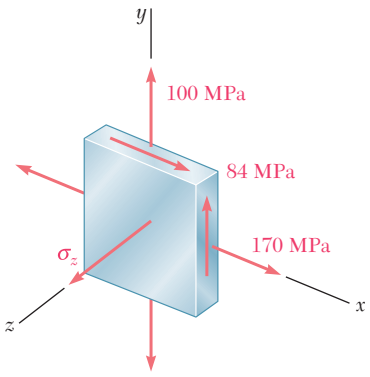
(*b*) $\sigma_z = +60$ MPa. Point *Z* is located between *A* and *B*. The largest of the 3 circles is still the circle through *A* and *B*, and we still have

$$\tau_{\max} = R = 91.0 \text{ MPa} \blacktriangleleft$$

(*c*) $\sigma_z = -60$ MPa. Point *Z* is now outside the circle through *A* and *B*. The largest circle is the circle through *Z* and *A*.

$$\tau_{\max} = \frac{1}{2}(ZH) = \frac{1}{2}(60 + 156) \qquad \tau_{\max} = 108.0 \text{ MPa} \blacktriangleleft$$

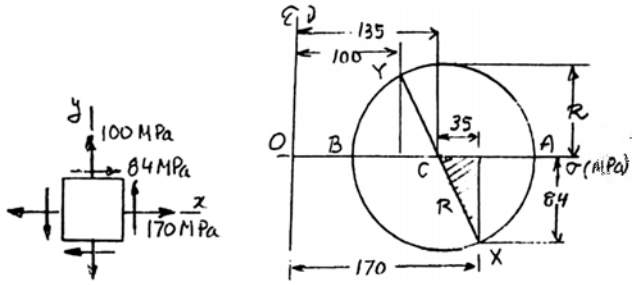
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PROBLEM 7.71

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = 0$, (*b*) $\sigma_z = +60$ MPa, (*c*) $\sigma_z = -60$ MPa.

SOLUTION



$$\sigma_{\text{ave}} = \frac{1}{2}(170 + 100) = 135$$

$$\frac{1}{2}(\sigma_x - \sigma_y) = \frac{1}{2}(170 - 100) = 35$$

$$R = \sqrt{(35)^2 + (84)^2} = 91$$

$$\sigma_A = 135 + 91 = 226 \text{ MPa}$$

$$\sigma_B = 135 - 91 = 44 \text{ MPa}$$

(*a*) $\sigma_z = 0$. Point *Z* corresponding to the *z* axis is located at *O*, outside the circle drawn through *A* and *B*. The largest of the 3 Mohr's circles is the circle through *O* and *A*. We have

$$\tau_{\max} = \frac{1}{2}(OA) = \frac{1}{2}\sigma_A = \frac{1}{2}(226) \qquad \tau_{\max} = 113.0 \text{ MPa} \blacktriangleleft$$

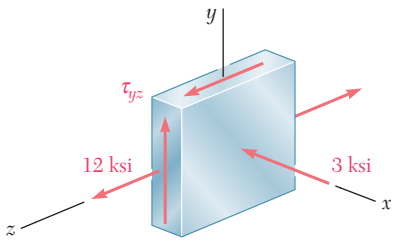
(*b*) $\sigma_z = +60$ MPa. Point *Z* is located between *B* and *A*. The largest of the 3 circles is the one drawn through *A* and *B*.

$$\tau_{\max} = R = 91.0 \text{ MPa} \blacktriangleleft$$

(*c*) $\sigma_z = -60$ MPa. Point *Z* is located outside the circle drawn through *A* and *B*. The largest of the 8 Mohr's circles is the circle through *Z* and *A*. We have

$$\tau_{\max} = \frac{1}{2}(ZA) = \frac{1}{2}(60 + 226) \qquad \tau_{\max} = 143.0 \text{ MPa} \blacktriangleleft$$

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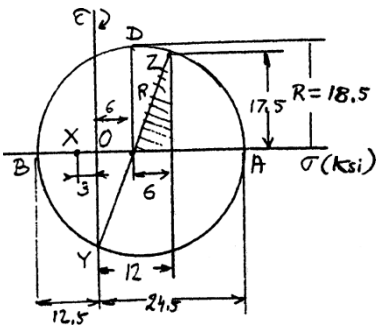


PROBLEM 7.72

For the state of stress shown, determine the maximum shearing stress when (*a*) $\tau_{yz} = 17.5$ ksi, (*b*) $\tau_{yz} = 8$ ksi, (*c*) $\tau_{yz} = 0$.

SOLUTION

(*a*) $\tau_{yz} = 17.5$ ksi $\quad \sigma_x = -3$ ksi



$$R = \sqrt{(6)^2 + (17.5)^2} = 18.5$$

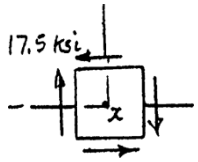
$$\sigma_A = 6 + 18.5 = 24.5$$

$$\sigma_B = 6 - 18.5 = -12.5$$

$$\sigma_{\max} = \sigma_A = 24.5 \text{ ksi}$$

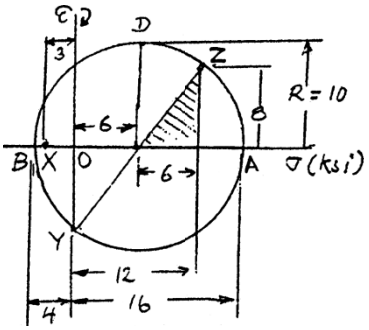
$$\sigma_{\min} = \sigma_B = -12.5 \text{ ksi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$



$\tau_{\max} = 18.50$ ksi ◀

(*b*) $\tau_{yz} = 8$ ksi $\quad \sigma_x = -3$ ksi



$$R = \sqrt{(6)^2 + (8)^2} = 10$$

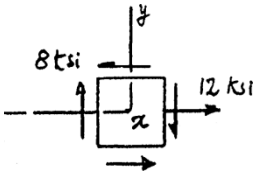
$$\sigma_A = 6 + 10 = 16$$

$$\sigma_B = 6 - 10 = -4$$

$$\sigma_{\max} = \sigma_A = 16 \text{ ksi}$$

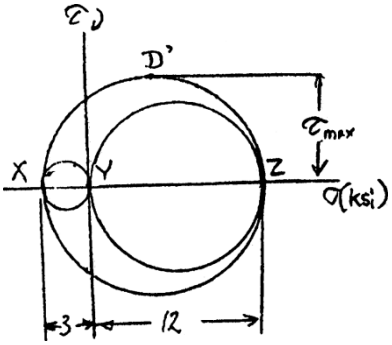
$$\sigma_{\min} = \sigma_B = -4 \text{ ksi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$



$\tau_{\max} = 10.00$ ksi ◀

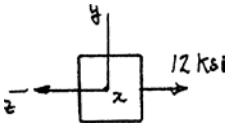
(*c*) $\tau_{yz} = 0$ $\quad \sigma_x = -3$ ksi



$$\sigma_{\max} = \sigma_z = 12 \text{ ksi}$$

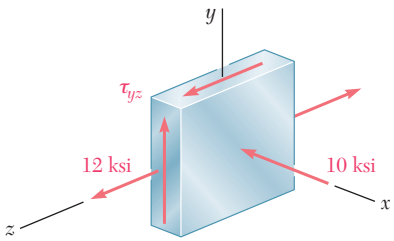
$$\sigma_{\min} = \sigma_x = -3 \text{ ksi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$



$\tau_{\max} = 7.50$ ksi ◀

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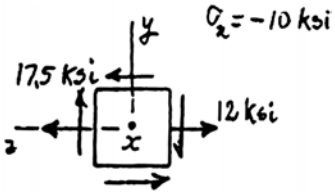


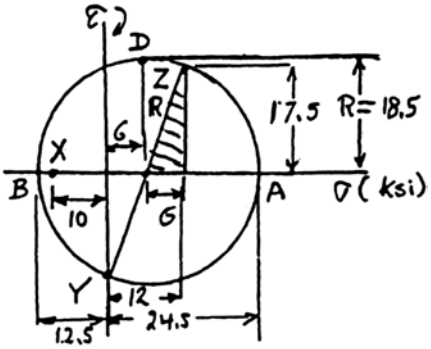
PROBLEM 7.73

For the state of stress shown, determine the maximum shearing stress when (*a*) $\tau_{yz} = 17.5$ ksi, (*b*) $\tau_{yz} = 8$ ksi, (*c*) $\tau_{yz} = 0$.

SOLUTION

(*a*) $\tau_{yz} = 17.5$ ksi





$$R = \sqrt{(6)^2 + (17.5)^2} = 18.5$$

$$\sigma_A = 6 + 18.5 = 24.5$$

$$\sigma_B = 6 - 18.5 = -12.5$$

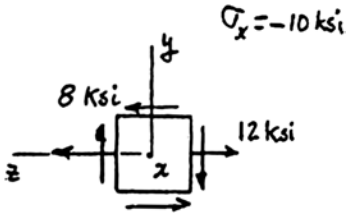
$$\sigma_{\max} = \sigma_A = 24.5 \text{ ksi}$$

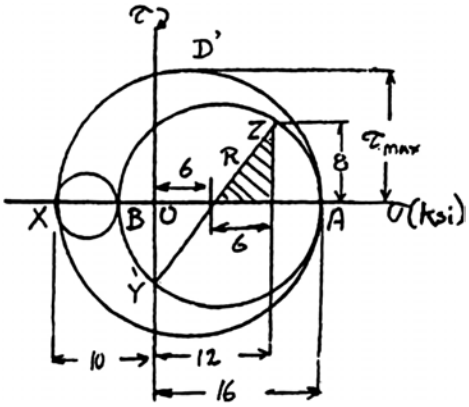
$$\sigma_{\min} = \sigma_B = -12.5 \text{ ksi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$$\tau_{\max} = 18.50 \text{ ksi} \blacktriangleleft$$

(*b*) $\tau_{yz} = 8$ ksi





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PROBLEM 7.73 *(Continued)*

$$R = \sqrt{(6)^2 + (8)^2} = 10$$

$$\sigma_A = 6 + 10 = 16$$

$$\sigma_B = 6 - 10 = -4$$

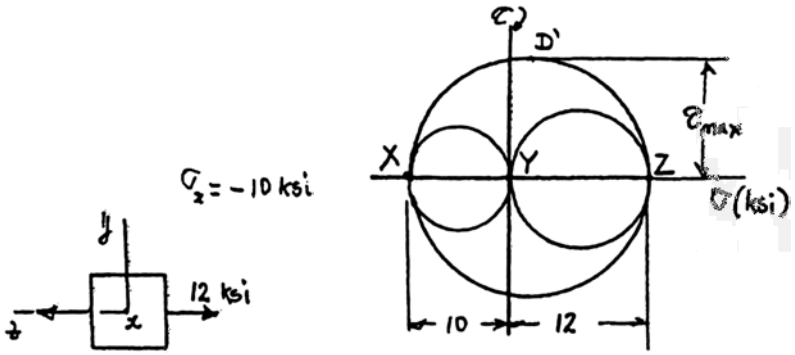
$$\sigma_{\max} = \sigma_A = 16 \text{ ksi}$$

$$\sigma_{\min} = \sigma_x = -10 \text{ ksi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$$\tau_{\max} = 13.00 \text{ ksi} \blacktriangleleft$$

(*c*) $\tau_{yz} = 0$



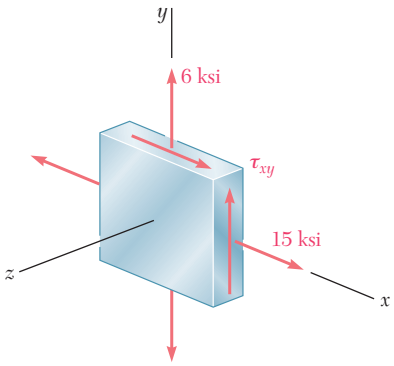
$$\sigma_{\max} = \sigma_z = 12 \text{ ksi}$$

$$\sigma_{\min} = \sigma_x = -10 \text{ ksi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$$\tau_{\max} = 11.00 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.74

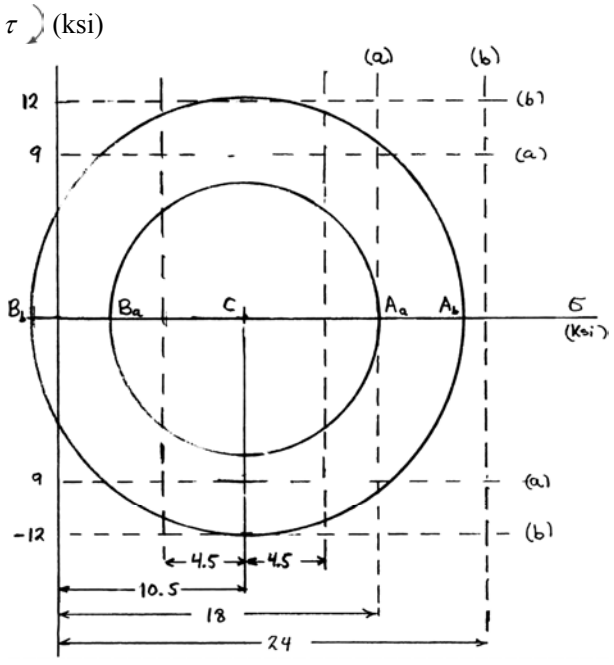
For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (*a*) 9 ksi, (*b*) 12 ksi.

SOLUTION

$$\sigma_x = 15 \text{ ksi} \qquad \sigma_y = 6 \text{ ksi}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 10.5 \text{ ksi}$$

$$u = \frac{\sigma_x - \sigma_y}{2} = 4.5 \text{ ksi}$$



(*a*) For $\tau_{\max} = 9$ ksi,
center of Mohr's circle lies at point *C*.
Lines marked (*a*) show the limits on $\tau_{\max}$.
Limit on $\sigma_{\max}$ is $\sigma_{\max} = 2\tau_{\max} = 18$ ksi.
The Mohr's circle $\sigma_a = \sigma_{\max}$ corresponds to point A_a.

$$R = \sigma_a - \sigma_{\text{ave}} = 18 - 10.5 = 7.5 \text{ ksi}$$

$$R = \sqrt{u^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm\sqrt{R^2 - u^2}$$

$$= \pm\sqrt{7.5^2 - 4.5^2}$$

$$= \pm 6.00 \text{ ksi} \qquad \tau_{xy} = \pm 6.00 \text{ ksi} \blacktriangleleft$$

(*b*) For $\tau_{\max} = 12$ ksi,
center of Mohr's circle lies at point *C*.

$$R = 12 \text{ ksi}$$

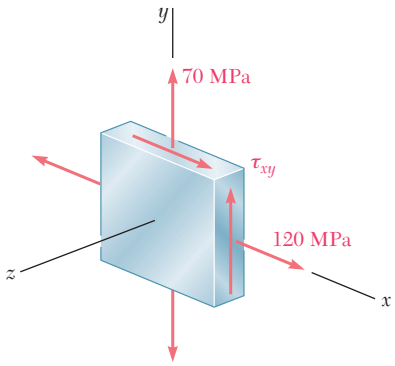
$$\tau_{xy} = \pm\sqrt{R^2 - u^2}$$

$$= \pm 11.24 \text{ ksi} \qquad \tau_{xy} = \pm 11.24 \text{ ksi} \blacktriangleleft$$

Checking, $\sigma_a = 10.5 + 12 = 22.5$ ksi
$\sigma_b = 10.5 - 12 = -1.5$ ksi
$\sigma_c = 0$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 12 \text{ ksi} \quad \text{(o.k.)}$$

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PROBLEM 7.75

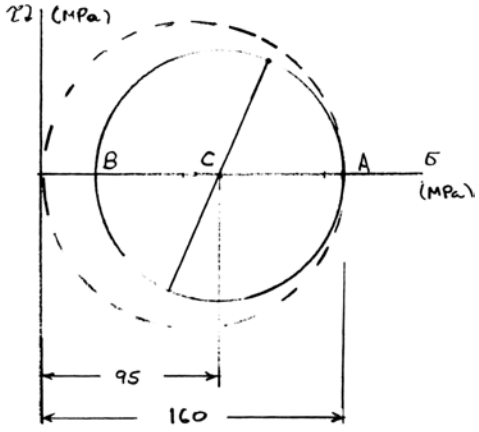
For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is 80 MPa.

SOLUTION

$$\sigma_x = 120 \text{ MPa} \qquad \sigma_y = 70 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 95 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{120 - 70}{2} = 25 \text{ MPa}$$



Assume $\sigma_{\min} = 0$

$$\sigma_{\max} = 2\tau_{\max} = 160 \text{ MPa}$$

$$\sigma_a = \sigma_{\max} = \sigma_{\text{ave}} + R$$

$$R = \sigma_{\max} - \sigma_{\text{ave}} = 160 - 95 = 65 \text{ MPa}$$

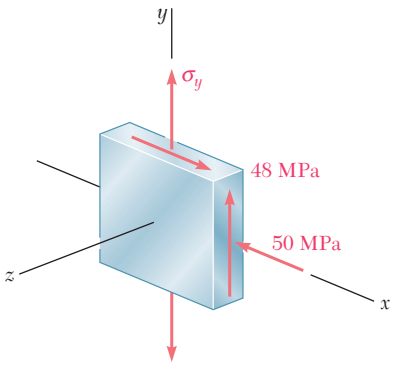
$$R^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$\tau_{xy}^2 = R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 = 65^2 - 25^2 = 60^2$$

$$\tau_{xy} = \pm 60.0 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_a - 2R = 160 - 130 = 30 \text{ MPa} \geq 0 \quad \text{(o.k.)}$$

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PROBLEM 7.76

For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 73 MPa.

SOLUTION

$$\sigma_x = -50 \text{ MPa}, \quad \tau_{xy} = 48 \text{ MPa}$$

Let $$u = \frac{\sigma_y - \sigma_x}{2} \qquad \sigma_y = 2u + \sigma_x$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$$

$$R = \sqrt{u^2 + \tau_{xy}^2} \qquad u = \pm\sqrt{R^2 - \tau_{xy}^2}$$

Case (1) $\tau_{\max} = R = 73$ MPa, $\quad u = \pm\sqrt{73^2 - 48^2} = \pm 55$ MPa

(1a) $u = +55$ MPa $\quad \sigma_y = 2u + \sigma_x = 60$ MPa

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 5 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 78 \text{ MPa}, \quad \sigma_b = \sigma_{\text{ave}} - R = -68 \text{ MPa}$$

$$\sigma_a = 0 \quad \sigma_{\max} = 78 \text{ MPa}, \quad \sigma_{\min} = -68 \text{ MPa}, \quad \tau_{\max} = 73 \text{ MPa}$$

(1b) $u = -55$ MPa $\quad \sigma_y = 2u + \sigma_x = -160$ MPa (reject)

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -105 \text{ MPa}, \quad \sigma_a = \sigma_{\text{ave}} + R = -32 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -178 \text{ MPa}, \quad \sigma_c = 0, \quad \sigma_{\max} = 0$$

$$\sigma_{\min} = -178 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 89 \text{ MPa} \neq 73 \text{ MPa}$$

$\sigma_y = 60.0$ MPa ◀

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PROBLEM 7.76 *(Continued)*

Case (2) Assume $\sigma_{\max} = 0$. $\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 73$ MPa

$$\sigma_{\min} = -146 \text{ MPa} = \sigma_b$$

$$\sigma_b = \sigma_{\text{ave}} - R = \sigma_x + u - \sqrt{u^2 + \tau_{xy}^2}$$

$$\sqrt{u^2 + \tau_{xy}^2} = -\sigma_x + u - \sigma_b$$

$$u^2 + \tau_{xy}^2 = (\sigma_x - \sigma_b)^2 + 2(\sigma_x - \sigma_b)u + u^2$$

$$2u = \frac{\tau_{xy}^2 - (\sigma_x - \sigma_b)^2}{\sigma_x - \sigma_b} = \frac{(48)^2 - (-50 + 146)^2}{-50 + 146} = -72 \text{ MPa}$$

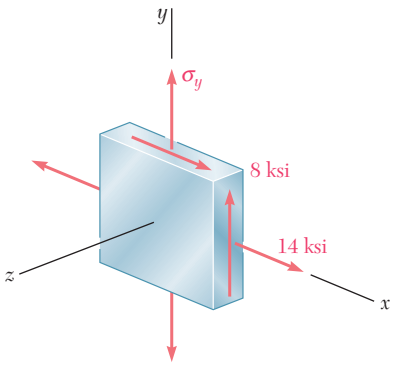
$$u = -36 \text{ MPa} \qquad \sigma_y = 2u + \sigma_x = -122 \text{ MPa}$$

$$R = \sqrt{u^2 + \tau_{xy}^2} = 60 \text{ MPa}$$

$$\sigma_a = \sigma_b + 2R = -146 + 120 = -26 \text{ MPa} \quad \text{(o.k.)}$$

$\sigma_y = -122.0$ MPa ◀

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PROBLEM 7.77

For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 10 ksi.

SOLUTION

$$\sigma_x = 14 \text{ ksi}, \quad \tau_{xy} = 8 \text{ ksi}, \quad \tau_{\max} = 10 \text{ ksi}$$

Let $$u = \frac{\sigma_y - \sigma_x}{2} \qquad \sigma_y = 2u + \sigma_x$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$$

$$R = \sqrt{u^2 + \tau_{xy}^2} \qquad u = \pm\sqrt{R^2 - \tau_{xy}^2}$$

Case (1) $$\tau_{\max} = R = 10 \text{ ksi}, \qquad u = \pm 6 \text{ ksi}$$

(1a) $u = +6\text{ksi}$ $\sigma_y = 2u + \sigma_x = 26$ ksi (reject)

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 20 \text{ ksi}, \quad \sigma_a = \sigma_{\text{ave}} + R = 30 \text{ ksi}, \quad \sigma_b = \sigma_{\text{ave}} - R = 10 \text{ ksi}$$

$$\sigma_{\max} = 30 \text{ ksi}, \quad \sigma_{\min} = 0, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 15 \text{ ksi} \neq 7.5 \text{ ksi}$$

(1b) $u = -6$ ksi $\sigma_y = 2u + \sigma_x = 2$ ksi

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 8 \text{ ksi}, \quad \sigma_a = \sigma_{\text{ave}} + R = 18 \text{ ksi}, \quad \sigma_b = \sigma_{\text{ave}} - R = -2 \text{ ksi}$$

$$\sigma_{\max} = 18 \text{ ksi}, \quad \sigma_{\min} = -2 \text{ ksi}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 10 \text{ ksi} \quad \text{(o.k.)}$$

$$\sigma_y = 2.00 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.77 *(Continued)*

Case (2) Assume $\sigma_{\min} = 0$. $\sigma_{\max} = 2\tau_{\max} = 20 \text{ ksi} = \sigma_a$

$$\sigma_a = \sigma_{\text{ave}} + R = \sigma_x + u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_a - \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$$

$$(\sigma_a - \sigma_x - u)^2 = u^2 + \tau_{xy}^2$$

$$(\sigma_a - \sigma_x)^2 - 2(\sigma_a - \sigma_x)u + u^2 = u^2 + \tau_{xy}^2$$

$$2u = \frac{(\sigma_a - \sigma_x)^2 - \tau_{xy}^2}{\sigma_a - \sigma_x} = \frac{(20 - 14)^2 - 8^2}{20 - 14} = -4.6667 \text{ ksi}$$

$$u = -2.3333 \text{ ksi} \quad \sigma_y = 2u + \sigma_x = 9.3333 \text{ ksi}$$

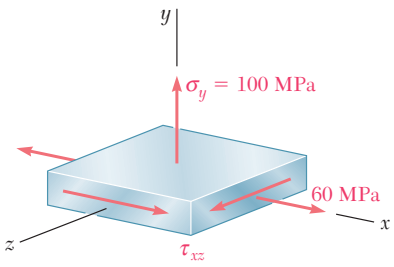
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 11.6667 \text{ ksi} \quad R = \sqrt{u^2 + \tau_{xy}^2} = 8.3333 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 20 \text{ ksi} \qquad \sigma_b = \sigma_{\text{ave}} - R = 3.3334 \text{ ksi}$$

$$\sigma_{\max} = 20 \text{ ksi}, \quad \sigma_{\min} = 0, \quad \tau_{\max} = 10 \text{ ksi}$$

$\sigma_y = 9.33 \text{ ksi}$ ◀

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PROBLEM 7.78

For the state of stress shown, determine the range of values of τ_{xz} for which the maximum shearing stress is equal to or less than 60 MPa.

SOLUTION

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_z = 0, \quad \sigma_y = 100 \text{ MPa}$$

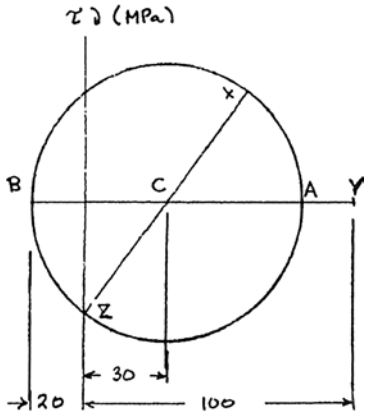
For Mohr's circle of stresses in *zx* plane,

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_z) = 30 \text{ MPa}$$

$$u = \frac{\sigma_x - \sigma_z}{2} = 30 \text{ MPa}$$

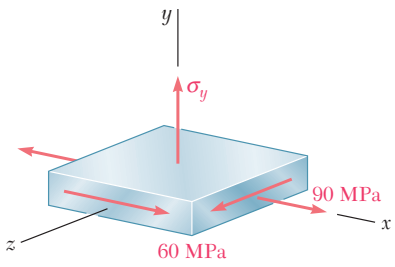
Assume $\sigma_{\max} = \sigma_y = 100$ MPa

$$\sigma_{\min} = \sigma_b = \sigma_{\max} - 2\tau_{\max}$$
$$= 100 - (2)(60) = -20 \text{ MPa}$$
$$R = \sigma_{\text{ave}} - \sigma_b$$
$$= 30 - (-20) = 50 \text{ MPa}$$
$$\sigma_a = \sigma_{\text{ave}} + R$$
$$= 30 + 50 = 80 \text{ MPa} < \sigma_y$$
$$R = \sqrt{u^2 + \tau_{xz}^2}$$
$$\tau_{xz} = \pm\sqrt{R^2 - u^2}$$
$$= \pm\sqrt{50^2 - 30^2} = \pm 40 \text{ MPa}$$



$$-40.0 \text{ MPa} \le \tau_{xz} \le 40.0 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.79

For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 80 MPa.

SOLUTION

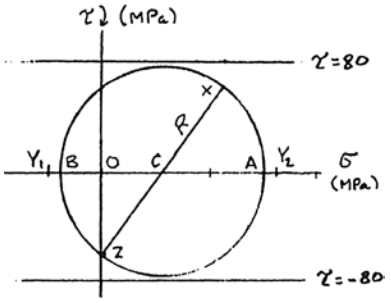
$$\sigma_x = 90 \text{ MPa}, \quad \sigma_z = 0, \quad \tau_{xz} = 60 \text{ MPa}$$

Mohr's circle of stresses in *zx* plane:

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_z) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{zx}^2} = \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 120 \text{ MPa}, \quad \sigma_b = \sigma_{\text{ave}} - R = -30 \text{ MPa}$$



Assume $\sigma_{\max} = \sigma_a = 120$ MPa.

$$\sigma_y = \sigma_{\min} = \sigma_{\max} - 2\tau_{\max}$$

$$= 120 - (2)(80)$$

$$\sigma_y = -40.0 \text{ MPa} \blacktriangleleft$$

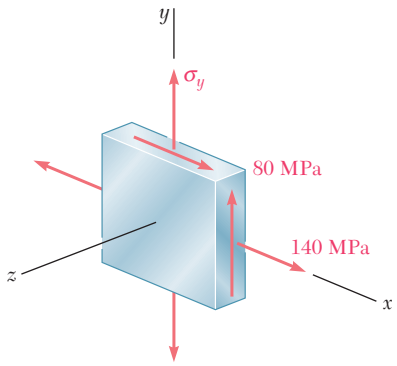
Assume $\sigma_{\min} = \sigma_b = -30$ MPa.

$$\sigma_y = \sigma_{\max} = \sigma_{\min} + 2\tau_{\max}$$

$$= -30 + (2)(80)$$

$$\sigma_y = 130.0 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.80*

For the state of stress of Prob. 7.69, determine (*a*) the value of σ_y for which the maximum shearing stress is as small as possible, (*b*) the corresponding value of the shearing stress.

SOLUTION

Let $$u = \frac{\sigma_x - \sigma_y}{2} \qquad \sigma_y = \sigma_x - 2u$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x - u$$

$$R = \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{\text{ave}} - R = \sigma_x - u - \sqrt{u^2 + \tau_{xy}^2}$$

Assume $\tau_{\max}$ is the in-plane shearing stress. $\tau_{\max} = R$

Then $\tau_{\max\,(\text{in-plane})}$ is minimum if $u = 0$.

$$\sigma_y = \sigma_x - 2u = \sigma_x = 140 \text{ MPa}, \qquad \sigma_{\text{ave}} = \sigma_x - u = 140 \text{ MPa}$$

$$R = |\tau_{xy}| = 80 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 140 + 80 = 220 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 140 - 80 = 60 \text{ MPa}$$

$$\sigma_{\max} = 220 \text{ MPa}, \quad \sigma_{\min} = 0, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 110 \text{ MPa}$$

Assumption is incorrect.

Assume $$\sigma_{\max} = \sigma_a = \sigma_{\text{ave}} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_{\min} = 0 \qquad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = \frac{1}{2}\sigma_a$$

$$\frac{d\sigma_a}{du} = -1 + \frac{u}{\sqrt{u^2 + \tau_{xy}^2}} \neq 0 \qquad \text{(no minimum)}$$

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PROBLEM 7.80* *(Continued)*

Optimum value for u occurs when $\tau_{\max\,(\text{out-of-plane})} = \tau_{\max\,(\text{in-plane})}$

$$\frac{1}{2}(\sigma_a + R) = R \quad \text{or} \quad \sigma_a = R \quad \text{or} \quad \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$$

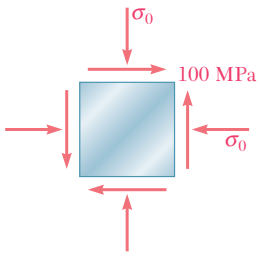
$$(\sigma_x - u)^2 = \sigma_x^2 - 2u\sigma_x + \cancel{u^2} = \cancel{u^2} + \tau_{xy}^2$$

$$2u = \frac{\sigma_x^2 - \tau_{xy}^2}{\sigma_x} = \frac{140^2 - 80^2}{140} = 94.286 \text{ MPa} \qquad u = 47.143 \text{ MPa}$$

(*a*) $\sigma_y = \sigma_x - 2u = 140 - 94.286$ $\qquad \sigma_y = 45.7 \text{ MPa}$ ◀

(*b*) $R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{\max} = 92.857 \text{ MPa}$ $\qquad \tau_{\max} = 92.857 \text{ MPa}$ ◀

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PROBLEM 7.81

The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\sigma_0 = 200$ MPa, (*b*) $\sigma_0 = 240$ MPa, (*c*) $\sigma_0 = 280$ MPa. If yield does not occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_{\text{ave}} = -\sigma_0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(*a*) $\underline{\sigma_0 = 200 \text{ MPa}}$ $\quad \sigma_{\text{ave}} = -200$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -100 \text{ MPa}, \qquad \sigma_b = \sigma_{\text{ave}} - R = -300 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 264.56 \text{ MPa} < 325 \text{ MPa} \qquad \text{(No yielding)}$$

$$F.S. = \frac{325}{264.56} \qquad F.S. = 1.228 \blacktriangleleft$$

(*b*) $\underline{\sigma_0 = 240 \text{ MPa}}$ $\quad \sigma_{\text{ave}} = -240$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -140 \text{ MPa}, \qquad \sigma_b = \sigma_{\text{ave}} - R = -340 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 295.97 \text{ MPa} < 325 \text{ MPa} \qquad \text{(No yielding)}$$

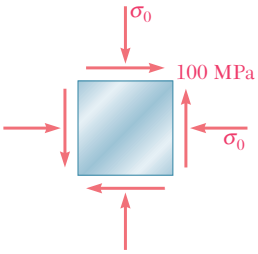
$$F.S. = \frac{325}{295.97} \qquad F.S. = 1.098 \blacktriangleleft$$

(*c*) $\underline{\sigma_0 = 280 \text{ MPa}}$ $\quad \sigma_{\text{ave}} = -280$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -180 \text{ MPa}, \qquad \sigma_b = \sigma_{\text{ave}} - R = -380 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 329.24 \text{ MPa} > 325 \text{ MPa} \qquad \text{(Yielding occurs)} \blacktriangleleft$$

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PROBLEM 7.82

Solve Prob. 7.81, using the maximum-shearing-stress criterion.

PROBLEM 7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\sigma_0 = 200$ MPa, (*b*) $\sigma_0 = 240$ MPa, (*c*) $\sigma_0 = 280$ MPa. If yield does not occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_{\text{ave}} = -\sigma_0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(*a*) $\underline{\sigma_0 = 200 \text{ MPa}}$: $\sigma_{\text{ave}} = -200$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -100 \text{ MPa} \qquad \sigma_b = \sigma_{\text{ave}} - R = -300 \text{ MPa}$$

$$\sigma_{\max} = 0, \quad \sigma_{\min} = -300 \text{ MPa}$$

$$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 300 \text{ MPa} < 325 \text{ MPa} \qquad \text{(No yielding)}$$

$$F.S. = \frac{325}{300} \qquad F.S. = 1.083 \blacktriangleleft$$

(*b*) $\underline{\sigma_0 = 240 \text{ MPa}}$: $\sigma_{\text{ave}} = -240$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -140 \text{ MPa}, \qquad \sigma_b = \sigma_{\text{ave}} - R = -340 \text{ MPa}$$

$$\sigma_{\max} = 0, \qquad \sigma_{\min} = -340 \text{ MPa}$$

$$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 340 \text{ MPa} > 325 \text{ MPa} \qquad \text{(Yielding occurs)} \blacktriangleleft$$

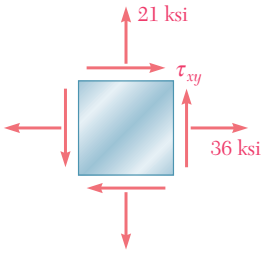
(*c*) $\underline{\sigma_0 = 280 \text{ MPa}}$: $\sigma_{\text{ave}} = -280$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -180 \text{ MPa}, \qquad \sigma_b = \sigma_{\text{ave}} - R = -380 \text{ MPa}$$

$$\sigma_{\max} = 0, \quad \sigma_{\min} = -380 \text{ MPa}$$

$$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 380 \text{ MPa} > 325 \text{ MPa} \qquad \text{(Yielding occurs)} \blacktriangleleft$$

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PROBLEM 7.83

The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 45$ ksi. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\tau_{xy} = 9$ ksi, (*b*) $\tau_{xy} = 18$ ksi, (*c*) $\tau_{xy} = 20$ ksi. If yield does not occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_x = 36 \text{ ksi}, \quad \sigma_y = 21 \text{ ksi}, \quad \sigma_z = 0$$

For stresses in *xy* plane, $\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 28.5$ ksi

$$\frac{\sigma_x - \sigma_y}{2} = 7.5 \text{ ksi}$$

(*a*) $\tau_{xy} = 9$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (9)^2} = 11.715$ ksi

$\sigma_a = \sigma_{\text{ave}} + R = 40.215$ ksi, $\quad \sigma_b = \sigma_{\text{ave}} - R = 16.875$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 34.977 \text{ ksi} < 45 \text{ ksi}$ (No yielding)

$F.S. = \frac{45}{39.977}$ $\quad F.S. = 1.287$ ◀

(*b*) $\tau_{xy} = 18$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (18)^2} = 19.5$ ksi

$\sigma_a = \sigma_{\text{ave}} + R = 48$ ksi, $\quad \sigma_b = \sigma_{\text{ave}} - R = 9$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 44.193 \text{ ksi} < 45 \text{ ksi}$ (No yielding)

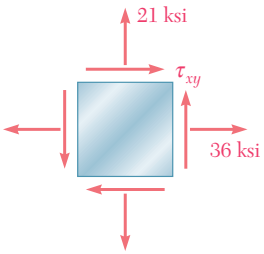
$F.S. = \frac{45}{44.193}$ $\quad F.S. = 1.018$ ◀

(*c*) $\tau_{xy} = 20$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (20)^2} = 21.36$ ksi

$\sigma_a = \sigma_{\text{ave}} + R = 49.86$ ksi, $\quad \sigma_b = \sigma_{\text{ave}} - R = 7.14$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 46.732 \text{ ksi} > 45 \text{ ksi}$ (Yielding occurs) ◀

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PROBLEM 7.84

Solve Prob. 7.83, using the maximum-shearing-stress criterion.

PROBLEM 7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 45$ ksi. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\tau_{xy} = 9$ ksi, (*b*) $\tau_{xy} = 18$ ksi, (*c*) $\tau_{xy} = 20$ ksi. If yield does not occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_x = 36 \text{ ksi}, \quad \sigma_y = 21 \text{ ksi}, \quad \sigma_z = 0$$

For stress in *xy* plane, $\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 28.5 \text{ ksi}$ $\quad \frac{\sigma_x - \sigma_y}{2} = 7.5 \text{ ksi}$

(*a*) $\tau_{xy} = 9$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 11.715$ ksi

$$\sigma_a = \sigma_{\text{ave}} + R = 40.215 \text{ ksi}, \quad \sigma_b = \sigma_{\text{ave}} - R = 16.875 \text{ ksi}$$

$$\tau_{\max} = 34.977 \text{ ksi}, \quad \sigma_{\min} = 0$$

$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 40.215 \text{ ksi} < 45 \text{ ksi}$ (No yielding)

$F.S. = \frac{45}{40.215}$ $F.S. = 1.119$ ◀

(*b*) $\tau_{xy} = 18$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 19.5$ ksi

$$\sigma_a = \sigma_{\text{ave}} + R = 48 \text{ ksi}, \quad \sigma_b = \sigma_{\text{ave}} - R = 9 \text{ ksi}$$

$\sigma_{\max} = 48$ ksi $\quad \sigma_{\min} = 0$

$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 48 \text{ ksi} > 45 \text{ ksi}$ (Yielding occurs) ◀

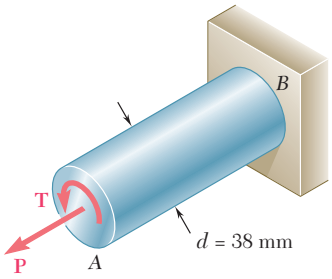
(*c*) $\tau_{xy} = 20$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 21.36$ ksi

$$\sigma_a = \sigma_{\text{ave}} + R = 49.86 \text{ ksi} \quad \sigma_b = \sigma_{\text{ave}} - R = 7.14 \text{ ksi}$$

$\tau_{\max} = 49.86$ ksi $\quad \sigma_{\min} = 0$

$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 49.86 \text{ ksi} > 45 \text{ ksi}$ (Yielding occurs) ◀

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PROBLEM 7.85

The 38-mm-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 240$ kN.

SOLUTION

$$P = 240 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(38)^2 = 1.13411 \times 10^3 \text{ mm}^2 = 1.13411 \times 10^{-3} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{240 \times 10^3}{1.13411 \times 10^{-3}} = 211.62 \times 10^6 \text{ Pa} = 211.62 \text{ MPa}$$

$$\sigma_y = 0$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$2\tau_{\max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_y$$

$$4\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2$$

$$\tau_{xy} = \frac{1}{2}\sqrt{\sigma_Y^2 - \sigma_x^2} = \frac{1}{2}\sqrt{250^2 - 211.62^2}$$

$$= 66.553 \text{ MPa} = 66.553 \times 10^6 \text{ Pa}$$

From torsion:

$$\tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$$

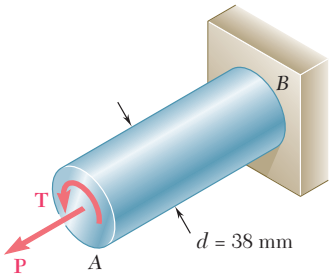
$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}\left(\frac{38}{2}\right)^4 = 204.71 \times 10^3 \text{ mm}^4 = 204.71 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2} d = 19 \times 10^{-3} \text{ m}$$

$$T = \frac{(204.71 \times 10^{-9})(66.553 \times 10^6)}{19 \times 10^{-3}}$$

$$= 717 \text{ N} \cdot \text{m}$$ ◀

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PROBLEM 7.86

Solve Prob. 7.85, using the maximum-distortion-energy criterion.

PROBLEM 7.85 The 38-mm-diameter shaft *AB* is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 240$ kN.

SOLUTION

$$P = 240 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(38)^2 = 1.13411 \times 10^3 \text{ mm}^2 = 1.13411 \times 10^{-3} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{240 \times 10^3}{1.13411 \times 10^{-3}} = 211.62 \times 10^6 \text{ Pa} = 211.62 \text{ MPa}$$

$$\sigma_y = 0$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R = \frac{1}{2}\sigma_x + \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{\text{ave}} - R = \frac{1}{2}\sigma_x - \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b = \frac{1}{4}\sigma_x^2 + \sigma_x\sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2$$

$$+ \frac{1}{4}\sigma_x^2 - \sigma_x\sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 - \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2$$

$$= \sigma_x^2 + 3\tau_{xy}^2 = \sigma_Y^2$$

$$\tau_{xy}^2 = \frac{1}{3}\left(\sigma_Y^2 - \sigma_x^2\right)$$

$$\tau_{xy} = \frac{1}{\sqrt{3}}\sqrt{250^2 - 211.62^2} = 76.848 \text{ MPa} = 76.848 \times 10^6 \text{ Pa}$$

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PROBLEM 7.86 ***(Continued)***

From torsion, $\tau_{xy} = \dfrac{Tc}{J}$ $\qquad T = \dfrac{J\tau_{xy}}{c}$

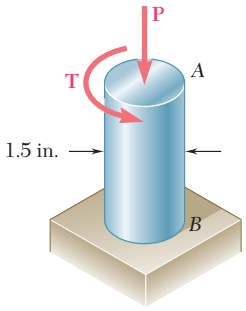
$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{38}{2}\right)^4 = 204.71 \times 10^3\ \text{mm}^4 = 204.71 \times 10^{-9}\ \text{m}^4$$

$$c = \frac{1}{2}d = 19 \times 10^{-3}\ \text{m}$$

$$T = \frac{(204.71 \times 10^{-9})(76.848 \times 10^6)}{19 \times 10^{-3}}$$

$$= 828\ \text{N} \cdot \text{m}$$ ◀

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PROBLEM 7.87

The 1.5-in.-diameter shaft *AB* is made of a grade of steel with a 42-ksi tensile yield stress. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 60$ kips.

SOLUTION

$$P = 60 \text{ kips}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$

$$\sigma_x = -\frac{P}{A} = -\frac{60}{1.76715} = -33.953 \text{ ksi}$$

$$\sigma_y = 0$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$2\tau_{\max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_Y$$

$$4\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2$$

$$\tau_{xy} = \frac{1}{2}\sqrt{\sigma_Y^2 - \sigma_x^2} = \frac{1}{2}\sqrt{42^2 - 33.953^2}$$

$$= 12.3612 \text{ ksi}$$

From torsion,

$$\tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$$

$$c = \frac{1}{2}d = 0.75 \text{ in.}$$

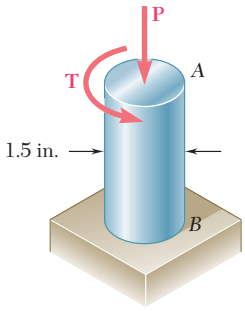
$$J = \frac{\pi}{2}c^4 = 0.49701 \text{ in}^4$$

$$T = \frac{(0.49701)(12.3612)}{0.75}$$

$$= 8.19 \text{ kip} \cdot \text{in.}$$

◀

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PROBLEM 7.88

Solve Prob. 7.87, using the maximum-distortion-energy criterion.

PROBLEM 7.87 The 1.5-in.-diameter shaft *AB* is made of a grade of steel with a 42-ksi tensile yield stress. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 60$ kips.

SOLUTION

$$P = 60 \text{ kips}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$

$$\sigma_x = -\frac{p}{A} = -\frac{60}{1.76715} = -33.953 \text{ ksi}$$

$$\sigma_y = 0$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R$$

$$\sigma_b = \sigma_{\text{ave}} - R$$

$$\begin{aligned}\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b &= (\sigma_{\text{ave}} + R)^2 + (\sigma_{\text{ave}} - R)^2 - (\sigma_{\text{ave}} + R)(\sigma_{\text{ave}} - R)\\ &= \sigma_{\text{ave}}^2 + 2\sigma_{\text{ave}}R + R^2 + \sigma_{\text{ave}}^2 - 2\sigma_{\text{ave}}R + R^2 - \sigma_{\text{ave}}^2 + R^2\\ &= \sigma_{\text{ave}}^2 + 3R^2\\ &= \frac{1}{4}\sigma_x^2 + 3\left(\frac{1}{4}\sigma_x^2 + \tau_{xy}^2\right) = \sigma_x^2 + 3\tau_{xy}^2 = \sigma_Y^2\end{aligned}$$

$$3\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2$$

$$\tau_{xy} = \frac{1}{\sqrt{3}}\left(\sigma_Y^2 - \sigma_x^2\right) = \frac{1}{\sqrt{3}}\sqrt{42^2 - 33.953^2}$$

$$= 14.2734 \text{ ksi}$$

From torsion,

$$\tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$$

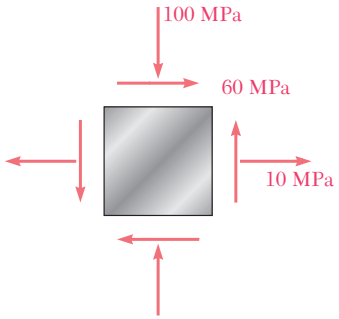
$$c = \frac{1}{2}d = 0.75 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.75)^4 = 0.49701 \text{ in}^4$$

$$T = \frac{(0.49701)(14.2734)}{0.75} = 9.46 \text{ kip} \cdot \text{in.}$$

◀

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PROBLEM 7.89

The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 80$ MPa and $\sigma_{UC} = 200$ MPa and using Mohr's criterion, determine whether rupture of the casting will occur.

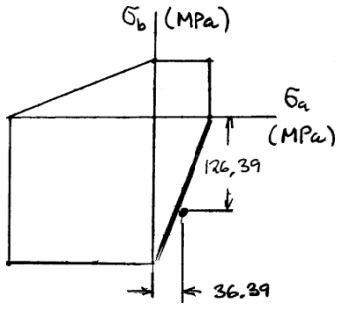
SOLUTION

$$\sigma_x = 10 \text{ MPa},$$
$$\sigma_y = -100 \text{ MPa},$$
$$\tau_{xy} = 60 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{10 - 100}{2} = -45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(55)^2 + (60)^2} = 81.39 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = -45 + 81.39 = 36.39 \text{ MPa}$$
$$\sigma_b = \sigma_{\text{ave}} - R = -45 - 81.39 = -126.39 \text{ MPa}$$



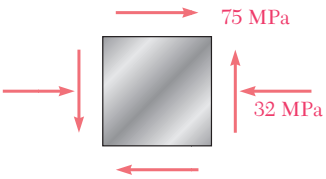
Equation of 4th quadrant of boundary:

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{36.39}{80} - \frac{(-126.39)}{200} = 1.087 > 1$$

Rupture will occur. ◀

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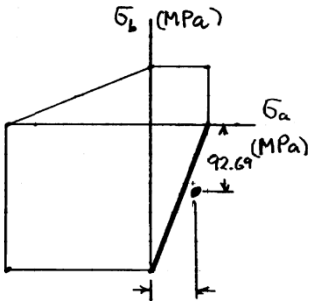


PROBLEM 7.90

The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 80$ MPa and $\sigma_{UC} = 200$ MPa and using Mohr's criterion, determine whether rupture of the casting will occur.

SOLUTION

$$\sigma_x = -32 \text{ MPa},$$
$$\sigma_y = 0,$$
$$\tau_{xy} = 75 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -16 \text{ MPa}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(16)^2 + (75)^2} = 76.69 \text{ MPa}$$



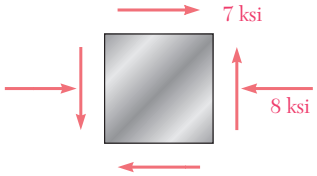
$$\sigma_a = \sigma_{\text{ave}} + R = -16 + 76.69 = 60.69 \text{ MPa}$$
$$\sigma_b = \sigma_{\text{ave}} - R = -16 - 76.69 = -92.69 \text{ MPa}$$

Equation of 4th quadrant of boundary:

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{60.69}{80} - \frac{(-92.69)}{200} = 1.222 > 1$$

Rupture will occur. ◀

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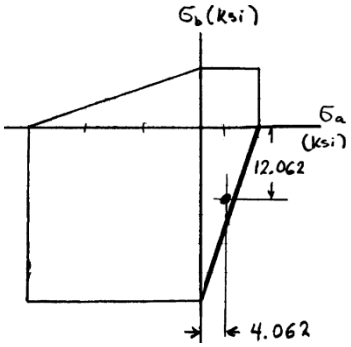


PROBLEM 7.91

The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 30$ ksi and using Mohr's criterion, determine whether rupture of the casting will occur.

SOLUTION

$$\sigma_x = -8 \text{ ksi},$$
$$\sigma_y = 0,$$
$$\tau_{xy} = 7 \text{ ksi}$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -4 \text{ ksi}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + 7^2} = 8.062 \text{ ksi}$$
$$\sigma_a = \sigma_{\text{ave}} + R = -4 + 8.062 = 4.062 \text{ ksi}$$
$$\sigma_b = \sigma_{\text{ave}} - R = -4 - 8.062 = -12.062 \text{ ksi}$$

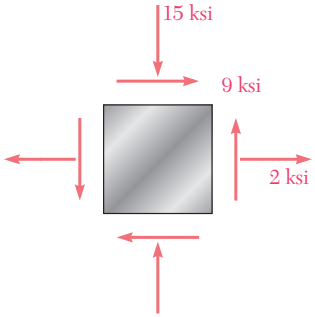


Equation of 4th quadrant of boundary:

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{4.062}{10} - \frac{(-12.062)}{30} = 0.808 < 1$$

No rupture. ◀

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PROBLEM 7.92

The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used σ_{UT} = 10 ksi and σ_{UC} = 30 ksi and using Mohr's criterion, determine whether rupture of the casting will occur.

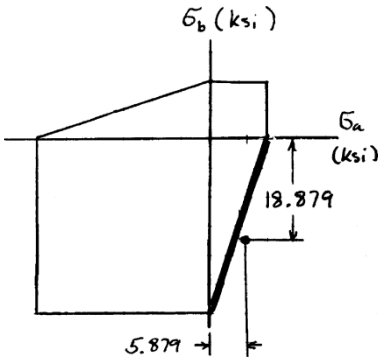
SOLUTION

$$\sigma_x = 2 \text{ ksi},$$
$$\sigma_y = -15 \text{ ksi},$$
$$\tau_{xy} = 9 \text{ ksi}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{8.5^2 + 9^2} = 12.379 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 5.879 \text{ ksi}$$
$$\sigma_b = \sigma_{\text{ave}} - R = -18.879 \text{ ksi}$$



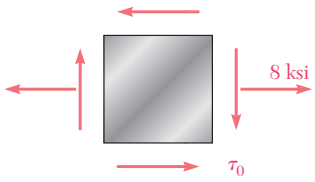
Equation of 4th quadrant of boundary:

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{5.879}{10} - \frac{(-18.879)}{30} = 1.217 > 1$$

Rupture will occur. ◀

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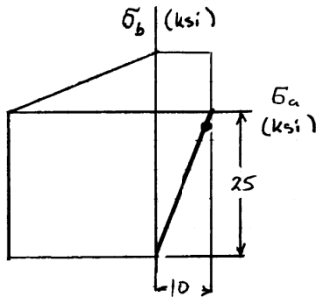


PROBLEM 7.93

The state of plane stress shown will occur at a critical point in an aluminum casting that is made of an alloy for which $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 25$ ksi. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.

SOLUTION

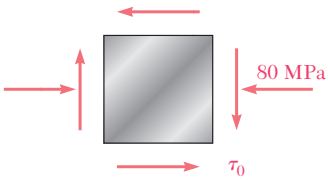
$$\sigma_x = 8 \text{ ksi},$$
$$\sigma_y = 0,$$
$$\tau_{xy} = \tau_0$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 4 \text{ ksi}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + \tau_0^2}$$
$$\tau_0 = \pm\sqrt{R^2 - 4^2}$$
$$\sigma_a = \sigma_{\text{ave}} + R = (4 + R) \text{ ksi}$$
$$\sigma_b = \sigma_{\text{ave}} - R = (4 - R) \text{ ksi}$$



Since $|\sigma_{\text{ave}}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{4 + R}{10} - \frac{4 - R}{25} = 1$$
$$\left(\frac{1}{10} + \frac{1}{25}\right)R = 1 - \frac{4}{10} + \frac{4}{25}$$
$$R = 5.429 \text{ ksi} \qquad \tau_0 = \pm\sqrt{5.429^2 - 4^2} \qquad \tau_0 = \pm 3.67 \text{ ksi} \blacktriangleleft$$

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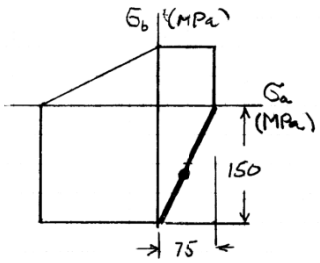


PROBLEM 7.94

The state of plane stress shown will occur at a critical point in a pipe made of an aluminum alloy for which $\sigma_{UT} = 75$ MPa and $\sigma_{UC} = 150$ MPa. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.

SOLUTION

$$\sigma_x = -80 \text{ MPa},$$
$$\sigma_y = 0,$$
$$\tau_{xy} = -\tau_0$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -40 \text{ MPa}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + \tau_0^2} \text{ MPa}$$



$$\sigma_a = \sigma_{\text{ave}} + R$$
$$\sigma_b = \sigma_{\text{ave}} - R$$
$$\tau_0 = \pm\sqrt{R^2 - 40^2}$$

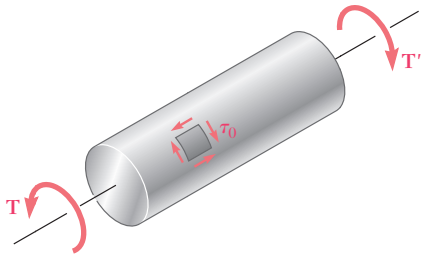
Since $|\sigma_{\text{ave}}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{-40 + R}{75} - \frac{-40 - R}{150} = 1$$
$$\frac{R}{75} + \frac{R}{150} = 1 + \frac{40}{75} - \frac{40}{150} = 1.2667$$

$$R = 63.33 \text{ MPa}, \qquad \tau_0 = \pm\sqrt{63.33^2 - 40^2}$$

$$\tau_0 = \pm 8.49 \text{ MPa} \blacktriangleleft$$

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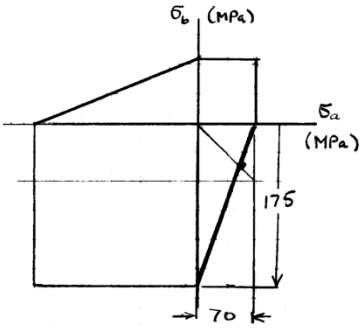


PROBLEM 7.95

The cast-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 175$ MPa. Knowing that the magnitude T of the applied torques is slowly increased and using Mohr's criterion, determine the shearing stress τ_0 that should be expected at rupture.

SOLUTION

$$\sigma_x = 0$$
$$\sigma_y = 0$$
$$\tau_{xy} = -\tau_0$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + \tau_{xy}^2} = |\tau_{xy}|$$
$$\sigma_a = \sigma_{\text{ave}} + R = R$$
$$\sigma_b = \sigma_{\text{ave}} - R = -R$$

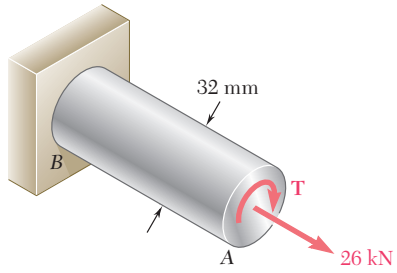


Since $|\sigma_{\text{ave}}| < R$, stress point lies in 4th quadrant. Equation of boundary of 4th quadrant is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{R}{70} - \frac{-R}{175} = 1$$
$$\left(\frac{1}{70} + \frac{1}{175}\right)R = 1$$
$$R = 50 \text{ MPa}$$
$$\tau_0 = R$$

$$\tau_0 = 50.0 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.96

The cast-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 60$ MPa and $\sigma_{UC} = 120$ MPa. Using Mohr's criterion, determine the magnitude of the torque **T** for which failure should be expected.

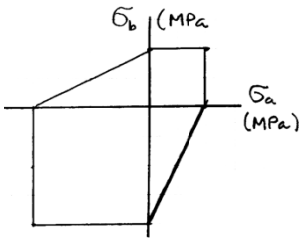
SOLUTION

$$P = 26\times10^{3}\text{ N} \qquad A = \frac{\pi}{4}(32)^2 = 804.25\text{ mm}^2 = 804.25\times10^{-6}\text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{26\times10^{3}}{804.25\times10^{-6}} = 32.328\times10^{6}\text{ Pa} = 32.328\text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(32.328 + 0) = 16.164\text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{1}{2}(32.328 - 0) = 16.164\text{ MPa}$$



$$\sigma_a = \sigma_{\text{ave}} + R = 16.164 + R\text{ MPa}$$
$$\sigma_b = \sigma_{\text{ave}} - R = 16.164 - R\text{ MPa}$$

Since $|\sigma_{\text{ave}}| < R$, stress point lies in the 4th quadrant. Equation of the 4th quadrant is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1 \qquad \frac{16.164 + R}{60} - \frac{16.164 - R}{120} = 1$$

$$\left(\frac{1}{60} + \frac{1}{120}\right)R = 1 - \frac{16.164}{60} + \frac{16.164}{120} \qquad R = 34.612\text{ MPa}$$

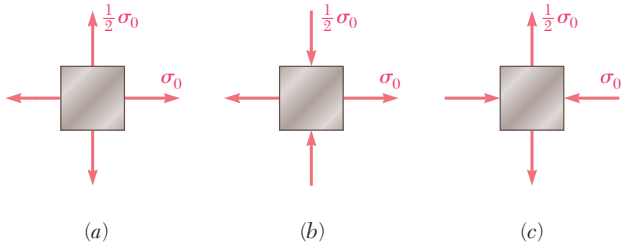
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right) + \tau_{xy}^2} \qquad \tau_{xy} = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{34.612^2 - 16.164^2} = 30.606\text{ MPa}$$

$$= 30.606\times10^{6}\text{ Pa}$$

For torsion, $\tau_{xy} = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$ where $c = \dfrac{1}{2}d = 16\text{ mm} = 16\times10^{-3}\text{ m}$

$$T = \frac{\pi}{2}c^3\tau_{xy} = \frac{\pi}{2}(16\times10^{-3})^3(30.606\times10^{6}) \qquad T = 196.9\text{ N}\cdot\text{m} \blacktriangleleft$$

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PROBLEM 7.97

A machine component is made of a grade of cast iron for which $\sigma_{UT} = 8$ ksi and $\sigma_{UC} = 20$ ksi. For each of the states of stress shown, and using Mohr's criterion, determine the normal stress σ_0 at which rupture of the component should be expected.

SOLUTION

(*a*) $\sigma_a = \sigma_0$

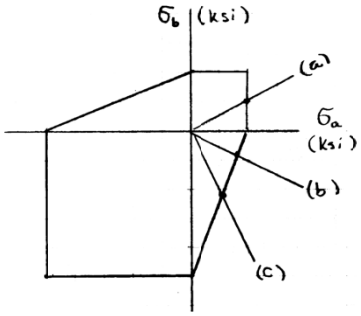
$\sigma_b = \dfrac{1}{2}\sigma_0$

Stress point lies in 1st quadrant.

$$\sigma_a = \sigma_0 = \sigma_{UT} \qquad \sigma_0 = 8.00 \text{ ksi} \blacktriangleleft$$

(*b*) $\sigma_a = \sigma_0$

$\sigma_b = -\dfrac{1}{2}\sigma_0$



Stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{\sigma_0}{8} - \frac{-\frac{1}{2}\sigma_0}{20} = 1 \qquad \sigma_0 = 6.67 \text{ ksi} \blacktriangleleft$$

(*c*) $\sigma_a = \dfrac{1}{2}\sigma_0, \quad \sigma_b = -\sigma_0, \quad$ 4th quadrant

$$\frac{\frac{1}{2}\sigma_0}{8} - \frac{-\sigma_0}{20} = 1 \qquad \sigma_0 = 8.89 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.98

A spherical pressure vessel has an outer diameter of 3 m and a wall thickness of 12 mm. Knowing that for the steel used $\sigma_{all} = 80$ MPa, $E = 200$ GPa, and $\nu = 0.29$, determine (*a*) the allowable gage pressure, (*b*) the corresponding increase in the diameter of the vessel.

SOLUTION

$$r = \frac{1}{2}d - t = \frac{1}{2}(3) - 12 \times 10^{-3} = 1.488 \text{ m}$$

$$\sigma_1 = \sigma_2 = \sigma_{all} = 80 \times 10^6 \text{ Pa}$$

(*a*) $$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$p = \frac{2t\sigma_1}{r} = \frac{(2)(12 \times 10^{-3})(80 \times 10^6)}{1.488}$$

$$p = 1.290 \times 10^6 \text{ Pa} \qquad p = 1.290 \text{ MPa} \blacktriangleleft$$

(*b*) $$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \upsilon\sigma_2)$$

$$= \frac{1-\upsilon}{E}\sigma_1 = \frac{1 - 0.29}{200 \times 10^9}(80 \times 10^6) = 284 \times 10^{-6}$$

$$\Delta d = d\varepsilon_1 = (3)(284 \times 10^{-6}) = 852 \times 10^{-6} \text{ m} \qquad \Delta d = 0.852 \text{ mm} \blacktriangleleft$$

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PROBLEM 7.99

A spherical gas container having an inner diameter of 5 m and a wall thickness of 24 mm is made of steel for which $E = 200$ GPa and $\nu = 0.29$. Knowing that the gage pressure in the container is increased from zero to 1.8 MPa, determine (*a*) the maximum normal stress in the container, (*b*) the corresponding increase in the diameter of the container.

SOLUTION

$$p = 1.8 \text{ MPa}$$

$$r = \frac{1}{2}d - t = \frac{1}{2}(5) - 24 \times 10^{-3} = 2.476 \text{ m}$$

(*a*) $$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(1.8)(2.476)}{(2)(24 \times 10^{-3})} = 92.850 \text{ MPa}$$

$$\sigma = 92.9 \text{ MPa} \blacktriangleleft$$

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \upsilon\sigma_2) = \frac{1-\upsilon}{E}\sigma_1 = \frac{1 - 0.29}{200 \times 10^9}(92.85 \times 10^6) = 329.6\ \mu$$

(*b*) $$\Delta d = d\varepsilon_1 = (5)(329.6 \times 10^{-6}) = 1.648 \times 10^{-3} \text{ m}$$

$$\Delta d = 1.648 \text{ mm} \blacktriangleleft$$

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PROBLEM 7.100

The maximum gage pressure is known to be 1150 psi in a spherical steel pressure vessel having a 10-in. outer diameter and a 0.25-in. wall thickness. Knowing that the ultimate stress in the steel used is $\sigma_U = 60$ ksi, determine the factor of safety with respect to tensile failure.

SOLUTION

$$r = \frac{d}{2} - t$$

$$= \frac{10 \text{ in.}}{2} - 0.25 \text{ in.}$$

$$= 4.75 \text{ in.}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$= \frac{(1150 \text{ psi})(4.75 \text{ in.})}{2(0.25 \text{ in.})}$$

$$= 10.925 \text{ ksi}$$

$$\text{F.S.} = \frac{\sigma_U}{\sigma_{\max}} = \frac{60 \text{ ksi}}{10.925 \text{ ksi}}$$

F.S. = 5.49 ◀

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PROBLEM 7.101

A spherical pressure vessel of 750-mm outer diameter is to be fabricated from a steel having an ultimate stress $\sigma_U = 400$ MPa. Knowing that a factor of safety of 4.0 is desired and that the gage pressure can reach 4.2 MPa, determine the smallest wall thickness that should be used.

SOLUTION

$$r = \frac{1}{2}d - t$$
$$= \frac{1}{2}(0.750\text{ m}) - t$$
$$= 0.375t\text{ (m)}$$

We have $$\sigma_{\max} = \sigma_1 = \sigma_2 = \frac{pr}{2t}$$

and $$\text{F.S.} = \frac{\sigma_U}{\sigma_{\max}}$$

Combining these two equations gives

$$\text{F.S.} = \frac{2t\sigma_U}{pr}$$

or $$2\sigma_U t = (\text{F.S.})pr$$

Substituting for r gives

$$2(400 \times 10^6\text{ Pa})t = (4)(4.2 \times 10^6\text{ Pa})(0.375 - t)$$
$$816.80 \times 10^6 t = 6.30 \times 10^6$$
$$t = 7.71 \times 10^{-3}\text{ m}$$

$$t = 7.71\text{ mm} \blacktriangleleft$$

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PROBLEM 7.102

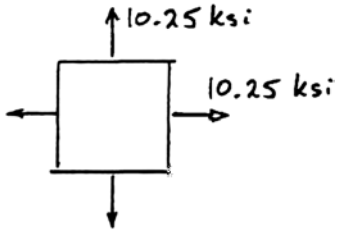
A spherical gas container made of steel has a 20-ft outer diameter and a wall thickness of $\frac{7}{16}$ in. Knowing that the internal pressure is 75 psi, determine the maximum normal stress and the maximum shearing stress in the container.

SOLUTION

$$d = 20 \text{ ft} = 240 \text{ in.}$$

$$t = \frac{7}{16} \text{ in.} = 0.4375 \text{ in.}$$

$$r = \frac{1}{2}d - t = 119.56 \text{ in.}$$



$$\sigma = \frac{pr}{2t} = \frac{(75)(119.56)}{(2)(0.4375)} = 10.25 \times 10^3 \text{ psi} \qquad \sigma = 10.25 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\max} = 10.25 \text{ ksi}$$

$$\sigma_{\min} \approx 0 \quad \text{(Neglecting small radial stress)}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 5.12 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.103

A basketball has a 300-mm outer diameter and a 3-mm wall thickness. Determine the normal stress in the wall when the basketball is inflated to a 120-kPa gage pressure.

SOLUTION

$$r = \frac{1}{2}d - t$$

$$= \frac{1}{2}(300\text{ mm}) - 3$$

$$= 147\text{ mm} \qquad \text{or} \qquad 147 \times 10^{-3}\text{ m}$$

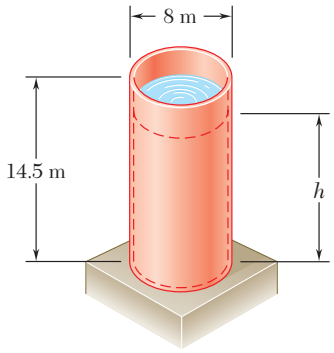
$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$= \frac{(120 \times 10^3\text{ Pa})(147 \times 10^{-3}\text{ m})}{2(3 \times 10^{-3}\text{ m})}$$

$$= 2.9400 \times 10^6\text{ Pa}$$

$$\sigma = 2.94\text{ MPa} \blacktriangleleft$$

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PROBLEM 7.104

The unpressurized cylindrical storage tank shown has a 5-mm wall thickness and is made of steel having a 400-MPa ultimate strength in tension. Determine the maximum height h to which it can be filled with water if a factor of safety of 4.0 is desired. (Density of water = 1000 kg/m^3.)

SOLUTION

$$d_0 = 8 \text{ m}$$

$$t = 5 \text{ mm} = 0.005 \text{ m}$$

$$r = \frac{1}{2}d - t = 4 - 0.005 = 3.995 \text{ m}$$

$$\sigma_{\text{all}} = \frac{\sigma_U}{\text{F.S.}} = \frac{400 \text{ MPa}}{4.0} = 100 \text{ MPa} = 100 \times 10^6 \text{ Pa}$$

$$\sigma_{\text{all}} = \frac{pr}{t}$$

$$p = \frac{t\sigma_{\text{all}}}{r} = \frac{(0.005 \text{ m})(100 \times 10^6 \text{ Pa})}{3.995 \text{ m}} = 125.156 \times 10^3 \text{ Pa}$$

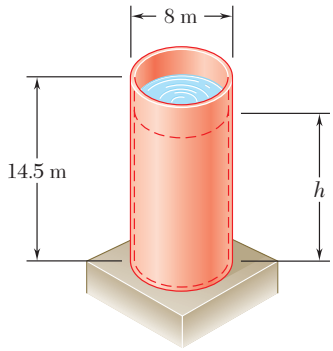
but

$$p = \rho g h,$$

$$h = \frac{p}{\rho g} = \frac{125.156 \times 10^3 \text{ Pa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = 12.7580 \text{ m}$$

$$h = 12.76 \text{ m} \blacktriangleleft$$

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PROBLEM 7.105

For the storage tank of Prob. 7.104, determine the maximum normal stress and the maximum shearing stress in the cylindrical wall when the tank is filled to capacity ($h = 14.5$ m).

PROBLEM 7.104 The unpressurized cylindrical storage tank shown has a 5-mm wall thickness and is made of steel having a 400-MPa ultimate strength in tension. Determine the maximum height h to which it can be filled with water if a factor of safety of 4.0 is desired. (Density of water = 1000 kg/m^3.)

SOLUTION

$$d_0 = 8 \text{ m}$$

$$t = 5 \text{ mm} = 0.005 \text{ m}$$

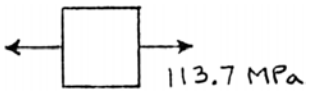
$$r = \frac{1}{2}d - t = 4 - 0.005 = 3.995 \text{ m}$$

$$p = \rho g h = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(14.5 \text{ m})$$

$$= 142.245 \times 10^3 \text{ Pa}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(142.245 \times 10^3 \text{ Pa})(3.995 \text{ m})}{0.005 \text{ m}}$$

$$= 113.654 \times 10^6 \text{ Pa}$$



$$\sigma_{max} = \sigma_1 \qquad \sigma_{max} = 113.7 \text{ MPa} \blacktriangleleft$$

$$\sigma_{min} \approx 0$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) \qquad \tau_{max} = 56.8 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.106

The bulk storage tank shown in Photo 7.3 has an outer diameter of 3.3 m and a wall thickness of 18 mm. At a time when the internal pressure of the tank is 1.5 MPa, determine the maximum normal stress and the maximum shearing stress in the tank.

SOLUTION

$$r = \frac{d}{2} - t = \frac{3.3}{2} - 18 \times 10^{-3} = 1.632 \text{ m}, \qquad t = 18 \times 10^{-3} \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.5 \times 10^6 \text{ Pa})(1.632 \text{ m})}{18 \times 10^{-3} \text{ m}} = 136 \times 10^6 \text{ Pa}$$

$$\sigma_{\max} = \sigma_1 = 136 \times 10^6 \text{ Pa} \qquad \sigma_{\max} = 136.0 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = -p \approx 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 68 \times 10^6 \text{ Pa} \qquad \tau_{\max} = 68.0 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.107

A standard-weight steel pipe of 12-in. nominal diameter carries water under a pressure of 400 psi. (*a*) Knowing that the outside diameter is 12.75 in. and the wall thickness is 0.375 in., determine the maximum tensile stress in the pipe. (*b*) Solve part *a*, assuming an extra-strong pipe is used of 12.75-in. outside diameter and 0.5-in. wall thickness.

SOLUTION

(*a*) $d_0 = 12.75 \text{ in.} \quad t = 0.375 \text{ in.} \quad r = \frac{1}{2} d_0 - t = 6.00 \text{ in.}$

$$\sigma = \frac{pr}{t} = \frac{(400)(6.00)}{0.375} = 6400 \text{ psi} \qquad \sigma = 6.40 \text{ ksi} \blacktriangleleft$$

(*b*) $d_0 = 12.75 \text{ in.} \quad t = 0.500 \text{ in.} \quad r = \frac{1}{2} d_0 - t = 5.875 \text{ in.}$

$$\sigma = \frac{pr}{t} = \frac{(400)(5.875)}{0.500} = 4700 \text{ psi} \qquad \sigma = 4.70 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.108

A cylindrical storage tank contains liquefied propane under a pressure of 1.5 MPa at a temperature of 38°C. Knowing that the tank has an outer diameter of 320 mm and a wall thickness of 3 mm, determine the maximum normal stress and the maximum shearing stress in the tank.

SOLUTION

$$r = \frac{d}{2} - t = \frac{320}{2} - 3 = 157 \text{ mm} = 157 \times 10^{-3} \text{ m}$$

$$t = 3 \times 10^{-3} \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.5 \times 10^6 \text{ Pa})(157 \times 10^{-3} \text{m})}{3 \times 10^{-3} \text{ m}} = 78.5 \times 10^6 \text{ Pa}$$

$$\sigma_{\max} = \sigma_1 = 78.5 \times 10^6 \text{ Pa} \qquad \sigma_{\max} = 78.5 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = -p \approx 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 39.25 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 39.3 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.109

Determine the largest internal pressure that can be applied to a cylindrical tank of 5.5-ft outer diameter and $\frac{5}{8}$-in. wall thickness if the ultimate normal stress of the steel used is 65 ksi and a factor of safety of 5.0 is desired.

SOLUTION

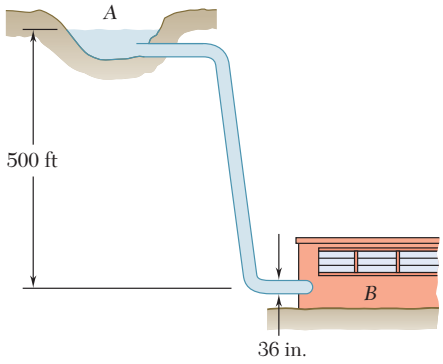
$$\sigma_1 = \frac{\sigma_U}{F.S.} = \frac{65\text{ ksi}}{5.0} = 13\text{ ksi} = 13 \times 10^3\text{ psi}$$

$$r = \frac{d}{2} - t = \frac{(5.5)(12)}{2} - 0.625 = 32.375\text{ in.}$$

$$\sigma_1 = \frac{pr}{t} \qquad p = \frac{t\sigma_1}{r} = \frac{(0.625)(13 \times 10^3)}{32.375}$$

$$p = 251\text{ psi} \blacktriangleleft$$

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PROBLEM 7.110

A steel penstock has a 36-in. outer diameter, a 0.5-in. wall thickness, and connects a reservoir at A with a generating station at B. Knowing that the specific weight of water is 62.4 lb/ft^3, determine the maximum normal stress and the maximum shearing stress in the penstock under static conditions.

SOLUTION

$$r = \frac{1}{2}d - t = \frac{1}{2}(36) - 0.5 = 17.5 \text{ in.}$$

$$p = rh = (62.4 \text{ lb/ft}^3)(500 \text{ ft}) = 31.2 \times 10^3 \text{ lb/ft}^2$$
$$= 216.67 \text{ psi}$$

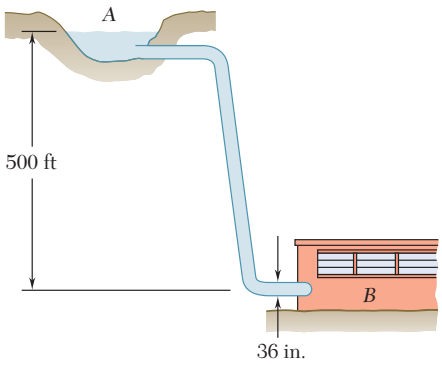
$$\sigma_1 = \frac{pr}{t} = \frac{(216.67)(17.5)}{0.5} = 7583 \text{ psi}$$

$$\sigma_{\max} = \sigma_1 = 7583 \text{ psi} \qquad \sigma_{\max} = 7.58 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\min} \approx -p = -217 \text{ psi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 3900 \text{ psi} \qquad \tau_{\max} = 3.90 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.111

A steel penstock has a 36-in. outer diameter and connects a reservoir at A with a generating station at B. Knowing that the specific weight of water is 62.4 lb/ft^3 and that the allowable normal stress in the steel is 12.5 ksi, determine the smallest thickness that can be used for the penstock.

SOLUTION

$$p = \gamma h = (62.4 \text{ lb/ft}^3)(500 \text{ ft}) = 31.2 \times 10^3 \text{ lb/ft}^2$$
$$= 216.67 \text{ psi}$$

$$\sigma_1 = 12.5 \text{ ksi} = 12.5 \times 10^3 \text{ psi}$$

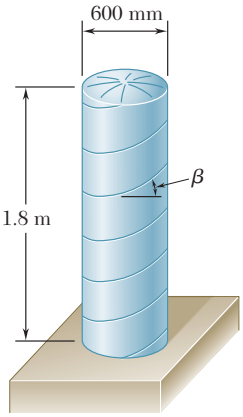
$$r = \frac{1}{2}d - t = 18 - t$$

$$\sigma_1 = \frac{pr}{t}, \quad \frac{r}{t} = \frac{\sigma_1}{p}$$

$$\frac{18 - t}{t} = \frac{12.5 \times 10^3}{216.67} = 57.692$$

$$\frac{18}{t} = 58.692 \qquad t = 0.307 \text{ in.} \blacktriangleleft$$

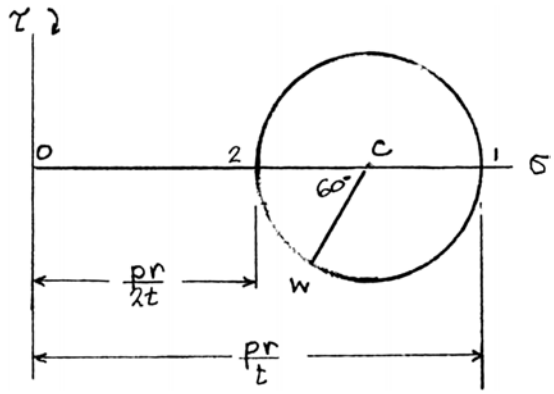
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PROBLEM 7.112

The cylindrical portion of the compressed-air tank shown is fabricated of 8-mm-thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Knowing that the allowable stress normal to the weld is 75 MPa, determine the largest gage pressure that can be used in the tank.

SOLUTION



$$r = \frac{1}{2}d - t = \frac{1}{2}(600) - 6 = 292 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{1}{2}\frac{pr}{t}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

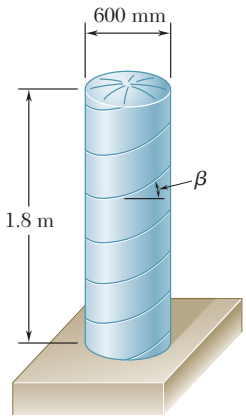
$$\sigma_w = \sigma_{\text{ave}} - R\cos 60°$$

$$= \frac{5}{8}\frac{pr}{t}$$

$$p = \frac{8}{5}\frac{\sigma_w t}{r}$$

$$p = \frac{8}{5}\frac{(75)(8)}{292} = 3.29 \text{ MPa} \blacktriangleleft$$

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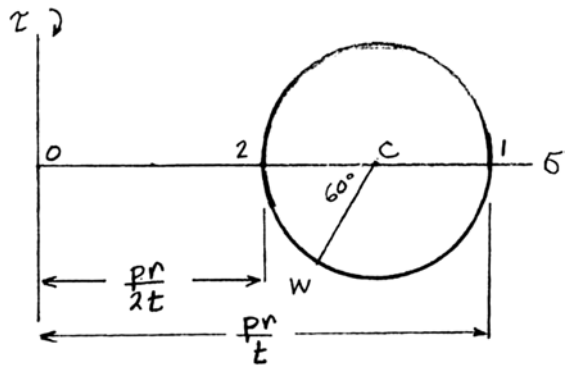


PROBLEM 7.113

For the compressed-air tank of Prob. 7.112, determine the gage pressure that will cause a shearing stress parallel to the weld of 30 MPa.

PROBLEM 7.112 The cylindrical portion of the compressed-air tank shown is fabricated of 8-mm-thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Knowing that the allowable stress normal to the weld is 75 MPa, determine the largest gage pressure that can be used in the tank.

SOLUTION



$$r = \frac{1}{2}d - t = \frac{1}{2}(600) - 8 = 292 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{1}{2}\frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\tau_w = R\sin 60°$$

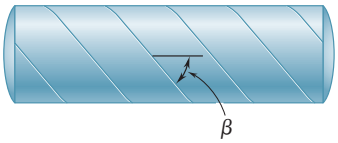
$$= \frac{\sqrt{3}}{8}\frac{pr}{t}$$

$$p = \frac{8}{\sqrt{3}}\frac{\tau_w t}{R}$$

$$p = \frac{8}{\sqrt{3}}\frac{(30)(8)}{292} = 3.80 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.114



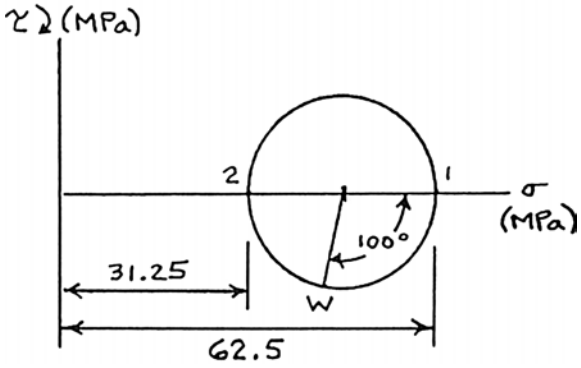
The steel pressure tank shown has a 750-mm inner diameter and a 9-mm wall thickness. Knowing that the butt-welded seams form an angle $\beta = 50°$ with the longitudinal axis of the tank and that the gage pressure in the tank is 1.5 MPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$r = \frac{d}{2} = 375 \text{ mm} = 0.375 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.5 \times 10^6 \text{ Pa} \times 0.375 \text{ m})}{0.009 \text{ m}} = 62.5 \times 10^6 \text{ Pa} = 62.5 \text{ MPa}$$

$$\sigma_2 = \frac{1}{2}\sigma_1 = 31.25 \text{ MPa} \qquad 2\beta = 100°$$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = 46.875 \text{ MPa}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 15.625 \text{ MPa}$$

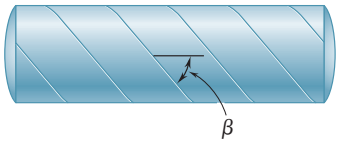
(*a*) $\sigma_w = \sigma_{\text{ave}} + R\cos 100°$

$$\sigma_w = 44.2 \text{ MPa} \blacktriangleleft$$

(*b*) $\tau_w = R\sin 100°$

$$\tau_w = 15.39 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.115

The pressurized tank shown was fabricated by welding strips of plate along a helix forming an angle β with a transverse plane. Determine the largest value of β that can be used if the normal stress perpendicular to the weld is not to be larger than 85 percent of the maximum stress in the tank.

SOLUTION

$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

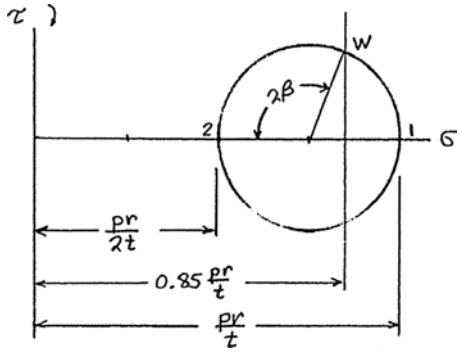
$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{\text{ave}} - R\cos 2\beta$$

$$0.85\frac{pr}{t} = \left(\frac{3}{4} - \frac{1}{4}\cos 2\beta\right)\frac{pr}{t}$$

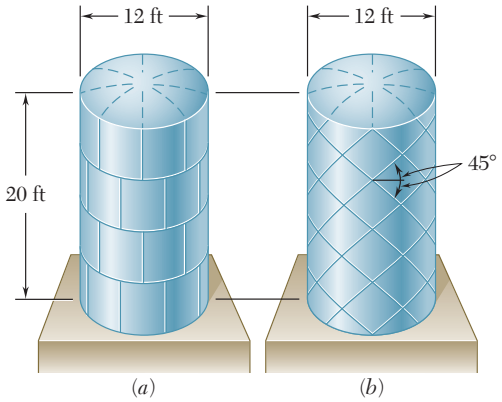
$$\cos 2\beta = -4\left(0.85 - \frac{3}{4}\right) = -0.4$$

$$2\beta = 113.6°$$



$$\beta = 56.8° \blacktriangleleft$$

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PROBLEM 7.116

Square plates, each of 0.5-in. thickness, can be bent and welded together in either of the two ways shown to form the cylindrical portion of a compressed-air tank. Knowing that the allowable normal stress perpendicular to the weld is 12 ksi, determine the largest allowable gage pressure in each case.

SOLUTION

$$d = 12\,\text{ft} = 144\,\text{in.} \quad r = \frac{1}{2}d - t = 71.5\,\text{in.}$$

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$$

(*a*)

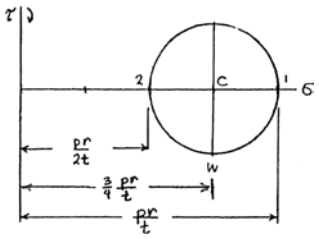
$$\sigma_1 = 12\,\text{ksi}$$

$$p = \frac{\sigma_1 t}{r} = \frac{(12)(0.5)}{71.5} = 0.0839\,\text{ksi}$$

$$p = 83.9\,\text{psi} \blacktriangleleft$$

(*b*)

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$



$$R = \frac{\sigma_1 + \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\beta = \pm 45°$$

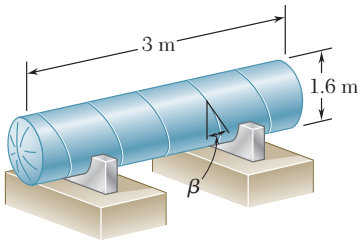
$$\sigma_w = \sigma_{\text{ave}} + R\cos\beta$$

$$= \frac{3}{4}\frac{pr}{t}$$

$$p = \frac{4}{3}\frac{\sigma_w t}{r} = \left(\frac{4}{3}\right)\frac{(12)(0.5)}{71.5} = 0.1119\,\text{ksi}$$

$$p = 111.9\,\text{psi} \blacktriangleleft$$

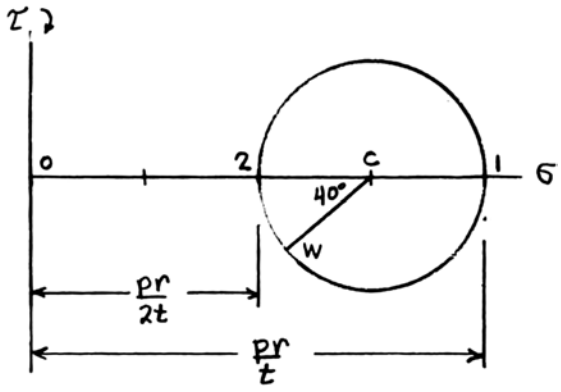
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PROBLEM 7.117

The pressure tank shown has a 0.375-in. wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 85 psi, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION



$$d = 5 \text{ ft} = 60 \text{ in.}$$

$$r = \frac{1}{2}d - t = 30 - 0.375 = 29.625 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(85)(29.625)}{0.375} = 6715 \text{ psi}$$

$$\sigma_2 = \frac{1}{2}\sigma_1 = 3357.5 \text{ psi}$$

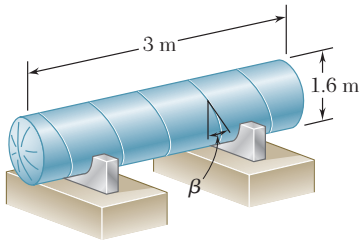
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = 5036.2 \text{ psi}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 1678.75 \text{ psi}$$

(*a*) $\sigma_w = \sigma_{\text{ave}} - R\cos 40° = 3750 \text{ psi}$ ◀

(*b*) $\tau_w = R\sin 40° = 1079 \text{ psi}$ ◀

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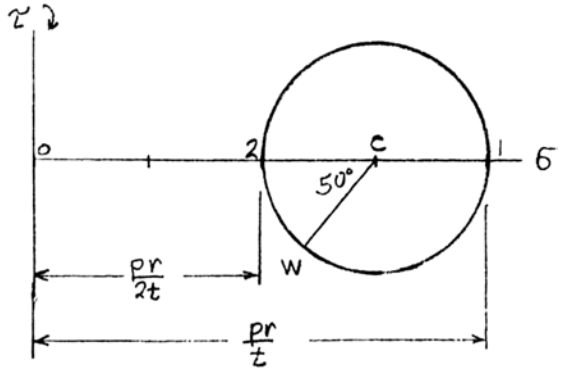


PROBLEM 7.118

For the tank of Prob. 7.117, determine the largest allowable gage pressure, knowing that the allowable normal stress perpendicular to the weld is 18 ksi and the allowable shearing stress parallel to the weld is 10 ksi.

PROBLEM 7.117 The pressure tank shown has a 0.375-in. wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 85 psi, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION



$$d = 5 \text{ ft} = 60 \text{ in.}$$

$$r = \frac{1}{2}d - t = 30 - 0.375 = 29.625 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{pr}{2t}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{\text{ave}} - R\cos 50°$$

$$= \left(\frac{3}{4} - \frac{1}{4}\cos 50°\right)\frac{pr}{t}$$

$$= 0.58930\frac{pr}{t}$$

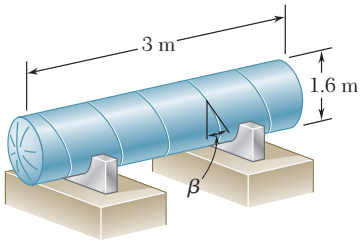
$$p = \frac{\sigma_w t}{0.5893r} = \frac{(18)(0.375)}{(0.58930)(29.625)} = 0.38664 \text{ ksi} = 387 \text{ psi}$$

$$\tau_w = R\sin 50° = 0.191511\frac{pr}{t}$$

$$p = \frac{\tau_w t}{0.191511r} = \frac{(10)(0.375)}{(0.191511)(29.625)} = 0.66097 \text{ ksi} = 661 \text{ psi}$$

Allowable gage pressure is the smaller value. $p = 387$ psi ◀

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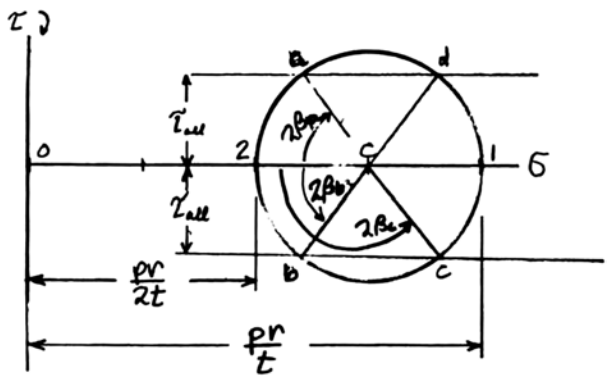


PROBLEM 7.119

For the tank of Prob. 7.117, determine the range of values of β that can be used if the shearing stress parallel to the weld is not to exceed 1350 psi when the gage pressure is 85 psi.

PROBLEM 7.117 The pressure tank shown has a 0.375-in. wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 85 psi, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION



$$d = 5 \text{ ft} = 60 \text{ in.}$$

$$r = \frac{1}{2}d - t = 30 - \frac{3}{8} = 29.625 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(85)(29.625)}{0.375} = 6715 \text{ psi}$$

$$\sigma_2 = \frac{1}{2}\sigma_1 = 3357.5 \text{ psi}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 1678.75$$

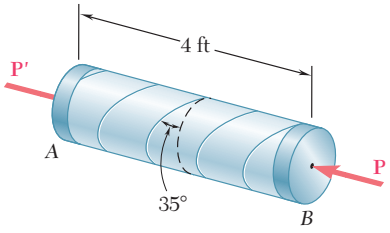
$$\tau_w = R \sin 2\beta = \tau_{\text{all}}$$

$$\sin 2\beta_a = \frac{\tau_w}{R} = \frac{1350}{1678.75} = 0.80417$$

$$\left.\begin{aligned} 2\beta_a &= -53.53° & \beta_a &= -26.8° \\ 2\beta_b &= +53.53° & \beta_b &= 26.8° \end{aligned}\right\} \quad -26.8° \le \beta \le 26.8° \quad \blacktriangleleft$$

$$\left.\begin{aligned} 2\beta_c &= -53.53° + 180° = 126.47° & \beta_c &= 63.2° \\ 2\beta_d &= 53.53° + 180° = 233.53° & \beta_d &= 116.8° \end{aligned}\right\} \quad 63.2° \le \beta \le 116.8° \quad \blacktriangleleft$$

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PROBLEM 7.120

A pressure vessel of 10-in. inner diameter and 0.25-in. wall thickness is fabricated from a 4-ft section of spirally-welded pipe AB and is equipped with two rigid end plates. The gage pressure inside the vessel is 300 psi and 10-kip centric axial forces **P** and **P′** are applied to the end plates. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$r = \frac{1}{2}d = \frac{1}{2}(10) = 5 \text{ in.} \qquad t = 0.25 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(300)(5)}{0.25} = 6000 \text{ psi} = 6 \text{ ksi}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(300)(5)}{(2)(0.25)} = 3000 \text{ psi} = 3 \text{ ksi}$$

$$r_0 = r + t = 5 + 0.25 = 5.25 \text{ in.}$$

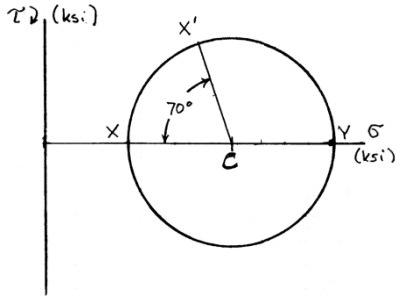
$$A = \pi\left(r_0^2 - r^2\right) = \pi(5.25^2 - 5.00^2) = 8.0503 \text{ in}^2$$

$$\sigma = -\frac{P}{A} = -\frac{10 \times 10^3}{8.0803} = -1242 \text{ psi} = -1.242 \text{ ksi}$$

Total stresses. Longitudinal: $\sigma_x = 3 - 1.242 = 1.758$ ksi

Circumferential: $\sigma_y = 6$ ksi

Shear: $\tau_{xy} = 0$



Plotted points for Mohr's circle:

X: (1.758, 0)

Y: (6, 0)

C: (3.879)

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 3.879 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

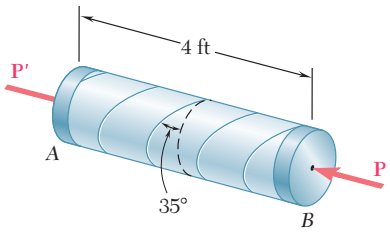
$$= \sqrt{\left[\frac{(1.758 - 6)}{2}\right]^2 + 0} = 2.121 \text{ ksi}$$

(*a*) $\sigma_{x'} = \sigma_{\text{ave}} + R\cos 70° = 3.879 - 2.121\cos 70°$ $\qquad \sigma_{x'} = 3.15 \text{ ksi}$ ◀

(*b*) $|\tau_{xy}| = R\sin 70° = 2.121\sin 70°$ $\qquad |\tau_{x'y'}| = 1.993 \text{ ksi}$ ◀

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PROBLEM 7.121



Solve Prob. 7.120, assuming that the magnitude P of the two forces is increased to 30 kips.

PROBLEM 7.120 A pressure vessel of 10-in. inner diameter and 0.25-in. wall thickness is fabricated from a 4-ft section of spirally-welded pipe AB and is equipped with two rigid end plates. The gage pressure inside the vessel is 300 psi and 10-kip centric axial forces **P** and **P′** are applied to the end plates. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$r = \frac{1}{2}d = \frac{1}{2}(10) = 5 \text{ in.} \qquad t = 0.25 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(300)(5)}{0.25} = 6000 \text{ psi} = 6 \text{ ksi}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(300)(5)}{(2)(0.25)} = 3000 \text{ psi} = 3 \text{ ksi}$$

$$r_0 = r + t = 5 + 0.25 = 5.25 \text{ in.}$$

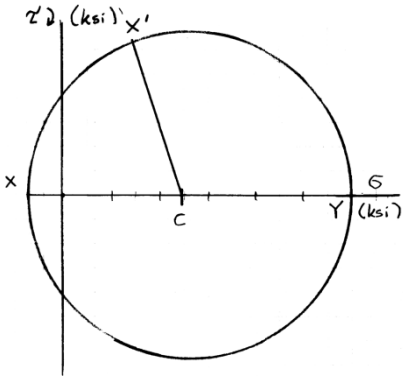
$$A = \pi\left(r_0^2 - r^2\right) = \pi(5.25^2 - 5^2) = 8.0503 \text{ in}^2$$

$$\sigma = -\frac{P}{A} = -\frac{30 \times 10^3}{8.0503} = -3727 \text{ psi} = -3.727 \text{ ksi}$$

Total stresses. Longitudinal: $\sigma_x = 3 - 3.727 = -0.727$ ksi

Circumferential: $\sigma_y = 6$ ksi

Shear: $\tau_{xy} = 0$



Plotted points for Mohr's circle:

X: (−0.727, 0)

Y: (6, 0)

C: (2.6365, 0)

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 2.6365 \text{ ksi}$$

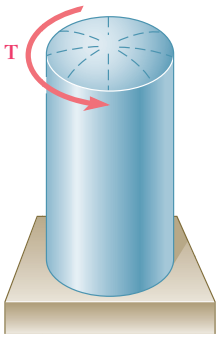
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-0.727 - 6}{2}\right)^2 + 0} = 3.3635 \text{ ksi}$$

(*a*) $\sigma_{x'} = \sigma_{\text{ave}} - R\cos 70° = 2.6365 - 3.3635 \cos 70°$ $\qquad \sigma_{x'} = 1.486$ ksi ◀

(*b*) $|\tau_{x'y'}| = R \sin 70° = 3.3635 \sin 70°$ $\qquad |\tau_{x'y'}| = 3.16$ ksi ◀

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PROBLEM 7.122

A torque of magnitude $T = 12\ \text{kN} \cdot \text{m}$ is applied to the end of a tank containing compressed air under a pressure of 8 MPa. Knowing that the tank has a 180-mm inner diameter and a 12-mm wall thickness, determine the maximum normal stress and the maximum shearing stress in the tank.

SOLUTION

$$d = 180\ \text{mm} \quad r = \frac{1}{2}d = 90\ \text{mm} \quad t = 12\ \text{mm}$$

Torsion:

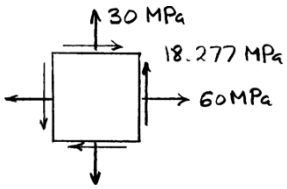
$$c_1 = 90\ \text{mm} \quad c_2 = 90 + 12 = 102\ \text{mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 66.968 \times 10^6\ \text{mm}^4 = 66.968 \times 10^{-6}\ \text{m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(12 \times 10^3)(102 \times 10^{-3})}{66.968 \times 10^{-6}} = 18.277\ \text{MPa}$$

Pressure:

$$\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60\ \text{MPa} \quad \sigma_2 = \frac{pr}{2t} = 30\ \text{MPa}$$



Summary of stresses:

$$\sigma_x = 60\ \text{MPa}, \quad \sigma_y = 30\ \text{MPa}, \quad \tau_{xy} = 18.277\ \text{MPa}$$

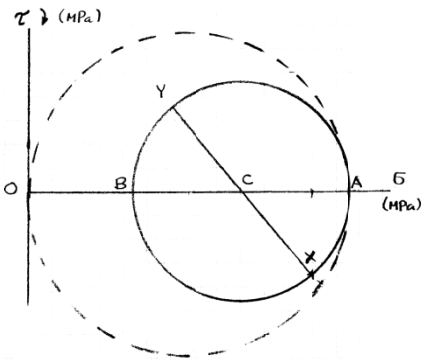
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 45\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 23.64\ \text{MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 68.64\ \text{MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 21.36\ \text{MPa}$$

$$\sigma_c \approx 0$$

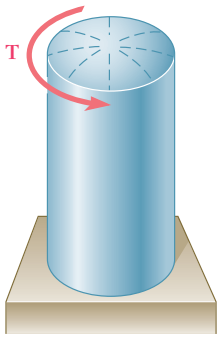


$$\sigma_{\max} = 68.6\ \text{MPa} \blacktriangleleft$$

$$\sigma_{\min} = 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 34.3\ \text{MPa} \blacktriangleleft$$

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PROBLEM 7.123

The tank shown has a 180-mm inner diameter and a 12-mm wall thickness. Knowing that the tank contains compressed air under a pressure of 8 MPa, determine the magnitude T of the applied torque for which the maximum normal stress is 75 MPa.

SOLUTION

$$r = \frac{1}{2}d = \left(\frac{1}{2}\right)(180) = 90 \text{ mm} \qquad t = 12 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 30 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_y) = 45 \text{ MPa}$$

$$\sigma_{\max} = 75 \text{ MPa}$$

$$R = \sigma_{\max} - \sigma_{\text{ave}} = 30 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau_{xy}^2} = \sqrt{15^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{R^2 - 15^2} = \sqrt{30^2 - 15^2} = 25.98 \text{ MPa}$$

$$= 25.98 \times 10^6 \text{ Pa}$$

Torsion:

$$c_1 = 90 \text{ mm}$$

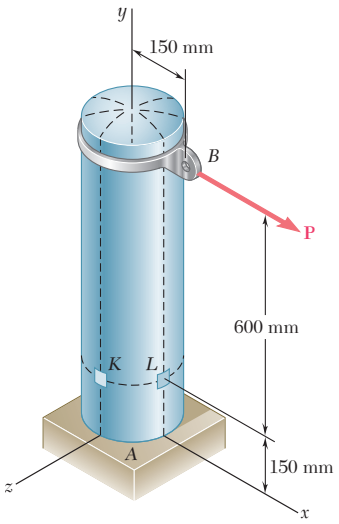
$$c_2 = 90 + 12 = 102 \text{ mm}$$

$$J = \frac{T}{2}\left(c_2^4 - c_1^4\right) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$$

$$\tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c} = \frac{(66.968 \times 10^{-6})(25.98 \times 10^6)}{102 \times 10^{-3}} = 17.06 \times 10^3 \text{ N} \cdot \text{m}$$

$$T = 17.06 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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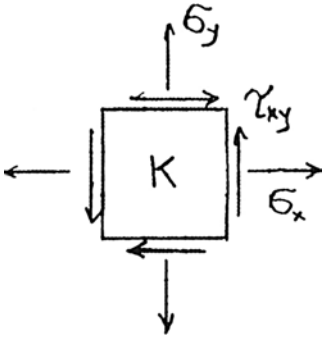


PROBLEM 7.124

The compressed-air tank *AB* has a 250-mm outside diameter and an 8-mm wall thickness. It is fitted with a collar by which a 40-kN force **P** is applied at *B* in the horizontal direction. Knowing that the gage pressure inside the tank is 5 MPa, determine the maximum normal stress and the maximum shearing stress at point *K*.

SOLUTION

Consider element at point *K*.



Stresses due to internal pressure:

$$p = 5 \text{ MPa} = 5\times10^{6} \text{ Pa}$$

$$r = \frac{1}{2}d - t = \frac{250}{2} - 8 = 117 \text{ mm}$$

$$\sigma_x = \frac{pr}{t} = \frac{(5\times10^{6})(117\times10^{-3})}{(8\times10^{-3})} = 73.125 \text{ MPa}$$

$$\sigma_y = \frac{pr}{2t} = \frac{(5\times10^{6})(117\times10^{-3})}{(2)(8\times10^{-3})} = 36.563 \text{ MPa}$$

Stress due to bending moment: Point *K* is on the neutral axis.

$$\sigma_y = 0$$

Stress due to transverse shear:

$$V = P = 40\times10^{3} \text{ N}$$

$$c_2 = \frac{1}{2}d = 125 \text{ mm}$$

$$c_1 = c_2 - t = 117 \text{ mm}$$

$$Q = \frac{2}{3}\left(c_2^3 - c_1^3\right) = \frac{2}{3}(125^3 - 117^3)$$

$$= 234.34\times10^{3} \text{ mm}^3 = 234.34\times10^{-6} \text{ m}^3$$

$$I = \frac{\pi}{4}\left(c_2^4 - c_1^4\right) = \frac{\pi}{4}(125^4 - 117^4)$$

$$= 44.573\times10^{6} \text{ mm}^4 = 44.573\times10^{-6} \text{ m}^4$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{PQ}{I(2t)} = \frac{(40\times10^{3})(234.34\times10^{-6})}{(44.573\times10^{-6})(16\times10^{-3})}$$

$$= 13.1436\times10^{6} \text{ Pa} = 13.1436 \text{ MPa}$$

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PROBLEM 7.124 ***(Continued)***

Total stresses: $\sigma_x = 73.125 \text{ MPa}, \quad \sigma_y = 36.563 \text{ MPa}, \quad \tau_{xy} = 13.1436 \text{ MPa}$

Mohr's circle: $\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 54.844 \text{ MPa}$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(18.281)^2 + (13.1436)^2} = 22.516 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 77.360 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 32.328 \text{ MPa}$$

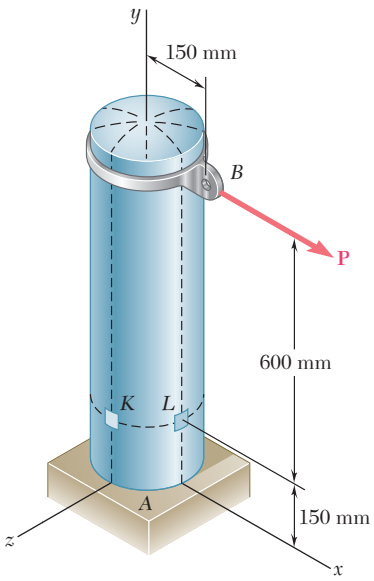
Principal stresses: $\sigma_a = 77.4 \text{ MPa}, \quad \sigma_b = 32.3 \text{ MPa}$ ◀

The 3rd principal stress is the radial stress. $\sigma_z \approx 0$ ◀

$\sigma_{\max} = 77.4 \text{ MPa}, \quad \sigma_{\min} = 0$ $\sigma_{\max} = 77.4 \text{ MPa}$ ◀

Maximum shearing stress: $\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$ $\tau_{\max} = 38.7 \text{ MPa}$ ◀

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PROBLEM 7.125

In Prob. 7.124, determine the maximum normal stress and the maximum shearing stress at point *L*.

PROBLEM 7.124 The compressed-air tank *AB* has a 250-mm outside diameter and an 8-mm wall thickness. It is fitted with a collar by which a 40-kN force **P** is applied at *B* in the horizontal direction. Knowing that the gage pressure inside the tank is 5 MPa, determine the maximum normal stress and the maximum shearing stress at point *K*.

SOLUTION

Consider element at point *L*.

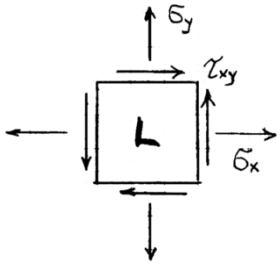
Stresses due to internal pressure:

$$p = 5 \text{ MPa} = 5\times10^{6} \text{ Pa}$$

$$r = \frac{1}{2}d - t = \frac{250}{2} - 8 = 117 \text{ mm}$$

$$\sigma_x = \frac{pr}{t} = \frac{(5\times10^{6})(117\times10^{-3})}{8\times10^{-3}} = 73.125 \text{ MPa}$$

$$\sigma_y = \frac{pr}{2t} = \frac{(5\times10^{3})(117\times10^{-3})}{(2)(8\times10^{-3})} = 36.563 \text{ MPa}$$



Stress due to bending moment: $M = (40 \text{ kN})(600 \text{ mm}) = 24{,}000 \text{ N}\cdot\text{m}$

$$c_2 = \frac{1}{2}d = 125 \text{ mm}$$

$$c_1 = c_2 - t = 125 - 8 = 117 \text{ mm}$$

$$I = \frac{\pi}{4}\left(c_2^4 - c_1^4\right) = \frac{\pi}{4}(125^4 - 117^4)$$

$$= 44.573\times10^{6} \text{ mm}^4 = 44.573\times10^{-6} \text{ m}^4$$

$$\sigma_y = -\frac{Mc}{I} = -\frac{(24{,}000)(125\times10^{-3})}{44.573\times10^{-6}} = -67.305 \text{ MPa}$$

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PROBLEM 7.125 *(Continued)*

Stress due to transverse shear: Point L lies in a plane of symmetry.

$$\tau_{xy} = 0$$

Total stresses: $\sigma_x = 73.125 \text{ MPa}, \quad \sigma_y = -30.742 \text{ MPa}, \quad \tau_{xy} = 0$

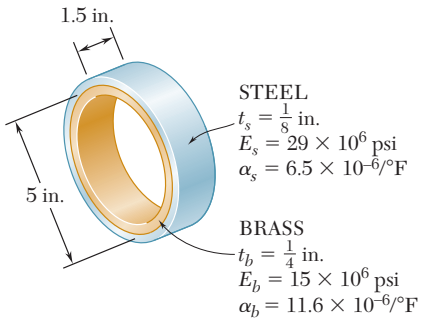
Principal stresses: Since $\tau_{xy} = 0$, σ_x and σ_y are principal stresses. The 3rd principal stress is in the radial direction, $\sigma_z \approx 0$.

$$\sigma_{\max} = 73.125 \text{ MPa}, \quad \sigma_{\min} = 0, \quad \sigma_a = 73.1 \text{ MPa}, \quad \sigma_b = -30.7 \text{ MPa}, \quad \sigma_z = 0$$

Maximum stress: $\sigma_{\max} = 73.1 \text{ MPa}$ ◀

Maximum shearing stress: $\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$ $\tau_{\max} = 51.9 \text{ MPa}$ ◀

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PROBLEM 7.126

A brass ring of 5-in. outer diameter and 0.25-in. thickness fits exactly inside a steel ring of 5-in. inner diameter and 0.125-in. thickness when the temperature of both rings is 50°F. Knowing that the temperature of both rings is then raised to 125°F, determine (*a*) the tensile stress in the steel ring, (*b*) the corresponding pressure exerted by the brass ring on the steel ring.

SOLUTION

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring. Internal pressure p: $$\sigma_s = \frac{pr}{t_s} \tag{1}$$

Corresponding strain: $$\varepsilon_{sp} = \frac{\sigma_s}{E_s} = \frac{pr}{E_s t_s}$$

Strain due to temperature change: $$\varepsilon_{sT} = \alpha_s \Delta T$$

Total strain: $$\varepsilon_s = \frac{pr}{E_s t_s} + \alpha_s \Delta T$$

Change in length of circumference:

$$\Delta L_s = 2\pi r \varepsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring. External pressure p: $$\sigma_b = -\frac{pr}{t_b}$$

Corresponding strains: $$\varepsilon_{bp} = -\frac{pr}{E_b t_b}, \quad \varepsilon_{bT} = \alpha_b \Delta T$$

Change in length of circumference:

$$\Delta L_b = 2\pi r \varepsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b, $$\frac{pr}{E_s t_s} + \alpha_s \Delta T = -\frac{pr}{E_b t_b} + \alpha_b \Delta T$$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s) \Delta T \tag{2}$$

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PROBLEM 7.126 ***(Continued)***

Data: $\Delta T = 125°\text{F} - 50°\text{F} = 75°\text{F}$

$$r = \frac{1}{2}d = \frac{1}{2}(5) = 2.5 \text{ in.}$$

From Equation (2), $$\left[\frac{2.5}{(29\times10^6)(0.125)} + \frac{2.5}{(15\times10^6)(0.25)}\right]p = (11.6-6.5)(10^{-6})(75)$$

$$1.35632\times10^{-6}\,p = 382.5\times10^{-6}$$

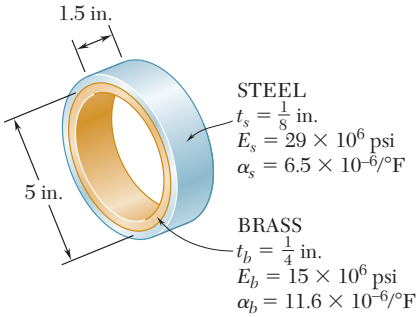
$$p = 282.0 \text{ psi}$$

From Equation (1), $$\sigma_s = \frac{pr}{t_s} = \frac{(282.0)(2.5)}{0.125} = 5.64\times10^3 \text{ psi}$$

(*a*) $\sigma_s = 5.64$ ksi ◀

(*b*) $p = 282$ psi ◀

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PROBLEM 7.127

Solve Prob. 7.126, assuming that the brass ring is 0.125 in. thick and the steel ring is 0.25 in. thick.

PROBLEM 7.126 A brass ring of 5-in. outer diameter and 0.25-in. thickness fits exactly inside a steel ring of 5-in. inner diameter and 0.125-in. thickness when the temperature of both rings is 50°F. Knowing that the temperature of both rings is then raised to 125°F, determine (*a*) the tensile stress in the steel ring, (*b*) the corresponding pressure exerted by the brass ring on the steel ring.

SOLUTION

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring. Internal pressure p:

$$\sigma_s = \frac{pr}{t_s} \tag{1}$$

Corresponding strain:

$$\varepsilon_{sp} = \frac{\sigma_s}{E_s} = \frac{pr}{E_s t_s}$$

Strain due to temperature change:

$$\varepsilon_{sT} = \alpha_s \Delta T$$

Total strain:

$$\varepsilon_s = \frac{pr}{E_s t_s} + \alpha_s \Delta T$$

Change in length of circumference:

$$\Delta L_s = 2\pi r \varepsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring. External pressure p:

$$\sigma_b = -\frac{pr}{t_b}$$

Corresponding strains:

$$\varepsilon_{bp} = -\frac{pr}{E_b t_b}, \quad \varepsilon_{bT} = \alpha_b \Delta T$$

Change in length of circumference:

$$\Delta L_b = 2\pi r \varepsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b,

$$\frac{pr}{E_s t_s} + \alpha_s \Delta T = -\frac{pr}{E_b t_b} + \alpha_b \Delta T$$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s) \Delta T \tag{2}$$

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PROBLEM 7.127 *(Continued)*

Data: $\Delta T = 125°\text{F} - 50°\text{F} = 75°\text{F}$

$$r = \frac{1}{2}d = \frac{1}{2}(5) = 2.5 \text{ in.}$$

From Equation (2), $$\left[\frac{2.5}{(29\times10^6)(0.25)} + \frac{2.5}{(15\times10^6)(0.125)}\right]p = (11.6 - 6.5)(10^{-6})(75)$$

$$1.67816\times10^{-6}\,p = 382.5\times10^{-6}$$

$$p = 227.93 \text{ psi}$$

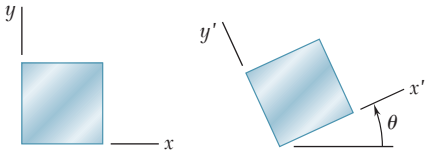
From Equation (1), $$\sigma_s = \frac{pr}{t_s} = \frac{(227.93)(2.5)}{0.25} = 2279 \text{ psi}$$

(*a*) $\sigma_s = 2.28$ ksi ◀

(*b*) $p = 228$ psi ◀

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PROBLEM 7.128



For the given state of plane strain, use the method of Sec. 7.7A to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = -800\mu, \quad \varepsilon_y = +450\mu, \quad \gamma_{xy} = +200\mu, \quad \theta = 25°$$

SOLUTION

$$\theta = -25°$$

$$\frac{\varepsilon_x + \varepsilon_y}{2} = -175\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -625\mu \qquad \frac{\gamma_{xy}}{2} = +100\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= 175\mu + (-625\mu)\cos(-50°) + (100\mu)\sin(-50°) \qquad \varepsilon_{x'} = -653\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

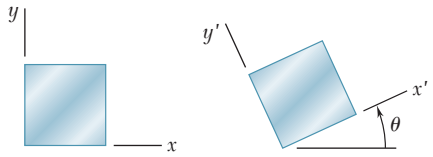
$$= -175\mu - (-625\mu)\cos(-50°) - (100\mu)\sin(-50°) \qquad \varepsilon_{y'} = 303\mu \blacktriangleleft$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= -(-800\mu - 450\mu)\sin(-50°) + (+200\mu)\cos(-50°) \qquad \gamma_{x'y'} = -829\mu \blacktriangleleft$$

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PROBLEM 7.129



For the given state of plane strain, use the method of Sec. 7.7A to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = +240\mu, \quad \varepsilon_y = +160\mu, \quad \gamma_{xy} = +150\mu, \quad \theta = 60°$$

SOLUTION

$$\theta = -60°$$

$$\frac{\varepsilon_x + \varepsilon_y}{2} = +200\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = 40\mu \qquad \frac{\gamma_{xy}}{2} = 75\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= 200 + 40\cos(-120°) + 75\sin(-120°) \qquad \varepsilon_{x'} = 115.0\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

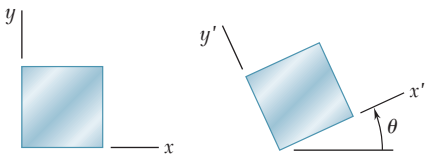
$$= 200 - 40\cos(-120°) - 75\sin(-120°) \qquad \varepsilon_{y'} = 285\mu \blacktriangleleft$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= -(240 - 160)\sin(-120°) + 150\cos(-120°) \qquad \gamma_{x'y'} = -5.72\mu \blacktriangleleft$$

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PROBLEM 7.130



For the given state of plane strain, use the method of Sec. 7.7A to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = -500\mu, \quad \varepsilon_y = +250\mu, \quad \gamma_{xy} = 0, \quad \theta = 15°$$

SOLUTION

$$\theta = +15°$$

$$\frac{\varepsilon_x + \varepsilon_y}{2} = -125\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -375\mu \qquad \frac{\gamma_{xy}}{2} = 0$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= -125\mu + (-375\mu)\cos 30° + 0 \qquad \varepsilon_{x'} = -450\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

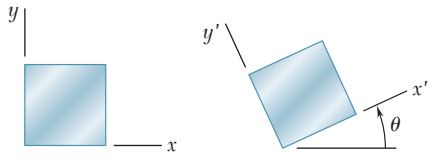
$$= -125\mu - (-375\mu)\cos 30° - 0 \qquad \varepsilon_{y'} = 199.8\mu \blacktriangleleft$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= -(-500\mu - 250\mu)\sin 30° + 0 \qquad \gamma_{x'y'} = 375\mu \blacktriangleleft$$

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PROBLEM 7.131



For the given state of plane strain, use the method of Sec 7.7A to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = 0, \quad \varepsilon_y = +320\mu, \quad \gamma_{xy} = -100\mu, \quad \theta = 30°$$

SOLUTION

$$\theta = +30°$$

$$\frac{\varepsilon_x + \varepsilon_y}{2} = 160\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -160\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= 160 - 160\cos 60° - \frac{100}{2}\sin 60° \qquad \varepsilon_{x'} = +36.7\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

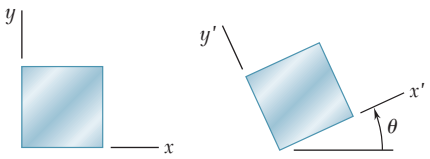
$$= 160 + 160\cos 60° + \frac{100}{2}\sin 60° \qquad \varepsilon_{y'} = +283\mu \blacktriangleleft$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= -(0 - 320)\sin 60° - 100\cos 60° \qquad \gamma_{x'y'} = +227\mu \blacktriangleleft$$

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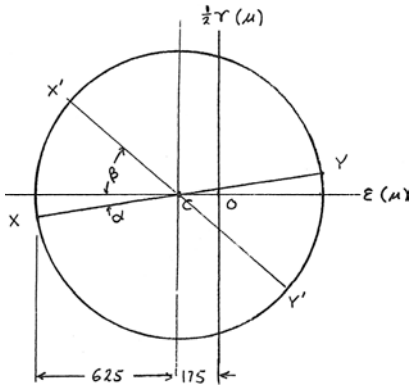
PROBLEM 7.132



For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = -800\mu, \quad \varepsilon_y + = 450\mu, \quad \gamma_{xy} = +200\mu, \quad \theta = 25°$$

SOLUTION



Plotted points:

X: $(-800\mu,\ -100\mu)$

Y: $(+450\mu,\ +100\mu)$

C: $(-175\mu,\ 0)$

$$\tan\alpha = \frac{100}{625} \qquad \alpha = 9.09°$$

$$R = \sqrt{(625\mu)^2 + (100\mu)^2} = 632.95\mu$$

$$\beta = 2\theta - \alpha = 50° - 9.09° = 40.91°$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} - R\cos\beta = -175\mu - 632.95\mu\cos 40.91°$$

$$\varepsilon_{x'} = -653\mu \blacktriangleleft$$

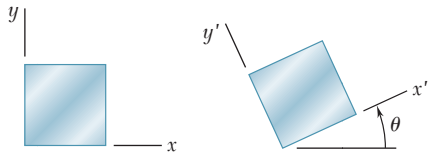
$$\varepsilon_{y'} = \varepsilon_{\text{ave}} + R\cos\beta = -175\mu + 632.95\mu\cos 40.91°$$

$$\varepsilon_{y'} = 303\mu \blacktriangleleft$$

$$\frac{1}{2}\gamma_{x'y'} = -R\sin\beta = -632.95\mu\sin 40.91° \qquad \gamma_{x'y'} = -829\mu \blacktriangleleft$$

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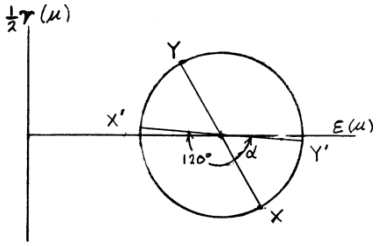
PROBLEM 7.133



For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = +240\mu, \quad \varepsilon_y = +160\mu, \quad \gamma_{xy} = +150\mu, \quad \theta = 60°$$

SOLUTION



Plotted points for Mohr's circle:

$$X: (+240\,\mu, -75\,\mu)$$
$$Y: (+160\,\mu, 75\,\mu)$$
$$C: (+200\,\mu, 0)$$

$$\tan\alpha = \frac{75}{40} = 1.875 \quad \alpha = 61.93°$$

$$R = \sqrt{(40\mu)^2 + (75\mu)^2} = 85\mu$$

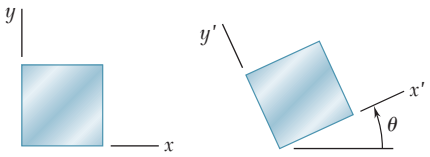
$$\beta = 2\theta - \alpha = -120° - 61.93° = -181.93°$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} + R\cos\beta = 200\mu + (85\mu)\cos(-181.93°) \qquad \varepsilon_{x'} = 115.0\,\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \varepsilon_{\text{ave}} - R\cos\beta = 200\mu - (85\mu)\cos(-181.93°) \qquad \varepsilon_{y'} = 285\,\mu \blacktriangleleft$$

$$\frac{1}{2}\gamma_{x'y'} = -R\sin\beta = -85\sin(-181.93°) = -2.86\mu \qquad \gamma_{x'y'} = -5.72\,\mu \blacktriangleleft$$

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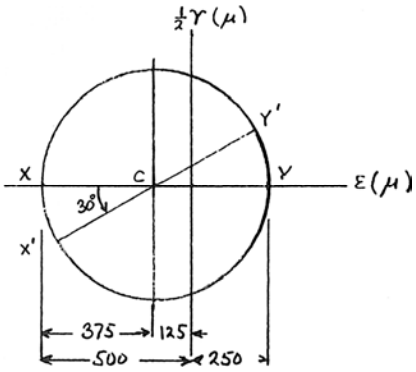


PROBLEM 7.134

For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = -500\mu, \quad \varepsilon_y = +250\mu, \quad \gamma_{xy} = 0, \quad \theta = 15°$$

SOLUTION



Plotted points:

X: $(-500\mu, 0)$

Y: $(+250\mu, 0)$

C: $(-125\mu, 0)$

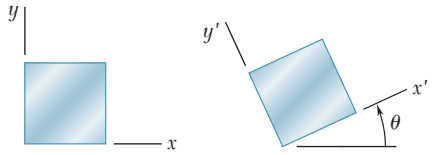
$R = 375\mu$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} + R\cos 2\theta = -125 - 375\cos 30° \qquad \varepsilon_{x'} = -450\,\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \varepsilon_{\text{ave}} + R\cos 2\theta = -125 + 375\cos 30° \qquad \varepsilon_{y'} = 199.8\,\mu \blacktriangleleft$$

$$\frac{1}{2}\gamma_{x'y'} = R\sin 2\theta = 375\sin 30° \qquad \gamma_{x'y'} = 375\,\mu \blacktriangleleft$$

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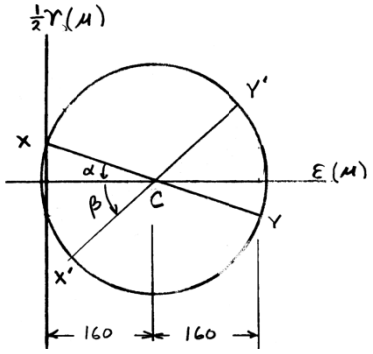
PROBLEM 7.135

For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = 0, \quad \varepsilon_y = +320\mu, \quad \gamma_{xy} = -100\mu, \quad \theta = 30°$$

SOLUTION

Plotted points for Mohr's circle:



$$X: (0, 50\mu)$$
$$Y: (320\mu, -50\mu)$$
$$C: (160\mu, 0)$$

$$\tan\alpha = \frac{50}{160} \quad \alpha = 17.35°$$

$$R = \sqrt{(160\mu)^2 + (50\mu)^2} = 167.63\mu$$

$$\beta = 2\theta - \alpha = 60° - 17.35° = 42.65°$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} - R\cos\beta = 160\mu - (167.63\mu)\cos 42.65° \qquad \varepsilon_{x'} = 36.7\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \varepsilon_{\text{ave}} + R\cos\beta = 160\mu + (167.63\mu)\cos 42.65° \qquad \varepsilon_{y'} = 283\mu \blacktriangleleft$$

$$\frac{1}{2}\gamma_{x'y'} = -R\sin\beta = (167.63\mu)\sin 42.65° \qquad \gamma_{x'y'} = 227\mu \blacktriangleleft$$

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PROBLEM 7.136

The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$.)

$$\varepsilon_x = -260\mu, \quad \varepsilon_y = -60\mu, \quad \gamma_{xy} = +480\mu$$

SOLUTION

For Mohr's circle of strain, plot points:

$$X\text{: } (-260\mu, -240\mu)$$
$$Y\text{: } (-60\mu, 240\mu)$$
$$C\text{: } (-160\mu, 0)$$

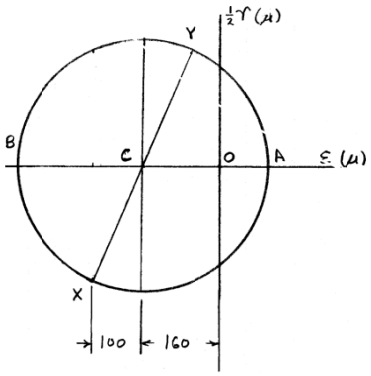
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{480}{-260 + 60} = -2.4$$

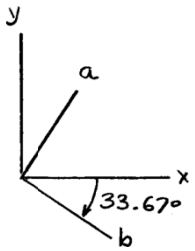
$$2\theta_p = -67.38°$$

$\theta_b = -33.7°$ ◀

$\theta_a = 56.3°$ ◀

$$R = \sqrt{(100\mu)^2 + (240\mu)^2}$$
$$R = 260\mu$$





(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = -160\mu + 260\mu$ $\varepsilon_a = 100\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = -160\mu - 260\mu$ $\varepsilon_b = -420\mu$ ◀

(*b*) $\frac{1}{2}\gamma_{\max\,(\text{in-plane})} = R \quad \gamma_{\max\,(\text{in-plane})} = 2R$ $\gamma_{\max\,(\text{in-plane})} = 520\mu$ ◀

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(-260 - 60)$$
$$= 160\mu$$

$\varepsilon_{\max} = 160\mu \quad \varepsilon_{\min} = -420\mu$ $\varepsilon_c = 160\mu$ ◀

(*c*) $\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 160\mu + 420\mu$ $\gamma_{\max} = 580\mu$ ◀

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PROBLEM 7.137

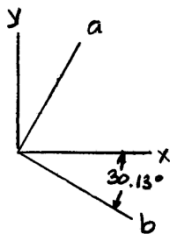
The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$.)

$$\varepsilon_x = -600\mu, \quad \varepsilon_y = -400\mu, \quad \gamma_{xy} = +350\mu$$

SOLUTION

Plotted points for Mohr's circle:

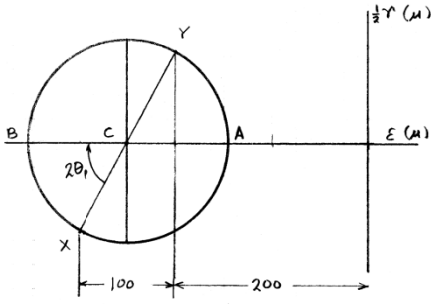
$$X: (-600\mu, -175\mu)$$
$$Y: (-400\mu, +175\mu)$$
$$C: (-500\mu, 0)$$



$$\tan 2\theta_p = -\frac{175}{100}$$
$$2\theta_p = -60.26°$$

$$\theta_b = -30.1° \blacktriangleleft$$
$$\theta_a = 59.9° \blacktriangleleft$$

$$R = \sqrt{(100\mu)^2 + (175\mu)^2}$$
$$= 201.6\mu$$



(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = -500\mu + 201.6\mu$ $\quad \varepsilon_a = -298\mu \blacktriangleleft$

$\varepsilon_b = \varepsilon_{\text{ave}} - R = -500\mu - 201.6\mu$ $\quad \varepsilon_b = -702\mu \blacktriangleleft$

(*b*) $\gamma_{\max\,(\text{in-plane})} = 2R$ $\quad \gamma_{\max\,(\text{in-plane})} = 403\mu \blacktriangleleft$

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(-600\mu - 400\mu)$$

$$\varepsilon_c = 500\mu \blacktriangleleft$$

$$\varepsilon_{\max} = 500\mu \quad \varepsilon_{\min} = -702\mu$$

(*c*) $\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 500\mu + 702\mu$ $\quad \gamma_{\max} = 1202\mu \blacktriangleleft$

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PROBLEM 7.138

The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$.)

$$\varepsilon_x = +160\mu, \quad \varepsilon_y = -480\mu, \quad \gamma_{xy} = -600\mu$$

SOLUTION

(*a*) For Mohr's circle of strain, plot points:

$$X\text{: } (160\mu,\ 300\mu)$$
$$Y\text{: } (-480\mu, -300\mu)$$
$$C\text{: } (-160\mu,\ 0)$$

(*a*) $$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-300}{320} = -0.9375$$

$$2\theta_p = -43.15^\circ \quad \theta_p = -21.58^\circ \quad \text{and} \quad -21.58 + 90 = 68.42^\circ$$

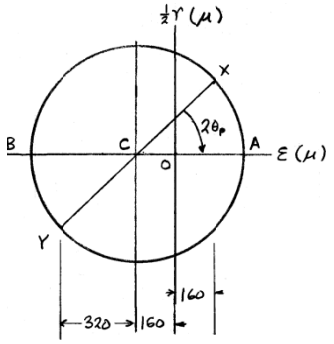
$\theta_a = -21.6^\circ$ ◀

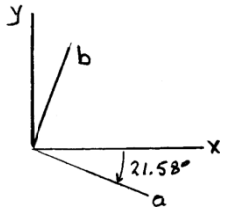
$\theta_b = 68.4^\circ$ ◀

$$R = \sqrt{(320\mu)^2 + (300\mu)^2} = 438.6\mu$$

$$\varepsilon_a = \varepsilon_{\text{ave}} + R = -160\mu + 438.6\mu \qquad \varepsilon_a = +279\mu \blacktriangleleft$$

$$\varepsilon_b = \varepsilon_{\text{ave}} - R = -160\mu - 438.6\mu \qquad \varepsilon_b = -599\mu \blacktriangleleft$$





(*b*) $$\frac{1}{2}\gamma_{(\text{max, in-plane})} = R \quad \gamma_{(\text{max, in-plane})} = 2R \qquad \gamma_{(\text{max, in-plane})} = 877\mu \blacktriangleleft$$

(*c*) $$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(160\mu - 480\mu) \qquad \varepsilon_c = 160.0\mu \blacktriangleleft$$

$$\varepsilon_{\max} = 278.6\mu \quad \varepsilon_{\min} = -598.6\mu$$

$$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 278.6\mu + 598.6\mu \qquad \gamma_{\max} = 877\mu \blacktriangleleft$$

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PROBLEM 7.139

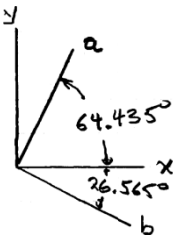
The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$.)

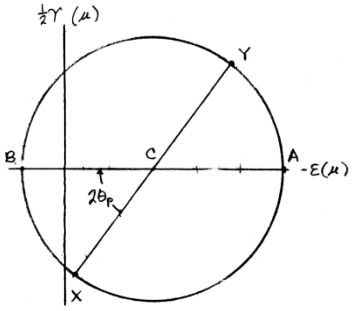
$$\varepsilon_x = +30\mu, \quad \varepsilon_y = +570\mu, \quad \gamma_{xy} = +720\mu$$

SOLUTION

Plotted points for Mohr's circle:

$$X\text{: } (30\mu, -360\mu)$$
$$Y\text{: } (570\mu, +360\mu)$$
$$C\text{: } (300\mu, 0)$$





$$\tan 2\theta_p = \frac{-360}{270} = -1.3333$$
$$2\theta_p = -53.13°$$

(*a*) $\theta_b = -26.6°$ ◀

$\theta_a = 64.4°$ ◀

$$R = \sqrt{(270\mu)^2 + (360\mu)^2} = 450\mu$$

$$\varepsilon_a = \varepsilon_{\text{ave}} + R = 300\mu + 450\mu \qquad \varepsilon_a = 750\mu \blacktriangleleft$$

$$\varepsilon_b = \varepsilon_{\text{ave}} - R = 300\mu - 450\mu \qquad \varepsilon_b = -150.0\mu \blacktriangleleft$$

(*b*) $\gamma_{\text{max (in-plane)}} = 2R$ $\qquad \gamma_{\text{max (in-plane)}} = 900\mu$ ◀

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{1/3}{2/3}(750\mu - 150\mu) \qquad \varepsilon_c = -300\mu \blacktriangleleft$$

$$\varepsilon_{\max} = \varepsilon_a = 750\mu, \quad \varepsilon_{\min} = \varepsilon_c = -300\mu$$

(*c*) $\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 750\mu - (-300\mu)$ $\qquad \gamma_{\max} = 1050\mu$ ◀

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PROBLEM 7.140

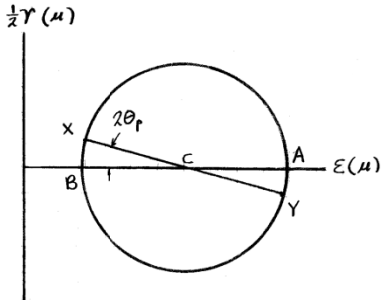
For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

$$\varepsilon_x = +60\mu, \quad \varepsilon_y = +240\mu, \quad \gamma_{xy} = -50\mu$$

SOLUTION

Plotted points:

$$X: (60\mu, 25\mu)$$
$$Y: (240\mu, -25\mu)$$
$$C: (150\mu, 0)$$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-50}{60 - 240} = 0.277778$$

$$2\theta_p = 15.52°$$

$$\theta_a = 97.8° \blacktriangleleft$$

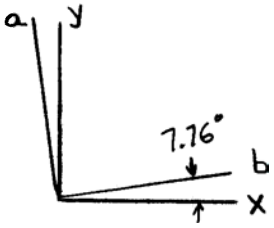
$$\theta_b = 7.8° \blacktriangleleft$$

$$R = \sqrt{(90\mu)^2 + (25\mu)^2} = 93.4\mu$$

(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = 150\mu + 93.4\mu$ $\qquad \varepsilon_a = 243\mu \blacktriangleleft$

$\varepsilon_b = \varepsilon_{\text{ave}} - R = 150\mu - 93.4\mu$ $\qquad \varepsilon_b = 56.6\mu \blacktriangleleft$

(*b*) $\gamma_{\text{max (in-plane)}} = 2R$ $\qquad \gamma_{\text{max (in-plane)}} = 186.8\mu \blacktriangleleft$



(*c*) $\varepsilon_c = 0, \quad \varepsilon_{\max} = 243.4\ \mu, \quad \varepsilon_{\min} = 0$ $\qquad \varepsilon_c = 0 \blacktriangleleft$

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min}$ $\qquad \gamma_{\max} = 243\,\mu \blacktriangleleft$

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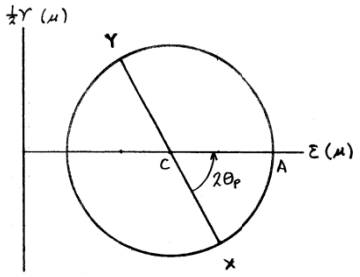
PROBLEM 7.141

For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

$$\varepsilon_x = +400\mu, \quad \varepsilon_y = +200\mu, \quad \gamma_{xy} = 375\mu$$

SOLUTION

Plotted points for Mohr's circle:



$$X\text{: } (+400\mu, -187.5\mu)$$
$$Y\text{: } (+200\mu, +187.5\mu)$$
$$C\text{: } (+300\mu, 0)$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{375}{400 - 200} = 1.875$$

$$2\theta_p = 61.93°$$

$$\theta_a = 31.0° \blacktriangleleft$$

$$\theta_b = 121.0° \blacktriangleleft$$

$$R = \sqrt{(100\mu)^2 + (187.5\mu)^2} = 212.5\mu$$

(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = 300\mu + 212.5\mu$

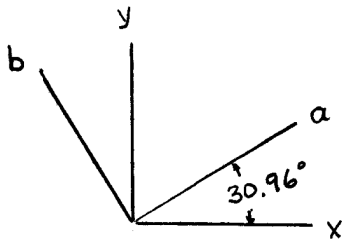
$$\varepsilon_a = 513\mu \blacktriangleleft$$

$\varepsilon_b = \varepsilon_{\text{ave}} - R = 300\mu - 212.5\mu$

$$\varepsilon_b = 87.5\mu \blacktriangleleft$$

(*b*) $\gamma_{\text{max (in-plane)}} = 2R$

$$\gamma_{\text{max (in-plane)}} = 425\mu \blacktriangleleft$$



(*c*) $\varepsilon_c = 0 \quad \varepsilon_{\max} = 512.5\mu \quad \varepsilon_{\min} = 0$

$$\varepsilon_c = 0 \blacktriangleleft$$

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min}$

$$\gamma_{\max} = 513\mu \blacktriangleleft$$

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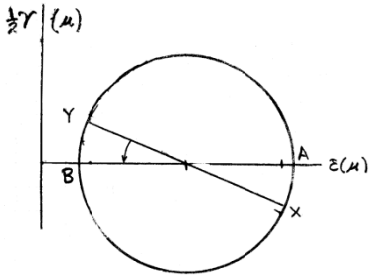
PROBLEM 7.142

For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

$$\varepsilon_x = +300\mu, \quad \varepsilon_y = +60\mu, \quad \gamma_{xy} = +100\mu$$

SOLUTION

$$X\text{: } (300\mu, -50\mu)$$
$$Y\text{: } (60\mu, 50\mu)$$
$$C\text{: } (180\mu, 0)$$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{100}{300 - 60}$$
$$2\theta_p = 22.62°$$

$\theta_a = 11.3°$ ◀

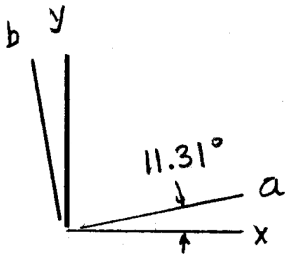
$\theta_b = 101.3°$ ◀

$$R = \sqrt{(120\mu)^2 + (50\mu)^2} = 130\mu$$

(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = 180\mu + 130\mu$ $\varepsilon_a = 310\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = 180\mu - 130\mu$ $\varepsilon_b = 50.0\mu$ ◀

(*b*) $\gamma_{\text{max (in-plane)}} = 2R$ $\gamma_{\text{max (in-plane)}} = 260\mu$ ◀



(*c*) $\varepsilon_c = 0, \quad \varepsilon_{\max} = 310\mu, \quad \varepsilon_{\min} = 0$ $\varepsilon_c = 0$ ◀

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min}$ $\gamma_{\max} = 310\mu$ ◀

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PROBLEM 7.143

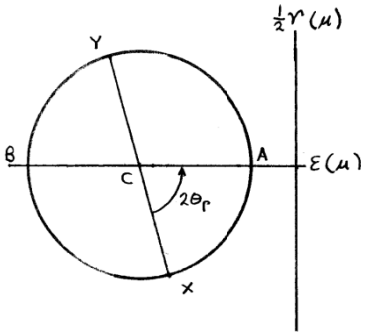
For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

$$\varepsilon_x = -180\mu, \quad \varepsilon_y = -260\mu, \quad \gamma_{xy} = +315\mu$$

SOLUTION

Plotted points for Mohr's circle:

$$X\text{: } (-180\mu, -157.5\mu)$$
$$Y\text{: } (-260\mu, +157.5\mu)$$
$$C\text{: } (-220\mu, 0)$$



(*a*) $\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{315}{80} = 3.9375$

$2\theta_p = 75.75°$

$\theta_a = 37.9°$ ◀

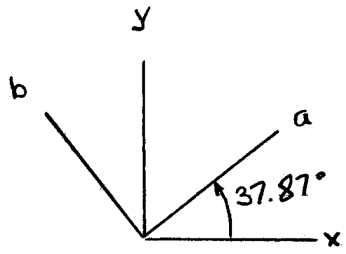
$\theta_b = 127.9°$ ◀

$R = \sqrt{(40\mu)^2 + (157.5\mu)^2} = 162.5\mu$

$\varepsilon_a = \varepsilon_{\text{ave}} + R = -220\mu + 162.5\mu$ $\qquad \varepsilon_a = -57.5\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = -220\mu - 162.5$ $\qquad \varepsilon_b = -383\mu$ ◀

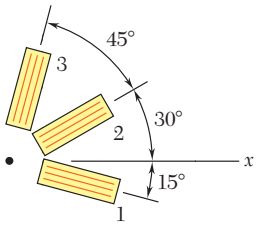
(*b*) $\gamma_{\text{max (in-plane)}} = 2R = 325\mu$ ◀



(*c*) $\varepsilon_c = 0, \quad \varepsilon_{\max} = 0, \quad \varepsilon_{\min} = -382.5\mu$ $\qquad \varepsilon_c = 0$ ◀

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 0 + 382.5\mu$ $\qquad \gamma_{\max} = 383\mu$ ◀

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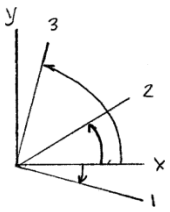


PROBLEM 7.144

Determine the strain ε_x, knowing that the following strains have been determined by use of the rosette shown:

$$\varepsilon_1 = +480\mu \quad \varepsilon_2 = -120\mu \quad \varepsilon_3 = +80\mu$$

SOLUTION



$$\theta_1 = -15°$$
$$\theta_2 = 30°$$
$$\theta_3 = 75°$$

$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1 \cos\theta_1 = \varepsilon_1$$

$$0.9330\varepsilon_x + 0.06699\varepsilon_y - 0.25\gamma_{xy} = 480\mu \tag{1}$$

$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2 = \varepsilon_2$$

$$0.75\varepsilon_x + 0.25\varepsilon_y + 0.4330\gamma_{xy} = -120\mu \tag{2}$$

$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3 = \varepsilon_3$$

$$0.06699\varepsilon_x + 0.9330\varepsilon_y + 0.25\gamma_{xy} = 80\mu \tag{3}$$

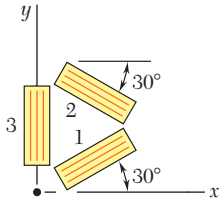
Solving (1), (2), and (3) simultaneously,

$$\varepsilon_x = 253\mu, \qquad \varepsilon_y = 307\mu, \qquad \gamma_{xy} = -893\mu$$

$$\varepsilon_x = 253\mu \blacktriangleleft$$

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PROBLEM 7.145



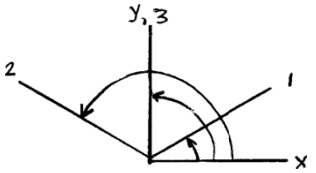
The strains determined by the use of the rosette shown during the test of a machine element are

$$\varepsilon_1 = +600\mu \quad \varepsilon_2 = +450\mu \quad \varepsilon_3 = -75\mu$$

Determine (*a*) the in-plane principal strains, (*b*) the in-plane maximum shearing strain.

SOLUTION

$$\theta_1 = 30°$$
$$\theta_2 = 150°$$
$$\theta_3 = 90°$$



$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1 \cos\theta_1 = \varepsilon_1$$

$$0.75\varepsilon_x + 0.25\varepsilon_y + 0.43301\gamma_{xy} = 600\mu \tag{1}$$

$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2 = \varepsilon_2$$

$$0.75\varepsilon_x + 0.25\varepsilon_y - 0.43301\gamma_{xy} = 450\mu \tag{2}$$

$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3 = \varepsilon_3$$

$$0 + \varepsilon_y + 0 = -75\mu \tag{3}$$

Solving (1), (2), and (3) simultaneously,

$$\varepsilon_x = 725\mu, \quad \varepsilon_y = -75\mu, \quad \gamma_{xy} = 173.21\mu$$

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 325\mu$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{725+75}{2}\right)^2 + \left(\frac{173.21}{2}\right)^2} = 409.3\mu$$

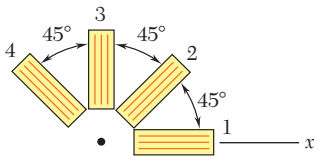
(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = 734\,\mu$ $\qquad \varepsilon_a = 734\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = -84.3\mu$ $\qquad \varepsilon_b = -84.3\mu$ ◀

(*b*) $\gamma_{\text{max (in-plane)}} = 2R = 819\mu$ $\qquad \gamma_{\text{max (in-plane)}} = 819\mu$ ◀

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PROBLEM 7.146



The rosette shown has been used to determine the following strains at a point on the surface of a crane hook:

$$\varepsilon_1 = +420 \times 10^{-6} \text{ in./in.} \quad \varepsilon_2 = -45 \times 10^{-6} \text{ in./in.} \quad \varepsilon_4 = +165 \times 10^{-6} \text{ in./in.}$$

(*a*) What should be the reading of gage 3? (*b*) Determine the principal strains and the maximum in-plane shearing strain.

SOLUTION

(*a*) Gages 2 and 4 are 90° apart.

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_2 + \varepsilon_4)$$

$$\varepsilon_{\text{ave}} = \frac{1}{2}(-45\times10^{-6} + 165\times10^{-6}) = 60\times10^{-6} \text{ in./in.}$$

Gages 1 and 3 are also 90° apart.

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

$$\varepsilon_3 = 2\varepsilon_{\text{ave}} - \varepsilon_1 = (2)(60\times10^{-6}) - 420\times10^{-6}$$

$$\varepsilon_3 = -300\times10^{-6} \text{ in./in.} \blacktriangleleft$$

(*b*) $\varepsilon_x = \varepsilon_1 = 420\times10^{-6} \text{ in./in.} \quad \varepsilon_y = \varepsilon_3 = -300\times10^{-6} \text{ in./in.}$

$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = (2)(-45\times10^{-6}) - 420\times10^{-6} - (-300\times10^{-6})$$

$$= -210\times10^{-6} \text{ in./in.}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{420\times10^{-6} - (-300\times10^{-6})}{2}\right)^2 + \left(\frac{-210\times10^{-6}}{2}\right)^2}$$

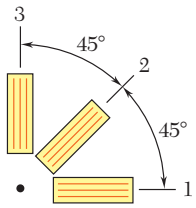
$$= 375\times10^{-6} \text{ in./in.}$$

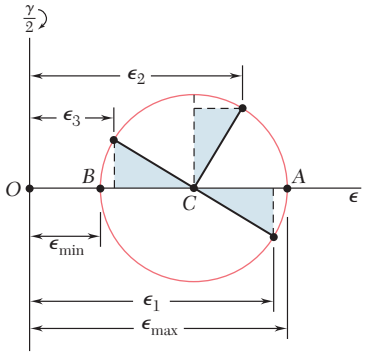
$$\varepsilon_a = \varepsilon_{\text{ave}} + R = 60\times10^{-6} + 375\times10^{-6} \qquad \varepsilon_a = 435\times10^{-6} \text{ in./in.} \blacktriangleleft$$

$$\varepsilon_b = \varepsilon_{\text{ave}} - R = 60\times10^{-6} - 375\times10^{-6} \qquad \varepsilon_b = -315\times10^{-6} \text{ in./in.} \blacktriangleleft$$

$$\gamma_{\text{max (in-plane)}} = 2R \qquad \gamma_{\text{max (in-plane)}} = 750\times10^{-6} \text{ in./in.} \blacktriangleleft$$

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PROBLEM 7.147

Using a 45° rosette, the strains ε_1, ε_2, and ε_3 have been determined at a given point. Using Mohr's circle, show that the principal strains are:

$$\varepsilon_{\max,\min} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3) \pm \frac{1}{\sqrt{2}}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{\frac{1}{2}}$$

(*Hint:* The shaded triangles are congruent.)

SOLUTION

Since gage directions 1 and 3 are 90° apart,

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

Let $u = \varepsilon_1 - \varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_1 - \varepsilon_3)$

$$v = \varepsilon_2 - \varepsilon_{\text{ave}} = \varepsilon_2 - \frac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

$$R^2 = u^2 + v^2$$

$$= \frac{1}{4}(\varepsilon_1 - \varepsilon_3)^2 + \varepsilon_2^2 - \varepsilon_2(\varepsilon_1 + \varepsilon_3) + \frac{1}{4}(\varepsilon_1 + \varepsilon_3)^2$$

$$= \frac{1}{4}\varepsilon_1^2 - \cancel{\frac{1}{2}\varepsilon_1\varepsilon_3} + \frac{1}{4}\varepsilon_3^2 + \varepsilon_2^2 - \varepsilon_2\varepsilon_1 - \varepsilon_2\varepsilon_3 + \frac{1}{4}\varepsilon_1^2 + \cancel{\frac{1}{2}\varepsilon_1\varepsilon_3} + \frac{1}{4}\varepsilon_3^2$$

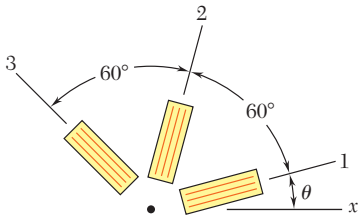
$$= \frac{1}{2}\varepsilon_1^2 - \varepsilon_2\varepsilon_1 + \varepsilon_2^2 - \varepsilon_2\varepsilon_3 + \frac{1}{2}\varepsilon_3^2$$

$$= \frac{1}{2}(\varepsilon_1 - \varepsilon_2)^2 + \frac{1}{2}(\varepsilon_2 - \varepsilon_3)^2$$

$$R = \frac{1}{\sqrt{2}}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{1/2}$$

$\varepsilon_{\max,\,\min} = \varepsilon_{\text{ave}} \pm R$ gives the required formula.

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PROBLEM 7.148

Show that the sum of the three strain measurements made with a 60° rosette is independent of the orientation of the rosette and equal to

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_{\text{avg}}$$

where ε_{avg} is the abscissa of the center of the corresponding Mohr's circle.

SOLUTION

$$\varepsilon_1 = \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \tag{1}$$

$$\begin{aligned}\varepsilon_2 &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 120°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 120°)\\ &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 120°\cos 2\theta - \sin 120°\sin 2\theta)\\ &\quad + \frac{\gamma_{xy}}{2}(\cos 120°\sin 2\theta + \sin 120°\cos 2\theta)\\ &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta - \frac{\sqrt{3}}{2}\sin 2\theta\right)\\ &\quad + \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta + \frac{\sqrt{3}}{2}\cos 2\theta\right)\end{aligned} \tag{2}$$

$$\begin{aligned}\varepsilon_3 &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 240°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 240°)\\ &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 240°\cos 2\theta - \sin 240°\sin 2\theta)\\ &\quad + \frac{\gamma_{xy}}{2}(\cos 240°\sin 2\theta + \sin 240°\cos 2\theta)\\ &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta + \frac{\sqrt{3}}{2}\sin 2\theta\right)\\ &\quad + \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta - \frac{\sqrt{3}}{2}\cos 2\theta\right)\end{aligned} \tag{3}$$

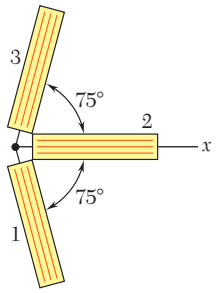
Adding (1), (2), and (3),

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_{\text{ave}} + 0 + 0$$

$$3\varepsilon_{\text{ave}} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \quad \blacktriangleleft$$

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PROBLEM 7.149



The strains determined by the use of the rosette attached as shown during the test of a machine element are

$$\varepsilon_1 = -93.1 \times 10^{-6} \text{ in./in.}$$
$$\varepsilon_2 = +385 \times 10^{-6} \text{ in./in.}$$
$$\varepsilon_3 = +210 \times 10^{-6} \text{ in./in.}$$

Determine (*a*) the orientation and magnitude of the principal strains in the plane of the rosette, (*b*) the maximum in-plane shearing strain.

SOLUTION

Use $\varepsilon_{x'} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$

where $\theta = -75°$ for gage 1,

$\theta = 0$ for gage 2,

and $\theta = +75°$ for gage 3.

$$\varepsilon_1 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos(-150°) + \frac{\gamma_{xy}}{2}\sin(-150°) \quad (1)$$

$$\varepsilon_2 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 0 + \frac{\gamma_{xy}}{2}\sin 0 \quad (2)$$

$$\varepsilon_3 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos(150°) + \frac{\gamma_{xy}}{2}\sin(150°) \quad (3)$$

From Eq. (2), $\varepsilon_x = \varepsilon_z = 385 \times 10^{-6}$ in./in.

Adding Eqs. (1) and (3),

$$\varepsilon_1 + \varepsilon_3 = (\varepsilon_x + \varepsilon_y) + (\varepsilon_x - \varepsilon_y)\cos 150°$$
$$= \varepsilon_x(1 + \cos 150°) + \varepsilon_y(1 - \cos 150°)$$
$$\varepsilon_y = \frac{\varepsilon_1 + \varepsilon_3 - \varepsilon_x(1 + \cos 150°)}{(1 - \cos 150°)}$$
$$= \frac{-93.1 \times 10^{-6} + 210 \times 10^{-6} - 385 \times 10^{-6}(1 + \cos 150°)}{1 - \cos 150°}$$
$$= 35.0 \times 10^{-6} \text{ in./in.}$$

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PROBLEM 7.149 ***(Continued)***

Subtracting Eq. (1) from Eq. (3),

$$\varepsilon_3 - \varepsilon_1 = \gamma_{xy} \sin 150°$$

$$\gamma_{xy} = \frac{\varepsilon_3 - \varepsilon_1}{\sin 150°} = \frac{210 \times 10^{-6} - (-93.1 \times 10^{-6})}{\sin 150°}$$

$$= 606.2 \times 10^{-6} \text{ in./in.}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{606.2 \times 10^{-6}}{385 \times 10^{-6} - 35.0 \times 10^{-6}} = 1.732$$

(*a*) $\theta_a = 30.0°, \quad \theta_b = 120.0°$ ◀

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = \frac{1}{2}(385 \times 10^{-6} + 35.0 \times 10^{-6})$$

$$= 210 \times 10^{-6} \text{ in./in.}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

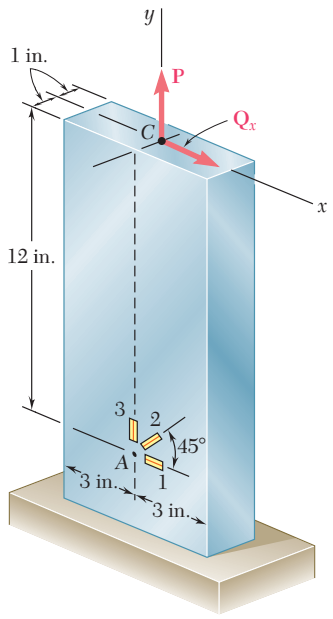
$$= \sqrt{\left(\frac{385 \times 10^{-6} - 35.0 \times 10^{-6}}{2}\right)^2 + \left(\frac{606.2}{2}\right)^2} = 350.0 \times 10^{-6}$$

$$\varepsilon_a = \varepsilon_{\text{ave}} + R = 210 \times 10^{-6} + 350.0 \times 10^{-6} \qquad \varepsilon_a = 560 \times 10^{-6} \text{ in./in.} \blacktriangleleft$$

$$\varepsilon_b = \varepsilon_{\text{ave}} - R = 210 \times 10^{-6} - 350.0 \times 10^{-6} \qquad \varepsilon_b = -140.0 \times 10^{-6} \text{ in./in.} \blacktriangleleft$$

(*b*) $$\frac{\gamma_{\text{max (in-plane)}}}{2} = R = 350.0 \times 10^{-6} \text{ in./in.} \qquad \gamma_{\text{max (in-plane)}} = 700 \times 10^{-6} \text{ in./in.} \blacktriangleleft$$

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PROBLEM 7.150

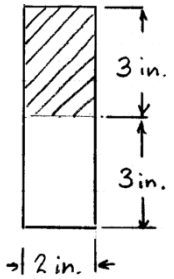
A centric axial force **P** and a horizontal force $\mathbf{Q}_x$ are both applied at point C of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point A indicates the following strains:

$$\varepsilon_1 = -60 \times 10^{-6} \text{ in./in.}$$
$$\varepsilon_2 = +240 \times 10^{-6} \text{ in./in.}$$
$$\varepsilon_3 = +200 \times 10^{-6} \text{ in./in.}$$

Knowing that $E = 29 \times 10^6$ psi and $v = 0.30$, determine the magnitudes of **P** and $\mathbf{Q}_x$.

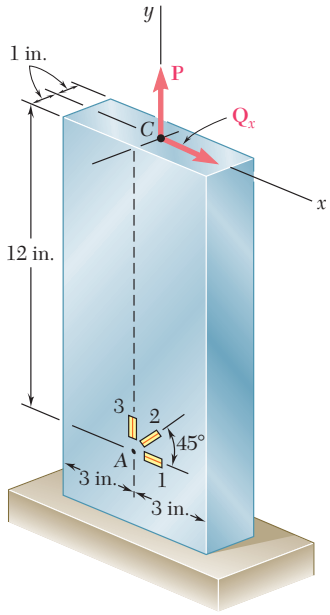
SOLUTION

$$\varepsilon_x = \varepsilon_1 = -60 \times 10^{-6}$$
$$\varepsilon_y = \varepsilon_3 = 200 \times 10^{-6}$$
$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = 340 \times 10^{-6}$$
$$\sigma_x = \frac{E}{1-v^2}(\varepsilon_x + v\varepsilon_y) = \frac{29}{1-(0.3)^2}[-60 + (0.3)(200)] = 0$$
$$\sigma_y = \frac{E}{1-v^2}(\varepsilon_y + v\varepsilon_x) = \frac{29}{1-(0.3)^2}[200 + (0.3)(-60)] = 5.8 \times 10^3 \text{ psi}$$
$$\frac{P}{A} = \sigma_y \qquad P = A\sigma_y = (2)(6)(5.8 \times 10^3)$$
$$= 69.6 \times 10^3 \text{ lb} \qquad P = 69.6 \text{ kips} \blacktriangleleft$$
$$G = \frac{E}{2(1+v)} = \frac{29 \times 10^6}{(2)(1.30)} = 11.1538 \times 10^6 \text{ psi}$$
$$\tau_{xy} = G\gamma_{xy} = (11.1538)(340) = 3.7923 \times 10^3 \text{ psi}$$
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$
$$\hat{Q} = A\bar{y} = (2)(3)(1.5) = 9 \text{ in}^3 \qquad t = 2 \text{ in.}$$
$$\tau_{xy} = \frac{V\hat{Q}}{It}$$
$$V = \frac{It\tau_{xy}}{\hat{Q}} = \frac{(36)(2)(3.7923 \times 10^3)}{9} = 30.338 \times 10^3 \text{ lb}$$
$$Q = V \qquad Q = 30.3 \text{ kips} \blacktriangleleft$$



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PROBLEM 7.151



Solve Prob. 7.150, assuming that the rosette at point A indicates the following strains:

$$\varepsilon_1 = -30 \times 10^{-6} \text{ in./in.}$$
$$\varepsilon_2 = +250 \times 10^{-6} \text{ in./in.}$$
$$\varepsilon_3 = +100 \times 10^{-6} \text{ in./in.}$$

PROBLEM 7.150 A centric axial force **P** and a horizontal force $\mathbf{Q}_x$ are both applied at point C of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point A indicates the following strains:

$$\varepsilon_1 = -60 \times 10^{-6} \text{ in./in.}$$
$$\varepsilon_2 = +240 \times 10^{-6} \text{ in./in.}$$
$$\varepsilon_3 = +200 \times 10^{-6} \text{ in./in.}$$

Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the magnitudes of **P** and $\mathbf{Q}_x$.

SOLUTION

$$\varepsilon_x = \varepsilon_1 = -30 \times 10^{-6}$$
$$\varepsilon_y = \varepsilon_3 = +100 \times 10^{-6}$$
$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = 430 \times 10^{-6}$$
$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{29}{1-(0.3)^2}[-30 + (0.3)(100)]$$
$$= 0$$
$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{29}{1-(0.3)^2}[100 + (0.3)(-30)]$$
$$= 2.9 \times 10^3 \text{ psi}$$
$$\frac{P}{A} = \sigma_y \quad P = A\sigma_y = (2)(6)(2.9 \times 10^3)$$
$$= 34.8 \times 10^3 \text{ lb} \qquad P = 34.8 \text{ kips} \blacktriangleleft$$

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PROBLEM 7.151 ***(Continued)***

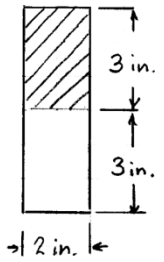
$$G = \frac{E}{2(1+\nu)} = \frac{29\times10^6}{(2)(1.30)} = 11.1538\times10^6 \text{ psi}$$

$$\tau_{xy} = G\gamma_{xy} = (11.1538)(430) = 4.7962\times10^3 \text{ psi}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$

$$\hat{Q} = A\bar{y} = (2)(3)(1.5) = 9 \text{ in}^3$$

$$t = 2 \text{ in.}$$



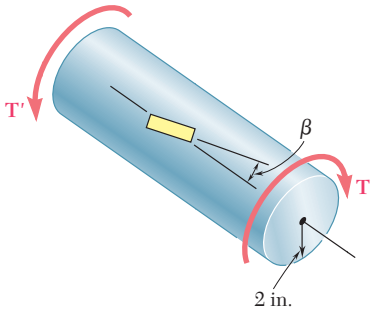
$$\tau_{xy} = \frac{V\hat{Q}}{It}$$

$$V = \frac{It\tau_{xy}}{\hat{Q}} = \frac{(36)(2)(4.7962\times10^3)}{9} = 38.37\times10^3 \text{ lb}$$

$$Q = V$$

$$Q = 38.4 \text{ kips} \blacktriangleleft$$

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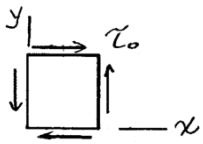


PROBLEM 7.152

A single strain gage is cemented to a solid 4-in.-diameter steel shaft at an angle $\beta = 25°$ with a line parallel to the axis of the shaft. Knowing that $G = 11.5 \times 10^6$ psi, determine the torque **T** indicated by a gage reading of 300×10^{-6} in./in.

SOLUTION

For torsion, $\sigma_x = \sigma_y = 0, \quad \tau = \tau_0$

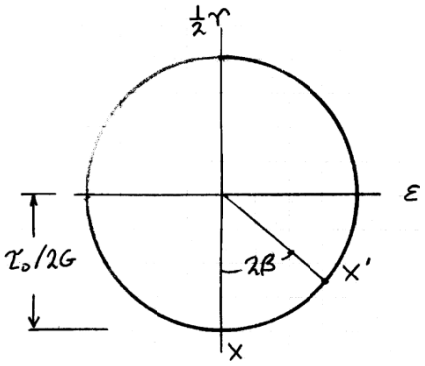


$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = 0$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 0$$

$$\gamma_{xy} = \frac{\tau_0}{G} \quad \frac{1}{2}\gamma_{xy} = \frac{\tau_0}{2G}$$

Draw the Mohr's circle for strain.



$$R = \frac{\tau_0}{2G}$$

$$\varepsilon_{x'} = R \sin 2\beta = \frac{\tau_0}{2G} \sin 2\beta$$

But $$\tau_0 = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2G\varepsilon_{x'}}{\sin 2\beta}$$

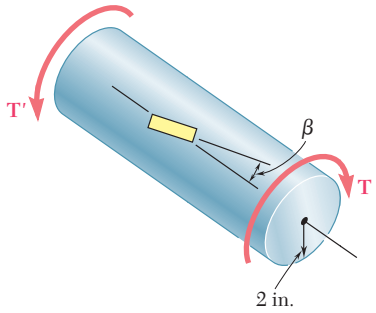
$$T = \frac{\pi c^3 G \varepsilon_{x'}}{\sin 2\beta}$$

$$= \frac{\pi(2)^3(11.5\times10^6)(300\times10^{-6})}{\sin 50°}$$

$$= 113.2 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$T = 113.2 \text{ kip} \cdot \text{in.} \blacktriangleleft$$

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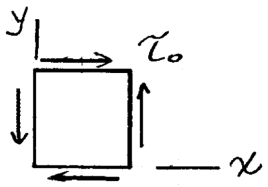
PROBLEM 7.153

Solve Prob. 7.152, assuming that the gage forms an angle $\beta = 35°$ with a line parallel to the axis of the shaft.

PROBLEM 7.152 A single gage is cemented to a solid 4-in.-diameter steel shaft at an angle $\beta = 25°$ with a line parallel to the axis of the shaft. Knowing that $G = 11.5 \times 10^6$ psi, determine the torque **T** indicated by a gage reading of 300×10^{-6} in./in.

SOLUTION

For torsion, $\sigma_x = 0, \quad \sigma_y = 0, \quad \tau_{xy} = \tau_0$



$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = 0$$

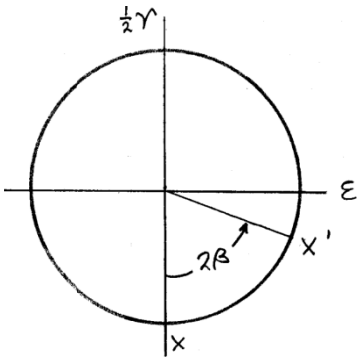
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 0$$

$$\gamma_{xy} = \frac{\tau_0}{G} \quad \frac{1}{2}\gamma_{xy} = \frac{\tau_0}{2G}$$

Draw Mohr's circle for strain.

$$R = \frac{\tau_0}{2G}$$

$$\varepsilon_{x'} = R \sin 2\beta = \frac{\tau_0}{2G} \sin 2\beta$$



But

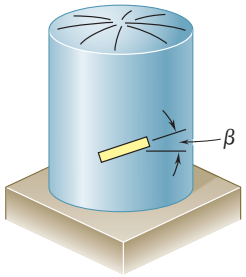
$$\tau_0 = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2G\varepsilon_{x'}}{\sin 2\beta}$$

$$T = \frac{\pi c^3 G\varepsilon_{x'}}{\sin 2\beta} = \frac{\pi(2)^3(11.5\times10^6)(300\times10^{-6})}{\sin 70°}$$

$$= 92.3\times10^3 \text{ lb}\cdot\text{in.}$$

$$T = 92.3 \text{ kip}\cdot\text{in.} \blacktriangleleft$$

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PROBLEM 7.154

A single strain gage forming an angle $\beta = 18°$ with a horizontal plane is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is 6 mm thick, has a 600-mm inside diameter, and is made of a steel with $E = 200$ GPa and $\nu = 0.30$. Determine the pressure in the tank indicated by a strain gage reading of 280μ.

SOLUTION

$$\sigma_x = \sigma_1 = \frac{pr}{t}$$

$$\sigma_y = \frac{1}{2}\sigma_x, \quad \sigma_z \approx 0$$

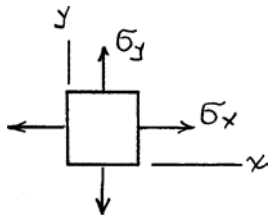
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \left(1 - \frac{\nu}{2}\right)\frac{\sigma_x}{E}$$

$$= 0.85\frac{\sigma_x}{E}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \left(\frac{1}{2} - \nu\right)\frac{\sigma_x}{E}$$

$$= 0.20\frac{\sigma_x}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$



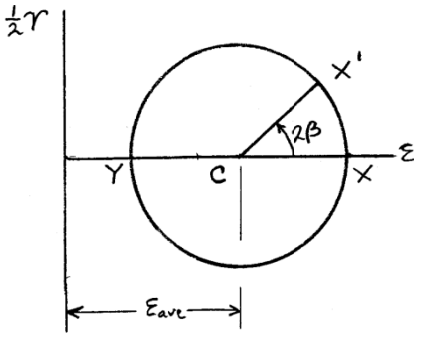
Draw Mohr's circle for strain.

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\frac{\sigma_x}{E}$$

$$R = \frac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325\frac{\sigma_x}{E}$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} + R\cos 2\beta = (0.525 + 0.325\cos 2\beta)\frac{\sigma_x}{E}$$

$$p = \frac{t\sigma_x}{r} = \frac{tE\varepsilon_{x'}}{r(0.525 + 0.325\cos 2\beta)}$$



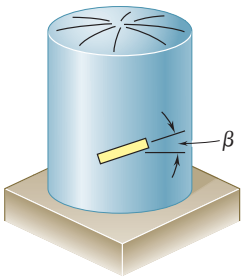
Data: $r = \frac{1}{2}d = \frac{1}{2}(600) = 300 \text{ mm} = 0.300 \text{ m}$

$$t = 6\times10^{-3}\text{ m} \quad E = 200\times10^{9}\text{ Pa}, \quad \varepsilon_{x'} = 280\times10^{-6} \quad \beta = 18°$$

$$p = \frac{(6\times10^{-3})(200\times10^{9})(280\times10^{-6})}{(0.300)(0.525 + 0.325\cos 36°)} = 1.421\times10^{6}\text{ Pa}$$

$$p = 1.421 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.155

Solve Prob. 7.154, assuming that the gage forms an angle $\beta = 35°$ with a horizontal plane.

PROBLEM 7.154 A single strain gage forming an angle $\beta = 18°$ with a horizontal plane is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is 6 mm thick, has a 600-mm inside diameter, and is made of a steel with $E = 200$ GPa and $\nu = 0.30$. Determine the pressure in the tank indicated by a strain gage reading of 280μ.

SOLUTION

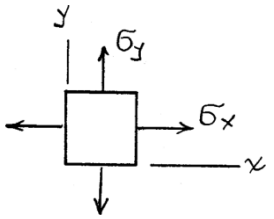
$$\sigma_x = \sigma_1 = \frac{pr}{t}$$

$$\sigma_y = \frac{1}{2}\sigma_x, \quad \sigma_z \approx 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \left(1 - \frac{\nu}{2}\right)\frac{\sigma_x}{E} = 0.85\frac{\sigma_x}{E}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \left(\frac{1}{2} - \nu\right)\frac{\sigma_x}{E} = 0.20\frac{\sigma_x}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$



Draw Mohr's circle for strain.

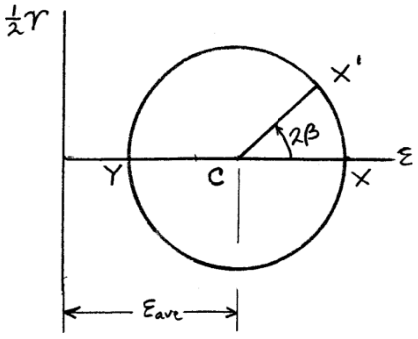
$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\frac{\sigma_x}{E}$$

$$R = \frac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325\frac{\sigma_x}{E}$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} + R\cos 2\beta$$

$$= (0.525 + 0.325\cos 2\beta)\frac{\sigma_x}{E}$$

$$p = \frac{t\sigma_x}{r} = \frac{tE\varepsilon_{x'}}{r(0.525 + 0.325\cos 2\beta)}$$



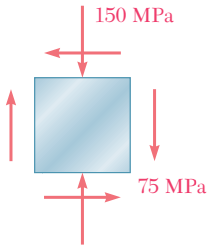
Data:

$$r = \frac{1}{2}d = \frac{1}{2}(600) = 300 \text{ mm} = 0.300 \text{ m}$$

$$t = 6\times10^{-3}\text{ m} \quad E = 200\times10^{9}\text{ Pa}, \quad \varepsilon_{x'} = 280\times10^{-6} \quad \beta = 35°$$

$$p = \frac{(6\times10^{-3})(200\times10^{9})(280\times10^{-6})}{(0.300)(0.525 + 0.325\cos 70°)} = 1.761\times10^{6}\text{ Pa} \qquad p = 1.761\text{ MPa} \blacktriangleleft$$

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PROBLEM 7.156

The given state of plane stress is known to exist on the surface of a machine component. Knowing that $E = 200\,\text{GPa}$ and $G = 77.2\,\text{GPa}$, determine the direction and magnitude of the three principal strains (*a*) by determining the corresponding state of strain [use Eq. (2.43) and Eq. (2.38)] and then using Mohr's circle for strain, (*b*) by using Mohr's circle for stress to determine the principal planes and principal stresses and then determining the corresponding strains.

SOLUTION

(*a*) $\sigma_x = 0, \quad \sigma_y = -150\times10^6\ \text{Pa}, \quad \tau_{xy} = -75\times10^6\ \text{Pa}$

$$E = 200\times10^9\ \text{Pa} \quad G = 77\times10^9\ \text{Pa}$$

$$G = \frac{E}{2(1+\nu)} \qquad \nu = \frac{E}{2G} - 1 = 0.2987$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{200\times10^9}[0 + (0.2987)(150\times10^6)]$$
$$= 224\mu$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{200\times10^9}[(-150\times10^6) - 0]$$
$$= -750\mu$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-75\times10^6}{77\times10^9} = -974\,\mu$$

$$\frac{\gamma_{xy}}{2} = -487.0\mu$$

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = -263\mu$$

$$\varepsilon_x - \varepsilon_y = 974\mu$$

$$\tan 2\theta_a = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-974}{974} = -1.000$$

$$2\theta_a = -45.0°$$

$\theta_a = -22.5°$ ◀

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 689\mu$$

$\theta_b = 67.5°$ ◀

$$\varepsilon_a = \varepsilon_{\text{ave}} + R$$

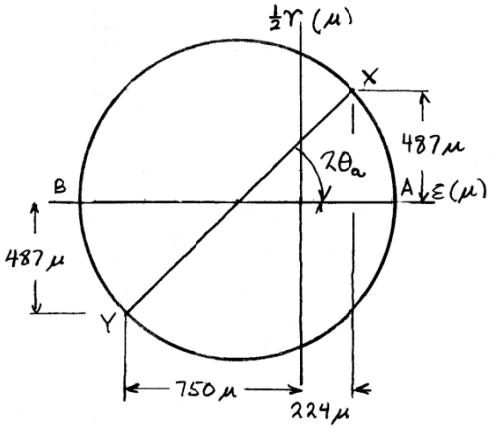
$\varepsilon_a = 426\,\mu$ ◀

$$\varepsilon_b = \varepsilon_{\text{ave}} - R$$

$\varepsilon_b = -952\,\mu$ ◀

$$\varepsilon_c = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{(0.2987)(0 - 150\times10^6)}{200\times10^9}$$

$\varepsilon_c = -224\,\mu$ ◀



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PROBLEM 7.156 *(Continued)*

(*b*) $\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -75 \text{ MPa}$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0+150}{2}\right)^2 + 75^2}$$

$$= 106.07 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 31.07 \text{ MPa}$$

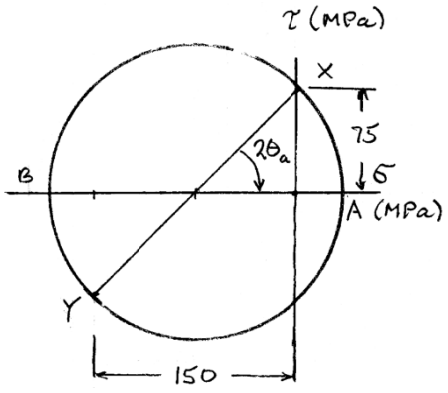
$$\sigma_b = \sigma_{\text{ave}} - R = -181.07 \text{ MPa}$$

$$\varepsilon_a = \frac{1}{E}(\sigma_a - \nu\sigma_b)$$

$$= \frac{1}{200\times10^9}[31.07\times10^6 - (0.2987)(-181.07\times10^6)]$$

$$= 426\times10^{-6}$$

$\varepsilon_a = 426\,\mu$ ◀



$$\tan 2\theta_a = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1.000$$

$2\theta_a = -45°$

$\theta_a = -22.5°$ ◀

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PROBLEM 7.157

The following state of strain has been determined on the surface of a cast-iron machine part:

$$\varepsilon_x = -720\mu \quad \varepsilon_y = -400\mu \quad \gamma_{xy} = +660\mu$$

Knowing that $E = 69$ GPa and $G = 28$ GPa, determine the principal planes and principal stresses (*a*) by determining the corresponding state of plane stress [use Eq. (2.36), Eq. (2.43), and the first two equations of Prob. 2.73] and then using Mohr's circle for stress, (*b*) by using Mohr's circle for strain to determine the orientation and magnitude of the principal strains and then determining the corresponding stresses.

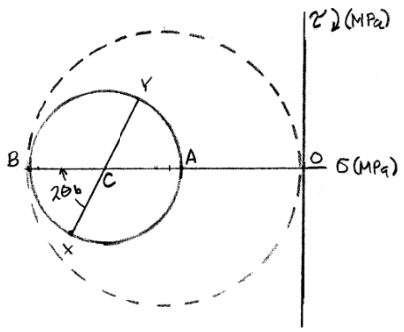
SOLUTION

The 3rd principal stress is $\sigma_z = 0$.

$$G = \frac{E}{2(1+v)} \quad v = \frac{E}{2G} - 1 = \frac{69}{56} - 1 = 0.2321$$

$$\frac{E}{1-v^2} = \frac{69}{1-(0.232)^2} = 72.93 \text{ GPa}$$

(*a*)
$$\sigma_x = \frac{E}{1-v^2}(\varepsilon_x + v\varepsilon_y)$$
$$= (72.93\times10^9)[-720\times10^{-6} + (0.232)(-400\times10^{-6})]$$
$$= -59.28 \text{ MPa}$$
$$\sigma_y = \frac{E}{1-v^2}(\varepsilon_y + v\varepsilon_x)$$
$$= (72.93\times10^9)[-400\times10^{-6} + (0.2321)(-720\times10^{-6})]$$
$$= -41.36 \text{ MPa}$$
$$\tau_{xy} = G\gamma_{xy} = (28\times10^9)(660\times10^{-6})$$
$$= 18.48 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -50.32 \text{ MPa}$$



$$\tan 2\theta_b = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -2.0625$$
$$2\theta_b = -64.1°, \qquad \theta_b = -32.1°, \quad \theta_a = 57.9° \blacktriangleleft$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 20.54 \text{ MPa}$$
$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = -29.8 \text{ MPa} \blacktriangleleft$$
$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -70.9 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.157 *(Continued)*

(*b*) $\varepsilon_{\text{ave}} = \dfrac{1}{2}(\varepsilon_x + \varepsilon_y) = -560\,\mu$

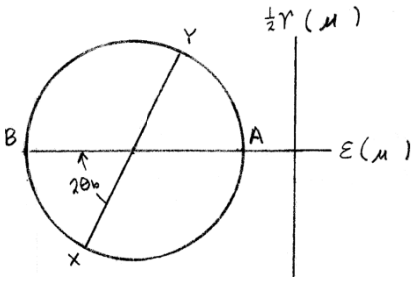
$\tan 2\theta_b = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = -2.0625$

$2\theta_b = -64.1°, \quad \theta_b = -32.1°, \quad \theta_a = 57.9°$ ◀

$R = \sqrt{\left(\dfrac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\dfrac{\gamma_{xy}}{2}\right)^2} = 366.74\,\mu$

$\varepsilon_a = \varepsilon_{\text{ave}} + R = -193.26\,\mu$

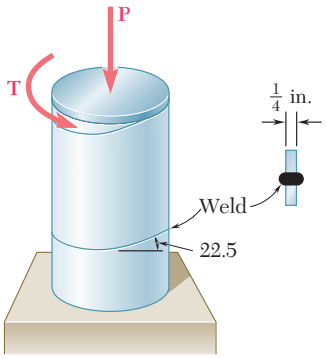
$\varepsilon_b = \varepsilon_{\text{ave}} - R = -926.74\,\mu$



$\sigma_a = \dfrac{E}{1-\nu^2}(\varepsilon_a + \nu\varepsilon_b)$ $\qquad \sigma_a = -29.8$ MPa ◀

$\sigma_b = \dfrac{E}{1-\nu^2}(\varepsilon_b + \nu\varepsilon_a)$ $\qquad \sigma_b = -70.9$ MPa ◀

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PROBLEM 7.158

A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix that forms an angle of 22.5° with a plane perpendicular to the axis of the pipe. Knowing that a 40-kip axial force **P** and an 80-kip · in. torque **T**, each directed as shown, are applied to the pipe, determine the normal and in-plane shearing stresses in directions, respectively, normal and tangential to the weld.

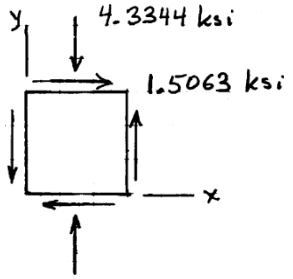
SOLUTION

$$d_2 = 12 \text{ in.}, \quad c_2 = \frac{1}{2} d_2 = 6 \text{ in.}, \quad t = 0.25 \text{ in.}$$

$$c_1 = c_2 - t = 5.75 \text{ in.}$$

$$A = \pi\left(c_2^2 - c_1^2\right) = \pi(6^2 - 5.75^2) = 9.2284 \text{ in}^2$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(6^4 - 5.75^4) = 318.67 \text{ in}^4$$

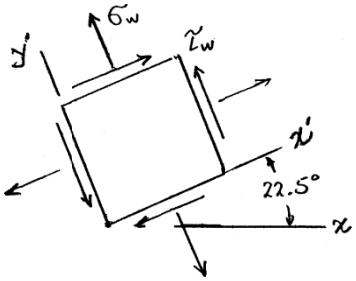


Stresses:

$$\sigma = -\frac{P}{A} = -\frac{40}{9.2284} = -4.3344 \text{ ksi}$$

$$\tau = \frac{Tc_2}{J} = \frac{(80)(6)}{318.67} = 1.5063 \text{ ksi}$$

$$\sigma_x = 0, \quad \sigma_y = -4.3344 \text{ ksi}, \quad \tau_{xy} = 1.5063 \text{ ksi}$$



Choose the x' and y' axes, respectively, tangential and normal to the weld.

Then $\sigma_w = \sigma_{y'}$ and $\tau_w = \tau_{x'y'}$ $\theta = 22.5°$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{(-4.3344)}{2} - \frac{[-(-4.3344)]}{2}\cos 45° - 1.5063 \sin 45°$$

$$= -4.76 \text{ ksi}$$

$\sigma_w = -4.76$ ksi ◀

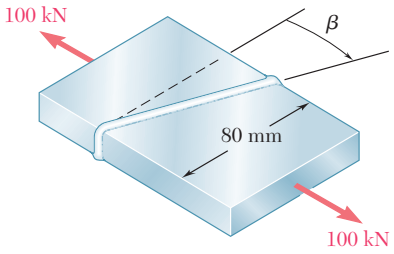
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{[-(-4.3344)]}{2}\sin 45° + 1.5063 \cos 45°$$

$$= -0.467 \text{ ksi}$$

$\tau_w = -0.467$ ksi ◀

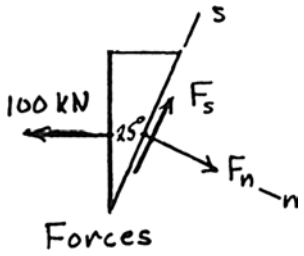
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PROBLEM 7.159

Two steel plates of uniform cross section 10×80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that $\beta = 25°$, determine (*a*) the in-plane shearing stress parallel to the weld, (*b*) the normal stress perpendicular to the weld.

SOLUTION



Area of weld:

$$A_w = \frac{(10 \times 10^{-3})(80 \times 10^{-3})}{\cos 25°}$$
$$= 882.7 \times 10^{-6}\ \text{m}^2$$

(*a*) $\Sigma F_s = 0$: $F_s - 100 \sin 25° = 0$ $F_s = 42.26$ kN

$$\tau_w = \frac{F_s}{A_w} = \frac{42.26 \times 10^3}{882.7 \times 10^{-6}} = 47.9 \times 10^6\ \text{Pa}$$

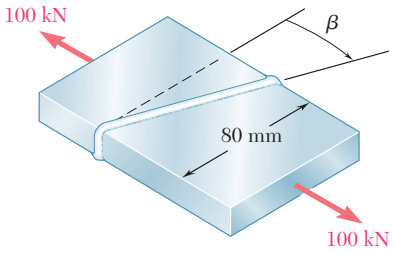
$\tau_w = 47.9$ MPa ◀

(*b*) $\Sigma F_n = 0$: $F_n - 100 \cos 25° = 0$ $F_n = 90.63$ kN

$$\sigma_w = \frac{F_n}{A_w} = \frac{90.63 \times 10^3}{882.7 \times 10^{-6}} = 102.7 \times 10^6\ \text{Pa}$$

$\sigma_w = 102.7$ MPa ◀

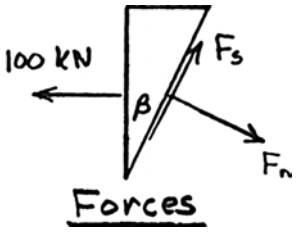
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PROBLEM 7.160

Two steel plates of uniform cross section 10×80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (*a*) the angle β, (*b*) the corresponding normal stress perpendicular to the weld.

SOLUTION



Area of weld:

$$A_w = \frac{(10 \times 10^{-3})(80 \times 10^{-3})}{\cos\beta} = \frac{800 \times 10^{-6}}{\cos\beta}\ \text{m}^2$$

(*a*) $\Sigma F_s = 0$: $F_s - 100\sin\beta = 0$ $\quad F_s = 100\sin\beta\ \text{kN} = 100 \times 10^3 \sin\beta\ \text{N}$

$$\tau_w = \frac{F_s}{A_w} \quad 30 \times 10^6 = \frac{100 \times 10^3 \sin\beta}{800 \times 10^{-6}/\cos\beta} = 125 \times 10^6 \sin\beta\cos\beta$$

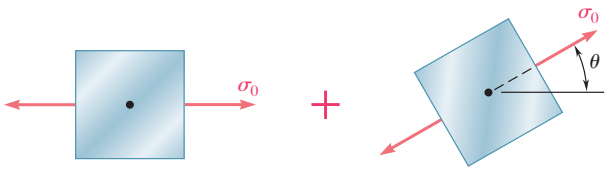
$$\sin\beta\cos\beta = \frac{1}{2}\sin 2\beta = \frac{30 \times 10^6}{125 \times 10^6} = 0.240 \qquad \beta = 14.34° \blacktriangleleft$$

(*b*) $\Sigma F_n = 0$: $F_n - 100\cos\beta = 0$ $\quad F_n = 100\cos 14.34° = 96.88\ \text{kN}$

$$A_w = \frac{800 \times 10^{-6}}{\cos 14.34} = 825.74 \times 10^{-6}\ \text{m}^2$$

$$\sigma = \frac{F_n}{A_w} = \frac{96.88 \times 10^3}{825.74 \times 10^{-6}} = 117.3 \times 10^6\ \text{Pa} \qquad \sigma = 117.3\ \text{MPa} \blacktriangleleft$$

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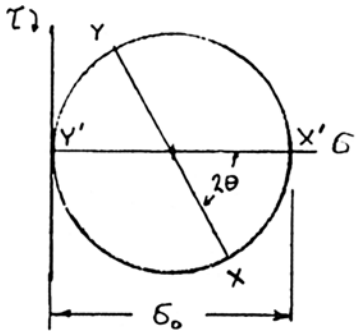


PROBLEM 7.161

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

SOLUTION

Mohr's circle for 2nd stress state:



$$\sigma_x = \frac{1}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\sigma_y = \frac{1}{2}\sigma_0 - \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\tau_{xy} = \frac{1}{2}\sigma_0 \sin 2\theta$$

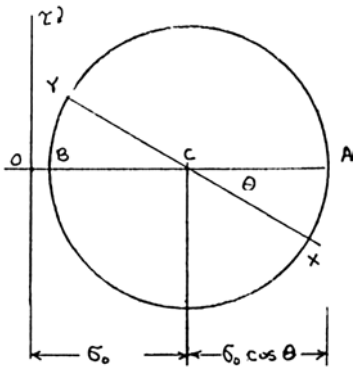
Resultant stresses:

$$\sigma_x = \sigma_0 + \frac{1}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta = \frac{3}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\sigma_y = 0 + \frac{1}{2}\sigma_0 - \frac{1}{2}\sigma_0 \cos 2\theta = \frac{1}{2}\sigma_0 - \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\tau_{xy} = 0 + \frac{1}{2}\sigma_0 \sin 2\theta = \frac{1}{2}\sigma_0 \sin 2\theta$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_0$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\sigma_0 \sin 2\theta}{\sigma_0 + \sigma_0 \cos 2\theta}$$

$$= \frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$$

$$\theta_p = \frac{1}{2}\theta \blacktriangleleft$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{1}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta\right)^2 + \left(\frac{1}{2}\sigma_0 \sin 2\theta\right)^2}$$

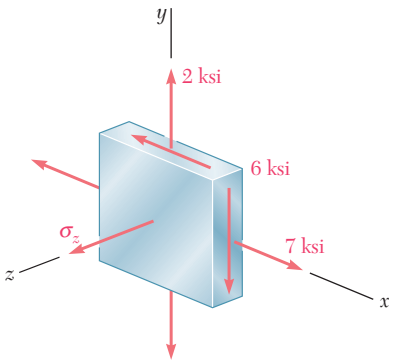
$$= \frac{1}{2}\sigma_0\sqrt{1 + 2\cos 2\theta + \cos^2 2\theta + \sin^2 2\theta} = \frac{\sqrt{2}}{2}\sigma_0\sqrt{1 + \cos 2\theta} = \sigma_0 |\cos\theta|$$

$$\theta_p = \frac{(\theta + \pi)}{2} \blacktriangleleft$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = \sigma_0 + \sigma_0 \cos\theta \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = \sigma_0 - \sigma_0 \cos\theta \blacktriangleleft$$

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PROBLEM 7.162

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = +4$ ksi, (*b*) $\sigma_z = -4$ ksi, (*c*) $\sigma_z = 0$.

SOLUTION

$$\sigma_x = 7 \text{ ksi}, \qquad \sigma_y = 2 \text{ ksi}, \qquad \tau_{xy} = -6 \text{ ksi}$$

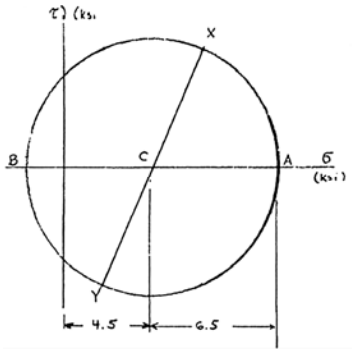
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 4.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{2.5^2 + (-6)^2} = 6.5 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 11 \text{ ksi}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -2 \text{ ksi}$$



(*a*) $\sigma_z = 4 \text{ ksi}, \quad \sigma_a = 11 \text{ ksi}, \quad \sigma_b = -2 \text{ ksi}$

$$\sigma_{\max} = 11 \text{ ksi}, \quad \sigma_{\min} = -2 \text{ ksi}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 6.50 \text{ ksi} \blacktriangleleft$$

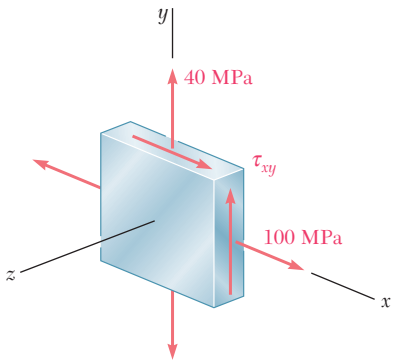
(*b*) $\sigma_z = -4 \text{ ksi}, \quad \sigma_a = 11 \text{ ksi}, \quad \sigma_b = -2 \text{ ksi}$

$$\sigma_{\max} = 11 \text{ ksi}, \quad \sigma_{\min} = -4 \text{ ksi}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 7.50 \text{ ksi} \blacktriangleleft$$

(*c*) $\sigma_z = 0, \quad \sigma_a = 11 \text{ ksi}, \quad \sigma_b = -2 \text{ ksi}$

$$\sigma_{\max} = 11 \text{ ksi}, \quad \sigma_{\min} = -2 \text{ ksi}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 6.50 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.163

For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (*a*) 60 MPa, (*b*) 78 MPa.

SOLUTION

$$\sigma_x = 100 \text{ MPa}, \quad \sigma_y = 40 \text{ MPa}, \quad \sigma_z = 0$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 70 \text{ MPa}$$

(*a*) $\tau_{\max} = 60$ MPa.

If σ_z is $\sigma_{\min}$, then $\sigma_{\max} = \sigma_{\min} + 2\tau_{\max}$.

$$\sigma_{\max} = 0 + (2)(60) = 120 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R$$

$$R = \sigma_{\max} - \sigma_{\text{ave}} = 120 - 70 = 50 \text{ MPa}$$

$$\sigma_b = \sigma_{\max} - 2R = 20 \text{ MPa} > 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + \tau_{xy}^2} = 50 \text{ MPa}$$

$$\tau_{xy} = \sqrt{50^2 - 30^2} \qquad \tau_{xy} = 40.0 \text{ MPa} \blacktriangleleft$$

(*b*) $\tau_{\max} = 78$ MPa.

If σ_z is $\sigma_{\min}$, then $\sigma_{\max} = \sigma_{\min} + 2\tau_{\max} = 0 + (2)(78) = 156$ MPa.

$$\sigma_{\max} = \sigma_{\text{ave}} + R$$

$$R = \sigma_{\max} - \sigma_{\text{ave}} = 156 - 70 = 86 \text{ MPa} > \tau_{\max} = 78 \text{ MPa}$$

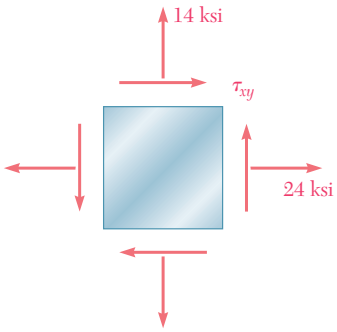
Set $R = \tau_{\max} = 78$ MPa.

$$\sigma_{\min} = \sigma_{\text{ave}} - R = -8 \text{ MPa} < 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{78^2 - 30^2} \qquad \tau_{xy} = 72.0 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.164

The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 30$ ksi. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\tau_{xy} = 6$ ksi, (*b*) $\tau_{xy} = 12$ ksi, (*c*) $\tau_{xy} = 14$ ksi. If yield does not occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_x = 24 \text{ ksi} \quad \sigma_y = 14 \text{ ksi} \quad \sigma_z = 0$$

For stresses in *xy*-plane,

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 19 \text{ ksi} \qquad \frac{\sigma_x - \sigma_y}{2} = 5 \text{ ksi}$$

(*a*) $\tau_{xy} = 6$ ksi

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(5)^2 + (6)^2} = 7.810 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 26.810 \text{ ksi}, \quad \sigma_b = \sigma_{\text{ave}} - R = 11.190 \text{ ksi}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 23.324 \text{ ksi} < 30 \text{ ksi} \qquad \text{(No yielding)}$$

$$F.S. = \frac{30}{23.324} \qquad F.S. = 1.286 \blacktriangleleft$$

(*b*) $\tau_{xy} = 12$ ksi

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(5)^2 + (12)^2} = 13 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 32 \text{ ksi}, \quad \sigma_b = \sigma_{\text{ave}} - R = 6 \text{ ksi}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 29.462 \text{ ksi} < 30 \text{ ksi} \qquad \text{(No yielding)}$$

$$F.S. = \frac{30}{29.462} \qquad F.S. = 1.018 \blacktriangleleft$$

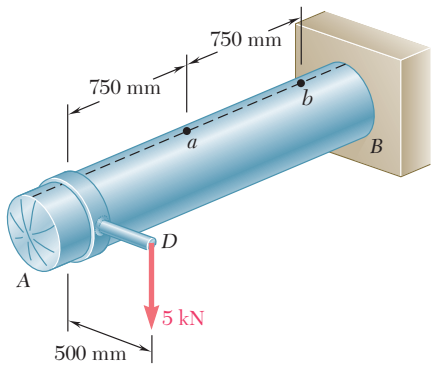
(*c*) $\tau_{xy} = 14$ ksi

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(5)^2 + (14)^2} = 14.866 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 33.866, \quad \sigma_b = \sigma_{\text{ave}} - R = 4.134 \text{ ksi}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 32.00 \text{ ksi} > 30 \text{ ksi} \qquad \text{(Yielding occurs)} \blacktriangleleft$$

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PROBLEM 7.165

The compressed-air tank *AB* has an inner diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure inside the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at point *a* on the top of the tank.

SOLUTION

Internal pressure:

$$r = \frac{1}{2}d = 225\,\text{mm} \quad t = 6\,\text{mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45\ \text{MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 22.5\ \text{MPa}$$

Torsion:

$$c_1 = 225\ \text{mm}, \quad c_2 = 225 + 6 = 231\ \text{mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 446.9 \times 10^6\ \text{mm}^4 = 446.9 \times 10^{-6}\ \text{m}^4$$

$$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500\ \text{N} \cdot \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}}$$

$$= 1.29224 \times 10^6\ \text{Pa} = 1.29224\ \text{MPa}$$

Transverse shear:

$$\tau = 0 \text{ at point } a.$$

Bending:

$$I = \frac{1}{2}J = 223.45 \times 10^{-6}\ \text{m}^4, \quad c = 231 \times 10^{-3}\ \text{m}$$

At point *a*,

$$M = (5 \times 10^3)(750 \times 10^{-3}) = 3750\ \text{N} \cdot \text{m}$$

$$\sigma = \frac{Mc}{I} = \frac{(3750)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 3.8767\ \text{MPa}$$

Total stresses (MPa).

Longitudinal: $\sigma_x = 22.5 + 3.8767 = 26.377\ \text{MPa}$

Circumferential: $\sigma_y = 45\ \text{MPa}$

Shear: $\tau_{xy} = 1.29224\ \text{MPa}$

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PROBLEM 7.165 ***(Continued)***

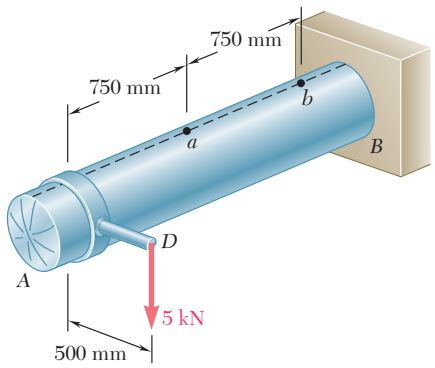
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 35.688 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 9.4007 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = 45.1 \text{ MPa} \qquad \sigma_{\max} = 45.1 \text{ MPa} \blacktriangleleft$$

$$\tau_{\max(\text{in-plane})} = R = 9.40 \text{ MPa} \qquad \tau_{\max(\text{in-plane})} = 9.40 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.166

For the compressed-air tank and loading of Prob. 7.165, determine the maximum normal stress and the maximum in-plane shearing stress at point *b* on the top of the tank.

PROBLEM 7.165 The compressed-air tank *AB* has an inner diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure inside the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at point *a* on the top of the tank.

SOLUTION

Internal pressure:

$$r = \frac{1}{2}d = 225 \text{ mm} \quad t = 6 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 22.5 \text{ MPa}$$

Torsion:

$$c_1 = 225 \text{ mm}, \quad c_2 = 225 + 6 = 231 \text{ mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 446.9 \times 10^6 \text{ mm}^4 = 446.9 \times 10^{-6} \text{ m}^4$$

$$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500 \text{ N} \cdot \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.29224 \times 10^6 \text{ Pa} = 1.29224 \text{ MPa}$$

Transverse shear:

$$\tau = 0 \text{ at point } b.$$

Bending:

$$I = \frac{1}{2}J = 223.45 \times 10^{-6} \text{ m}^4, \quad c = 231 \times 10^{-3} \text{ m}$$

At point *b*,

$$M = (5 \times 10^3)(2 \times 750 \times 10^{-3}) = 7500 \text{ N} \cdot \text{m}$$

$$\sigma = \frac{Mc}{I} = \frac{(7500)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 7.7534 \text{ MPa}$$

Total stresses (MPa).

Longitudinal: $\sigma_x = 22.5 + 7.7534 = 30.253 \text{ MPa}$

Circumferential: $\sigma_y = 45 \text{ MPa}$

Shear: $\tau_{xy} = 1.29224 \text{ MPa}$

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PROBLEM 7.166 ***(Continued)***

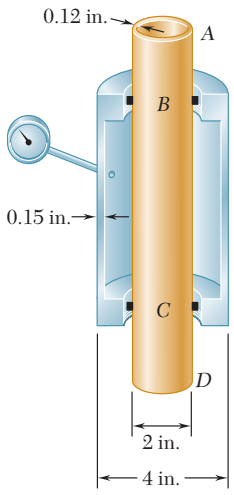
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 37.626 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7.4859 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = 45.1 \text{ MPa} \qquad \sigma_{\max} = 45.1 \text{ MPa} \blacktriangleleft$$

$$\tau_{\max\,(\text{in-plane})} = R = 7.49 \text{ MPa} \qquad \tau_{\max\,(\text{in-plane})} = 7.49 \text{ MPa} \blacktriangleleft$$

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PROBLEM 7.167

The brass pipe *AD* is fitted with a jacket used to apply a hydrostatic pressure of 500 psi to portion *BC* of the pipe. Knowing that the pressure inside the pipe is 100 psi, determine the maximum normal stress in the pipe.

SOLUTION

The only stress to be considered is the hoop stress. This stress can be obtained by applying

$$\sigma_1 = \frac{pr}{t}$$

Using successively the inside and outside pressures (the latter of which causes a compressive stress),

$$p_i = 100 \text{ psi}, \quad r_i = 1 - 0.12 = 0.88 \text{ in.}, \quad t = 0.12 \text{ in.}$$

$$(\sigma_{\max})_i = \frac{p_i r_i}{t} = \frac{(100)(0.88)}{0.12} = +733.33 \text{ psi}$$

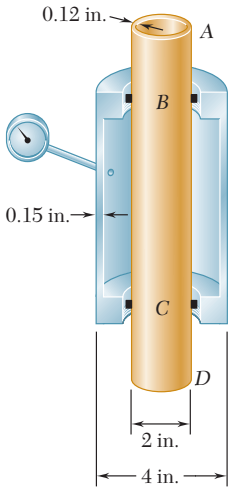
$$p_o = 500 \text{ psi}, \quad r_o = 1 \text{ in.}, \quad t = 0.12 \text{ in.}$$

$$(\sigma_{\max})_o = -\frac{p_o r_o}{t} = -\frac{(500)(1)}{0.12} = -4166.7 \text{ psi}$$

$$\sigma_{\max} = +733.33 - 4166.7 = -3433.4 \text{ psi}$$

$$\sigma_{\max} = 3.43 \text{ ksi (compression)} \blacktriangleleft$$

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PROBLEM 7.168

For the assembly of Prob. 7.167, determine the normal stress in the jacket (*a*) in a direction perpendicular to the longitudinal axis of the jacket, (*b*) in a direction parallel to that axis.

PROBLEM 7.167 The brass pipe *AD* is fitted with a jacket used to apply a hydrostatic pressure of 500 psi to portion *BC* of the pipe. Knowing that the pressure inside the pipe is 100 psi, determine the maximum normal stress in the pipe.

SOLUTION

(*a*) Hoop stress.

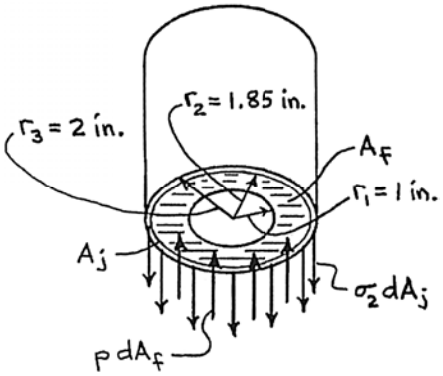
$$p = 500 \text{ psi}, \quad t = 0.15 \text{ in.}, \quad r = 2 - 0.15 = 1.85 \text{ in.}$$

$$(\sigma_1) = \frac{pr}{t} = \frac{(500)(1.85)}{0.15} = 6166.7 \text{ psi}$$

$$\sigma_1 = 6.17 \text{ ksi} \blacktriangleleft$$

(*b*) Longitudinal stress.

Free body of portion of jacket above a horizontal section, considering vertical forces only:



$$+\uparrow \sum F_y = 0:$$

$$\int_{A_f} p\, dA_f - \int_{A_j} \sigma_2\, dA_j = 0$$

$$pA_f - \sigma_2 A_j = 0$$

$$\sigma_2 = p\frac{A_f}{A_j} \qquad (1)$$

Areas :

$$A_f = \pi\left(r_2^2 - r_1^2\right) = \pi[(1.85)^2 - (1)^2] = 7.6105 \text{ in}^2$$

$$A_j = \pi\left(r_3^2 - r_2^2\right) = \pi[(2)^2 - (1.85)^2] = 1.81427 \text{ in}^2$$

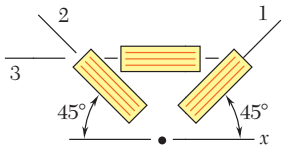
Recalling Eq. (1),

$$\sigma_2 = p\frac{A_f}{A_j} = (500)\frac{7.6105}{1.81427} = 2097.4 \text{ psi}$$

$$\sigma_2 = 2.10 \text{ ksi} \blacktriangleleft$$

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PROBLEM 7.169



Determine the largest in-plane normal strain, knowing that the following strains have been obtained by the use of the rosette shown:

$$\varepsilon_1 = -50 \times 10^{-6} \text{ in./in.}$$
$$\varepsilon_2 = +360 \times 10^{-6} \text{ in./in.}$$
$$\varepsilon_3 = +315 \times 10^{-6} \text{ in./in.}$$

SOLUTION

$$\theta_1 = 45°, \quad \theta_2 = -45°, \quad \theta_3 = 0$$

$$\varepsilon_x \cos^2 \theta_1 + \varepsilon_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1 = \varepsilon_1$$
$$0.5\varepsilon_x + 0.5\varepsilon_y + 0.5\gamma_{xy} = -50 \times 10^{-6} \tag{1}$$

$$\varepsilon_x \cos^2 \theta_2 + \varepsilon_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2 = \varepsilon_2$$
$$0.5\varepsilon_x + 0.5\varepsilon_y - 0.5\gamma_{xy} = 360 \times 10^{-6} \tag{2}$$

$$\varepsilon_x \cos^2 \theta_3 + \varepsilon_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3 = \varepsilon_3$$
$$\varepsilon_x + 0 + 0 = 315 \times 10^{-6} \tag{3}$$

From (3), $\varepsilon_x = 315 \times 10^{-6}$ in./in.

Eq. (1) – Eq. (2): $\gamma_{xy} = -50 \times 10^{-6} - 360 \times 10^{-6} = -410 \times 10^{-6}$ in./in.

Eq. (1) + Eq. (2): $\varepsilon_x + \varepsilon_y = \varepsilon_1 + \varepsilon_2$

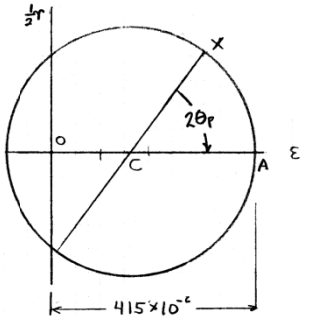
$$\varepsilon_y = \varepsilon_1 + \varepsilon_2 - \varepsilon_x = -50 \times 10^{-6} + 360 \times 10^{-6} - 315 \times 10^{-6} = -5 \times 10^{-6} \text{ in./in.}$$

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 155 \times 10^{-6} \text{ in./in.}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \sqrt{\left(\frac{315 \times 10^{-6} + 5 \times 10^{-6}}{2}\right)^2 + \left(\frac{-410 \times 10^{-6}}{2}\right)^2}$$

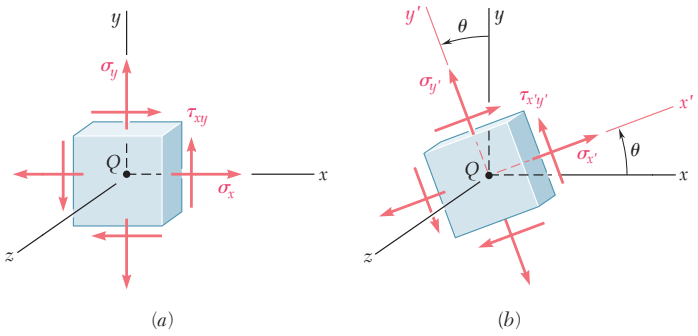
$$= 260 \times 10^{-6} \text{ in./in.}$$



$$\varepsilon_{\max} = \varepsilon_{\text{ave}} + R = 155 \times 10^{-6} + 260 \times 10^{-6}$$

$$\varepsilon_{\max} = 415 \times 10^{-6} \text{ in./in.} \blacktriangleleft$$

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PROBLEM 7.C1

A state of plane stress is defined by the stress components σ_x, σ_y, and τ_{xy} associated with the element shown in Fig. P7.C1*a*. (*a*) Write a computer program that can be used to calculate the stress components $\sigma_{x'}\sigma_{y'}$, and $\tau_{x'y'}$ associated with the element after it has rotated through an angle θ about the z axis (Fig. P.7C1*b*). (*b*) Use this program to solve Probs. 7.13 through 7.16.

SOLUTION

Program following equations:

Equation (7.5), Page 427: $$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Equation (7.7), Page 427: $$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

Equation (7.6), Page. 427: $$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

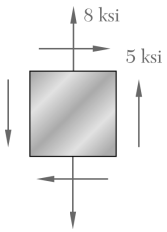
Enter $\sigma_x, \sigma_y, \tau_{xy}$ and θ

Print values obtained for $\sigma_{x'}, \sigma_{y'}$ and $\tau_{x'y'}$

Problem Outputs

Problem 7.13

$$\sigma_x = 0 \text{ ksi}$$
$$\sigma_x = 8 \text{ ksi}$$
$$\tau_{xy} = 5 \text{ ksi}$$

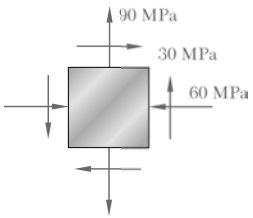


Rotation of element (+ counterclockwise) $\theta = -25°$	Rotation of element (+ counterclockwise) $\theta = 10°$
$\sigma_{x'} = -2.40$ ksi	$\sigma_{x'} = 1.95$ ksi
$\sigma_{y'} = 10.40$ ksi	$\sigma_{y'} = 6.05$ ksi
$\tau_{x'y'} = 0.15$ ksi	$\tau_{x'y'} = 6.07$ ksi

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PROBLEM 7.C1 *(Continued)*

Problem 7.14



$$\sigma_x = -60 \text{ MPa}$$
$$\sigma_y = 90 \text{ MPa}$$
$$\tau_{xy} = 30 \text{ MPa}$$

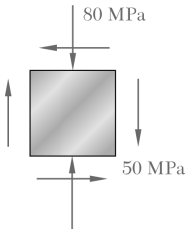
Rotation of Element (+ counterclockwise) $\theta = -25°$	Rotation of Element (+ counterclockwise) $\theta = 10°$
$\sigma_{x'} = -56.19$ MPa	$\sigma_{x'} = -45.22$ MPa
$\sigma_{y'} = 86.19$ MPa	$\sigma_{y'} = 75.22$ MPa
$\tau_{x'y'} = -38.17$ MPa	$\tau_{x'y'} = 53.84$ MPa

Problem 7.15

$$\sigma_x = 8 \text{ ksi}$$
$$\sigma_y = -12 \text{ ksi}$$
$$\tau_{xy} = -6 \text{ ksi}$$

Rotation of Element (+ counterclockwise) $\theta = -25°$	Rotation of Element (+ counterclockwise) $\theta = 10°$
$\sigma_{x'} = 9.02$ ksi	$\sigma_{x'} = 5.34$ ksi MPa
$\sigma_{y'} = -13.02$ ksi	$\sigma_{y'} = -9.34$ ksi MPa
$\tau_{x'y'} = 3.80$ ksi	$\tau_{x'y'} = -9.06$ ksi MPa

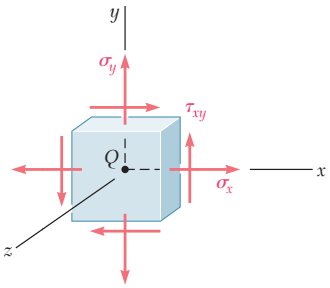
Problem 7.16



$$\sigma_x = 0 \text{ MPa}$$
$$\sigma_y = -80 \text{ MPa}$$
$$\tau_{xy} = -50 \text{ MPa}$$

Rotation of Element (+ counterclockwise) $\theta = -25°$	Rotation of Element (+ counterclockwise) $\theta = 10°$
$\sigma_{x'} = 24.01$ MPa	$\sigma_{x'} = -19.51$ MPa
$\sigma_{y'} = -104.01$ MPa	$\sigma_{y'} = -60.49$ MPa
$\tau_{x'y'} = -1.50$ MPa	$\tau_{x'y'} = -60.67$ MPa

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PROBLEM 7.C2

A state of plane stress is defined by the stress components $\sigma_x, \sigma_y,$ and τ_{xy} associated with the element shown in Fig. P7.C1*a*. (*a*) Write a computer program that can be used to calculate the principal axes, the principal stresses, the maximum in-plane shearing stress, and the maximum shearing stress. (*b*) Use this program to solve Probs. 7.5, 7.9, 7.68, and 7.69.

SOLUTION

Program following equations:

Equation (7.10) $$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2}: \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Equation (7.14) $$\sigma_{\max} = \sigma_{\text{ave}} + R$$
$$J_{\min} = \sigma_{\text{ave}} - R$$

Equation (7.12) $$\theta_p = \tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Equation (7.15) $$\theta_s = \tan^{-1} - \frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Shearing stress: If $\sigma_{\max} > 0$ and $\sigma_{\min} < 0$:

Then $$\tau_{\max(\text{in-plane})} = R; \quad \tau_{\max(\text{out-of-plane})} = R$$

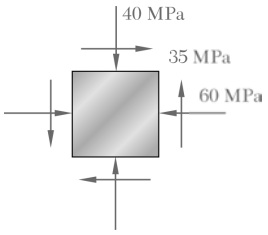
If $\sigma_{\max} > 0$ and $\sigma_{\min} > 0$:

Then $$\tau_{\max(\text{in-plane})} = R; \quad \tau_{\max(\text{out-of-plane})} = \frac{1}{2}\sigma_{\max}$$

If $\sigma_{\max} < 0$ and $\sigma_{\min} < 0$:

Then $$\tau_{\max(\text{in-plane})} = R; \quad \tau_{\max(\text{out-of-plane})} = \frac{1}{2}|\sigma_{\min}|$$

Program Outputs



Problems 7.5 and 7.9

$\sigma_x = -60.00$ MPa
$\sigma_y = -40.00$ MPa
$\tau_{xy} = 35.00$ MPa

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PROBLEM 7.C2 *(Continued)*

Program Outputs (*Continued*)

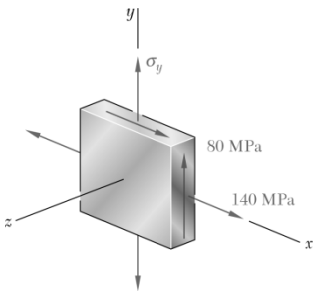
Angle between *xy* axes and principal axes (+ counterclockwise):

$$\theta_p = -37.03° \quad \text{and} \quad 52.97°$$
$$\sigma_{\max} = -13.60 \text{ MPa}$$
$$\sigma_{\min} = -86.40 \text{ MPa}$$

Angle between *xy* axis and planes of maximum in-plane shearing stress (+ counterclockwise):

$$\theta_s = 7.97° \quad \text{and} \quad 97.97°$$
$$\tau_{\max\,(\text{in-plane})} = 36.40 \text{ MPa}$$
$$\tau_{\max} = 43.20 \text{ MPa}$$

Problem 7.68



$$\sigma_x = 140.00 \text{ MPa}$$
$$\sigma_y = 40.00 \text{ MPa}$$
$$\tau_{xy} = 80.00 \text{ MPa}$$

Angle between *xy* axes and principal axes (+ counterclockwise):

$$\theta_p = 29.00° \quad \text{and} \quad 119°$$
$$\sigma_{\max} = -184.34 \text{ MPa}$$
$$\sigma_{\min} = -4.34 \text{ MPa}$$

Angle between *xy* axis and planes of maximum in-plane in-plane shearing stress (+ counterclockwise):

$$\theta_s = 74.00° \quad \text{and} \quad 164.00°$$
$$\tau_{\max\,(\text{in-plane})} = 94.34 \text{ MPa}$$
$$\tau_{\max\,(\text{out-of-plane})} = 94.34 \text{ MPa}$$
$$\sigma_x = 140.00 \text{ MPa}$$
$$\sigma_y = 120.00 \text{ MPa}$$
$$\tau_{xy} = 80.00 \text{ MPa}$$

Angle between *xy* axes and principal axes (+ counterclockwise):

$$\theta_p = 41.44° \quad \text{and} \quad 131.44°$$
$$\sigma_{\max} = 210.62 \text{ MPa}$$
$$\sigma_{\min} = 49.38 \text{ MPa}$$

Angle between *xy* axis and planes of maximum in-plane in-plane shearing stress (+ counterclockwise):

$$\theta_s = 86.44° \quad \text{and} \quad 176.44°$$
$$\tau_{\max\,(\text{in-plane})} = 80.62 \text{ MPa}$$
$$\tau_{\max\,(\text{out-of-plane})} = 105.31 \text{ MPa}$$

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PROBLEM 7.C2 *(Continued)*

Program Outputs (*Continued*)

Problem 7.69

$$\sigma_x = 140.00 \text{ MPa}$$
$$\sigma_y = 20.00 \text{ MPa}$$
$$\tau_{xy} = 80.00 \text{ MPa}$$

Angle between *xy* axes and principal axes (+ counterclockwise):

$$\theta_p = 26.57° \quad \text{and} \quad 116.57°$$
$$\sigma_{\max} = 180.00 \text{ MPa}$$
$$\sigma_{\min} = -20.00 \text{ MPa}$$

Angle between *xy* axis and planes of maximum in-plane in-plane shearing stress (+ counterclockwise):

$$\theta_s = 71.57° \quad \text{and} \quad 161.57°$$
$$\tau_{\max\,(\text{in-plane})} = 100.00 \text{ MPa}$$
$$\tau_{\max\,(\text{out-of-plane})} = 100.00 \text{ MPa}$$
$$\sigma_x = 140.00 \text{ MPa}$$
$$\sigma_y = 140.00 \text{ MPa}$$
$$\tau_{xy} = 80.00 \text{ MPa}$$

Angle between *xy* axes and principal axes (+ counterclockwise):

$$\theta_p = 45° \quad \text{and} \quad 135.00°$$
$$\sigma_{\max} = 220.00 \text{ MPa}$$
$$\sigma_{\min} = 60.00 \text{ MPa}$$

Angle between *xy* axis and planes of maximum in-plane in-plane shearing stress (+ counterclockwise):

$$\theta_s = 90.00° \quad \text{and } 180.00°$$
$$\tau_{\max\,(\text{in-plane})} = 80.00 \text{ MPa}$$
$$\tau_{\max\,(\text{out-of-plane})} = 110.00 \text{ MPa}$$

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PROBLEM 7.C3

(*a*) Write a computer program that, for a given state of plane stress and a given yield strength of a ductile material, can be used to determine whether the material will yield. The program should use both the maximum shearing-strength criterion and the maximum-distortion-energy criterion. It should also print the values of the principal stresses and, if the material does not yield, calculate the factor of safety. (*b*) Use this program to solve Probs. 7.81, 7.82, and 7.164.

SOLUTION

Principal stresses.

$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2}; \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R$$

$$\sigma_b = \sigma_{\text{ave}} - R$$

Maximum-shearing-stress criterion. $\tau_y = \frac{1}{2}\sigma_y$

If σ_a and τ_b have same sign, $\tau_{\max} = \frac{1}{2}\sigma_a$

If $\tau_{\max} > \tau_y$, yielding occurs.

If $\tau_{\max} < \tau_y$, no yielding occurs, and factor of safety $= \dfrac{\tau_y}{\tau_{\max}}$

Maximum-distortion-energy criterion.

Compute radical $= \sqrt{\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2}$

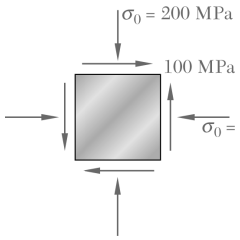
If radical $> \sigma_y$, yielding occurs.

If radical $< \sigma_y$, no yielding occurs, and factor of safety $= \dfrac{\sigma_y}{\text{Radical}}$

Program Outputs

Problems 7.81a and 7.82a Yield strength = 325 MPa

$\sigma_x = -200.00$ MPa
$\sigma_y = -200.00$ MPa
$\tau_{xy} = 100.00$ MPa
$\sigma_{\max} = -100.00$ MPa
$\sigma_{\min} = -300.00$ MPa



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PROBLEM 7.C3 *(Continued)*

Program Outputs (*Continued*)

Using the maximum-shearing-stress criterion,

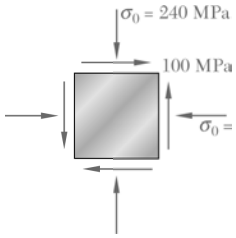
material will not yield.

$$F.S. = 1.083$$

Using the maximum-distortion-energy criterion,

material will not yield.

$$F.S. = 1.228$$



Problems 7.81b and 7.82b Yield strength = 325 MPa

$$\sigma_x = -240.00 \text{ MPa}$$
$$\sigma_y = -240.00 \text{ MPa}$$
$$\tau_{xy} = 100.00 \text{ MPa}$$
$$\sigma_{\max} = -140.00 \text{ MPa}$$
$$\sigma_{\min} = -340.00 \text{ MPa}$$

Using the maximum-shearing-stress criterion,

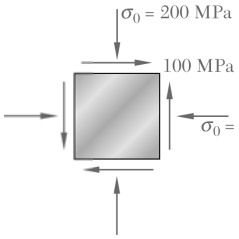
material will yield.

Using the maximum-distortion-energy criterion,

material will not yield.

$$F.S. = 1.098$$

Problems 7.81c and 7.82c Yield strength = 325 MPa



$$\sigma_x = -280.00 \text{ MPa}$$
$$\sigma_y = -280.00 \text{ MPa}$$
$$\tau_{xy} = 100.00 \text{ MPa}$$
$$\sigma_{\max} = -180.00 \text{ MPa}$$
$$\sigma_{\min} = -380.00 \text{ MPa}$$

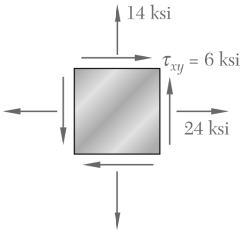
Using the maximum-shearing-stress criterion,

material will yield.

Using the maximum-distortion-energy criterion,

material will yield.

Problem 7.164a Yield strength = 30 ksi



$$\sigma_x = 24.00 \text{ ksi}$$
$$\sigma_y = 14.00 \text{ ksi}$$
$$\tau_{xy} = 6.00 \text{ ksi}$$
$$\sigma_{\max} = 26.81 \text{ ksi}$$
$$\sigma_{\min} = 11.19 \text{ ksi}$$

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PROBLEM 7.C3 *(Continued)*

Program Outputs (*Continued*)

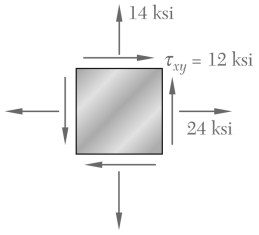
(a) Using the maximum-shearing-stress criterion, material will not yield.

$$F.S. = 1.119$$

(b) Using the maximum-distortion-energy criterion, material will not yield.

$$F.S. = 1.286$$

Problem 7.164b Yield strength = 30 ksi



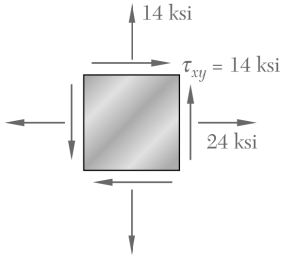
$$\sigma_x = 24.00 \text{ ksi}$$
$$\sigma_y = 14.00 \text{ ksi}$$
$$\tau_{xy} = 12.00 \text{ ksi}$$
$$\sigma_{\max} = 32.00 \text{ ksi}$$
$$\sigma_{\min} = 6.00 \text{ ksi}$$

(a) Using the maximum-shearing-stress criterion, material will yield.

(b) Using the maximum-distortion-energy criterion, material will not yield.

$$F.S. = 1.018$$

Problem 7.164c Yield strength = 30 ksi



$$\sigma_x = 24.00 \text{ ksi}$$
$$\sigma_y = 14.00 \text{ ksi}$$
$$\tau_{xy} = 14.00 \text{ ksi}$$
$$\sigma_{\max} = 33.87 \text{ ksi}$$
$$\sigma_{\min} = 4.13 \text{ ksi}$$

(a) Using the maximum-shearing-stress criterion, material will yield.

(b) Using the maximum-distortion-energy criterion, material will yield.

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PROBLEM 7.C4

(*a*) Write a computer program based on Mohr's fracture criterion for brittle materials that, for a given state of plane stress and given values of the ultimate strength of the material in tension and compression, can be used to determine whether rupture will occur. The program should also print the values of the principal stresses. (*b*) Use this program to solve Probs. 7.89 and 7.90 and to check the answers to Probs. 7.93 and 7.94.

SOLUTION

Principal stresses.

$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R$$

$$\sigma_b = \sigma_{\text{ave}} - R$$

Mohr's fracture criterion.

If σ_a and σ_b have same sign, and

$\sigma_a < \sigma_{\text{UT}}$ and $\sigma_b < \sigma_{\text{UC}}$, no failure;

$\sigma_a < \sigma_{\text{UT}}$ or $\sigma_b > \sigma_{\text{UC}}$, failure.

If $\sigma_a > 0$ and $\sigma_b < 0$:

Consider fourth quadrant of Figure 7.47.

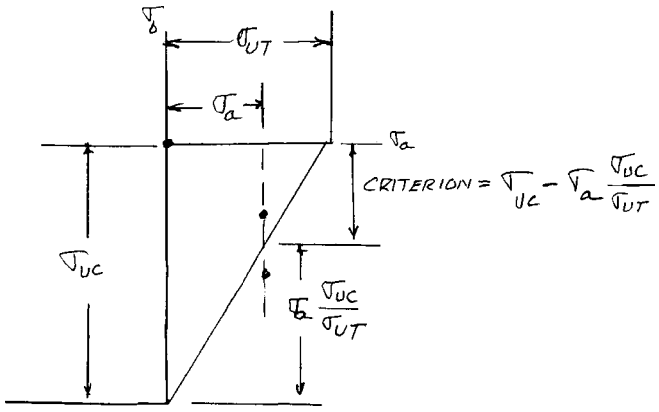
For no rupture to occur, point (σ_a, σ_b) must lie within Mohr's envelope (Figure 7.47).

If $\sigma_b >$ Criterion,

then rupture occurs.

If $\sigma_b <$ Criterion,

then no rupture occurs.

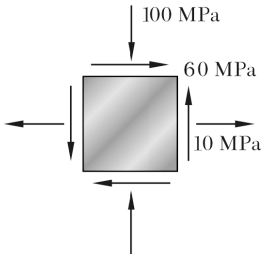


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PROBLEM 7.C4 *(Continued)*

Program Outputs

Problem 7.89



$\sigma_x = 10.00$ MPa

$\sigma_y = -100.00$ MPa

$\tau_{xy} = 60$ MPa

Ultimate strength in tension $= 80$ MPa

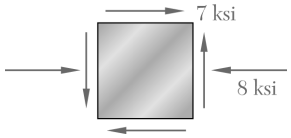
Ultimate strength in compression $= 200$ MPa

$\sigma_{\max} = \sigma_a \quad 36.39$ MPa

$\sigma_{\min} = \sigma_b \quad -126.39$ MPa

Rupture will occur

Problem 7.90



$\sigma_x = -32.00$ MPa

$\sigma_y = 0.00$ MPa

$\tau_{xy} = 75.00$ MPa

Ultimate strength in tension $= 80$ MP

Ultimate strength in compression $= 200$ MP

$\sigma_{\max} = \sigma_a \quad 60.69$ MPa

$\sigma_{\min} = \sigma_b \quad -92.69$ MPa

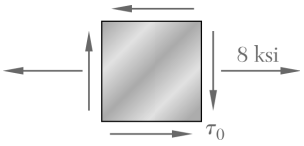
Rupture will not occur.

To check answers to the following problems, we check for rupture using given answers and an adjacent value.

Answer:

Rupture occurs for $\tau_0 = 3.67$ ksi.

Problem 7.93



$\sigma_x = 8.00$ ksi

$\sigma_y = 0.00$ ksi

$\rightarrow \tau_{xy} = 3.67$ ksi $\leftarrow$

Ultimate strength in tension $= 10$ ksi

Ultimate strength in compression $= 25$ ksi

$\sigma_{\max} = \sigma_a \quad 9.43$ ksi

$\sigma_{\min} = \sigma_b \quad -1.43$ ksi

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PROBLEM 7.C4 *(Continued)*

Program Outputs (*Continued*)

Rupture will not occur.

$$\sigma_x = 8.00 \text{ ksi}$$
$$\sigma_y = 0.00 \text{ ksi}$$
$$\rightarrow \tau_{xy} = 3.68 \text{ ksi} \leftarrow$$

Ultimate strength in tension $= 10$ ksi

Ultimate strength in compression $= 25$ ksi

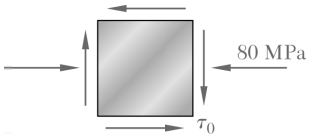
$$\sigma_{\max} = \sigma_a \quad 9.44 \text{ ksi}$$
$$\sigma_{\min} = \sigma_b \; -1.44 \text{ ksi}$$

Rupture will occur.

Answer:

Rupture occurs for $\tau_0 = 49.1$ MPa.

Problem 7.94



$$\sigma_x = -80.00 \text{ MPa}$$
$$\sigma_y = 0.00 \text{ MPa}$$
$$\rightarrow \tau_{xy} = 49.10 \text{ MPa} \leftarrow$$

Ultimate strength in tension $= 75$ MPa

Ultimate strength in compression $= 150$ MPa

$$\sigma_{\max} = \sigma_a = \; 23.33 \text{ MPa}$$
$$\sigma_{\min} = \sigma_b = -103.33 \text{ MPa}$$

Rupture will not occur.

$$\sigma_x = -80.00 \text{ MPa}$$
$$\sigma_y = 0.00 \text{ MPa}$$
$$\rightarrow \tau_{xy} = 49.20 \text{ MPa} \leftarrow$$

Ultimate strength in tension $= 75$ MPa

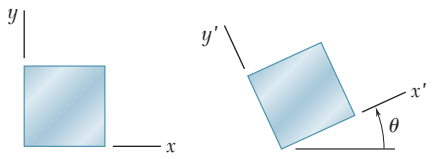
Ultimate strength in compression $= 150$ MPa

$$\sigma_{\max} = \sigma_a = 23.41 \text{ MPa}$$
$$\sigma_{\min} = \sigma_b = -103.41 \text{ MPa}$$

Rupture will occur.

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PROBLEM 7.C5



A state of plane strain is defined by the strain components ε_x, ε_y, and γ_{xy} associated with the x and y axes. (*a*) Write a computer program that can be used to calculate the strain components $\varepsilon_{x'}$, $\varepsilon_{y'}$, and $\gamma_{x'y'}$ associated with the frame of reference $x'y'$ obtained by rotating the x and y axes through an angle θ. (*b*) Use this program to solve Probs 7.129 and 7.131.

SOLUTION

Program following equations:

Equation (7.44): $$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta$$

Equation (7.45): $$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta - \frac{1}{2}\gamma_{xy}\cos 2\theta$$

Equation (7.46): $$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

Enter ε_x, ε_y, γ_{xy}, and θ.

Print values obtained for $\varepsilon_{x'}$, $\varepsilon_{y'}$, and $\gamma_{xy'}$.

Program Outputs

Problem 7.129

$\varepsilon_x = 240$ micro meters
$\varepsilon_y = 160$ micro meters
$\gamma_{xy} = 150$ micro radians

Rotation of element, in degrees (+ counterclockwise):

$\theta = -60°$
$\varepsilon_{x'} = 115.05$ micro meters
$\varepsilon_{y'} = 284.95$ micro meters
$\gamma_{x'y'} = -5.72$ micro radians

Problem 7.131

$\varepsilon_x = 0$ micro meters
$\varepsilon_y = 320$ micro meters
$\gamma_{xy} = -100$ micro radians

Rotation of element, in degrees (+ counterclockwise):

$\theta = 30°$
$\varepsilon_{x'} = 36.70$ micro meters
$\varepsilon_{y'} = 283.30$ micro meters
$\gamma_{x'y'} = 227.13$ micro radians

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PROBLEM 7.C6

A state of strain is defined by the strain components $\varepsilon_x, \varepsilon_y,$ and γ_{xy} associated with the x and y axes. (*a*) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (*b*) Use this program to solve Probs 7.136 through 7.139.

SOLUTION

Program following equations:

Equation (7.50): $$\varepsilon_{\text{ave}} = \frac{\varepsilon_x + \varepsilon_y}{2} \quad R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Equation (7.51): $$\varepsilon_{\max} = \varepsilon_{\text{ave}} + R \quad \varepsilon_{\min} = \varepsilon_{\text{ave}} - R$$

Equation (7.52): $$\theta_p = \tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

Shearing strains: Maximum in-plane shearing strain

$$\gamma_{\max\,(\text{in-plane})} = 2R$$

Calculate out-of-plane shearing strain and check whether it is the maximum shearing strain.

Let $\varepsilon_a = \varepsilon_{\max}$

$\varepsilon_b = \varepsilon_{\min}$

Calculate $$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b)$$

If $\varepsilon_a > \varepsilon_b > \varepsilon_c$, $\gamma_{\text{out-of-plane}} = \varepsilon_a - \varepsilon_c$

If $\varepsilon_a > \varepsilon_c > \varepsilon_b$, $\gamma_{\text{out-of-plane}} = \varepsilon_a - \varepsilon_b = 2R$

If $\varepsilon_c > \varepsilon_a > \varepsilon_b$, $\gamma_{\text{out-of-plane}} = \varepsilon_c - \varepsilon_b$

Program Printout

Problem 7.136

$\varepsilon_x = -260$ micro meters

$\varepsilon_y = -60$ micro meters

$\gamma_{xy} = 480$ micro radians

$\nu = 0.333$

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PROBLEM 7.C6 *(Continued)*

Program Printout (*Continued*)

Angle between *xy* axes and principal axes (+ = counterclockwise):

$\theta_p = -33.69°$

$\varepsilon_a = 100.00$ micro meters

$\varepsilon_b = -420.00$ micro meters

$\varepsilon_c = 159.98$ micro meters

$\gamma_{\max(\text{in-plane})} = 520.00$ microradians

$\gamma_{\max} = 579.98$ microradians

Problem 7.137

$\varepsilon_x = -600$ micrometers

$\varepsilon_y = -400$ micrometers

$\gamma_{xy} = 350$ microradians

$\nu = 0.333$

Angle between *xy* axes and principal axes (+ = counterclockwise):

$\theta_p = -30.13°$

$\varepsilon_a = -298.44$ micrometers

$\varepsilon_b = -701.56$ micrometers

$\varepsilon_c = 500.00$ micrometers

$\gamma_{\max(\text{in-plane})} = 403.11$ microradians

$\gamma_{\max} = 1201.56$ microradians

Problem 7.138

$\varepsilon_x = 160$ micrometers

$\varepsilon_y = -480$ micrometers

$\gamma_{xy} = -600.00$ microradians

$\nu = 0.333$

Angle between *xy* axes and principal axes (+ = counterclockwise):

$\theta_p = -21.58°$

$\varepsilon_a = 278.63$ micrometers

$\varepsilon_b = -598.63$ micrometers

$\varepsilon_c = 159.98$ micrometers

$\gamma_{\max(\text{in-plane})} = 877.27$ microradians

$\gamma_{\max} = 877.27$ microradians

Problem 7.139

$\varepsilon_x = 30$ micrometers

$\varepsilon_y = 570$ micrometers

$\gamma_{xy} = 720$ microradians

$\nu = 0.333$

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PROBLEM 7.C6 *(Continued)*

Angle between *xy* axes and principal axes (+ = counterclockwise):

$$\theta_p = -26.57°$$

$$\varepsilon_a = 750.00 \text{ micrometers}$$

$$\varepsilon_b = -150.00 \text{ micrometers}$$

$$\varepsilon_c = -300.00 \text{ micrometers}$$

$$\gamma_{\max(\text{in-plane})} = 900.00 \text{ microradians}$$

$$\gamma_{\max} = 1050.00 \text{ microradians}$$

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PROBLEM 7.C7

A state of plane strain is defined by the strain components ε_x, ε_y, and γ_{xy} measured at a point. (*a*) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the magnitude of the shearing strain. (*b*) Use this program to solve Probs 7.140 through 7.143.

SOLUTION

Program following equations:

Equation (7.50) $$\varepsilon_{\text{ave}} = \frac{\varepsilon_x + \varepsilon_y}{2} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Equation (7.51) $$\varepsilon_{\max} = \varepsilon_{\text{ave}} + R \quad \varepsilon_{\min} = \varepsilon_{\text{ave}} - R$$

Equation (7.52) $$\theta_p = \tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

Shearing strains: Maximum in-plane shearing strain

$$\gamma_{xy(\text{in-plane})} = 2R$$

Calculate out-of-plane-shearing strain and check whether it is the maximum shearing strain.

Let $\varepsilon_a = \varepsilon_{\max}$

$\varepsilon_b = \varepsilon_{\min}$

$\varepsilon_c = 0$ (Plain strain)

If $\varepsilon_a > \varepsilon_b > \varepsilon_c$, $\gamma_{\text{out-of-plane}} = \varepsilon_a - \varepsilon_c$

If $\varepsilon_a > \varepsilon_c > \varepsilon_b$, $\gamma_{\text{out-of-plane}} = \varepsilon_a - \varepsilon_b = 2R$

If $\varepsilon_c > \varepsilon_a > \varepsilon_b$, $\gamma_{\text{out-of-plane}} = \varepsilon_c - \varepsilon_b$

Program Printout

Problem 7.140

$\varepsilon_x = 60$ micrometers

$\varepsilon_y = 240$ micrometers

$\gamma_{xy} = -50$ microradians

$\nu = 0.000$

Angle between *xy* axes and principal axes (+ = counterclockwise):

$\theta_p = 7.76$ and $-82.24°$

$\varepsilon_a = 243.41$ micrometers

$\varepsilon_b = 56.59$ micrometers

$\varepsilon_c = 0.00$ micrometers

$\gamma_{\max(\text{in-plane})} = 186.82$ microradians

$\gamma_{\max} = 243.41$ microradians

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PROBLEM 7.C7 *(Continued)*

Program Printout (*Continued*)

Problem 7.141

$\varepsilon_x = 400$ micrometers
$\varepsilon_y = 200$ micrometers
$\gamma_{xy} = 375$ microradians
$\nu = 0.000$

Angle between *xy* axes and principal axes (+ = counterclockwise):

$\theta_p = 30.96$ and $-59.04°$
$\varepsilon_a = 512.50$ micrometers
$\varepsilon_b = 87.50$ micrometers
$\varepsilon_c = 0.00$ micrometers
$\gamma_{\max(\text{in-plane})} = 425.00$ microradians
$\gamma_{\max} = 512.50$ microradians

Problem 7.142

$\varepsilon_x = 300$ micrometers
$\varepsilon_y = 60$ micrometers
$\gamma_{xy} = 100$ microradians
$\nu = 0.000$

Angle between *xy* axes and principal axes (+ = counterclockwise):

$\theta_p = 11.31$ and $-78.69°$
$\varepsilon_a = 310.00$ micrometers
$\varepsilon_b = 50.00$ micrometers
$\varepsilon_c = 0.00$ micrometers
$\gamma_{\max(\text{in-plane})} = 260.00$ microradians
$\gamma_{\max} = 310.00$ microradians

Problem 7.143

$\varepsilon_x = -180$ micrometers
$\varepsilon_y = -260$ micrometers
$\gamma_{xy} = 315$ microradians
$\nu = 0.000$

Angle between *xy* axes and principal axes (+ = counterclockwise):

$\theta_p = 37.87$ and $-52.13°$
$\varepsilon_a = -57.50$ micrometers
$\varepsilon_b = -382.50$ micrometers
$\varepsilon_c = 0.00$ micrometers
$\gamma_{\max(\text{in-plane})} = 325.00$ microradians
$\gamma_{\max} = 382.50$ microradians

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PROBLEM 7.C8

A rosette consisting of three gages forming, respectively, angles of θ_1, θ_2, and θ_3 with the x axis is attached to the free surface of a machine component made of a material with a given Poisson's ratio v. (*a*) Write a computer program that, for given readings ε_1, ε_2, and ε_3 of the gages, can be used to calculate the strain components associated with the x and y axes and to determine the orientation and magnitude of the three principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (*b*) Use this program to solve Probs 7.144, 7.145, 7.146, and 7.169.

SOLUTION

For $n = 1$ to 3, enter θ_n and ε_n.

Enter: $NU = V$

Solve Equation (7.60) for ε_x, ε_y, and γ_{xy} using method of determinates or any other method.

Enter
$$\varepsilon_{\text{ave}} = \frac{\varepsilon_x + \varepsilon_y}{2}; \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \gamma_{xy}^2}$$
$$\varepsilon_a = \varepsilon_{\max} = \varepsilon_{\text{ave}} + R$$
$$\varepsilon_b = \varepsilon_{\max} - \varepsilon_{\text{avg}} - R$$
$$\varepsilon_c = -\frac{V}{1-V}(\varepsilon_a + \varepsilon_b)$$
$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

Shearing strains: Maximum in-plane shearing strain

$$\gamma_{\max\,(\text{in-plane})} = 2R$$

Calculate out-of-plane shearing strain, and check whether it is the maximum shearing strain.

If $\varepsilon_c < \varepsilon_b$, $\gamma_{\text{out-of-plane}} = \varepsilon_a - \varepsilon_c$

If $\varepsilon_c > \varepsilon_a$, $\gamma_{\text{out-of-plane}} = \varepsilon_c - \varepsilon_b$

Otherwise, $\gamma_{\text{out-of-plane}} = 2R$

Problem Outputs

Problem 7.144

Gage	Theta Degrees	Epsilon Micro Meters
1	–15	480
2	30	–120
3	75	80

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PROBLEM 7.C8 *(Continued)*

Program Outputs (*Continued*)

$$\varepsilon_x = 253.21 \text{ micrometers}$$
$$\varepsilon_y = 306.79 \text{ micrometers}$$
$$\gamma_{xy} = -892.82 \text{ microradians}$$
$$\varepsilon_a = 727.21 \text{ micrometers}$$
$$\varepsilon_b = -167.21 \text{ micrometers}$$
$$\gamma_{\text{max (in-plane)}} = 894.43 \text{ microradians}$$

Problem 7.145

Gage	Theta Degrees	Epsilon Micro Meters
1	30	600
2	–30	450
3	90	–75

$$\varepsilon_x = 725.00 \text{ micrometers}$$
$$\varepsilon_y = -75.000 \text{ micrometers}$$
$$\gamma_{xy} = 173.205 \text{ microradians}$$
$$\varepsilon_a = 734.268 \text{ micrometers}$$
$$\varepsilon_b = -84.268 \text{ micrometers}$$
$$\gamma_{\text{max (in-plane)}} = 818.535 \text{ microradians}$$

Problem 7.146

Observe that Gage 3 is orientated along the *y* axis. Therefore,

enter θ_4 and ε_4 as θ_3 and ε_3, the value of ε_y that is obtained is also the expected reading of Gage 3.

Gage	Theta Degrees	Epsilon in./in.
1	0	420
2	45	–45
4	135	165

$$\varepsilon_x = 420.00 \text{ in./in.}$$
$$\varepsilon_y = -300.00 \text{ in./in.}$$
$$\gamma_{xy} = -210.00 \text{ microradians}$$
$$\varepsilon_a = 435.00 \text{ in./in.}$$
$$\varepsilon_b = -315.00 \text{ in./in.}$$
$$\gamma_{\text{max (in-plane)}} = 750.00 \text{ microradians}$$

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PROBLEM 7.C8 *(Continued)*

Program Outputs (*Continued*)

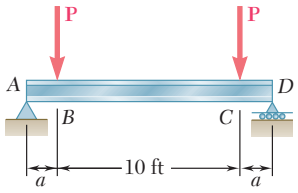
Problem 7.169

Gage	Theta Degrees	Epsilon in./in.
1	45	–50
2	–45	360
3	0	315

$$\varepsilon_x = 315.000 \text{ in./in.}$$
$$\varepsilon_y = -5.000 \text{ in./in.}$$
$$\gamma_{xy} = -410.000 \text{ microradians}$$
$$\varepsilon_a = 415.048 \text{ in./in.}$$
$$\varepsilon_b = -105.048 \text{ in./in.}$$
$$\gamma_{\max\ (\text{in-plane})} = 520.096 \text{ microradians}$$

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CHAPTER 8

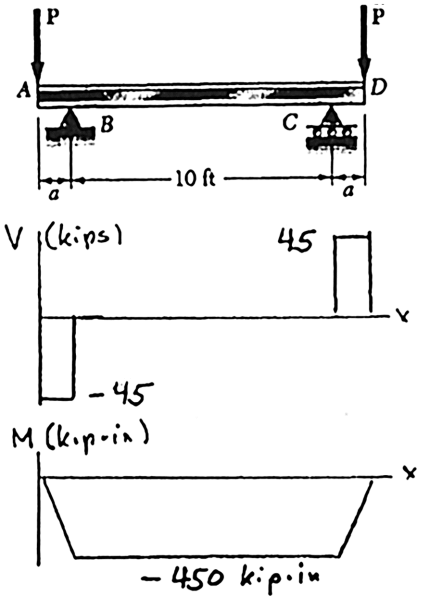


PROBLEM 8.1

A W10×39 rolled-steel beam supports a load **P** as shown. Knowing that $P = 45$ kips, $a = 10$ in., and $\sigma_{all} = 18$ ksi, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION

$$|V|_{max} = 90 \text{ kips}$$
$$|M|_{max} = (45)(10) = 450 \text{ kip} \cdot \text{in.}$$



For W10 × 39 rolled steel section,

$$d = 9.92 \text{ in.}, \quad b_f = 7.99 \text{ in.}, \quad t_f = 0.530 \text{ in.},$$
$$t_w = 0.315 \text{ in.}, \quad I_x = 209 \text{ in}^4 \quad S_x = 42.1 \text{ in}^3$$
$$c = \frac{1}{2}d = 4.96 \text{ in.} \qquad y_b = c - t_f = 4.43 \text{ in.}$$

(*a*) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{450}{42.1} \qquad \sigma_m = 10.69 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{4.43}{4.96}\right)(10.69) = 9.55 \text{ ksi}$$
$$A_f = b_f t_f = 4.2347 \text{ in}^2$$
$$\bar{y}_f = \frac{1}{2}(c + y_b) = 4.695 \text{ in.}$$
$$Q_b = A_f \bar{y}_f = 19.8819 \text{ in}^3$$
$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(45)(19.8819)}{(209)(0.315)} = 13.5898 \text{ ksi}$$
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 14.4043 \text{ ksi}$$

(*b*) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 19.18 \text{ ksi} \qquad \sigma_{max} = 19.18 \text{ ksi} \blacktriangleleft$$

(*c*) Since $\sigma_{max} > \sigma_{all}$ (= 18 ksi), W10 × 39 is not acceptable. ◀

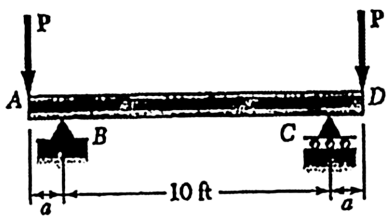
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PROBLEM 8.2

Solve Prob. 8.1, assuming that $P = 22.5$ kips and $a = 20$ in.

PROBLEM 8.1 A $W10 \times 39$ rolled-steel beam supports a load **P** as shown. Knowing that P = 45 kips, $a = 10$ in., and $\sigma_{all} = 18$ ksi, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

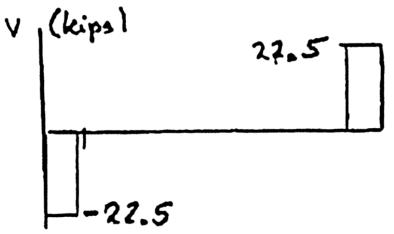
SOLUTION

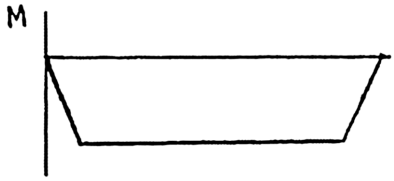


$$|V|_{max} = 22.5 \text{ kips}$$
$$|M|_{max} = (22.5)(20) = 450 \text{ kip} \cdot \text{in.}$$

For $W10 \times 39$ rolled steel section,

$$d = 9.92 \text{ in.}, \quad b_f = 7.99 \text{ in.}, \quad t_f = 0.530 \text{ in.},$$
$$t_w = 0.315 \text{ in.}, \quad I_x = 209 \text{ in}^4, \quad S_x = 42.1 \text{ in}^3$$
$$c = \frac{1}{2}d = 4.96 \text{ in.} \quad y_b = c - t_f = 4.43 \text{ in.}$$





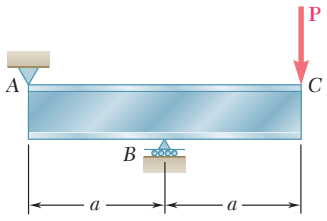
(*a*) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{450}{42.1}$$ $\sigma_m = 10.69$ ksi ◀

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{4.43}{4.96}\right)(10.69) = 9.55 \text{ ksi}$$
$$A_f = b_f t_f = 4.2347 \text{ in}^2$$
$$\bar{y}_f = \frac{1}{2}(c + y_b) = 4.695 \text{ in.}$$
$$Q_b = A_f \bar{y}_f = 19.8819 \text{ in}^3$$
$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(22.5)(19.8819)}{(209)(0.315)} = 6.7949 \text{ ksi}$$
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 8.3049 \text{ ksi}$$

(*b*) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 13.08 \text{ ksi}$$ $\sigma_{max} = 13.08$ ksi ◀

(*c*) Since $\sigma_{max} < \sigma_{all}$ (= 18 ksi), $W10 \times 39$ is acceptable. ◀

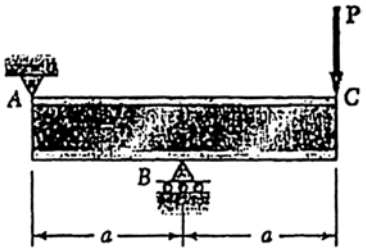
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PROBLEM 8.3

An overhanging W920×449 rolled-steel beam supports a load **P** as shown. Knowing that $P = 700$ kN, $a = 2.5$ m, and $\sigma_{all} = 100$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION



$$|V|_{max} = 700 \text{ kN} = 700 \times 10^3 \text{ N}$$

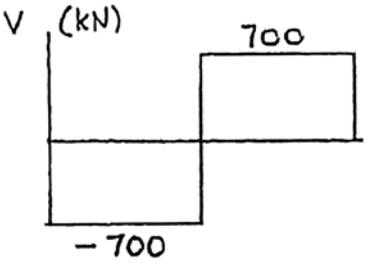
$$|M|_{max} = (700 \times 10^3)(2.5) = 1.75 \times 10^6 \text{ N} \cdot \text{m}$$

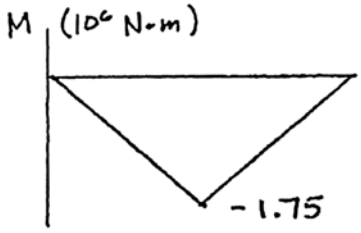
For W920×449 rolled steel beam,

$$d = 947 \text{ mm}, \quad b_f = 424 \text{ mm}, \quad t_f = 42.7 \text{ mm},$$

$$t_w = 24.0 \text{ mm}, \quad I_x = 8780 \times 10^6 \text{ mm}^4, \quad S_x = 18{,}500 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2} d = 473.5 \text{ mm}, \quad y_b = c - t_f = 430.8 \text{ mm}$$





(*a*) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{1.75 \times 10^6}{18{,}500 \times 10^{-6}}$$

$$\sigma_m = 94.595 \text{ MPa}$$

$$\sigma_m = 94.6 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{430.8}{473.5}(94.595) = 86.064 \text{ MPa}$$

$$A_f = b_f t_t = 18.1048 \times 10^3 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 452.15 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 8186.1 \times 10^3 \text{ mm}^3 = 8186.1 \times 10^{-6} \text{ m}^3$$

$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(700 \times 10^3)(8186.1 \times 10^{-6})}{(8780 \times 10^{-6})(24.0 \times 10^{-3})} = 27.194 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{86.064}{2}\right)^2 + 27.194^2} = 50.904 \text{ MPa}$$

(*b*) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 93.9 \text{ MPa}$$

$$\sigma_{max} = 93.9 \text{ MPa} \blacktriangleleft$$

(*c*) Since 94.6 MPa $< \sigma_{all}$ (= 100 MPa), W920×449 is acceptable. ◀

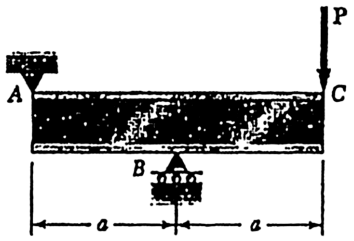
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PROBLEM 8.4

Solve Prob. 8.3, assuming that $P = 850$ kN and $a = 2.0$ m.

PROBLEM 8.3 An overhanging W920 × 449 rolled-steel beam supports a load **P** as shown. Knowing that $P = 700$ kN, $a = 2.5$ m, and $\sigma_{all} = 100$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION



$$|V|_{max} = 850 \text{ kN} = 850 \times 10^3 \text{ N}$$

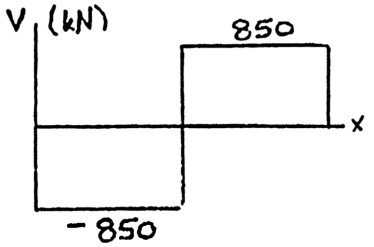
$$|M|_{max} = (850 \times 10^3)(2.0) = 1.70 \times 10^6 \text{ N} \cdot \text{m}$$

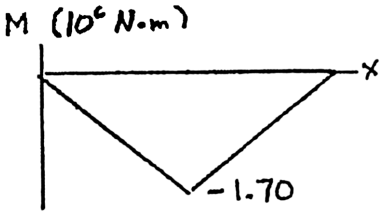
For W920 × 449 rolled steel section,

$$d = 947 \text{ mm}, \quad b_f = 424 \text{ mm}, \quad t_f = 42.7 \text{ mm},$$

$$t_w = 24.0 \text{ mm}, \quad I_x = 8780 \times 10^6 \text{ mm}^4, \quad S_x = 18{,}500 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2} d = 473.5 \text{ mm} \qquad y_b = c - t_f = 430.8 \text{ mm}$$





(*a*) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{1.70 \times 10^6}{18{,}500 \times 10^{-6}}$$

$\sigma_m = 91.892$ MPa

$\sigma_m = 91.9$ MPa ◀

$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{430.8}{473.5}(91.892) = 83.605 \text{ MPa}$$

$$A_f = b_f t_f = 18.1048 \times 10^3 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 452.15 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 8186.1 \times 10^3 \text{ mm}^3 = 8186.1 \times 10^{-6} \text{ m}^3$$

$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(850 \times 10^3)(8186.1 \times 10^{-6})}{(8780 \times 10^{-6})(24.0 \times 10^{-3})} = 33.021 \text{ MPa}$$

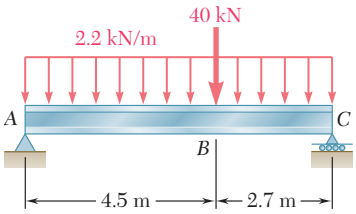
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{83.605}{2}\right)^2 + 33.021^2} = 53.271 \text{ MPa}$$

(*b*) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 95.1 \text{ MPa}$$

$\sigma_{max} = 95.1$ MPa ◀

(*c*) Since 95.1 MPa < σ_{all} (= 100 MPa), W920 × 449 is acceptable. ◀

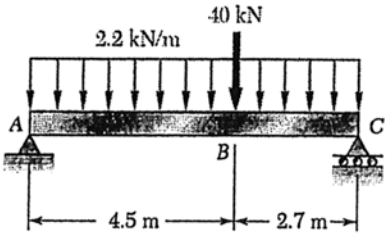
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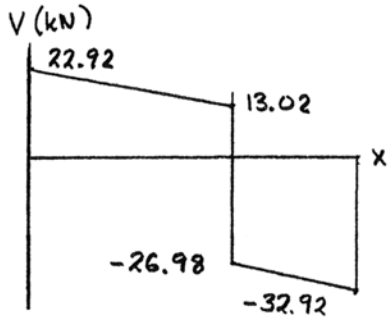


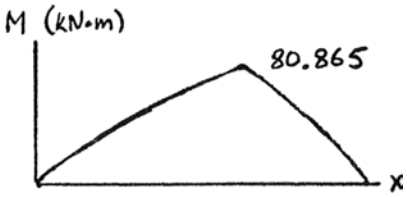
PROBLEM 8.5

(*a*) Knowing that $\sigma_{all} = 160$ MPa and $\tau_{all} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION







$$+\sum M_C = 0: \quad -7.2R_A + (2.2)(7.2)(3.6) + (40)(2.7) = 0$$

$$R_A = 22.92 \text{ kN}$$

$$V_A = R_A = 22.92 \text{ kN}$$

$$V_B^- = 22.92 - (2.2)(4.5) = 13.02 \text{ kN}$$

$$V_B^+ = 13.02 - 40 = -26.98 \text{ kN}$$

$$V_C = -26.98 - (2.2)(2.7) = -32.92 \text{ kN}$$

$$M_A = 0$$

$$M_B = 0 + \frac{1}{2}(22.92 + 13.02)(4.5) = 80.865 \text{ kN} \cdot \text{m}$$

$$M_C = 0$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{80.865 \times 10^3}{165 \times 10^6} = 490 \times 10^{-6} \text{ m}^3$$

$$= 490 \times 10^3 \text{ mm}^3$$

Shape	S (10^3 mm^3)	
W360 × 39	578	
W310 × 38.7	547	←
W250 × 44.8	531	
W200 × 52	511	

(*a*) Use W310 × 38.7. ◀

$d = 310$ mm $\quad t_f = 9.65$ mm

$t_w = 5.84$ mm

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PROBLEM 8.5 *(Continued)*

(*b*) $\sigma_m = \dfrac{M_B}{S} = \dfrac{80.865 \times 10^3}{547 \times 10^{-6}} = 147.834 \times 10^6 \text{ Pa}$

$\sigma_m = 147.8 \text{ MPa}$ ◀

$\tau_m = \dfrac{|V|_{\max}}{dt_w} = \dfrac{32.92 \times 10^3}{(310 \times 10^{-3})(5.84 \times 10^{-3})} = 18.1838 \times 10^6 \text{ Pa}$

$\tau_m = 18.18 \text{ MPa}$ ◀

$c = \dfrac{1}{2}d = 155 \text{ mm} \qquad y_b = c - t_f = 155 - 9.65 = 145.35 \text{ mm}$

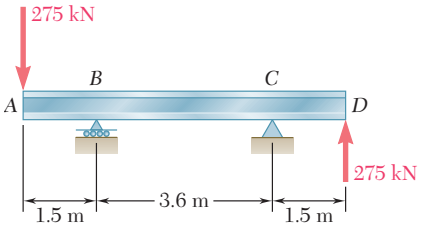
$\sigma_b = \dfrac{y_b}{c}\sigma_m = \left(\dfrac{145.35}{155}\right)(147.834) = 138.630 \text{ MPa}$

At point *B*, $\tau_w = \dfrac{V}{dt_w} = \dfrac{(26.98 \times 10^3)}{(310 \times 10^{-3})(5.84 \times 10^{-3})} = 14.9028 \text{ MPa}$

$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(69.315)^2 + (14.9028)^2} = 70.899 \text{ MPa}$

$\sigma_{\max} = \dfrac{\sigma_b}{2} + R = 69.315 + 70.899$ $\sigma_{\max} = 140.2 \text{ MPa}$ ◀

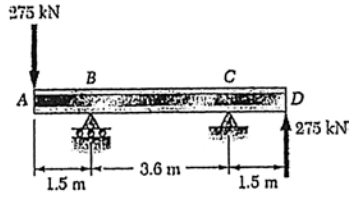
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PROBLEM 8.6

(*a*) Knowing that $\sigma_{all} = 160$ MPa and $\tau_{all} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION

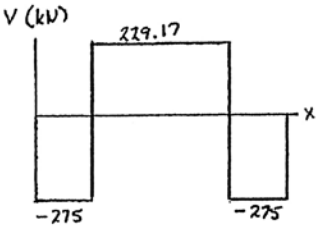


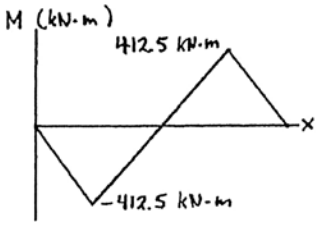
$$R_B = 504.17 \text{ kN} \uparrow \qquad R_C = 504.17 \downarrow$$

$$|V|_{max} = 275 \text{ kN} \qquad |M|_{max} = 412.5 \text{ kN} \cdot \text{m}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{412.5 \times 10^2}{160 \times 10^6} = 2578 \times 10^{-6} \text{ m}^3$$

$$= 2578 \times 10^3 \text{ mm}^3$$





→

Shape	$S_x(10^3 \text{ mm}^3)$
W760×147	4410
W690×125	3490
W530×150	3720
W460×158	3340
W360×216	3800

(*a*) Use W690 × 125. ◀

$d = 678$ mm $\quad t_f = 16.3$ mm $\quad t_w = 11.7$ mm

(*b*) $$\sigma_m = \frac{|M|_{max}}{S} = \frac{412.5 \times 10^3}{3490 \times 10^{-6}} = 118.195 \times 10^6 \text{ Pa}$$ $\sigma_m = 118.2$ MPa ◀

$$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V|_{max}}{dt_w} = \frac{275 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 34.667 \times 10^6 \text{ Pa}$$ $\tau_m = 34.7$ MPa ◀

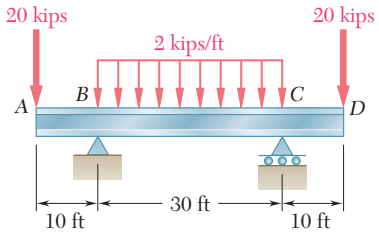
$$c = \frac{1}{2}d = \frac{67.8}{2} = 339 \text{ mm}, \quad t_f = 16.3 \text{ mm}, \quad y_b = c - t_f = 339 - 16.3 = 322.7 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{322.7}{339}\right)(118.195) = 112.512 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(56.256)^2 + (34.667)^2} = 66.080 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 56.256 + 66.080$$ $\sigma_{max} = 122.3$ MPa ◀

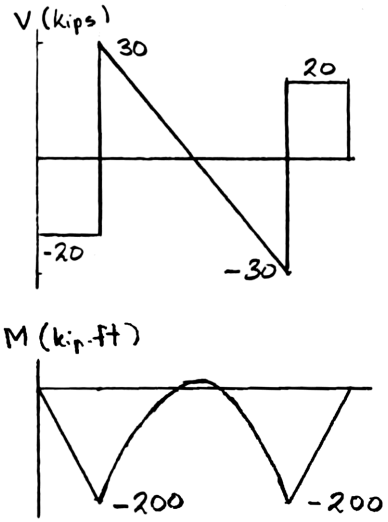
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PROBLEM 8.7

(*a*) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION



$$R_A = 50 \text{ kips} \uparrow \qquad R_D = 50 \text{ kips} \uparrow$$

$$|V|_{max} = 30 \text{ kips} \qquad |M|_{max} = 200 \text{ kip} \cdot \text{ft} = 2400 \text{ kip} \cdot \text{in.}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{2400}{24} = 100 \text{ in}^3$$

	Shape	S (in^3)
	W24 × 68	154
→	W21 × 62	127
	W18 × 76	146
	W16 × 77	134
	W12 × 96	131
	W10 × 112	126

(*a*) Use W21 × 62. ◀

$$d = 21.0 \text{ in.} \quad t_f = 0.615 \text{ in.} \quad t_w = 0.400 \text{ in.}$$

(*b*) $$\sigma_m = \frac{|M|_{max}}{S} = \frac{2400}{127} = 18.8976 \text{ ksi} \qquad \sigma_m = 18.9 \text{ ksi} \blacktriangleleft$$

$$\tau_m = \frac{|V|_{max}}{dt_w} = \frac{30}{(21.0)(0.400)} = 3.5714 \text{ ksi} \qquad \tau_m = 3.57 \text{ ksi} \blacktriangleleft$$

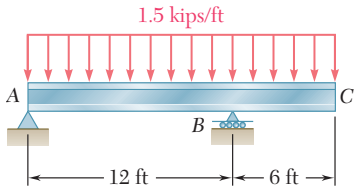
$$c = \frac{1}{2}d = \frac{21.0}{2} = 10.50 \text{ in.} \qquad y_b = c - t_f = 10.50 - 0.615 = 9.8850 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{9.8850}{10.50}\right)(18.8976) = 17.7907 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(8.8954)^2 + (3.5714)^2} = 9.5856 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.8954 + 9.5856 = 18.4810 \text{ ksi} \qquad \sigma_{max} = 18.48 \text{ ksi} \blacktriangleleft$$

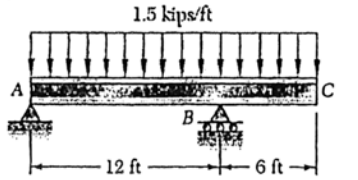
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PROBLEM 8.8

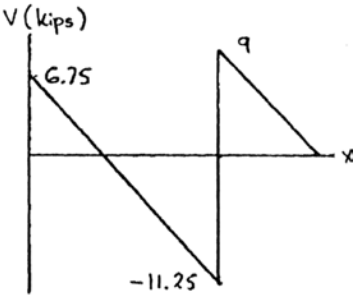
(*a*) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION

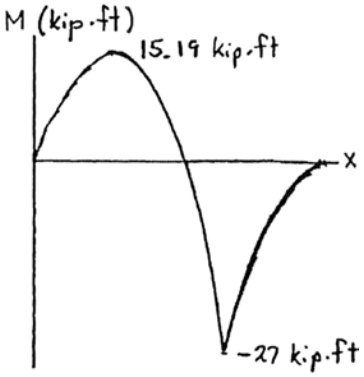


$$+\circlearrowleft \Sigma M_B = 0: \quad -12R_A + (1.5)(18)(3) = 0 \quad R_A = 6.75 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 12R_B + (1.5)(18)(9) = 0 \quad R_B = 20.25 \text{ kips} \uparrow$$



$$|V|_{max} = 11.25 \text{ kips}$$



$$|M|_{max} = 27 \text{ kip} \cdot \text{ft} = 324 \text{ kip} \cdot \text{in.}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{324}{24} = 13.5 \text{ in}^3$$

Shape	$S(\text{in}^3)$
W12 × 16	17.1
→ W10 × 15	13.8
W8 × 18	15.2
W6 × 20	13.4

(*a*) Use W10 × 15. ◀

$$d = 10.0 \text{ in.}$$
$$t_f = 0.270 \text{ in.}$$
$$t_w = 0.230 \text{ in.}$$

(*b*) $$\sigma_m = \frac{|M|_{max}}{S} = \frac{324}{13.8} = 23.478 \text{ ksi}$$

$$\sigma_m = 23.5 \text{ ksi} \blacktriangleleft$$

$$\tau_m = \frac{|V|_{max}}{dt_w} = \frac{11.25}{(10.0)(0.230)} = 4.8913 \text{ ksi}$$

$$\tau_m = 4.89 \text{ ksi} \blacktriangleleft$$

$$c = \frac{1}{2}d = \frac{10.0}{2} = 5.00 \text{ in.}$$

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PROBLEM 8.8 ***(Continued)***

$$y_b = c - t_f = 5.00 - 0.270 = 4.73 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{4.73}{5.00}\right)(23.478) = 22.210 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\frac{22.210}{2}\right)^2 + (4.8913)^2} = 12.1345 \text{ ksi}$$

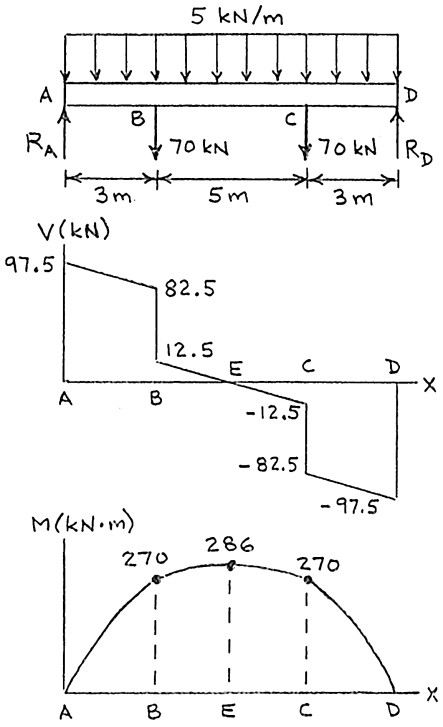
$$\sigma_{\max} = \frac{\sigma_b}{2} + R = \frac{22.210}{2} + 12.1345 \qquad \sigma_{\max} = 23.2 \text{ ksi} \blacktriangleleft$$

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PROBLEM 8.9

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design (use the loading of Prob. 5.73 and selected W530 × 92 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION



Reactions: $R_A = 97.5 \text{ kN} \uparrow \quad R_D = 97.5 \text{ kN} \uparrow$

$|V|_{max} = 97.5 \text{ kN}$

$|M|_{max} = 286 \text{ kN} \cdot \text{m}$

For W530 × 92 rolled-steel section,

$d = 533 \text{ mm}, \quad b_f = 209 \text{ mm}, \quad t_f = 15.6 \text{ mm},$

$t_w = 10.2 \text{ mm}, \quad c = \frac{1}{2}d = 266.5 \text{ mm}$

$I = 554 \times 10^6 \text{ mm}^4 \quad S = 2080 \times 10^3 \text{ mm}^3$

(*a*) $$\sigma_m = \frac{|M|_{max}}{S} = \frac{286 \times 10^3}{2080 \times 10^{-6}} = 137.5 \times 10^6 \text{ Pa}$$

$\sigma_m = 137.5 \text{ MPa}$ ◀

$y_b = c - t_f = 250.9 \text{ mm}$

$A_f = b_f t_f = 3260.4 \text{ mm}^2$

$\bar{y} = \frac{1}{2}(c + y_b) = 258.7 \text{ mm}$

$Q = A_f \bar{y} = 843.47 \times 10^3 \text{mm}^3$

At midspan: $V = 0 \quad \tau_b = 0$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{250.9}{266.5}(137.5) = 129.5 \text{ MPa}$$

$\sigma_{max} = 129.5 \text{ MPa}$ ◀

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PROBLEM 8.9 ***(Continued)***

At sections *B* and *C*:

$$\sigma_m = \frac{M}{S} = \frac{270 \times 10^3}{2080 \times 10^{-6}} = 129.808 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{250.9}{266.5}(129.808) = 122.209 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f\bar{y}}{It_w} = \frac{(82.5 \times 10^3)(3260.4 \times 10^{-6})(258.7 \times 10^{-3})}{(554 \times 10^{-6})(10.2 \times 10^{-3})}$$

$$= 12.3143 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 62.333 \text{ MPa}$$

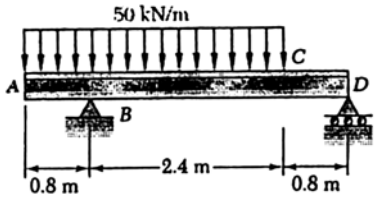
$$\sigma_{\max} = \frac{\sigma_b}{2} + R \qquad \sigma_{\max} = 123.4 \text{ MPa} \blacktriangleleft$$

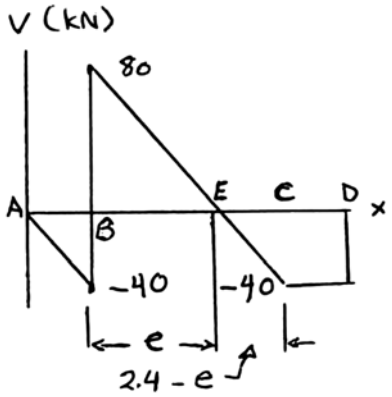
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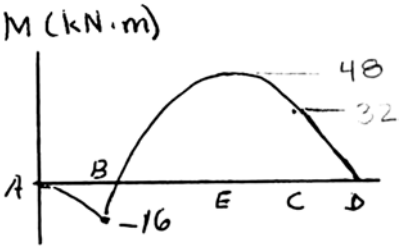
PROBLEM 8.10

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{\text{all}}$. For the selected design (use the loading of Prob. 5.74 and selected W250 × 28.4 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of a flange and the web.

SOLUTION







From Problem 5.74, $\sigma_{\text{all}} = 160$ MPa

$|M|_{\max} = 48$ kN · m at section *E*, which lies 1.6 m to the right of *B*.

$|V| = 0$

For W250 × 28.4 rolled-steel section,

$d = 259$ mm $\quad b_f = 102$ mm $\quad t_f = 10.0$ mm

$t_w = 6.35$ mm $\quad I = 40.1 \times 10^6$ mm^4

$S = 308 \times 10^3$ mm^3

$$c = \frac{1}{2} d = 129.5 \text{ mm}$$

(*a*) $$\sigma_m = \frac{|M|_{\max}}{S} = \frac{48 \times 10^3}{308 \times 10^{-6}}$$

$\sigma_m = 155.8$ MPa ◀

$$y_b = c - t_f = 119.5 \text{ mm}$$

$$A_f = \frac{1}{2} b_f t_f = 1020 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 124.5 \text{ mm}$$

$$Q = A_f \bar{y} = 126.99 \times 10^3 \text{ mm}^3 = 126.99 \times 10^{-6} \text{ m}^3$$

At section *E*, $V = 0$ $\quad M = M_{\max}$

$$\sigma_b = \frac{y_b}{c} \sigma_m = 143.8 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It_w} = 0$$

(*b*) $\sigma_{\max} = \sigma_b$

$\sigma_{\max} = 143.8$ MPa ◀

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PROBLEM 8.10 *(Continued)*

At section *C*,

$$|V| = 40 \text{ kN} \qquad M = 32 \text{ kN} \cdot \text{m}$$

$$\sigma_b = \frac{My_b}{I} = \frac{(32 \times 10^3)(119.5 \times 10^{-3})}{40.0 \times 10^{-6}} = 95.6 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It_w} = \frac{(40 \times 10^3)(126.99 \times 10^{-6})}{(40.1 \times 10^{-6})(6.35 \times 10^{-3})} = 19.95 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{47.8^2 + 19.95^2} = 51.08 \text{ MPa}$$

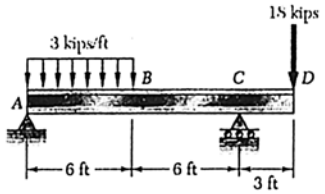
$$\sigma_{\max} = \frac{\sigma_b}{2} + R = 99.6 \text{ MPa} \quad \text{(less than value at section } E\text{)}$$

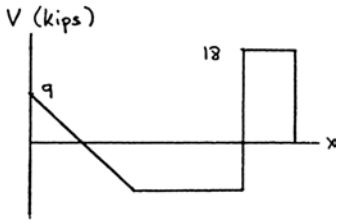
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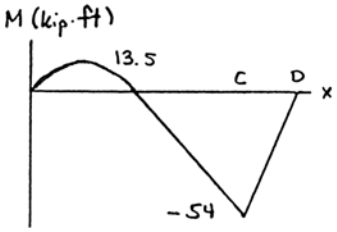
PROBLEM 8.11

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{\text{all}}$. For the selected design (use the loading of Prob. 5.75 and selected S12 × 31.8 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of a flange and the web.

SOLUTION







From Prob. 5.75, $\sigma_{\text{all}} = 24$ ksi

$|M|_{\max} = 54 \text{ kip} \cdot \text{ft} = 648 \text{ kip} \cdot \text{in.}$ at C.

At C, $|V| = 18$ kips

For S12 × 31.8 rolled-steel shape,

$d = 12.0$ in., $b_f = 5.00$ in., $t_f = 0.544$ in.,

$t_w = 0.350$ in. $I_z = 217 \text{ in}^4$, $S_z = 36.2 \text{ in}^3$

$$c = \frac{1}{2}d = 6.00 \text{ in.}$$

$$\sigma_m = \frac{|M|}{S_z} = \frac{648}{36.2} = 17.9006 \text{ ksi}$$

(*a*) $\sigma_m = 17.90$ ksi ◀

$$y_b = c - t_f = 5.456 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = 16.2776 \text{ ksi} \qquad \frac{\sigma_b}{2} = 8.1388 \text{ ksi}$$

$$A_f = b_f t_f = 2.72 \text{ in}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 5.728 \text{ in.}$$

$$Q = A_f \bar{y} = 15.5802 \text{ in}^3$$

$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(18)(15.5802)}{(217)(0.350)} = 3.6925 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.1388^2 + 3.6925^2} = 8.9373 \text{ ksi}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R = 8.1388 + 8.9373$$

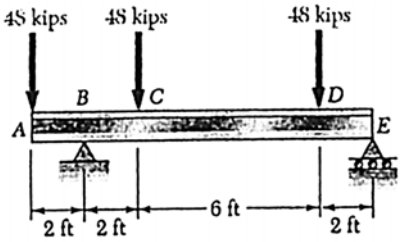
(*b*) $\sigma_{\max} = 17.08$ ksi ◀

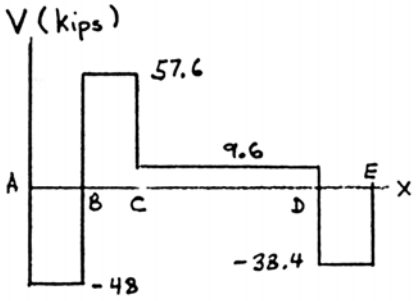
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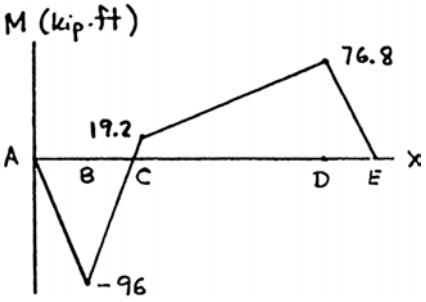
PROBLEM 8.12

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design (use the loading of Prob. 5.76 and selected S15 × 42.9 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION







From Prob. 5.76, $\sigma_{all} = 24$ ksi

$|M|_{max} = 96 \text{ kip} \cdot \text{ft} = 1152 \text{ kip} \cdot \text{in.}$ at *D*.

At *D*, $|V| = 38.4$ kips

For S15 × 42.9 shape,

$d = 15.0$ in., $b_f = 5.50$ in. $t_f = 0.622$ in.,

$t_w = 0.411$ in., $I_z = 446 \text{ in}^4$, $S_z = 59.4 \text{ in}^3$

$$c = \frac{1}{2}d = 7.5 \text{ in.}$$

$$\sigma_m = \frac{|M|}{S} = \frac{1152}{59.4} = 19.3939 \text{ ksi} \qquad (a) \qquad \sigma_m = 19.39 \text{ ksi} \blacktriangleleft$$

$$y_b = c - t_f = 6.878 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = 17.7855 \text{ ksi} \qquad \frac{\sigma_b}{2} = 8.8928 \text{ ksi}$$

$$A_f = b_f t_f = 3.421 \text{ in}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 7.189 \text{ in.}$$

$$Q = A_f \bar{y} = 24.594 \text{ in}^3$$

$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(57.6)(24.594)}{(446)(0.411)} = 7.7281 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.8928^2 + 7.7281^2} = 11.7816 \text{ ksi}$$

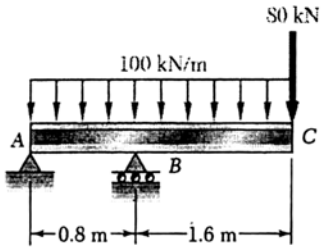
$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.8928 + 11.7816 \qquad (b) \qquad \sigma_{max} = 20.7 \text{ ksi} \blacktriangleleft$$

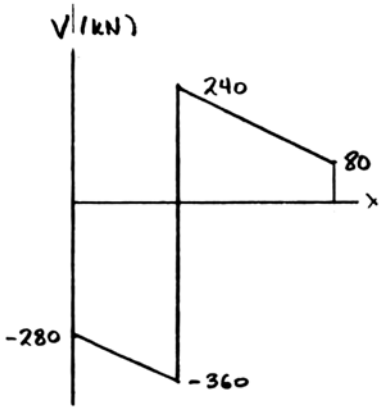
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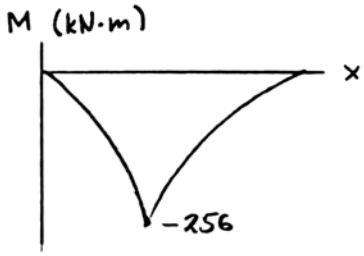
PROBLEM 8.13

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design (use the loading of Prob. 5.77 and selected S510 × 98.2 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION







From Problem 5.77, $\sigma_{all} = 160$ MPa

$|M|_{max} = 256$ kN · m at point *B*

$|V| = 360$ kN at *B*

For S510 × 98.2 rolled-steel section,

$d = 508$ mm, $b_f = 159$ mm, $t_f = 20.2$ mm

$t_w = 12.8$ mm, $I_x = 495 \times 10^6$ mm^4, $S_x = 1950 \times 10^3$ mm^3

$$c = \frac{1}{2}d = 254 \text{ mm}$$

(*a*) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{256 \times 10^3}{1950 \times 10^{-6}} = 131.3 \text{ MPa}$$ ◀

$$y_b = c - t_f = 233.8$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = 120.9 \text{ MPa} \qquad \frac{\sigma_b}{2} = 60.45 \text{ MPa}$$

$$A_f = b_f t_f = 3212 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 243.9 \text{ mm}$$

$$Q = A_f \bar{y} = 783.4 \times 10^3 \text{ mm}^3$$

$$\tau_b = \frac{VQ}{It_w} = \frac{(360 \times 10^3)(783.4 \times 10^{-6})}{(495 \times 10^{-6})(12.8 \times 10^{-3})} = 44.5 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{60.45^2 + 44.5^2} = 75.06 \text{ MPa}$$

(*b*) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 60.45 + 75.06 = 135.5 \text{ MPa}$$

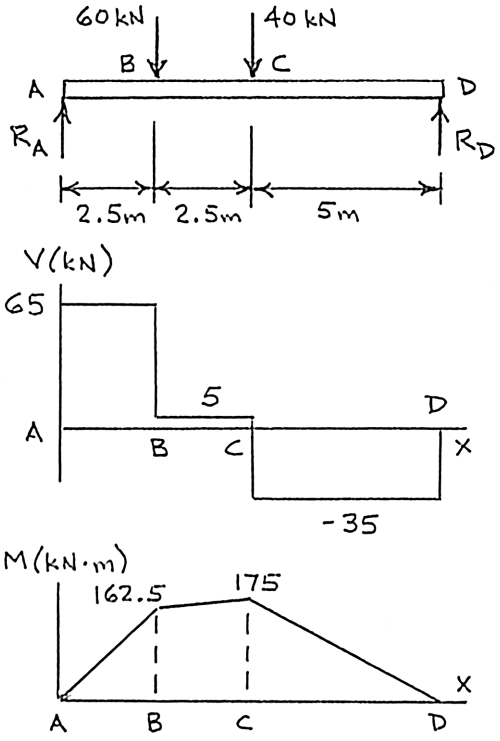
$\sigma_{max} = 135.5$ MPa ◀

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PROBLEM 8.14

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design (use the loading of Prob. 5.78 and selected S460 × 81.4 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION



Reactions: $R_A = 65 \text{ kN} \uparrow \qquad R_D = 35 \text{ kN} \uparrow$

$$|V|_{max} = 65 \text{ kN}$$
$$|M|_{max} = 175 \text{ kN} \cdot \text{m}$$

For S460 × 81.4 rolled-steel section,

$$d = 457 \text{ mm}, \quad b_f = 152 \text{ mm}, \quad t_f = 17.6 \text{ mm},$$
$$t_w = 11.7 \text{ mm}, \quad c = \frac{1}{2}d = 228.5 \text{ mm}$$
$$I = 333 \times 10^6 \text{ mm}^4 \quad S = 1460 \times 10^3 \text{ mm}^3$$

(*a*) $$\sigma_m = \frac{|M|_{max}}{S} = \frac{175 \times 10^3}{1460 \times 10^{-6}} = 119.863 \times 10^6 \text{ Pa}$$

$$\sigma_m = 119.9 \text{ MPa} \blacktriangleleft$$

(*b*) $$y_b = c - t_f = 210.9 \text{ mm}$$
$$A_f = b_f t_f = 2675.2 \text{ mm}^2$$
$$\bar{y} = \frac{1}{2}(c + y_b) = 219.7 \text{ mm}$$
$$Q = A_f \bar{y} = 587.74 \times 10^3 \text{ mm}^3$$

At section *C*,
$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{210.9}{228.5}(119.863) = 110.631 \text{ MPa}$$
$$\tau_b = \frac{VQ}{It} = \frac{VA_f\bar{y}}{It_w} = \frac{(35 \times 10^3)(2675.2 \times 10^{-6})(219.7 \times 10^{-3})}{(333 \times 10^{-6})(11.7 \times 10^{-3})} = 5.2799 \text{ MPa}$$
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 55.567 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R \qquad \sigma_{max} = 110.9 \text{ MPa} \blacktriangleleft$$

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PROBLEM 8.14 ***(Continued)***

At section *B*,

$$\sigma_m = \frac{M}{S} = \frac{162.5 \times 10^3}{1460 \times 10^{-6}} = 111.301 \text{ MPa}$$

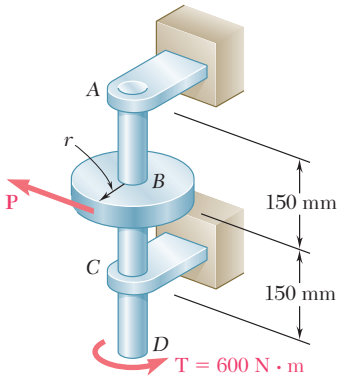
$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{210.9}{228.5}(111.301) = 102.728 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f\bar{y}}{It_w} = \frac{(65 \times 10^3)(2675.2 \times 10^{-6})(219.7 \times 10^{-3})}{(333 \times 10^{-6})(11.7 \times 10^{-3})} = 9.8055 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 52.292 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R \qquad \sigma_{\max} = 103.7 \text{ MPa} \blacktriangleleft$$

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PROBLEM 8.15

Determine the smallest allowable diameter of the solid shaft *ABCD*, knowing that $\tau_{all} = 60$ MPa and that the radius of disk *B* is $r = 80$ mm.

SOLUTION

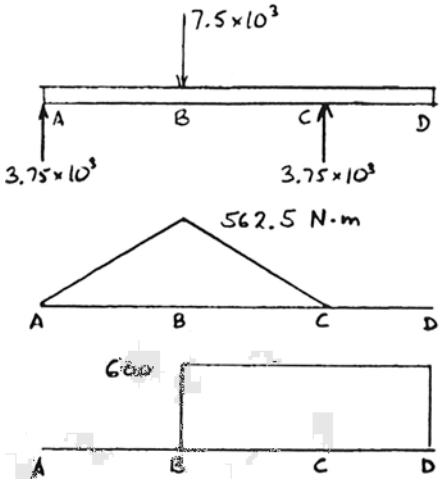
$$\Sigma M_{axis} = 0: \quad T - Pr = 0 \quad P = \frac{T}{r} = \frac{600}{80 \times 10^{-3}} = 7.5 \times 10^{3}$$

$$R_A = R_C = \frac{1}{2}P$$

$$= 3.75 \times 10^{3}\text{N}$$

$$M_B = (3.75 \times 10^{3})(150 \times 10^{-3})$$

$$= 562.5 \text{ N} \cdot \text{m}$$



Bending moment: (See sketch).

Torque: (See sketch).

Critical section lies at point *B*.

$$M = 562.5 \text{ N} \cdot \text{m}, \qquad T = 600 \text{ N} \cdot \text{m}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M^2 + T^2}\right)_{max}}{\tau_{all}}$$

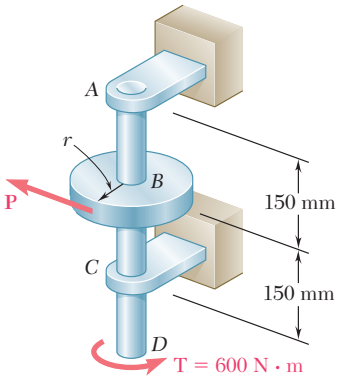
$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{2}{\pi}\frac{\sqrt{(562.5)^2 + (600)^2}}{60 \times 10^{6}}$$

$$= 8.726 \times 10^{-6}\text{m}^3$$

$$c = 20.58 \times 10^{-3}\text{m} \qquad d = 2c = 41.2 \times 10^{-3}\text{m}$$

$$d = 41.2 \text{ mm} \blacktriangleleft$$

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PROBLEM 8.16

Determine the smallest allowable diameter of the solid shaft *ABCD*, knowing that $\tau_{all} = 60$ MPa and that the radius of disk *B* is $r = 120$ mm.

SOLUTION

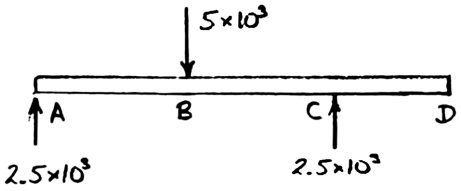
$$\Sigma M_{AD} = 0, \qquad T - Pr = 0 \qquad P = \frac{T}{r} = \frac{600}{120 \times 10^{-3}} = 5 \times 10^3 \text{ N}$$

$$R_A = R_C = \frac{1}{2}P$$

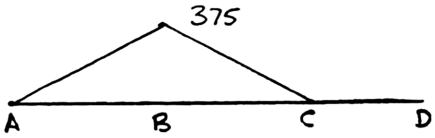
$$= 2.5 \times 10^3 \text{ N}$$

$$M_B = (2.5 \times 10^3)(0.150 \times 10^{-3})$$

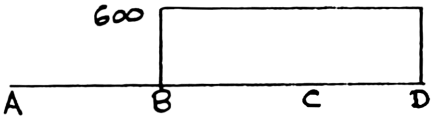
$$= 375 \text{ N} \cdot \text{m}$$



Bending moment: (See sketch).



Torque: (See sketch).



Critical section lies at point *B*.

$$M = 375 \text{ N} \cdot \text{m}, \qquad T = 600 \text{ N} \cdot \text{m}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M^2 + T^2}\right)_{\max}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{2}{\pi}\frac{\sqrt{375^2 + 600^2}}{60 \times 10^6} = 7.5073 \times 10^{-6} \text{ m}^3$$

$$c = 19.5807 \times 10^{-3} \text{ m} \qquad d = 2c = 39.2 \times 10^{-3} \text{ m} = 39.2 \text{ mm}$$ ◀

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PROBLEM 8.17

Using the notation of Sec. 8.2 and neglecting the effect of shearing stresses caused by transverse loads, show that the maximum normal stress in a circular shaft can be expressed as follows:

$$\sigma_{\max} = \frac{c}{J}\left[\left(M_y^2 + M_z^2\right)^{1/2} + \left(M_y^2 + M_z^2 + T^2\right)^{1/2}\right]_{\max}$$

SOLUTION

Maximum bending stress:

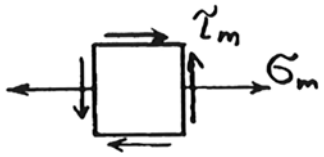
$$\sigma_m = \frac{|M|c}{I} = \frac{\sqrt{M_y^2 + M_z^2}\,c}{I}$$

Maximum torsional stress:

$$\tau_m = \frac{Tc}{J}$$

$$\frac{\sigma_m}{2} = \frac{\sqrt{M_y^2 + M_z^2}\,c}{2I} = \frac{c}{J}\sqrt{M_y^2 + M_z^2}$$

Using Mohr's circle,



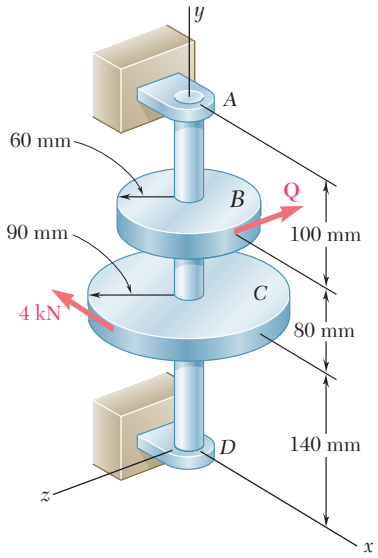
$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_m^2} = \sqrt{\frac{c^2}{J^2}\left(M_y^2 + M_z^2\right) + \frac{T^2c^2}{J^2}}$$

$$= \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$\sigma_{\max} = \frac{\sigma_m}{2} + R = \frac{c}{J}\sqrt{M_y^2 + M_z^2} + \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$= \frac{c}{J}\left[\left(M_y^2 + M_z^2\right)^{1/2} + \left(M_y^2 + M_z^2 + T^2\right)^{1/2}\right] \blacktriangleleft$$

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PROBLEM 8.18

The 4-kN force is parallel to the *x* axis, and the force **Q** is parallel to the *z* axis. The shaft *AD* is hollow. Knowing that the inner diameter is half the outer diameter and that $\tau_{all} = 60$ MPa, determine the smallest permissible outer diameter of the shaft.

SOLUTION

$$\Sigma M_y = 0: \quad 60\times10^{-3}Q - (90\times10^{-3})(4\times10^{3}) = 0$$

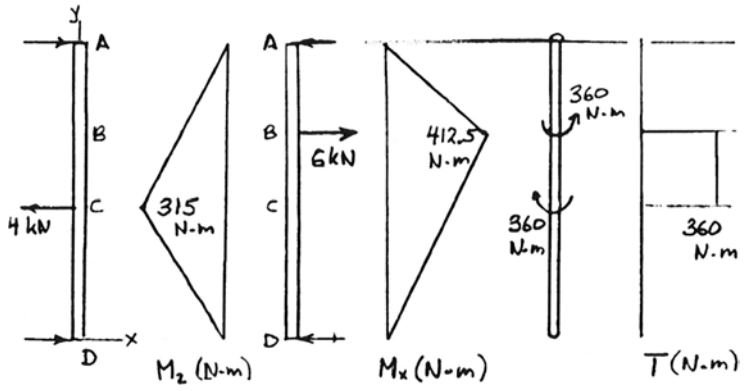
$$Q = 6\times10^{3}\,\text{N} = 6\,\text{kN}$$

Bending moment and torque diagrams.

In *xy* plane: $(M_z)_{max} = 315\ \text{N}\cdot\text{m}$ at *C*.

In *yz* plane: $(M_x)_{max} = 412.5\ \text{N}\cdot\text{m}$ at *B*.

About *z* axis: $T_{max} = 360\ \text{N}\cdot\text{m}$ between *B* and *C*.



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PROBLEM 8.18 *(Continued)*

At *B*,

$$M_z = \left(\frac{100}{180}\right)(315) = 175 \text{ N} \cdot \text{m}$$

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{175^2 + 412.5^2 + 360^2} = 574.79 \text{ N} \cdot \text{m}$$

At *C*,

$$M_x = \left(\frac{140}{220}\right)(412.5) = 262.5 \text{ N} \cdot \text{m}$$

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{315^2 + 262.5^2 + 360^2} = 545.65 \text{ N} \cdot \text{m}$$

Largest value is $574.79 \text{ N} \cdot \text{m}$.

$$\tau_{\max} = \frac{\max\sqrt{M_x^2 + M_z^2 + T^2}\, c}{J}$$

$$\frac{J}{c} = \frac{\max\sqrt{M_x^2 + M_z^2 + T^2}}{\tau_{\max}} = \frac{574.79}{60 \times 10^6}$$

$$= 9.5798 \times 10^{-6} \text{m}^3 = 9.5798 \times 10^3 \text{mm}^3$$

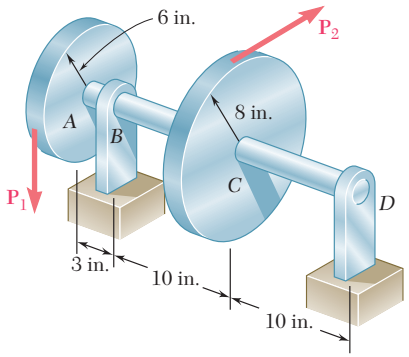
$$\frac{J}{c} = \frac{\frac{\pi}{2}\left(c_o^4 - c_i^4\right)}{c_o} = \frac{\pi}{2} c_o^3 \left(1 - \frac{c_i^4}{c_o^4}\right) = \frac{\pi}{2} c_o^3 \left[1 - \left(\frac{1}{2}\right)^4\right]$$

$$= 1.47262 c_o^3$$

$$1.47262 c_o^3 = 9.5798 \times 10^3 \quad c_o = 18.67 \text{ mm}$$

$$d_o = 2c_o \qquad d_o = 37.3 \text{ mm} \blacktriangleleft$$

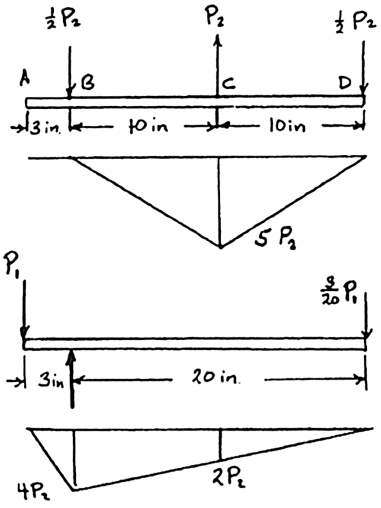
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PROBLEM 8.19

The vertical force $\mathbf{P_1}$ and the horizontal force $\mathbf{P_2}$ are applied as shown to disks welded to the solid shaft *AD*. Knowing that the diameter of the shaft is 1.75 in. and that $\tau_{all} = 8$ ksi, determine the largest permissible magnitude of the force $\mathbf{P_2}$.

SOLUTION



Let P_2 be in kips.

$$\sum M_{\text{shaft}} = 0: \qquad 6P_1 - 8P_2 = 0 \qquad P_1 = \frac{4}{3}P_2$$

Torque over portion *ABC*: $T = 8P_2$

Bending in horizontal plane: $M_{Cy} = 10 \cdot \frac{1}{2}P_2 = 5P_2$

Bending in vertical plane: $M_{Bz} = 3P_1$

$$= 3 \cdot \frac{4}{3}P_2$$

$$= 4P_1$$

Critical point is just to the left of point *C*.

$$T = 8P_2 \qquad M_y = 5P_2 \qquad M_z = 2P_2$$

$$d = 1.75 \text{ in.} \qquad c = \frac{1}{2}d = 0.875 \text{ in.}$$

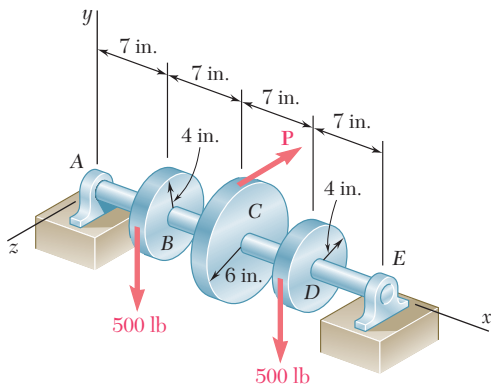
$$J = \frac{\pi}{2}(0.875)^4 = 0.92077 \text{ in}^4$$

$$\tau_{all} = \frac{c}{J}\sqrt{T^2 + M_y^2 + M_z^2}$$

$$8 = \frac{0.875}{0.92077}\sqrt{(8P_2)^2 + (5P_2)^2 + (2P_2)^2} = 9.164P_2$$

$$P_2 = 0.873 \text{ kips} \qquad P_2 = 873 \text{ lb} \blacktriangleleft$$

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PROBLEM 8.20

The two 500-lb forces are vertical and the force **P** is parallel to the *z* axis. Knowing that $\tau_{all} = 8$ ksi, determine the smallest permissible diameter of the solid shaft *AE*.

SOLUTION

$$\Sigma M_x = 0: \quad (4)(500) - 6P + (4)(500) = 0$$

$$P = 666.67 \text{ lb}$$

Torques:

$$AB: \quad T = 0$$

$$BC: \quad T = -(4)(500) = -2000 \text{ lb} \cdot \text{in.}$$

$$CD: \quad T = 4(500) = 2000 \text{ lb} \cdot \text{in.}$$

$$DE: \quad T = 0$$

Critical sections are on either side of disk *C*.

$$T = 2000 \text{ lb} \cdot \text{in.}$$

$$M_z = 3500 \text{ lb} \cdot \text{in.}$$

$$M_y = 4667 \text{ lb} \cdot \text{in.}$$

$$\tau_{all} = \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3$$

$$= \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{all}}$$

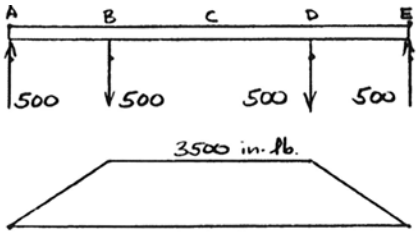
$$= \frac{\sqrt{4667^2 + 3500^2 + 2000^2}}{8 \times 10^3}$$

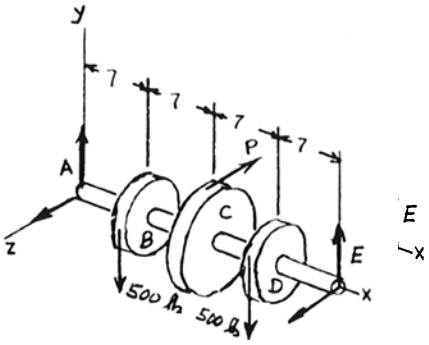
$$= 0.77083 \text{ in}^3$$

$$c = 0.7888 \text{ in.} \qquad d = 2c$$

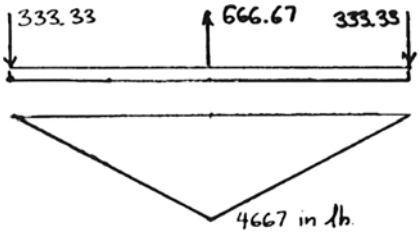
$$d = 1.578 \text{ in.} \blacktriangleleft$$

Forces in vertical plane:



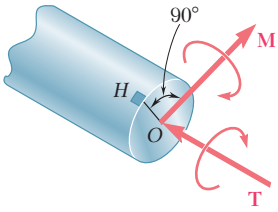


Forces in horizontal plane:

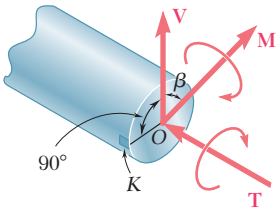


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PROBLEM 8.21



(*a*)



(*b*)

It was stated in Sec. 8.2 that the shearing stresses produced in a shaft by the transverse loads are usually much smaller than those produced by the torques. In the preceding problems, their effect was ignored and it was assumed that the maximum shearing stress in a given section occurred at point *H* (Fig. P8.21*a*) and was equal to the expression obtained in Eq. (8.5), namely,

$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2}$$

Show that the maximum shearing stress at point *K* (Fig. P8.21*b*), where the effect of the shear *V* is greatest, can be expressed as

$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + \left(\frac{2}{3}cV + T\right)^2}$$

where β is the angle between the vectors **V** and **M**. It is clear that the effect of the shear *V* cannot be ignored when $\tau_K \geq \tau_H$. (*Hint:* Only the component of **M** along **V** contributes to the shearing stress at *K*.)

SOLUTION

Shearing stress at point *K*.

Due to *V*: For a semicircle, $Q = \frac{2}{3}c^3$

For a circle cut across its diameter, $t = d = 2c$

For a circular section, $I = \frac{1}{2}J$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(V)\left(\frac{2}{3}c^3\right)}{\left(\frac{1}{2}J\right)(2c)} = \frac{2}{3}\frac{Vc^2}{J}$$

Due to *T*: $\tau_{xy} = \frac{Tc}{J}$

Since these shearing stresses have the same orientation,

$$\tau_{xy} = \frac{c}{J}\left(\frac{2}{3}Vc + T\right)$$

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PROBLEM 8.21 *(Continued)*

Bending stress at point K: $\sigma_x = \dfrac{Mu}{I} = \dfrac{2Mu}{J}$

Where u is the distance between point K and the neutral axis,

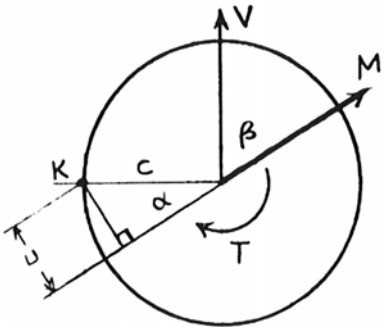
$$u = c \sin \alpha = c \sin\left(\frac{\pi}{2} - \beta\right) = c \cos \beta$$

$$\sigma_x = \frac{2Mc \cos \beta}{J}$$

Using Mohr's circle,

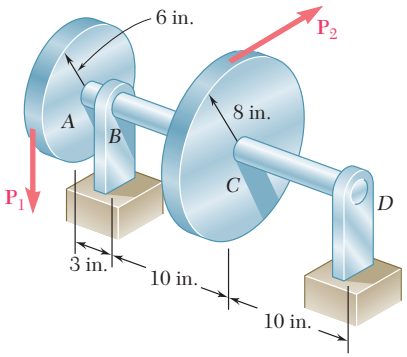
$$\tau_K = R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{c}{J}\sqrt{(M \cos \beta)^2 + \left(\frac{2}{3}Vc + T\right)^2}$$



Cross section

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PROBLEM 8.22

Assuming that the magnitudes of the forces applied to disks *A* and *C* of Prob. 8.19 are, respectively, $P_1 = 1080$ lb and $P_2 = 810$ lb, and using the expressions given in Prob. 8.21, determine the values τ_H and τ_K in a section (*a*) just to the left of *B*, (*b*) just to the left of *C*.

SOLUTION

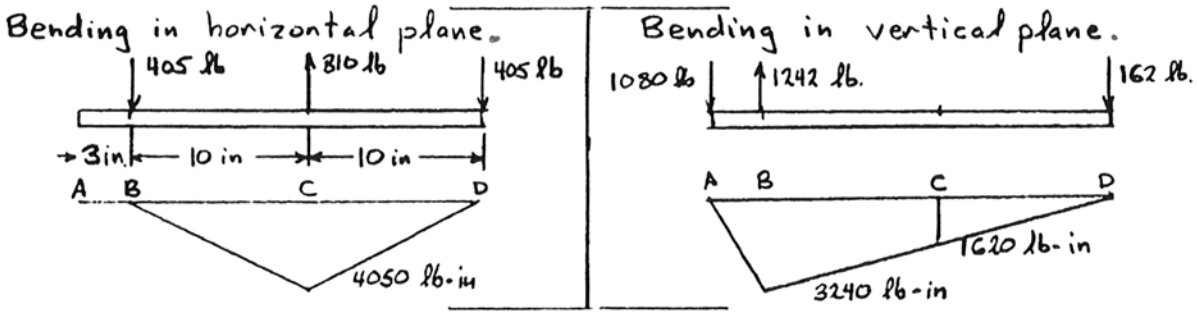
From Prob. 8.19, shaft diameter = 1.75 in.

$$c = \frac{1}{2}d = 0.875 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = 0.92077$$

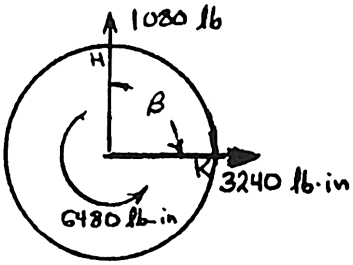
Torque over portion *ABC*:

$$T = (6)(1080) = (8)(810) = 6480 \text{ lb} \cdot \text{in.}$$



(*a*) Just to the left of point *B*: $V = 1080$ lb $M = 3240$ lb · in.

$\beta = 90°$ $T = 6480$ lb · in.



$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2} = \frac{0.875}{0.92077}\sqrt{(3240)^2 + (6480)^2}$$

$$\tau_H = 6880 \text{ psi} \blacktriangleleft$$

$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + \left(\tfrac{2}{3}V_C + T\right)^2} = \frac{c}{J}\left[\frac{2}{3}V_C + T\right]$$

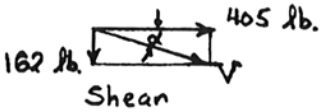
$$= \frac{0.875}{0.92077}\left[\left(\frac{2}{3}\right)(1080)(0.875) + 6480\right]$$

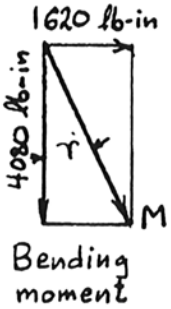
$$\tau_K = 6760 \text{ psi} \blacktriangleleft$$

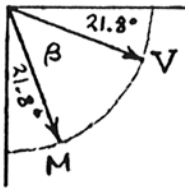
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PROBLEM 8.22 *(Continued)*

(*b*) Just to the left of point *C*:







$$V = \sqrt{(162)^2 + (405)^2} = 436.2 \text{ lb}$$

$$\alpha = \tan^{-1}\frac{162}{405} = 21.80°$$

$$M = \sqrt{(1620)^2 + (4050)^2} = 4362 \text{ lb} \cdot \text{in.}$$

$$\gamma = \tan^{-1}\frac{1620}{4050} = 21.80°$$

$$\beta = 90° - 21.8° - 21.8° = 46.4°$$

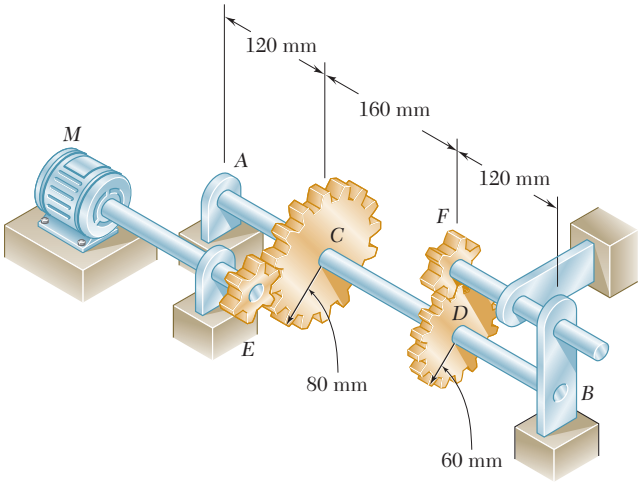
$$\tau_H = \frac{0.875}{0.92077}\sqrt{(6480)^2 + (4362)^2} \qquad \tau_H = 7420 \text{ psi} \blacktriangleleft$$

$$\frac{2}{3}V_C + T = \left(\frac{2}{3}\right)(436.2)(0.875) + 6480 = 6734 \text{ lb} \cdot \text{in.}$$

$$M\cos\beta = 4362\cos 46.4° = 3008 \text{ lb} \cdot \text{in.}$$

$$\tau_K = \frac{0.875}{0.92077}\sqrt{(3008)^2 + (6734)^2} \qquad \tau_K = 7010 \text{ psi} \blacktriangleleft$$

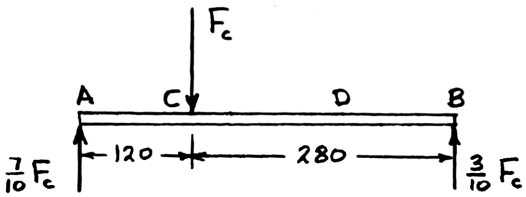
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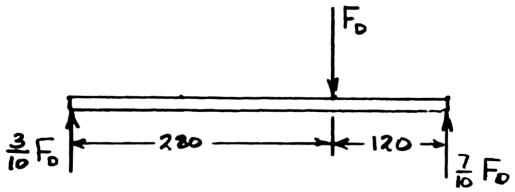
PROBLEM 8.23

The solid shaft *AB* rotates at 600 rpm and transmits 80 kW from the motor *M* to a machine tool connected to gear *F*. Knowing that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft *AB*.

SOLUTION









$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{80 \times 10^3}{(2\pi)(10)} = 1273.24 \text{ N} \cdot \text{m}$$

Gear *C*: $F_C = \dfrac{T}{r_C}$

$$F_C = \frac{1273.24}{80 \times 10^{-3}} = 15.9155 \times 10^3 \text{N}$$

Gear *D*: $F_D = \dfrac{T}{r_D}$

$$F_D = \frac{1273.24}{60 \times 10^{-3}} = 21.221 \times 10^3 \text{ N}$$

Forces in vertical plane:

$$M_{Cz} = (120 \times 10^{-3})\left(\frac{7}{10}F_c\right) = 1336.90 \text{ N} \cdot \text{m}$$

$$M_{Dz} = \frac{120}{280}M_{Cz} = 572.96 \text{ N} \cdot \text{m}$$

Forces in horizontal plane:

$$M_{Dy} = (120 \times 10^{-3})\left(\frac{7}{10}F_D\right) = 1782.56 \text{ N.m}$$

$$M_{Cy} = \frac{120}{280}M_{Dy} = 763.95 \text{ N} \cdot \text{m}$$

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PROBLEM 8.23 *(Continued)*

At C, $\sqrt{M_y^2 + M_z^2 + T^2} = 1998.01\ \text{N} \cdot \text{m}$

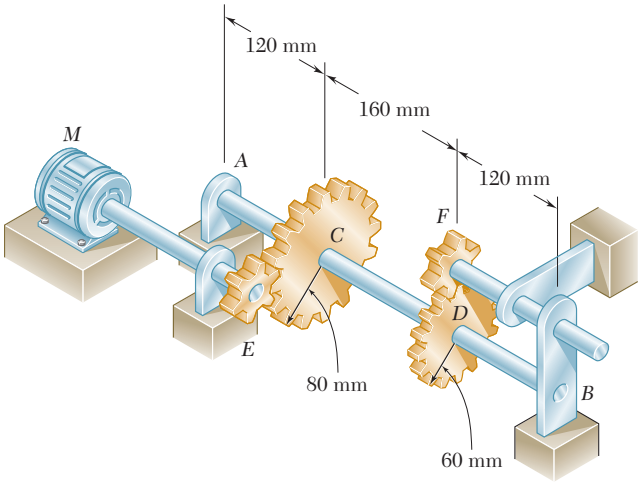
At D, $\sqrt{M_y^2 + M_z^2 + T^2} = 2264.3\ \text{N} \cdot \text{m}$

$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}}{\tau_{\text{all}}} = \frac{2264.3}{60 \times 10^6} = 37.738 \times 10^{-6}\ \text{m}^3$$

$$c = 28.855 \times 10^{-3}\,\text{m} \qquad d = 2c = 57.7 \times 10^{-3}\,\text{m} \qquad d = 57.7\ \text{mm} \blacktriangleleft$$

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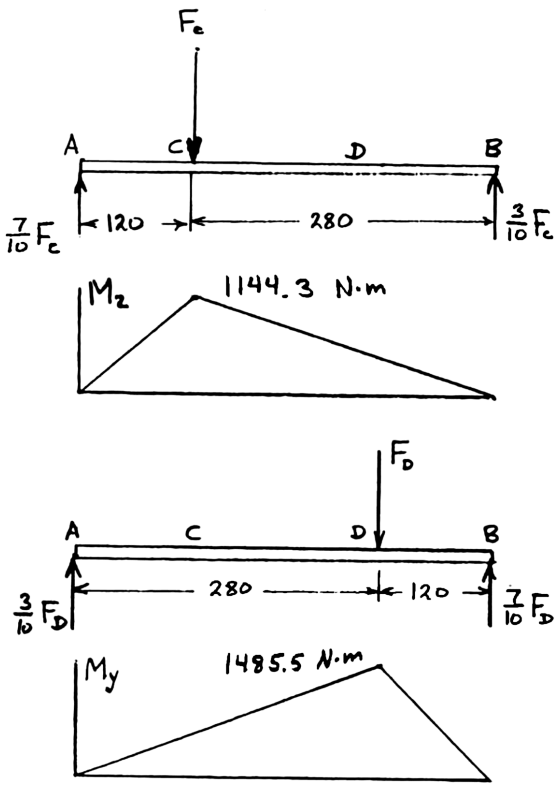


PROBLEM 8.24

Solve Prob. 8.23, assuming that shaft *AB* rotates at 720 rpm.

PROBLEM 8.23 The solid shaft *AB* rotates at 600 rpm and transmits 80 kW from the motor *M* to a machine tool connected to gear *F*. Knowing that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft *AB*.

SOLUTION



$$f = \frac{720 \text{ rpm}}{60 \text{ sec/min}} = 12 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{80 \times 10^3}{(2\pi)(12)} = 1061.03 \text{ N} \cdot \text{m}$$

Gear *C*: $F_C = \dfrac{I}{r_C}$

$$F_C = \frac{1061.03}{80 \times 10^{-3}} = 13.2629 \times 10^3 \text{ N}$$

Gear *D*: $F_D = \dfrac{T}{r_D}$

$$F_D = \frac{1061.03}{60 \times 10^{-3}} = 17.6838 \times 10^3 \text{ N}$$

Forces in vertical plane:

$$M_{Cz} = (120 \times 10^{-3})\left(\frac{7}{10} F_C\right) = 1114.08 \text{ N} \cdot \text{m}$$

$$M_{Dz} = \frac{120}{280} M_{Cz} = 477.46 \text{ N.m}$$

Forces in horizontal plane:

$$M_{Dy} = (120 \times 10^{-3})\left(\frac{7}{10} F_D\right) = 1485.44 \text{ N} \cdot \text{m}$$

$$M_{Cy} = \frac{120}{280} M_{Dy} = 636.62 \text{ N} \cdot \text{m}$$

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PROBLEM 8.24 *(Continued)*

At C, $\sqrt{M_y^2 + M_z^2 + T^2} = 1665.01\ \text{N} \cdot \text{m}$

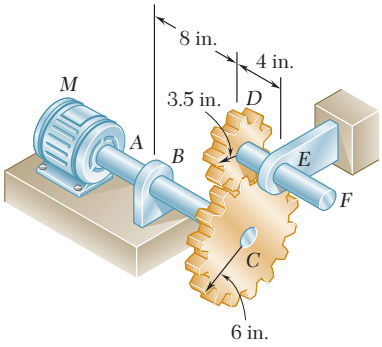
At D, $\sqrt{M_y^2 + M_z^2 + T^2} = 1886.87\ \text{N} \cdot \text{m}$

$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}}{\tau_{\text{all}}} = \frac{1886.87}{60 \times 10^6} = 31.448 \times 10^{-6}\ \text{m}^3$$

$$c = 27.153 \times 10^{-3}\ \text{m} \qquad d = 2c = 54.3 \times 10^{-3} \qquad d = 54.3\ \text{mm} \blacktriangleleft$$

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PROBLEM 8.25

The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 20 hp from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 7.5$ ksi, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.

SOLUTION

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in.} \cdot \text{lb/s}$$

$$240 \text{ rpm} = \frac{240}{60} = 4 \text{ Hz}$$

(*a*) Shaft *ABC*:

$$T = \frac{P}{2\pi f} = \frac{132 \times 10^3}{(2\pi)(4)} = 5252 \text{ lb} \cdot \text{in.}$$

Gear *C*:

$$F_{CD} = \frac{T}{r_C} = \frac{5252}{6} = 875.4 \text{ lb}$$

Bending moment at *B*:

$$M_B = (8)(875.4) = 7003 \text{ lb} \cdot \text{in.}$$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{(5252)^2 + (7003)^2}}{7500} = 1.1671 \text{ in}^3$$

$$c = 0.9057 \text{ in.} \quad d = 2c \qquad d = 1.811 \text{ in.} \blacktriangleleft$$

(*b*) Shaft *DEF*:

$$T = r_D F_{CD} = (3.5)(875.4) = 3064 \text{ lb} \cdot \text{in.}$$

Bending moment at *E*:

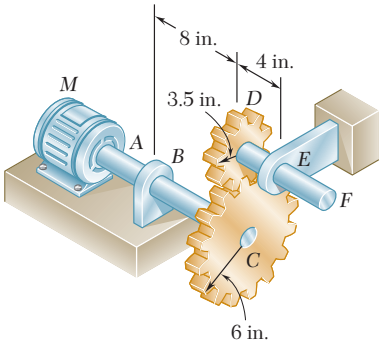
$$M_E = (4)(875.4) = 3502 \text{ lb} \cdot \text{in.}$$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{(3502)^2 + (3064)^2}}{7500} = 0.6204 \text{ in}^3$$

$$c = 0.7337 \text{ in.} \quad d = 2c \qquad d = 1.467 \text{ in.} \blacktriangleleft$$

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PROBLEM 8.26

Solve Prob. 8.25, assuming that the motor rotates at 360 rpm.

PROBLEM 8.25 The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 20 hp from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 7.5$ ksi, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.

SOLUTION

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in.} \cdot \text{lb/s}$$

$$360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

(*a*) Shaft *ABC*:

$$T = \frac{P}{2\pi f} = \frac{132 \times 10^3}{(2\pi)(6)} = 3501 \text{ lb} \cdot \text{in.}$$

Gear *C*:

$$F_{CD} = \frac{T}{r_C} = \frac{3501}{6} = 583.6 \text{ lb}$$

Bending moment at *B*: $M_B = (8)(583.6) = 4669 \text{ lb} \cdot \text{in.}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{4669^2 + 3501^2}}{7500} = 0.77806 \text{ in}^3$$

$$c = 0.791 \text{ in.} \qquad d = 2c \qquad d = 1.582 \text{ in.} \blacktriangleleft$$

(*b*) Shaft *DEF*:

$$T = r_D F_{CD} = (3.5)(583.6) = 2043 \text{ lb} \cdot \text{in.}$$

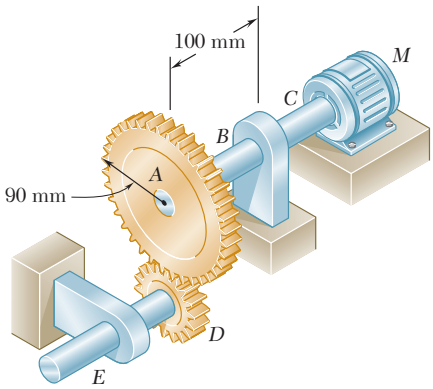
Bending moment at *E*: $M_E = (4)(583.6) = 2334 \text{ lb} \cdot \text{in.}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{2334^2 + 2043^3}}{7500} = 0.41362 \text{ in}^3$$

$$c = 0.6410 \text{ in.} \qquad d = 2c \qquad d = 1.282 \text{ in.} \blacktriangleleft$$

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PROBLEM 8.27

The solid shaft *ABC* and the gears shown are used to transmit 10 kW from the motor *M* to a machine tool connected to gear *D*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft *ABC*.

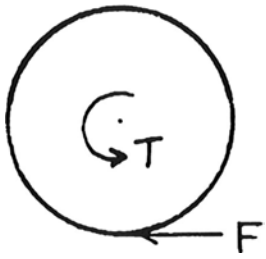
SOLUTION

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/ min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N} \cdot \text{m}$$

Gear *A*: $Fr_A - T = 0$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$



Bending moment at *B*:

$$M_B = L_{AB}F = (100 \times 10^{-3})(4421) = 442.1 \text{ N} \cdot \text{m}$$

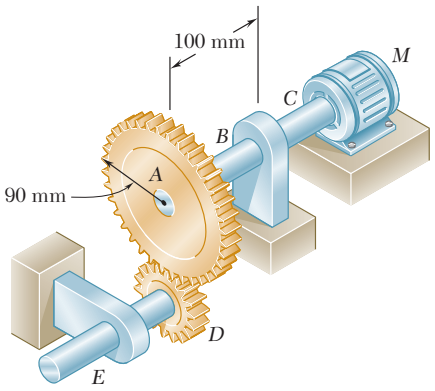
$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{(2)\sqrt{442.1^2 + 397.89^2}}{\pi(60 \times 10^6)} = 6.3108 \times 10^{-6} \text{m}^3$$

$$c = 18.479 \times 10^{-3} \text{m} \qquad d = 2c = 37.0 \times 10^{-3} \text{m} \qquad d = 37.0 \text{ mm} \blacktriangleleft$$

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PROBLEM 8.28

Assuming that shaft *ABC* of Prob. 8.27 is hollow and has an outer diameter of 50 mm, determine the largest permissible inner diameter of the shaft.

PROBLEM 8.27 The solid shaft *ABC* and the gears shown are used to transmit 10 kW from the motor *M* to a machine tool connected to gear *D*. Knowing that the motor rotates at 240 rpm and that $\tau_{\text{all}} = 60$ MPa, determine the smallest permissible diameter of shaft *ABC*.

SOLUTION

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N} \cdot \text{m}$$

Gear *A*: $Fr_A - T = 0$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$



Bending moment at *B*:

$$M_B = L_{AB}F = (100 \times 10^{-3})(4421) = 442.1 \text{ N} \cdot \text{m}$$

$$\tau_{\text{all}} = \frac{c_o}{J}\sqrt{M^2 + T^2} \qquad c_o = \frac{1}{2}d_o = 25 \times 10^{-3} \text{m}$$

$$\frac{J}{c_o} = \frac{\pi}{2}\frac{(c_o^4 - c_i^4)}{c_o} = \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}}$$

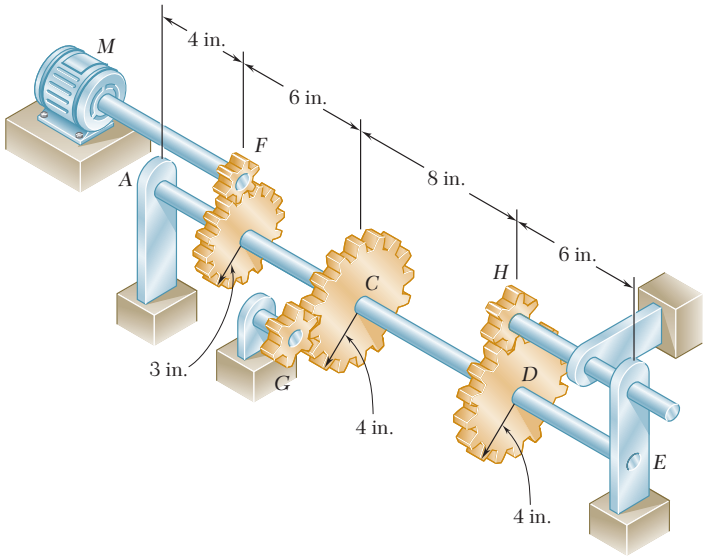
$$c_i^4 = c_o^4 - \frac{2c_o\sqrt{M^2 + T^2}}{\pi\tau_{\text{all}}}$$

$$= (25 \times 10^{-3})^4 - \frac{(2)(25 \times 10^{-3})\sqrt{442.1^2 + 397.89^2}}{\pi(60 \times 10^6)}$$

$$= 390.625 \times 10^{-9} - 157.772 \times 10^{-9} = 232.85 \times 10^{-9}$$

$$c_i = 21.967 \times 10^{-3} \text{m} \qquad d_i = 2c_i = 43.93 \times 10^{-3} \text{m} \qquad d = 43.9 \text{ mm} \blacktriangleleft$$

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PROBLEM 8.29

The solid shaft *AE* rotates at 600 rpm and transmits 60 hp from the motor *M* to machine tools connected to gears *G* and *H*. Knowing that $\tau_{all} = 8$ ksi and that 40 hp is taken off at gear *G* and 20 hp is taken off at gear *H*, determine the smallest permissible diameter of shaft *AE*.

SOLUTION

$$60 \text{ hp} = (60)(6600) = 396 \times 10^3 \text{ in.} \cdot \text{lb/sec}$$

$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

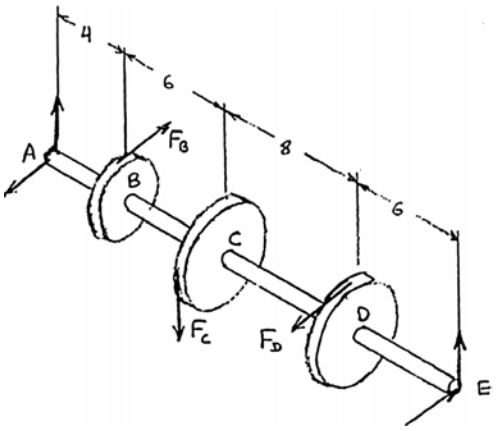
Torque on gear *B*:

$$T_B = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(10)} = 6302.5 \text{ lb} \cdot \text{in.}$$

Torques on gears *C* and *D*:

$$T_C = \frac{40}{60} T_B = 4201.7 \text{ lb} \cdot \text{in.}$$

$$T_D = \frac{20}{60} T_B = 2100.8 \text{ lb} \cdot \text{in.}$$



Shaft torques:

AB: $T_{AB} = 0$
BC: $T_{BC} = 6302.5 \text{ lb} \cdot \text{in.}$
CD: $T_{CD} = 2100.8 \text{ lb} \cdot \text{in.}$
DE: $T_{DE} = 0$

Gear forces:

$$F_B = \frac{T_B}{r_B} = \frac{6302.5}{3} = 2100.8 \text{ lb}$$

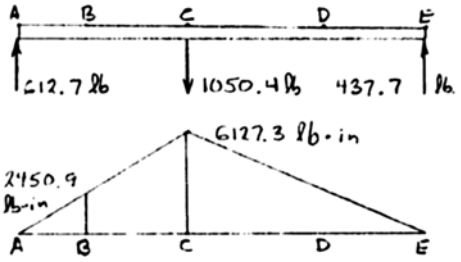
$$F_C = \frac{T_C}{r_C} = \frac{4201.7}{4} = 1050.4 \text{ lb}$$

$$F_D = \frac{T_D}{r_D} = \frac{2100.8}{4} = 525.2 \text{ lb}$$

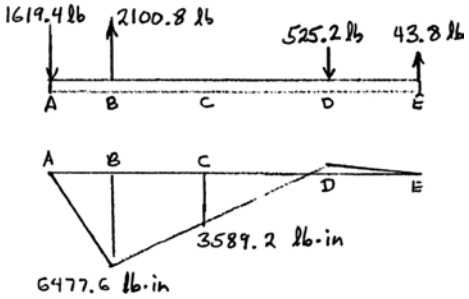
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PROBLEM 8.29 *(Continued)*

Forces in vertical plane:



Forces in horizontal plane:



$$\text{At } B^+,\ \sqrt{M_z^2 + M_y^2 + T^2} = \sqrt{2450.9^2 + 6477.6^2 + 6302.5^2}$$

$$= 9364 \text{ lb} \cdot \text{in.}$$

$$\text{At } C^-,\ \sqrt{M_z^2 + M_y^2 + T^2} = \sqrt{6127.3^2 + 3589.2^2 + 6302.5^2}$$

$$= 9495 \text{ lb} \cdot \text{in.} \quad \text{(maximum)}$$

$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)_{\max}$$

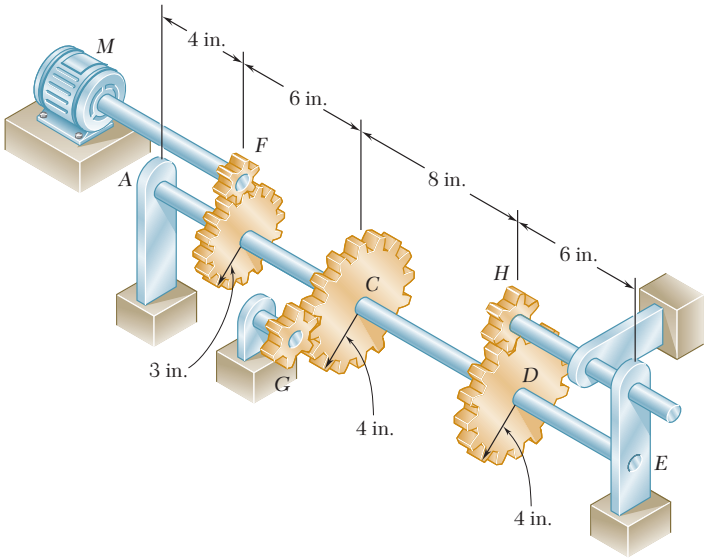
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)}{\tau_{\text{all}}}$$

$$= \frac{9495}{8 \times 10^3} = 1.1868 \text{ in}^3$$

$$c = 0.911 \text{ in.} \qquad d = 2c$$

$$d = 1.822 \text{ in.} \blacktriangleleft$$

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PROBLEM 8.30

Solve Prob. 8.29, assuming that 30 hp is taken off at gear *G* and 30 hp is taken off at gear *H*.

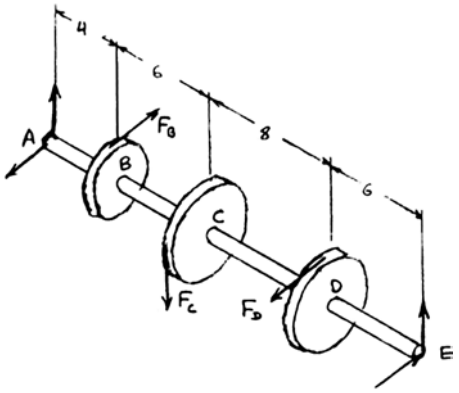
PROBLEM 8.29 The solid shaft *AE* rotates at 600 rpm and transmits 60 hp from the motor *M* to machine tools connected to gears *G* and *H*. Knowing that $\tau_{all} = 8$ ksi and that 40 hp is taken off at gear *G* and 20 hp is taken off at gear *H*, determine the smallest permissible diameter of shaft *AE*.

SOLUTION

$$60 \text{ hp} = (60)(6600)$$
$$= 396 \times 10^3 \text{ in.} \cdot \text{lb/sec}$$
$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

Torque on gear *B*:

$$T_B = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(10)} = 6302.5 \text{ lb} \cdot \text{in.}$$



Torques on gears *C* and *D*:

$$T_C = \frac{30}{60} T_B = 3151.3 \text{ lb} \cdot \text{in.}$$
$$T_D = \frac{30}{60} T_B = 3151.3 \text{ lb} \cdot \text{in.}$$

Shaft torques:

$$AB: \quad T_{AB} = 0$$
$$BC: \quad T_{BC} = 6302.5 \text{ lb} \cdot \text{in.}$$
$$CD: \quad T_{CD} = 3151.3 \text{ lb} \cdot \text{in.}$$
$$DE: \quad T_{DE} = 0$$

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PROBLEM 8.30 *(Continued)*

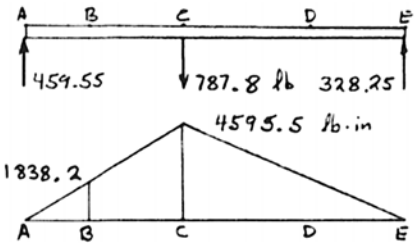
Gear forces:

$$F_B = \frac{T_B}{r_B} = \frac{6302.5}{3} = 2100.8 \text{ lb}$$

$$F_C = \frac{T_C}{r_C} = \frac{3151.3}{4} = 787.8 \text{ lb}$$

$$F_D = \frac{T_D}{r_D} = \frac{3151.3}{4} = 787.8 \text{ lb}$$

Forces in vertical plane:



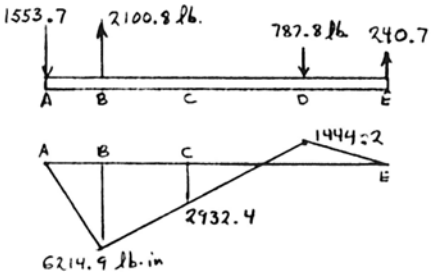
At B^+, $\sqrt{M_z^2 + M_y^2 + T^2} = \sqrt{1838.2^2 + 6214.9^2 + 6302.5^2}$

$= 9040.2 \text{ lb} \cdot \text{in.}$ (maximum)

At C^-, $\sqrt{M_z^2 + M_y^2 + T^2} = \sqrt{4595.5^2 + 2932.4^2 + 6302.5^2}$

$= 8333.0 \text{ lb} \cdot \text{in.}$

Forces in horizontal plane:



$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)_{\max}$$

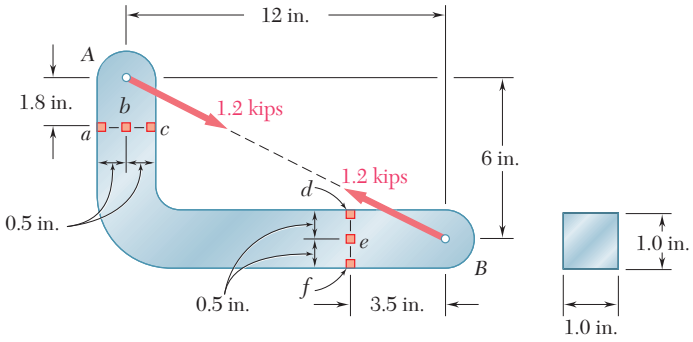
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)}{\tau_{\text{all}}}$$

$$= \frac{9040.3}{8 \times 10^3} = 1.1300 \text{ in}^3$$

$c = 0.8960$ in. $\quad d = 2c$

$d = 1.792$ in. ◀

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PROBLEM 8.31

Two 1.2-kip forces are applied to an L-shaped machine element *AB* as shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Let β be the slope angle of line *AB*.

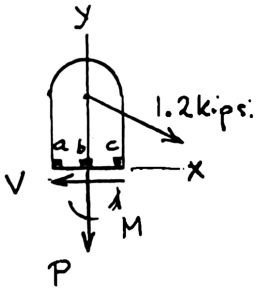
$$\tan\beta = \frac{6}{12} \qquad \beta = 26.565^\circ$$

Draw a free body sketch of the portion of the machine element lying above section abc.

$$P = -(1.2)\sin\beta = -0.53666 \text{ kips}$$

$$V = 1.2\cos\beta = 1.07331 \text{ kips}$$

$$M = (1.8)(1.2\cos\beta) = 1.93196 \text{ kip} \cdot \text{in.}$$



Section properties:

$$A = (1.0)^2 = 1.0 \text{ in}^2$$

$$I = \frac{1}{12}(1.0)(1.0)^3 = 0.083333 \text{ in}^4$$

$$c = 0.5 \text{ in.}$$

(*a*) Point *a*:

$$\sigma = \frac{P}{A} - \frac{Mx}{I} = \frac{-0.53666}{1.0} - \frac{(1.93196)(-0.5)}{0.083333}$$

$\sigma = 11.06$ ksi ◀

$\tau = 0$ ◀

(*b*) Point *b*:

$$\sigma = \frac{P}{A} = \frac{-0.53666}{1.0}$$

$\sigma = -0.537$ ksi ◀

$$Q = (0.5)(1.0)(0.25) = 0.125 \text{ in}^3$$

$$\tau = \frac{VQ}{It} = \frac{(1.07331)(0.125)}{(0.083333)(1.0)}$$

$\tau = 1.610$ ksi ◀

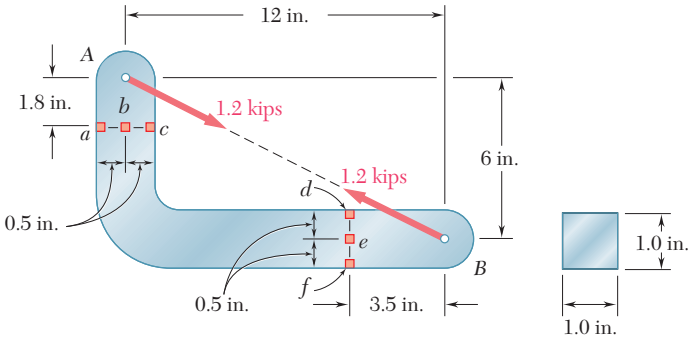
(*c*) Point *c*:

$$\sigma = \frac{P}{A} - \frac{Mx}{I} = \frac{-0.53660}{1.0} - \frac{(1.93196)(0.5)}{0.083333}$$

$\sigma = -12.13$ ksi ◀

$\tau = 0$ ◀

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PROBLEM 8.32

Two 1.2-kip forces are applied to an L-shaped machine element *AB* as shown. Determine the normal and shearing stresses at (*a*) point *d*, (*b*) point *e*, (*c*) point *f*.

SOLUTION

Let β be the slope angle of line *AB*.

$$\tan\beta = \frac{6}{12} \qquad \beta = 26.535^\circ$$

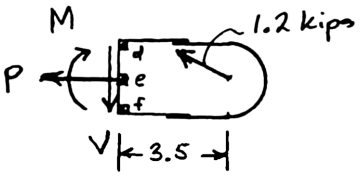
Draw a free body sketch of the portion of the machine element lying to the right of section *def*.

$$P = -(1.2)\cos\beta$$

$$= -1.07331 \text{ kips}$$

$$V = 1.2\sin\beta = 0.53666 \text{ kips}$$

$$M = (3.5)(1.2\sin\beta) = 1.87831 \text{ kip} \cdot \text{in.}$$



Section properties:

$$A = (1.0)^2 = 1.0 \text{ in}^2$$

$$I = \frac{1}{12}(1.0)(1.0)^3 = 0.083333 \text{ in}^4$$

$$c = 0.5 \text{ in.}$$

(*a*) Point *d*:

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{-1.07331}{1.0} - \frac{(1.87831)(0.5)}{0.083333}$$

$$\sigma = -12.34 \text{ ksi} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

(*b*) Point *e*:

$$\sigma = \frac{P}{A} = \frac{-1.07331}{1.0} \qquad \sigma = -1.073 \text{ ksi} \blacktriangleleft$$

$$Q = (0.5)(1.0)(0.25) = 0.125 \text{ in}^3$$

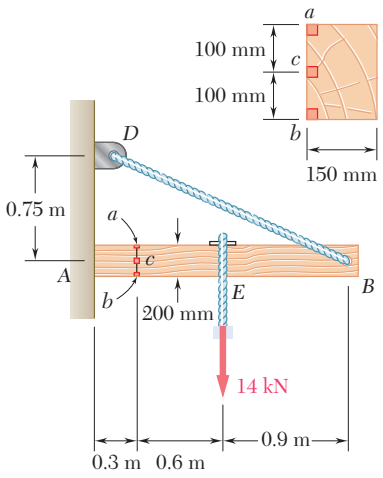
$$\tau = \frac{VQ}{It} = \frac{(0.53666)(0.125)}{(0.083333)(1.0)} \qquad \tau = 0.805 \text{ ksi} \blacktriangleleft$$

(*c*) Point *f*:

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{-1.07331}{1.0} - \frac{(1.87831)(-0.5)}{0.083333} \qquad \sigma = 10.20 \text{ ksi} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

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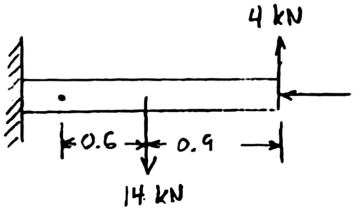
PROBLEM 8.33

The cantilever beam *AB* has a rectangular cross section of 150 × 200 mm. Knowing that the tension in the cable *BD* is 10.4 kN and neglecting the weight of the beam, determine the normal and shearing stresses at the three points indicated.

SOLUTION

$$\overline{DB} = \sqrt{0.75^2 + 1.8^2}$$

$$= 1.95 \text{ m}$$



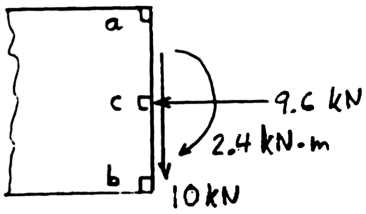
Vertical component of T_{DB}: $\left(\frac{0.75}{1.95}\right)(10.4) = 4 \text{ kN}$

Horizontal components of T_{DB}: $\left(\frac{1.8}{1.95}\right)(10.4) = 9.6 \text{ kN}$

At section containing points *a*, *b*, and *c*,

$$P = -9.6 \text{ kN} \qquad V = 14 - 4 = 10 \text{ kN}$$

$$M = (1.5)(4) - (0.6)(14) = -2.4 \text{ kN} \cdot \text{m}$$



Section properties:

$$A = (0.150)(0.200) = 0.030 \text{ m}^2$$

$$I = \frac{1}{12}(0.150)(0.200)^3 = 100 \times 10^{-6} \text{ m}^2$$

$$c = 0.100 \text{ m}$$

At point *a*, $\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{9.6 \times 10^3}{0.030} + \frac{(2.4 \times 10^3)(0.100)}{100 \times 10^{-6}} = 2.08 \text{ MPa}$ $\quad \sigma_x = 2.08 \text{ MPa}$ ◀

$\tau_{xy} = 0$ ◀

At point *b*, $\sigma_x = -\frac{P}{A} + \frac{Mc}{I} = -\frac{9.6 \times 10^3}{0.030} - \frac{(2.4 \times 10^3)(0.100)}{100 \times 10^{-6}} = -2.72 \text{ MPa}$ $\quad \sigma_x = -2.72 \text{ MPa}$ ◀

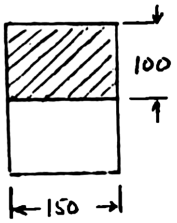
$\tau_{xy} = 0$ ◀

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PROBLEM 8.33 *(Continued)*

At point *c*,

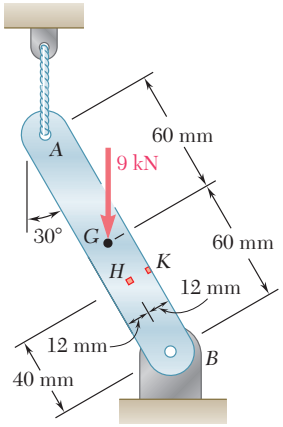
$$\sigma_x = -\frac{P}{A} = -\frac{9.6 \times 10^3}{0.030} = -0.320 \text{ MPa}$$ ◀



$$Q = (150)(100)(50) = 750 \times 10^3 \text{ mm}^3 = 750 \times 10^{-6} \text{ m}^3$$

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(10 \times 10^3)(750 \times 10^{-6})}{(100 \times 10^{-6})(0.150)} = -0.500 \text{ MPa}$$ ◀

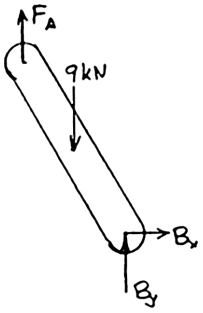
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PROBLEM 8.34

Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (*a*) point H, (*b*) point K.

SOLUTION



$$\pm \rightarrow \Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(120 \sin 30°) - 9(60 \sin 30°) = 0$$

$$B_y = 4.5 \text{ kN}$$

At the section containing points H and K,



$$P = 4.5 \cos 30° = 3.897 \text{ kN}$$

$$V = 4.5 \sin 30° = 2.25 \text{ kN}$$

$$M = (4.5 \times 10^3)(40 \times 10^{-3} \sin 30°)$$

$$= 90 \text{ N} \cdot \text{m}$$

$$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4$$

$$= 11.52 \times 10^{-9} \text{ m}^4$$

(*a*) At point H,

$$\sigma_x = -\frac{P}{A} = -\frac{3.897 \times 10^3}{240 \times 10^{-6}}$$

$$\sigma = -16.24 \text{ MPa} \blacktriangleleft$$

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25 \times 10^3}{240 \times 10^{-6}}$$

$$\tau = 14.06 \text{ MPa} \blacktriangleleft$$

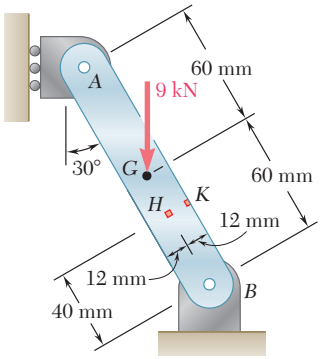
(*b*) At point K,

$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{3.897 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$$

$$\sigma = 110.0 \text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

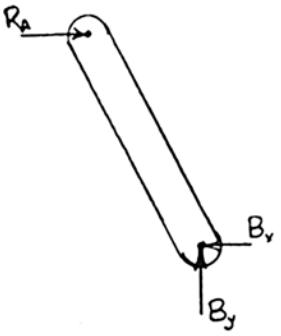
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PROBLEM 8.35

Member *AB* has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (*a*) point *H*, (*b*) point *K*.

SOLUTION



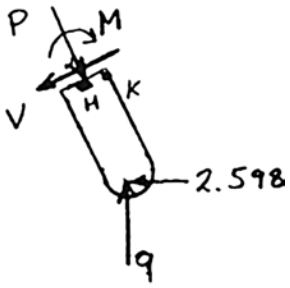
$$+\circlearrowleft \Sigma M_B = 0: \quad (120\cos 30°)R_A - (60\sin 30°)(9) = 0$$

$$R_A = 2.598 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 9 = 0 \qquad B_y = 9 \text{ kN} \uparrow$$

$$\rightarrow + \Sigma F_x = 0: \quad 2.598 - B_x = 0 \quad B_x = 2.598 \text{ kN} \leftarrow$$

At the section containing points *H* and *K*,



$$P = 9\cos 30° + 2.598 \sin 30° = 9.093 \text{ kN}$$

$$V = 9\sin 30° - 2.598\cos 30° = 2.25 \text{ kN}$$

$$M = (9\times10^{3})(40\times10^{-3}\sin 30°) - (2.598\times10^{3})(40\times10^{-3}\cos 30°)$$

$$= 90 \text{ N}\cdot\text{m}$$

$$A = 10\times240 = 240 \text{ mm}^2 = 240\times10^{-6}\text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52\times10^{3}\text{ mm}^4 = 11.52\times10^{-9}\text{ m}^4$$

(*a*) At point *H*,

$$\sigma_x = -\frac{P}{A} = -\frac{9.093\times10^{3}}{240\times10^{-6}} \qquad \sigma = -37.9 \text{ MPa} \blacktriangleleft$$

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25\times10^{3}}{240\times10^{-6}} \qquad \tau = 14.06 \text{ MPa} \blacktriangleleft$$

(*b*) At point *K*,

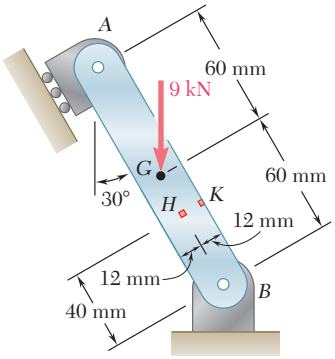
$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I}$$

$$= -\frac{9.093\times10^{3}}{240\times10^{-6}} - \frac{(90)(12\times10^{-3})}{11.52\times10^{-9}}$$

$$\sigma = -131.6 \text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

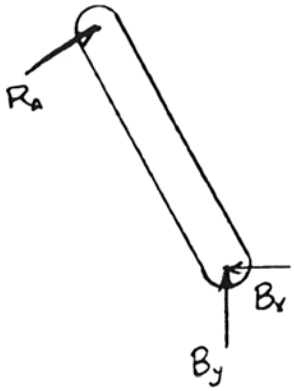
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PROBLEM 8.36

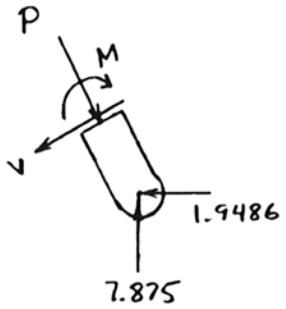
Member *AB* has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (*a*) point *H*, (*b*) point *K*.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad (9)(60\sin 30°) - 120R_A = 0$$
$$R_A = 2.25 \text{ kN}$$
$$\rightarrow + \Sigma F_x = 0: \quad 2.25\cos 30° - B_x = 0$$
$$B_x = 1.9486 \text{ kN} \leftarrow$$
$$+\uparrow \Sigma F_y = 0: \quad 2.25\sin 30° - 9 + B_y = 0$$
$$B_y = 7.875 \text{ kN} \uparrow$$

At the section containing points *H* and *K*,



$$P = 7.875\cos 30° + 1.9486\sin 30° = 7.794 \text{ kN}$$
$$V = 7.875\sin 30° - 1.9486\cos 30° = 2.25 \text{ kN}$$
$$M = (7.875\times10^{3})(40\times10^{-3}\sin 30°) - (1.9486\times10^{3})(40\times10^{-3}\cos 30°)$$
$$= 90 \text{ N}\cdot\text{m}$$
$$A = 10\times 24 = 240 \text{ mm}^2 = 240\times10^{-6}\text{ m}^2$$
$$I = \frac{1}{12}(10)(24)^3 = 11.52\times10^{3}\text{ mm}^4 = 11.52\times10^{-9}\text{ m}^4$$

(*a*) At point *H*,

$$\sigma_x = -\frac{P}{A} = -\frac{7.794\times10^{3}}{240\times10^{-6}}$$

$\sigma_x = -32.5$ MPa ◀

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25\times10^{3}}{240\times10^{-6}}$$

$\tau_{xy} = 14.06$ MPa ◀

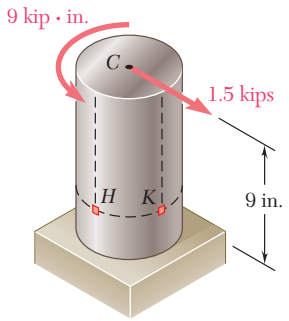
(*b*) At point *K*,

$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{7.794\times10^{3}}{240\times10^{-6}} - \frac{(90)(12\times10^{-3})}{11.52\times10^{-9}}$$

$\sigma_x = -126.2$ MPa ◀

$\tau_{xy} = 0$ ◀

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PROBLEM 8.37

A 1.5-kip force and a 9-kip · in. couple are applied at the top of the 2.5-in.-diameter cast-iron post shown. Determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.

SOLUTION

Diameter $= 2.5$ in.

At the section containing points *H* and *K*,

$$P = 0 \qquad V = 1.5 \text{ kips}$$

$$T = 9 \text{ kip} \cdot \text{in.} \qquad M = (1.5)(9) = 13.5 \text{ kip} \cdot \text{in.}$$

$$d = 2.5 \text{ in.} \qquad c = \frac{1}{2}d = 1.25 \text{ in.}$$

$$A = \pi c^2 = 4.909 \text{ in}^2 \qquad I = \frac{\pi}{4}c^4 = 1.9175 \text{ in}^4 \qquad J = 2I = 3.835 \text{ in}^4$$

For a semicircle, $$Q = \frac{2}{3}c^3 = 1.3021 \text{ in}^3$$

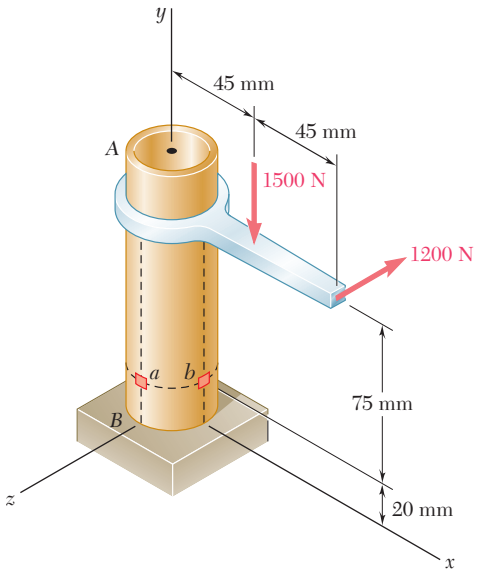
(*a*) At point *H*, $\sigma_H = 0$ ◀

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(9)(1.25)}{3.835} + \frac{(1.5)(1.3021)}{(1.9175)(2.5)} = 2.934 + 0.407 = 3.34 \text{ ksi} \qquad \tau_H = 3.34 \text{ ksi} \blacktriangleleft$$

(*b*) At point *K*, $$\sigma_K = -\frac{Mc}{I} = -\frac{(13.5)(1.25)}{1.9175} = -8.80 \text{ ksi} \qquad \sigma_K = -8.80 \text{ ksi} \blacktriangleleft$$

$$\tau_K = \frac{Tc}{J} = \frac{(9)(1.25)}{3.835} = 2.93 \text{ ksi} \qquad \tau_K = 2.93 \text{ ksi} \blacktriangleleft$$

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PROBLEM 8.38

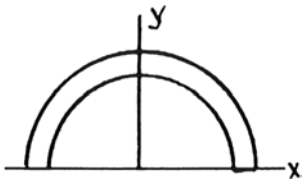
Two forces are applied to the pipe *AB* as shown. Knowing that the pipe has inner and outer diameters equal to 35 and 42 mm, respectively, determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*.

SOLUTION

$$c_o = \frac{d_o}{2} = 21\text{ mm}, \quad c_i = \frac{d_i}{2} = 17.5\text{ mm} \quad A = \pi\left(c_o^2 - c_i^2\right) = 423.33\text{ mm}^2$$

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) = 158.166 \times 10^3 \text{mm}^4 \quad I = \frac{1}{2}J = 79.083 \times 10^3 \text{mm}^4$$

For semicircle with semicircular cutout,



$$Q = \frac{2}{3}\left(c_o^3 - c_i^3\right) = 2.6011 \times 10^3 \text{mm}^3$$

At the section containing points *a* and *b*,

$$P = -1500\text{ N} \quad V_z = -1200\text{ N} \quad V_x = 0$$

$$M_z = -(45 \times 10^{-3})(1500) = -67.5\text{ N} \cdot \text{m}$$

$$M_x = -(75 \times 10^{-3})(1200) = -90\text{ N} \cdot \text{m}$$

$$T = (90 \times 10^{-3})(1200) = 108\text{ N} \cdot \text{m}$$

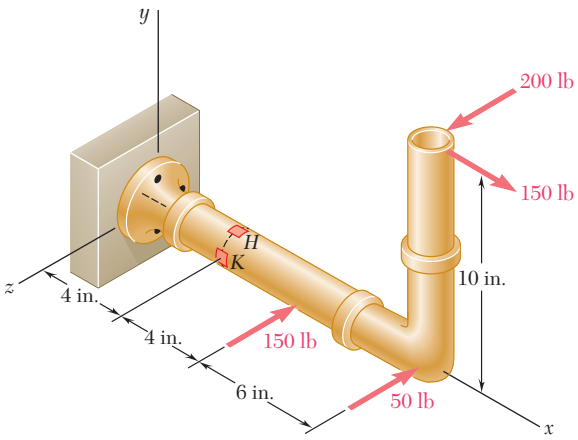
(*a*) $$\sigma = \frac{P}{A} - \frac{M_x c}{I} = \frac{-1500}{423.33 \times 10^{-6}} - \frac{(-90)(21 \times 10^{-3})}{79.083 \times 10^{-9}}$$ $\sigma = 20.4$ MPa ◀

$$\tau = \frac{Tc}{J} + \frac{V_x Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + 0$$ $\tau = 14.34$ MPa ◀

(*b*) $$\sigma = \frac{P}{A} + \frac{M_z c}{I} = \frac{-1500}{423.33 \times 10^{-6}} + \frac{(-67.5)(21 \times 10^{-3})}{79.083 \times 10^{-9}}$$ $\sigma = -21.5$ MPa ◀

$$\tau = \frac{Tc}{J} + \frac{|V_z| Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + \frac{(1200)(2.6011 \times 10^{-6})}{(79.083 \times 10^{-9})(7 \times 10^{-3})}$$ $\tau = 19.98$ MPa ◀

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PROBLEM 8.39

Several forces are applied to the pipe assembly shown. Knowing that the pipe has inner and outer diameters equal to 1.61 in. and 1.90 in., respectively, determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.

SOLUTION

Section properties:

$$P = 150 \text{ lb}$$

$$T = (200 \text{ lb})(10 \text{ in.}) = 2000 \text{ lb} \cdot \text{in.}$$

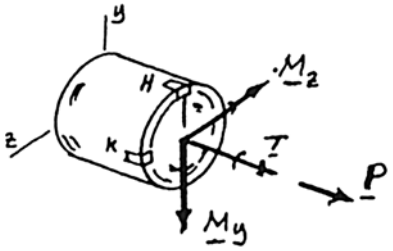
$$M_z = (150 \text{ lb})(10 \text{ in.}) = 1500 \text{ lb} \cdot \text{in.}$$

$$M_y = (200 \text{ lb} - 50 \text{ lb})(10 \text{ in.}) - (150 \text{ lb})(4 \text{ in.})$$

$$= 900 \text{ lb} \cdot \text{in.}$$

$$V_z = 200 - 150 - 50 = 0$$

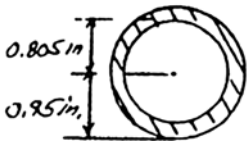
$$V_y = 0$$



$$A = \pi(0.95^2 - 0.805^2) = 0.79946 \text{ in}^2$$

$$I = \frac{\pi}{4}(0.95^4 - 0.805^4) = 0.30989 \text{ in}^4$$

$$J = 2I = 0.61979 \text{ in}^4$$



(*a*) Point *H*:

$$\sigma_H = \frac{P}{A} + \frac{M_z c}{I} = \frac{150 \text{ lb}}{0.79946 \text{ in}^2} + \frac{(1500 \text{ lb} \cdot \text{in.})(0.95 \text{ in.})}{0.30989 \text{ in}^4}$$

$$= 187.6 \text{ psi} + 4593 \text{ psi}$$

$$\sigma_H = 4.79 \text{ ksi} \blacktriangleleft$$

$$\tau_H = \frac{Tc}{J} = \frac{(2000 \text{ lb} \cdot \text{in.})(0.95 \text{ in.})}{0.61979 \text{ in}^4} = 3065.6 \text{ psi}$$

$$\tau_H = 3.07 \text{ ksi} \blacktriangleleft$$

(*b*) Point *K*:

$$\sigma_K = \frac{P}{A} + \frac{M_y c}{I} = \frac{150 \text{ lb}}{0.79946 \text{ in}^2} - \frac{(900 \text{ lb} \cdot \text{in.})(0.95 \text{ in.})}{0.30989 \text{ in}^4}$$

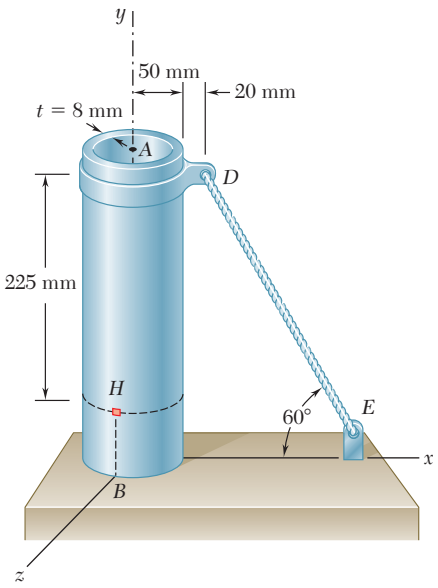
$$= 187.6 \text{ psi} - 2759 \text{ psi}$$

$$\sigma_K = -2.57 \text{ ksi} \blacktriangleleft$$

$$\tau_K = \frac{Tc}{J} = \text{ same as for } \tau_H$$

$$\tau_K = 3.07 \text{ ksi} \blacktriangleleft$$

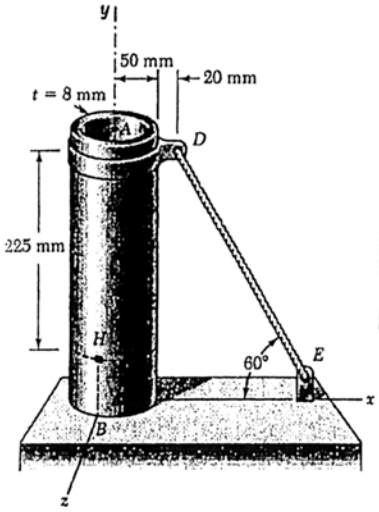
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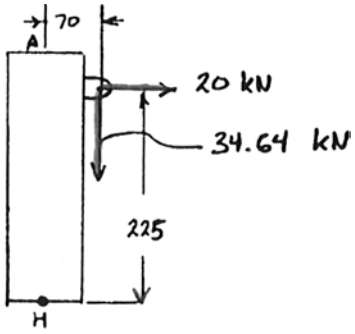


PROBLEM 8.40

The steel pipe *AB* has a 100-mm outer diameter and an 8-mm wall thickness. Knowing that the tension in the cable is 40 kN, determine the normal and shearing stresses at point *H*.

SOLUTION





Vertical force:

$$40\cos 30^\circ = 34.64 \text{ kN}$$

Horizontal force:

$$40\sin 30^\circ = 20 \text{ kN}$$

Point *H* lies on neutral axis of bending.

Section properties:

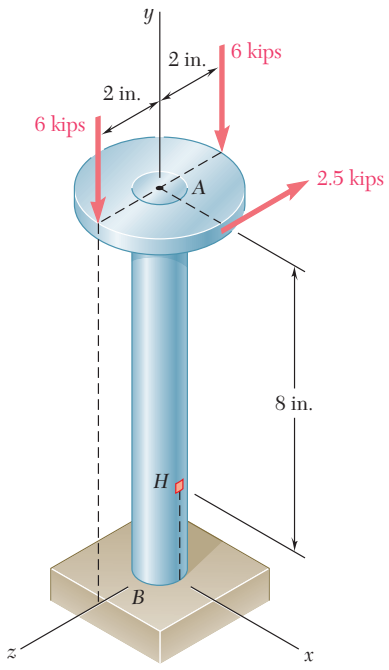
$$d_o = 100 \text{ mm}, \qquad c_o = \frac{1}{2}d_o = 50 \text{ mm}, \qquad c_i = c_o - t = 42 \text{ mm},$$

$$A = \pi(c_o^2 - c_i^2) = 2.312 \times 10^3 \text{ mm}^2 = 2.312 \times 10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} = -\frac{34.64 \times 10^3}{2.312 \times 10^{-6}} \qquad \sigma = -14.98 \text{ MPa} \blacktriangleleft$$

For thin pipe, $\tau = 2\dfrac{V}{A} = \dfrac{(2)(20 \times 10^3)}{2.314 \times 10^{-3}}$ $\qquad \tau = 17.29 \text{ MPa} \blacktriangleleft$

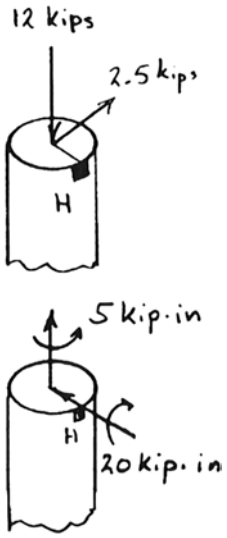
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PROBLEM 8.41

Three forces are applied to a 4-in.-diameter plate that is attached to the solid 1.8-in. diameter shaft *AB*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.

SOLUTION



At the section containing point *H*,

$$P = 12 \text{ kips} \quad \text{(compression)}$$

$$V = 2.5 \text{ kips}$$

$$T = (2)(2.5) = 5 \text{ kip} \cdot \text{in.}$$

$$M = (8)(2.5) = 20 \text{ kip} \cdot \text{in.}$$

$$d = 1.8 \text{ in.} \quad c = \frac{1}{2}d = 0.9 \text{ in.}$$

$$A = \pi c^2 = 2.545 \text{ in}^2$$

$$I = \frac{\pi}{4}c^4 = 0.5153 \text{ in}^4$$

$$J = 2I = 1.0306 \text{ in}^4$$

For a semicircle,

$$Q = \frac{2}{3}c^3 = 0.486 \text{ in}^3$$

Point *H* lies on neutral axis of bending.

$$\sigma_H = \frac{P}{A} = -\frac{12}{2.545} = -4.715 \text{ ksi}$$

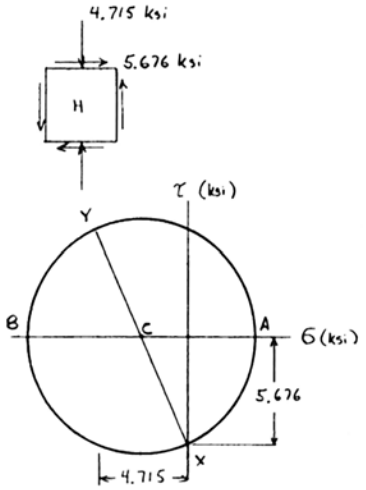
$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(5)(0.9)}{1.0306} + \frac{(2.5)(0.486)}{(0.5153)(1.8)}$$

$$= 5.676 \text{ ksi}$$

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PROBLEM 8.41 *(Continued)*

Use Mohr's circle.



$$\sigma_{\text{ave}} = \frac{1}{2}(-4.715)$$
$$= -2.3575 \text{ ksi}$$
$$R = \sqrt{\left(\frac{4.715}{2}\right)^2 + 5.676^2}$$
$$= 6.1461 \text{ ksi}$$

(*a*) $\sigma_a = \sigma_{\text{ave}} + R$ $\quad \sigma_a = 3.79 \text{ ksi}$ ◀

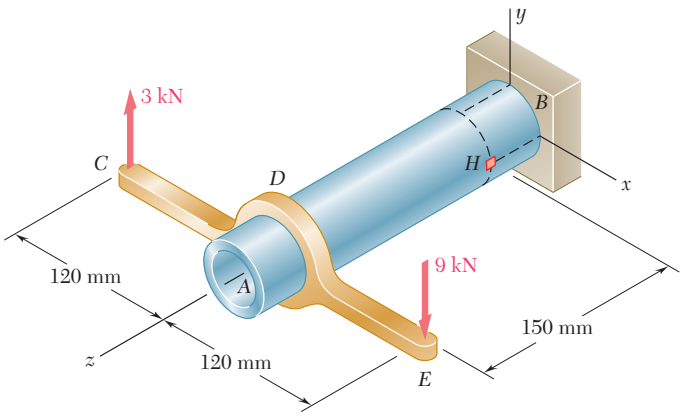
$\sigma_b = \sigma_{\text{ave}} - R$ $\quad \sigma_b = -8.50 \text{ ksi}$ ◀

$\tan 2\theta_p = \dfrac{(2)(5.676)}{4.715} = 2.408$ $\quad \theta_a = 33.7°$ ◀

$\theta_b = 123.7°$ ◀

(*b*) $\tau_{\max} = R$ $\quad \tau_{\max} = 6.15 \text{ ksi}$ ◀

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PROBLEM 8.42

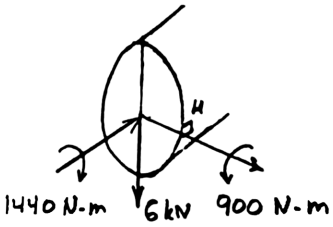
The steel pipe *AB* has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that the arm *CDE* is rigidly attached to the pipe, determine the principal stresses, principal planes, and the maximum shearing stress at point *H*.

SOLUTION

Replace the forces at *C* and *E* by an equivalent force-couple system at *D*.

$$F_D = 9 - 3 = 6\ \text{kN}\downarrow$$

$$T_D = (9\times10^{3})(120\times10^{-3}) + (3\times10^{3})(120\times10^{-3}) = 1440\ \text{N}\cdot\text{m}$$



At the section containing point *H*,

$$P = 0 \qquad V = 6\ \text{kN} \qquad T = 1440\ \text{N}\cdot\text{m}$$

$$M = (6\times10^{3})(150\times10^{-3}) = 900\ \text{N}\cdot\text{m}$$

Section properties: $d_o = 72\ \text{mm}$ $c_o = \frac{1}{2}d_o = 36\ \text{mm}$ $c_i = c_o - t = 31\ \text{mm}$

$$A = \pi(c_o^2 - c_i^2) = 1.0524\times10^{3}\ \text{mm}^2 = 1.0524\times10^{-3}\ \text{m}^2$$

$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 593.84\times10^{-3}\ \text{mm}^4 = 593.84\times10^{-9}\ \text{m}^4$$

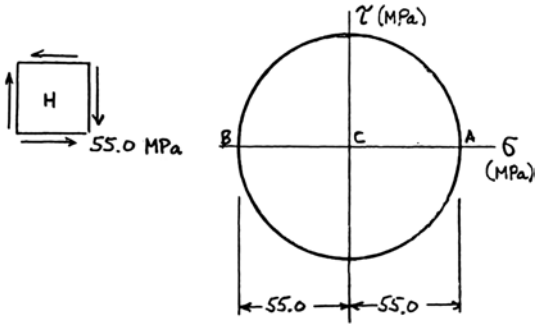
$$J = 2I = 1.1877\times10^{-6}\ \text{m}^4$$

For half-pipe, $$Q = \frac{2}{3}(c_o^3 - c_i^3) = 11.243\times10^{3}\ \text{mm}^3 = 11.243\times10^{-6}\ \text{m}^3$$

At point *H*, point *H* lies on the neutral axis of bending. $\sigma_H = 0$.

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(1440)(36\times10^{-3})}{1.1877\times10^{-6}} + \frac{(6\times10^{3})(11.243\times10^{-6})}{(593.84\times10^{-9})(10\times10^{3})} = 55.0\ \text{MPa}$$

Use Mohr's circle.



$$\sigma_c = 0$$

$$R = 55.0\ \text{MPa}$$

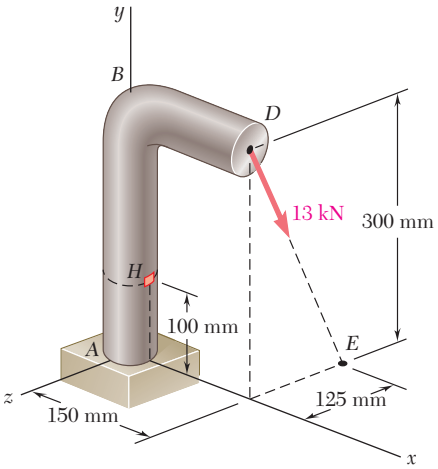
$$\sigma_a = \sigma_c + R \qquad \sigma_a = 55.0\ \text{MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_c - R \qquad \sigma_b = -55.0\ \text{MPa} \blacktriangleleft$$

$$\theta_a = -45°,\ \theta_b = +45° \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 55.0\ \text{MPa} \blacktriangleleft$$

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PROBLEM 8.43

A 13-kN force is applied as shown to the 60-mm-diameter cast-iron post *ABD*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.

SOLUTION

$$DE = \sqrt{125^2 + 300^2} = 325 \text{ mm}$$

At point *D*,

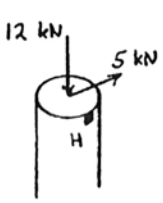
$$F_x = 0$$

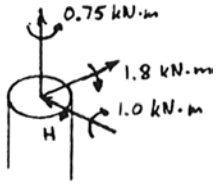
$$F_y = -\left(\frac{300}{325}\right)(13) = -12 \text{ kN}$$

$$F_z = -\left(\frac{125}{300}\right)(13) = -5 \text{ kN}$$

Moment of equivalent force-couple system at *C*, the centroid of the section containing point *H*:

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.150 & 0.200 & 0 \\ 0 & -12 & -5 \end{vmatrix} = -1.00\vec{i} + 0.75\vec{j} - 1.8\vec{k} \text{ kN} \cdot \text{m}$$





Section properties:

$$d = 60 \text{ mm} \quad c = \frac{1}{2}d = 30 \text{ mm}$$

$$A = \pi c^2 = 2.8274 \times 10^3 \text{ mm}^2$$

$$I = \frac{\pi}{4}c^4 = 636.17 \times 10^3 \text{ mm}^4$$

$$J = 2I = 1.2723 \times 10^6 \text{ mm}^4$$

For a semicircle,

$$Q = \frac{2}{3}c^3 = 18.00 \times 10^3 \text{ mm}^3$$

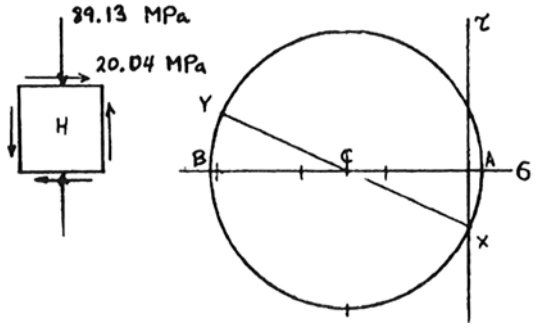
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PROBLEM 8.43 *(Continued)*

At point H,

$$\sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{12 \times 10^3}{2.8274 \times 10^{-3}} - \frac{(1.8 \times 10^3)(30 \times 10^{-3})}{636.17 \times 10^{-9}} = -89.13 \text{ MPa}$$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(0.75 \times 10^3)(30 \times 10^{-3})}{1.2723 \times 10^{-6}} + \frac{(5 \times 10^3)(18.00 \times 10^{-6})}{(636.17 \times 10^{-9})(60 \times 10^{-3})} = 20.04 \text{ MPa}$$



(*a*)

$$\sigma_{\text{ave}} = \frac{\sigma_H}{2} = -44.565 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 48.863 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 4.30 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -93.4 \text{ MPa} \blacktriangleleft$$

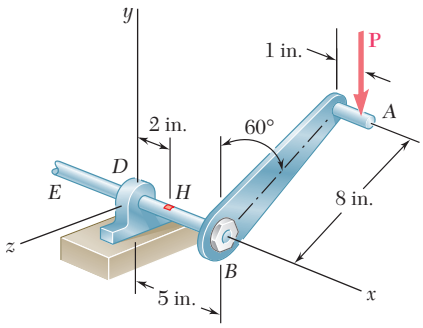
$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = 0.4497$$

$$\theta_a = 12.1^\circ, \quad \theta_b = 102.1^\circ \blacktriangleleft$$

(*b*)

$$\tau_{\max} = R = 48.9 \text{ MPa} \blacktriangleleft$$

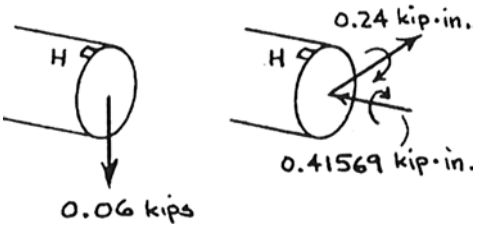
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PROBLEM 8.44

A vertical force **P** of magnitude 60 lb is applied to the crank at point *A*. Knowing that the shaft *BDE* has a diameter of 0.75 in., determine the principal stresses and the maximum shearing stress at point *H* located at the top of the shaft, 2 in. to the right of support *D*.

SOLUTION



Force-couple system at the centroid of the section containing point *H*:

$$F_x = 0, \qquad V_y = -0.06 \text{ kips}, \qquad V_z = 0$$

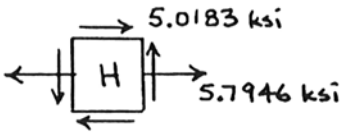
$$M_z = -(5 - 2 + 1)(0.06) = -0.24 \text{ kip} \cdot \text{in.}$$

$$M_x = -(8 \sin 60°)(0.06) = -0.41569 \text{ kip} \cdot \text{in.}$$

$$d = 0.75 \text{ in.} \qquad c = \frac{1}{2} d = 0.375 \text{ in.}$$

$$I = \frac{\pi}{4} c^4 = \frac{\pi}{4} (0.375)^4 = 15.5316 \times 10^{-3} \text{ in}^4$$

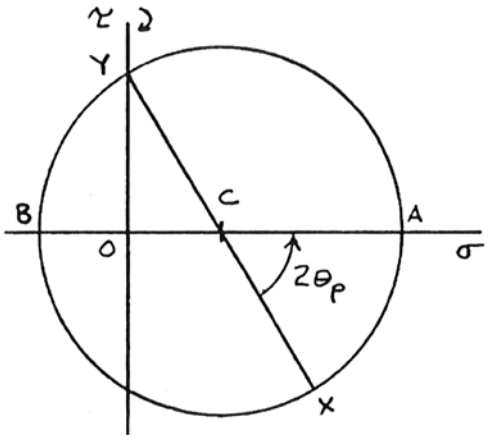
$$J = 2I = 31.063 \times 10^{-3} \text{ in}^4$$



At point *H*,

$$\sigma_H = -\frac{M_z y}{I_z} = -\frac{(-0.24)(0.375)}{15.5316 \times 10^{-3}} = 5.7946 \text{ ksi}$$

$$\tau_H = \frac{Tc}{J} = \frac{(0.41569)(0.375)}{31.063 \times 10^{-3}} = 5.0183 \text{ ksi}$$



Use Mohr's circle. $\sigma_{\text{ave}} = \frac{1}{2} \sigma_H = 2.8973 \text{ ksi}$

$$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 5.7946 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_{\max} = 8.69 \text{ ksi} \blacktriangleleft$$

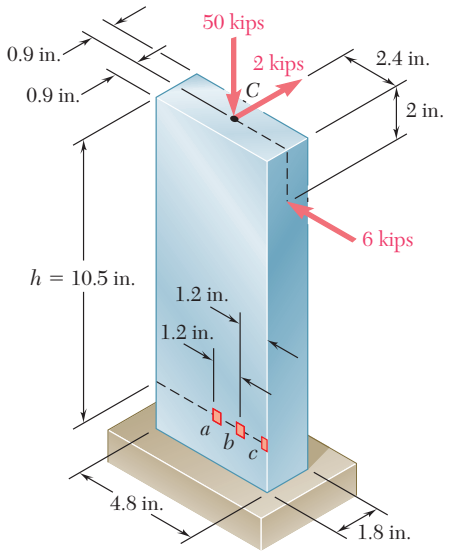
$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_{\min} = -2.90 \text{ ksi} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{2(5.0183)}{5.7946} = 1.7321$$

$$\theta_a = 30.0° \qquad \theta_b = 120.0°$$

$$\tau_{\max} = R \qquad \tau_{\max} = 5.79 \text{ ksi} \blacktriangleleft$$

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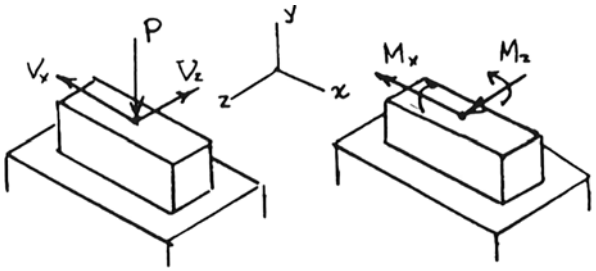


PROBLEM 8.45

Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Calculate forces and couples at section containing points *a*, *b*, and *c*.



Forces Couples

$$h = 10.5 \text{ in.}$$

$$P = 50 \text{ kips} \quad V_x = 6 \text{ kips} \quad V_z = 2 \text{ kips}$$

$$M_z = (10.5 - 2)(6) = 51 \text{ kip} \cdot \text{in.}$$

$$M_x = (10.5)(2) = 21 \text{ kip} \cdot \text{in.}$$

Section properties.

$$A = (1.8)(4.8) = 8.64 \text{ in}^2$$

$$I_x = \frac{1}{12}(4.8)(1.8)^3 = 2.3328 \text{ in}^4$$

$$I_z = \frac{1}{12}(1.8)(4.8)^3 = 16.5888 \text{ in}^4$$

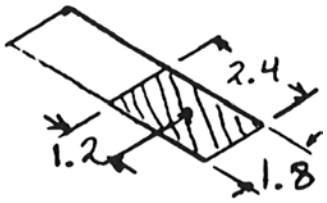
Stresses.

$$\sigma = -\frac{P}{A} + \frac{M_z x}{I_z} + \frac{M_x z}{I_x} \qquad \tau = \frac{V_x Q}{I_z t}$$

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PROBLEM 8.45 *(Continued)*

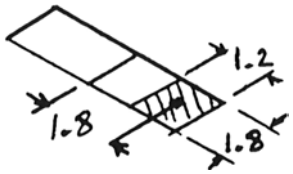
(*a*) Point *a*: $x = 0, \quad z = 0.9 \text{ in.}, \quad Q = (1.8)(2.4)(1.2) = 5.184 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + 0 + \frac{(21)(0.9)}{2.3328} \qquad \sigma = 2.31 \text{ ksi} \blacktriangleleft$$

$$\tau = \frac{(6)(5.184)}{(16.5888)(1.8)} \qquad \tau = 1.042 \text{ ksi} \blacktriangleleft$$

(*b*) Point *b*: $x = 1.2 \text{ in.}, \quad z = 0.9 \text{ in.}, \quad Q = (1.8)(1.2)(1.8) = 3.888 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + \frac{(51)(1.2)}{16.5888} + \frac{(21)(0.9)}{2.3328} \qquad \sigma = 6.00 \text{ ksi} \blacktriangleleft$$

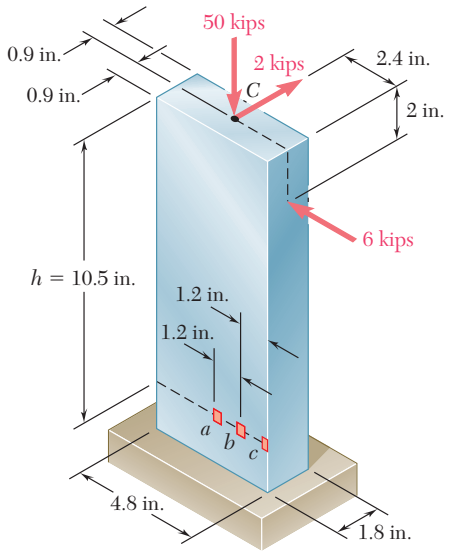
$$\tau = \frac{(6)(3.888)}{(16.5888)(1.8)} = 0.781 \text{ ksi} \qquad \tau = 0.781 \text{ ksi} \blacktriangleleft$$

(*c*) Point *c*: $x = 2.4 \text{ in.}, \quad z = 0.9 \text{ in.}, \quad Q = 0$

$$\sigma = -\frac{50}{8.64} + \frac{(51)(2.4)}{16.5888} + \frac{(21)(0.9)}{2.3328} \qquad \sigma = 9.69 \text{ ksi} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

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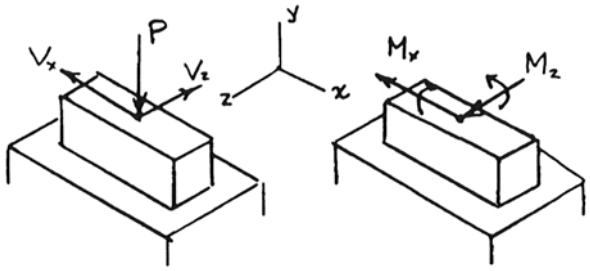
PROBLEM 8.46

Solve Prob. 8.45, assuming that $h = 12$ in.

PROBLEM 8.45 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Calculate forces and couples at section containing points *a*, *b*, and *c*.



Forces Couples

$h = 12$ in.

$P = 50$ kips $\quad V_x = 6$ kips $\quad V_z = 2$ kips

$M_z = (12 - 2)(6) = 60$ kip · in.

$M_x = (12)(2) = 24$ kip · in.

Section properties.

$$A = (1.8)(4.8) = 8.64 \text{ in}^2$$

$$I_x = \frac{1}{12}(4.8)(1.8)^3 = 2.3328 \text{ in}^4$$

$$I_z = \frac{1}{12}(1.8)(4.8)^3 = 16.5888 \text{ in}^4$$

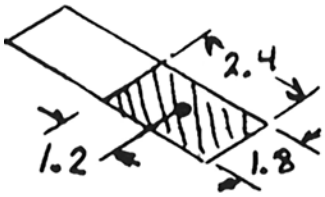
Stresses.

$$\sigma = -\frac{P}{A} + \frac{M_z x}{I_z} + \frac{M_x z}{I_x} \qquad \tau = \frac{V_x Q}{I_z t}$$

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PROBLEM 8.46 *(Continued)*

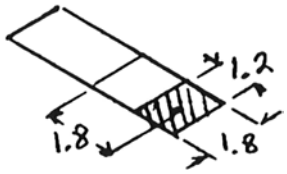
(*a*) Point *a*: $x = 0, \quad z = 0.9 \text{ in.}, \quad Q = (1.8)(2.4)(1.2) = 5.184 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + 0 + \frac{(24)(0.9)}{2.3328} \qquad \sigma = 3.47 \text{ ksi} \blacktriangleleft$$

$$\tau = \frac{(6)(5.184)}{(16.5888)(1.8)} \qquad \tau = 1.042 \text{ ksi} \blacktriangleleft$$

(*b*) Point *b*: $x = 1.2 \text{ in.}, \quad z = 0.9 \text{ in.}, \quad Q = (1.8)(1.2)(1.8) = 3.888 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + \frac{(60)(1.2)}{16.5888} + \frac{(24)(0.9)}{2.3328} \qquad \sigma = 7.81 \text{ ksi} \blacktriangleleft$$

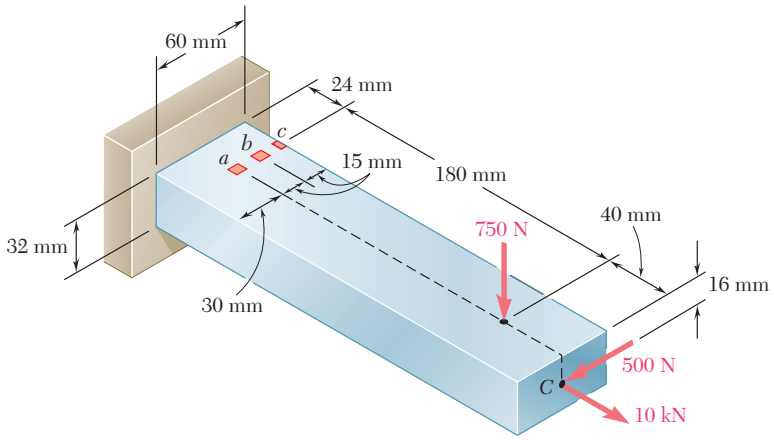
$$\tau = \frac{(6)(3.888)}{(16.5888)(1.8)} \qquad \tau = 0.781 \text{ ksi} \blacktriangleleft$$

(*c*) Point *c*: $x = 2.4 \text{ in.}, \quad z = 0.9 \text{ in.}, \quad Q = 0$

$$\sigma = -\frac{50}{8.64} + \frac{(60)(2.4)}{16.5888} + \frac{(24)(0.9)}{2.3328} \qquad \sigma = 12.15 \text{ ksi} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

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PROBLEM 8.47

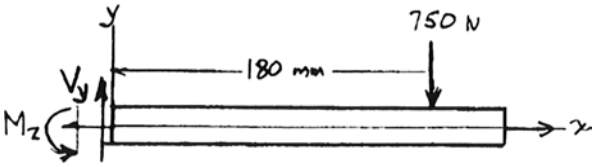
Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

$$A = (60)(32) = 1920 \text{ mm}^2$$
$$= 1920 \times 10^{-6} \text{ m}^2$$
$$I_z = \frac{1}{12}(60)(32)^3 = 163.84 \times 10^3 \text{ mm}^4$$
$$= 163.84 \times 10^{-9} \text{ m}^4$$
$$I_y = \frac{1}{12}(32)(60)^3$$
$$= 579 \times 10^3 \text{ mm}^4$$
$$= 576 \times 10^{-9} \text{ m}^4$$

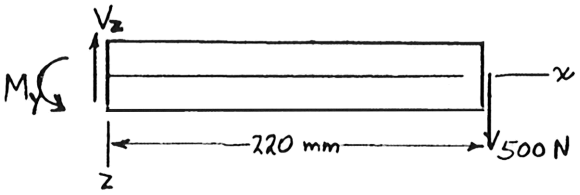
At the section containing points *a*, *b*, and *c*,

$$P = 10 \text{ kN}$$
$$V_y = 750 \text{ N},$$
$$V_z = 500 \text{ N}$$
$$M_z = (180 \times 10^{-3})(750)$$
$$= 135 \text{ N} \cdot \text{m}$$



Side View

$$M_y = (220 \times 10^{-3})(500)$$
$$= 110 \text{ N} \cdot \text{m}$$
$$T = 0$$
$$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$$
$$\tau = \frac{V_z Q}{I_y t}$$

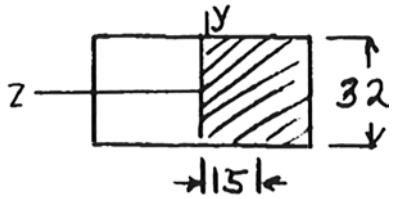


Top View

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PROBLEM 8.47 *(Continued)*

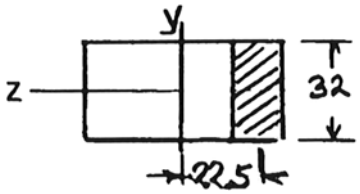
(*a*) Point *a*: $y = 16$ mm, $z = 0$, $Q = A\bar{z} = (32)(30)(15) = 14.4 \times 10^3 \text{ mm}^3$



$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - 0 \qquad \sigma = 18.39 \text{ MPa} \blacktriangleleft$$

$$\tau = \frac{(500)(14.4 \times 10^{-6})}{(576 \times 10^{-9})(32 \times 10^{-3})} \qquad \tau = 0.391 \text{ MPa} \blacktriangleleft$$

(*b*) Point *b*: $y = 16$ mm, $z = -15$ mm, $Q = A\bar{z} = (32)(15)(22.5) = 10.8 \times 10^3 \text{ mm}^3$



$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-15 \times 10^{-3})}{576 \times 10^{-9}} \qquad \sigma = 21.3 \text{ MPa} \blacktriangleleft$$

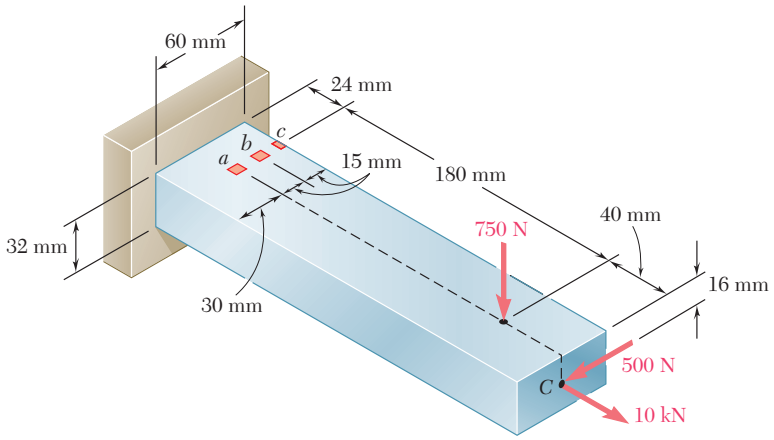
$$\tau = \frac{(500)(10.8 \times 10^{-6})}{(576 \times 10^{-9})(32 \times 10^{-3})} \qquad \tau = 0.293 \text{ MPa} \blacktriangleleft$$

(*c*) Point *c*: $y = 16$ mm, $z = -30$ mm, $Q = 0$

$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-30 \times 10^{-6})}{576 \times 10^{-9}} \qquad \sigma = 24.1 \text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

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PROBLEM 8.48

Solve Prob. 8.47, assuming that the 750-N force is directed vertically upward.

PROBLEM 8.47 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

$$A = (60)(32) = 1920 \text{ mm}^2$$
$$= 1920 \times 10^{-6} \text{ m}^2$$
$$I_z = \frac{1}{12}(60)(32)^3$$
$$= 163.84 \times 10^3 \text{ mm}^4$$
$$= 163.84 \times 10^{-9} \text{ m}^4$$
$$I_y = \frac{1}{12}(32)(60)^3$$
$$= 576 \times 10^3 \text{ mm}^4$$
$$= 576 \times 10^{-9} \text{ m}^4$$

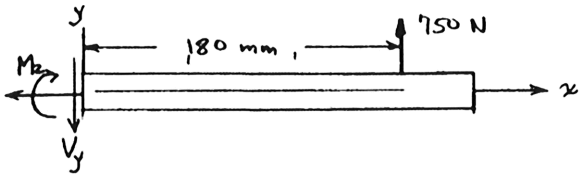
At the section containing points *a*, *b*, and *c*,

$$P = 10 \text{ kN} \qquad T = 0$$
$$V_y = 750 \text{ N}$$
$$V_z = 500 \text{ N}$$
$$M_z = (180 \times 10^{-3})(750)$$
$$= 135 \text{ N} \cdot \text{m}$$

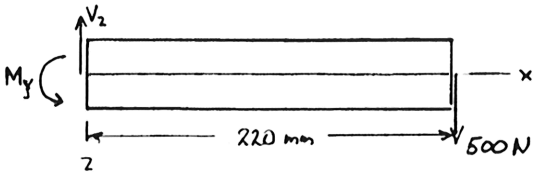
$$M_y = (220 \times 10^{-3})(500)$$
$$= 110 \text{ N} \cdot \text{m}$$

$$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$$

$$\tau = \frac{V_z Q}{I_y t}$$



Side View

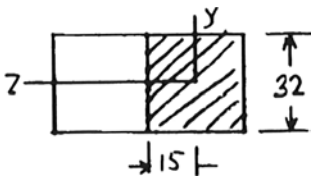


Top View

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PROBLEM 8.48 *(Continued)*

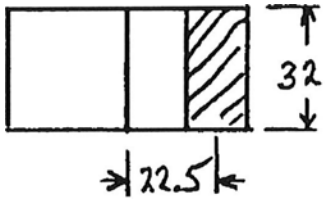
(*a*) Point *a*: $y = 16$ mm, $z = 0$, $Q = A\bar{z} = (32)(30)(15) = 14.4 \times 10^3 \text{ mm}^3$



$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} - \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} = 0 \qquad \sigma = -7.98 \text{ MPa} \blacktriangleleft$$

$$\tau = \frac{(500)(14.4 \times 10^{-6})}{(163.84 \times 10^{-9})(32 \times 10^{-3})} \qquad \tau = 0.391 \text{ MPa} \blacktriangleleft$$

(*b*) Point *b*: $y = 16$ mm, $z = -15$ mm, $Q = A\bar{z} = (32)(15)(22.5) = 10.8 \times 10^3 \text{ mm}^3$



$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} - \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-15 \times 10^{-3})}{576 \times 10^{-9}} \qquad \sigma = -5.11 \text{ MPa} \blacktriangleleft$$

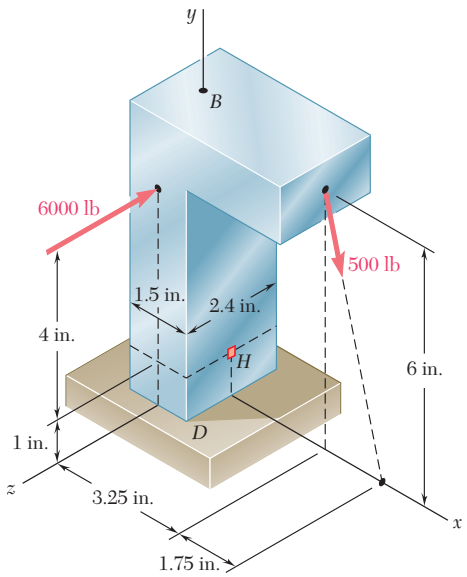
$$\tau = \frac{(500)(10.8 \times 10^{-6})}{(163.84 \times 10^{-9})(32 \times 10^{-9})} \qquad \tau = 0.293 \text{ MPa} \blacktriangleleft$$

(*c*) Point *c*: $y = 16$ mm, $z = -30$ mm, $Q = 0$

$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} - \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-30 \times 10^{-3})}{576 \times 10^{-9}} \qquad \sigma = -2.25 \text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

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PROBLEM 8.49

Two forces are applied to the small post *BD* as shown. Knowing that the vertical portion of the post has a cross section of 1.5 × 2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

SOLUTION

Components of 500-lb force:

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb}$$

Moment arm of 500-lb force:

$$\vec{r} = 3.25\vec{i} + (6 - 1)\vec{j}$$

Moment of 500-lb force:

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260\vec{k} \text{ lb} \cdot \text{in.}$$

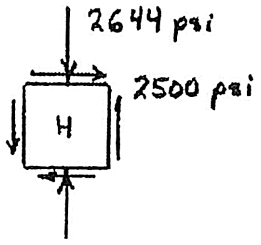
At the section containing point *H*, $P = -480$ lb $V_x = 140$ lb

$V_z = -6000$ lb, $M_z = -2260$ lb · in., $M_x = -(4)(6000) = -24{,}000$ lb · in.

$A = (1.5)(2.4) = 3.6 \text{ in}^2$ $I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$

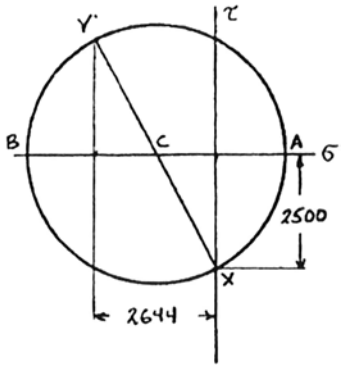
$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{6000}{3.6} = 2500 \text{ psi}$$



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PROBLEM 8.49 *(Continued)*



Use Mohr's circle.

$$\sigma_{\text{ave}} = -\frac{2644}{2} = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (2500)^2} = 2828 \text{ psi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 1506 \text{ psi} \blacktriangleleft$$

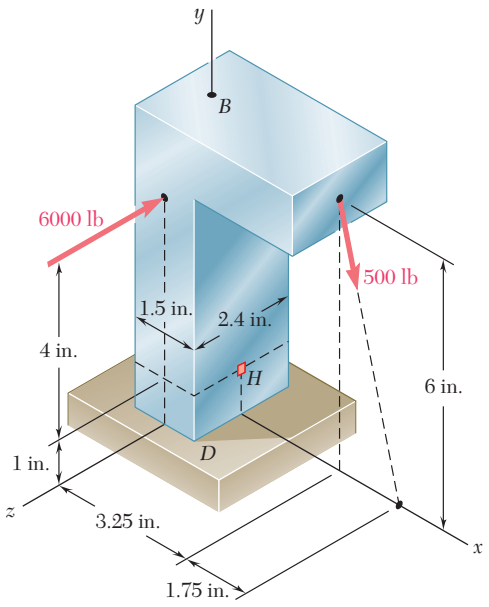
$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -4150 \text{ psi} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(2500)}{2644} = 1.891$$

$$\theta_a = 31.1°, \qquad \theta_b = 121.1° \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 2830 \text{ psi} \blacktriangleleft$$

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PROBLEM 8.50

Solve Prob. 8.49, assuming that the magnitude of the 6000-lb force is reduced to 1500 lb.

PROBLEM 8.49 Two forces are applied to the small post *BD* as shown. Knowing that the vertical portion of the post has a cross section of 1.5×2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

SOLUTION

Components of 500-lb force:

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb}$$

Moment arm of 500-lb force:

$$\vec{r} = 3.25\vec{i} + (6-1)\vec{j}$$

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260\vec{k} \text{ lb} \cdot \text{in.}$$

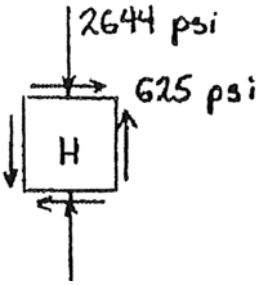
At the section containing point *H*, $P = -480$ lb $V_x = 140$ lb

$V_z = -1500$ lb, $M_z = -2260$ lb · in., $M_x = -(4)(1500) = -6000$ lb · in.

$A = (1.5)(2.4) = 3.6 \text{ in}^2$ $I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$

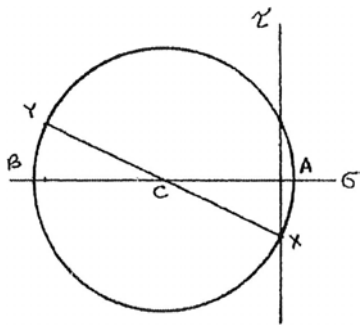
$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{1500}{3.6} = 625 \text{ psi}$$



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PROBLEM 8.50 ***(Continued)***



Use Mohr's circle.

$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_H = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (625)^2} = 1462.30 \text{ psi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 140.3 \text{ psi} \blacktriangleleft$$

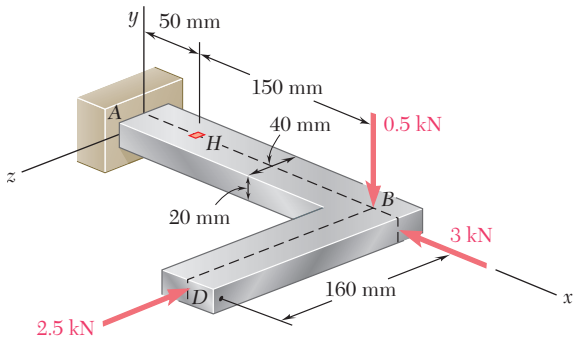
$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -2780 \text{ psi} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(625)}{2644} = 0.4728$$

$$\theta_a = 12.6° \qquad \theta_b = 102.6° \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 1462 \text{ psi} \blacktriangleleft$$

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PROBLEM 8.51

Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a 20×40-mm rectangle, determine the principal stresses and the maximum shearing stress at point *H*.

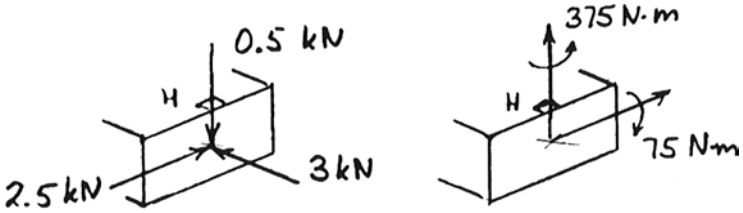
SOLUTION

Equivalent force-couple system at section containing point *H*:

$$F_x = -3\text{ kN}, \quad F_y = -0.5\text{ kN}, \quad F_z = -2.5\text{ kN}$$

$$M_x = 0, \quad M_y = (0.150)(2500) = 375\text{ N}\cdot\text{m}$$

$$M_z = -(0.150)(500) = -75\text{ N}\cdot\text{m}$$



$$A = (20)(40) = 800\text{ mm}^2$$
$$= 800 \times 10^{-6}\text{m}^2$$

$$I_z = \frac{1}{12}(40)(20)^3$$
$$= 26.667 \times 10^{3}\text{mm}^4$$
$$= 26.667 \times 10^{-9}\text{m}^4$$

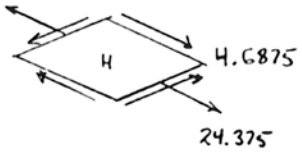
$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z}$$
$$= \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}}$$
$$= 24.375\text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{|V_z|}{A} = \frac{3}{2} \cdot \frac{2500}{800 \times 10^{-6}}$$
$$= 4.6875\text{ MPa}$$

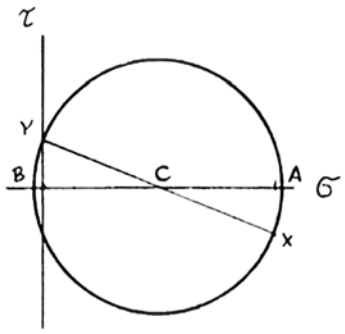
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PROBLEM 8.51 *(Continued)*

Use Mohr's circle.



$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_H$$
$$= 12.1875 \text{ MPa}$$
$$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (4.6875)^2}$$
$$= 13.0579 \text{ MPa}$$



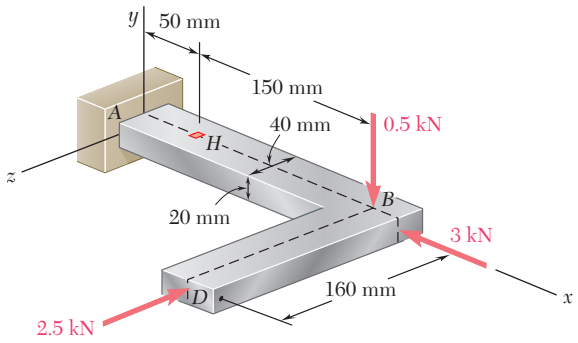
$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 25.2 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -0.87 \text{ MPa} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(4.6875)}{24.375} = 0.3846$$
$$\theta_a = 10.5°, \quad \theta_b = 100.5°$$

$$\tau_{\max} = R \qquad \tau_{\max} = 13.06 \text{ MPa} \blacktriangleleft$$

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PROBLEM 8.52

Solve Prob. 8.51, assuming that the magnitude of the 2.5-kN force is increased to 10 kN.

PROBLEM 8.51 Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a 20×40-mm rectangle, determine the principal stresses and the maximum shearing stress at point *H*.

SOLUTION

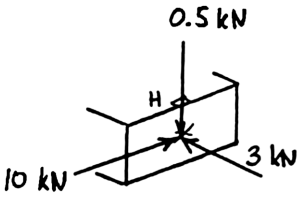
Equivalent force-couple system at section containing point *H*:

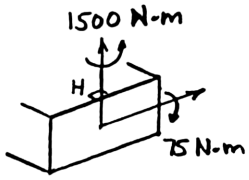
$$F_x = -3\text{ kN}, \quad F_y = -0.5\text{ kN}, \quad F_z = -10\text{ kN}$$

$$M_x = 0, M_y = (0.150)(10{,}000) = 1500\text{ N}\cdot\text{m}$$

$$M_z = -(0.150)(500) = -75\text{ N}\cdot\text{m}$$

$$A = (20)(40) = 800\text{ mm}^2 = 800 \times 10^{-6}\text{ m}^2$$

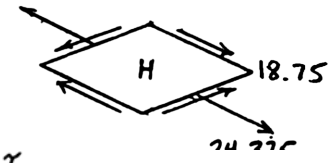


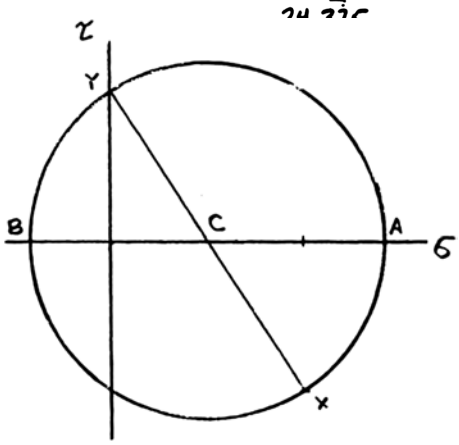


$$I_z = \frac{1}{12}(40)(20)^3 = 26.667 \times 10^3\text{ mm}^4 = 26.667 \times 10^{-9}\text{ m}^4$$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z} = \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}} = 24.375\text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{|V_z|}{A} = \frac{3}{2} \cdot \frac{10{,}000}{800 \times 10^{-6}} = 18.75\text{ MPa}$$





$$\sigma_c = \frac{1}{2}\sigma_H = 12.1875\text{ MPa}$$

$$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (18.75)^2} = 22.363\text{ MPa}$$

$$\sigma_a = \sigma_c + R \qquad \sigma_a = 34.6\text{ MPa} \blacktriangleleft$$

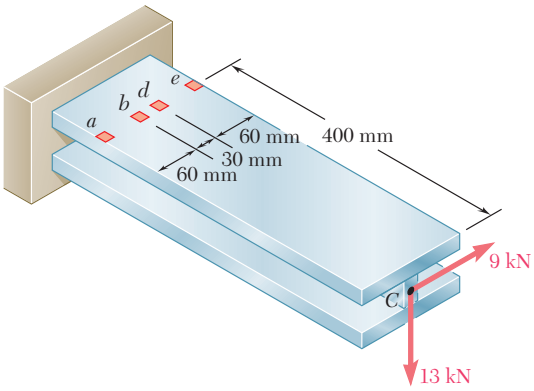
$$\sigma_b = \sigma_c - R \qquad \sigma_b = -10.18\text{ MPa} \blacktriangleleft$$

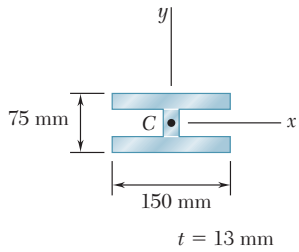
$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(18.75)}{24.375} = 1.5385$$

$$\theta_a = 28.5°, \quad \theta_b = 118.5°$$

$$\tau_{\max} = R \qquad \tau_{\max} = 22.4\text{ MPa} \blacktriangleleft$$

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PROBLEM 8.53

Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *a* and *b*.

SOLUTION

Equivalent force-couple system at section containing points *a* and *b*:

$$F_x = 9\text{ kN}, \quad F_y = -13\text{ kN}, \quad F_z = 0$$

$$M_x = (0.400)(13\times10^3) = 5200\text{ N}\cdot\text{m}$$

$$M_y = (0.400)(9\times10^3) = 3600\text{ N}\cdot\text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75-26)$$

$$= 4537\text{ mm}^2$$

$$= 4537\times10^{-6}\text{ m}^2$$

$$I_x = 2\left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5-6.5)^2\right] + \frac{1}{12}(13)(75-26)^3$$

$$= 3.9303\times10^6\text{ mm}^4$$

$$= 3.9303\times10^{-6}\text{ m}^4$$

$$I_y = 2\cdot\frac{1}{12}(13)(150)^3 + \frac{1}{12}(75-26)(13)^3$$

$$= 7.3215\times10^6\text{ mm}^4 = 7.3215\times10^{-6}\text{ mm}^4$$

For point *a*,

$$Q_x = 0, \quad Q_y = 0$$

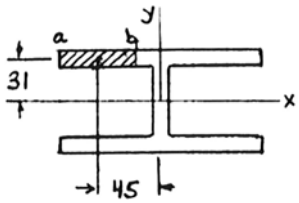
For point *b*,

$$A^* = (60)(13) = 780\text{ mm}^2$$

$$\bar{x} = -45\text{ mm} \quad \bar{y} = 31\text{ mm}$$

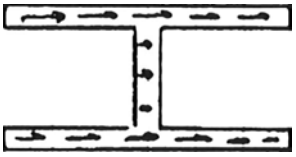
$$Q_x = A^*\bar{y} = 24.18\times10^3\text{ mm}^3 = 24.18\times10^{-6}\text{ m}^3$$

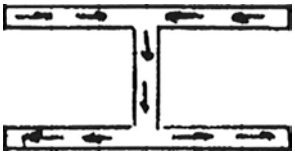
$$Q_y = A^*\bar{x} = -35.1\times10^{-3}\text{ mm}^3 = -35.1\times10^{-6}\text{ m}^3$$



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PROBLEM 8.53 ***(Continued)***





Direction of shearing stress for horizontal and for vertical components of shear:

At point *a*,

$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-75\times10^{-3})}{7.3215\times10^{-6}} \qquad \sigma = 86.5 \text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

At point *b*,

$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

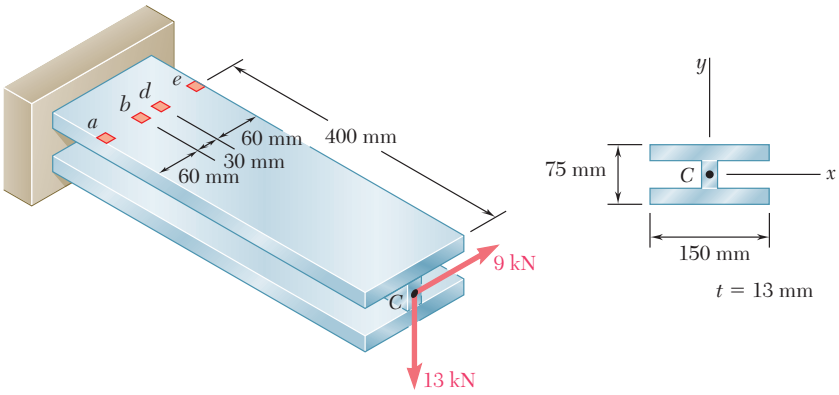
$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-15\times10^{-3})}{7.3215\times10^{-6}} \qquad \sigma = 57.0 \text{ MPa} \blacktriangleleft$$

$$\tau = \frac{|V_x||Q_y|}{I_y t} + \frac{|V_y||Q_x|}{I_x t}$$

$$= \frac{(9\times10^{3})(35.1\times10^{-6})}{(7.3215\times10^{-6})(13\times10^{-3})} + \frac{(13\times10^{3})(24.18\times10^{-6})}{(3.9303\times10^{-6})(13\times10^{-3})}$$

$$= 3.32 \text{ MPa} + 6.15 \text{ MPa} \qquad \tau = 9.47 \text{ MPa} \blacktriangleleft$$

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PROBLEM 8.54

Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *d* and *e*.

SOLUTION

Equivalent force-couple system at section containing points *d* and *e*.

$$F_x = 9\text{ kN}, \quad F_y = -13\text{ kN}, \quad F_z = 0$$

$$M_x = (0.400)(13\times10^3) = 5200\text{ N}\cdot\text{m}$$

$$M_y = (0.400)(9\times10^3) = 3600\text{ N}\cdot\text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75-26)$$

$$= 4537\text{ mm}^2$$

$$= 4537\times10^{-6}\text{ m}^2$$

$$I_x = 2\left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5-6.5)^2\right] + \frac{1}{12}(13)(75-26)^3$$

$$= 3.9303\times10^6\text{ mm}^4$$

$$= 3.9303\times10^{-6}\text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(13)(150)^3\right] + \frac{1}{12}(75-26)(13)^3$$

$$= 7.3215\times10^6\text{ mm}^4 = 7.3215\times10^{-6}\text{ m}^4$$

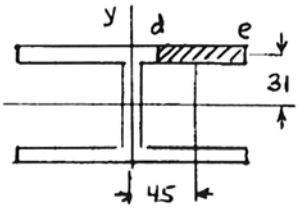
For point *d*,

$$A^* = (60)(13) = 780\text{ mm}^2$$

$$\bar{x} = 45\text{ mm} \quad \bar{y} = 31\text{ mm}$$

$$Q_x = A^*\bar{y} = 24.18\times10^3\text{ mm}^3 = 24.18\times10^{-6}\text{ m}^3$$

$$Q_y = A^*\bar{x} = 35.1\times10^3\text{ mm}^3 = 35.1\times10^{-6}\text{ m}^3$$



For point *e*, $Q_x = 0, \quad Q_y = 0$

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PROBLEM 8.54 *(Continued)*

At point *d*,

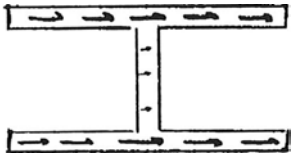
$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(15\times10^{-3})}{7.3215\times10^{-6}} \qquad \sigma = 42.2 \text{ MPa} \blacktriangleleft$$

Due to V_x:

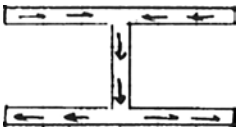
$$\tau = \frac{|V_x||Q_y|}{I_y t} = \frac{(9000)(35.1\times10^{-6})}{(7.3215\times10^{-6})(13\times10^{-3})}$$

$$= 3.32 \text{ MPa} \rightarrow$$



Due to V_y:

$$\tau = \frac{|V_y||Q_x|}{I_x t} = \frac{(13{,}000)(24.18\times10^{-6})}{(3.9303\times10^{-6})(13\times10^{-3})} = 6.15 \text{ MPa} \leftarrow$$



By superposition, the net value is $\tau = 2.83 \text{ MPa} \blacktriangleleft$

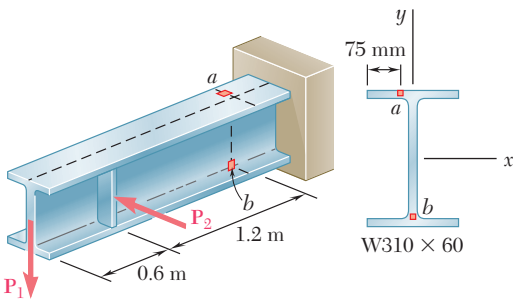
At point *e*,

$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(75\times10^{-3})}{7.3215\times10^{-6}}$$

$$\sigma = 12.74 \text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

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PROBLEM 8.55

Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$ are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $\mathbf{P}_1 = 25$ kN and $\mathbf{P}_2 = 24$ kN, determine the principal stresses and the maximum shearing stress at point *a*.

SOLUTION

At the section containing points *a* and *b*,

$$M_x = (1.8)(25) = 45 \text{ kN} \cdot \text{m}$$
$$M_y = -(1.2)(24) = -28.8 \text{ kN} \cdot \text{m}$$
$$V_x = -24 \text{ kN} \qquad V_y = -25 \text{ kN}$$

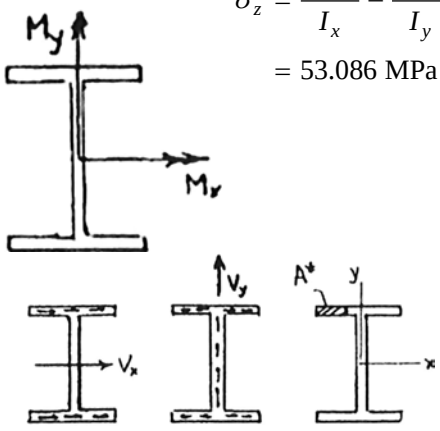
For W310 × 60 rolled-steel section,

$$d = 302 \text{ mm}, \quad b_f = 203 \text{ mm}, \quad t_f = 13.1 \text{ mm}, \quad t_w = 7.49 \text{ mm}$$
$$I_x = 128 \times 10^6 \text{ mm}^4 = 128 \times 10^{-6} \text{ m}^4, \quad I_y = 18.4 \times 10^6 \text{ mm}^4 = 18.4 \times 10^{-6} \text{ m}^4$$

Normal stress at point *a*: $x = -\dfrac{b_f}{2} + 75 = -26.5 \text{ mm}$

$$y = \frac{1}{2}d = 151 \text{ mm}$$

$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45 \times 10^3)(151 \times 10^{-3})}{128 \times 10^{-6}} - \frac{(-28.8 \times 10^3)(-26.5 \times 10^{-3})}{18.4 \times 10^{-6}}$$
$$= 53.086 \text{ MPa} - 41.478 \text{ MPa} = 11.608 \text{ MPa}$$



Shearing stress at point *a*:

$$\tau_{xz} = -\frac{V_x A^* \bar{x}}{I_y t_f} - \frac{V_y A^* \bar{y}}{I_x t_f}$$

$$A^* = (75 \times 10^{-3})(13.1 \times 10^{-3}) = 982.5 \times 10^{-6} \text{ m}^2$$

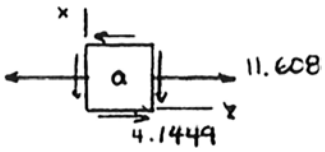
$$\bar{x} = -\frac{b_f}{2} + \frac{75}{2} = -64 \text{ mm}$$

$$\bar{y} = \frac{d}{2} - \frac{t_f}{2} = 144.45 \text{ mm}$$

$$\tau_{xz} = -\frac{(-24 \times 10^3)(982.5 \times 10^{-6})(-64 \times 10^{-3})}{(18.4 \times 10^{-6})(13.1 \times 10^{-3})} - \frac{(-25 \times 10^3)(982.5 \times 10^{-6})(144.45 \times 10^{-3})}{(128 \times 10^{-6})(13.1 \times 10^{-3})}$$
$$= -6.2609 \text{ MPa} + 2.1160 \text{ MPa} = -4.1449 \text{ MPa}$$

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PROBLEM 8.55 *(Continued)*



$$\sigma_{ave} = \frac{11.608}{2} = 5.804 \text{ MPa}$$

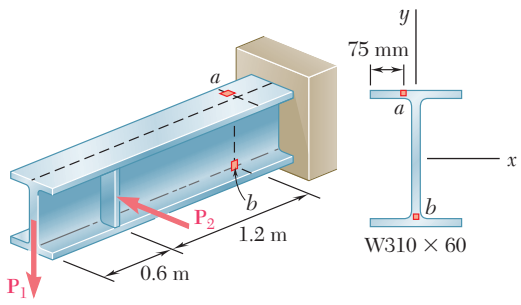
$$R = \sqrt{\left(\frac{11.608}{2}\right)^2 + (4.1449)^2} = 7.1321 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R \qquad \sigma_{max} = 12.94 \text{ MPa} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R \qquad \sigma_{min} = -1.328 \text{ MPa} \blacktriangleleft$$

$$\tau_{max} = R \qquad \tau_{max} = 7.13 \text{ MPa} \blacktriangleleft$$

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PROBLEM 8.56

Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$ are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $\mathbf{P}_1 = 25$ kN and $\mathbf{P}_2 = 24$ kN, determine the principal stresses and the maximum shearing stress at point *b*.

SOLUTION

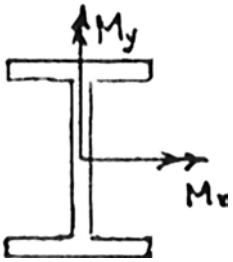
At the section containing points *a* and *b*,

$$M_x = (1.8)(25) = 45 \text{ kN} \cdot \text{m}$$
$$M_y = -(1.2)(24) = -28.8 \text{ kN} \cdot \text{m}$$
$$V_x = -24 \text{ kN}, \qquad V_y = -25 \text{ kN}$$

For W310 × 60 rolled-steel section,

$$d = 302 \text{ mm}, \quad b_f = 203 \text{ mm}, \quad t_f = 13.1 \text{ mm}, \quad t_w = 7.49 \text{ mm}$$
$$I_x = 128 \times 10^6 \text{ mm}^4 = 128 \times 10^{-6} \text{ m}^4, \quad I_y = 18.4 \times 10^6 \text{ mm}^4 = 18.4 \times 10^{-6} \text{ m}^4$$

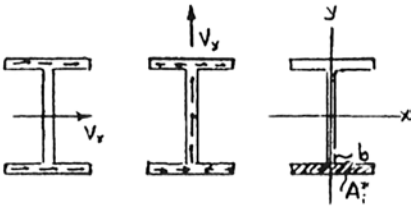
Normal stress at point *b*: $x \approx 0$, $y = -\frac{1}{2}d + t_f = -137.9$ mm



$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45 \times 10^3)(-137.9 \times 10^{-3})}{128 \times 10^{-6}} - 0$$
$$= -48.480 \text{ MPa}$$

Shearing stress at point *b*:

$$\tau_{yz} = -\frac{V_y A^* \bar{y}}{I_x t_w}$$

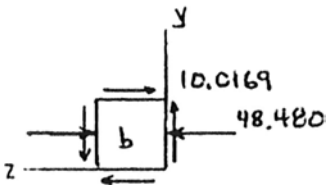


$$A^* = A_f = b_f t_f = 2659.3 \text{ mm}^2$$
$$= 2659.3 \times 10^{-6} \text{ m}^2$$
$$\bar{x} = 0, \qquad \bar{y} = -\frac{1}{2}d + \frac{1}{2}t_f = -144.45 \text{ mm}$$

$$\tau_{yz} = -\frac{(-25 \times 10^3)(2659.3 \times 10^{-6})(-144.45 \times 10^{-3})}{(128 \times 10^{-6})(7.49 \times 10^{-3})} = -10.0169 \text{ MPa}$$

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PROBLEM 8.56 ***(Continued)***



$$\sigma_{ave} = -\frac{48.480}{2} = -24.240 \text{ MPa}$$

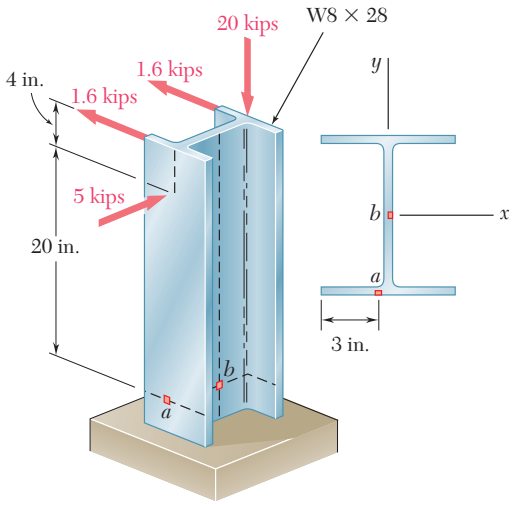
$$R = \sqrt{\left(\frac{48.48}{2}\right)^2 + (10.0169)^2} = 26.228 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R \qquad \sigma_{max} = 1.988 \text{ MPa} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R \qquad \sigma_{min} = -50.5 \text{ MPa} \blacktriangleleft$$

$$\tau_{max} = R \qquad \tau_{max} = 26.2 \text{ MPa} \blacktriangleleft$$

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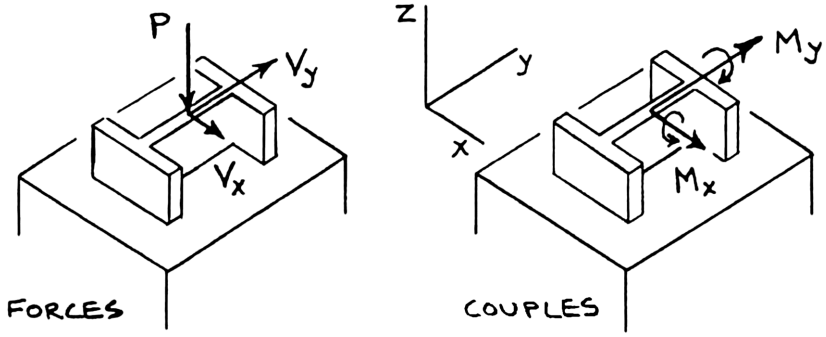


PROBLEM 8.57

Four forces are applied to a W8 × 28 rolled-steel beam as shown. Determine the principal stresses and maximum shearing stress at point *a*.

SOLUTION

Calculate forces and couples at section containing point of interest.



$$P = 20 \text{ kips} \qquad V_x = -3.2 \text{ kips} \qquad V_y = 5 \text{ kips}$$

$$M_x = -(20)(5) - (4.03)(20) = -180.6 \text{ kip} \cdot \text{in.}$$

$$M_y = -(20 + 4)(3.2) = -76.8 \text{ kip} \cdot \text{in.}$$

Section properties:

$$A = 8.24 \text{ in}^4 \qquad d = 8.06 \text{ in.} \qquad b_f = 6.54 \text{ in.} \qquad t_f = 0.465 \text{ in.}$$

$$t_w = 0.285 \text{ in.} \qquad I_x = 98.0 \text{ in}^4 \qquad I_y = 21.7 \text{ in}^4$$

Point *a*:

$$x_a = -\frac{6.54}{2} + 3 = -0.27 \text{ in.} \qquad y_a = -\frac{8.06}{2} = -4.03 \text{ in.}$$

$$\sigma_a = -\frac{P}{A} + \frac{M_x y_a}{I_x} - \frac{M_y x_a}{I_y}$$

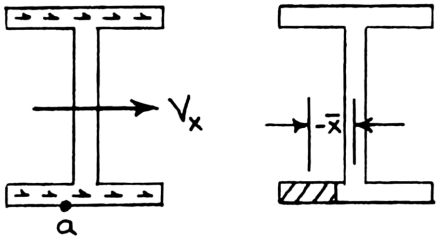
$$= -\frac{20}{8.24} + \frac{(-180.6)(-4.03)}{98.0} - \frac{(-76.8)(-0.27)}{21.7}$$

$$= -2.4272 + 7.4267 - 0.95558 = 4.0439 \text{ ksi}$$

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PROBLEM 8.57 *(Continued)*

Shearing stress at point *a* due to V_x:



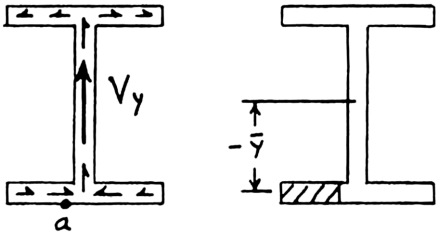
$$A = (3)(0.465) = 1.395 \text{ in}^2$$

$$\bar{x} = -\frac{6.54}{2} + \frac{3}{2} = -1.77 \text{ in.}$$

$$Q = -A\bar{x} = 2.4692 \text{ in}^3$$

$$\tau_{xz} = \frac{V_x Q}{I_y t} = \frac{(-3.2)(2.4692)}{(21.7)(0.465)} = -0.78306 \text{ ksi}$$

Shearing stress at point *a* due to V_y:



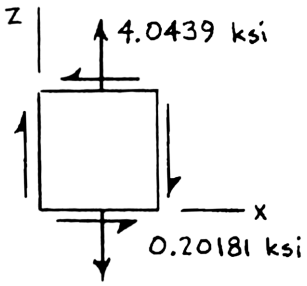
$$A = (3)(0.465) = 1.395 \text{ in}^2$$

$$\bar{y} = -\frac{8.06}{2} + \frac{0.465}{2} = -3.7975 \text{ in.}$$

$$Q = -A\bar{y} = 5.2975 \text{ in}^3$$

$$\tau_{xz} = \frac{V_y Q}{I_x t} = \frac{(5)(5.2975)}{(98.0)(0.465)} = 0.58125 \text{ ksi}$$

Combined shearing stress: $\tau_a = -0.78306 + 0.58125 = 0.20181 \text{ ksi}$



$$\sigma_{ave} = \frac{4.0439 + 0}{2} = 2.0220 \text{ ksi}$$

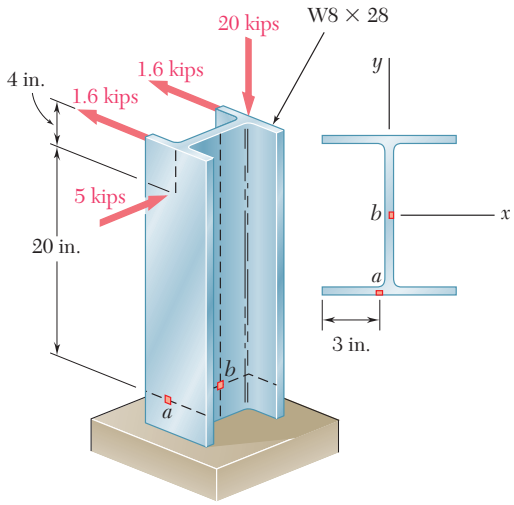
$$R = \sqrt{\left(\frac{4.0439 - 0}{2}\right)^2 + (0.20181)^2} = 2.0320 \text{ ksi}$$

$\sigma_{max} = \sigma_{ave} + R$ $\qquad \sigma_{max} = 4.05 \text{ ksi}$ ◄

$\sigma_{min} = \sigma_{ave} - R$ $\qquad \sigma_{min} = -0.010 \text{ ksi}$ ◄

$\tau_{max} = R$ $\qquad \tau_{max} = 2.03 \text{ ksi}$ ◄

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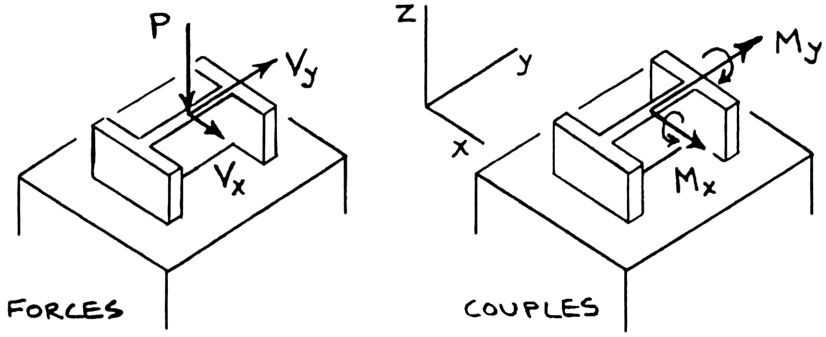


PROBLEM 8.58

Four forces are applied to a W8 × 28 rolled-steel beam as shown. Determine the principal stresses and maximum shearing stress at point *b*.

SOLUTION

Calculate forces and couples at section containing point of interest.



$$P = 20 \text{ kips} \qquad V_x = -3.2 \text{ kips} \qquad V_y = 5 \text{ kips}$$

$$M_x = -(20)(5) - (4.03)(20) = -180.6 \text{ kip} \cdot \text{in.}$$

$$M_y = -(20 + 4)(3.2) = -76.8 \text{ kip} \cdot \text{in.}$$

Section properties:

$$A = 8.24 \text{ in}^4 \qquad d = 8.06 \text{ in.} \qquad b_f = 6.54 \text{ in.} \qquad t_f = 0.465 \text{ in.}$$

$$t_w = 0.285 \text{ in.} \qquad I_x = 98.0 \text{ in}^4 \qquad I_y = 21.7 \text{ in}^4$$

Point *b*: $x_b = 0 \qquad y_b = 0$

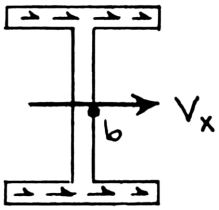
$$\sigma_b = -\frac{P}{A} + \frac{M_x y_b}{I_x} + \frac{M_y x_b}{I_y}$$

$$= -\frac{20}{8.24} + 0 + 0 = -2.4272 \text{ ksi}$$

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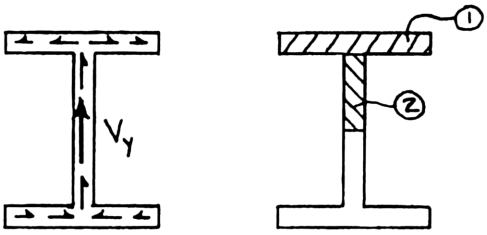
PROBLEM 8.58 *(Continued)*

Shearing stress at point *b* due to V_x:

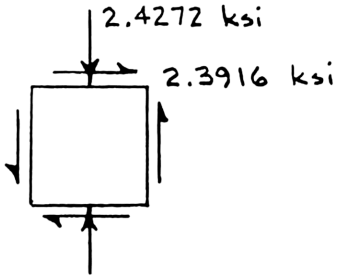


$$\tau_{xz} = 0$$

Shearing stress at point *b* due to V_y:



	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$
①	3.0411	3.7975	11.5486
②	1.01603	1.7825	1.81107
Σ			13.3597



$$Q = \Sigma A\bar{y} = 13.3597 \text{ in}^3$$

$$\tau_b = \frac{V_y Q}{I_x t_w} = \frac{(5)(13.3597)}{(98.0)(0.285)} = 2.3916 \text{ ksi}$$

$$\sigma_{\text{ave}} = \frac{-2.4272 + 0}{2} = -1.2136 \text{ ksi}$$

$$R = \sqrt{\left(\frac{-2.4272 - 0}{2}\right)^2 + (2.3916)^2}$$

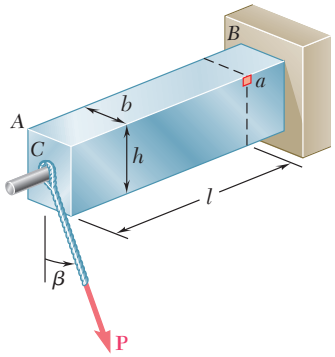
$$= 2.6819 \text{ ksi}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R \qquad \sigma_{\max} = 1.468 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R \qquad \sigma_{\min} = -3.90 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 2.68 \text{ ksi} \blacktriangleleft$$

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PROBLEM 8.59

A force **P** is applied to a cantilever beam by means of a cable attached to a bolt located at the center of the free end of the beam. Knowing that **P** acts in a direction perpendicular to the longitudinal axis of the beam, determine (*a*) the normal stress at point *a* in terms of *P*, *b*, *h*, *l*, and β, (*b*) the values of β for which the normal stress at *a* is zero.

SOLUTION

$$I_x = \frac{1}{12}bh^3 \quad I_y = \frac{1}{12}hb^3$$

$$\sigma = \frac{M_x(h/2)}{I_x} - \frac{M_y(b/2)}{I_y}$$

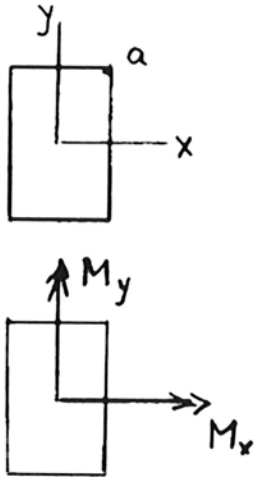
$$= \frac{6M_x}{bh^2} - \frac{6M_y}{hb^2}$$

$$\boldsymbol{P} = P\sin\beta\,\boldsymbol{i} - P\cos\beta\,\boldsymbol{j} \quad \boldsymbol{r} = l\,\boldsymbol{k}$$

$$\boldsymbol{M} = \boldsymbol{r}\times\boldsymbol{P} = l\,\boldsymbol{k}\times(P\sin\beta\boldsymbol{i} - P\cos\beta\,\boldsymbol{j})$$

$$= Pl\cos\beta\,\boldsymbol{i} + Pl\sin\beta\,\boldsymbol{j}$$

$$M_x = Pl\cos\beta \qquad M_y = Pl\sin\beta$$

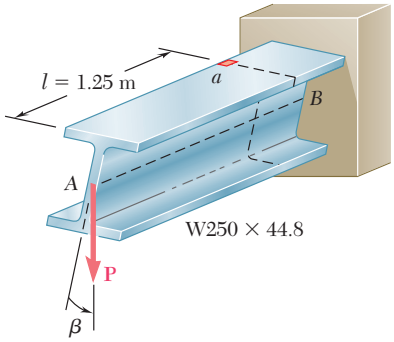


(*a*) $$\sigma = \frac{6\,Pl\cos\beta}{bh^2} - \frac{6\,Pl\sin\beta}{hb^2} = \frac{6\,Pl}{bh}\left[\frac{\cos\beta}{h} - \frac{\sin\beta}{b}\right]$$ ◀

(*b*) $$\sigma = 0 \qquad \frac{\cos\beta}{h} - \frac{\sin\beta}{b} = 0 \qquad \tan\beta = \frac{b}{h}$$

$$\beta = \tan^{-1}\left(\frac{b}{h}\right)$$ ◀

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PROBLEM 8.60

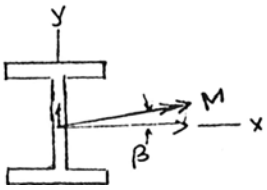
A vertical force **P** is applied at the center of the free end of cantilever beam *AB*. (*a*) If the beam is installed with the web vertical ($\beta = 0$) and with its longitudinal axis *AB* horizontal, determine the magnitude of the force **P** for which the normal stress at point *a* is +120 MPa. (*b*) Solve part *a*, assuming that the beam is installed with $\beta = 3°$.

SOLUTION

For W250 × 44.8 rolled-steel section,

$$S_x = 531 \times 10^3 \text{ mm}^3 = 531 \times 10^{-6} \text{ m}^3$$
$$S_y = 94.2 \times 10^3 \text{ mm}^3 = 94.2 \times 10^{-6} \text{ m}^3$$

At the section containing point *a*,



$$M_x = Pl\cos\beta, \qquad M_y = Pl\sin\beta$$

Stress at *a*:

$$\sigma = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{Pl\cos\beta}{S_x} + \frac{Pl\sin\beta}{S_y}$$

Allowable load. $$P_{\text{all}} = \frac{\sigma_{\text{all}}}{l}\left[\frac{\cos\beta}{S_x} + \frac{\sin\beta}{S_y}\right]^{-1}$$

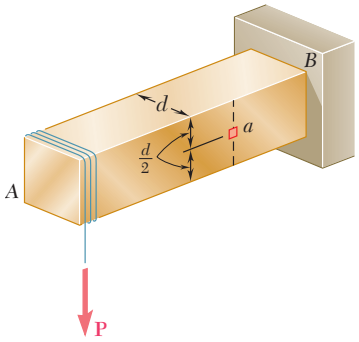
(*a*) $\beta = 0$: $$P_{\text{all}} = \frac{120 \times 10^6}{1.25}\left[\frac{1}{531 \times 10^{-6}} + 0\right]^{-1} = 51.0 \times 10^3 \text{ N}$$

$$P_{\text{all}} = 51.0 \text{ kN} \blacktriangleleft$$

(*b*) $\beta = 3°$: $$P_{\text{all}} = \frac{120 \times 10^6}{1.25}\left[\frac{\cos 3°}{531 \times 10^{-6}} + \frac{\sin 3°}{94.2 \times 10^{-6}}\right]^{-1} = 39.4 \times 10^3 \text{ N}$$

$$P_{\text{all}} = 39.4 \text{ kN} \blacktriangleleft$$

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PROBLEM 8.61*

A 5-kN force **P** is applied to a wire that is wrapped around bar *AB* as shown. Knowing that the cross section of the bar is a square of side $d = 40$ mm, determine the principal stresses and the maximum shearing stress at point *a*.

SOLUTION

Bending: Point *a* lies on the neutral axis.

$$\sigma = 0$$

Torsion:
$$\tau = \frac{T}{c_1 a b^2} \quad \text{where} \quad a = b = d$$

and $c_1 = 0.208$ for a square section.

Since
$$T = \frac{Pd}{2}, \quad \tau_T = \frac{P}{0.416 d^2} = 2.404 \frac{P}{d^2}.$$

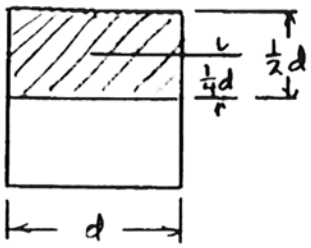
Transverse shear:

$$V = P \qquad I = \frac{1}{12} d^4$$

$$A = \frac{1}{2} d^2 \qquad \bar{y} = \frac{1}{4} d \qquad Q = A\bar{y} = \frac{1}{8} d^3$$

$$t = d$$

$$\tau_V = \frac{VQ}{It} = 1.5 \frac{P}{d^2}$$

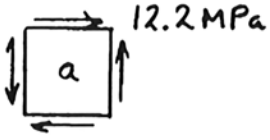


By superposition,

$$\tau = \tau_T + \tau_V = 3.904 \frac{P}{d^2}$$

$$\tau = \frac{(3.904)(5 \times 10^3)}{(40 \times 10^{-3})^2}$$

$$= 12.2 \times 10^6 \text{ Pa} = 12.2 \text{ MPa}$$



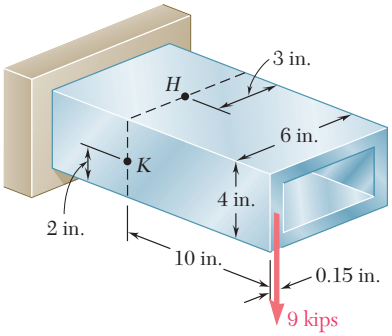
By Mohr's circle,

$$\sigma_{\max} = 12.2 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = -12.2 \text{ MPa} \blacktriangleleft$$

$$\tau_{\max} = 12.2 \text{ MPa} \blacktriangleleft$$

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PROBLEM 8.62*

Knowing that the structural tube shown has a uniform wall thickness of 0.3 in., determine the principal stresses, principal planes, and maximum shearing stress at (*a*) point *H*, (*b*) point *K*.

SOLUTION

At the section containing points *H* and *K*,

$$V = 9 \text{ kips} \quad M = (9)(10) = 90 \text{ kip} \cdot \text{in.}$$
$$T = (9)(3 - 0.15) = 25.65 \text{ kip} \cdot \text{in.}$$

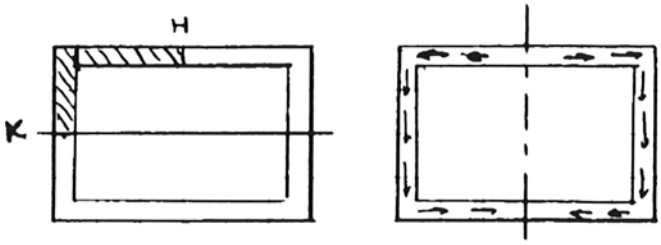
Torsion:



$$\mathcal{A} = (5.7)(3.7) = 21.09 \text{ in}^2$$
$$\tau = \frac{T}{2t\mathcal{A}} = \frac{25.65}{(2)(0.3)(21.09)} = 2.027 \text{ ksi}$$

Transverse shear:

$$Q_H = 0$$
$$Q_K = (3)(2)(1) - (2.7)(1.7)(0.85) = 2.0985 \text{ in}^3$$
$$I = \frac{1}{12}(6)(4)^3 - \frac{1}{12}(5.4)(3.4)^3 = 14.3132 \text{ in}^4$$
$$\tau_H = 0 \qquad \tau_K = \frac{VQ_K}{It} = \frac{(9)(2.0985)}{(14.3132)(0.3)} = 4.398 \text{ ksi}$$



Bending:

$$\sigma_H = \frac{Mc}{I} = \frac{(90)(2)}{14.3132} = 12.576 \text{ ksi}, \qquad \sigma_K = 0$$

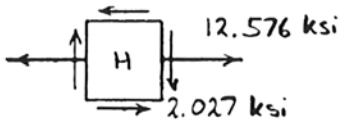
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PROBLEM 8.62* *(Continued)*

(*a*) Point *H*:

$$\sigma_{ave} = \frac{12.576}{2} = 6.288 \text{ ksi}$$

$$R = \sqrt{\left(\frac{12.576}{2}\right)^2 + (2.027)^2} = 6.607 \text{ ksi}$$



$$\sigma_{max} = \sigma_{ave} + R \qquad \sigma_{max} = 12.90 \text{ ksi} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R \qquad \sigma_{min} = -0.32 \text{ ksi} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau}{\sigma} = -0.3224 \qquad \theta_p = -8.9°, 81.1° \blacktriangleleft$$

$$\tau_{max} = R = 6.61 \text{ ksi} \qquad \tau_{max} = 6.61 \text{ ksi} \blacktriangleleft$$

(*b*) Point *K*:

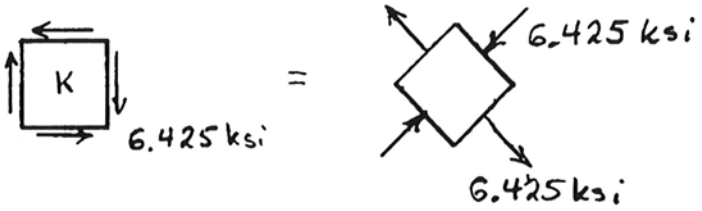
$$\sigma = 0 \qquad \tau = 2.027 + 4.398 = 6.425 \text{ ksi}$$

$\sigma_{max} = 6.43$ ksi ◀

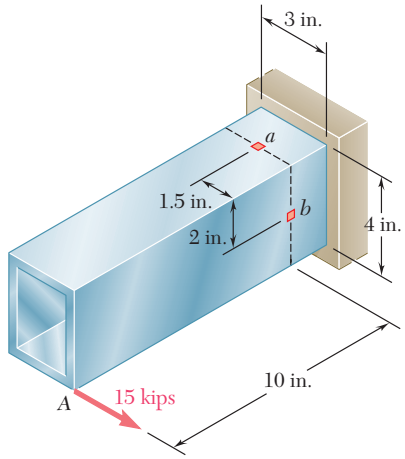
$\sigma_{min} = -6.43$ ksi ◀

$\theta = \pm 45°$ ◀

$\tau_{max} = 6.43$ ksi ◀



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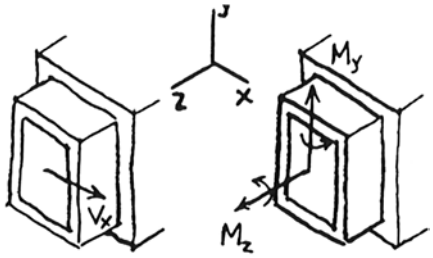
PROBLEM 8.63*

The structural tube shown has a uniform wall thickness of 0.3 in. Knowing that the 15-kip load is applied 0.15 in. above the base of the tube, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

Calculate forces and couples at section containing points *a* and *b*.

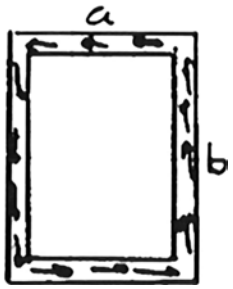
$$V_x = 15 \text{ kips}$$
$$M_z = (2 - 0.15)(15) = 27.75 \text{ kip} \cdot \text{in.}$$
$$M_y = (10)(15) = 150 \text{ kip} \cdot \text{in.}$$



Forces Couples

Shearing stresses due to torque $T = M_z$:

$$\mathcal{A} = [3 - (2)(0.15)][4 - (2)(0.15)] = 9.99 \text{ in}^2$$
$$q = \frac{M_z}{2\mathcal{A}} = \frac{27.75}{(2)(9.99)} = 1.3889 \text{ kip/in.}$$



At point *a*, $t = 0.3$ in. $\tau_a = \dfrac{q}{t} = \dfrac{1.3889}{0.3} = 4.630$ ksi ←

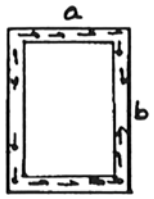
At point *b*, $t = 0.3$ in. $\tau_b = \dfrac{q}{t} = \dfrac{1.3889}{0.3} = 4.630$ ksi ↑

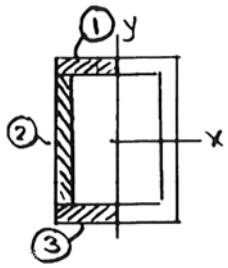
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PROBLEM 8.63* *(Continued)*

Shearing stresses due to V_x:

At point *a*,





Part	A(in^2)	$\bar{x}$(in.)	$A\bar{x}$(in^3)
①	0.45	–0.75	–0.3375
②	1.02	–1.35	–1.377
③	0.45	–0.75	–0.3375
Σ			–2.052

$$Q = \left|\Sigma A\bar{x}\right| = 2.052 \text{ in}^3$$

$$t = (2)(0.3) = 0.6 \text{ in.}$$

$$I_y = \frac{1}{12}(4)(3)^3 - \frac{1}{12}(3.4)(2.4)^3 = 5.0832$$

$$\tau = \frac{V_x Q}{I_y t} = \frac{(15)(2.052)}{(5.0832)(0.6)} = 10.092 \text{ ksi}$$

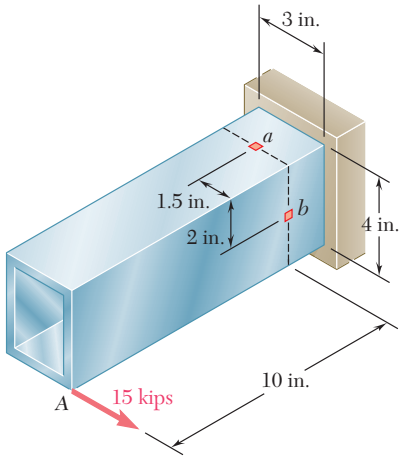
At point *b*, $\tau_b = 0$

Combined shearing stresses.

(*a*) At point *a*, $\tau_a = 4.630 \leftarrow +10.092 \rightarrow = 5.46 \text{ ksi} \rightarrow$ $\tau_a = 5.46 \text{ ksi}$ ◀

(*b*) At point *b*, $\tau_b = 4.630 \uparrow + 0 = 4.63 \text{ ksi} \uparrow$ $\tau_b = 4.63 \text{ ksi}$ ◀

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PROBLEM 8.64*

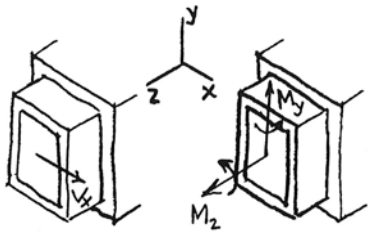
For the tube and loading of Prob. 8.63, determine the principal stresses and the maximum shearing stress at point *b*.

PROBLEM 8.63* The structural tube shown has a uniform wall thickness of 0.3 in. Knowing that the 15-kip load is applied 0.15 in. above the base of the tube, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

Calculate forces and couples at section containing point *b*.

$$V_x = 15 \text{ kips}$$
$$M_z = (2 - 0.15)(15) = 27.75 \text{ kip} \cdot \text{in.}$$
$$M_y = (10)(15) = 150 \text{ kip} \cdot \text{in.}$$

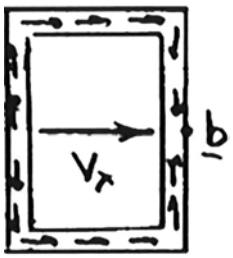


Forces Couples

$$I_y = \frac{1}{12}(4)(3)^3 - \frac{1}{12}(3.4)(2.4)^3 = 5.0832 \text{ in}^4$$
$$\sigma_b = -\frac{M_y x_b}{I_y} = -\frac{(150)(1.5)}{5.0832} = -44.26 \text{ ksi}$$

Shearing stress at point *b* due to torque M_z:

$$\mathcal{A} = [3 - (2)(0.15)][4 - (2)(0.15)] = 9.99 \text{ in}^2$$
$$q = \frac{M_z}{2\mathcal{A}} = \frac{27.75}{(2)(9.99)} = 1.38889 \text{ kip/in.}$$
$$\tau = \frac{q}{t} = \frac{1.38889}{0.3} = 4.630 \text{ ksi}$$



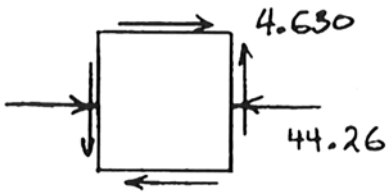
Shearing at point *b* due to V_x:

$$\tau = 0$$

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PROBLEM 8.64* *(Continued)*

Calculation of principal stresses and maximum shearing stress.



$$\sigma_{ave} = \frac{-44.26 + 0}{2} = -22.13 \text{ ksi}$$

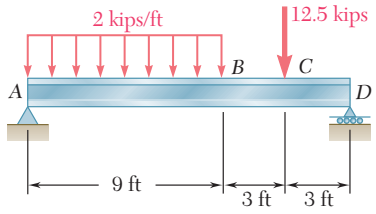
$$R = \sqrt{\left(-\frac{44.26 - 0}{2}\right)^2 + (4.630)^2} = 22.61 \text{ ksi}$$

$$\sigma_{max} = \sigma_{ave} + R \qquad \sigma_{max} = 0.48 \text{ ksi} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R \qquad \sigma_{min} = -44.7 \text{ ksi} \blacktriangleleft$$

$$\tau_{max} = R \qquad \tau_{max} = 22.6 \text{ ksi} \blacktriangleleft$$

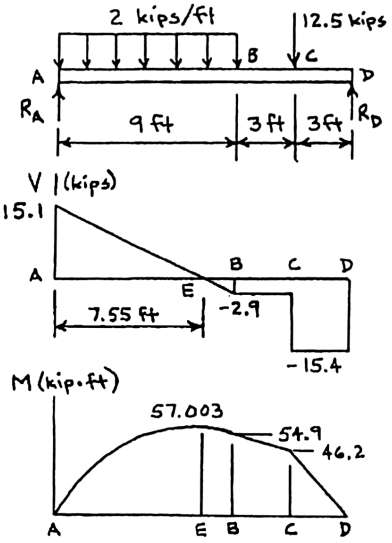
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PROBLEM 8.65

(*a*) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION



$$+\circlearrowleft \Sigma M_D = 0: \quad -15R_A + (10.5)(9)(2) + (3)(12.5) = 0$$

$$R_A = 15.1 \text{ kips} \uparrow \qquad R_D = 15.4 \text{ kips} \uparrow$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{(57.003 \times 12 \text{ kip} \cdot \text{in.})}{24 \text{ ksi}} = 28.502 \text{ in}^3$$

	Shape	S (in^3)	
	W16 × 26	38.4	
→	W14 × 22	29.0	(*a*) Use W14 × 22. ◀
	W12 × 26	33.4	
	W10 × 30	32.4	
	W8 × 35	31.2	

For W14 × 22, $A_{web} = dt_w = (13.7)(0.230) = 3.151 \text{ in}^2$

(*b*) Point *E*:

$$\sigma_m = \frac{M_E}{S} = \frac{(57.003)(12)}{29.0} = 23.587 \text{ ksi} \qquad \sigma_m = 23.6 \text{ ksi} \blacktriangleleft$$

$$c = \frac{1}{2}d = 6.85 \text{ in.} \qquad y_b = c - t_f = 6.515 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{6.515}{6.85}\right)(23.587) = 22.433 \text{ ksi}$$

Since $\tau_m = 0$, $\sigma_{max} = \sigma_b = 22.4 \text{ ksi} \blacktriangleleft$

Point *C*:

$$\sigma_m = \frac{M_C}{S} = \frac{(46.2)(12)}{29.0} = 19.1172 \text{ ksi} \qquad \sigma_m = 19.12 \text{ ksi}$$

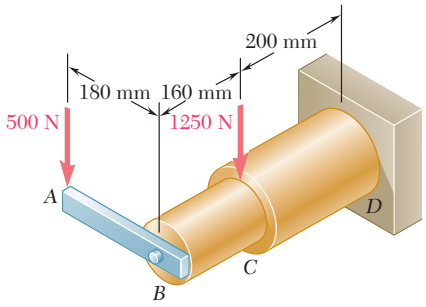
$$\tau_m = \frac{|V|}{A_{web}} = \frac{15.4}{3.151} = 4.8873 \text{ ksi} \qquad \tau_m = 4.89 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{6.515}{6.85}\right)(19.1172) = 18.1823 \text{ ksi} \qquad R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m{}^2} = 10.3216 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R \qquad \sigma_{max} = 19.41 \text{ ksi} \blacktriangleleft$$

For σ_{max}, point *B* controls; $\sigma_{max} = 22.4 \text{ ksi} \blacktriangleleft$

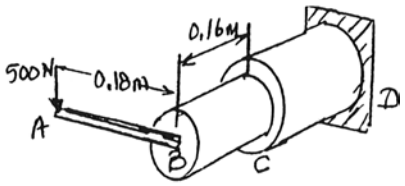
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PROBLEM 8.66

Neglecting the effect of fillets and of stress concentrations, determine the smallest permissible diameters of the solid rods *BC* and *CD*. Use $\tau_{all} = 60$ MPa.

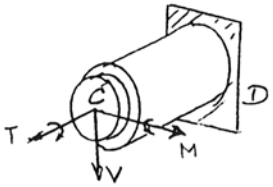
SOLUTION



$$\tau_{all} = 60 \times 10^6 \text{ Pa}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{all}} \qquad d = 2c$$



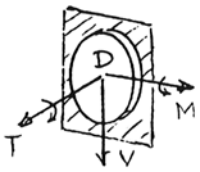
Bending moments and torques.

Just to the left of *C*:

$$M = (500)(0.16) = 80 \text{ N} \cdot \text{m}$$

$$T = (500)(0.18) = 90 \text{ N} \cdot \text{m}$$

$$\sqrt{M^2 + T^2} = 120.416 \text{ N} \cdot \text{m}$$

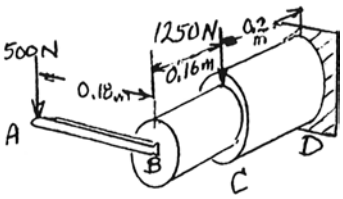


Just to the left of *D*:

$$T = 90 \text{ N} \cdot \text{m}$$

$$M = (500)(0.36) + (1250)(0.2) = 430 \text{ N} \cdot \text{m}$$

$$\sqrt{M^2 + T^2} = 439.32 \text{ N} \cdot \text{m}$$



Smallest permissible diameter d_{BC}.

$$c^3 = \frac{(2)(120.416)}{\pi(60 \times 10^6)} = 1.27765 \times 10^{-6} \text{ m}^3$$

$$c = 0.01085 \text{ m} = 10.85 \text{ mm}$$

$$d_{BC} = 21.7 \text{ mm} \blacktriangleleft$$

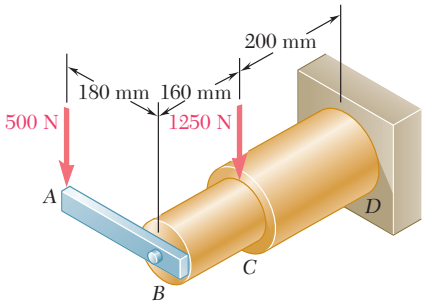
Smallest permissible diameter d_{CD}.

$$c^3 = \frac{(2)(439.32)}{\pi(60 \times 10^6)} = 4.6613 \times 10^{-6} \text{ m}^3$$

$$c = 0.01670 \text{ m} = 16.7 \text{ mm}$$

$$d_{CD} = 33.4 \text{ mm} \blacktriangleleft$$

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PROBLEM 8.67

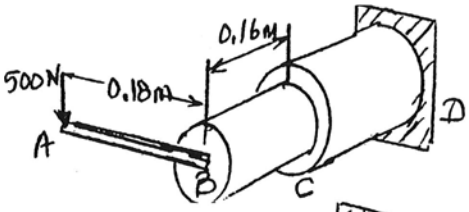
Knowing that rods *BC* and *CD* are of diameter 24 mm and 36 mm, respectively, determine the maximum shearing stress in each rod. Neglect the effect of fillets and of stress concentrations.

SOLUTION

Over *BC*: $c = \frac{1}{2}d = 12 \text{ mm} = 0.012 \text{ m}$

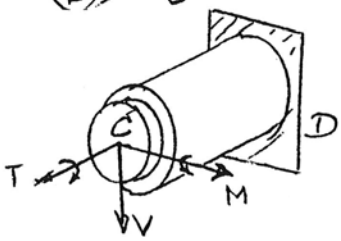
Over *CD*: $c = \frac{1}{2}d = 18 \text{ mm} = 0.018 \text{ m}$

$$\tau = \frac{\sqrt{M^2 + T^2}\,c}{J} \qquad \frac{2}{\pi}\frac{\sqrt{M^2 + T^2}}{c^3}$$

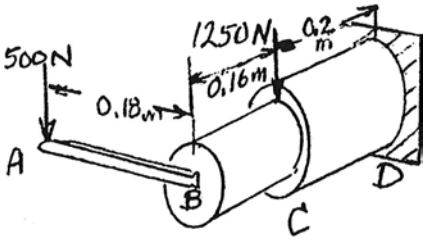


Bending moments and torques.

Just to the left of *C*:

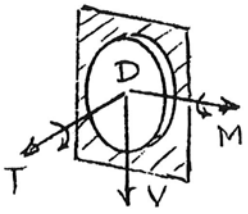


$$M = (500)(0.16) = 80 \text{ N} \cdot \text{m}$$
$$T = (500)(0.18) = 90 \text{ N} \cdot \text{m}$$
$$\sqrt{M^2 + T^2} = 120.416 \text{ N} \cdot \text{m}$$



Just to the left of *D*:

$$T = 90 \text{ N} \cdot \text{m}$$
$$M = (500)(0.36) + (1250)(0.2)$$
$$= 430 \text{ N} \cdot \text{m}$$
$$\sqrt{M^2 + T^2} = 439.32 \text{ N} \cdot \text{m}$$



Maximum shearing stress in portion *BC*.

$$\tau_{\max} = \frac{(2)(120.416)}{\pi(0.012)^3} = 44.36 \times 10^6 \text{ Pa}$$

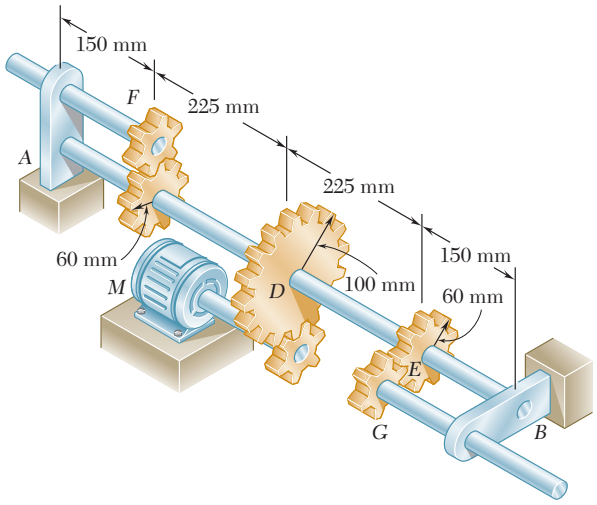
$\tau_{\max} = 44.4 \text{ MPa}$ ◀

Maximum shearing stress in portion *CD*.

$$\tau_{\max} = \frac{(2)(439.32)}{\pi(0.018)^3} = 47.96 \times 10^6 \text{ Pa}$$

$\tau_{\max} = 48.0 \text{ MPa}$ ◀

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PROBLEM 8.68

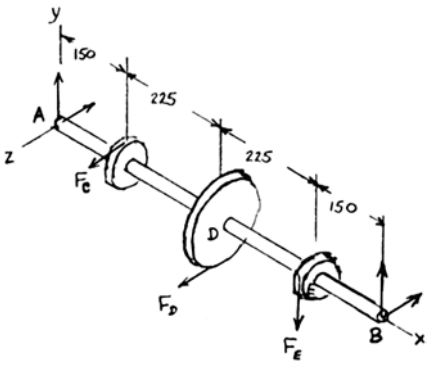
The solid shaft *AB* rotates at 450 rpm and transmits 20 kW from the motor *M* to machine tools connected to gears *F* and *G*. Knowing that $\tau_{all} = 55$ MPa and assuming that 8 kW is taken off at gear *F* and 12 kW is taken off at gear *G*, determine the smallest permissible diameter of shaft *AB*.

SOLUTION

$$f = \frac{450}{60} = 7.5 \text{ Hz}$$

Torque applied at *D*:

$$T_D = \frac{P}{2\pi f} = \frac{20 \times 10^3}{(2\pi)(7.5)}$$
$$= 424.41 \text{ N} \cdot \text{m}$$



Torques on gears *C* and *E*:

$$T_C = \frac{8}{20} T_D = 169.76 \text{ N} \cdot \text{m}$$

$$T_E = \frac{12}{20} T_D = 254.65 \text{ N} \cdot \text{m}$$

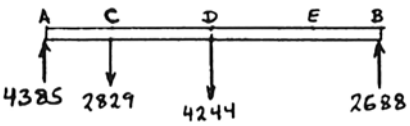
Forces on gears:

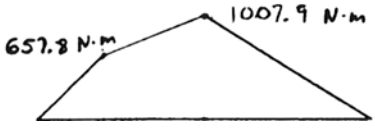
$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100 \times 10^{-3}} = 4244 \text{ N}$$

$$F_C = \frac{T_C}{r_C} = \frac{169.76}{60 \times 10^{-3}} = 2829 \text{ N}$$

$$F_E = \frac{T_E}{r_E} = \frac{254.65}{60 \times 10^{-3}} = 4244 \text{ N}$$

Forces in horizontal plane:





Torques in various parts:

AC: $T = 0$

CD: $T = 169.76 \text{ N} \cdot \text{m}$

DE: $T = 254.65 \text{ N} \cdot \text{m}$

EB: $T = 0$

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PROBLEM 8.68 *(Continued)*

Critical point lies just to the right of *D*.

$$T = 254.65\ \text{N} \cdot \text{m}$$

$$M_y = 1007.9\ \text{N} \cdot \text{m}$$

$$M_z = 318.3\ \text{N} \cdot \text{m}$$

$$\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max} = 1087.2\ \text{N} \cdot \text{m}$$

$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}}{\tau_{\text{all}}}$$

$$= \frac{1087.2}{55 \times 10^6}$$

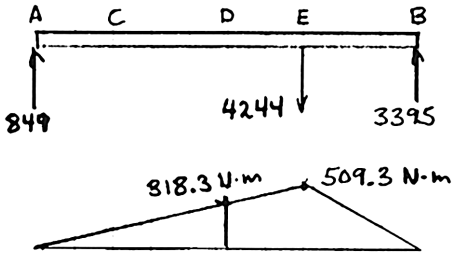
$$= 19.767 \times 10^{-3}\ \text{m}^3$$

$$c = 23.26 \times 10^{-3}\ \text{m}$$

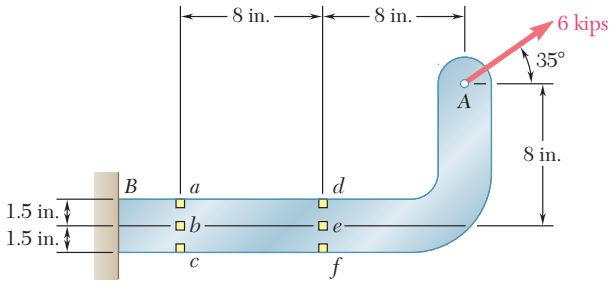
$$d = 2c = 46.5 \times 10^{-3}\ \text{m}$$

$$d = 46.5\ \text{mm} \blacktriangleleft$$

Forces in vertical plane:



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PROBLEM 8.69

A 6-kip force is applied to the machine element *AB* as shown. Knowing that the uniform thickness of the element is 0.8 in., determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Thickness = 0.8 in.

At the section containing points *a*, *b*, and *c*,

$$P = 6\cos 35° = 4.9149 \text{ kips} \qquad V = 6\sin 35° = 3.4415 \text{ kips}$$

$$M = (6\sin 35°)(16) - (6\cos 35°)(8) = 15.744 \text{ kip} \cdot \text{in.}$$

$$A = (0.8)(3.0) = 2.4 \text{ in}^2$$

$$I = \frac{1}{12}(0.8)(3.0)^3 = 1.80 \text{ in}^4$$

(*a*) At point *a*,

$$\sigma_x = \frac{P}{A} - \frac{Mc}{I} = \frac{4.9149}{2.4} - \frac{(15.744)(1.5)}{1.80}$$

$\sigma_x = -11.07$ ksi ◀

$\tau_{xy} = 0$ ◀

(*b*) At point *b*,

$$\sigma_x = \frac{P}{A} = \frac{4.9149}{2.4}$$

$\sigma_x = 2.05$ ksi ◀

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2} \cdot \frac{3.4415}{2.4}$$

$\tau_{xy} = 2.15$ ksi ◀

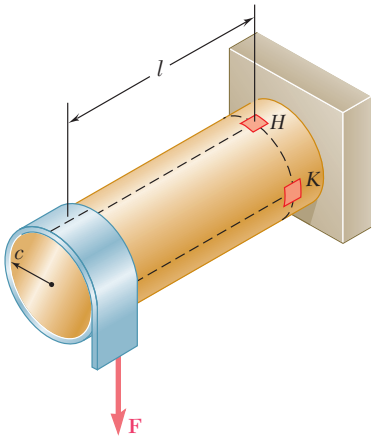
(*c*) At point *c*,

$$\sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{4.9149}{2.4} + \frac{(15.744)(1.5)}{1.80}$$

$\sigma_x = 15.17$ ksi ◀

$\tau_{xy} = 0$ ◀

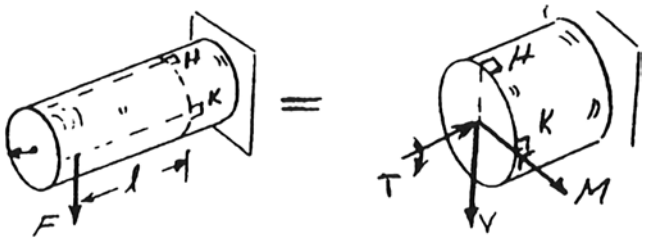
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PROBLEM 8.70

A thin strap is wrapped around a solid rod of radius $c = 20$ mm as shown. Knowing that $l = 100$ mm and $F = 5$ kN, determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.

SOLUTION



At the section containing points *H* and *K*,

$$T = Fc \qquad M = Fl \qquad V = F$$

$$J = \frac{\pi}{2}c^4 \qquad I = \frac{\pi}{4}c^4$$

Point *H*:

$$\sigma = \frac{Mc}{I} = \frac{Flc}{\frac{\pi}{4}c^4} \qquad \sigma = \frac{4Fl}{\pi c^2}$$

$$\tau = \frac{Tc}{J} = \frac{Fc^2}{\frac{\pi}{4}c^4} \qquad \tau = \frac{2F}{\pi c^2}$$

Point *K*: Point *K* lies on the neutral axis. $\sigma = 0$

Due to torque:

$$\tau = \frac{Tc}{J} = \frac{2F}{\pi c^2}$$

Due to shear: For a semicircle, $Q = \frac{2}{3}c^3, \quad t = d = 2c$

$$\tau = \frac{VQ}{It} = \frac{F\frac{2}{3}c^3}{\frac{\pi}{4}c^4(2c)} = \frac{4F}{3\pi c^2}$$

Combined:

$$\tau = \frac{2F}{\pi c^2} + \frac{4F}{3\pi c^2} \qquad \tau = \frac{10F}{3\pi c^2}$$

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PROBLEM 8.70 *(Continued)*

Data: $F = 5 \text{ kN} = 5 \times 10^3 \text{ N}, \quad l = 100 \text{ mm} = 0.100 \text{ m}$

$c = 20 \text{ mm} = 0.020 \text{ m}$

(*a*) Point *H*:

$$\sigma = \frac{(4)(5 \times 10^3)(0.100)}{\pi (0.020)^3} \qquad \sigma = 79.6 \text{ MPa} \blacktriangleleft$$

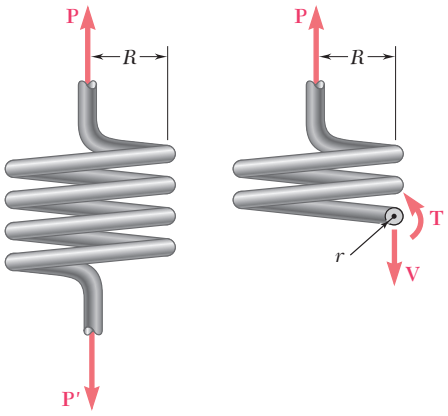
$$\tau = \frac{(2)(5 \times 10^3)}{\pi (0.020)^2} \qquad \tau = 7.96 \text{ MPa} \blacktriangleleft$$

(*b*) Point *K*:

$$\sigma = 0 \blacktriangleleft$$

$$\tau = \frac{(10)(5 \times 10^3)}{3\pi (0.020)^2} \qquad \tau = 13.26 \text{ MPa} \blacktriangleleft$$

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PROBLEM 8.71

A close-coiled spring is made of a circular wire of radius r that is formed into a helix of radius R. Determine the maximum shearing stress produced by the two equal and opposite forces **P** and **P′**. (*Hint:* First determine the shear **V** and the torque **T** in a transverse cross section.)

SOLUTION

$$+\uparrow \Sigma F_y = 0: \qquad P - V = 0 \qquad V = P$$

$$+\circlearrowleft \Sigma M_C = 0: \qquad T - PR = 0 \qquad T = PR$$

Shearing stress due to T.

$$\tau_T = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2PR}{\pi r^3}$$

Shearing stress due to V.

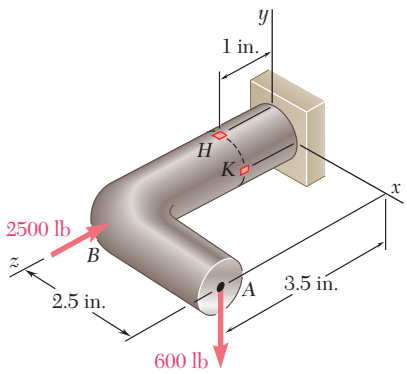
For semicircle,
$$Q = \frac{2}{3}r^3, \quad t = d = 2r$$

For solid circular section,
$$I = \frac{1}{2}J = \frac{\pi}{4}r^3$$

$$\tau_V = \frac{VQ}{It} = \frac{V\left(\frac{2}{3}r^3\right)}{\frac{\pi}{4}r^4(2r)} = \frac{4V}{3\pi r^2} = \frac{4P}{3\pi r^2}$$

By superposition,
$$\tau_{\max} = \tau_T + \tau_V \qquad \tau_{\max} = P(2R + 4r/3)/\pi r^3 \blacktriangleleft$$

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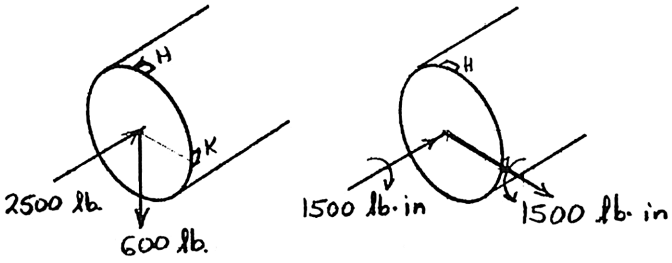
PROBLEM 8.72

Forces are applied at points *A* and *B* of the solid cast-iron bracket shown. Knowing that the bracket has a diameter of 0.8 in., determine the principal stresses and the maximum shearing stress at (*a*) point *H*, (*b*) point *K*.

SOLUTION

At the section containing points *H* and *K*,

$$P = 2500 \text{ lb} \quad \text{(compression)}$$
$$V_y = -600 \text{ lb}$$
$$V_x = 0$$
$$M_x = (3.5-1)(600) = 1500 \text{ lb}\cdot\text{in.}$$
$$M_y = 0$$
$$M_z = -(2.5)(600) = -1500 \text{ lb}\cdot\text{in.}$$



Forces Couples

$$c = \frac{1}{2}d = 0.4 \text{ in.}$$
$$A = \pi c^2 = 0.50265 \text{ in}^2$$
$$I = \frac{\pi}{4}c^4 = 20.106\times10^{-3} \text{ in}^4$$
$$J = 2I = 40.212\times10^{-3} \text{ in}^4$$

For semicircle,

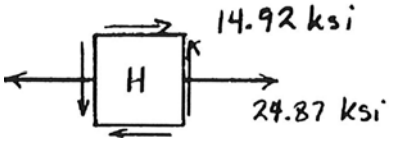
$$Q = \frac{2}{3}c^3$$
$$= 42.667\times10^{-3} \text{ in}^3$$

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PROBLEM 8.72 *(Continued)*

(*a*) At point *H*,

$$\sigma_H = \frac{P}{A} + \frac{Mc}{I} = -\frac{2500}{0.50265} + \frac{(1500)(0.4)}{20.106\times10^{-3}}$$
$$= 24.87\times10^{3}\ \text{psi}$$
$$\tau_H = \frac{Tc}{J} = \frac{(1500)(0.4)}{40.212\times10^{-3}} = 14.92\times10^{3}\ \text{psi}$$

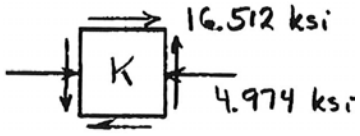


$$\sigma_{\text{ave}} = \frac{24.87}{2} = 12.435\ \text{ksi}$$
$$R = \sqrt{\left(\frac{24.87}{2}\right)^2 + (14.92)^2} = 19.423\ \text{ksi}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R \qquad \sigma_{\max} = 31.9\ \text{ksi} \blacktriangleleft$$
$$\sigma_{\min} = \sigma_{\text{ave}} - R \qquad \sigma_{\min} = -6.99\ \text{ksi} \blacktriangleleft$$
$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 19.42\ \text{ksi} \blacktriangleleft$$

(*b*) At point *K*,

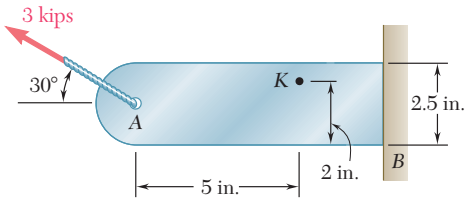
$$\sigma_K = \frac{P}{A} = -\frac{2500}{0.50265} = -4.974\times10^{3}\ \text{psi}$$
$$\tau_K = \frac{Tc}{J} + \frac{VQ}{It}$$
$$= \frac{(1500)(0.4)}{40.212\times10^{-3}} + \frac{(600)(42.667\times10^{-3})}{(20.106\times10^{-3})(0.8)}$$
$$= 16.512\times10^{3}\ \text{psi}$$



$$\sigma_{\text{ave}} = -\frac{4.974}{2} = -2.487\ \text{ksi}$$
$$R = \sqrt{\left(-\frac{4.974}{2}\right)^2 + (16.512)^2} = 16.698\ \text{ksi}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R \qquad \sigma_{\max} = 14.21\ \text{ksi} \blacktriangleleft$$
$$\sigma_{\min} = \sigma_{\text{ave}} - R \qquad \sigma_{\min} = -19.18\ \text{ksi} \blacktriangleleft$$
$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 16.70\ \text{ksi} \blacktriangleleft$$

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PROBLEM 8.73

Knowing that the bracket *AB* has a uniform thickness of $\frac{5}{8}$ in., determine (*a*) the principal planes and principal stresses at point *K*, (*b*) the maximum shearing stress at point *K*.

SOLUTION

Resolve the 3-kip force **F** at point *A* into *x* and *y* components.

$$F_x = -F\cos 30° = -(3)\cos 30° = -2.598 \text{ kips}$$

$$F_y = F\sin 30° = (3)\sin 30° = 1.5 \text{ kips}$$

At the section containing points *H* and *K*,

$$P = -F_x = 2.598 \text{ kips}, \quad V = F_y = 1.5 \text{ kips}$$

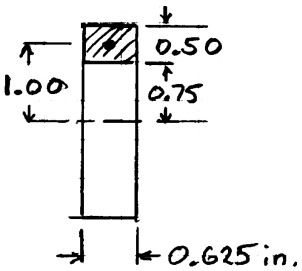
$$M = (5)(1.5) = 7.5 \text{ kip} \cdot \text{in.}$$

Section properties. $t = \frac{5}{8}$ in. $= 0.625$ in., $A = \left(\frac{5}{8}\right)(2.5) = 1.5625 \text{ in}^2$,

$$I = \frac{1}{12}(0.625)(2.5)^3 = 0.8138 \text{ in}^4, \qquad c = 1.25 \text{ in.}$$

At point *k*, $y = 0.75$ in.

$$Q = (0.625)(0.50)(1.00) = 0.3125 \text{ in}^3$$

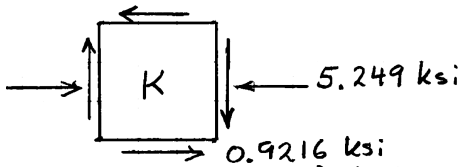


Stresses at point *K*.

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{2.598}{1.5625} - \frac{(7.5)(0.75)}{0.8138} = -5.249 \text{ ksi}$$

$$\tau = \frac{VQ}{It} = \frac{(1.5)(0.3125)}{(0.8138)(0.625)} = 0.9216 \text{ ksi}$$

$$\sigma_x = -5.249 \text{ ksi}, \quad \sigma_y = 0, \quad \tau_{xy} = -0.9216 \text{ ksi}$$



Mohr's circle.

$$X: (\sigma_x, -\tau_{xy}) = (-5.249 \text{ ksi}, 0.9216 \text{ ksi})$$

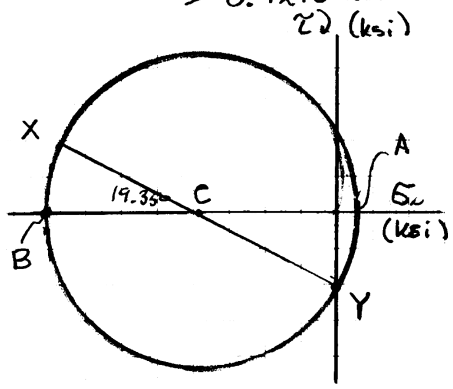
$$Y: (\sigma_y, \tau_{xy}) = (0, -0.9216 \text{ ksi})$$

$$C: (\sigma_{ave}, 0) = (-2.6245 \text{ ksi}, 0)$$

$$\frac{\sigma_x - \sigma_y}{2} = -2.6245 \text{ ksi}$$

$$R = \sqrt{(2.6245)^2 + (0.9216)^2} = 2.7816 \text{ ksi}$$

$$\tan 2\theta_p = \frac{0.9216}{2.6245} = 0.35112 \quad 2\theta_p = 19.35°$$

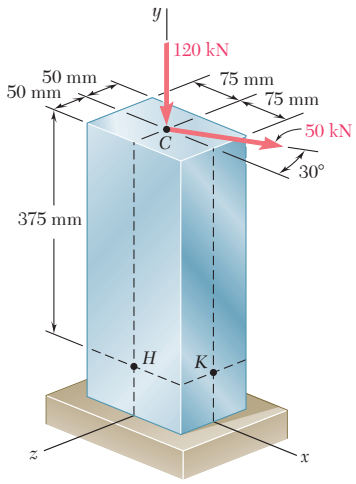


(*a*) $\sigma_{max} = \sigma_{ave} + R = -2.6245 + 2.7816$ $\quad \sigma_{max} = 0.157 \text{ ksi at } 80.3°$ ◀

(*b*) $\sigma_{min} = \sigma_{ave} - R = -2.6245 - 2.7816$ $\quad \sigma_{min} = -5.41 \text{ ksi at } 9.7°$ ◀

$\tau_{max} = R = 2.7816 \text{ ksi}$ $\quad \tau_{max} = 2.78 \text{ ksi}$ ◀

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PROBLEM 8.74

For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

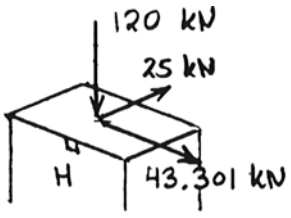
SOLUTION

Components of force at point *C*:

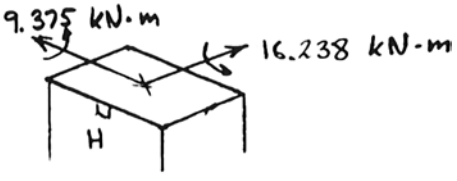
$$F_x = 50 \cos 30° = 43.301 \text{ kN}$$
$$F_z = -50 \sin 30° = -25 \text{ kN}$$
$$F_y = -120 \text{ kN}$$

Section forces and couples at the section containing points *H* and *K*:

$$P = 120 \text{ kN} \qquad \text{(compression)}$$
$$V_x = 43.301 \text{ kN}$$
$$V_z = -25 \text{ kN}$$
$$M_x = -(25)(0.375) = -9.375 \text{ kN} \cdot \text{m}$$
$$M_y = 0$$
$$M_z = -(43.301)(0.375) = -16.238 \text{ kN} \cdot \text{m}$$
$$A = (100)(150) = 15 \times 10^3 \text{ mm}^2 = 15 \times 10^{-3} \text{ m}^2$$
$$I_x = \frac{1}{12}(150)(100)^3 = 12.5 \times 10^6 \text{ mm}^4 = 12.5 \times 10^{-6} \text{ m}^4$$



Forces



Couples

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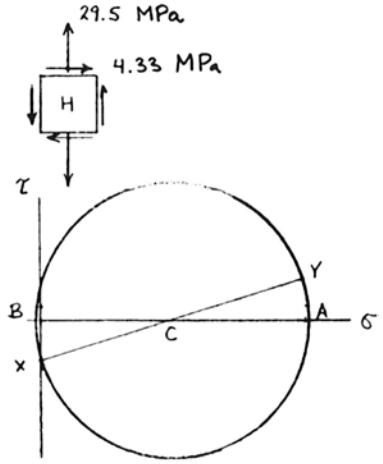
PROBLEM 8.74 *(Continued)*

Stresses at point *H*:

$$\sigma_H = -\frac{P}{A} - \frac{M_x z}{I_x} = -\frac{(120\times10^3)}{15\times10^{-3}} - \frac{(-9.375\times10^3)(50\times10^{-3})}{12.5\times10^{-6}} = 29.5 \text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{V_x}{A} = \frac{3}{2}\frac{43.301\times10^3}{15\times10^{-3}} = 4.33 \text{ MPa}$$

Use Mohr's circle.



$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_H = 14.75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{29.5}{2}\right)^2 + 4.33^2} = 15.37 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 30.1 \text{ MPa} \blacktriangleleft$$

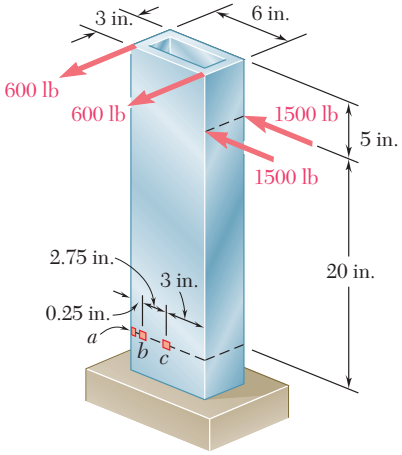
$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -0.62 \text{ MPa} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{-\sigma_H} = -0.2936$$

$$\theta_a = -8.2^\circ \quad \theta_b = 81.8^\circ \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 15.37 \text{ MPa} \blacktriangleleft$$

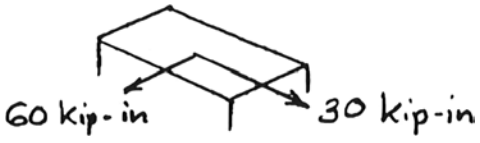
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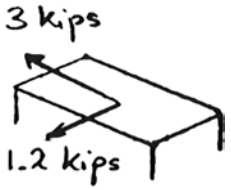
PROBLEM 8.75

Knowing that the structural tube shown has a uniform wall thickness of 0.25 in., determine the normal and shearing stresses at the three points indicated.

SOLUTION

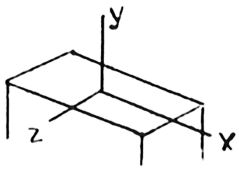


Bending moment



Shear

$$b_o = 6 \text{ in.} \quad b_i = b_o - 2t = 5.5 \text{ in.}$$
$$h_o = 3 \text{ in.} \quad h_i = h_o - 2t = 2.5 \text{ in.}$$
$$I_x = \frac{1}{12}(b_o h_o^3 - b_i h_i^3) = 6.3385 \text{ in}^4$$
$$I_z = \frac{1}{12}(h_o b_o^3 - h_i b_i^3) = 19.3385 \text{ in}^4$$



Normal stresses.

$$\sigma = \frac{M_z x}{I_z} - \frac{M_x z}{I_x}$$

At a: $\dfrac{(60)(-3)}{19.3385} - \dfrac{(30)(1.5)}{6.3385}$ $\quad \sigma_a = -16.41 \text{ ksi}$ ◀

At b: $\dfrac{(60)(-2.75)}{19.3385} - \dfrac{(30)(1.5)}{6.3385}$ $\quad \sigma_b = -15.63 \text{ ksi}$ ◀

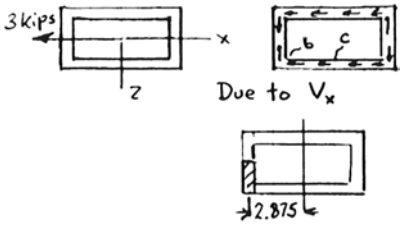
At c: $\dfrac{(60)(0)}{19.3385} - \dfrac{(30)(1.5)}{6.3385}$ $\quad \sigma_c = -7.10 \text{ ksi}$ ◀

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PROBLEM 8.75 *(Continued)*

Shearing stresses.

Point *a* is an outside corner. $\tau_a = 0$ ◀



At point *b*,

$$Q_{zb} = (1.5)(0.25)(2.875) = 1.0781 \text{ in}^3$$

$$\tau_{b,v_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(1.0781)}{(19.3385)(0.25)} = 0.66899 \text{ ksi}$$

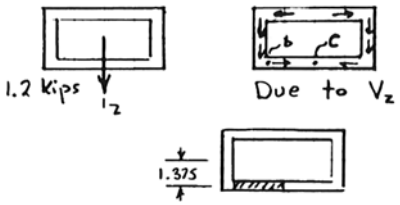
At point *c*,



$$Q_{zc} = Q_{zb} + (2.75)(0.25)\left(\frac{2.75}{2}\right) = 2.0234 \text{ in}^3$$

$$\tau_{c,v_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(2.0234)}{(19.3385)(0.25)} = 1.256 \text{ ksi}$$

At point *b*,



$$Q_{xb} = (2.75)(0.25)(1.375) = 0.9453 \text{ in}^3$$

$$\tau_{b,v_z} = \frac{V_z Q_y}{I_x t} = \frac{(1.2)(0.9453)}{(6.3385)(0.25)} = 0.71585 \text{ ksi}$$

At point *c*, (symmetry axis)

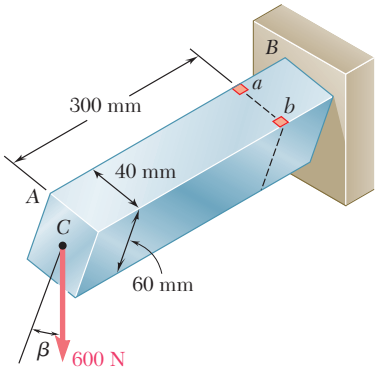
$$\tau_{c,v_z} = 0$$

Net shearing stress at points *b* and *c*:

$$\tau_b = 0.71585 - 0.66899 \qquad \tau_b = 0.0469 \text{ ksi} \blacktriangleleft$$

$$\tau_c = 1.256 \qquad \tau_c = 1.256 \text{ ksi} \blacktriangleleft$$

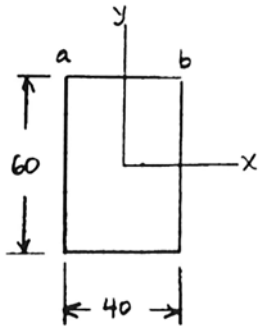
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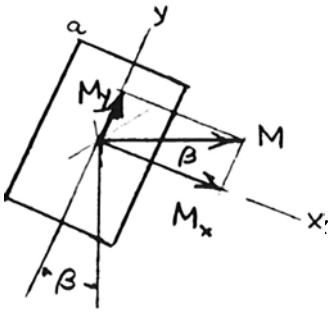
PROBLEM 8.76

The cantilever beam *AB* will be installed so that the 60-mm side forms an angle β between 0 and 90° with the vertical. Knowing that the 600-N vertical force is applied at the center of the free end of the beam, determine the normal stress at point *a* when (*a*) $\beta = 0$, (*b*) $\beta = 90°$. (*c*) Also, determine the value of β for which the normal stress at point *a* is a maximum and the corresponding value of that stress.

SOLUTION



$$S_x = \frac{1}{6}(40)(60)^2 = 24 \times 10^3 \text{ mm}^3$$
$$= 24 \times 10^{-6} \text{ m}^3$$
$$S_y = \frac{1}{6}(60)(40)^2 = 16 \times 10^3 \text{ mm}^3$$
$$= 16 \times 10^{-3} \text{ m}^3$$
$$M = Pl = (600)(300 \times 10^{-3}) = 180 \text{ N} \cdot \text{m}$$
$$M_x = M \cos\beta = 180 \cos\beta$$
$$M_y = M \sin\beta = 180 \sin\beta$$



$$\sigma_a = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{180\cos\beta}{24 \times 10^{-6}} + \frac{180\sin\beta}{16 \times 10^{-6}}$$
$$= (7.5 \times 10^6)\left(\cos\beta + \frac{3}{2}\sin\beta\right) \text{Pa}$$
$$= 7.5\left(\cos\beta + \frac{3}{2}\sin\beta\right) \text{MPa}$$

(*a*) $\beta = 0$. $\sigma_a = 7.50$ MPa $\sigma = 7.50$ MPa ◀

(*b*) $\beta = 90°$. $\sigma_a = 11.25$ MPa $\sigma = 11.25$ MPa ◀

(*c*) Maximum. $\dfrac{d\sigma_a}{d\beta} = 7.5\left(-\sin\beta + \frac{3}{2}\cos\beta\right) = 0$

$$\sin\beta = \frac{3}{2}\cos\beta \qquad \tan\beta = \frac{3}{2} \qquad \beta = 56.3° \blacktriangleleft$$

$$\sigma_a = 7.5\left(\cos 56.3° + \frac{3}{2}\sin 56.3°\right) \qquad \sigma = 13.52 \text{ MPa} \blacktriangleleft$$

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PROBLEM 8.C1

Let us assume that the shear V and the bending moment M have been determined in a given section of a rolled-steel beam. Write a computer program to calculate in that section, from the data available in Appendix C, (*a*) the maximum normal stress σ_m, (*b*) the principal stress $\sigma_{\max}$ at the junction of a flange and the web. Use this program to solve parts *a* and *b* of the following problems:

(1) Prob. 8.1 (Use $V = 45$ kips and $M = 450$ kip · in.)

(2) Prob. 8.2 (Use $V = 22.5$ kips and $M = 450$ kip · in.)

(3) Prob. 8.3 (Use $V = 700$ kN and $M = 1750$ kN · m.)

(4) Prob. 8.4 (Use $V = 850$ kN and $M = 1700$ kN · m.)

SOLUTION

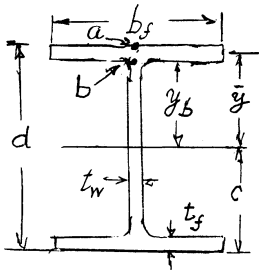
We enter the given values of V and M and obtain from Appendix C the values of d, b_f, t_f, t_w, I, and S for the given WF shape.

We compute

$$c = \frac{d}{2}, \quad y_b = c - t_f$$

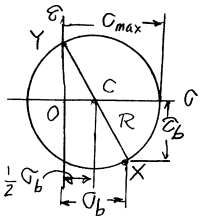
$$\overline{y} = c - \frac{1}{2}t_f, \quad \sigma_a = \frac{M}{S}, \quad \sigma_b = \sigma_a\left(\frac{y_b}{c}\right)$$

$$Q = b_f t_f \overline{y}, \qquad \tau_b = \frac{VQ}{It_w}$$



From Mohr's circle, $\sigma_{\max} = \frac{1}{2}\sigma_b + R$

$$\sigma_{\max} = \frac{1}{2}\sigma_b + \sqrt{\left(\frac{1}{2}\sigma_b\right)^2 + \tau_b^2}$$



Program Outputs

Problem 8.1	Problem 8.2
Given Data:	Given Data:
$V = 45$ kips, $M = 450$ kip · in.	$V = 22.5$ kips, $M = 450$ kip · in.
$d = 9.92$ in., $b_f = 7.985$ in.	$d = 9.92$ in., $b_f = 7.985$ in.
$t_f = 0.530$ in., $t_w = 0.315$ in.	$t_f = 0.530$ in., $t_w = 0.315$ in.
$I = 209.00$ in.^{4}, $S = 42.10$ in.^{4}	$I = 209.00$ in.^{4}, $S = 42.10$ in.^{4}
Answers:	Answers:
(a) $\sigma_A = 10{,}688.84$ psi	(a) $\sigma_A = 10{,}688.84$ psi
(b) $\sigma_m = 19{,}169.08$ psi	(b) $\sigma_m = 13{,}073.82$ psi

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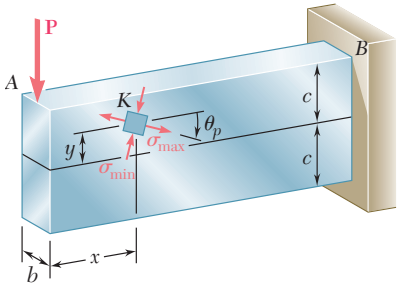
PROBLEM 8.C1 *(Continued)*

Program Outputs (*Continued*)

Problem 8.3	Problem 8.4
Given Data:	Given Data:
$V = 700$ kN, $M = 1750$ kN · m	$V = 850$ kN, $M = 1700$ kN · m
$d = 9.30$ mm, $b_f = 423$ mm	$d = 930$ mm, $b_f = 423$ mm
$t_f = 43$ mm, $t_w = 24$ mm	$t_f = 43$ mm, $t_w = 24$ mm
$I = 8470\ (10^6\ \text{mm}^4)$	$I = 8470\ (10^6 \text{mm}^4)$
$S = 18{,}200\ (10^3 \text{mm}^3)$	$S = 18{,}200\ (10^3 \text{mm}^3)$
Answers:	Answers:
(a) $\sigma_A = 96.15$ MPa	(a) $\sigma_A = 93.40$ MPa
(b) $\sigma_m = 95.39$ MPa	(b) $\sigma_m = 96.56$ MPa

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PROBLEM 8.C2



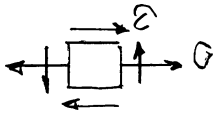
A cantilever beam *AB* with a rectangular cross section of width *b* and depth 2*c* supports a single concentrated load **P** at its end *A*. Write a computer program to calculate, for any values of *x*/*c* and *y*/*c*, (*a*) the ratios σ_{max}/σ_m and σ_{min}/σ_m, where σ_{max} and σ_{min} are the principal stresses at Point *K*(*x*, *y*) and σ_m the maximum normal stress in the same transverse section, (*b*) the angle θ_p that the principal planes at *K* form with a transverse and a horizontal plane through *K*. Use this program to check the values shown in Fig. 8.8 and to verify that σ_{max} exceeds σ_m if $x \leqslant 0.544c$, as indicated in the second footnote on Page 560.

SOLUTION

Since the distribution of the normal stresses is linear, we have $\sigma = \sigma_m \left(\frac{y}{c}\right)$ (1)

where $\sigma_m = \frac{MC}{I} = \frac{Pxc}{I}$ (2)

We use Equation (8.4), Page 498: $\tau = \frac{3}{2}\frac{P}{A}\left(1 - \frac{y^2}{c^2}\right)$ (3)



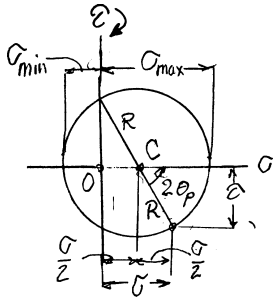
Dividing (3) by (2), $\frac{\tau}{\sigma_m} = \frac{3}{2}\frac{I}{A}\frac{1-\left(\frac{y}{c}\right)^2}{xc}$

or, since $\frac{I}{A} = \frac{\frac{1}{12}b(2c)^3}{b(2c)} = \frac{1}{3}c^2, \quad \frac{\tau}{\sigma_m} = \frac{1}{2}\frac{1-\left(\frac{y}{c}\right)^2}{\frac{x}{c}}$ (4)

Letting $X = \frac{x}{c}$ and $Y = \frac{y}{c}$, Equations (1) and (4) yield

$$\sigma = \sigma_m Y$$

$$\tau = \sigma_m \frac{1-Y^2}{2X}$$



Using Mohr's circle, we calculate

$$R = \sqrt{\left(\frac{1}{2}\sigma\right)^2 + \tau^2} = \frac{1}{2}\sigma_m\sqrt{Y^2 + \left(\frac{1-Y^2}{X}\right)^2}$$

$$\frac{\sigma_{max}}{\sigma_m} = \frac{1}{2}Y + R$$

$$\frac{\sigma_{min}}{\sigma_m} = \frac{1}{2}Y - R$$

◀

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PROBLEM 8.C2 *(Continued)*

$$\tan 2\theta_P = \frac{\tau}{\frac{\sigma}{2}} = \frac{1-Y^2}{2X(\frac{y}{2})} = \frac{1-Y^2}{XY}$$

$$\theta_P = \frac{1}{2}\tan^{-1}\left(\frac{1-Y^2}{XY}\right) \text{ CW}$$ ◀

Note:

For $y > 0$, the angle θ_P is CW, which is opposite to what was arbitrarily assumed in Figure P8.C2.

Program Outputs

For $\frac{x}{c} = 2$,

$\frac{x}{c}$	$\frac{\sigma_{\min}}{\sigma_m}$	$\frac{\sigma_{\max}}{\sigma_m}$	θ CW
1.0	0.000	1.000	0.00
0.8	–0.010	0.810	6.34
0.6	–0.040	0.640	14.04
0.4	–0.090	0.490	23.20
0.2	–0.160	0.360	33.69
0.0	–0.250	0.250	45.00
–0.2	–0.360	0.160	–33.69
–0.4	–0.490	0.090	–23.20
–0.6	–0.640	0.040	–14.04
–0.8	–0.810	0.010	–6.34
–1.0	–1.000	0.000	–0.00

For $\frac{x}{c} = 8$,

$\frac{y}{c}$	$\frac{\sigma_{\min}}{\sigma_m}$	$\frac{\sigma_{\max}}{\sigma_m}$	θ CW
1.0	0.000	1.000	0.00
0.8	–0.001	0.801	1.61
0.6	–0.003	0.603	3.80
0.4	–0.007	0.407	7.35
0.2	–0.017	0.217	15.48
0.0	–0.062	0.063	45.00
–0.2	–0.217	0.017	–15.48
–0.4	–0.407	0.007	–7.35
–0.6	–0.603	0.003	–3.80
–0.8	–0.801	0.001	–1.61
–1.0	–1.000	0.000	–0.00

To check that $\sigma_{\max} > \sigma_m$ if $x \leqslant 0.544c$, we run the program for $\frac{x}{c} = 0.544$ and for $\frac{x}{c} = 0.545$ and observe that $\frac{\sigma_{\max}}{\sigma_m}$ exceeds 1 for several values of $\frac{y}{c}$ in the first case, but does not exceed 1 in the second case.

For $\frac{x}{c} = 0.544$,

$\frac{y}{c}$	$\frac{\sigma_{\min}}{\sigma_m}$	$\frac{\sigma_{\max}}{\sigma_m}$	θ CW
0.30	–0.700	0.9997	39.92
0.31	–0.690	1.0001	39.72
0.32	–0.680	1.0004	39.51
0.33	–0.670	1.0005	39.30
0.34	–0.660	1.0005	39.09
0.35	–0.650	1.0003	38.88
0.36	–0.640	1.0000	38.66
0.37	–0.630	0.9996	38.44
0.38	–0.619	0.9990	38.21
0.39	–0.608	0.9983	37.98
0.40	–0.598	0.9975	37.74

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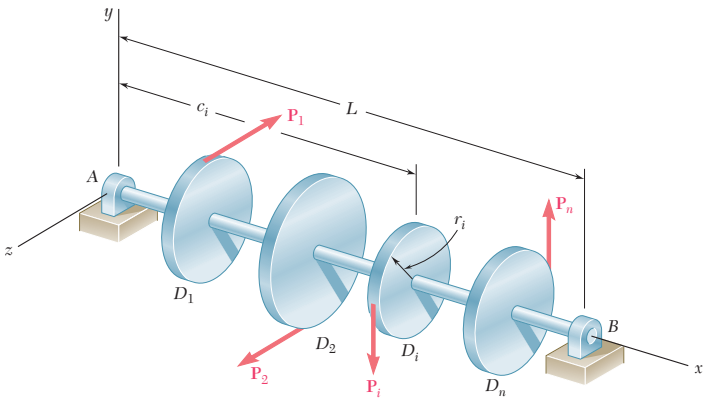
PROBLEM 8.C2 *(Continued)*

Program Outputs (*Continued*)

For $\frac{x}{c} = 0.545$,

$\frac{y}{c}$	$\frac{\sigma_{\min}}{\sigma_m}$	$\frac{\sigma_{\max}}{\sigma_m}$	θ
0.30	–0.698	0.9982	39.91
0.31	–0.689	0.9986	39.71
0.32	–0.679	0.9989	39.50
0.33	–0.669	0.9990	39.29
0.34	–0.659	0.9990	39.08
0.35	–0.649	0.9988	38.87
0.36	–0.639	0.9986	38.65
0.37	–0.628	0.9982	38.42
0.38	–0.618	0.9976	38.20
0.39	–0.607	0.9970	37.96
0.40	–0.596	0.9962	37.73

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PROBLEM 8.C3

Disks $D_1, D_2, \ldots, D_n$ are attached as shown in Fig. 8.C3 to the solid shaft *AB* of length *L*, uniform diameter *d*, and allowable shearing stress τ_{all}. Forces $\mathbf{P}_1$, $\mathbf{P}_2$, . . . , $\mathbf{P}_n$ of known magnitude (except for one of them) are applied to the disks, either at the top or bottom of its vertical diameter, or at the left or right end of its horizontal diameter. Denoting by r_i the radius of disk D_i and by c_i its distance from the support at *A*, write a computer program to calculate (*a*) the magnitude of the unknown force $\mathbf{P}_i$, (*b*) the smallest permissible value of the diameter *d* of shaft *AB*. Use this program to solve Prob. 8.18.

SOLUTION

1. Determine the unknown force P_i by equating to zero the sum of their torques T_i about the *x* axis.

2. Determine the components $(F_y)_i$ and $(F_z)_i$ of all forces.

3. Determine the components A_y and A_z of reaction at *A* by summing moments about axes $Bz' \,//\, z$ and $By' \,//\, y$:

$$\Sigma M_{z'} = 0: \quad -A_y L - \Sigma(F_y)_i (L - c_i) = 0, \qquad A_y = -\frac{1}{L}\Sigma(F_y)_i (L - c_i)$$

$$\Sigma M_{y'} = 0: \quad A_z L + \Sigma(F_z)_i (L - c_i) = 0, \qquad A_z = -\frac{1}{L}\Sigma(F_z)_i (L - c_i)$$

4. Determine $(M_y)_i$, $(M_z)_i$ and torque T_i just to the left of disk D_i:

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k \langle c_i - c_k \rangle^1$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k \langle c_i - c_k \rangle^1$$

$$T_i = \sum_k T_k \langle c_i - c_k \rangle^0$$

where < > indicates a singularity function.

5. The minimum diameter *d* required to the left of D_i is obtained by first computing $(J/c)_i$ from Equation (8.7).

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{\tau_{all}}$$

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PROBLEM 8.C3 *(Continued)*

6. Recalling that $J = \frac{1}{2}\pi c^4$ and, thus, that $\left(\frac{J}{c}\right)_i = \frac{1}{2}\pi c_i^3$, we have $c_i = \frac{2}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$ and $d_i = \frac{4}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$ ◀

   This is the required diameter just to the left of disk D_i.

7. The required diameter just to the right of disk D_i is obtained by replacing T_i with T_{i+1} in the above computation.

8. The smallest permissible value of the diameter of the shaft is the largest of the values obtained for D_i.

Program Output

Problem 8.19

Length of shaft = 28 in.

τ(ksi) = 8

For Disk 1,

Force = 0.500 kips
Radius of disk = 4.0 in.
Distance from A = 7.0 in.

For Disk 2,

Force = 0.000 kips
Radius of disk = 6.0 in.
Distance from A = 14.0 in.

For Disk 3,

Force = 0.500 kips
Radius of disk = 4.0 in.
Distance from A = 21.0 in.
Unknown force = –0.667 kips
A_Y = 0.500 kips, A_Z = 0.333 kips
B_Y = 0.500 kips, B_Z = 0.333 kips

Just to the left of Disk 1:

M_Y = 2.3333 kip · in.
M_Z = –3.5000 kip · in.
T = 0.0000 kip · in.

Diameter must be at least 1.389 in.

Just to the right of Disk 1:

T = 2.00 kip · in.

Diameter must be at least 1.437 in.

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PROBLEM 8.C3 *(Continued)*

Program Output (*Continued*)

Just to the left of Disk 2:

$M_Y = 4.6667$ kip · in.

$M_Z = -3.5000$ kip · in.

$T = 2.0000$ kip · in.

Diameter must be at least 1.578 in. ◀

Just to the right of Disk 2:

$T = -2.00$ kip · in.

Diameter must be at least 1.578 in.

Just to the left of Disk 3:

$M_Y = 2.3333$ kip · in.

$M_Z = -3.5000$ kip · in.

$T = -2.0000$ kip · in.

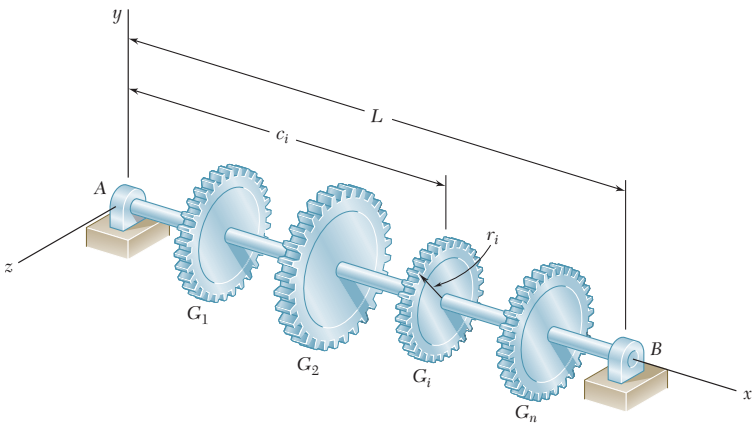
Diameter must be at least 1.437 in.

Just to the right of Disk 3:

$T = 0.00$ kip · in.

Diameter must be at least 1.389 in.

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PROBLEM 8.C4

The solid shaft *AB* of length *L*, uniform diameter *d*, and allowable shearing stress τ_{all} rotates at a given speed expressed in rpm (Fig. 8.C4). Gears G_1, G_2, . . . , G_n are attached to the shaft and each of these gears meshes with another gear (not shown), either at the top or bottom of its vertical diameter, or at the left or right end of its horizontal diameter. One of these gears is connected to a motor and the rest of them to various machine tools. Denoting by r_i the radius of disk G_i, by c_i its distance from the support at *A*, and by P_i the power transmitted to that gear (+sign) or taken of that gear (–sign), write a computer program to calculate the smallest permissible value of the diameter *d* of shaft *AB*. Use this program to solve Probs 8.27 and 8.68.

SOLUTION

1. Enter *w* in rpm and determine frequency $f = w/60$.
2. For each gear, determine the torque $T_i = P_i/2\pi f$, where P_i is the power input (+) or output (–) at the gear.
3. For each gear, determine the force $F_i = T_i/r_i$ exerted on the gear and its components $(F_y)_i$ and $(F_z)_i$.
4. Determine the components A_y and A_z of reaction at *A* by summing moments about axes $Bz' \,//\, z$ and $By' \,//\, y$:

$$\Sigma M_{z'} = 0: \quad -A_y L - \Sigma (F_y)_i (L - c_i) = 0, \quad A_y = -\frac{1}{L}\Sigma (F_y)_i (L - c_i)$$

$$\Sigma M_{y'} = 0: \quad A_z L + \Sigma (F_z)_i (L - c_i) = 0, \quad A_z = -\frac{1}{L}\Sigma (F_z)_i (L - c_i)$$

5. Determine $(M_y)_i, (M_z)_i$, and torque T_i just to the left of gear G_i:

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k \langle c_i - c_k \rangle^1$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k \langle c_i - c_k \rangle^1$$

$$T_i = \sum_k T_k \langle c_i - c_k \rangle^0$$

where $\langle\rangle$ indicates a singularity function.

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PROBLEM 8.C4 *(Continued)*

6. The minimum diameter d required to the left of G_i is obtained by first computing $(J/c)_i$ from Equation (8.7).

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{\left(M_y\right)_i^2 + \left(M_z\right)_i^2 + T_i^2}}{\tau_{\text{all}}}$$

7. Recalling that $J = \frac{1}{2}\pi c^4$ and, thus, that $\left(\frac{J}{c}\right)_i = \frac{1}{2}\pi c_i^3$,

   we have $c_i = \frac{2}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$ and $d_i = \frac{4}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$ ◀

   This is the required diameter just to the left of gear G_i.

8. The required diameter just to the right of gear G_i is obtained by replacing T_i with T_{i+1} in the above computation.
9. The smallest permissible value of the diameter of the shaft is the largest of the values obtained for d_i.

Program Outputs

Problem 8.25

ω = 450 rpm
Number of Gears: 3
Length of shaft = 750 mm
τ = 55 MPa

For Gear 1,
Power input = –8.00 kW
Radius of gear = 60 mm
Distance from *A* in mm = 150

For Gear 2,
Power input = 20.00 kW
Radius of gear = 100 mm
Distance from *A* in mm = 375

For Gear 3,
Power input = –12.00 kW
Radius of gear = 60 mm
Distance from *A* in mm = 600
A_Y = –0.849 kN, A_Z = 4.386
B_Y = –3.395 kN, B_Z = 2.688

Just to the left of Gear 1:
M_Y = 657.84 Nm
M_Z = 127.32 Nm
T = 0.00 Nm

Problem 8.27

ω = 600 rpm
Number of Gears: 3
Length of shaft = 24 in.
τ = 8 ksi

For Gear 1,
Power input = 60.00 hp
Radius of gear = 3.00 in.
Distance from *A* in inches = 4.0
F_Y = 0
F_z = 2.100845

For Gear 2,
Power input = –40.00 hp
Radius of gear = 4.00 in.
Distance from *A* in inches = 10.0
F_Y = 1.050423
F_Z = 0

For Gear 3,
Power input = –20.00 hp
Radius of gear = 4.00 in.
Distance from *A* in inches = 18.0
F_Y = 0
F_Z = –0.5252113

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PROBLEM 8.C4 *(Continued)*

Diameter must be at least 39.59 mm.

Just to the right of Gear 1:

$T = -169.77$ N · m

Diameter must be at least 40.00 mm.

Just to the left of Gear 2:

$M_Y = 1007.98$ N · m

$M_Z = 318.31$ N · m

$T = -169.77$ N · m

Diameter must be at least 46.28 mm.

Just to the right of Gear 2:

$T = 254.65$ N · m

Diameter must be at least 46.52 mm. ◀

Just to the left of Gear 3:

$M_Y = 403.19$ N · m

$M_Z = 509.30$ N · m

$T = 254.65$ N · m

Diameter must be at least 40.13 mm.

Just to the right of Gear 3:

$T = 0.00$ N · m

Diameter must be at least 39.18 mm.

$A_Y = -0.6127$ kips, $A_Z = -1.6194$ kips

$B_Y = -0.4377$ kips, $B_Z = 0.438$ kips

Just to the left of Gear 1:

$M_Y = -6.478$ kip · in.

$M_Z = 2.451$ kip · in.

$T = 0.000$ kip · in.

Diameter must be at least 1.640 in.

Just to the right of Gear 1:

$T = 6.3025$ kip · in.

Diameter must be at least 1.813 in.

Just to the left of Gear 2:

$M_Y = -3.589$ kip · in.

$M_Z = 6.127$ kip · in.

$T = 6.303$ kip · in.

Diameter must be at least 1.822 in. ◀

Just to the right of Gear 2:

$T = 2.1008$ kip · in.

Diameter must be at least 1.677 in.

Just to the left of Gear 3:

$M_Y = 0.263$ kip · in.

$M_Z = 2.626$ kip · in.

$T = 2.101$ kip · in.

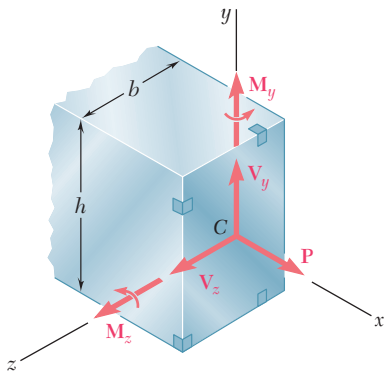
Diameter must be at least 1.290 in.

Just to the right of Gear 3:

$T = 0.000$ kip · in.

Diameter must be at least 1.189 in.

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PROBLEM 8.C5

Write a computer program that can be used to calculate the normal and shearing stresses at points with given coordinates *y* and *z* located on the surface of a machine part having a rectangular cross section. The internal forces are known to be equivalent to the force-couple system shown. Write the program so that the loads and dimensions can be expressed in either SI or U.S. customary units. Use this program to solve (*a*) Prob. 8.45*b*, (*b*) Prob. 8.47*a*.

SOLUTION

Enter: b and h

Program: $A = bh \qquad I_y = \dfrac{b^3 h}{12} \qquad I_z = \dfrac{bh^3}{12}$

For point on surface, enter y and z.

Note: y and z must satisfy one of following:

$$y^2 = \frac{h^2}{4} \quad \text{and} \quad z^2 \le \frac{b^2}{4} \tag{1}$$

or $$z^2 = \frac{b^2}{4} \quad \text{and} \quad y^2 \le \frac{h^2}{4} \tag{2}$$

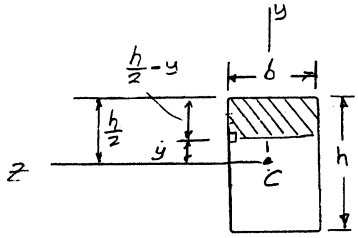
If either (1) and (2) are satisfied, compute

$$\sigma = \frac{P}{A} + \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$

If $z^2 = b^2/4$, then point is on vertical surface and

$$Q_z = b\left(\frac{h}{2} - y\right)\left(\frac{h}{2} + y\right)\frac{1}{2} = b\left(\frac{h^2}{8} - \frac{y^2}{2}\right)$$

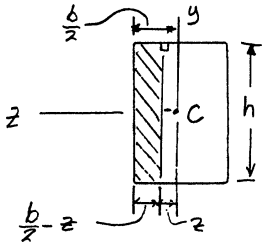
$$\gamma = \frac{v_y Q_z}{I_z b}$$



If $y^2 = h^2/4$, the point is on horizontal surface, and

$$Q_y = h\left(\frac{b}{2} - z\right)\left(\frac{b}{2} + z\right)\frac{1}{2} = h\left(\frac{b^2}{8} - \frac{z^2}{2}\right)$$

$$\gamma = \frac{V_z Q_y}{I_y h}$$



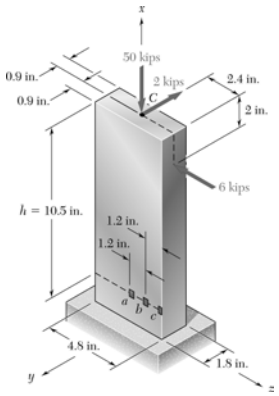
Force-couple system: $P = -50$ kips $\quad V_z = -6$ kips $\quad V_y = -2$ kips

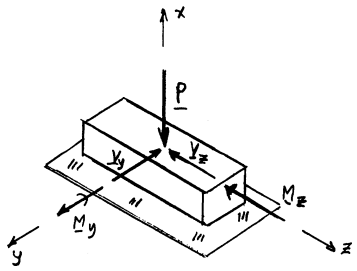
$$M_y = (6 \text{ kips})(8.5 \text{ in.}) = 51 \text{ kip} \cdot \text{in.}$$

$$M_z = (2 \text{ kips})(10.5 \text{ in.}) = 21 \text{ kip} \cdot \text{in.}$$

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PROBLEM 8.C5 *(Continued)*





Problem 8.45*b* Force-Couple at Centroid

$P = -50.00$ kips

$M_Y = 51.00$ kip · in. $M_Z = -21.00$ kip · in.

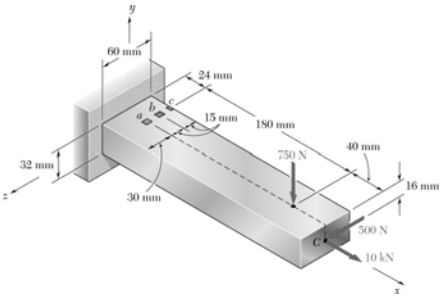
$V_Y = -2.00$ kips $V_Z = -6.00$ kips

+ +

At point of coordinates: $y = 0.90$ in. $z = 1.20$ in.

$\sigma = 6.004$ ksi

$\tau = -0.781$ ksi



Force-Couple System

$P = 10$ kN $V_y = -750$ N $V_z = 500$ N

$M_y = (500 \text{ N})(220 \text{ mm}) = 110 \text{ N} \cdot \text{m}$

$M_z = (750 \text{ N})(180 \text{ mm}) = 135 \text{ N} \cdot \text{m}$

Problem 8.47*a* Force-Couple at Centroid

$P = 10000.00$ N

$M_Y = 110.00$ N · m $M_Z = -135.00$ N · m

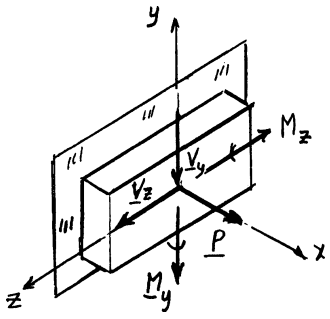
$V_Y = 750.00$ N $V_Z = 500.00$ N

+ +

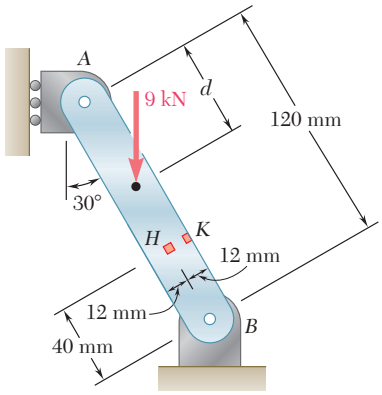
At point of coordinates: $y = 16.00$ mm $x = 0.00$ mm

$\sigma = 18.392$ MPa

$\tau = 0.391$ MPa



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PROBLEM 8.C6

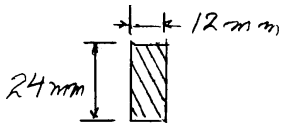
Member *AB* has a rectangular cross section of 10×24 mm. For the loading shown, write a computer program that can be used to determine the normal and shearing stresses at Points *H* and *K* for values of *d* from 0 to 120 mm, using 15-mm increments. Use this program to solve Prob. 8.35.

SOLUTION

Cross section:

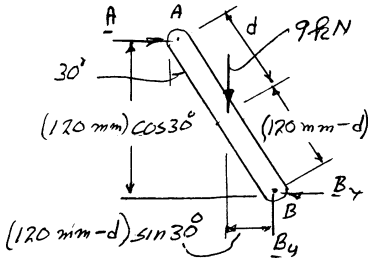
Enter

$$A = (0.010\text{ m})(0.024\text{ m}) = 240 \times 10^{-6}\text{ m}^2$$
$$I = (0.010\text{ m})(0.024\text{ m})^3/12 = 138.24 \times 10^{-9}\text{ m}^4$$
$$R = 0.5(0.029\text{ m}) = 12 \times 10^{-3}\text{ m}$$



Compute reaction at *A*.

$$+\circlearrowleft \Sigma M_B = 0: \quad (9\text{ kN})(120 - d)\sin 30° - A\,(120)\cos 30° = 0$$
$$A = (9\text{ kN})\frac{(120\text{ mm} - d)}{120\text{ mm}}\tan 30°$$

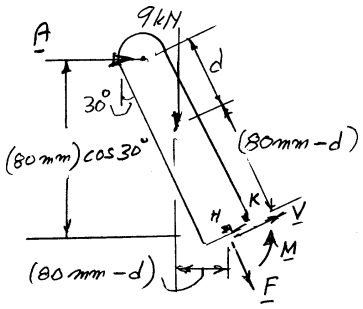


Free Body: From *A* to section containing Points *H* and *K*,

Define: If $d < 80$ mm, Then STP = 1 Else STP = 0

Program force-couple system.

$$F = -A\sin 30° - (9\text{ kN})\cos 30°\,(\text{Step})$$
$$V = -A\cos 30° + (9\text{ kN})\sin 30°\,(\text{Step})$$
$$M = A(80\text{ mm})\cos 30° - (9\text{ kN})(80\text{ mm} - d)\sin 30°\,(\text{STP})$$



At Point *H*,

$$\sigma_H = +F/A$$
$$\tau_H = \frac{3}{2}V/A$$

At Point *K*,

$$\sigma_K = +F/A - Mc/I$$
$$\tau_K = 0$$

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PROBLEM 8.C6 *(Continued)*

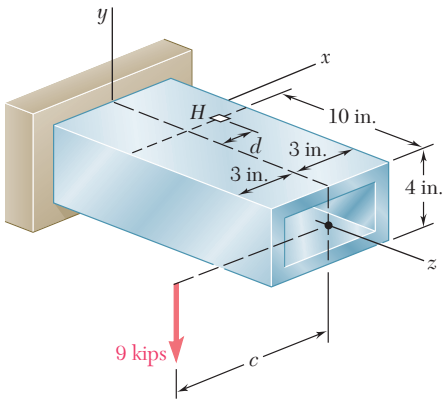
Program Output

Problem 8.35

	Stresses in MPa			
d (mm)	σ_H	τ_H	σ_K	τ_K
0.0	–43.30	0.00	–43.30	0.00
15.0	–41.95	3.52	–65.39	0.00
30.0	–40.59	7.03	–87.47	0.00
45.0	–39.24	10.55	–109.55	0.00
60.0	–37.89	14.06	–131.64	0.00
75.0	–36.54	17.58	–153.72	0.00
90.0	–2.71	–7.03	–96.46	0.00
105.0	–1.35	–3.52	–48.23	0.00
120.0	0.00	0.00	0.00	0.00

◀

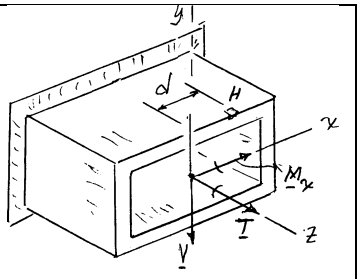
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PROBLEM 8.C7*

The structural tube shown has a uniform wall thickness of 0.3 in. A 9-kip force is applied at a bar (not shown) that is welded to the end of the tube. Write a computer program that can be used to determine, for any given value of c, the principal stresses, principal planes, and maximum shearing stress at Point H for values of d from -3 in. to 3 in., using one-inch increments. Use this program to solve Prob. 8.62*a*.

SOLUTION

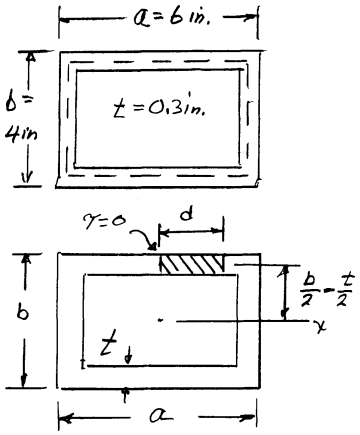


Force-couple system.

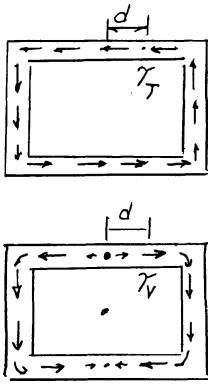
Enter:

$$V = 9 \text{ kips} \downarrow$$
$$M_x = (9 \text{ kips})(10 \text{ in.})$$
$$= 90 \text{ kips} \cdot \text{in.}$$
$$T = (9 \text{ kips})c$$

Area enclosed.

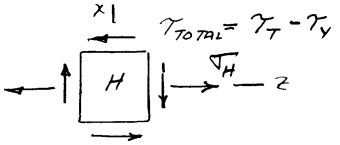


$$a = (a - t)(b - t)$$
$$\tau_T = \frac{T}{2ta} = \frac{9c}{2ta}$$
$$\tau_T = \text{Shearing stress due to torsion}$$
$$Q = dt\left(\frac{b}{2} - \frac{t}{2}\right)$$
$$\bar{I} = ab^3/12 - (a - 2t)(b - 2t)^3/12$$
$$\tau_V = \frac{VQ}{It}$$
$$\tau_V = \text{Shearing stress due to } V$$
$$\tau_{\text{total}} = \tau_T - \tau_Y$$



Bending:

$$\sigma_H = \frac{M_x\left(\frac{b}{2}\right)}{I}$$



Principal stresses:

$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_H; \quad R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{\text{total}}^2}$$
$$\sigma_{\max} = \sigma_{\text{ave}} + R; \quad \sigma_{\min} = \sigma_{\text{ave}} - R;$$
$$\theta_P = \frac{1}{2}\tan^{-1}\left(\frac{\tau_{\text{total}}}{\sigma_{\text{ave}}}\right); \quad \tau_{\max} = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{\text{total}}^2}$$

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PROBLEM 8.C7* *(Continued)*

Rectangular tube of uniform thickness $t = 0.3$ in.

Outside dimensions.

Horizontal width: $a = 6$ in.

Vertical depth: $b = 4$ in.

Vertical load: $P = 9$ kips;

Line of action at $x = -c$

Find normal and shearing stresses at point H. $(x = d,\ y = b/2)$

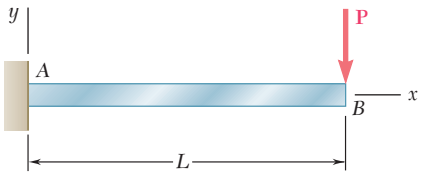
Problem 8.62a

Program output for value of $c = 2.85$ in.

d in.	σ ksi	τ_V ksi	τ_T ksi	τ_{Total} ksi	$\sigma_{\max}$ ksi	$\sigma_{\min}$ ksi	$\tau_{\max}$ ksi	θ_P Degrees
–3.00	12.58	–3.49	–2.03	–5.52	14.65	–2.08	8.36	–18.49
–2.00	12.58	–2.33	–2.03	–4.35	13.94	–1.36	7.65	–16.00
–1.00	12.58	–1.16	–2.03	–3.19	13.34	–0.76	7.05	–12.78
→ 0.00	12.58	0.00	–2.03	–2.03	12.89	–0.32	6.61	–8.73 ←
1.00	12.58	1.16	–2.03	–0.86	12.63	–0.06	6.35	–3.89
2.00	12.58	2.33	–2.03	0.30	12.58	–0.01	6.30	1.36
3.00	12.58	3.49	–2.03	1.46	12.74	–0.17	6.46	6.46

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CHAPTER 9



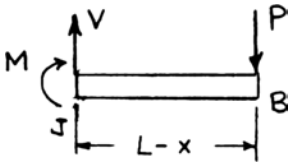
PROBLEM 9.1

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION

$$+\circlearrowleft \Sigma M_J = 0: \quad -M - P(L-x) = 0$$

$$M = -P(L-x)$$



$$EI\frac{d^2y}{dx^2} = -P(L-x) = -PL + Px$$

$$EI\frac{dy}{dx} = -PLx + \frac{1}{2}Px^2 + C_1$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right]: \qquad 0 = -0 + 0 + C_1 \qquad C_1 = 0$$

$$EIy = -\frac{1}{2}PLx^2 + \frac{1}{6}Px^3 + C_1x + C_2$$

$$[x = 0,\ y = 0]: \qquad 0 = -0 + 0 + 0 + C_2 \qquad C_2 = 0$$

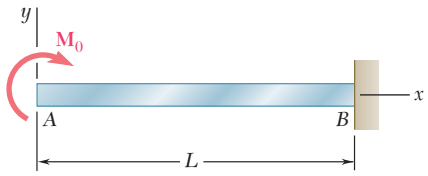
(*a*) Elastic curve. $$y = -\frac{Px^2}{6EI}(3L - x) \blacktriangleleft$$

$$\frac{dy}{dx} = -\frac{Px}{2EI}(2L - x)$$

(*b*) y at $x = L$. $$y_B = -\frac{PL^2}{6EI}(3L - L) = -\frac{PL^3}{3EI} \qquad y_B = \frac{PL^3}{3EI} \downarrow \blacktriangleleft$$

(*c*) $\frac{dy}{dx}$ at $x = L$. $$\left.\frac{dy}{dx}\right|_B = -\frac{PL}{2EI}(2L - L) = -\frac{PL^2}{2EI} \qquad \theta_B = \frac{PL^2}{2EI} \blacktriangleleft$$

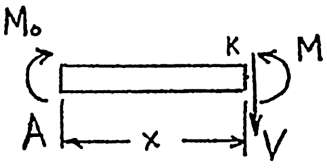
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PROBLEM 9.2

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION



$$+\circlearrowleft \Sigma M_K = 0: \quad -M_0 + M = 0$$

$$M = M_0$$

$$EI\frac{d^2y}{dx} = M = M_0$$

$$EI\frac{dy}{dx} = M_0x + C_1$$

$$\left[x = L, \frac{dy}{dx} = 0\right]: \qquad 0 = M_0L + C_1 \qquad C_1 = -M_0L$$

$$EIy = \frac{1}{2}M_0x^2 + C_1x + C_2$$

$$[x = L, y = 0] \qquad 0 = \frac{1}{2}M_0L^2 - M_0L^2 + C_2 \qquad C_2 = \frac{1}{2}M_0L^2$$

(*a*) Elastic curve:

$$y = \frac{M_0}{2EI}(x^2 - 2Lx + L^2) \blacktriangleleft$$

$$y = \frac{M_0}{2EI}(L - x)^2 \blacktriangleleft$$

(*b*) y at $x = 0$:

$$y_A = \frac{M_0}{2EI}(L - 0)^2$$

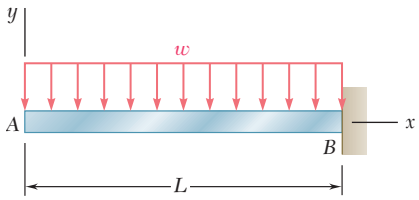
$$y_A = \frac{M_0L^2}{2EI} \uparrow \blacktriangleleft$$

(*c*) $\frac{dy}{dx}$ at $x = 0$:

$$\frac{dy}{dx} = -\frac{M_0}{EI}(L - x) = -\frac{M_0}{EI}(L - 0) = -\frac{M_0L}{EI}$$

$$\theta_A = \frac{M_0L}{EI} \blacktriangleleft$$

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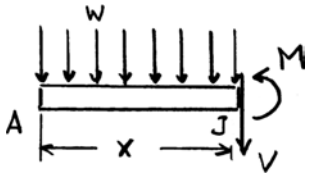


PROBLEM 9.3

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION

$$+\circlearrowleft \Sigma M_J = 0: \quad (wx)\frac{x}{2} + M = 0$$



$$M = -\frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + C_1$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right]: \quad 0 = -\frac{1}{6}wL^3 + C_1 \quad C_1 = \frac{1}{6}wL^3$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}wL^3$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wL^3x + C_2$$

$$[x = L,\ y = 0] \quad 0 = -\frac{1}{24}wL^4 + \frac{1}{6}wL^4 + C_2 = 0$$

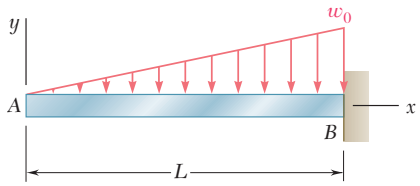
$$C_2 = \left(\frac{1}{24} - \frac{1}{6}\right)wL^4 = -\frac{3}{24}wL^4$$

(*a*) Elastic curve. $y = -\dfrac{w}{24EI}(x^4 - 4L^3x + 3L^4)$ ◀

(*b*) y at $x = 0$. $y_A = -\dfrac{3wL^4}{24EI} = -\dfrac{wL^4}{8EI}$ $y_A = \dfrac{wL^4}{8EI} \downarrow$ ◀

(*c*) $\dfrac{dy}{dx}$ at $x = 0$. $\left.\dfrac{dy}{dx}\right|_A = \dfrac{wL^3}{6EI}$ $\theta_A = \dfrac{wL^3}{6EI}$ ◀

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PROBLEM 9.4

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION

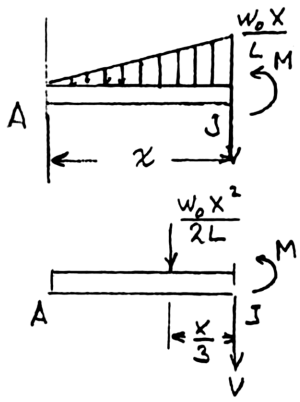
Use Free body *AJ*.

$$+\circlearrowleft \Sigma M_J = 0: \quad M + \frac{w_0 x^2}{2L} \cdot \frac{x}{3} = 0$$

$$M = -\frac{1}{6}\frac{w_0 x^3}{L}$$

$[x = L, y = 0]$

$$\left[x = L, \frac{dy}{dx} = 0\right]$$



$$\frac{d^2y}{dx} = -\frac{1}{6}\frac{w_0 x^3}{L}$$

$$EI\frac{dy}{dx} = -\frac{1}{24}\frac{w_0 x^4}{L} + C_1$$

$$EIy = -\frac{1}{120}\frac{w_0 x^5}{L} + C_1 x + C_2$$

$$\left[x = L, \frac{dy}{dx} = 0\right]: \quad -\frac{1}{24}w_0 L^3 + C_1 = 0$$

$$C_1 = \frac{1}{24}w_0 L^3$$

$[x = L, y = 0]$ $\quad EIy = -\frac{1}{120}w_0 L^4 + \frac{1}{24}w_0 L^4 + C_2 = 0$

$$C_2 = \frac{1}{30}w_0 L^4$$

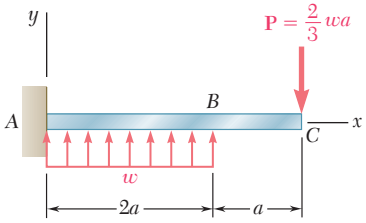
(*a*) Elastic curve. $\quad y = -\frac{w_0}{120EIL}(x^5 - 5L^4 x + 4L^5)$ ◀

$$\frac{dy}{dx} = \frac{w_0}{24EIL}(-x^4 + L^4)$$

(*b*) y@ $x = 0$: $\quad y_A = -\frac{w_0 L^4}{30EI}$ $\quad y_A = \frac{w_0 L^4}{30EI} \downarrow$ ◀

(*c*) $\frac{dy}{dx}$@ $x = 0$: $\quad \left.\frac{dy}{dx}\right|_A = \frac{w_0 L^3}{24EI}$ $\quad \theta_A = \frac{w_0 L^3}{24EI}$ ◀

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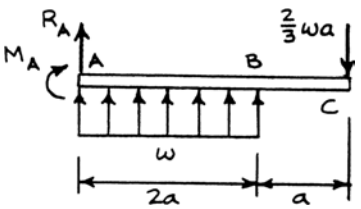
PROBLEM 9.5

For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.

SOLUTION

Using *ABC* as a free body,

FBD *ABC*:



$$+\uparrow \Sigma F_y = 0: \quad R_A + 2wa - \frac{2}{3}wa = 0$$

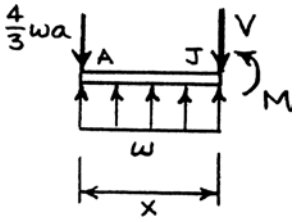
$$R_A = -\frac{4}{3}wa = \frac{4}{3}wa \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + (2wa)(a) - \left(\frac{2}{3}wa\right)(3a) = 0$$

$$M_A = 0$$

Using *AJ* as a free body,

FBD *AJ*:



$$+\circlearrowleft \Sigma M_J = 0: \quad M + \left(\frac{4}{3}wa\right)(x) - (wx)\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{2}wx^2 - \frac{4}{3}wax$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{2}wx^2 - \frac{4}{3}wax$$

$$EI\frac{dy}{dx} = \frac{1}{6}wx^3 - \frac{2}{3}wax^2 + C_1$$

$$\left[x = 0, \ \frac{dy}{dx} = 0\right]: \quad 0 = 0 - 0 + C_1 \quad \therefore \quad C_1 = 0$$

$$EIy = \frac{1}{24}wx^4 - \frac{2}{9}wax^3 + C_2$$

$$[x = 0, \ y = 0]: \quad 0 = 0 - 0 + C_2 \quad \therefore \quad C_2 = 0$$

(*a*) Elastic curve over *AB*.

$$\frac{dy}{dx} = \frac{w}{6EI}(x^3 - 4ax^2)$$

$$y = \frac{w}{72EI}(3x^4 - 16ax^3) \blacktriangleleft$$

(*b*) y at $x = 2a$.

$$y_B = -\frac{10wa^4}{9EI}$$

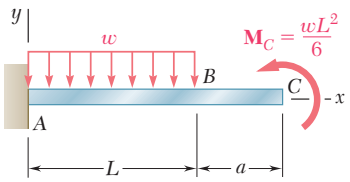
$$y_B = \frac{10wa^4}{9EI} \downarrow \blacktriangleleft$$

(*c*) $\frac{dy}{dx}$ at $x = 2a$.

$$\left(\frac{dy}{dx}\right)_B = -\frac{4wa^3}{3EI}$$

$$\theta_B = \frac{4wa^3}{3EI} \blacktriangleleft$$

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PROBLEM 9.6

For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.

SOLUTION

Using *ABC* as a free body,

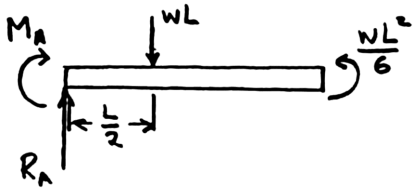
$$+\uparrow \Sigma F_y = 0: \quad R_A - wL = 0 \quad R_A = wL$$

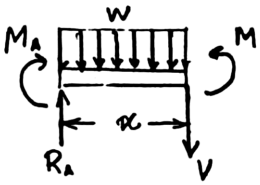
$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (wL)\left(\frac{L}{2}\right) + \frac{wL^2}{6} = 0$$

$$M_A = -\frac{1}{3}wL^2$$

$[x = 0, y = 0]$

$\left[x = 0, \dfrac{dy}{dx} = 0\right]$





Using *AJ* as a free body (portion *AB* only),

$$+\circlearrowleft M_J = 0: \quad M + (wx)\left(\frac{x}{2}\right) - R_A x - M_A = 0$$

$$M = -\frac{1}{2}wx^2 + R_A x + M_A$$

$$= -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$$

$$EI\frac{d^2y}{dx^2} = -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{2}wLx^2 - \frac{1}{3}wLx + C_1$$

$\left[x = 0, \dfrac{dy}{dx} = 0\right]$: $\quad -0 + 0 - 0 + C_1 = 0 \qquad C_1 = 0$

$$EI_y = -\frac{1}{24}wx^4 + \frac{1}{6}wLx^3 - \frac{1}{6}wLx^2 + C_2$$

$[x = 0, y = 0]$: $\quad -0 + 0 - 0 + C_2 = 0 \qquad C_2 = 0$

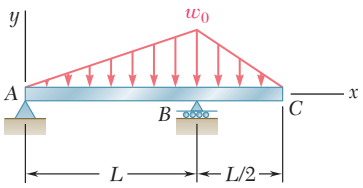
(*a*) Elastic curve over *AB*. $\qquad y = \dfrac{w}{24EI}(-x^4 + 4Lx^3 - 4L^2x^2)$ ◀

$$\frac{dy}{dx} = \frac{w}{6EI}(-x^3 + 2Lx^2 - L^2x)$$

(*b*) *y* at $x = L$: $\qquad y_B = -\dfrac{wL^4}{24EI}$ $\qquad y_B = \dfrac{wL^4}{24EI} \downarrow$ ◀

(*c*) $\dfrac{dy}{dx}$ at $x = L$: $\qquad \left.\dfrac{dy}{dx}\right|_B = 0$ $\qquad \theta_B = 0$ ◀

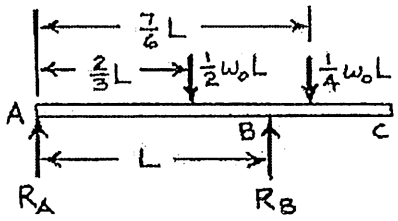
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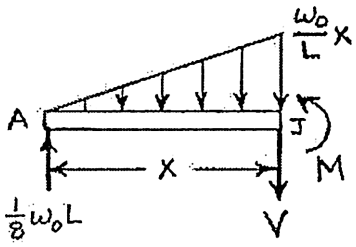


PROBLEM 9.7

For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at midspan, (*c*) the slope at *B*.

SOLUTION





Reactions:

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \left(\frac{1}{2}w_0 L\right)\left(\frac{1}{3}L\right) - \left(\frac{1}{4}w_0 L\right)\left(\frac{1}{6}L\right) = 0$$

$$R_A = \frac{1}{8}w_0 L$$

Boundary conditions: $[x = 0, y = 0]$ $[x = L, y = 0]$

For portion *AB* only, $(0 \le x < L)$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{8}w_0 L x + \frac{1}{2}\left(\frac{w_0}{L}x\right)(x)\left(\frac{x}{3}\right) + M = 0$$

$$M = \frac{1}{8}w_0 L x - \frac{1}{6}\frac{w_0}{L}x^3$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{8}w_0 L x - \frac{1}{6}\frac{w_0}{L}x^3$$

$$EI\frac{dy}{dx} = \frac{1}{16}w_0 L x^2 - \frac{1}{24}\frac{w_0}{L}x^4 + C_1$$

$$EIy = \frac{1}{48}w_0 L x^3 - \frac{1}{120}\frac{w_0}{L}x^5 + C_1 x + C_2$$

$[x = 0, \ y = 0]$: $\quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L, \ y = 0]$: $\quad 0 = \frac{1}{48}w_0 L^4 - \frac{1}{120}w_0 L^4 + C_1 L \qquad C_1 = -\frac{1}{80}w_0 L^3$

(*a*) Elastic curve.

$$y = \frac{w_0}{EIL}\left(\frac{1}{48}L^2x^3 - \frac{1}{120}x^5 - \frac{1}{80}L^4 x\right) \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{w_0}{EIL}\left(\frac{1}{16}L^2x^2 - \frac{1}{24}x^4 - \frac{1}{80}L^4\right)$$

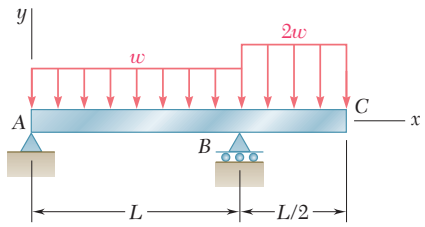
(*b*) y at $x = \frac{L}{2}$.

$$y_{L/2} = \frac{w_0}{EIL}\left(\frac{L^5}{384} - \frac{L^5}{3840} - \frac{L^5}{160}\right) = -\frac{15w_0L^4}{3840EI} \qquad y_{L/2} = \frac{w_0L^4}{256EI}\downarrow \blacktriangleleft$$

(*c*) $\frac{dy}{dx}$ at $x = L$.

$$\left.\frac{dy}{dx}\right|_B = \frac{w_0}{EIL}\left(\frac{L^4}{16} - \frac{L^4}{24} - \frac{L^4}{80}\right) = +\frac{2w_0L^3}{240EI} \qquad \theta_B = \frac{w_0L^3}{120EI}\measuredangle \blacktriangleleft$$

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PROBLEM 9.8

For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the slope at *A*, (*c*) the slope at *B*.

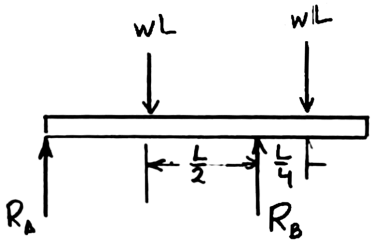
SOLUTION

Using free body *ABC*,

$[x = 0, y = 0] \quad [x = L, y = 0]$

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + (wL)\left(\frac{L}{2}\right) - (wL)\left(\frac{L}{4}\right) = 0$$

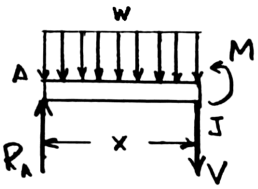
$$R_A = \frac{1}{4}wL$$



For portion *AB*, $(0 < x < L)$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - R_A x + (wx)\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{4}wLx - \frac{1}{2}wx^2$$



$$EI\frac{d^2y}{dx^2} = \frac{1}{4}wLx - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{1}{8}wLx^2 - \frac{1}{6}wx^3 + C_1$$

$$EIy = \frac{1}{24}wLx^3 - \frac{1}{24}wx^4 + C_1x + C_2$$

$[x = 0, y = 0]$: $\quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L, y = 0]$: $\quad 0 = \frac{1}{24}wL^4 - \frac{1}{24}wL^4 + C_1L + 0 = 0 \qquad C_1 = 0$

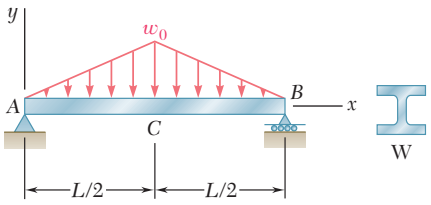
(*a*) Elastic curve $(0 \le x \le L)$. $\qquad y = \frac{w}{24EI}(Lx^3 - x^4)$ ◀

$$\frac{dy}{dx} = \frac{w}{24EI}(3Lx^2 - 4x^3)$$

(*b*) $\frac{dy}{dx}$ at $x = 0$. $\qquad \left.\frac{dy}{dx}\right|_A = 0 \qquad \theta_A = 0$ ◀

(*c*) $\frac{dy}{dx}$ at $x = L$. $\qquad \left.\frac{dy}{dx}\right|_B = -\frac{wL^3}{24EI} \qquad \theta_B = \frac{wL^3}{24EI}$ ↘ ◀

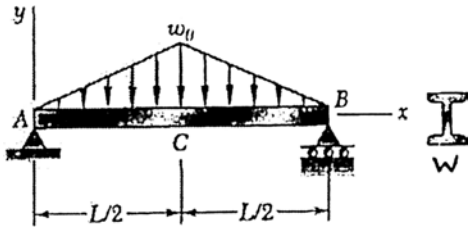
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PROBLEM 9.9

Knowing that beam AB is a $W10 \times 33$ rolled shape and that $w_0 = 3$ kips/ft, $L = 12$ ft, and $E = 29 \times 10^6$ psi, determine (*a*) the slope at A, (*b*) the deflection at C.

SOLUTION



Use symmetry boundary conditions at C.

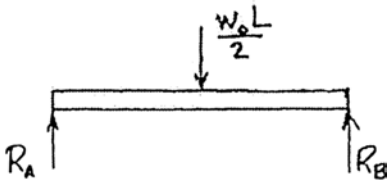
Using free body ACB and symmetry,

$$R_A = R_B = \frac{1}{4} w_0 L$$

For $0 < x < \frac{L}{2}$, $\quad w = \frac{2w_0 x}{L}$

$[x = 0, \ y = 0] \quad [x = L, \ y = 0]$

$$\left[x = \frac{L}{2}, \ \frac{dy}{dx} = 0\right]$$

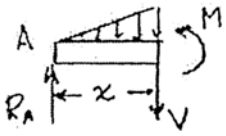


$$\frac{dV}{dx} = -w = -\frac{2w_0 x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{L} + R_A = \frac{w_0}{L}\left(\frac{1}{4}L^2 - x^2\right)$$

$$M = \frac{w_0}{L}\left(\frac{1}{4}L^2 x - \frac{1}{3}x^3\right) + C_M$$

But $M = 0$ at $x = 0$; hence, $C_M = 0$



$$EI\frac{d^2y}{dx^2} = \frac{w_0}{L}\left(\frac{1}{4}L^2 x - \frac{1}{3}x^3\right)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L}\left(\frac{1}{8}L^2 x^2 - \frac{1}{12}x^4\right) + C_1$$

$\left[x = \frac{L}{2}, \ \frac{dy}{dx} = 0\right]$

$$0 = \frac{w_0}{L}\left(\frac{1}{32}L^4 - \frac{1}{192}L^4\right) + C_1 = 0 \qquad C_1 = -\frac{5}{192}w_0 L^3$$

$$EIy = \frac{w_0}{L}\left(\frac{1}{24}L^2 x^3 - \frac{1}{120}x^5\right) - \frac{5}{192}w_0 L^3 x + C_2$$

$[x = 0, \ y = 0]$ $\qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

Elastic curve.

$$y = \frac{w_0}{EIL}\left(\frac{1}{24}L^2 x^3 - \frac{1}{60}x^5 - \frac{5}{192}L^4 x\right)$$

$$\frac{dy}{dx} = \frac{w_0}{EIL}\left(\frac{1}{8}L^2 x^2 - \frac{1}{12}x^4 - \frac{5}{192}L^4\right)$$

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PROBLEM 9.9 *(Continued)*

Data: $w_0 = 3$ kips/ft, $E = 29 \times 10^6$ psi, $I = 171\,\text{in}^4$

$$EI = (29 \times 10^6)(171) = 4.959 \times 10^9\,\text{lb} \cdot \text{in}^2 = 34.438 \times 10^3\,\text{kip} \cdot \text{ft},$$

$$L = 12\ \text{ft}$$

(*a*) Slope at $x = 0$.

$$\frac{dy}{dx} = \frac{3}{(34.438 \times 10^3)(12)}\left[-\left(\frac{5}{192}\right)(12)^4\right] = -3.92 \times 10^3$$

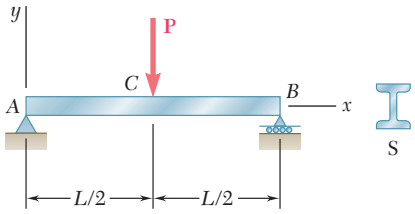
$$\theta_A = 3.92 \times 10^{-3}\ \text{rad} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at $x = 6$ ft.

$$y_C = \frac{3}{(34.438 \times 10^3)(12)}\left[\left(\frac{1}{24}\right)(12)^2(6)^3 - \frac{1}{60}(6)^5 - \frac{5}{192}(12)^4(6)\right] = -15.0531 \times 10^{-3}\ \text{ft}$$

$$y_C = 0.1806\ \text{in.} \downarrow \blacktriangleleft$$

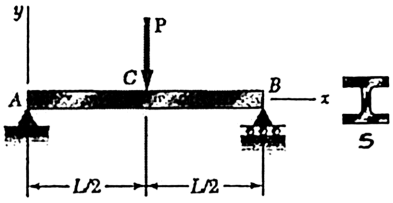
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PROBLEM 9.10

Knowing that beam AB is an S200 × 34 rolled shape and that $P = 60$ kN, $L = 2$ m, and $E = 200$ GPa, determine (*a*) the slope at A, (*b*) the deflection at C.

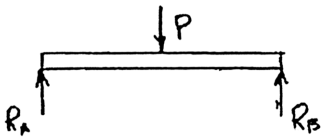
SOLUTION

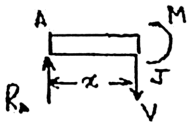


Use symmetry boundary conditions at C.

By symmetry, $R_A = R_B = \frac{1}{2}P$

Using free body AJ, $\left(0 \le x \le \frac{L}{2}\right)$





$+\circlearrowleft \Sigma M_J = 0: \quad M - R_A x = 0$

$$M = R_A x = \frac{1}{2}Px$$

$$EI\frac{d^2y}{dy^2} = \frac{1}{2}Px$$

$$EI\frac{dy}{dx} = \frac{1}{4}Px^2 + C_1$$

$$EIy = \frac{1}{12}Px^3 + C_1x + C_2$$

$[x = 0, y = 0]$ $\quad 0 = 0 + 0 + C_2 \quad C_2 = 0$

$\left[x = \frac{L}{2}, \frac{dy}{dx} = 0\right]$ $\quad 0 = \frac{1}{4}P\left(\frac{L}{2}\right)^2 + C_1 \quad C_1 = -\frac{1}{16}PL^2$

Elastic curve.

$$y = \frac{PL}{48EI}(4x^3 - 3L^2x)$$

$$\frac{dy}{dx} = \frac{PL}{16EI}(4x^2 - L^2)$$

Slope at $x = 0$.

$$\left.\frac{dy}{dx}\right|_A = -\frac{PL^2}{16EI} \qquad \theta_A = \frac{PL^2}{16EI} \measuredangle$$

Deflection at $x = \frac{L}{2}$.

$$y_C = -\frac{PL^3}{48EI} \qquad y_C = \frac{PL^3}{48EI} \downarrow$$

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PROBLEM 9.10 *(Continued)*

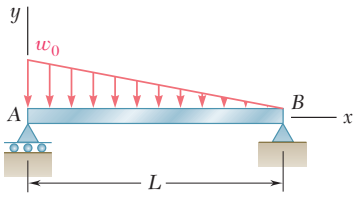
Data: $P = 60 \times 10^3$ N, $I = 26.9 \times 10^6$ mm^4 $= 26.9 \times 10^{-6}$ m^4

$E = 200 \times 10^9$ Pa $EI = 5.38 \times 10^6$ N · m^2 $L = 2$ m

(*a*) $$\theta_A = \frac{(60 \times 10^3)(2)^2}{(16)(5.38 \times 10^6)}$$ $\theta_A = 2.79 \times 10^{-3}$ rad ◀

(*b*) $$y_C = \frac{(60 \times 10^3)(2)^3}{(48)(5.38 \times 10^6)} = 1.859 \times 10^{-3} \text{ m}$$ $y_C = 1.859$ mm ↓ ◀

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PROBLEM 9.11

For the beam and loading shown, (*a*) express the magnitude and location of the maximum deflection in terms of w_0, L, E, and I. (*b*) Calculate the value of the maximum deflection, assuming that beam AB is a W18 × 50 rolled shape and that $w_0 = 4.5$ kips/ft, $L = 18$ ft, and $E = 29 \times 10^6$ psi.

SOLUTION

$$\frac{dV}{dx} = -w = -\frac{w_0}{L}(L - x)$$

$[x = 0, y = 0] \quad [x = L, y = 0]$

$$V = -\frac{w_0}{L}\left(Lx - \frac{1}{2}x^2\right) + C_V = \frac{dM}{dx}$$

$$M = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + C_V x + C_M$$

$[x = 0, M = 0]$

$$0 = 0 + 0 + 0 + C_M \qquad C_M = 0$$

$[x = L, M = 0]$

$$0 = -\frac{w_0}{L}\left(\frac{1}{2}L^3 - \frac{1}{6}L^3\right) + C_V L \quad C_V = \frac{1}{3}w_0 L$$

$$EI\frac{d^2y}{dx^2} = M = \frac{w_0}{L}\left(\frac{1}{3}L^2x - \frac{1}{2}Lx^2 + \frac{1}{6}x^3\right)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L}\left(\frac{1}{6}L^2x^2 - \frac{1}{6}Lx^3 + \frac{1}{24}x^4\right) + C_1$$

$$EIy = \frac{w_0}{L}\left(\frac{1}{18}L^2x^3 - \frac{1}{24}Lx^4 + \frac{1}{120}x^5\right) + C_1x + C_2$$

$[x = 0, y = 0]$

$$0 = 0 - 0 + 0 + 0 + C_2 \qquad C_2 = 0$$

$[x = L, y = 0]$

$$0 = \frac{w_0}{L}\left\{\frac{1}{18}L^5 - \frac{1}{24}L^5 + \frac{1}{120}L^5\right\} + C_1L + 0$$

$$C_1 = -\frac{1}{45}w_0L^3$$

$$y = \frac{w_0}{EIL}\left(\frac{1}{18}L^2x^3 - \frac{1}{24}Lx^4 + \frac{1}{120}x^5 - \frac{1}{45}L^4x\right)$$

$$\frac{dy}{dx} = \frac{w_0}{EIL}\left(\frac{1}{6}L^2x^2 - \frac{1}{6}Lx^3 + \frac{1}{24}x^4 - \frac{1}{45}L^4\right)$$

To find location of maximum deflection, set $\dfrac{dy}{dx} = 0$.

$$f = \frac{1}{6}L^2x_m^2 - \frac{1}{6}Lx_m^3 + \frac{1}{24}x_m^4 - \frac{1}{45}L^4 = 0$$

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PROBLEM 9.11 *(Continued)*

Let $z = \dfrac{x_m}{L}$

$$f(z) = \frac{1}{6}z^2 - \frac{1}{6}z^3 + \frac{1}{24}z^4 - \frac{1}{45}$$

$$\frac{df}{dz} = \frac{1}{3}z - \frac{1}{2}z^2 + \frac{1}{6}z^3$$

By Newton–Raphson method,

$$z = z_0 - \frac{f(z_0)}{df/dz}$$

$$z = \cancel{0.5}, \cancel{0.4805}, 0.4807, 0.4807 \qquad x_m = 0.481L \blacktriangleleft$$

$$y_m = \frac{w_0 L^4}{EI}\left\{\frac{1}{18}(0.4807)^3 - \frac{1}{24}(0.4807)^4 + \frac{1}{120}(0.4807)^5 - \frac{1}{45}(0.4807)\right\}$$

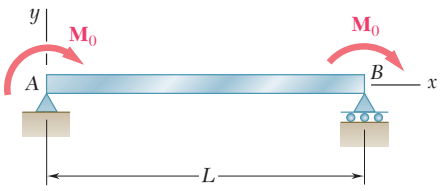
$$= -0.00652\frac{w_0 L^4}{EI} \qquad y_m = 0.00652\frac{w_0 L^4}{EI} \downarrow \blacktriangleleft$$

Data: $w_0 = 4.5 \text{ kips/ft} = \dfrac{4500}{12} = 375 \text{ lb/in.}, \qquad L = 18 \text{ ft} = 216 \text{ in.}$

$I = 800 \text{ in}^4$ for W18 × 50

$$y_m = \frac{(0.00652)(375)(216)^4}{(29 \times 10^6)(800)} = 0.229 \text{ in.} \blacktriangleleft$$

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PROBLEM 9.12

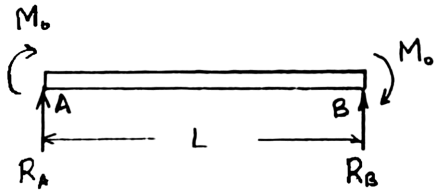
(*a*) Determine the location and magnitude of the maximum absolute deflection in *AB* between *A* and the center of the beam. (*b*) Assuming that beam *AB* is a W460 × 113, $M_0 = 224$ kN · m, and $E = 200$ GPa, determine the maximum allowable length *L* of the beam if the maximum deflection is not to exceed 1.2 mm.

SOLUTION

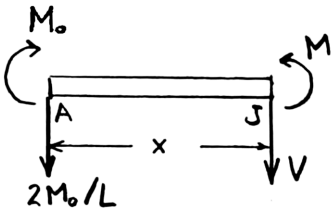
Using *AB* as a free body,

$$\Sigma M_B = 0: \quad -2M_0 - R_A L = 0$$

$$R_A = -\frac{2M_0}{L}$$



Using portion *AJ* as a free body,



$$\Sigma M_J = 0: \quad -M_0 + \frac{2M_0}{L}x + M = 0$$

$$M = \frac{M_0}{L}(L - 2x)$$

$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}(L - 2x)$$

$$EI\frac{dy}{dx} = \frac{M_0}{L}(Lx - x^2) + C_1$$

$$EIy = \frac{M_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{3}x^3\right) + C_1x + C_2$$

$[x = 0, y = 0]$ $\quad 0 = 0 - 0 + 0 + C_2$ $\quad C_2 = 0$

$[x = L, y = 0]$ $\quad 0 = \frac{M_0}{L}\left(\frac{1}{2}L^3 - \frac{1}{3}L^3\right) + C_1L + 0$ $\quad C_1 = -\frac{1}{6}M_0L^2$

$$y = \frac{M_0}{EIL}\left(\frac{1}{2}Lx^2 - \frac{1}{3}x^3 - \frac{1}{6}L^2x\right)$$

$$\frac{dy}{dx} = \frac{M_0}{EIL}\left(Lx - x^2 - \frac{1}{6}L^2\right)$$

To find location of maximum deflection, set $\frac{dy}{dx} = 0$

$$x_m^2 - Lx_m - \frac{1}{6}L^2 = 0$$

$$x_m = \frac{L - \sqrt{L^2 - (4)\left(\frac{1}{6}L^2\right)}}{2} = \frac{1}{2}\left(1 - \frac{\sqrt{3}}{3}\right)L = 0.21132L$$

$$x_m = 0.211L \blacktriangleleft$$

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PROBLEM 9.12 ***(Continued)***

$$y_m = \frac{M_0 L^2}{EI}\left\{\left(\frac{1}{2}\right)(0.21132)^2 - \left(\frac{1}{3}\right)(0.21132)^3 - \left(\frac{1}{6}\right)(0.21132)\right\}$$

$$= -0.0160375\,\frac{M_0 L^2}{EI}$$

$$|y_m| = 0.0160375\frac{M_0 L^2}{EI} \qquad |y_m| = 0.01604\frac{M_0 L^2}{EI} \blacktriangleleft$$

Solving for L,

$$L = \left\{\frac{EI|y_m|}{0.0160375 M_0}\right\}^{1/2}$$

Data:

$$E = 200 \times 10^9 \text{ Pa},$$

$$I = 554 \times 10^6 \text{ mm}^4$$

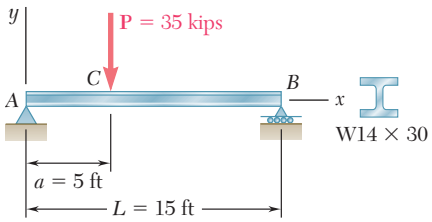
$$= 554 \times 10^{-6} \text{ m}^4$$

$$|y_m| = 1.2 \text{ mm} = 1.2 \times 10^3 \text{ m},$$

$$M_0 = 224 \times 10^3 \text{ N} \cdot \text{m}$$

$$L = \left\{\frac{(200 \times 10^9)(554 \times 10^{-6})(1.2 \times 10^{-3})}{(0.0160375)(224 \times 10^3)}\right\}^{1/2} = 6.08 \text{ m} \blacktriangleleft$$

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PROBLEM 9.13

For the beam and loading shown, determine the deflection at point C. Use $E = 29 \times 10^6$ psi.

$[x = 0,\ y = 0]$ $\qquad$ $[x = L,\ y = 0]$

$[x = a,\ y = y]$

$\left[x = a,\ \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$

SOLUTION

Let $\quad b = L - a.$

Reactions: $\quad R_A = \dfrac{Pb}{L} \uparrow,$

$R_B = \dfrac{Pa}{L} \uparrow$

Bending moments:

$$0 < x < a: \quad M = \frac{Pb}{L}x$$

$$a < x < L: \quad M = \frac{P}{L}[bx - L(x - a)]$$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}(bx)$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left(\frac{1}{2}bx^2\right) + C_1 \qquad (1)$$

$$EIy = \frac{P}{L}\left(\frac{1}{6}bx^3\right) + C_1x + C_2 \qquad (2)$$

$a < x < L$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}[bx - L(x - a)]$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left[\frac{1}{2}bx^2 - \frac{1}{2}L(x - a)^2\right] + C_3 \qquad (3)$$

$$EIy = \frac{P}{L}\left[\frac{1}{6}bx^3 - \frac{1}{6}L(x - a)^3\right] + C_3x + C_4 \qquad (4)$$

$[x = 0,\ y = 0]$ Eq. (2): $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$\left[x = a, \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$ Eqs. (1) and (3): $\quad \dfrac{P}{L}\left(\cancel{\dfrac{1}{2}ba^2}\right) + C_1 = \dfrac{P}{L}\left[\cancel{\dfrac{1}{2}ba^2} + 0\right] + C_3 \quad \therefore \quad C_3 = C_1$

$[x = a,\ y = y]$ Eqs. (2) and (4):

$$\frac{P}{L}\left(\cancel{\frac{1}{6}ba^3}\right) + \cancel{C_1a} + C_2 = \frac{P}{L}\left[\cancel{\frac{1}{6}ba^3} + 0\right] + \cancel{C_1a} + C_4 \qquad C_4 = C_2 = 0$$

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PROBLEM 9.13 *(Continued)*

$$[x = L,\ y = 0] \quad \text{Eq. (4):} \qquad \frac{P}{L}\left[\frac{1}{6}bL^3 - \frac{1}{6}L(L-a)^3\right] + C_3L = 0$$

$$C_1 = C_3 = \frac{P}{L}\left[\frac{1}{6}(L-a)^3 - \frac{1}{6}bL^2\right] = \frac{P}{L}\left(\frac{1}{6}b^3 - \frac{1}{6}bL^2\right)$$

Make $x = a$ in Eq. (2).

$$y_C = \frac{P}{EIL}\left[\frac{1}{6}ba^3 + \frac{1}{6}b^3a - \frac{1}{6}bL^2a\right] = \frac{P(ba^3 + b^3a - L^2ab)}{6EIL}$$

Data: $P = 35$ kips, $E = 29 \times 10^6$ psi

$= 29 \times 10^3$ kips/in^2

$L = 15$ ft, $a = 5$ ft, $b = 10$ ft

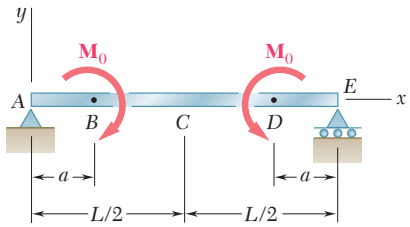
$I = 291$ in^4, $EI = 8.439 \times 10^6$ kip · in^2

$= 58.604 \times 10^3$ kip · ft^2

$$y_C = \frac{35}{(6)(58.604 \times 10^3)(15)}[(10)(5)^3 + (10^3)(5) - (15)^2(5)(10)]$$

$= -33.179 \times 10^{-3}$ ft $= -0.398$ in. $\qquad y_C = 0.398$ in. ↓ ◀

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PROBLEM 9.14

Knowing that beam *AE* is a W360 × 101 rolled shape and that $M_0 = 310$ kN · m, $L = 2.4$ m, $a = 0.5$ m, and $E = 200$ GPa, determine (*a*) the equation of the elastic curve for portion *BD*, (*b*) the deflection at point *C*.

SOLUTION

Use continuity boundary condition at *B* and symmetry boundary condition at *C*.

$[x = 0, y = 0]$

$[x = a, y = y]$

$\left[x = a, \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$

$\left[x = \dfrac{L}{2}, \dfrac{dy}{dx} = 0\right]$

From statics, $R_A = R_B = 0$

$0 \le x \le a$

$$M = 0$$

$$EI\frac{d^2y}{dx^2} = 0$$

$$EI\frac{dy}{dx} = C_1$$

$$EIy = C_1x + C_2$$

$a \le x \le L - a$

$$M = M_0$$

$$EI\frac{d^2y}{dx^2} = M_0$$

$$EI\frac{dy}{dx} = M_0x + C_3$$

$$EIy = \frac{1}{2}M_0x^2 + C_3x + C_4$$

$[x = 0, y = 0]$ $\quad 0 = 0 + C_2 \quad C_2 = 0$

$\left[x = \dfrac{L}{2}, \dfrac{dy}{dx} = 0\right]$ $\quad 0 = \dfrac{1}{2}M_0L + C_3 \quad C_3 = -\dfrac{1}{2}M_0L$

$\left[x = a, \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$ $\quad C_1 = M_0a + C_3 = -M_0\left(\dfrac{1}{2}L - a\right)$

$[x = a, y = y]$ $\quad -M_0\left(\dfrac{1}{2}L - a\right)a + 0 = \dfrac{1}{2}M_0a^2 - \dfrac{1}{2}M_0La + C_4$

$$C_4 = \frac{1}{2}M_0a^2$$

(*a*) Elastic curve ($a \le x \le L - a$).

$$EIy = \frac{1}{2}M_0x^2 - \frac{1}{2}M_0Lx + \frac{1}{2}M_0a^2$$

$$y = \frac{M_0}{2EI}(x^2 - Lx + a^2)$$ ◀

(*b*) Deflection at $x = \dfrac{L}{2}$.

$$y_C = \frac{M_0}{2EI}\left[\left(\frac{L}{2}\right)^2 - (L)\left(\frac{L}{2}\right) + a^2\right]$$

$$= -\frac{M_0}{8EI}(L^2 - 4a^2)$$

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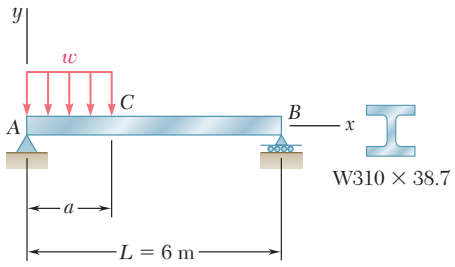
PROBLEM 9.14 *(Continued)*

Data:

$$M_0 = 310 \times 10^3 \text{ N} \cdot \text{m},$$
$$L = 2.4 \text{ m},$$
$$a = 0.5 \text{ m},$$
$$E = 200 \times 10^9 \text{ Pa}$$
$$I = 301 \times 10^6 \text{ mm}^4$$
$$= 301 \times 10^{-6} \text{ m}^4$$
$$EI = 60.2 \times 10^6 \text{ N} \cdot \text{m}^2$$
$$y_C = -\frac{310 \times 10^3}{(8)(60.2 \times 10^6)}[(2.4)^2 - (4)(0.5)^2]$$
$$= -3.06 \times 10^{-3}$$

$$y_C = 3.06 \text{ mm} \downarrow \blacktriangleleft$$

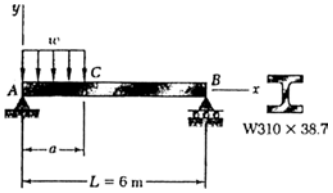
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PROBLEM 9.15

For the beam and loading shown, knowing that $a = 2$ m, $w = 50$ kN/m, and $E = 200$ GPa, determine (*a*) the slope at support *A*, (*b*) the deflection at point *C*.

SOLUTION



Using ACB as a free body and noting that $L = 3a$,

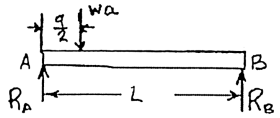
$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - (wa)\left(\frac{a}{2}\right) = 0$$

$$R_B = (wa)\frac{a}{2L} = \frac{1}{6}wa$$

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

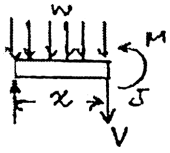
$[x = a,\ y = y]$

$$\left[x = a,\ \frac{dy}{dx} = \frac{dy}{dx}\right]$$



$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wa = 0 \qquad R_A = \frac{5}{6}wa$$

$0 \le x \le a$



$$+\circlearrowleft \Sigma M_J = 0:$$

$$M - R_A x + (wx)\left(\frac{x}{2}\right) = 0$$

$$M = R_A x - \frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = R_A x - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$$

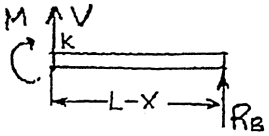
$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x + C_2$$

$$[x = 0,\ y = 0] \quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$$

$a \le x \le L$



$$+\circlearrowleft \Sigma M_K = 0:$$

$$-M + R_B(L - x) = 0$$

$$M = R_B(L - x)$$

$$EI\frac{d^2y}{dx^2} = R_B(L - x)$$

$$EI\frac{dy}{dx} = -\frac{1}{2}R_B(L - x)^2 + C_3$$

$$EIy = \frac{1}{6}R_B(L - x)^3 + C_3 x + C_4$$

$$[x = L,\ y = 0] \quad 0 = 0 + C_3 L + C_4 \qquad C_4 = -C_3 L$$

$$EIy = \frac{1}{6}R_B(L - x)^2 - C_3(L - x)$$

$$EI\frac{dy}{dx} = -\frac{1}{2}R_B(L - x)^2 + C_3$$

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PROBLEM 9.15 *(Continued)*

$$\left[x = a,\ \frac{dy}{dx} = \frac{dy}{dx}\right] \qquad \frac{1}{2}R_A a^2 - \frac{1}{6}wa^3 + C_1 = -\frac{1}{2}R_B(2a)^2 + C_3$$

$$C_3 = C_1 + \frac{1}{2}R_A a^2 - \frac{1}{6}wa^3 + \frac{1}{2}R_B(2a)^2 = C_1 + \frac{7}{12}wa^3$$

$$\left[x = a,\ y = y\right] \qquad \frac{1}{6}R_A a^3 - \frac{1}{24}wa^4 + C_1 a = \frac{1}{6}R_B(2a)^3 - \left(C_1 + \frac{7}{12}wa^3\right)(2a)$$

$$3C_1 a = -\frac{1}{6}R_A a^3 + \frac{1}{24}wa^4 + \frac{1}{6}R_B(2a)^3 - \frac{7}{12}wa^2(2a) = -\frac{25}{24}wa^4$$

$$C_1 = -\frac{25}{72}wa^3$$

For $0 \le x \le a$, $\quad EIy = \frac{5}{36}wax^3 - \frac{1}{24}wx^4 - \frac{25}{72}wa^3x$

$$EI\frac{dy}{dx} = \frac{5}{12}wax^2 - \frac{1}{6}wx^3 - \frac{25}{72}wa^3$$

Data: $\quad w = 50 \times 10^3$ N/m, $\quad a = 2$ m, $\quad E = 200 \times 10^9$ Pa

$$I = 84.9 \times 10^6\ \text{mm}^4 = 84.9 \times 10^{-6}\ \text{m}^4, \quad EI = 16.98 \times 10^6\ \text{N} \cdot \text{m}^2$$

(*a*) Slope at $x = 0$.

$$16.98 \times 10^6 \left.\frac{dy}{dx}\right|_A = 0 - 0 - \frac{25}{72}(50 \times 10^3)(2)^3$$

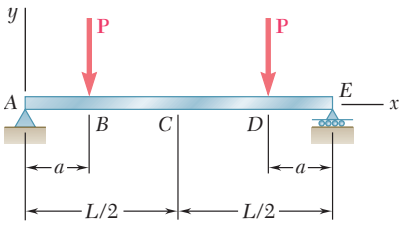
$$\left.\frac{dy}{dx}\right|_A = \theta_A = -8.18 \times 10^{-3} \qquad \theta_A = 8.18 \times 10^{-3}\ \text{rad} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at $x = 2$ m.

$$EIy_C = \frac{5}{36}wa^4 - \frac{1}{24}wa^4 - \frac{25}{72}wa^4 = -\frac{1}{4}wa^4$$

$$16.98 \times 10^6 y_C = -\frac{1}{4}(50 \times 10^3)(2)^4 \quad y_C = -11.78 \times 10^{-3}\ \text{m} \qquad y_C = 11.78\ \text{mm} \downarrow \blacktriangleleft$$

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PROBLEM 9.16

Knowing that beam AE is an $S200 \times 27.4$ rolled shape and that $P = 17.5$ kN, $L = 2.5$ m, $a = 0.8$ m and $E = 200$ GPa, determine (*a*) the equation of the elastic curve for portion BD, (*b*) the deflection at the center C of the beam.

SOLUTION

Consider portion ABC only. Apply symmetry about C.

Reactions: $R_A = R_E = P$

Boundary conditions: $[x = 0,\ y = 0]$, $[x = a,\ y = y]$, $\left[x = a,\ \frac{dy}{dx} = \frac{dy}{dx}\right]$, $\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right]$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = M = Px$$

$$EI\frac{dy}{dx} = \frac{1}{2}Px^2 + C_1 \qquad (1)$$

$$EIy = \frac{1}{6}Px^3 + C_1x + C_2 \qquad (2)$$

$$[x = 0,\ y = 0] \rightarrow C_2 = 0$$

$a < x < L - a$

$$EI\frac{d^2y}{dx^2} = M = Pa$$

$$EI\frac{dy}{dx} = Pax + C_3$$

$$EIy = \frac{1}{2}Pax^2 + C_3x + C_4$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right] \rightarrow C_3 = -\frac{1}{2}PaL$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right] \qquad \frac{1}{2}Pa^2 + C_1 = Pa^2 - \frac{1}{2}PaL \qquad C_1 = \frac{1}{2}Pa^2 - \frac{1}{2}PaL$$

$$\left[x = \frac{L}{2},\ y = y\right] \qquad \frac{1}{6}Pa^3 + \left(\frac{1}{2}Pa^2 - \frac{1}{2}PaL\right)a = \frac{1}{2}Pa^3 - \frac{1}{2}Pa^2L + C_4$$

$$C_4 = \frac{1}{6}Pa^3$$

(*a*) Elastic curve for portion BD.

$$y = \frac{1}{EI}\left(\frac{1}{2}Pax^2 + C_3x + C_4\right)$$

$$y = \frac{P}{EI}\left(\frac{1}{2}ax^2 - \frac{1}{2}aLx + \frac{1}{6}a^3\right) \blacktriangleleft$$

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PROBLEM 9.16 *(Continued)*

For deflection at C,

set $$x = \frac{L}{2}.$$

$$y_C = \frac{P}{EI}\left(\frac{1}{8}aL^2 - \frac{1}{4}aL^2 + \frac{1}{6}a^3\right)$$

$$= -\frac{Pa}{EI}\left(\frac{1}{8}L^2 - \frac{1}{6}a^2\right)$$

Data: $I = 23.9 \times 10^6\ \text{mm}^4 = 23.9 \times 10^{-6}\ \text{m}^4$,

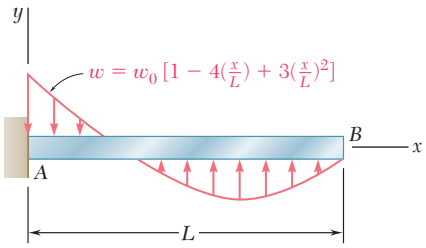
$E = 200 \times 10^9\ \text{Pa}$

$P = 17.5 \times 10^3\ \text{N}$,

$L = 2.5\ \text{m}, \quad a = 0.8\ \text{m}$

(*b*) $$y_C = -\frac{(17.5 \times 10^3)(0.8)}{(200 \times 10^9)(23.9 \times 10^6)}\left\{\frac{2.5^2}{8} - \frac{0.8^2}{6}\right\} = -1.976 \times 10^{-3}\ \text{m} \qquad y_C = 1.976\ \text{mm} \downarrow \blacktriangleleft$$

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PROBLEM 9.17

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the free end.

SOLUTION

Boundary conditions are shown at right.

$[x = 0,\ y = 0]$ $\qquad$ $[x = L,\ V = 0]$

$[x = 0,\ \frac{dy}{dx} = 0]$ $\qquad$ $[x = L,\ M = 0]$

$$\frac{dV}{dx} = -w = -w_0\left[1 - 4\left(\frac{x}{L}\right) + 3\left(\frac{x}{L}\right)^2\right]$$

$$V = -w_0\left[x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right] + C_V$$

$$[x = L, V = 0]:\ 0 = -w_0[L - 2L + L] + C_V = 0 \qquad C_V = 0$$

$$\frac{dM}{dx} = V = -w_0\left[x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right]$$

$$M = -w_0\left[\frac{x^2}{2} - \frac{2x^3}{3L} + \frac{x^4}{4L^2}\right] + C_M$$

$$[x = L,\ M = 0]:\ \ 0 = -w_0\left[\frac{1}{2}L^2 - \frac{2}{3}L^2 + \frac{1}{4}L^2\right] + C_M \qquad C_M = \frac{1}{12}w_0L^2$$

$$EI\frac{d^2y}{dx} = M = -w_0\left[\frac{1}{2}x^2 - \frac{2}{3}\frac{x^3}{L} + \frac{1}{4}\frac{x^4}{L^2} - \frac{1}{12}L^2\right]$$

$$EI\frac{dy}{dx} = -w_0\left[\frac{1}{6}x^3 - \frac{1}{6}\frac{x^4}{L} + \frac{1}{20}\frac{x^5}{L^2} - \frac{1}{12}L^2x\right] + C_1$$

$$[x = 0, \frac{dy}{dx} = 0] \qquad C_1 = 0$$

$$EIy = -w_0\left[\frac{1}{24}x^4 - \frac{1}{30}\frac{x^5}{L} + \frac{1}{120}\frac{x^6}{L^2} - \frac{1}{24}L^2x^2\right] + C_2$$

$$[x = 0,\ y = 0] \qquad C_2 = 0$$

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PROBLEM 9.17 ***(Continued)***

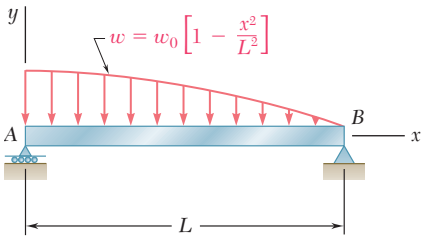
(*a*) Elastic curve.

$$y = -\frac{w_0}{EIL^2}\left(\frac{1}{24}L^2x^4 - \frac{1}{30}Lx^5 + \frac{1}{120}x^6 - \frac{1}{24}L^4x^2\right) \blacktriangleleft$$

(*b*) Deflection at $x = L$.

$$y_B = -\frac{w_0}{EIL^2}\left(\frac{1}{24}L^6 - \frac{1}{30}L^6 + \frac{1}{120}L^6 - \frac{1}{24}L^6\right) = -\frac{w_0L^4}{40EI} \qquad y_B = \frac{w_0L^4}{40EI}\downarrow \blacktriangleleft$$

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PROBLEM 9.18

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection at the midpoint of the span.

SOLUTION

$$w = w_0\left[1 - \frac{x^2}{L^2}\right] = \frac{w_0}{L^2}[L^2 - x^2]$$

$$\frac{dV}{dx} = -w = \frac{w_0}{L^2}[x^2 - L^2]$$

$$\frac{dM}{dx} = V = \frac{w_0}{L^2}\left[\frac{x^3}{3} - L^2x\right] + C_1$$

$$M = \frac{w_0}{L^2}\left[\frac{1}{12}x^4 - \frac{1}{2}L^2x^2\right] + C_1x + C_2$$

$[x = 0, M = 0]$: $0 = 0 - 0 + 0 + C_2 \quad \therefore \quad C_2 = 0$

$[x = L, M = 0]$: $0 = \frac{w_0}{L^2}\left[\frac{1}{12}L^4 - \frac{1}{2}L^4\right] + C_1L \quad \therefore \quad C_1 = \frac{5}{12}w_0L$

$$EI\frac{d^2y}{dx^2} = M = \frac{w_0}{L^2}\left[\frac{1}{12}x^4 - \frac{1}{2}L^2x^2 + \frac{5}{12}L^3x\right]$$

$$EI\frac{dy}{dx} = \frac{w_0}{L^2}\left[\frac{1}{60}x^5 - \frac{1}{6}L^2x^3 + \frac{5}{24}L^3x^2\right] + C_3$$

$$EIy = \frac{w_0}{L^2}\left[\frac{1}{360}x^6 - \frac{1}{24}L^2x^4 + \frac{5}{72}L^3x^3\right] + C_3x + C_4$$

$[x = 0, y = 0]$: $0 = 0 - 0 + 0 + 0 + C_4 \quad \therefore \quad C_4 = 0$

$[x = L, y = 0]$: $0 = \frac{w_0}{L^2}\left[\frac{1}{360}L^6 - \frac{1}{24}L^6 + \frac{5}{72}L^6\right] + C_3L \quad \therefore \quad C_3 = -\frac{11}{360}w_0L^3$

(*a*) Elastic curve. $y = w_0(x^6 - 15L^2x^4 + 25L^3x^3 - 11L^5x)/360EIL^2$ ◀

$$\frac{dy}{dx} = w_0(6x^5 - 60L^2x^3 + 75L^3x^2 - 11L^5)/360EIL^2$$

(*b*) Slope at end *A*. Set $x = 0$ in dy/dx. $\left.\frac{dy}{dx}\right|_A = -\frac{11}{360}\frac{w_0L^3}{EI}$ $\theta_A = \frac{11}{360}\frac{w_0L^3}{EI}$ ◀

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PROBLEM 9.18 ***(Continued)***

(*c*) Deflection at midpoint (say, point *C*). Set $x = \frac{L}{2}$ in deflection equation.

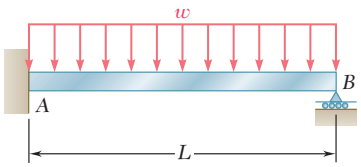
$$y_C = w_0\left(\frac{1}{64}L^6 - \frac{15}{16}L^6 + \frac{25}{8}L^6 - \frac{11}{2}L^6\right)\Big/360EIL^2$$

$$= w_0\left(\frac{1}{64}L^6 - \frac{60}{64}L^6 + \frac{200}{64}L^6 - \frac{352}{64}L^6\right)\Big/360EIL^2$$

$$= -\frac{211}{23{,}040}\frac{w_0L^4}{EI}$$

$$y_C = 0.00916\frac{w_0L^4}{EI}\downarrow \blacktriangleleft$$

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PROBLEM 9.19

For the beam and loading shown, determine the reaction at the roller support.

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

SOLUTION

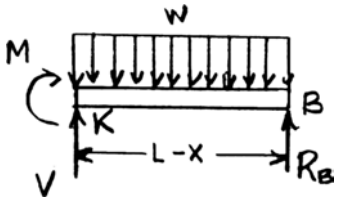
Reactions are statically indeterminate.

Boundary conditions are shown above.

Using free body *KB*,

$$+\circlearrowleft M_K = 0: \quad R_B(L-x) - w(L-x)\left(\frac{L-x}{2}\right) - M = 0$$

$$M = R_B(L-x) - \frac{1}{2}w(L-x)^2$$



$$EI\frac{d^2y}{dx^2} = R_B(L-x) - \frac{1}{2}w(L-x)^2$$

$$EI\frac{dy}{dx} = -\frac{1}{2}R_B(L-x)^2 + \frac{1}{6}w(L-x)^3 + C_1$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right]: \quad 0 = -\frac{1}{2}R_BL^2 + \frac{1}{6}wL^3 + C_1$$

$$C_1 = \frac{1}{2}R_BL^2 - \frac{1}{6}wL^3$$

$$EI_y = \frac{1}{6}R_B(L-x)^3 - \frac{1}{24}w(L-x)^4 + C_1x + C_2$$

$$[x = 0,\ y = 0]: \quad 0 = \frac{1}{6}R_BL^3 - \frac{1}{24}wL^3 + C_2$$

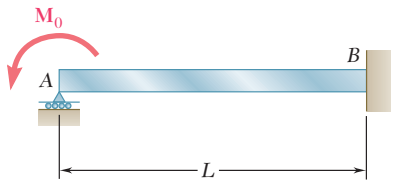
$$C_2 = -\frac{1}{6}R_BL^3 + \frac{1}{24}wL^4$$

$$[x = L,\ y = 0]: \quad 0 = 0 - 0 + C_1L + C_2$$

$$\frac{1}{2}R_BL^3 - \frac{1}{6}wL^4 - \frac{1}{6}R_BL^3 + \frac{1}{24}wL^4 = 0$$

$$R_B = \frac{3}{8}wL \uparrow \blacktriangleleft$$

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PROBLEM 9.20

For the beam and loading shown, determine the reaction at the roller support.

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

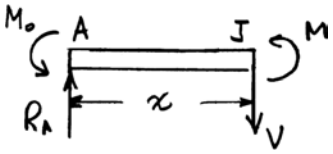
$$\left[x = L,\ \frac{dy}{dx} = 0\right]$$

SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown above.

Using free body *AJ*,



$$+\circlearrowleft \Sigma M_J = 0: \quad M_0 - R_A x + M = 0$$

$$M = R_A x - M_0$$

$$EI\frac{d^2y}{dx^2} = R_A x - M_0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0 x + C_1$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right] \quad 0 = \frac{1}{2}R_A L^2 - M_0 L + C_1$$

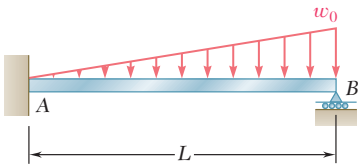
$$C_1 = M_0 L - \frac{1}{2}R_A L^2$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0 x^3 + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\quad C_2 = 0$

$[x = L,\ y = 0]$ $\quad 0 = \frac{1}{6}R_A L^3 - \frac{1}{2}M_0 L^2 + \left(M_0 L - \frac{1}{2}R_A L^2\right)L + 0$ $\quad R_A = \frac{3}{2}\frac{M_0}{L} \uparrow$ ◀

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PROBLEM 9.21

For the beam and loading shown, determine the reaction at the roller support.

$[x = 0,\ y = 0]$ $\qquad$ $[x = L,\ y = 0]$

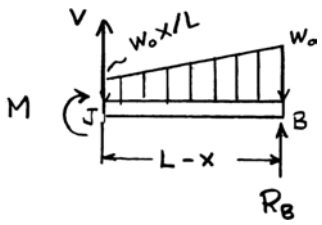
$$\left[x = 0,\ \frac{dy}{dx} = 0\right]$$

SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown above.

Using free body *JB*,



$$+\circlearrowleft \Sigma M_J = 0: \quad -M + R_B(L-x) + \frac{1}{2}w_0(L-x)\frac{2}{3}(L-x)$$

$$+\frac{1}{2}\frac{w_0x}{L}(L-x)\frac{1}{3}(L-x) = 0$$

$$M = R_B(L-x) - \frac{w_0}{6L}[2L(L-x)^2 + x(L-x)^2]$$

$$= R_B(L-x) - \frac{w_0}{6L}[2L^3 - 4L^2x + 2Lx^2 + xL^2 - 2Lx^2 + x^3]$$

$$= R_B(L-x) - \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3)$$

$$EI\frac{d^2y}{dx^2} = R_B(L-x) - \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3)$$

$$EI\frac{dy}{dx} = R_B\left(Lx - \frac{1}{2}x^2\right) - \frac{w_0}{6L}\left(\frac{1}{4}x^4 - \frac{3}{2}L^2x^2 + 2L^3x\right) + C_1$$

$$EIy = R_B\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) - \frac{w_0}{6L}\left(\frac{1}{20}x^5 - \frac{1}{2}L^2x^3 + L^3x^2\right) + C_1x + C_2$$

$$[x = 0,\ y = 0] \quad \rightarrow C_2 = 0$$

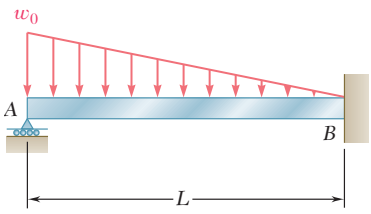
$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \rightarrow C_1 = 0$$

$$[x = L,\ y = 0] \quad 0 = R_BL^3\left(\frac{1}{2} - \frac{1}{6}\right) - \frac{w_0L^4}{6}\left(\frac{1}{20} - \frac{1}{2} + 1\right)$$

$$\frac{1}{3}R_B = \left(\frac{1}{6}\right)\left(\frac{11}{20}\right)w_0L$$

$$R_B = \frac{11}{40}w_0L \uparrow \blacktriangleleft$$

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PROBLEM 9.22

For the beam and loading shown, determine the reaction at the roller support.

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$$\left[x = L,\ \frac{dy}{dx} = 0\right]$$

SOLUTION

Reactions are statically indeterminate.

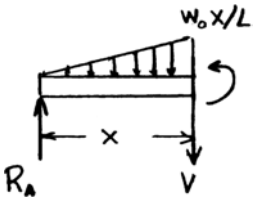
Boundary conditions are shown above.

$$w = \frac{w_0}{L}(L - x)$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L}(L - x)$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L}\left(Lx - \frac{1}{2}x^2\right) + R_A$$

$$M = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + R_A x$$



$$EI\frac{d^2y}{dx^2} = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + R_A x$$

$$EI\frac{dy}{dx} = -\frac{w_0}{L}\left(\frac{1}{6}Lx^3 - \frac{1}{24}x^4\right) + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = -\frac{w_0}{L}\left(\frac{1}{24}Lx^4 - \frac{1}{120}x^5\right) + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\quad 0 = 0 + 0 + 0 + C_2 \quad C_2 = 0$

$$\left[x = L,\ \frac{dy}{dx} = 0\right] \quad -\frac{w_0}{L}\left(\frac{1}{6}L^4 - \frac{1}{24}L^4\right) + \frac{1}{2}R_A L^2 + C_1 = 0$$

$$C_1 = \frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2$$

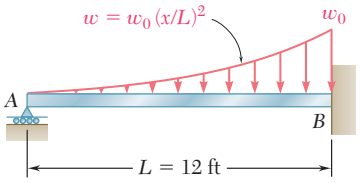
$$[x = L,\ y = 0] \quad -\frac{w_0}{L}\left(\frac{1}{24}L^5 - \frac{1}{120}L^5\right) + \frac{1}{6}R_A L^3 + \left(\frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2\right)L = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_A = \left(\frac{1}{8} - \frac{1}{24} + \frac{1}{120}\right)w_0 L$$

$$\frac{1}{3}R_A = \frac{11}{120}w_0 L \qquad R_A = \frac{11}{40}w_0 L \uparrow \blacktriangleleft$$

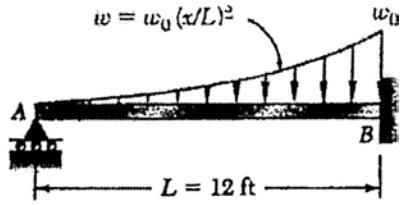
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PROBLEM 9.23



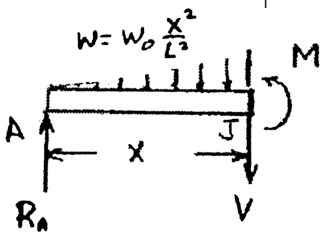
For the beam shown, determine the reaction at the roller support when $w_0 = 6$ kips/ft.

SOLUTION



$[x = 0, y = 0]$ $[x = L, y = 0]$

$\left[x = L, \dfrac{dy}{dx} = 0\right]$



Reactions are statically indeterminate.

Boundary conditions are shown at left.

$$w = w_0 \frac{x^2}{L^2}$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L^2} x^2$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L^2}\frac{x^3}{3} + R_A$$

$$M = -\frac{w_0}{L^2}\frac{x^4}{12} + R_A x$$

$$EI\frac{d^2y}{dx^2} = -\frac{w_0}{L^2}\frac{x^4}{12} + R_A x$$

$$EI\frac{dy}{dx} = -\frac{w_0}{L^2}\frac{x^5}{60} + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = -\frac{w_0}{L^2}\frac{x^6}{360} + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\quad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$\left[x = L,\ \dfrac{dy}{dx} = 0\right]$ $\quad -\dfrac{1}{60}w_0L^3 + \dfrac{1}{2}R_AL^2 + C_1 = 0 \qquad C_1 = \dfrac{1}{60}w_0L^3 - \dfrac{1}{2}R_AL^2$

$[x = L,\ y = 0]$ $\quad -\dfrac{1}{360}w_0L^4 + \dfrac{1}{6}R_AL^3 + \left(\dfrac{1}{60}w_0L^4 - \dfrac{1}{2}R_AL^3\right)L = 0$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_A = \left(\frac{1}{60} - \frac{1}{360}\right)w_0L$$

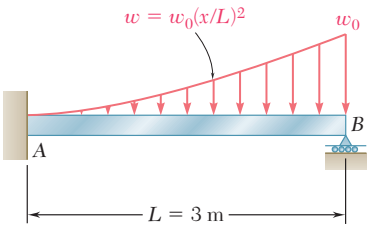
$$\frac{1}{3}R_A = \frac{1}{72}w_0L \qquad R_A = \frac{1}{18}w_0L$$

Data: $w_0 = 6$ kips/ft, $L = 12$ ft

$$R_A = \frac{1}{18}(6)(12)$$

$R_A = 4.00$ kips ↑ ◀

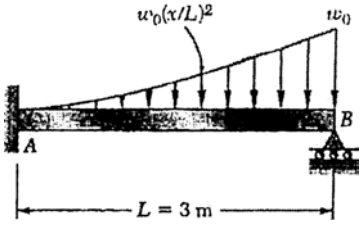
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PROBLEM 9.24

For the beam shown, determine the reaction at the roller support when $w_0 = 15$ kN/m.

SOLUTION



Reactions are statically indeterminate.

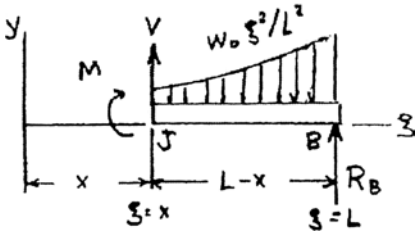
Boundary conditions are shown at left.

Using free body *JB*,

$+\circlearrowleft \Sigma M_J = 0$:

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$[x = 0,\ \dfrac{dy}{dx} = 0]$



$$-M + \int_x^L \frac{w_0}{L^2}\xi^2(\xi - x)d\xi + R_B(L - x) = 0$$

$$M = \frac{w_0}{L^2}\int_x^L \xi^2(\xi - x)d\xi - R_B(L - x)$$

$$= \frac{w_0}{L^2}\left(\frac{1}{4}\xi^4 - \frac{1}{3}x\xi^3\right)\Big|_x^L - R_B(L - x)$$

$$= \frac{w_0}{L^2}\left(\frac{1}{4}L^4 - \frac{1}{3}L^3x + \frac{1}{12}x^4\right) - R_B(L - x)$$

$$EI\frac{d^2y}{dx^2} = \frac{w_0}{L^2}\left(\frac{1}{4}L^4 - \frac{1}{3}L^3x + \frac{1}{12}x^4\right) - R_B(L - x)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L^2}\left(\frac{1}{4}L^4x - \frac{1}{6}L^3x^2 + \frac{1}{60}x^5\right) - R_B\left(Lx - \frac{1}{2}x^2\right) + C_1$$

$$EIy = \frac{w_0}{L^2}\left(\frac{1}{8}L^4x^2 - \frac{1}{18}L^3x^3 + \frac{1}{360}x^6\right) - R_B\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + C_1x + C_2$$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$ $\quad 0 = 0 + 0 + C_1 \quad C_1 = 0$

$[x = 0,\ y = 0]$ $\quad 0 = 0 + 0 + 0 + C_2 \quad C_2 = 0$

$[x = L,\ y = 0]$ $\quad \left(\dfrac{1}{8} - \dfrac{1}{18} + \dfrac{1}{360}\right)w_0L^4 - \left(\dfrac{1}{2} - \dfrac{1}{6}\right)R_BL^3 = 0$

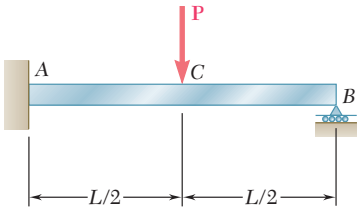
$$\frac{13}{180}w_0L^4 - \frac{1}{3}R_BL^3 = 0 \qquad R_B = \frac{13}{60}w_0L$$

Data: $w_0 = 15$ kN/m $\qquad L = 3$ m

$$R_B = \frac{13}{60}(15)(3) = 9.75 \text{ kN}$$

$R_B = 9.75$ kN ↑ ◀

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$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

PROBLEM 9.25

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

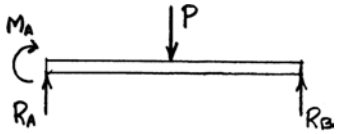
SOLUTION

Reactions are statically indeterminate.

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \quad R_A = P - R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + \frac{1}{2}PL - R_B L = 0$$

$$M_A = R_B L - \frac{1}{2}PL$$



$$\underline{0 < x < \frac{1}{2}L:}$$

$$M = M_A + R_A x$$

$$EI\frac{d^2y}{dx^2} = M_A + R_A x$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$$\underline{\frac{1}{2}L < x < L:}$$

$$M = M_A + R_A x - P\left(x - \frac{1}{2}L\right)$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\left(x - \frac{1}{2}L\right)$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\left(x - \frac{1}{2}L\right)^2 + C_3$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left(x - \frac{L}{2}\right)^3 + C_3 x + C_4$$

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PROBLEM 9.25 ***(Continued)***

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \qquad 0 + 0 + C_1 = 0 \qquad C_1 = 0$$

$$[x = 0,\ y = 0] \quad 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$$

$$\frac{1}{2}M_A L + \frac{1}{8}R_A L^2 + 0 = \frac{1}{2}M_A L + \frac{1}{8}R_A L^2 - 0 + C_3 \qquad C_3 = 0$$

$$\left[x = \frac{L}{2},\ y = y\right]$$

$$\frac{1}{8}M_A L^2 + \frac{1}{48}R_A L^3 + 0 + 0 = \frac{1}{8}M_A L^2 + \frac{1}{48}R_A L^3 - 0 + 0 + C_4 \quad C_4 = 0$$

$$[x = L,\ y = 0]$$

$$\frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{48}PL^3 + 0 + 0 = 0$$

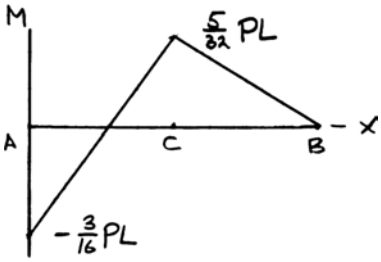
$$\frac{1}{2}\left(R_B L - \frac{1}{2}P\right)L^3 + \frac{1}{6}(P - R_B)L^3 - \frac{1}{48}PL^3 = 0 \qquad R_B = \frac{5}{16}P \uparrow \blacktriangleleft$$

$$R_A = P - \frac{5}{16}P \qquad R_A = \frac{7}{16}P \uparrow$$

$$M_A = \frac{5}{16}PL - \frac{1}{2}PL \qquad M_A = -\frac{3}{16}PL \blacktriangleleft$$

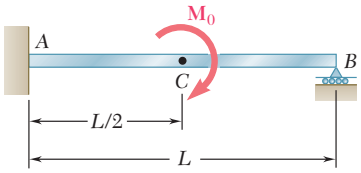
$$M_C = R_B\left(\frac{L}{2}\right) = \left(\frac{5}{16}P\right)\left(\frac{L}{2}\right) \qquad M_C = \frac{5}{32}PL \blacktriangleleft$$

$$M_B = 0 \blacktriangleleft$$



Bending moment diagram

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PROBLEM 9.26

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

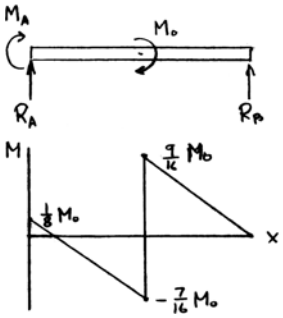
SOLUTION

Reactions are statically indeterminate.

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B = 0 \quad R_A = -R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - M_0 + R_B L = 0 \quad M_A = R_B L - M_0$$

$$\underline{0 < x < \frac{L}{2}}$$



$$M = R_B x + M_A = -M_0 + R_B L - R_B x$$

$$EI\frac{d^2y}{dx^2} = -M_0 + R_B(L - x)$$

$$EI\frac{dy}{dx} = -M_0 x + R_B\left(Lx - \frac{1}{2}x^2\right) + C_1$$

$$EIy = -\frac{1}{2}M_0 x^2 + R_B\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + C_1 x + C_2$$

$$\underline{\frac{L}{2} < x < L}$$

$$M = R_B(L - x)$$

$$EI\frac{d^2y}{dx^2} = R_B(L - x)$$

$$EI\frac{dy}{dx} = R_B\left(Lx - \frac{1}{2}x^2\right) + C_3$$

$$EIy = R_B\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + C_3 x + C_4$$

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PROBLEM 9.26 *(Continued)*

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \quad 0 + 0 + C_1 = 0 \qquad C_1 = 0$$

$$[x = 0,\ y = 0] \quad 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$$

$$-M_0\frac{L}{2} + \cancel{R_B\left(\frac{1}{2}L^2 - \frac{1}{6}L^2\right)} = \cancel{R_B\left(\frac{1}{2}L^2 - \frac{1}{6}L^2\right)} + C_3 \qquad C_3 = -\frac{M_0 L}{2}$$

$$\left[x = \frac{L}{2},\ y = y\right]$$

$$-\frac{1}{2}M_0\left(\frac{L}{2}\right)^2 + \cancel{R_B\left(\frac{1}{8}L^3 - \frac{1}{48}L^3\right)} = \cancel{R_B\left(\frac{1}{8}L^3 - \frac{1}{48}L^3\right)} + C_3\frac{L}{2} + C_4$$

$$C_4 = -\frac{1}{8}M_0L^2 - \frac{1}{2}C_3L$$

$$= \left(-\frac{1}{8} + \frac{1}{4}\right)M_0L^2 = \frac{1}{8}M_0L^2$$

$$[x = L,\ y = 0]$$

$$R_B\left(\frac{1}{2}L^3 - \frac{1}{6}L^3\right) + \frac{M_0L}{2}L + \frac{1}{8}M_0L^2 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_BL^3 = \left(\frac{1}{2} - \frac{1}{8}\right)M_0L^2 \quad \frac{1}{3}R_B = \frac{3}{8}\frac{M_0}{L}$$

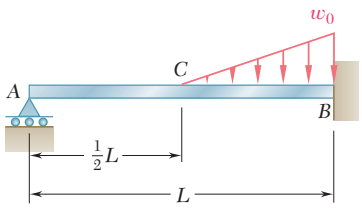
$$R_B = \frac{9}{8}\frac{M_0}{L} \uparrow \blacktriangleleft$$

$$M_A = \frac{9}{8}M_0 - M_0 = \frac{1}{8}M_0 \qquad M_A = \frac{1}{8}M_0 \blacktriangleleft$$

$$M_{C^-} = -M_0 + \frac{9}{8}\frac{M_0}{L} = -\frac{7}{16}M_0 \qquad M_{C^-} = -\frac{7}{16}M_0 \blacktriangleleft$$

$$M_{C^+} = R_B\left(L - \frac{L}{2}\right) = \frac{9}{8}\frac{M_0}{L}\left(\frac{L}{2}\right) = \frac{9}{16}M_0 \qquad M_{C^+} = \frac{9}{16}M_0 \blacktriangleleft$$

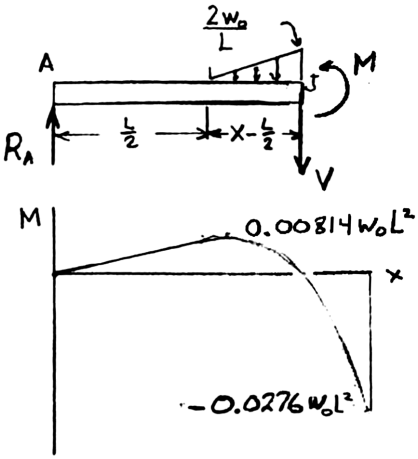
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PROBLEM 9.27

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION



Reactions are statically indeterminate.

$$0 \le x \le \frac{L}{2}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x \tag{1}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1 \tag{2}$$

$$EIy = \frac{1}{6}R_A x^3 + C_1 x + C_2 \tag{3}$$

$$\frac{L}{2} \le x \le L$$

$$+\circlearrowleft \Sigma M_J = 0:$$

$$-R_A x + \frac{1}{2}\frac{2w_0}{L}\left(x-\frac{L}{2}\right)\left(x-\frac{L}{2}\right)\frac{1}{3}\left(x-\frac{L}{2}\right) + M = 0$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{3}\frac{w_0}{L}\left(x-\frac{L}{2}\right)^3 \tag{4}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{12}\frac{w_0}{L}\left(x-\frac{L}{2}\right)^4 + C_3 \tag{5}$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{60}\frac{w_0}{L}\left(x-\frac{L}{2}\right)^5 + C_3 x + C_4 \tag{6}$$

$[x = 0, y = 0]$ $\quad 0 = 0 + 0 + C_2$ $\quad C_2 = 0$

$\left[x = \frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}\right]$ $\quad \frac{1}{2}R_A\left(\frac{L}{2}\right)^2 + C_1 = \frac{1}{2}R_A\left(\frac{L}{2}\right)^2 - 0 + C_3$ $\quad C_1 = C_3$

$\left[x = \frac{L}{2}, y = y\right]$ $\quad \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} + \cancel{C_1\frac{L}{2}} + C_2 = \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} - 0 + \cancel{C_3\frac{L}{2}} + C_4$ $\quad C_4 = C_2 = 0$

$\left[x = L, \frac{dy}{dx} = 0\right]$ $\quad \frac{1}{2}R_A L^2 - \frac{1}{12}\frac{w_0}{L}\left(\frac{L}{2}\right)^4 + C_3 = 0$ $\quad C_3 = \frac{1}{192}w_0L^3 - \frac{1}{2}R_A L^2$

$[x = L, y = 0]$ $\quad \frac{1}{6}R_A L^3 - \frac{1}{60}\frac{w_0}{L}\left(\frac{L}{2}\right)^5 + \frac{1}{96}w_0L^4 - \frac{1}{2}R_A L^3 + 0 = 0$

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PROBLEM 9.27 *(Continued)*

$$\left(\frac{1}{2}-\frac{1}{6}\right)R_A L^3 = \left(\frac{1}{192}-\frac{1}{1920}\right)w_0 L^4 \qquad \frac{1}{3}R_A = \frac{3}{640}w_0 L \qquad R_A = \frac{9}{640}w_0 L \uparrow \blacktriangleleft$$

From (1), with $x = \frac{L}{2}$, $\qquad M_C = R_A\frac{L}{2} = \frac{9}{1280}w_0 L^2 \qquad M_C = 0.007031 w_0 L^2 \blacktriangleleft$

From (4), with $x = L$, $\qquad M_B = \frac{9}{640}w_0 L^2 - \frac{1}{3}\frac{w_0}{L}\left(\frac{L}{2}\right)^3 = -\frac{53}{1920}w_0 L^2$

$$M_B = -0.0276 w_0 L^2 \blacktriangleleft$$

Location of maximum positive M in portion CB:

$$\frac{dM}{dx} = R_A - \frac{w_0}{L}\left(x_m - \frac{L}{2}\right)^2 = 0$$

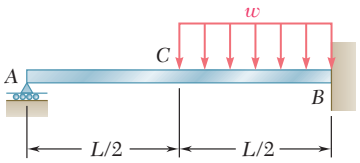
$$x_m - \frac{L}{2} = \sqrt{\frac{R_A L}{w_0}} = \sqrt{\frac{9}{640}}L = 0.1186L$$

$$x_m = 0.5L + 0.1186L = 0.6186L$$

From (4), with $x = x_m$, $\qquad M_m = R_A(0.6186L) - \frac{1}{3}\frac{w_0}{L}(0.1186L)^3$

$$M_m = 0.00814 w_0 L^2 \blacktriangleleft$$

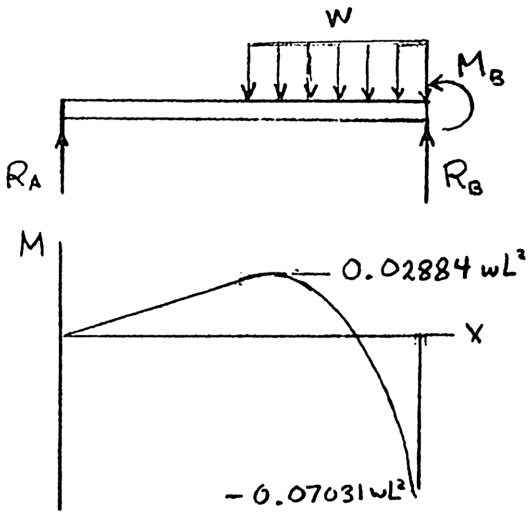
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PROBLEM 9.28

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION



Reactions are statically indeterminate.

$$0 < x < \frac{L}{2}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x \tag{1}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1 \tag{2}$$

$$EIy = \frac{1}{6}R_A x^3 + C_1 x + C_2 \tag{3}$$

$$\frac{L}{2} < x < L$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2}w\left(x - \frac{L}{2}\right)^2 \tag{4}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}w\left(x - \frac{L}{2}\right)^3 + C_3 \tag{5}$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}w\left(x - \frac{L}{2}\right)^4 + C_3 x + C_4 \tag{6}$$

$\left[x = 0,\ y = 0\right]$ $\quad 0 = 0 + 0 + C_2$ $\quad C_2 = 0$

$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$ $\quad \frac{1}{2}R_A\left(\frac{L}{2}\right)^2 + C_1 = \frac{1}{2}R_A\left(\frac{L}{2}\right)^2 + 0 + C_3$ $\quad C_1 = C_3$

$\left[x = \frac{L}{2},\ y = y\right]$ $\quad \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 + C_1\frac{L}{2} + C_2 = \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 - 0 + C_3\frac{L}{2} + C_4$ $\quad C_2 = C_4 = 0$

$\left[x = L,\ \frac{dy}{dx} = 0\right]$ $\quad \frac{1}{2}R_x L^2 - \frac{1}{6}w\left(\frac{L}{2}\right)^3 + C_3 = 0$ $\quad C_3 = \frac{1}{48}wL^3 - \frac{1}{2}R_A L^2$

$\left[x = L,\ y = 0\right]$ $\quad \frac{1}{6}R_A L^2 - \frac{1}{24}w\left(\frac{L}{2}\right)^4 + \left(\frac{1}{48}wL^3 - \frac{1}{2}R_A L^2\right)L + 0 = 0$

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PROBLEM 9.28 *(Continued)*

$$\left(\frac{1}{2}-\frac{1}{6}\right)R_A L^3=\left(\frac{1}{48}-\frac{1}{384}\right)wL^4 \qquad \frac{1}{3}R_A=\frac{7}{384}wL \qquad R_A=\frac{7}{128}wL \uparrow \blacktriangleleft$$

From (1), with $x=\frac{L}{2}$, $\quad M_C=R_A\left(\frac{L}{2}\right)=\frac{7}{256}wL^2 \qquad M_C=0.0273wL^2 \blacktriangleleft$

From (4), with $x=L$, $\quad M_B=R_AL-\frac{1}{2}w\left(\frac{L}{2}\right)^2=\left(\frac{7}{128}-\frac{1}{8}\right)wL-\frac{9}{128}wL^2$

$$M_B=-0.0703wL \blacktriangleleft$$

Location of maximum positive M:

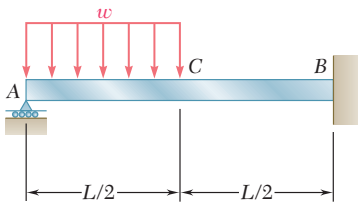
$$\frac{L}{2}<x<L \qquad V_m=R_A-w\left(x_m-\frac{L}{2}\right)=0 \qquad x_m-\frac{L}{2}=\frac{R_A}{w}=\frac{7}{128}L$$

$$x_m=\frac{L}{2}+\frac{7}{128}L=\frac{71}{128}L$$

From (4), with $x=x_m$, $\quad M_m=R_Ax_m-\frac{1}{2}w\left(x_m-\frac{L}{2}\right)^2$

$$=\left(\frac{7}{128}wL\right)\left(\frac{71}{128}L\right)-\frac{1}{2}w\left(\frac{7}{128}L\right)^2 \qquad M_m=0.0288wL^2 \blacktriangleleft$$

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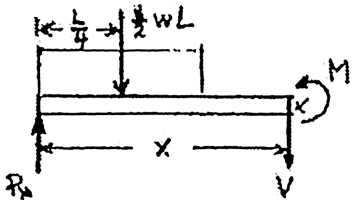


PROBLEM 9.29

Determine the reaction at the roller support and the deflection at point *C*.

SOLUTION

Reactions are statically indeterminate.



$$0 < x < \frac{L}{2}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x + C_2$$

$$\frac{L}{2} < x < L \qquad \text{(See free body diagram.)}$$

$$+\circlearrowleft \sum M_K = 0: \; -R_A x + \frac{1}{2}wL\left(x - \frac{L}{4}\right) + M = 0$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2}wL\left(x - \frac{1}{4}L\right)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{4}wL\left(x - \frac{L}{4}\right)^2 + C_3$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{12}wL\left(x - \frac{L}{4}\right)^3 + C_3 x + C_4$$

$$[x = 0, \; y = 0]: \qquad 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$\left[x = \frac{L}{2}, \; \frac{dy}{dx} = \frac{dy}{dx}\right]: \qquad \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} - \frac{1}{6}w\left(\frac{L}{2}\right)^3 + C_1 = \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} - \frac{1}{4}wL\left(\frac{L}{4}\right)^2 + C_3$$

$$C_1 = C_3 + \frac{1}{48}wL^3 - \frac{1}{64}wL^3 = C_3 + \frac{1}{192}wL^3$$

$$\left[x = \frac{L}{2}, \; y = y\right]: \quad \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} - \frac{1}{24}w\left(\frac{L}{2}\right)^4 + \left(\cancel{C_3} + \frac{1}{192}wL^3\right)\frac{L}{2} = \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 - \frac{1}{12}wL\left(\frac{L}{4}\right)^3 + \cancel{C_3\frac{L}{2}} + C_4$$

$$C_4 = -\frac{1}{384}wL^4 + \frac{1}{384}wL^4 + \frac{1}{768}wL^4 = \frac{1}{768}wL^4$$

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PROBLEM 9.29 *(Continued)*

$$\left[x = L,\ \frac{dy}{dx} = 0\right]: \qquad \frac{1}{2}R_A L^2 - \frac{1}{4}wL\left(\frac{3L}{4}\right)^2 + C_3 = 0 \qquad C_3 = \frac{9}{64}wL^3 - \frac{1}{2}R_A L^2$$

$$[x = L,\ y = 0]: \qquad \frac{1}{6}R_A L^3 - \frac{1}{12}wL\left(\frac{3L}{4}\right)^3 + \left(\frac{9}{64}wL^3 - \frac{1}{2}R_A L^2\right)L + \frac{1}{768}wL^4 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_A L^3 = \left(\frac{9}{64} - \frac{27}{768} + \frac{1}{768}\right)wL^4 \qquad \frac{1}{3}R_A = \frac{41}{384}wL \qquad R_A = \frac{41}{128}wL \uparrow \blacktriangleleft$$

$$C_3 = \frac{9}{64}wL^3 - \frac{1}{2}\frac{41}{128}wL^3 = -\frac{5}{256}wL^3$$

$$C_1 = -\frac{5}{256}wL^3 + \frac{1}{192}wL^3 = -\frac{11}{768}wL^3$$

Deflection at *C*. $\left(y \text{ at } x = \frac{L}{2}\right)$

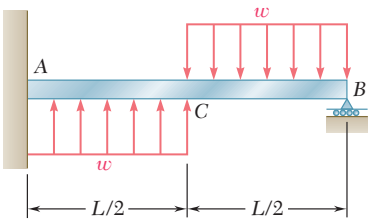
$$y_C = \frac{wL^4}{EI}\left\{\frac{1}{6}\cdot\frac{41}{128}\cdot\left(\frac{1}{2}\right)^3 - \frac{1}{24}\cdot\left(\frac{1}{2}\right)^4 - \frac{11}{768}\cdot\frac{1}{2} + 0\right\}$$

$$= \left(\frac{41}{6144} - \frac{1}{384} - \frac{11}{1536}\right)\frac{wL^4}{EI} = -\frac{19}{6144}\frac{wL^4}{EI} \qquad y_C = \frac{19}{6144}\frac{wL^4}{EI} \downarrow \blacktriangleleft$$

or $$y_C = \frac{wL^4}{EI}\left\{\frac{1}{6}\cdot\frac{41}{128}\left(\frac{1}{2}\right)^3 - \frac{1}{12}\cdot\left(\frac{1}{4}\right)^3 + \frac{5}{256}\cdot\frac{1}{2} + \frac{1}{768}\right\}$$

$$= \left(\frac{41}{6144} - \frac{1}{\cancel{768}} - \frac{5}{512} + \frac{1}{\cancel{768}}\right)\frac{wL^4}{EI} = -\frac{19}{6144}\frac{wL^4}{EI}$$

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PROBLEM 9.30

Determine the reaction at the roller support and the deflection at point *C*.

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right]$$

$$\left[x = \frac{L}{2},\ y = y\right]$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$$

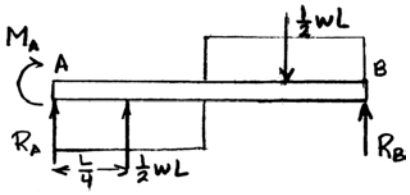
SOLUTION

Reactions are statically indeterminate.

$$+\uparrow \Sigma F_y = 0: \quad R_A + \frac{1}{2}wL - \frac{1}{2}wL + R_B = 0 \quad R_A = -R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - \left(\frac{1}{2}wL\right)\frac{L}{2} + R_B L = 0$$

$$M_A = R_B L - \frac{1}{4}wL^2$$



From *A* to *C*: $0 < x \le \frac{L}{2}$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x + \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{6}wx^3 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{24}wx^4 + C_1 x + C_2$$

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PROBLEM 9.30 *(Continued)*

From *C* to *B*: $\frac{L}{2} \le x < L$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x + \frac{1}{2}wL\left(x - \frac{L}{4}\right) - \frac{1}{2}w\left(x - \frac{L}{2}\right)^2$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{4}wL\left(x - \frac{L}{4}\right)^2 - \frac{1}{6}w\left(x - \frac{L}{2}\right)^3 + C_3$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{12}wL\left(x - \frac{L}{4}\right)^3 - \frac{1}{24}w\left(x - \frac{L}{2}\right)^4 + C_3 x + C_4$$

$\left[x = 0,\ \frac{dy}{dx} = 0\right]$ $\quad 0 + 0 + 0 + C_1 = 0 \qquad C_1 = 0$

$[x = 0,\ y = 0]$ $\quad 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$$

$$\cancel{M_A\frac{L}{2}} - \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} + \frac{1}{6}w\left(\frac{L}{2}\right)^3 = \cancel{M_A\frac{L}{2}} + \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} + \frac{1}{4}wL\left(\frac{L}{4}\right)^2 - 0 + C_3$$

$$C_3 = \left(\frac{1}{48} - \frac{1}{64}\right)wL^3 = \frac{1}{192}wL^3$$

$$\left[x = \frac{L}{2},\ y = y\right]$$

$$\cancel{\frac{1}{2}M_A\left(\frac{L}{2}\right)^2} + \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} + \frac{1}{24}w\left(\frac{L}{2}\right)^4$$

$$= \cancel{\frac{1}{2}M_A\left(\frac{L}{2}\right)^2} + \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} + \frac{1}{12}wL\left(\frac{L}{4}\right)^3$$

$$- 0 + \frac{1}{192}wL^3\left(\frac{L}{2}\right) + C_4$$

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PROBLEM 9.30 *(Continued)*

$$C_4 = \left(\frac{1}{384} - \frac{1}{768} - \frac{1}{384}\right)wL^4 = -\frac{1}{768}wL^4$$

$[x = L,\ y = 0]$

$$\frac{1}{2}M_AL^2 + \frac{1}{6}R_AL^3 + \frac{1}{12}wL\left(\frac{3L}{4}\right)^3 - \frac{1}{24}w\left(\frac{L}{2}\right)^4 + \frac{1}{192}wL^3(L) - \frac{1}{768}wL^4 = 0$$

$$\frac{1}{2}\left(R_BL - \frac{1}{4}wL^2\right)L^2 + \frac{1}{6}(-R_B)L^3 + \left(\frac{27}{768} - \frac{1}{384} + \frac{1}{192} - \frac{1}{768}\right)wL^4 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_BL^3 = \left(\frac{1}{8} - \frac{7}{192}\right)wL^4 \quad \frac{1}{3}R_B = \frac{17}{192}wL \qquad R_B = \frac{17}{64}wL \uparrow \blacktriangleleft$$

$$R_A = -R_B = -\frac{17}{64}wL$$

$$M_A = R_BL - \frac{1}{4}wL^2 = \left(\frac{17}{64} - \frac{1}{4}\right)wL^2 = \frac{1}{64}wL^2$$

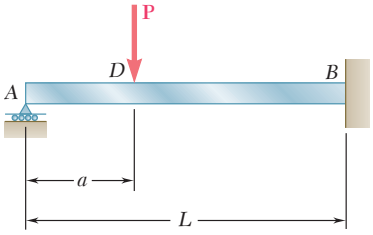
Deflection at C. $\left(y \text{ at } x = \frac{L}{2}\right)$

$$EIy_C = \frac{1}{2}M_A\left(\frac{L}{2}\right)^2 + \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 + \frac{1}{24}w\left(\frac{L}{2}\right)^4$$

$$= \frac{1}{2}\left(\frac{1}{64}wL^2\right)\left(\frac{L}{2}\right)^2 + \frac{1}{6}\left(-\frac{17}{64}wL\right)\left(\frac{L}{2}\right)^3 + \frac{1}{24}w\left(\frac{L}{2}\right)^4$$

$$= \left(\frac{1}{512} - \frac{17}{3072} + \frac{1}{384}\right)wL^4 = -\frac{1}{1024}wL^4 \qquad y_C = \frac{1}{1024}\frac{wL^4}{EI} \downarrow \blacktriangleleft$$

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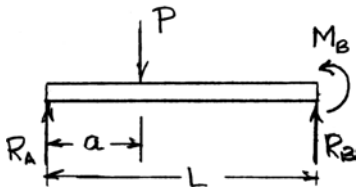
PROBLEM 9.31

Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.

SOLUTION

$\underline{0 \le x \le a}$:

$$M = R_A x$$



$$EI\frac{d^2y}{dx^2} = M = R_A x$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$\underline{a \le x \le L}$:

$$M = R_A x - P(x-a)$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - P(x-a)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{2}P(x-a)^2 + C_3$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{6}P(x-a)^3 + C_3 x + C_4$$

$[x = 0,\ y = 0]$: $\quad 0 + 0 + C_2 = 0 \qquad \therefore \quad C_2 = 0$

$\left[x = a,\ \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$:

$$\frac{1}{2}R_A a^2 + C_1 = \frac{1}{2}R_A a^2 - 0 + C_3 \qquad \therefore \quad C_1 = C_3$$

$[x = a,\ y = y]$:

$$\frac{1}{6}R_A a^3 + C_1 a + 0 = \frac{1}{6}R_A a^3 - 0 + C_1 a + C_4 \qquad \therefore \quad C_4 = 0$$

$\left[x = L,\ \dfrac{dy}{dx} = 0\right]$:

$$\frac{1}{2}R_A L^2 - \frac{1}{2}P(L-a)^2 + C_3 = 0 \qquad \therefore \quad C_3 = \frac{1}{2}P(L-a)^2 - \frac{1}{2}R_A L^2$$

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PROBLEM 9.31 *(Continued)*

$$[x = L,\ y = 0]:$$

$$\frac{1}{6}R_A L^3 - \frac{1}{6}P(L-a)^3 + \left[\frac{1}{2}P(L-a)^2 - \frac{1}{2}R_A L^2\right](L) + 0 = 0$$

$$R_A = \frac{P}{2L^3}(2L^3 - 3aL^2 + a^3) = \frac{P}{2L^3}\left(2L^3 - L^3 + \frac{L^3}{9}\right) \qquad R_A = \frac{14}{27}P \uparrow \blacktriangleleft$$

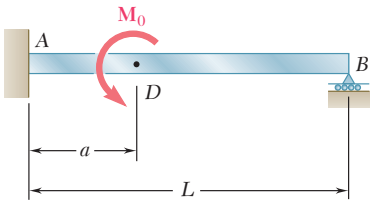
Deflection at *D*. $\left(y \text{ at } x = a = \dfrac{L}{3}\right)$

$$y_D = \frac{1}{EI}\left\{\frac{1}{6}R_A\left(\frac{L}{3}\right)^3 + C_1\left(\frac{L}{3}\right)\right\}$$

$$= \frac{1}{EI}\left\{\frac{1}{6}\left(\frac{14}{27}P\right)\left(\frac{L}{3}\right)^3 + \left[\frac{1}{2}P\left(L - \frac{L}{3}\right)^2 - \frac{1}{2}\left(\frac{14}{27}P\right)L^2\right]\left(\frac{L}{3}\right)\right\}$$

$$= -\frac{20}{2187}\frac{PL^3}{EI} \qquad y_D = \frac{20}{2187}\frac{PL^3}{EI} \downarrow \blacktriangleleft$$

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PROBLEM 9.32

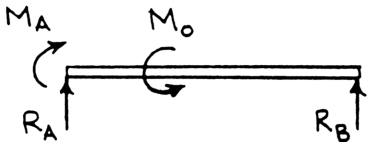
Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.

SOLUTION

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B = 0 \quad R_A = -R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_0 - M_A + R_B L = 0 \quad M_A = R_B L + M_0$$

$0 \le x \le a$:



$$M = M_A + R_A x$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$a \le x \le L$:

$$M = M_A + R_A x - M_0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - M_0$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - M_0 x + C_3$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{2}M_0 x^2 + C_3 x + C_4$$

$$\left[x = 0, \ \frac{dy}{dx} = 0\right]: \quad 0 + 0 + C_1 = 0 \qquad \therefore \quad C_1 = 0$$

$$[x = 0, \ y = 0]: \quad 0 + 0 + 0 + C_2 = 0 \qquad \therefore \quad C_2 = 0$$

$$\left[x = a, \ \frac{dy}{dx} = \frac{dy}{dx}\right]:$$

$$M_A a + \frac{1}{2}R_A a^2 = M_A a + \frac{1}{2}R_A a^2 - M_0 a + C_3 \qquad \therefore \quad C_3 = M_0 a$$

$$[x = a, \ y = y]:$$

$$\frac{1}{2}M_A a^2 + \frac{1}{6}R_A a^3 = \frac{1}{2}M_A a^2 + \frac{1}{6}R_A a^3 - \frac{1}{2}M_0 a^2 + (M_0 a)(a) + C_4 \qquad \therefore \quad C_4 = -\frac{1}{2}M_0 a^2$$

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PROBLEM 9.32 *(Continued)*

$$[x = L,\ y = 0]:$$

$$\frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{2}M_0 L^2 + (M_0 a)(L) - \frac{1}{2}M_0 a^2 = 0$$

$$\frac{1}{2}(R_B L + M_0)L^2 + \frac{1}{6}(-R_B)L^3 - \frac{1}{2}M_0 L^2 + M_0 aL - \frac{1}{2}M_0 a^2 = 0$$

$$R_B = \frac{3M_0 a}{2L^3}(a - 2L) = \frac{3M_0}{2L^3}\left(\frac{L}{3}\right)\left(\frac{L}{3} - 2L\right) = -\frac{5M_0}{6L} \qquad R_B = \frac{5M_0}{6L} \downarrow \blacktriangleleft$$

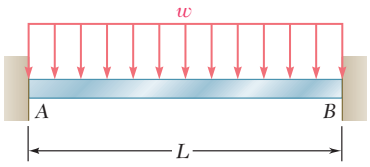
Deflection at *D*. $\left(y \text{ at } x = a = \dfrac{L}{3}\right)$

$$y_D = \frac{1}{EI}\left\{\frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3\right\}$$

$$= \frac{1}{EI}\left\{\frac{1}{2}\left(-\frac{5M_0}{6L}L + M_0\right)\left(\frac{L}{3}\right)^2 + \frac{1}{6}\left(+\frac{5M_0}{6L}\right)\left(\frac{L}{3}\right)^3\right\}$$

$$= -\frac{7M_0 L^2}{486EI} \qquad y_D = \frac{7M_0 L^2}{486EI} \uparrow \blacktriangleleft$$

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$[x = 0,\ y = 0]$ $\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

$[x = L,\ y = 0]$ $\left[x = L,\ \dfrac{dy}{dx} = 0\right]$

PROBLEM 9.33

Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

SOLUTION

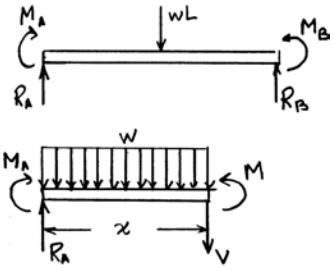
Reactions are statically indeterminate.

By symmetry, $R_B = R_A; \quad M_B = M_A$

$$\frac{dy}{dx} = 0 \quad \text{at} \quad x = \frac{L}{2}$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wL = 0 \qquad R_B = R_A = \frac{1}{2}wL \uparrow \ \blacktriangleleft$$

Over entire beam, $M = M_A + R_A x - \dfrac{1}{2}wx^2$



$$EI\frac{d^2y}{dx^2} = M_A + \frac{1}{2}wLx - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{4}wLx^2 - \frac{1}{6}wx^3 + C_1$$

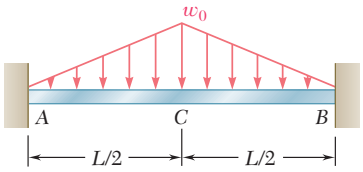
$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \quad 0 + 0 - 0 + C_1 = 0 \qquad C_1 = 0$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right] \qquad \frac{1}{2}M_A L + \frac{1}{16}wL^3 - \frac{1}{48}wL^3 + 0 = 0$$

$$M_A = -\frac{1}{12}wL^2 \ \blacktriangleleft$$

$$M = -\frac{1}{12}wL^2 + \frac{1}{2}wLx - \frac{1}{2}wx^2 \qquad M = w[6x(L - x) - L^2]/12 \ \blacktriangleleft$$

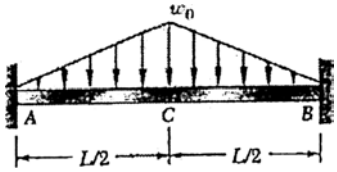
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PROBLEM 9.34

Determine the reaction at *A* and draw the bending moment diagram for the beam and loading shown.

SOLUTION



Reactions are statically indeterminate.

Because of symmetry, $\frac{dy}{dx} = 0$ and $V = 0$ at $x = \frac{L}{2}$.

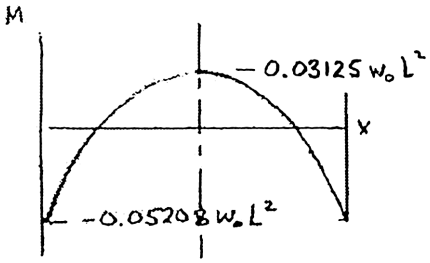
$[x = 0,\ y = 0]$ | $[x = L,\ y = 0]$

$\left[x = 0,\ \frac{dy}{dx} = 0\right]$ | $\left[x = L,\ \frac{dy}{dx} = 0\right]$

$\left[x = \frac{L}{2},\ V = 0\right]$

$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right]$

Use portion *AC* of beam. $\left(0 < x \le \frac{L}{2}\right)$



$$\frac{dV}{dx} = -w = -2\frac{w_0}{L}x$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L}x^2 + R_A \qquad (1)$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{3}\frac{w_0}{L}x^3 + R_Ax + M_A \qquad (2)$$

$$EI\frac{dy}{dx} = -\frac{1}{12}\frac{w_0}{L}x^4 + \frac{1}{2}R_Ax^2 + M_Ax + C_1 \qquad (3)$$

$$EIy = -\frac{1}{60}\frac{w_0}{L}x^2 + \frac{1}{6}R_Ax^3 + \frac{1}{2}M_Ax^2 + C_1x + C_2 \qquad (4)$$

$\left[x = 0,\ \frac{dy}{dx} = 0\right]$: $\quad 0 = 0 + 0 + 0 + C_1 \quad C_1 = 0$

$[x = 0,\ y = 0]$: $\quad 0 = 0 + 0 + 0 + 0 + C_2 \quad C_2 = 0$

$\left[x = \frac{L}{2},\ V = 0\right]$: $\quad -\frac{w_0}{L}\left(\frac{L}{2}\right)^2 + R_A = 0 \quad R_A = \frac{wL}{4}$ ◀

$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right]$: $\quad -\frac{1}{12}\frac{w_0}{L}\left(\frac{L}{2}\right)^4 + \frac{1}{2}\left(\frac{1}{4}w_0L\right)\left(\frac{L}{2}\right)^2 + M_A\frac{L}{2} + 0 = 0$

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PROBLEM 9.34 ***(Continued)***

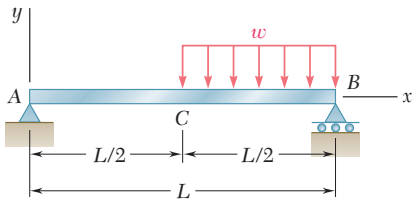
$$M_A = -2\left(\frac{1}{32} - \frac{1}{192}\right)w_0L^2 = -\frac{5}{96}w_0L^2 \qquad M_A = -0.0521w_0L^2 \blacktriangleleft$$

From (2), with $x = \dfrac{L}{2}$,

$$M_C = -\frac{1}{3}\frac{w_0}{L}\left(\frac{L}{2}\right)^3 + \left(\frac{1}{4}w_0L\right)\left(\frac{L}{2}\right) - \frac{5}{96}w_0L^{12}$$

$$= \left(-\frac{1}{24} + \frac{1}{8} - \frac{5}{96}\right)w_0L^2 = \frac{1}{32}w_0L^2 \qquad M_C = 0.0313w_0L^2 \blacktriangleleft$$

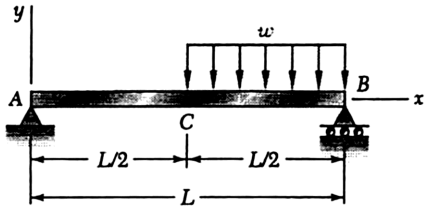
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PROBLEM 9.35

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.

SOLUTION



$$\frac{dV}{dx} = -w\left\langle x - \frac{L}{2}\right\rangle^0$$

$$\frac{dM}{dx} = V = R_A - w\left\langle x - \frac{L}{2}\right\rangle^1$$

$$M = \cancel{M_A} + R_A X - \frac{1}{2}w\left\langle x - \frac{L}{2}\right\rangle^2$$

$[x = 0, M = 0]$ $[x = L, M = 0]$ $\quad [x = L, M = 0] \quad R_A L - \frac{1}{2}w\left(\frac{L}{2}\right)^2 = 0$

$[x = 0, y = 0]$ $[x = L, y = 0]$ $\quad R_A = \frac{1}{8}wL$

$$EI\frac{d^2y}{dx^2} = \frac{1}{8}wLx - \frac{1}{2}w\left\langle x - \frac{L}{2}\right\rangle^2$$

$$EI\frac{dy}{dx} = \frac{1}{16}wLx^2 - \frac{1}{6}w\left\langle x - \frac{L}{2}\right\rangle^3 + C_1$$

$$EIy = \frac{1}{48}wLx^3 - \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4 + C_1x + C_2$$

$[x = 0, y = 0] \quad 0 = 0 + 0 + 0 + C_2 \quad C_2 = 0$

$[x = L, y = 0] \quad \frac{1}{48}wL^4 - \frac{1}{24}w\left(\frac{L}{2}\right)^4 + C_1L + 0 = 0$

$$C_1 = -\left(\frac{1}{48} - \frac{1}{24}\cdot\frac{1}{16}\right)wL^3 = -\frac{7}{384}wL^3$$

(*a*) Elastic curve.

$$EIy = \frac{1}{48}wLx^3 - \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4 - \frac{7}{384}wL^3x$$

$$y = \frac{w}{EI}\left\{\frac{1}{48}Lx^3 - \frac{1}{24}\left\langle x - \frac{L}{2}\right\rangle^4 - \frac{7}{384}L^3x\right\}$$ ◀

$$\frac{dy}{dx} = \frac{w}{EI}\left\{\frac{1}{16}Lx^2 - \frac{1}{6}\left\langle x - \frac{L}{2}\right\rangle^3 - \frac{7}{384}L^3\right\}$$

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PROBLEM 9.35 *(Continued)*

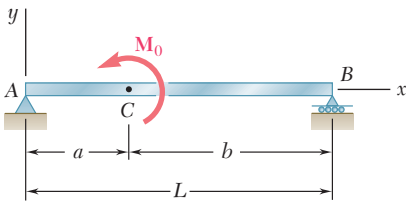
(*b*) Slope at *A*. ($x = 0$ in slope equation)

$$\theta_A = -\frac{7}{384}\frac{wL^3}{EI}$$ ◀

(*c*) Deflection at *C*. $\left(x = \frac{L}{2} \text{ in deflection equation}\right)$

$$y_C = \frac{wL^4}{EI}\left\{\frac{1}{48}\cdot\frac{1}{8} - \frac{7}{384}\cdot\frac{1}{2}\right\} = \left(\frac{1}{384} - \frac{7}{768}\right)\frac{wL^4}{EI} = -\frac{5}{768}\frac{wL^4}{EI}$$ ◀

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[$x = 0,\ y = 0$] [$x = L,\ y = 0$]

PROBLEM 9.36

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.

SOLUTION

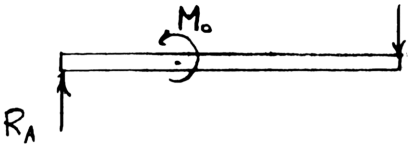
Reactions:

$$R_A = \frac{M_0}{L} \uparrow, \qquad R_B = \frac{M_0}{L} \downarrow$$

$$0 < x < a \qquad M = R_A x$$

$$a < x < L \qquad M = R_A x - M_0$$

Using singularity functions,



$$EI\frac{d^2y}{dx^2} = M = R_A x - M_0\langle x - a\rangle^0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0\langle x - a\rangle^1 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0\langle x - a\rangle^2 + C_1 x + C_2$$

[$x = 0,\ y = 0$] $\quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

[$x = L,\ y = 0$] $\quad \frac{1}{6}R_A L^3 - \frac{1}{2}M_0(L - a)^2 + C_1 L + 0 = 0$

$$C_1 L = -\frac{1}{6}\frac{M_0}{L}L^3 + \frac{1}{2}M_0 b^2$$

$$C_1 = \frac{M_0}{6L}(3b^2 - L^2)$$

(*a*) Elastic curve. $\quad y = \frac{1}{EI}\left\{\frac{1}{6}\frac{M_0}{L}x^3 - \frac{1}{2}M_0\langle x - a\rangle^2 + \frac{M_0}{6L}(3b^2 - L^2)x\right\}$

$$y = \frac{M_0}{6EIL}\left\{x^3 - 3L\langle x - a\rangle^2 + (3b^2 - L^2)x\right\} \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{M_0}{6EIL}\left\{3x^2 - 6L\langle x - a\rangle^1 + (3b^2 - L^2)\right\}$$

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PROBLEM 9.36 ***(Continued)***

(*b*) Slope at *A*. $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

$$\theta_A = \frac{M_0}{6EIL}\{0 - 0 + 3Lb^2 - L^3\} \qquad \theta_A = \frac{M_0}{6EIL}(3b^2 - L^2) \curvearrowright \blacktriangleleft$$

(*c*) Deflection at *C*. (y at $x = a$)

$$y_C = \frac{M_0}{6EIL}\{a^3 - 0 + (3b^2 - L^2)a\}$$

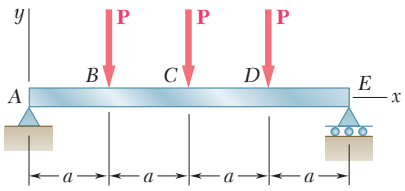
$$= \frac{M_0 a}{6EIL}\{a^2 + 3b^2 - (a + b)^2\}$$

$$= \frac{M_0 a}{6EIL}\{\cancel{a^2} + 3b^2 - \cancel{a^2} - 2ab - b^2\}$$

$$= \frac{M_0 a}{6EIL}\{2b^2 - 2ab\}$$

$$y_C = \frac{M_0 ab}{3EIL}(b - a) \uparrow \blacktriangleleft$$

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PROBLEM 9.37

For the beam and loading shown, determine the deflection at (*a*) point *B*, (*b*) point *C*, (*c*) point *D*.

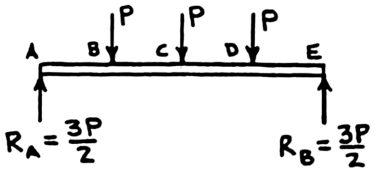
SOLUTION

$$\frac{dM}{dx} = V = \frac{3P}{2} - P\langle x-a\rangle^0 - P\langle x-2a\rangle^0 - P\langle x-3a\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = \frac{3P}{2}x - P\langle x-a\rangle^1 - P\langle x-2a\rangle^1 - P\langle x-3a\rangle^1$$

$$EI\frac{dy}{dx} = \frac{3P}{4}x^2 - \frac{1}{2}P\langle x-a\rangle^2 - \frac{1}{2}P\langle x-2a\rangle^2 - \frac{1}{2}P\langle x-3a\rangle^2 + C_1$$

$$EIy = \frac{P}{4}x^3 - \frac{1}{6}P\langle x-a\rangle^3 - \frac{1}{6}P\langle x-2a\rangle^3 - \frac{1}{6}P\langle x-3a\rangle^3 + C_1x + C_2$$



$[x = 0, y = 0]$: $\quad 0 - 0 - 0 - 0 + 0 + C_2 = 0 \qquad \therefore \quad C_2 = 0$

$[x = 4a, y = 0]$: $\quad 16Pa^3 - \frac{9}{2}Pa^3 - \frac{4}{3}Pa^3 - \frac{1}{6}Pa^3 + 4aC_1 = 0 \qquad \therefore \quad C_1 = -\frac{5}{2}Pa^2$

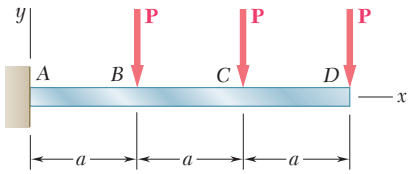
Elastic curve. $\quad y = \frac{P}{12EI}[3x^3 - 2\langle x-a\rangle^3 - 2\langle x-2a\rangle^3 - 2\langle x-3a\rangle^3 - 30a^2x]$

(*a*) $\quad x = a$: $\quad y_B = \frac{Pa^3}{12EI}[3 - 0 - 0 - 0 - 30] \qquad y_B = \frac{9Pa^3}{4EI}\downarrow$ ◀

(*b*) $\quad x = 2a$: $\quad y_C = \frac{Pa^3}{12EI}[24 - 2 - 0 - 0 - 60] \qquad y_C = \frac{19Pa^3}{6EI}\downarrow$ ◀

(c) $\quad x = 3a$: $\quad y_D = \frac{Pa^3}{12EI}[81 - 16 - 2 - 0 - 90] \qquad y_D = \frac{9Pa^3}{4EI}\downarrow$ ◀

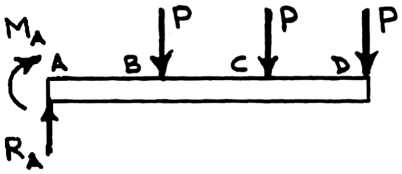
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PROBLEM 9.38

For the beam and loading shown, determine the deflection at (*a*) point *B*, (*b*) point *C*, (*c*) point *D*.

SOLUTION



$$+\uparrow \Sigma F_y = 0: \quad R_A - P - P - P = 0 \qquad R_A = 3P$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - Pa - P(2a) - P(3a) = 0 \qquad M_A = -6Pa$$

$$\frac{dM}{dx} = V = 3P - P\langle x-a\rangle^0 - P\langle x-2a\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = 3Px - P\langle x-a\rangle^1 - P\langle x-2a\rangle^1 - 6Pa$$

$$EI\frac{dy}{dx} = \frac{3}{2}Px^2 - \frac{1}{2}P\langle x-a\rangle^2 - \frac{1}{2}P\langle x-2a\rangle^2 - 6Pax + C_1$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right]: \quad 0 - 0 - 0 - 0 + C_1 = 0 \qquad \therefore \quad C_1 = 0$$

$$EIy = \frac{1}{2}Px^3 - \frac{1}{6}P\langle x-a\rangle^3 - \frac{1}{6}P\langle x-2a\rangle^3 - 3Pax^2 + C_2$$

$$[x = 0,\ y = 0]: \quad 0 - 0 - 0 - 0 + C_2 = 0 \qquad \therefore \quad C_2 = 0$$

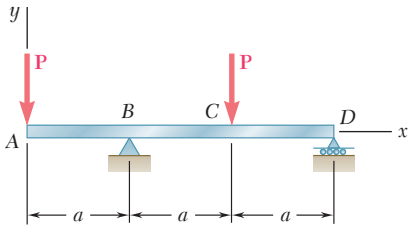
Elastic curve. $y = \dfrac{P}{6EI}[3x^3 - \langle x-a\rangle^3 - \langle x-2a\rangle^3 - 18ax^2]$

(*a*) $x = a$: $y_B = \dfrac{Pa^3}{6EI}[3 - 0 - 0 - 18]$ $\qquad y_B = \dfrac{5Pa^3}{2EI} \downarrow$ ◀

(*b*) $x = 2a$: $y_C = \dfrac{Pa^3}{6EI}[24 - 1 - 0 - 72]$ $\qquad y_C = \dfrac{49Pa^3}{6EI} \downarrow$ ◀

(*c*) $x = 3a$: $y_D = \dfrac{Pa^3}{6EI}[81 - 8 - 1 - 162]$ $\qquad y_D = \dfrac{15Pa^3}{4EI} \downarrow$ ◀

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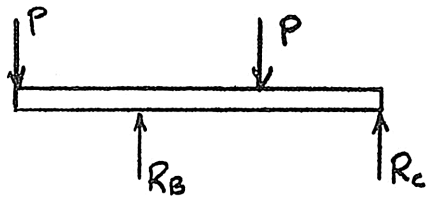


$[x = a,\ y = 0]$ $[x = 3a,\ y = 0]$

PROBLEM 9.39

For the beam and loading shown, determine (*a*) the deflection at end *A*, (*b*) the deflection at point *C*, (*c*) the slope at end *D*.

SOLUTION



Reactions: $R_B = 2P \uparrow, \quad R_D = 0$

$(0 < x < a)$: $V = -P$

$(a < x < 2a)$: $V = -P + 2P$

$(2a < x < 3a)$: $V = -P + 2P - P$

Using singularity functions,

$$\frac{dM}{dx} = V = -P + 2P\langle x - a\rangle^0 - P\langle x - 2a\rangle^0$$

$$M = -Px + 2P\langle x - a\rangle^1 - P\langle x - 2a\rangle^1 + M_A$$

But $M = 0$ at $x = 0$ $\quad M_A = 0$

$$EI\frac{d^2y}{dx^2} = M = -Px + 2P\langle x - a\rangle^1 - P\langle x - 2a\rangle^1 \tag{1}$$

$$EI\frac{dy}{dx} = -\frac{1}{2}Px^2 + P\langle x - a\rangle^2 - \frac{1}{2}P\langle x - 2a\rangle^2 + C_1 \tag{2}$$

$$EIy = -\frac{1}{6}Px^3 + \frac{1}{3}P\langle x - a\rangle^3 - \frac{1}{6}P\langle x - 2a\rangle^3 + C_1x + C_2 \tag{3}$$

$[x = a,\ y = 0]$ $\quad -\frac{1}{6}Pa^3 + 0 - 0 + C_1a + C_2 = 0 \quad aC_1 + C_2 = \frac{1}{6}Pa^3$ (4)

$[x = 3a,\ y = 0]$ $\quad -\frac{1}{6}P(3a)^3 + \frac{1}{3}P(2a)^3 - \frac{1}{6}Pa^3 + C_1(3a) + C_2 = 0 \quad 3aC_1 + C_2 = 2Pa^2$ (5)

Eq (5) – Eq (4) $\quad 2C_1a + \frac{11}{6}Pa^2 \quad C_1 + \frac{11}{12}Pa^2$

$$C_2 = \frac{1}{6}Pa^2 - aC_1 = -\frac{3}{4}Pa^3$$

$$y = \frac{P}{EI}\left\{-\frac{1}{6}x^3 + \frac{1}{3}\langle x - a\rangle^3 - \frac{1}{6}\langle x - 2a\rangle^3 + \frac{11}{12}a^2x - \frac{3}{4}a^3\right\}$$

$$\frac{dy}{dx} = \frac{P}{EI}\left\{-\frac{1}{2}x^2 + \langle x - a\rangle^2 - \frac{1}{2}\langle x - 2a\rangle^2 + \frac{11}{12}a^2\right\}$$

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PROBLEM 9.39 *(Continued)*

(*a*) Deflection at *A*. (y at $x = 0$)

$$y_A = \frac{Pa^3}{EI}\left\{-0 + 0 - 0 + 0 - \frac{3}{4}\right\} = -\frac{3}{4}\frac{Pa^3}{EI} \qquad y_A = \frac{3}{4}\frac{Pa^3}{EI} \downarrow \blacktriangleleft$$

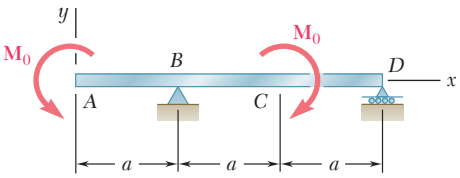
(*b*) Deflection at *C*. (y at $x = 2a$)

$$y_C = \frac{Pa^3}{EI}\left\{-\frac{1}{6}(2)^3 + \frac{1}{3}(1)^3 - 0 + \frac{11}{12}(2) - \frac{3}{4}\right\} \qquad y_C = \frac{1}{12}\frac{Pa^3}{EI} \uparrow \blacktriangleleft$$

(*c*) Slope at D. $\left(\frac{dy}{dx} \text{ at } x = 3a\right)$

$$\theta_D = \frac{Pa^2}{EI}\left\{-\frac{1}{2}(3)^2 + (2)^2 - \frac{1}{2}(1)^2 + \frac{11}{12}\right\} = -\frac{1}{12}\frac{Pa^2}{EI} \qquad \theta_D = \frac{1}{12}\frac{Pa^2}{EI} \blacktriangleleft$$

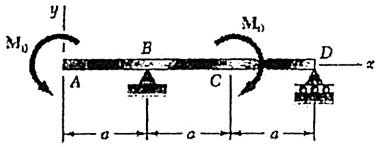
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PROBLEM 9.40

For the beam and loading shown, determine (*a*) the deflection end *A*, (*b*) the deflection at point *C*, (*c*) the slope at end *D*.

SOLUTION



$[x = a,\ y = 0]$ $[x = 3a,\ y = 0]$

Since loads self-equilibrate,

$$R_B = 0, \qquad R_D = 0$$

$$(0 < x < 2a): \quad M = -M_0$$

$$(2a < x < 3a): \quad M = -M_0 + M_0 = 0$$

Using singularity functions,

$$EI\frac{d^2y}{dx^2} = M = -M_0 + M_0\langle x - 2a\rangle^0$$

$$EI\frac{dy}{dx} = -M_0x + M_0\langle x - 2a\rangle^1 + C_1$$

$$EIy = -\frac{1}{2}M_0x^2 + \frac{1}{2}M_0\langle x - 2a\rangle^2 + C_1x + C_2$$

$[x = 3a,\ y = 0]$ $\quad -\frac{1}{2}M_0(3a)^2 + \frac{1}{2}M_0a^2 + C_1(3a) + C_2 = 0 \quad 3aC_1 + C_2 = 4M_0a^2$

$[x = a,\ y = 0]$ $\quad -\frac{1}{2}M_0a^2 + 0 + C_1a + C_2 = 0 \quad aC_1 + C_2 = \frac{1}{2}M_0a^2$

Subtracting, $\quad 2aC_1 = \frac{7}{2}M_0a^2 \quad C_1 = \frac{7}{4}M_0a$

$$C_2 = \frac{1}{2}M_0a^2 - aC_1 = -\frac{5}{4}M_0a^2$$

$$y = \frac{M_0}{EI}\left\{-\frac{1}{2}x^2 + \frac{1}{2}\langle x - 2a\rangle^2 + \frac{7}{4}ax - \frac{5}{4}a^2\right\}$$

$$\frac{dy}{dx} = \frac{M_0}{EI}\left\{-x + \langle x - a\rangle^1 + \frac{7}{4}a\right\}$$

(*a*) Deflection at *A*. $\quad (y \text{ at } x = 0)$

$$y_A = \frac{M_0a^2}{EI}\left\{-0 + 0 + 0 - \frac{5}{4}\right\} = -\frac{5}{4}\frac{M_0a^2}{EI}, \qquad y_A = \frac{5}{4}\frac{M_0a^2}{EI}\downarrow \blacktriangleleft$$

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PROBLEM 9.40 *(Continued)*

(*b*) Deflection at *C*. (y at $x = 2a$)

$$y_C = \frac{M_0 a^2}{EI}\left\{-\frac{1}{2}(2)^2 + 0 + \frac{7}{4}(2) - \frac{5}{4}\right\} = \frac{1}{4}\frac{M_0 a^2}{EI}$$

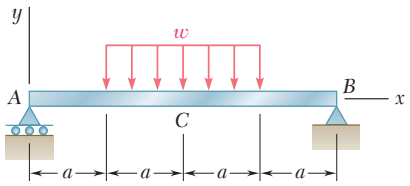
$$y_C = \frac{1}{4}\frac{M_0 a^2}{EI} \uparrow \blacktriangleleft$$

(*c*) Slope at *D*. $\left(\frac{dy}{dx} \text{ at } x = 3a\right)$

$$\theta_D = \frac{M_0 a}{EI}\left\{-3 + 1 + \frac{7}{4}\right\} = -\frac{1}{4}\frac{M_0 a}{EI},$$

$$\theta_D = \frac{1}{4}\frac{M_0 a}{EI} \blacktriangleleft$$

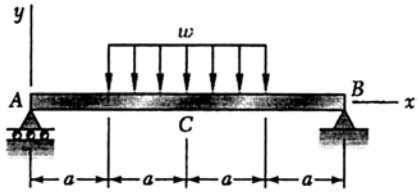
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PROBLEM 9.41

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the midpoint *C*.

SOLUTION



By symmetry,

$$R_A = R_B$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - 2wa = 0 \qquad R_A = wa$$

$$w(x) = w\langle x-a\rangle^0 - w\langle x-3a\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w\langle x-a\rangle^0 + w\langle x-3a\rangle^0$$

$$\frac{dM}{dx} = V = R_A - w\langle x-a\rangle^1 + w\langle x-3a\rangle^1$$

$$M = M_A + R_A x - \frac{1}{2}w\langle x-a\rangle^2 + \frac{1}{2}\langle x-3a\rangle^2 \qquad \text{with } M_A = 0$$

$$EI\frac{d^2y}{dx^2} = M = wax - \frac{1}{2}w\langle x-a\rangle^2 + \frac{1}{2}w\langle x-3a\rangle^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}wax^2 - \frac{1}{6}w\langle x-a\rangle^3 + \frac{1}{6}w\langle x-3a\rangle^3 + C_1$$

$$EIy = \frac{1}{6}wax^3 - \frac{1}{24}w\langle x-a\rangle^4 + \frac{1}{24}w\langle x-3a\rangle^4 + C_1x + C_2$$

$[x = 0, y = 0]$: $\quad 0 - 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = 4a, y = 0]$: $\quad \frac{1}{6}wa(4a)^3 - \frac{1}{24}w(3a)^4 + \frac{1}{24}(a)^4 + C_1(4a) = 0$

$$4C_1 = wa^3\left\{-\frac{64}{6} + \frac{81}{24} - \frac{1}{24}\right\} = -\frac{22}{3}wa^3 \qquad C_1 = -\frac{11}{6}wa^3$$

(*a*) Equation of elastic curve.

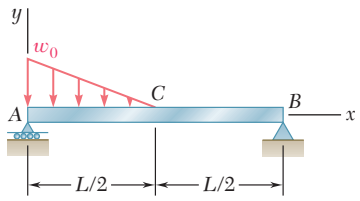
$$y = \frac{w}{EI}\left\{\frac{1}{6}ax^3 - \frac{1}{24}\langle x-a\rangle^4 + \frac{1}{24}\langle x-3a\rangle^4 - \frac{11}{6}a^3x\right\} \blacktriangleleft$$

(*b*) Deflection at *C*. (y at $x = 2a$)

$$y_C = \frac{wa^4}{EI}\left\{\frac{1}{6}(2)^3 - \frac{1}{24}(1)^4 + 0 - \frac{11}{6}(2)\right\} = -\frac{19}{8}\frac{wa^4}{EI}$$

$$y_C = \frac{19}{8}\frac{wa^4}{EI}\downarrow \blacktriangleleft$$

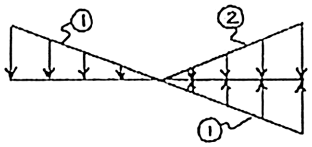
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PROBLEM 9.42

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the midpoint *C*.

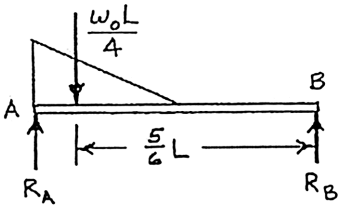
SOLUTION



Distributed loads: $k = \dfrac{2w_0}{L}$

(1) $w_1(x) = w_0 - kx$ (2) $w_2(x) = k\left\langle x - \dfrac{L}{2}\right\rangle^1$

$+\circlearrowleft \Sigma M_B = 0$: $\left(\dfrac{w_0 L}{4}\right)\left(\dfrac{5}{6}L\right) - R_A L = 0$ $R_A = \dfrac{5}{24}w_0 L \uparrow$



$$w(x) = w_0 - kx + k\left\langle x - \frac{L}{2}\right\rangle^1 = w_0 - \frac{2w_0}{L}x + \frac{2w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^1$$

$$\frac{dV}{dx} = -w = -w_0 + \frac{2w_0}{L}x - \frac{2w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^1$$

$$\frac{dM}{dx} = V = \frac{5}{24}w_0 L - w_0 x + \frac{w_0}{L}x^2 - \frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^2$$

$$EI\frac{d^2y}{dx^2} = M = \frac{5}{24}w_0 L x - \frac{1}{2}w_0 x^2 + \frac{1}{3}\frac{w_0}{L}x^3 - \frac{1}{3}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^3$$

$$EI\frac{dy}{dx} = \frac{5}{48}w_0 L x^2 - \frac{1}{6}w_0 x^3 + \frac{1}{12}\frac{w_0}{L}x^4 - \frac{1}{12}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^4 + C_1$$

$$EIy = \frac{5}{144}w_0 L x^3 - \frac{1}{24}w_0 x^4 + \frac{1}{60}\frac{w_0}{L}x^5 - \frac{1}{60}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^5 + C_1 x + C_2$$

$[x = 0,\ y = 0]$: $C_2 = 0$

$[x = L,\ y = 0]$: $\dfrac{5}{144}w_0 L^4 - \dfrac{1}{24}w_0 L^4 + \dfrac{1}{60}w_0 L^4 - \dfrac{1}{60}\dfrac{w_0}{L}\left(\dfrac{L}{2}\right)^5 + C_1 L = 0$ $C_1 = -\dfrac{53}{5760}w_0 L^3$

(*a*) Equation of elastic curve.

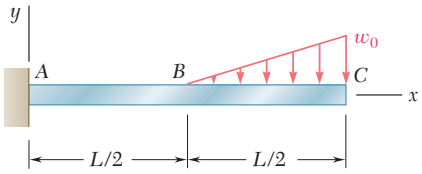
$$y = w_0\left[96x^5 - 96\left\langle x - \frac{L}{2}\right\rangle^5 - 240Lx^4 + 200L^2x^3 - 53L^4x\right] / 5760EIL \blacktriangleleft$$

(*b*) Deflection at *C*. $\left(y \text{ at } x = \dfrac{L}{2}\right)$

$$y_C = \frac{w_0 L^4}{5760EI}\left(\frac{96}{32} - 0 - \frac{240}{16} + \frac{200}{8} - \frac{53}{2}\right) = -\frac{3w_0 L^4}{1280EI}$$

$y_C = \dfrac{3w_0 L^4}{1280EI} \downarrow \blacktriangleleft$

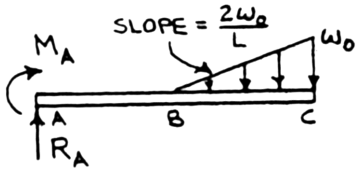
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PROBLEM 9.43

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at point *B*, (*c*) the deflection at point *C*.

SOLUTION



$$+\uparrow \Sigma F_Y = 0: \quad R_A - \frac{1}{2}(w_0)\left(\frac{L}{2}\right) = 0 \qquad R_A = \frac{1}{4}w_0 L$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - \frac{1}{2}(w_0)\left(\frac{L}{2}\right)\left(\frac{5L}{6}\right) = 0 \qquad M_A = -\frac{5}{24}w_0 L^2$$

Using singularity functions,

$$w = \frac{2w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^1$$

$$\frac{dV}{dx} = -w = -\frac{2w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^1$$

$$\frac{dM}{dx} = V = \frac{1}{4}w_0 L - \frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^2$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{5}{24}w_0 L^2 + \frac{1}{4}w_0 Lx - \frac{1}{3}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^3$$

$$EI\frac{dy}{dx} = -\frac{5}{24}w_0 L^2 X + \frac{1}{8}w_0 Lx^2 - \frac{1}{12}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^4 + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right]: \quad -0 + 0 - 0 + C_1 = 0 \qquad \therefore \quad C_1 = 0$$

$$EIy = -\frac{5}{48}w_0 L^2 x^2 + \frac{1}{24}w_0 Lx^3 - \frac{1}{60}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^5 + C_2$$

$$[x = 0, y = 0]: \quad -0 + 0 - 0 + C_2 = 0 \qquad \therefore \quad C_2 = 0$$

(*a*) Elastic curve.

$$y = \frac{w_0}{EIL}\left[-\frac{5}{48}L^3x^2 + \frac{1}{24}L^2x^3 - \frac{1}{60}\left\langle x - \frac{L}{2}\right\rangle^5\right] \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{w_0}{EIL}\left[-\frac{5}{24}L^3x + \frac{1}{8}L^2x^2 - \frac{1}{12}\left\langle x - \frac{L}{2}\right\rangle^4\right]$$

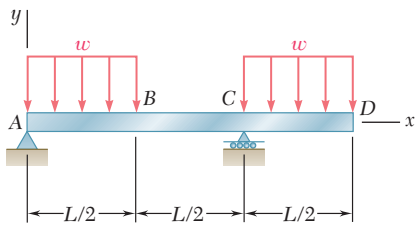
(*b*) Deflection at *B*. $\left(y \text{ at } x = \frac{L}{2}\right)$

$$y_B = \frac{w_0 L^4}{EI}\left[-\frac{5}{48}\left(\frac{1}{2}\right)^2 + \frac{1}{24}\left(\frac{1}{2}\right)^3 - 0\right] = -\frac{4}{192}\frac{w_0 L^4}{EI} \qquad y_B = \frac{1}{48}\frac{w_0 L^4}{EI}\downarrow \blacktriangleleft$$

(*c*) Deflection at *C*. $(y \text{ at } x = L)$

$$y_C = \frac{w_0 L^4}{EI}\left[-\frac{5}{48}(1)^2 + \frac{1}{24}(1)^3 - \frac{1}{60}\left(\frac{1}{2}\right)^5\right] = -\frac{121}{1920}\frac{w_0 L^4}{EI} \qquad y_C = \frac{121}{1920}\frac{w_0 L^4}{EI}\downarrow \blacktriangleleft$$

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PROBLEM 9.44

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at point *B*, (*c*) the deflection at point *D*.

SOLUTION

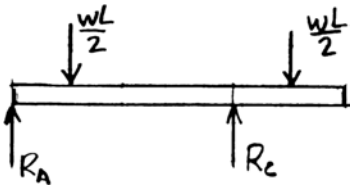
Use free body *ABCD* with the distributed loads replaced by equivalent concentrated loads.

$$+\circlearrowleft \Sigma M_C = 0: \quad -R_A L + \left(\frac{wL}{2}\right)\left(\frac{3L}{4}\right) - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) = 0$$

$$R_A = \frac{1}{4}wL$$

$$+\circlearrowleft \Sigma M_A = 0: \quad R_C L - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) - \left(\frac{wL}{2}\right)\left(\frac{5L}{4}\right) = 0$$

$$R_C = \frac{3}{4}wL$$



$$\frac{dV}{dx} = -w = -w + w\left\langle x - \frac{L}{2}\right\rangle^0 - w\langle x - L\rangle^0$$

Integrating and adding terms to account for the reactions,

$$\frac{dM}{dx} = V = -wx + w\left\langle x - \frac{L}{2}\right\rangle^1 - w\langle x - L\rangle^1 + R_A + R_C\langle x - L\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2 + \frac{1}{2}w\left\langle x - \frac{L}{2}\right\rangle^2 - \frac{1}{2}w\langle x - L\rangle^2 + R_A x + R_C\langle x - L\rangle^1$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}w\left\langle x - \frac{L}{2}\right\rangle^3 - \frac{1}{6}w\langle x - L\rangle^3 + \frac{1}{2}R_A x^2 + \frac{1}{2}R_C\langle x - L\rangle^2 + C_1$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4 - \frac{1}{24}w\langle x - L\rangle^4 + \frac{1}{6}R_A x^3 + \frac{1}{6}R_C\langle x - L\rangle^3 + C_1 x + C_2$$

$[x = 0,\ y = 0] \quad -0 + 0 - 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$$[x = L,\ y = 0] \quad -\frac{1}{24}wL^4 + \frac{1}{24}w\left(\frac{L}{2}\right)^4$$

$$-0 + \frac{1}{6}\left(\frac{wL}{4}\right)L^3 + 0 + C_1 L + 0 = 0 \qquad C_1 = -\frac{1}{384}wL^3$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4 + \frac{1}{24}w\langle x - L\rangle^4 + \frac{1}{6}\left(\frac{wL}{4}\right)x^3 + \frac{1}{6}\left(\frac{3wL}{4}\right)\langle x - L\rangle^3 - \frac{1}{384}wL^3 x$$

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PROBLEM 9.44 *(Continued)*

(*a*) Elastic curve.

$$y = \frac{w}{24EI}\left\{-x^4 + \left\langle x - \frac{L}{2}\right\rangle^4 - \langle x - L\rangle^4 + Lx^3 + 3L\langle x - L\rangle^3 - \frac{1}{16}L^3x\right\} \blacktriangleleft$$

(*b*) Deflection at *B*. $\left(y \text{ at } x = \frac{L}{2}\right)$

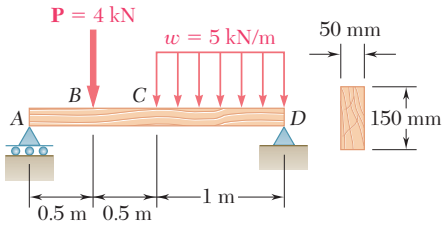
$$y_B = \frac{w}{24EI}\left\{-\left(\frac{L}{2}\right)^4 + 0 - 0 + (L)\left(\frac{L}{2}\right)^3 + 0 - \left(\frac{1}{16}L^3\right)\left(\frac{L}{2}\right)\right\} \qquad y_B = \frac{wL^4}{768EI} \uparrow \blacktriangleleft$$

(*c*) Deflection at *D*. $\left(y \text{ at } x = \frac{3L}{2}\right)$

$$y_D = \frac{w}{24EI}\left\{-\left(\frac{3L}{2}\right)^4 + L^4 - \left(\frac{L}{2}\right)^4 + (L)\left(\frac{3L}{2}\right)^3 + (3L)\left(\frac{L}{2}\right)^3 - \left(\frac{1}{16}L\right)\left(\frac{3L}{2}\right)\right\}$$

$$= -\frac{5wL^4}{256EI} \qquad y_D = \frac{5wL^4}{256EI} \downarrow \blacktriangleleft$$

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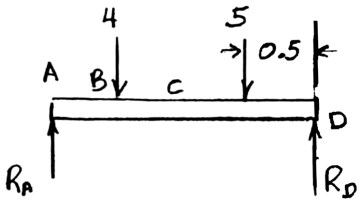


PROBLEM 9.45

For the timber beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 12$ GPa.

SOLUTION

Units: Forces in kN, lengths in meters.



$$I = \frac{1}{12}(50)(150)^3$$
$$= 14.0625 \times 10^6 \text{ mm}^4$$
$$= 14.0625 \times 10^{-6} \text{ m}^4$$
$$EI = (12 \times 10^9)(14.0625 \times 10^{-6})$$
$$= 168.75 \times 10^3 \text{ N} \cdot \text{m}^2$$
$$= 168.75 \text{ kN} \cdot \text{m}^2$$
$$+\circlearrowleft M_D = 0: \quad -2R_A + (1.5)(4) + (0.5)(5) = 0$$
$$R_A = 4.25 \text{ kN}$$

$$w(x) = 5\langle x-1\rangle^0 \quad \text{kN} \cdot \text{m} \qquad \text{kNm}$$
$$\frac{dV}{dx} = -w = -5\langle x-1\rangle^0 \qquad \text{kN/m}$$
$$\frac{dM}{dx} = V = -5\langle x-1\rangle^1 + 4.25 - 4\langle x-0.5\rangle^0 \qquad \text{kN}$$
$$EI\frac{d^2y}{dx^2} = M = -\frac{5}{2}\langle x-1\rangle^2 + 4.25x - 4\langle x-0.5\rangle^1 \qquad \text{kN} \cdot \text{m}$$
$$EI\frac{dy}{dx} = -\frac{5}{6}\langle x-1\rangle^3 + 2.125x^2 - 2\langle x-0.5\rangle^2 + C_1 \qquad \text{kN} \cdot \text{m}^2$$
$$EIy = -\frac{5}{24}\langle x-1\rangle^4 + \frac{2.125}{3}x^3 - \frac{2}{3}\langle x-0.5\rangle^3 + C_1x + C_2 \qquad \text{kN} \cdot \text{m}^3$$

$[x = 0, y = 0] \quad -0 + 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$$[x = 2 \text{ m}, y = 0] \quad -\left(\frac{5}{24}\right)(1)^4 + \left(\frac{2.125}{3}\right)(2)^3 - \left(\frac{2}{3}\right)(1.5)^3 + 2C_1 = 0$$

$$C_1 = -1.60417 \text{ kN} \cdot \text{m}^2$$

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PROBLEM 9.45 *(Continued)*

(*a*) Slope at end *A*. $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

$$EI\left(\frac{dy}{dx}\right)_A = -0 + 0 - 0 + C_1$$

$$\left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} = \frac{-1.60417}{168.75} = -9.51 \times 10^{-3} \qquad \theta_A = 9.51 \times 10^{-3}\,\text{rad} \measuredangle \blacktriangleleft$$

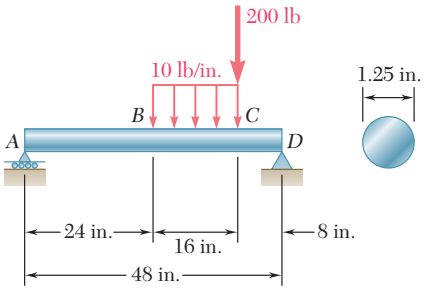
(*b*) Deflection at midpoint *C*. (y at $x = 1\,\text{m}$)

$$EIy_C = -0 + \left(\frac{2.125}{3}\right)(1)^3 - \left(\frac{2}{3}\right)(0.5)^3 + (-1.60417)(1)$$

$$= -979.17 \times 10^{-3}\,\text{kN} \cdot \text{m}^3$$

$$y_C = \frac{-979.17 \times 10^{-3}}{168.75} = -5.80 \times 10^{-3}\,\text{m} \qquad y_C = 5.80\,\text{mm} \downarrow \blacktriangleleft$$

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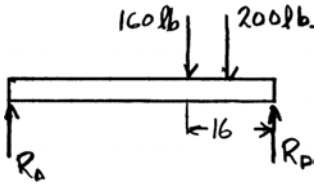
PROBLEM 9.46

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *B*. Use $E = 29 \times 10^6$ psi.

SOLUTION

Units: Forces in lbs; lengths in inches.

$$c = \frac{1}{2}d = \left(\frac{1}{2}\right)(1.25)$$
$$= 0.625 \text{ in.}$$
$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.625)^4$$
$$= 119.84 \times 10^{-3} \text{in}^4$$
$$EI = (29 \times 10^6)(119.84 \times 10^{-3})$$
$$= 3.4754 \times 10^6 \text{ lb} \cdot \text{in}^2$$



Use entire beam *ABCD* as free body.

$$+\circlearrowleft \Sigma M_D = 0: \quad -48R_A + (16)(160) + (8)(200) = 0 \qquad R_A = 86.667 \text{ lb}\uparrow$$

$$w(x) = 10\langle x-24\rangle^0 - 10\langle x-40\rangle^0 \quad \text{lb/in.}$$

$$\frac{dV}{dx} = -w = -10\langle x-24\rangle^0 + 10\langle x-40\rangle^0 \quad \text{lb/in.}$$

$$\frac{dM}{dx} = V = -10\langle x-24\rangle^1 + 10\langle x-40\rangle^1 + 86.667 - 200\langle x-40\rangle^0 \quad \text{lb} \cdot \text{in.}$$

$$EI\frac{d^2y}{dx^2} = M = -5\langle x-24\rangle^2 + 5\langle x-40\rangle^2 + 86.667x - 200\langle x-40\rangle^1 \quad \text{lb} \cdot \text{in.}$$

$$EI\frac{dy}{dx} = -\frac{5}{3}\langle x-24\rangle^3 + \frac{5}{3}\langle x-40\rangle^3 + 43.333x^2 - 100\langle x-40\rangle^2 + C_1 \quad \text{lb} \cdot \text{in}^2$$

$$EIy = -\frac{5}{12}\langle x-24\rangle^4 + \frac{5}{12}\langle x-40\rangle^4 + 14.4444x^3 - \frac{100}{3}\langle x-40\rangle^3 + C_1x + C_2 \quad \text{lb} \cdot \text{in}^3$$

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PROBLEM 9.46 *(Continued)*

$$[x = 0,\ y = 0] \qquad -0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = 48,\ y = 0] \qquad -\left(\frac{5}{12}\right)(24)^4 + \left(\frac{5}{12}\right)(8)^4 + (14.4444)(48)^3 - \left(\frac{100}{3}\right)(8)^3 + 48C_1 = 0$$

$$C_1 = -30.08 \times 10^3 \text{ lb} \cdot \text{in}^2$$

(*a*) Slope at end *A*. $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

$$EI\left(\frac{dy}{dx}\right)_A = -0 + 0 + 0 + C_1$$

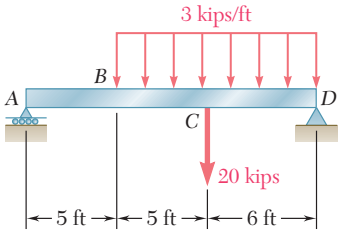
$$\left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} = \frac{-30.08 \times 10^3}{3.4754 \times 10^6} = -8.66 \times 10^{-3} \qquad \theta_A = 8.66 \times 10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at point *B*. (y at $x = 24$ in.)

$$EIy_B = -0 + 0 + (14.4444)(24)^3 - 0 + (-30.08 \times 10^3)(24)$$
$$= -522.24 \times 10^3 \text{ lb} \cdot \text{in}^3$$

$$y_B = \frac{-522.24 \times 10^3}{3.4754 \times 10^6} = -0.1503 \text{ in.} \qquad y_B = 0.1503 \text{ in.} \downarrow \blacktriangleleft$$

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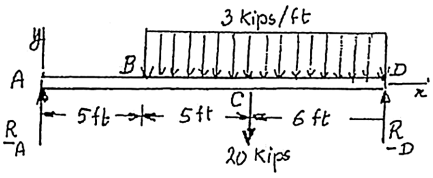




PROBLEM 9.47

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the point *C*. Use $E = 29 \times 10^6$ psi.

SOLUTION



Units: Forces in kips, lengths in feet.

$$E = 29 \times 10^3 \text{ ksi}, \quad I = 758 \text{ in}^4$$

$$EI = (29 \times 10^3)(758) = 21.982 \times 10^6 \text{ kip} \cdot \text{in}^2$$

$$= 152{,}650 \text{ kip} \cdot \text{ft}^2$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (20)(6) + (3 \times 11)(5.5) - R_A(16) = 0 \qquad R_A = 18.844 \text{ kips} \uparrow$$

Express the loading as a singularity function.

$$w(x) = 3\langle x - 5\rangle^0$$

$$V(x) = R_A - \int w\,dx - 20\langle x - 10\rangle^0 = 18.844 - 3\langle x - 5\rangle^1 - 20\langle x - 10\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M(x) = 18.844x - 1.5\langle x - 5\rangle^2 - 20\langle x - 10\rangle^1$$

$$EI\frac{dy}{dx} = 9.422x^2 - 0.5\langle x - 5\rangle^3 - 10\langle x - 10\rangle^2 + C_1$$

$$EIy = 3.141x^3 - 0.125\langle x - 5\rangle^4 - 3.333\langle x - 10\rangle^3 + C_1x + C_2$$

Boundary conditions:

$[x = 0,\ y = 0]$: $C_2 = 0$

$[x = 16,\ y = 0]$: $3.141(16)^3 - (0.125)(11)^4 - (3.3333)(6)^3 + 16C_1 = 0 \quad C_1 = -644.7 \text{ kip} \cdot \text{ft}^2$

$$EI\frac{dy}{dx} = 9.442x^2 - 0.5\langle x - 5\rangle^3 - 10\langle x - 10\rangle^2 - 644.7$$

$$EIy = 3.141x^3 - 0.125\langle x - 5\rangle^4 - 3.333\langle x - 10\rangle^3 - 644.7x$$

(*a*) Slope at *A*. $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

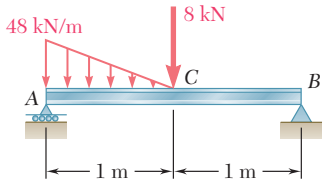
$$152{,}650\theta_A = 0 + 0 + 0 - 644.7 \qquad \theta_A = -4.22 \times 10^{-3} \text{ rad} \qquad \theta_A = 4.22 \times 10^{-3} \text{ rad} \ \searrow \blacktriangleleft$$

(*b*) Deflection at *C*. $(y \text{ at } x = 10 \text{ ft})$

$$152{,}650y_C = (3.141)(10)^3 - (0.125)(5)^4 + 0 - (644.7)(10) = -3384$$

$$y_C = -0.02217 \text{ ft} \qquad y_C = 0.266 \text{ in.} \downarrow \blacktriangleleft$$

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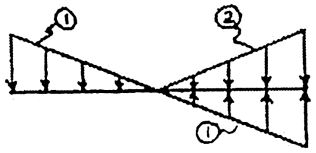




PROBLEM 9.48

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 200$ GPa.

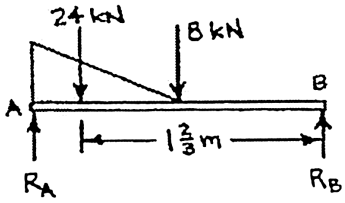
SOLUTION



Distributed loads: (1) $w_1(x) = w_0 - kx$ (2) $w_2 = k\langle x-1\rangle^1$

$$w_0 = 48\ \text{kN/m}, \quad k = 48\ \text{kN/m}^2$$

$$+\circlearrowleft \Sigma M_B = 0: -2R_A + (24)\left(1\frac{2}{3}\right) + (8)(1) = 0 \qquad R_A = 24\ \text{kN} \uparrow$$



$$w(x) = w_0 - kx + k\langle x-1\rangle^1 = 48 - 48x + 48\langle x-1\rangle^1$$

$$\frac{dV}{dx} = -w = -48 + 48x - 48\langle x-1\rangle^1 \quad \text{kN/m}$$

$$\frac{dM}{dx} = V = 24 - 48x + 24x^2 - 24\langle x-1\rangle^2 - 8\langle x-1\rangle^0 \quad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = 24x - 24x^2 + 8x^3 - 8\langle x-1\rangle^3 - 8\langle x-1\rangle^1 \quad \text{kN}\cdot\text{m}$$

$$EI\frac{dy}{dx} = 12x^2 - 8x^3 + 2x^4 - 2\langle x-1\rangle^4 - 4\langle x-1\rangle^2 + C_1 \quad \text{kN}\cdot\text{m}^2$$

$$EIy = 4x^3 - 2x^4 + \frac{2}{5}x^5 - \frac{2}{5}\langle x-1\rangle^5 - \frac{4}{3}\langle x-1\rangle^3 + C_1x + C_2 \quad \text{kN}\cdot\text{m}^3$$

$[x = 0,\ y = 0]$: $0 - 0 + 0 - 0 - 0 + 0 + C_2 = 0 \quad \therefore \quad C_2 = 0$

$[x = 2,\ y = 0]$: $4(2)^3 - 2(2)^4 + \frac{2}{5}(2)^5 - \frac{2}{5}(1)^5 - \frac{4}{3}(1)^3 + C_1(2) = 0 \quad \therefore \quad C_1 = -\frac{83}{15}\ \text{kN}\cdot\text{m}^2$

Data: $E = 200(10^6)\ \text{kN/m}^2 \quad I = 5.12(10^6)\ \text{mm}^4 = 5.12(10^{-6})\ \text{m}^4$

$$EI = (200\times10^6)(5.12\times10^{-6}) = 1024\ \text{kN}\cdot\text{m}^2$$

(*a*) Slope at *A*. $\left(\frac{dy}{dx} \quad \text{at} \quad x = 0\right)$

$$EI\theta_A = 0 - 0 + 0 - 0 - 0 - \frac{83}{15}\ \text{kN}\cdot\text{m}^2$$

$$\theta_A = -\frac{83/15}{1024} = -5.4036\times10^{-3}\ \text{rad} \qquad \theta_A = -5.40\times10^{-3}\ \text{rad} \ \blacktriangleleft$$

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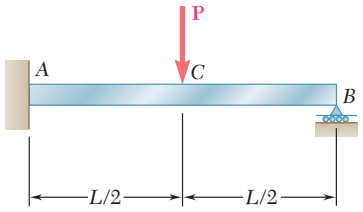
PROBLEM 9.48 *(Continued)*

(*b*) Deflection at *C*. (y at $x = 1\,\text{m}$)

$$EIy_C = 4(1)^3 - 2(1)^4 + \frac{2}{5}(1)^4 - 0 - 0 - \frac{83}{15}(1) = -3.1333\,\text{kN}\cdot\text{m}^3$$

$$y_C = -\frac{3.1333}{1024} = -3.0599 \times 10^{-3}\,\text{m} \qquad y_C = 3.06\,\text{mm}\downarrow \blacktriangleleft$$

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PROBLEM 9.49

For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *C*.

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

SOLUTION

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \qquad R_A = P - R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - P\frac{L}{2} + R_B L = 0 \qquad M_A = R_B L - \frac{1}{2}PL$$

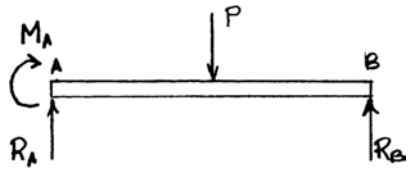
Reactions are statically indeterminate.

$$\frac{dM}{dx} = V = R_A - P\left\langle x - \frac{L}{2}\right\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\left\langle x - \frac{L}{2}\right\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\left\langle x - \frac{L}{2}\right\rangle^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left\langle x - \frac{L}{2}\right\rangle^3 + C_1 x + C_2$$



$\left[x = 0, \dfrac{dy}{dx} = 0\right]$ $\quad 0 + 0 + 0 + C_1 = 0 \qquad C_1 = 0$

$[x = 0, y = 0]$ $\quad 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$$[x = L,\ y = 0] \quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{6}P\left(\frac{L}{2}\right)^3 + 0 + 0 = 0$$

$$\frac{1}{2}\left(R_B L - \frac{1}{2}PL\right)L^2 + \frac{1}{6}(P - R_B)L^3 - \frac{1}{48}PL^3 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_B L^3 = \left(\frac{1}{4} - \frac{1}{6} + \frac{1}{48}\right)PL^3 \qquad \frac{1}{3}R_B = \frac{5}{48}P$$

$$(a)\ R_B = \frac{5}{16}P \uparrow \blacktriangleleft$$

$$R_A = P - \frac{5}{16}P = \frac{11}{16}P$$

$$M_A = \frac{5}{16}PL - \frac{1}{2}PL = -\frac{3}{16}PL$$

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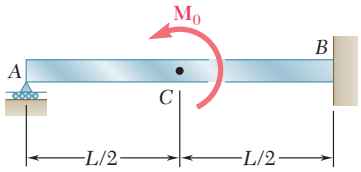
PROBLEM 9.49 *(Continued)*

(*b*) Deflection at *C*. $\left(y \text{ at } x = \frac{L}{2}\right)$

$$y_C = \frac{1}{EI}\left\{\frac{1}{2}M_A\left(\frac{L}{2}\right)^2 + \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 + 0 + 0 + 0\right\}$$

$$= \frac{PL^3}{EI}\left\{\left(\frac{1}{2}\right)\left(-\frac{3}{16}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{6}\right)\left(\frac{11}{16}\right)\left(\frac{1}{8}\right)\right\} = -\frac{7}{168}\frac{PL^3}{EI} \qquad y_C = \frac{7}{168}\frac{PL^3}{EI} \downarrow \blacktriangleleft$$

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PROBLEM 9.50

For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *C*.

$[x = 0,\ y = 0]$ $\left[x = L,\ \dfrac{dy}{dx} = 0\right]$ $[x = L,\ y = 0]$

SOLUTION

For $0 \le x \le \dfrac{1}{2}$, $M = R_A x$

For $\dfrac{L}{2} \le x \le L$, $M = R_A x - M_0$

Then

$$EI\frac{d^2y}{dx^2} = M = R_A x - M_0\left\langle x - \frac{L}{2}\right\rangle^0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0\left\langle x - \frac{L}{2}\right\rangle^1 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0\left\langle x - \frac{L}{2}\right\rangle^2 + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\quad 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$\left[x = L,\ \dfrac{dy}{dx} = 0\right]$ $\quad \dfrac{1}{2}R_A L^2 - M_0\left(\dfrac{L}{2}\right) + C_1 = 0 \qquad C_1 = \dfrac{1}{2}(M_0 L - R_A L^2)$

$[x = L,\ y = 0]$ $\quad \dfrac{1}{6}R_A L^3 - \dfrac{1}{2}M_0\left(\dfrac{L}{2}\right)^2 + \dfrac{1}{2}(M_0 L - R_A L^2)L + 0 = 0$

$$-\frac{1}{3}R_A L^3 + \frac{3}{8}M_0 L^2 = 0$$

(*a*) Reaction at *A*. $\qquad M_A = 0, \qquad R_A = \dfrac{9M_0}{8L} \uparrow$ ◀

$$C_1 = \frac{1}{2}\left[M_0 L - \left(\frac{9M_0}{8L}\right)(L^2)\right] = -\frac{1}{16}M_0 L$$

$$EIy = \frac{1}{6}\left(\frac{9M_0}{8L}\right)x^3 - \frac{1}{2}M_0\left\langle x - \frac{L}{2}\right\rangle^2 - \frac{1}{16}M_0 L x + 0$$

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PROBLEM 9.50 *(Continued)*

Elastic curve.

$$y = \frac{M_0}{EIL}\left\{\frac{9}{8}x^3 - \frac{1}{2}L\left\langle x - \frac{L}{2}\right\rangle^2 - \frac{1}{16}L^2x\right\}$$

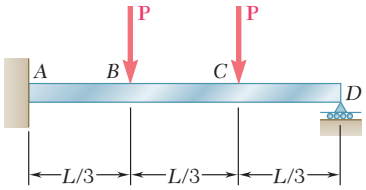
(*b*) Deflection at point *C*. $\left(y \text{ at } x = \frac{L}{2}\right)$

$$y_C = \frac{M_0}{EIL}\left\{\left(\frac{1}{6}\right)\left(\frac{9}{8}\right)\left(\frac{L}{2}\right)^3 - 0 - \left(\frac{1}{16}L^2\right)\left(\frac{L}{2}\right)\right\}$$

$$= -\frac{M_0L^2}{128EI}$$

$$y_C = \frac{M_0L^2}{128EI} \downarrow \blacktriangleleft$$

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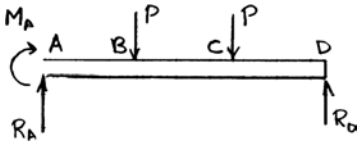


PROBLEM 9.51

For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *B*.

SOLUTION

$$+\uparrow \Sigma F_y = 0: \qquad R_A - P - P + R_D = 0 \qquad R_A = 2P - R_D$$



$$+\circlearrowleft \Sigma M_A = 0: \qquad -M_A - \frac{PL}{3} - \frac{2PL}{3} + R_D L = 0$$

$$M_A = R_D L - PL$$

$$\frac{dM}{dx} = V = R_A - P\left\langle x - \frac{L}{3}\right\rangle^0 - P\left\langle x - \frac{2L}{3}\right\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\left\langle x - \frac{L}{3}\right\rangle^1 - P\left\langle x - \frac{2L}{3}\right\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\left\langle x - \frac{L}{3}\right\rangle^2 - \frac{1}{2}P\left\langle x - \frac{2L}{3}\right\rangle^2 + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right] \quad 0 + 0 - 0 - 0 + C_1 = 0 \qquad C_1 = 0$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left\langle x - \frac{L}{3}\right\rangle^3 - \frac{1}{6}P\left\langle x - \frac{2L}{3}\right\rangle^3 + C_2$$

$$[x = 0,\ y = 0]\ 0 + 0 - 0 - 0 + C_2 = 0 \qquad C_2 = 0$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left\langle x - \frac{L}{3}\right\rangle^3 - \frac{1}{6}P\left\langle x - \frac{2L}{3}\right\rangle^3$$

$$[x = L,\ y = 0] \quad \frac{1}{2}(R_D L - PL)L^2 + \frac{1}{6}(2P - R_D)L^3 - \frac{1}{6}P\left(\frac{2L}{3}\right)^3 - \frac{1}{6}P\left(\frac{L}{3}\right)^3 = 0$$

$$\frac{1}{3}R_D L^3 - \frac{2}{9}PL^3 = 0$$

(*a*) Reaction at *D*. $\qquad R_D = \frac{2}{3}P\uparrow \blacktriangleleft$

$$M_A = \frac{2}{3}PL - PL = -\frac{1}{3}PL \qquad R_A = 2P - \frac{2}{3}P = \frac{4}{3}P$$

$$EIy = \frac{1}{2}\left(-\frac{1}{3}PL\right)x^2 + \frac{1}{6}\left(\frac{4}{3}P\right)x^2 - \frac{1}{6}P\left\langle x - \frac{L}{3}\right\rangle^3 - \frac{1}{6}\left\langle x - \frac{2L}{3}\right\rangle^3$$

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PROBLEM 9.51 *(Continued)*

Elastic curve.

$$y = \frac{P}{EI}\left\{-\frac{1}{6}Lx^2 + \frac{2}{9}x^3 - \frac{1}{6}\left\langle x - \frac{L}{3}\right\rangle^3 - \frac{1}{6}\left\langle x - \frac{2L}{3}\right\rangle^3\right\}$$

(*b*) Deflection at *B*.

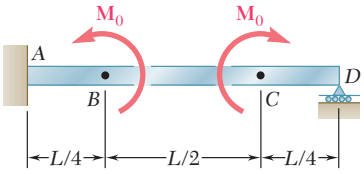
$$\left(y \text{ at } x = \frac{L}{3}\right)$$

$$y_B = \frac{P}{EI}\left\{-\left(\frac{1}{6}L\right)\left(\frac{L}{3}\right)^2 + \frac{2}{9}\left(\frac{L}{3}\right)^3 - 0 - 0\right\}$$

$$= -\frac{5PL^3}{486EI}$$

$$y_B = \frac{5PL^3}{486EI} \downarrow \blacktriangleleft$$

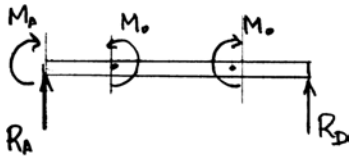
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PROBLEM 9.52

For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *B*.

SOLUTION



$$+\uparrow \Sigma F_y = 0: \quad R_A + R_D = 0 \qquad R_A = -R_D$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + M_0 - M_0 + RL = 0$$

$$M_A = R_D L$$

$$M(x) = M_A + R_A x - M_0\left\langle x - \frac{L}{4}\right\rangle^0 + M_0\left\langle x - \frac{3L}{4}\right\rangle^0$$

$$EI\frac{d^2y}{dx^2} = R_D L - R_D x - M_0\left\langle x - \frac{L}{4}\right\rangle^0 + M_0\left\langle x - \frac{3L}{4}\right\rangle^0$$

$$EI\frac{dy}{dx} = R_D L x - \frac{1}{2}R_D x^2 - M_0\left\langle x - \frac{L}{4}\right\rangle^1 + M_0\left\langle x - \frac{3L}{4}\right\rangle^1 + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right] \quad 0 - 0 - 0 + 0 + C_1 = 0$$

$$C_1 = 0$$

$$EIy = \frac{1}{2}R_D L x^2 - \frac{1}{6}R_D x^3 - \frac{1}{2}M_0\left\langle x - \frac{L}{4}\right\rangle^1 + \frac{1}{2}M_0\left\langle x - \frac{3L}{4}\right\rangle^1 + C_2$$

$$[x = 0, y = 0] \quad 0 - 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = L, y = 0] \quad \frac{1}{2}R_D L^3 - \frac{1}{6}R_D L^3 - \frac{1}{2}M_0\left(\frac{3L}{4}\right)^2 + \frac{1}{2}M_0\left(\frac{L}{4}\right)^2 = 0$$

(*a*) Reaction at *D*.

$$R_D = \frac{3M_0}{4L} \uparrow \blacktriangleleft$$

$$EIy = \frac{1}{2}\left(\frac{3M_0}{4L}\right)Lx^2 - \frac{1}{6}\left(\frac{3M_0}{4L}\right)x^3 - \frac{1}{2}M_0\left\langle x - \frac{L}{4}\right\rangle^2 + \frac{1}{2}M_0\left\langle x - \frac{3L}{4}\right\rangle^2$$

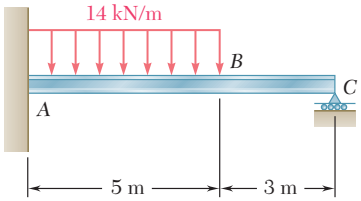
Elastic curve.

$$y = \frac{M_0}{EIL}\left\{\frac{3}{8}Lx^2 - \frac{1}{8}x^3 - \frac{1}{2}L\left\langle x - \frac{L}{4}\right\rangle^2 + \frac{1}{2}L\left\langle x - \frac{3L}{8}\right\rangle^2\right\}$$

(*b*) Deflection at point *B*. $\left(y \text{ at } x = \frac{L}{4}\right)$

$$y_B = \frac{M_0}{EIL}\left\{\left(\frac{3}{8}L\right)\left(\frac{L}{4}\right)^2 - \left(\frac{1}{8}\right)\left(\frac{L}{4}\right)^3 - 0 + 0\right\} \qquad y_B = \frac{11M_0L^2}{512EI} \uparrow \blacktriangleleft$$

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PROBLEM 9.53

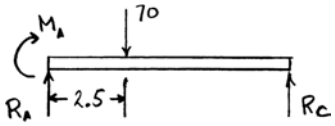
For the beam and loading shown, determine (*a*) the reaction at point *C*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.

$[x = 0,\ y = 0]$ $[x = 8,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

SOLUTION

Units: Forces in kN; lengths in m.



$$+\uparrow \Sigma F_y = 0: \quad R_A - 70 + R_C = 0$$

$$R_A = 70 - R_C \quad \text{kN}$$

$$+\circlearrowleft M_A = 0: \quad -M_A - (70)(2.5) + 8R_C = 0$$

$$M_A = 8R_C - 175 \quad \text{kN} \cdot \text{m}$$

Reactions are statically indeterminate.

$$w(x) = 14 - 14\langle x - 5\rangle^0 \qquad \text{kN/m}$$

$$\frac{dV}{dx} = -w = -14 + 14\langle x - 5\rangle^0 \qquad \text{kN/m}$$

$$\frac{dM}{dx} = V = R_A - 14x + 14\langle x - 5\rangle^1 \qquad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - 7x^2 + 7\langle x - 5\rangle^2 \qquad \text{kN} \cdot \text{m}$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{7}{3}x^3 + \frac{7}{3}\langle x - 5\rangle^3 + C_1 \qquad \text{kN} \cdot \text{m}^2$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{7}{12}x^4 + \frac{7}{12}\langle x - 5\rangle^4 + C_1 x + C_2 \qquad \text{kN} \cdot \text{m}^3$$

$\left[x = 0, \dfrac{dy}{dx} = 0\right]$ $\quad 0 + 0 + 0 + 0 + C_1 = 0 \quad C_1 = 0$

$[x = 0,\ y = 0]$ $\quad 0 + 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$

$[x = 8,\ y = 0]$ $\quad \frac{1}{2}M_A(8)^2 + \frac{1}{6}R_A(8)^3 - \frac{7}{12}(8)^4 + \frac{7}{12}(3)^4 + 0 + 0 = 0$

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PROBLEM 9.53 *(Continued)*

$$32(8R_C - 175) + \frac{512}{6}(70 - R_C) - \frac{28{,}105}{12} = 0$$

$$170.667R_C = 5600 - \frac{35{,}840}{6} + \frac{28{,}105}{12} = 1968.75 \qquad R_C = 11.536 \text{ kN} \uparrow$$

(*a*) Reaction at *C*. $R_C = 11.54 \text{ kN} \uparrow$ ◀

$$M_A = (8)(11.536) - 175 = -82.715 \text{ kN} \cdot \text{m}$$

$$R_A = 70 - 11.536 = 58.464 \text{ kN}$$

Data: $E = 200 \times 10^9 \text{ Pa} \qquad I = 216 \times 10^6 \text{ mm}^4 = 216 \times 10^{-6} \text{ m}^4$

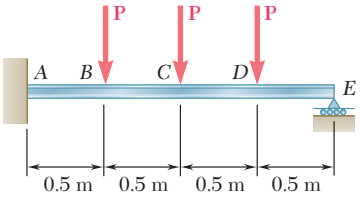
$$EI = (200 \times 10^9)(216 \times 10^{-6}) = 43.2 \times 10^6 \text{ N} \cdot \text{m}^2 = 43{,}200 \text{ kN} \cdot \text{m}^2$$

(*b*) Deflection at *B*. (y at $x = 5$ m)

$$EIy_B = \frac{1}{2}(-82.715)(5)^2 + \frac{1}{6}(58.464)(5)^3 - \frac{7}{12}(5)^4 = -180.52 \text{ kN} \cdot \text{m}^3$$

$$y_B = -\frac{180.52}{43{,}200} = -4.18 \times 10^{-3} \text{ m} \qquad y_B = 4.18 \text{ mm} \downarrow$$ ◀

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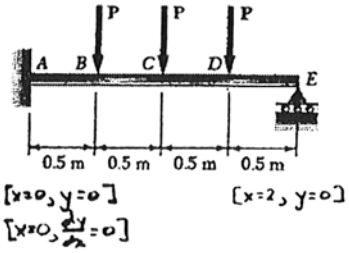




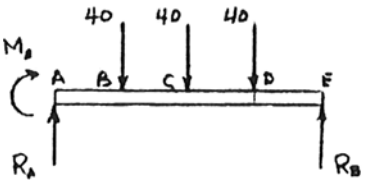
PROBLEM 9.54

For the beam shown and knowing that $P = 40$ kN, determine (*a*) the reaction at point *E*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION







Units: Forces in kN; lengths in m.

$$+\uparrow \sum F_y = 0: \quad R_A - 40 - 40 - 40 + R_E = 0$$

$$R_A = 120 - R_E \quad \text{kN}$$

$$+\circlearrowleft M_A = 0: \quad -M_A - 20 - 40 - 60 + 2R_B = 0$$

$$M_A = 2R_E - 120 \quad \text{kN} \cdot \text{m}$$

Reactions are statically indeterminate.

$$\frac{dM}{dx} = V = R_A - 40\langle x - 0.5\rangle^0 - 40\langle x - 1\rangle^0 - 40\langle x - 1.5\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - 40\langle x - 0.5\rangle^1 - 40\langle x - 1\rangle^1 - 40\langle x - 1.5\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - 20\langle x - 0.5\rangle^2 - 20\langle x - 1\rangle^2 - 20\langle x - 1.5\rangle^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{20}{3}\langle x - 0.5\rangle^3 - \frac{20}{3}\langle x - 1\rangle^3 - \frac{20}{3}\langle x - 1.5\rangle^3 + C_1 x + C_2$$

$$\left[x = 0, \ \frac{dy}{dx} = 0\right] \qquad 0 + 0 + 0 + 0 + 0 + C_1 = 0 \qquad C_1 = 0$$

$$[x = 0, \ y = 0] \qquad 0 + 0 + 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = 2, \ y = 0] \qquad \frac{1}{2}M_A(2)^2 + \frac{1}{6}R_A(2)^3 - \frac{20}{3}(1.5)^3 - \frac{20}{3}(1)^3 - \frac{20}{3}(0.5)^3 + 0 + 0 = 0$$

(*a*) Reaction at *E*.

$$\frac{1}{2}(2R_E - 120)(2)^2 + \frac{1}{6}(120 - R_E)(2)^3 = 30$$

$$2.66667\,R_E = 30 + 240 - 160 = 110 \qquad R_E = 41.25 \text{ kN} \uparrow \blacktriangleleft$$

$$M_A = (2)(41.25) - 120 = -37.5 \text{ kN} \cdot \text{m}$$

$$R_A = 120 - 41.25 = 78.25 \text{ kN}$$

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PROBLEM 9.54 *(Continued)*

Data: $E = 200 \times 10^9$ Pa, $I = 45.8 \times 10^6$ mm^4 $= 45.8 \times 10^{-6}$ m^4

$EI = (200 \times 10^9)(45.8 \times 10^{-6}) = 9.16 \times 10^6$ N · m^2 $= 9160$ kN · m^2

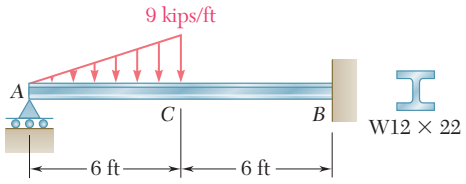
(*b*) Deflection at *C*. (y at $x = 1$ m)

$$EIy_C = \frac{1}{2}(-37.5)(1)^2 + \frac{1}{6}(78.75)(1)^3 - \frac{20}{3}(0.5)^3 - 0 - 0 + 0 + 0$$

$$= -6.4583 \text{ kN} \cdot \text{m}^3$$

$$y_C = -\frac{6.4583}{9160} = -0.705 \times 10^{-3} \text{ m} \qquad y_C = 0.705 \text{ mm} \downarrow \blacktriangleleft$$

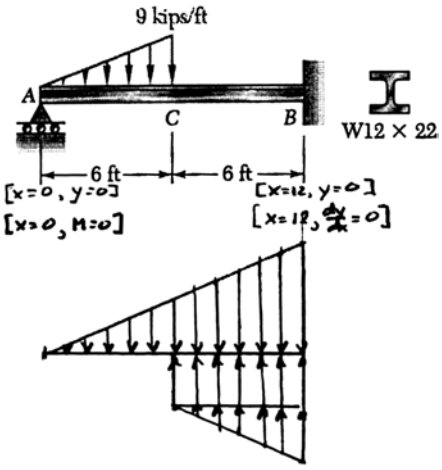
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PROBLEM 9.55

For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at point *C*. Use $E = 29 \times 10^6$ psi.

SOLUTION



Units: Forces in kips, lengths in ft.

$$k = \frac{9 \text{ kips/ft}}{6 \text{ ft}} = 1.5 \text{ kips/ft}^2$$

$$w(x) = 1.5x - 9\langle x-6\rangle^0 - 1.5\langle x-6\rangle^1$$

$$\frac{dV}{dx} = -w(x) = -1.5x + 9\langle x-6\rangle^0 + 1.5\langle x-6\rangle^1$$

$$\frac{dM}{dx} = V = R_A - 0.75x^2 + 9\langle x-6\rangle^1 + 0.75\langle x-6\rangle^2$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - 0.25x^3 + 4.5\langle x-6\rangle^2 + 0.25\langle x-6\rangle^3$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - 0.0625x^4 + 1.5\langle x-6\rangle^3 + 0.0625\langle x-6\rangle^4 + C_1 \qquad \text{kip} \cdot \text{ft}^2$$

$$EIy = \frac{1}{6}R_A x^3 - 0.0125x^5 + 0.375\langle x-6\rangle^4 + 0.0125\langle x-6\rangle^5 + C_1 x + C_2 \qquad \text{kip} \cdot \text{ft}^3$$

$$[x = 0,\ y = 0] \quad 0 + 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$\left[x = 12, \frac{dy}{dx} = 0\right] \quad \frac{1}{2}(R_A)(12)^2 - (0.0625)(12)^4 + (1.5)(6)^3 + (0.0625)(6)^4 + C_1 = 0$$

$$C_1 = 891 - 72R_A = 0 \quad \text{kip} \cdot \text{ft}^2$$

$$[x = 12,\ y = 0] \quad \frac{1}{6}R_A(12)^3 - (0.0125)(12)^5 + (0.375)(6)^4 + (0.0125)(6)^5 + (891 - 72R_A)(12) + 0 = 0$$

$$(864 - 288)R_A = 8164.8$$

$$R_A = 14.175 \text{ kips} \qquad R_A = 14.18 \text{ kips} \uparrow \blacktriangleleft$$

$$C_1 = 891 - (72)(14.175) = -129.6 \text{ kip} \cdot \text{ft}^2$$

Data: $E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$

$I = 156 \text{ in}^4$

$$EI = (29 \times 10^3)(156) = 4.524 \times 10^6 \text{ kip} \cdot \text{in}^2 = 31{,}417 \text{ kip} \cdot \text{ft}^2$$

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PROBLEM 9.55 *(Continued)*

(*b*) Deflection at *C*. (y at $x = 6$)

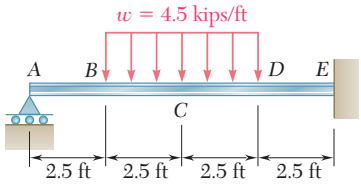
$$EIy_C = \frac{1}{6}(14.175)(6)^3 - (0.0125)(6)^5 + 0 + 0 - (129.6)(6) + 0$$

$$= -364.5 \text{ kip} \cdot \text{ft}^3$$

$$y_C = -\frac{364.5}{31{,}417} = -11.60 \times 10^{-3} \text{ ft}$$

$$y_C = 0.1392 \text{ in.} \downarrow \blacktriangleleft$$

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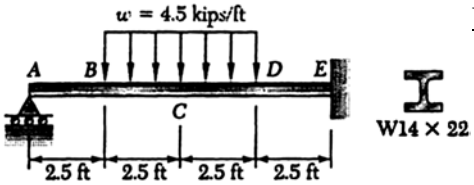




PROBLEM 9.56

For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at point *C*. Use $E = 29 \times 10^6$ psi.

SOLUTION



Units: Forces in kips, lengths in ft.

$$w(x) = 4.5\langle x - 2.5\rangle^0 - 4.5\langle x - 7.5\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -4.5\langle x - 2.5\rangle^0 + 4.5\langle x - 7.5\rangle^0 \text{ kip}^2/\text{ft}$$

$$\frac{dM}{dx} = V = R_A - 4.5\langle x - 2.5\rangle^1 + 4.5\langle x - 7.5\rangle^1 \quad \text{kips}$$

$[x = 0, y = 0]$ $[x = 10, y = 0]$

$[x = 0, M = 0]$ $\left[x = 10, \dfrac{dy}{dx} = 0\right]$

$$EI\frac{d^2y}{dx^2} = M = R_A x - 2.25\langle x - 2.5\rangle^2 + 2.25\langle x - 7.5\rangle^2 \qquad \text{kip} \cdot \text{ft}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{2.25}{3}\langle x - 2.5\rangle^3 + \frac{2.25}{3}\langle x - 7.5\rangle^3 + C_1 \qquad \text{kip} \cdot \text{ft}^2$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{2.25}{12}\langle x - 2.5\rangle^4 + \frac{2.25}{12}\langle x - 2.5\rangle^4 + C_1 x + C_2 \qquad \text{kip} \cdot \text{ft}^3$$

$[x = 0, y = 0]$ $\quad 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$\left[x = 10, \dfrac{dy}{dx} = 0\right]$ $\quad \dfrac{1}{2}R_A(10)^2 - \dfrac{2.25}{12}(7.5)^3 + \dfrac{2.25}{12}\langle 2.5\rangle^3 + C_1 = 0$

$$C_1 = 304.69 - 50R_A \qquad \text{kip} \cdot \text{ft}^2$$

$[x = 10, y = 0]$ $\quad \dfrac{1}{6}R_A(10)^3 - \dfrac{2.25}{12}(7.5)^4 + \dfrac{2.25}{12}(2.5)^4 + (304.69 - 50R_A)(10) + 0 = 0$

$$\left(500 - \frac{1000}{6}\right)R_A = 24{,}609 \qquad R_A = 7.3833 \qquad R_A = 7.38 \text{ kips} \uparrow \blacktriangleleft$$

$$C_1 = 304.69 - (50)(7.3833) = -64.45 \text{ kip} \cdot \text{ft}^2$$

Data: $E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}, \quad I = 199 \text{ in}^4$

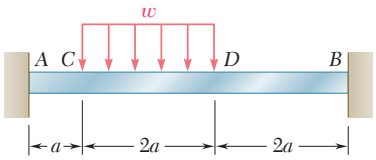
$$EI = (29 \times 10^3)(199) = 5.771 \times 10^6 \text{ kip} \cdot \text{in}^2 = 40{,}076 \text{ kip} \cdot \text{ft}^2$$

(*b*) Deflection at *C*. (y at $x = 5$ ft)

$$EIy_C = \frac{1}{6}(7.3833)(5)^3 - \frac{2.25}{12}(2.5)^4 + 0 - (64.45)(5) + 0 = -175.76 \text{ kip} \cdot \text{ft}^2$$

$$y_C = -\frac{175.76}{40{,}076} = -4.3856 \times 10^{-3} \text{ ft} \qquad y_C = 0.0526 \text{ in.} \downarrow \blacktriangleleft$$

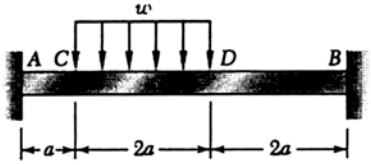
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PROBLEM 9.57

For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at point *D*.

SOLUTION



$$w(x) = w\langle x-a\rangle^0 - w\langle x-3a\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w\langle x-a\rangle^0 + w\langle x-3a\rangle^0$$

$$\frac{dM}{dx} = R_A - w\langle x-a\rangle^1 + w\langle x-3a\rangle^1$$

$[x = 0, y = 0]$ $[x = 5a, y = 0]$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - \frac{1}{2}w\langle x-a\rangle^2 + \frac{1}{2}w\langle x-3a\rangle^2$$

$\left[x = 0, \frac{dy}{dx} = 0\right]$ $\left[x = 5a, \frac{dy}{dx} = 0\right]$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{6}w\langle x-a\rangle^3 + \frac{1}{6}w\langle x-3a\rangle^3 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{24}w\langle x-a\rangle^4 + \frac{1}{24}\langle x-3a\rangle^4 + C_1 x + C_2$$

$\left[x = 0, \frac{dy}{dx} = 0\right]$ $0 + 0 + 0 + 0 + C_1 = 0$ $\quad C_1 = 0$

$[x = 0, y = 0]$ $0 + 0 + 0 + 0 + 0 + C_2 = 0$ $\quad C_2 = 0$

$\left[x = 5a, \frac{dy}{dx} = 0\right]$ $M_A(5a) + \frac{1}{2}R_A(5a)^2 - \frac{1}{6}w(4a)^3 + \frac{1}{6}w(2a)^3 + 0 = 0$

$$5M_A a + 12.5R_A a^2 = 9.3333wa^3 \qquad (1)$$

$[x = 5a, y = 0]$ $\frac{1}{2}M_A(5a)^2 + \frac{1}{6}R_A(5a)^3 - \frac{1}{24}w(4a)^4 + \frac{1}{24}(2a)^4 + 0 + 0 = 0$

$$12.5M_A a^2 + 20.8333R_A a^3 = 10wa^4 \qquad (2)$$

Solving (1) and (2) simultaneously,

$$M_A = -1.3333wa^2 \blacktriangleleft$$

$$R_A = 1.280wa \uparrow \blacktriangleleft$$

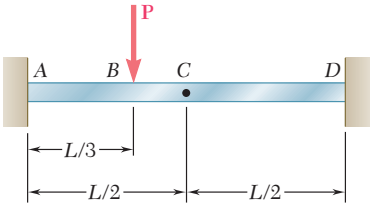
(*b*) Deflection at *D*. (y at $x = 3a$)

$$Y_D = \frac{1}{EI}\left\{\frac{1}{2}M_A(3a)^2 + \frac{1}{6}R_A(3a)^3 - \frac{1}{24}w(2a)^4 + 0 + 0 + 0\right\}$$

$$= \left[\frac{9}{2}(-1.3333) + \frac{1}{6}(1.28)(27) - \frac{1}{24}(16)\right]\frac{wa^4}{EI} = -0.907\frac{wa^4}{EI}$$

$$y_D = 0.907\frac{wa^4}{EI} \downarrow \blacktriangleleft$$

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PROBLEM 9.58

For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at midpoint *C*.

SOLUTION

$$\frac{dM}{dx} = V = R_A - P\left\langle x - \frac{L}{3}\right\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\left\langle x - \frac{L}{3}\right\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\left\langle x - \frac{L}{3}\right\rangle^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left\langle x - \frac{L}{3}\right\rangle^3 + C_1 x + C_2$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \quad 0 + 0 - 0 + C_1 = 0 \qquad \therefore\ C_1 = 0$$

$$[x = 0,\ y = 0] \quad 0 + 0 - 0 + C_2 = 0 \qquad \therefore\ C_2 = 0$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right] \quad M_A L + \frac{1}{2}R_A L^2 - \frac{1}{2}P\left(\frac{2L}{3}\right)^2 = 0 \tag{1}$$

$$[x = L,\ y = 0] \quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{6}P\left(\frac{2L}{3}\right)^3 = 0 \tag{2}$$

(*a*) Solving Eqs. (1) and (2) simultaneously,

$$R_A = \frac{20}{27}P \qquad M_A = -\frac{4}{27}PL$$

$$R_A = \frac{20}{27}P \uparrow \blacktriangleleft$$

$$M_A = \frac{4}{27}PL \circlearrowleft \blacktriangleleft$$

Elastic curve.

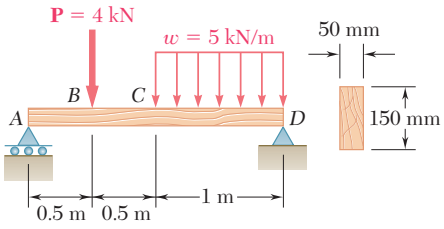
$$y = \frac{P}{EI}\left[-\frac{2}{27}Lx^2 + \frac{10}{81}x^3 - \frac{1}{6}\left\langle x - \frac{L}{3}\right\rangle^3\right]$$

(*b*) Deflection at midpoint *C*. $\left(y \text{ at } x = \frac{L}{2}\right)$

$$y_C = \frac{P}{EI}\left[-\frac{2}{27}L\left(\frac{L}{2}\right)^2 + \frac{10}{81}\left(\frac{L}{2}\right)^3 - \frac{1}{6}\left(\frac{L}{6}\right)^3\right] = -\frac{5PL^3}{1296EI}$$

$$y_C = \frac{5PL^3}{1296EI} \downarrow \blacktriangleleft$$

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PROBLEM 9.59

For the beam and loading of Prob. 9.45, determine the magnitude and location of the largest downward deflection.

SOLUTION

See solution to Prob.9.45 for the derivation of the equations used in the following:

$$EI = 168.75\ \text{kN}\cdot\text{m}^2 \qquad C_1 = -1.60417\ \text{kN}\cdot\text{m}^2 \qquad C_2 = 0$$

$$EI\frac{dy}{dx} = -\frac{5}{6}\langle x-1\rangle^3 + 2.125x^2 - 2\langle x-0.5\rangle^2 + C_1 \qquad \text{kN}\cdot\text{m}^2$$

$$EIy = -\frac{5}{24}\langle x-1\rangle^4 + \frac{2.125}{3}x^3 - \frac{2}{3}\langle x-0.5\rangle^2 + C_1x + C_2 \qquad \text{kN}\cdot\text{m}^3$$

Compute slope at C. $\left(\dfrac{dy}{dx}\text{ at } x = 1\,\text{in.}\right)$

$$EI\left(\frac{dy}{dx}\right)_c = 0 + (2.125)(1)^2 - 2(0.5)^2 - 1.60417 = 20.83\times10^{-3} \qquad \text{kN}\cdot\text{m}^2$$

Since the slope at C is positive, the largest deflection occurs in portion BC, where

$$EI\frac{dy}{dx} = 2.125x^2 - 2(x-0.5)^2 - 1.60417$$

$$EIy = \frac{2.125}{3}x^3 - \frac{2}{3}(x-0.5)^3 - 1.60417x$$

To find the location of the largest downward deflection, set $\dfrac{dy}{dx} = 0$.

$$2.125\,x_m^2 - 2(x_m^2 - x_m + 0.25) - 1.60417 = 0.125x_m^2 + 2x_m - 2.10417 = 0$$

$$x = 1.0521 - 0.0625x^2$$

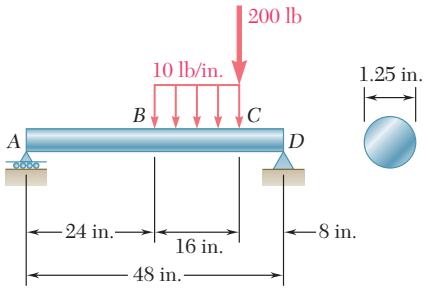
Solve by iteration. $x_m = 1,\ 0.989,\ 0.991,$ $\qquad x_m = 0.991\,\text{m}$ ◀

$$EIy_m = \left(\frac{2.125}{3}\right)(0.991)^3 - \frac{2}{3}(0.991-0.5)^3 - (1.60417)(0.991)$$

$$= -0.97927\ \text{kN}\cdot\text{m}^3$$

$$y_m = -\frac{0.97927}{168.75} = -5.80\times10^{-3}\,\text{m} \qquad y_m = 5.80\,\text{mm}\downarrow$$ ◀

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PROBLEM 9.60

For the beam and loading of Prob. 9.46, determine the magnitude and location of the largest downward deflection.

SOLUTION

See solution to Prob. 9.46 for the derivation of equations used in the following:

$$EI = 3.4754 \times 10^6 \text{ lb} \cdot \text{in}^2$$

$$C_1 = -30.08 \times 10^3 \text{ lb} \cdot \text{in}^2, \qquad C_2 = 0$$

$$EI\frac{dy}{dx} = -\frac{5}{3}\langle x-24\rangle^3 + \frac{5}{3}\langle x-40\rangle^3 + 43.333x^2 - 100\langle x-40\rangle^2 + C_1 \qquad \text{lb} \cdot \text{in}^2$$

$$EIy = -\frac{5}{12}\langle x-24\rangle^4 + \frac{5}{12}\langle x-40\rangle^4 + 14.4444x^3 - \frac{100}{3}\langle x-40\rangle^3 + C_1x + C_2 \qquad \text{lb} \cdot \text{in}^3$$

To find location of maximum $|y|$, set $\dfrac{dy}{dx} = 0$.

Assume $24 < x_m < 40$.

$$EI\frac{dy}{dx} = -\frac{5}{3}(x_m-24)^3 + 0 + 43.3333x^2 - 0 - 30.08 \times 10^3 = 0$$

$$f = -\frac{5}{3}(x_m-24)^3 + 43.3333x^2 - 30.08 \times 10^3$$

$$\frac{df}{dx_m} = -5(x_m-24)^2 + 86.667x$$

Solve by iteration:

x_m	$= 24$	26.46	26.35
f	$= -5.12 \times 10^3$	249	−14
df/dx	$= 2.08 \times 10^3$	2.26×10^3	

$$x_m = 26.4 \text{ in.} \blacktriangleleft$$

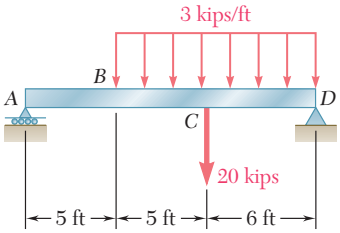
$$EIy_m = -\frac{5}{12}(26.35-24)^4 + 0 + (14.4444)(26.35)^3 - (30.08 \times 10^3)(26.35)$$

$$= -528.35 \times 10^3 \text{ lb} \cdot \text{in}^3$$

$$y_m = \frac{-528.35 \times 10^3}{3.4754 \times 10^6} = -0.1520 \text{ in.}$$

$$y_m = 0.1520 \text{ in.} \downarrow \blacktriangleleft$$

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PROBLEM 9.61

For the beam and loading of Prob. 9.47, determine the magnitude and location of the largest downward deflection.

SOLUTION

See solution to Prob. 9.47 for the derivation of following:

$$EI = 152{,}650 \text{ kip} \cdot \text{ft}^2$$

$$EI\frac{dy}{dx} = 9.422\,x^2 - 0.5\langle x-5\rangle^3 - 10\langle x-10\rangle^2 - 644.7 \qquad \text{kip} \cdot \text{ft}^2$$

$$EIy = 3.141x^3 - 0.125\langle x-5\rangle^4 - 3.333\langle x-10\rangle^3 - 644.7x \qquad \text{kip} \cdot \text{ft}^3$$

To find the location of maximum $|y|$, set $\dfrac{dy}{dx} = 0$.

Assume $5 \le x < 10$.

$$EI\frac{dy}{dx} = 9.422x^2 - 0.5(x-5)^3 - 644.7 = f(x) = 0 \tag{1}$$

$$\frac{df}{dx} = 18.844x - 1.5(x-5)^2$$

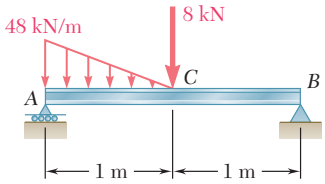
Solve Eq. (1) by iteration: $\quad x_{i+1} = x_i - \dfrac{f}{df/dx}$

x	9	8.406	8.397	$x_m = 8.40$ ft ◀
f	86.48	1.310	0.0415	
df/dx	145.6	141.0		

$$EIy_m = (3.141)(8.397)^3 - (0.125)(8.397-5)^4 - (644.7)(8.397) = -3570.5 \text{ kip} \cdot \text{ft}^3$$

$$152{,}650\,y_m = -3570.5 \qquad y_m = -0.02339 \text{ ft} \qquad y_m = 0.281 \text{ in.} \downarrow \blacktriangleleft$$

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PROBLEM 9.62

For the beam and loading of Prob. 9.48, determine the magnitude and location of the largest downward deflection.

SOLUTION

See solution to Prob. 9.48 for the derivation of the equations used in the following:

$$EI = 1024\ \text{kN} \cdot \text{m}^2$$

$$EI\frac{dy}{dx} = 12x^2 - 8x^3 + 2x^4 - 2\langle x-1\rangle^4 - 4\langle x-1\rangle^2 - \frac{83}{15} \quad \text{kN} \cdot \text{m}^2$$

$$EIy = 4x^3 - 2x^4 + \frac{2}{5}x^5 - \frac{2}{5}\langle x-1\rangle^5 - \frac{4}{3}\langle x-1\rangle^3 - \frac{83}{15}x \quad \text{kN} \cdot \text{m}^3$$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $0 < x < 1\,\text{m}$.

$$EI\frac{dy}{dx} = 12x^2 - 8x^3 + 2x^4 - \frac{83}{15} = 0$$

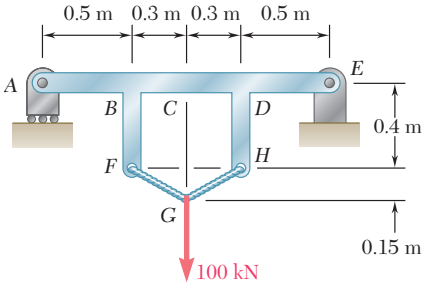
Solving: $x = 0.94166\ \text{m}$ $\qquad x_m = 0.942\ \text{m}$ ◀

$$EIy_m = 4(0.94166)^3 - 2(0.94166)^4 + \frac{2}{5}(0.94166)^5 - \frac{83}{15}(0.94166)$$

$$= -3.1469\ \text{kN} \cdot \text{m}^3$$

$$y_m = -\frac{3.1469}{1024} = -3.0731 \times 10^{-3}\ \text{m} \qquad y_m = 3.07\ \text{mm} \downarrow$$ ◀

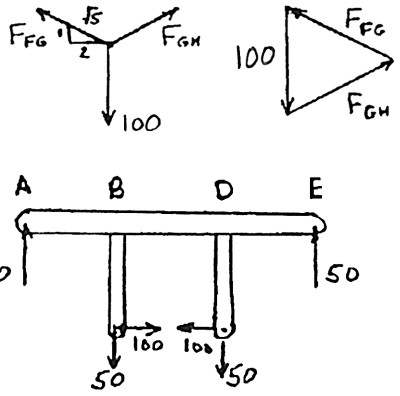
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PROBLEM 9.63

The rigid bars *BF* and *DH* are welded to the rolled-steel beam *AE* as shown. Determine for the loading shown (*a*) the deflection at point *B*, (*b*) the deflection at midpoint *C* of the beam. Use $E = 200$ GPa.

SOLUTION



Use joint *G* as a free body.

By symmetry, $F_{GH} = F_{FG}$

$$+\uparrow \sum F_y = 0: \quad 2F_{GHy} - 100 = 0 \qquad F_{GHy} = 50 \text{ kN}$$

$$F_{GHx} = 2F_{GHy} = 100 \text{ kN}.$$

Forces in kN; lengths in m.

$$V = 50 - 50\langle x - 0.5\rangle^0 - 50\langle x - 1.1\rangle^0 \quad \text{kN}$$

$$M = 50x - 50\langle x - 0.5\rangle^1 - 50\langle x - 1.1\rangle^0$$
$$+40\langle x - 0.5\rangle^0 - 40\langle x - 1.1\rangle^0 \quad \text{kN} \cdot \text{m}$$

$$EI\frac{dy}{dx} = 25x^2 - 25\langle x - 0.5\rangle^2 - 25\langle x - 1.1\rangle^2 - 40\langle x - 0.5\rangle^1 + 40\langle x - 1.1\rangle^1 + C_1 \quad \text{kN} \cdot \text{m}^2$$

$$EIy = \frac{25}{3}x^3 - \frac{25}{3}\langle x - 0.5\rangle^3 - \frac{25}{3}\langle x - 1.1\rangle^3 - 20\langle x - 0.5\rangle^2 + 20\langle x - 1.1\rangle^2 + C_1x + C_2 \quad \text{kN} \cdot \text{m}^3$$

$[x = 0,\ y = 0] \qquad C_2 = 0$

$[x = 1.6,\ y = 0]$

$$\left(\frac{25}{3}\right)(1.6)^3 - \left(\frac{25}{3}\right)(1.1)^3 - \left(\frac{25}{3}\right)(0.5)^3 - (20)(1.1)^2 + (20)(0.5)^2 + C_1(1.6) + 0 = 0$$

$$C_1 = -1.75 \text{ kN} \cdot \text{m}^3$$

For EIy_B, $x = 0.5\,\text{m}$

$$EIy_B = \left(\frac{25}{3}\right)(0.5)^3 - 0 - 0 + 0 - 0 - (1.75)(0.5) = 0.1667 \text{ kN} \cdot \text{m}^3$$

For EIy_C, $x = 0.8$ m

$$EIy_C = \left(\frac{25}{3}\right)(0.8)^3 - \left(\frac{25}{3}\right)(0.3)^3 - 0 - (20)(0.3)^2 - 0 - (1.75)(0.8) + 0$$
$$= -0.8417 \text{ kN} \cdot \text{m}^3$$

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PROBLEM 9.63 *(Continued)*

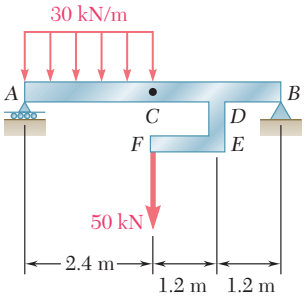
For W100 × 19.3 rolled-steel shape, $I = 4.70 \times 10^{6}\ \text{mm}^4 = 4.70 \times 10^{-6}\ \text{m}^4$

$$EI = (200 \times 10^{9})(4.70 \times 10^{-6}) = 940 \times 10^{3}\ \text{N} \cdot \text{m}^2 = 940\ \text{kN} \cdot \text{m}^2$$

(*a*) $y_B = \dfrac{0.1667}{940} = 0.1773 \times 10^{-3}\ \text{m}$ $\qquad y_B = 0.1773\ \text{mm} \uparrow$ ◀

(*b*) $y_C = \dfrac{0.8417}{940} = 0.895 \times 10^{-3}\ \text{m}$ $\qquad y_C = 0.895\ \text{mm} \uparrow$ ◀

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PROBLEM 9.64

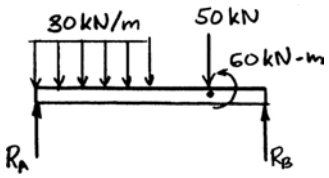
The rigid bar *DEF* is welded at point *D* to the rolled-steel beam *AB*. For the loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at midpoint *C* of the beam. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in meters.

$$+\circlearrowleft M_B = 0: \quad -4.8R_A + (30)(2.4)(3.6) + (50)(2.4) = 0$$

$$R_A = 79 \text{ kN} \uparrow$$



$$I = 212 \times 10^6 \text{ mm}^4 = 212 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(212 \times 10^{-6}) = 42.4 \times 10^6 \text{ N} \cdot \text{m}^2 = 42{,}400 \text{ kN} \cdot \text{m}^2$$

$$w(x) = 30 - 30\langle x - 2.4\rangle^0$$

$$\frac{dV}{dx} = -w = -30 + 30\langle x - 2.4\rangle^0 \qquad \text{kN/m}$$

$$\frac{dM}{dx} = V = 79 - 30x + 30\langle x - 2.4\rangle^1 - 50\langle x - 3.6\rangle^0 \qquad \text{kN}$$

$$EI\frac{d^2y}{dx} = M = 79x - 15x^2 + 15\langle x - 2.4\rangle^2 - 50\langle x - 3.6\rangle^1 - 60\langle x - 3.6\rangle^0 \qquad \text{kN} \cdot \text{m}$$

$$EI\frac{dy}{dx} = \frac{79}{2}x^2 - 5x^3 + 5\langle x - 2.4\rangle^3 - 25\langle x - 3.6\rangle^2 - 60\langle x - 3.6\rangle^1 + C_1 \qquad \text{kN} \cdot \text{m}^2$$

$$EIy = \frac{79}{6}x^3 - \frac{5}{4}x^4 + \frac{5}{4}\langle x - 2.4\rangle^4 - \frac{25}{3}\langle x - 3.6\rangle^3 - 30\langle x - 3.6\rangle^2 + C_1x + C_2 \qquad \text{kN} \cdot \text{m}^3$$

$[x = 0, y = 0]$ $\quad 0 - 0 + 0 - 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$

$[x = 4.8, y = 0]$

$$\left(\frac{79}{6}\right)(4.8)^3 - \left(\frac{5}{4}\right)(4.8)^4 + \left(\frac{5}{4}\right)(2.4)^4 - \left(\frac{25}{3}\right)(1.2)^3 - (30)(1.2)^2 + 4.8C_1 = 0$$

$$C_1 = -161.76 \text{ kN} \cdot \text{m}^2$$

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PROBLEM 9.64 *(Continued)*

(*a*) Slope at point *A*. $\left(\frac{dy}{dx} \text{ at } x = 0\right)$

$$EI\left(\frac{dy}{dx}\right)_A = 0 - 0 + 0 - 0 - 0 - 161.76$$

$$= -161.76 \text{ kN} \cdot \text{m}^2$$

$$\left(\frac{dy}{dx}\right)_A = \frac{-161.76}{42{,}400} = -3.82 \times 10^{-3} \qquad \theta_A = 3.82 \times 10^{-3} \text{ rad.} \curvearrowright \blacktriangleleft$$

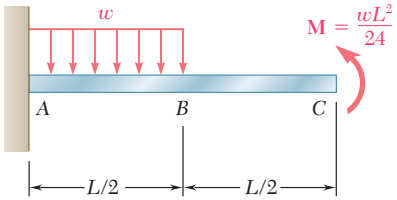
(*b*) Deflection at midpoint *C*. (y at $x = 2.4$)

$$EIy_C = \left(\frac{79}{6}\right)(2.4)^3 - \left(\frac{5}{4}\right)(2.4)^4 + 0 - 0 - 0 - (161.76)(2.4) + 0$$

$$= -247.68 \text{ kN} \cdot \text{m}^3$$

$$y_C = \frac{-247.68}{42{,}400} = -5.84 \times 10^{-3} \text{ m} \qquad y_C = 5.84 \text{ mm} \downarrow \blacktriangleleft$$

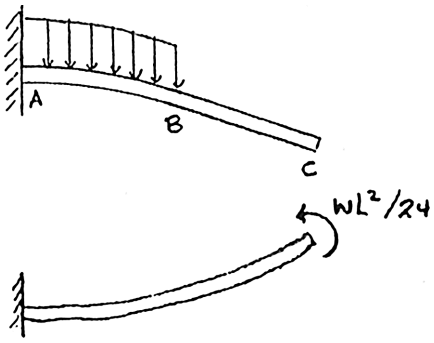
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PROBLEM 9.65

For the cantilever beam and loading shown, determine the slope and deflection at the free end.

SOLUTION



Loading I: Downward distributed load w applied to portion AB.

Case 2 of Appendix D applied to portion AB.

$$\theta'_B = -\frac{w(L/2)^3}{6EI} = -\frac{1}{48}\frac{wL^2}{EI}$$

$$y'_B = -\frac{w(L/2)^4}{8EI} = -\frac{1}{128}\frac{wL^4}{EI}$$

Portion BC remains straight.

$$\theta'_C = \theta'_B = -\frac{1}{48}\frac{wL^3}{EI}$$

$$y'_C = y'_B + \left(\frac{L}{2}\right)\theta'_B = -\frac{1}{128}\frac{wL^4}{EI} - \frac{1}{96}\frac{wL^4}{EI} = -\frac{7}{384}\frac{wL^4}{EI}$$

Loading II: Counterclockwise couple $\frac{wL^2}{24}$ applied at C.

Case 3 of Appendix D.

$$\theta''_C = \frac{(wL^2/24)L}{EI} = \frac{1}{24}\frac{wL^3}{EI}$$

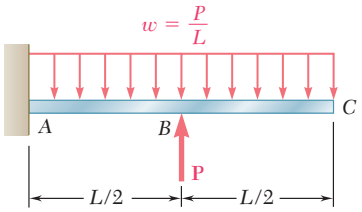
$$y''_C = \frac{(wL^2/24)L^2}{2EI} = \frac{1}{48}\frac{wL^4}{EI}$$

By superposition,

$$\theta_C = \theta'_C + \theta''_C = -\frac{1}{48}\frac{wL^3}{EI} + \frac{1}{24}\frac{wL^3}{EI} \qquad \theta_C = \frac{1}{48}\frac{wL^3}{EI}\curvearrowright \blacktriangleleft$$

$$y_C = y'_C + y''_C = -\frac{7}{384}\frac{wL^4}{EI} + \frac{1}{48}\frac{wL^4}{EI} \qquad y_C = \frac{1}{384}\frac{wL^4}{EI}\uparrow \blacktriangleleft$$

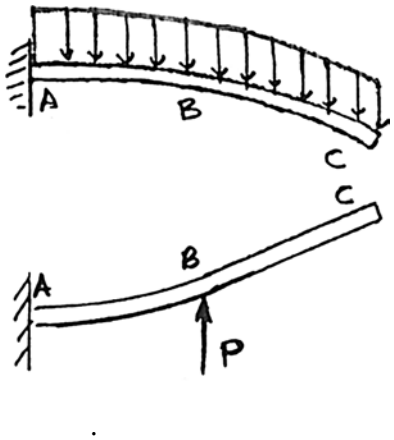
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PROBLEM 9.66

For the cantilever beam and loading shown, determine the slope and deflection at the end.

SOLUTION



Loading I: Uniformly distributed downward loading with $w = P/L$.

Case 2 of Appendix *D*.

$$\theta'_C = -\frac{(P/L)L^3}{6EI} = -\frac{1}{6}\frac{PL^2}{EI}$$

$$y'_C = -\frac{(P/L)L^4}{8EI} = -\frac{1}{8}\frac{PL^3}{EI}$$

Loading II: Upward concentrated load at *P* at point *B*.

Case 1 of Appendix *D* applied to portion *AB*.

$$\theta''_B = \frac{P(L/2)^2}{2EI} = \frac{1}{8}\frac{PL^2}{EI}$$

$$y''_B = \frac{P(L/2)^3}{3EI} = \frac{1}{24}\frac{PL^3}{EI}$$

Portion *BC* remains straight.

$$\theta''_C = \theta''_B = \frac{1}{8}\frac{PL^2}{EI}$$

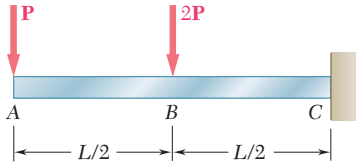
$$y''_C = y''_B + \frac{L}{2}\theta''_B = \frac{1}{24}\frac{PL^3}{EI} + \frac{1}{16}\frac{PL^3}{EI} = \frac{5}{48}\frac{PL^3}{EI}$$

By superposition,

$$\theta_C = \theta'_C + \theta''_C = -\frac{1}{6}\frac{PL^2}{EI} + \frac{1}{8}\frac{PL^2}{EI} = -\frac{1}{24}\frac{PL^2}{EI} \qquad \theta_C = \frac{PL^2}{24EI} \curvearrowleft \blacktriangleleft$$

$$y_C = y'_C + y''_C = -\frac{1}{8}\frac{PL^3}{EI} + \frac{5}{48}\frac{PL^2}{EI} = -\frac{1}{48}\frac{PL^3}{EI} \qquad y_C = \frac{PL^3}{48EI} \downarrow \blacktriangleleft$$

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PROBLEM 9.67

For the cantilever beam and loading shown, determine the slope and deflection at the free end.

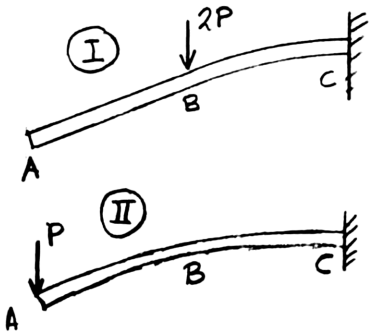
SOLUTION

Loading I: $2P$ downward at B.

Case 1 of Appendix D applied to portion BC.

$$\theta'_B = \frac{(2P)(L/2)^2}{2EI} = \frac{1}{4}\frac{PL^2}{EI}$$

$$y'_B = \frac{(2P)(L/2)^3}{3EI} = \frac{1}{2}\frac{PL^3}{EI}$$



AB remains straight.

$$\theta'_A = \theta'_B = \frac{1}{4}\frac{PL^2}{EI}$$

$$y'_A = y'_B - \left(\frac{L}{2}\right)\theta'_B$$

$$= -\frac{1}{12}\frac{PL^3}{EI} - \frac{1}{8}\frac{PL^3}{EI}$$

$$= -\frac{5}{24}\frac{PL^3}{EI}$$

Loading II: P downward at A.

Case 1 of Appendix D.

$$\theta''_A = \frac{PL^2}{2EI},$$

$$y''_A = -\frac{PL^3}{3EI}$$

By superposition,

$$\theta_A = \theta'_A + \theta''_A$$

$$= \frac{1}{4}\frac{PL^2}{EI} + \frac{1}{2}\frac{PL^2}{EI} = \frac{3}{4}\frac{PL^2}{EI}$$

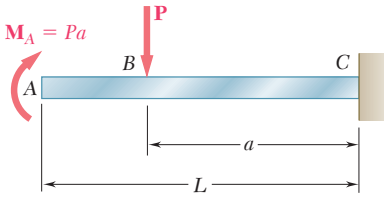
$$\frac{3PL^2}{4EI} \measuredangle \blacktriangleleft$$

$$y_A = y'_A + y''_A$$

$$= -\frac{5}{24}\frac{PL^3}{EI} = -\frac{1}{3}\frac{PL^3}{EI} = -\frac{13}{24}\frac{PL^3}{EI}$$

$$\frac{13PL^3}{24EI} \downarrow \blacktriangleleft$$

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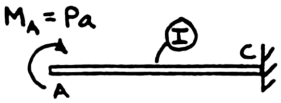
PROBLEM 9.68

For the cantilever beam and loading shown, determine the slope and deflection at the free end.

SOLUTION

Loading I: M_A at A. Case 3 of appendix D.

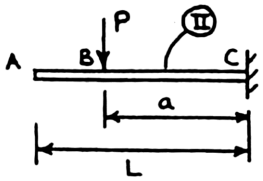
$$\theta'_A = -\frac{M_A L}{EI} \qquad y'_A = \frac{M_A L^2}{2EI}$$



with $M_A = Pa$

$$\theta'_A = -\frac{PaL}{EI}$$

$$y'_A = \frac{PaL^2}{2EI}$$



Loading II: P downward at B. Case 1 of appendix D applied to portion BC.

$$\theta''_B = \frac{Pa^2}{2EI} \qquad y''_B = -\frac{Pa^3}{3EI}$$

AB remains straight.

$$\theta''_A = \theta''_B = \frac{Pa^2}{2EI}$$

$$y''_A = y''_B - (L-a)\theta''_B$$

$$= -\frac{Pa^3}{3EI} - (L-a)\frac{Pa^2}{2EI} = -\frac{Pa^2 L}{2EI} + \frac{Pa^3}{6EI}$$

By superposition,

$$\theta_A = \theta'_A + \theta''_A = -\frac{PaL}{EI} + \frac{Pa^2}{2EI} = -\frac{Pa}{2EI}(2L-a)$$

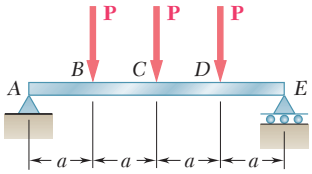
$$\frac{Pa}{2EI}(2L-a) \curvearrowright \blacktriangleleft$$

$$y_A = y'_A + y''_A = \frac{PaL^2}{2EI} - \frac{Pa^2 L}{2EI} + \frac{Pa^3}{6EI}$$

$$= \frac{Pa}{6EI}(3L^2 - 3aL + a^2)$$

$$\frac{Pa}{6EI}(3L^2 - 3aL + a^2) \uparrow \blacktriangleleft$$

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PROBLEM 9.69

For the beam and loading shown, determine (*a*) the deflection at point *C*, (*b*) the slope at end *A*.

SOLUTION

Loading I: Load at *B*. Case 5 in Appendix *D*.

$$L = 4a, \quad a = a, \quad b = 3a, \quad x = 2a$$

For $x > a$, replace x by $L - x$ and interchange a and b in expression for elastic curve given.

$$y = \frac{Pa}{6EIL}[(L-x)^3 - (L^2 - a^2)(L-x)]$$

$$y_C = \frac{Pa}{6EI(4a)}[(2a)^3 - (16a^2 - a^2)(2a)] = -\frac{11}{12}\frac{Pa^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(3a)(16a^2 - 9a^2)}{6EI(4a)} = -\frac{7}{8}\frac{Pa^2}{EI}$$

Loading II: Load at *C*. Case 4 of Appendix *D* with $L = 4a$.

$$y_C = -\frac{PL^3}{48EI} = -\frac{P(4a)^3}{48EI} = -\frac{4}{3}\frac{Pa^3}{EI}$$

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{P(4a)^2}{16EI} = -\frac{Pa^2}{EI}$$

Loading III: Load at *D*. Case 5 of Appendix *D*.

$$L = 4a, \quad a = 3a, \quad b = a, \quad x = 2a \text{ at point } C.$$

$$y_C = \frac{Pb}{6EIL}[x^3 - (L^2 - b^2)x]$$

$$= \frac{Pa}{16EI(4a)}[(2a)^3 - (16a^2 - a^2)(2a)] = -\frac{11}{12}\frac{Pa^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{Pa(16a^2 - a^2)}{6EI(4a)} = -\frac{5}{8}\frac{Pa^3}{EI}$$

(*a*) Deflection at *C*.

$$y_C = -\frac{11}{12}\frac{Pa^3}{EI} - \frac{4}{3}\frac{Pa^3}{EI} - \frac{11}{12}\frac{Pa^3}{EI} = -\frac{19}{6}\frac{Pa^3}{EI}$$

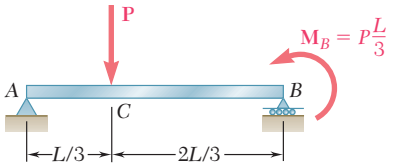
$$y_C = \frac{19}{6}\frac{Pa^3}{EI} \downarrow \blacktriangleleft$$

(*b*) Slope at *A*.

$$\theta_A = -\frac{7}{8}\frac{Pa^2}{EI} - \frac{Pa^2}{EI} - \frac{5}{8}\frac{Pa^2}{EI} = -\frac{5}{2}\frac{Pa^2}{EI}$$

$$\theta_A = \frac{5}{2}\frac{Pa^2}{EI} \curvearrowright \blacktriangleleft$$

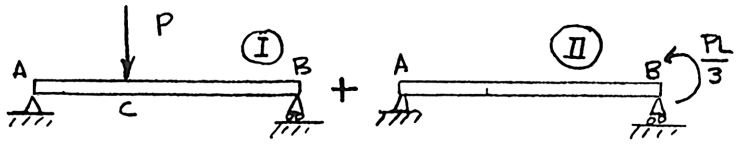
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PROBLEM 9.70

For the beam and loading shown, determine (*a*) the deflection at point *C*, (*b*) the slope at end *A*.

SOLUTION



Loading I: Case 5. $a = \frac{L}{3}, \quad b = \frac{2L}{3}, \quad P = P, \quad x = a$

$$y_C = -\frac{Pa^2b^2}{6EIL} = -\frac{P}{6EIL}\left(\frac{L}{3}\right)^2\left(\frac{2L}{3}\right)^2 = -\frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P}{6EIL}\left(\frac{2L}{3}\right)\left[L^2 - \left(\frac{2L}{3}\right)^2\right] = -\frac{5}{81}\frac{PL^2}{EI}$$

Loading II: Case 7. $M = -\frac{PL}{3}, \quad x = \frac{L}{3}$

$$y_C = -\frac{M}{6EIL}(x^3 - L^2x) = +\frac{PL/3}{6EIL}\left\{\left(\frac{L}{3}\right)^3 - L^2\left(\frac{L}{3}\right)\right\} = -\frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A = +\frac{ML}{6EI} = \frac{-(PL/3)L}{6EI} = -\frac{1}{18}\frac{PL^2}{EI}$$

(*a*) Deflection at *C*.

$$y_C = -\frac{4}{243}\frac{PL^3}{EI} - \frac{4}{243}\frac{PL^3}{EI} = -\frac{8}{243}\frac{PL^3}{EI}$$

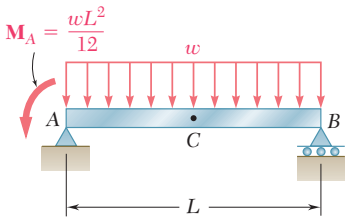
$$y_C = \frac{8}{243}\frac{PL^3}{EI} \downarrow \blacktriangleleft$$

(*b*) Slope at *A*.

$$\theta_A = -\frac{5}{81}\frac{PL^2}{EI} - \frac{1}{18}\frac{PL^2}{EI} = -\frac{19}{162}\frac{PL^2}{EI}$$

$$\theta_A = \frac{19}{162}\frac{PL^2}{EI} \blacktriangleleft$$

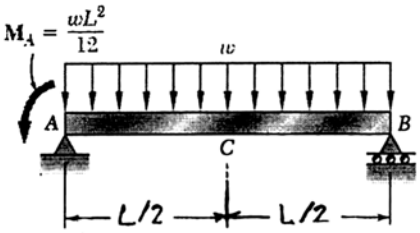
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PROBLEM 9.71

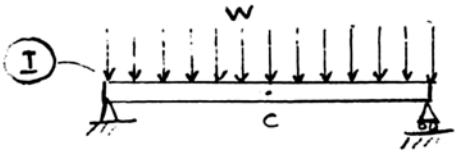
For the beam and loading shown, determine (*a*) the deflection at point *C*, (*b*) the slope at end *A*.

SOLUTION



Loading I: Case 6 in Appendix *D*.

$$y_C = -\frac{5}{384}\frac{wL^4}{EI}; \quad \theta_A = -\frac{1}{24}\frac{wL^3}{EI}$$



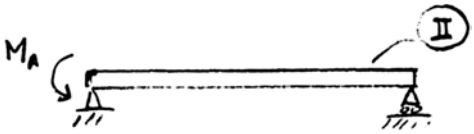
Loading II: Case 7 of Appendix *D*.

Note that center deflection is

$$y_C = -\frac{M_A}{6EIL}\left[\left(\frac{L}{2}\right)^3 - L^2\left(\frac{L}{2}\right)\right]$$

$$= \frac{1}{16}\frac{M_A L}{EI}$$

$$\theta_A = \frac{M_A L}{3EI}$$



with $M_A = \frac{wL^2}{12}$, $y_C = \frac{1}{192}\frac{wL^4}{EI}$, $\theta_A = \frac{1}{36}\frac{wL^3}{EI}$

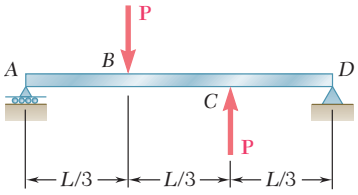
(*a*) Deflection at *C*. $y_C = -\frac{5}{384}\frac{wL^4}{EI} + \frac{1}{192}\frac{wL^4}{EI} = -\frac{1}{128}\frac{wL^3}{EI}$

$$y_C = \frac{1}{128}\frac{wL^4}{EI} \downarrow \blacktriangleleft$$

(*b*) Slope at *A*. $\theta_A = -\frac{1}{24}\frac{wL^3}{EI} + \frac{1}{36}\frac{wL^3}{EI} = -\frac{1}{72}\frac{wL^3}{EI}$

$$\theta_A = \frac{1}{72}\frac{wL^3}{EI} \blacktriangleleft$$

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PROBLEM 9.72

For the beam and loading shown, determine (*a*) the deflection at point *C*, (*b*) the slope at end *A*.

SOLUTION

Loading I: Downward load *P* at *B*.

Use Case 5 of Appendix *D* with

$$P = P, \quad a = \frac{L}{3}, \quad b = \frac{2L}{3}, \quad L = L, \quad x = \frac{2L}{3}$$

For $x < a$, given elastic curve is $y = \dfrac{Pb}{EIL}[x^3 - (L^2 - b^2)x]$

To obtain elastic curve for $x > a$, replace x by $L - x$ and interchange a and b to get

$$y = \frac{Pa}{6EIL}[(L - x)^3 - (L^2 - a^2)(L - x)] \text{ with } x = \frac{2L}{3} \text{ at point } C.$$

$$y_C = \frac{P(L/3)}{6EIL}\left[\left(\frac{L}{3}\right)^3 - \left(L^2 - \left(\frac{L}{3}\right)^2\right)\left(\frac{L}{3}\right)\right] = -\frac{7}{486}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(2L/3)[L^2 - (2L/3)^2]}{6EIL} = -\frac{5}{81}\frac{PL^2}{EI}$$

Loading II: Upward load at *C*. Use Case 5 of Appendix *D* with

$$P = -P, \quad a = \frac{2L}{3}, \quad b = \frac{L}{3}, \quad L = L, \quad x = a = \frac{2L}{3}$$

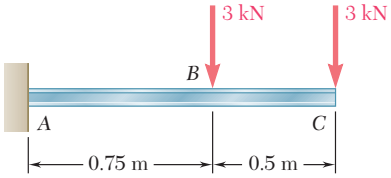
$$y_C = -\frac{(-P)(2L/3)^2(L/3)^2}{3EIL} = \frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{(-P)(L/3)(L^2 - (L/3)^2)}{6EIL} = \frac{4}{81}\frac{PL^2}{EI}$$

(*a*) Deflection at *C*. $y_C = -\dfrac{7}{486}\dfrac{PL^3}{EI} + \dfrac{4}{243}\dfrac{PL^3}{EI}$ $\qquad y_C = \dfrac{1}{486}\dfrac{PL^3}{EI} \uparrow$ ◀

(*b*) Slope at *A*. $\theta_A = -\dfrac{5}{81}\dfrac{PL^2}{EI} + \dfrac{4}{81}\dfrac{PL^2}{EI}$ $\qquad \theta_A = \dfrac{1}{81}\dfrac{PL^2}{EI}$ ↘ ◀

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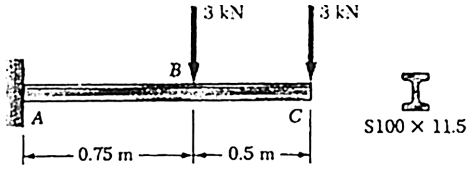


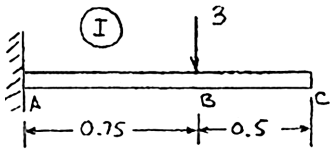


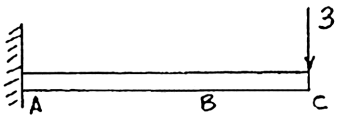
PROBLEM 9.73

For the cantilever beam and loading shown, determine the slope and deflection at end *C*. Use $E = 200$ GPa.

SOLUTION







Units: Forces in kN; lengths in m.

Loading I: Concentrated load at *B*.

Case 1 of Appendix *D* applied to portion *AB*.

$$\theta'_B = -\frac{PL^2}{2EI} = -\frac{(3)(0.75)^2}{2EI} = -\frac{0.84375}{EI}$$

$$y'_B = -\frac{PL^3}{3EI} = -\frac{(3)(0.75)^3}{3EI} = -\frac{0.421875}{EI}$$

Portion *BC* remains straight.

$$\theta'_C = \theta'_B = -\frac{0.84375}{EI}$$

$$y'_C = y'_B - (0.5)\theta'_B = -\frac{0.84375}{EI}$$

Loading II: Concentrated load at *C*. Case 1 of Appendix *D*.

$$\theta''_A = -\frac{PL^2}{2EI} = -\frac{(3)(1.25)^2}{2EI} = -\frac{2.34375}{EI}$$

$$y''_A = -\frac{PL^3}{3EI} = -\frac{(3)(1.25)^3}{3EI} = -\frac{1.953125}{EI}$$

By superposition,

$$\theta_A = \theta'_A + \theta''_A = -\frac{3.1875}{EI}$$

$$y_A = y'_A + y''_A = -\frac{2.796875}{EI}$$

Data:

$$E = 200 \times 10^9 \text{ Pa}, \quad I = 2.52 \times 10^6 \text{ mm}^4 = 2.52 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^4)(2.52 \times 10^{-6}) = 504 \times 10^3 \text{ N} \cdot \text{m}^2 = 504 \text{ kN} \cdot \text{m}^2$$

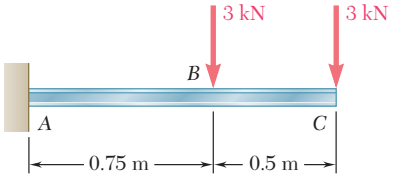
Slope at *C*.

$$\theta_C = -\frac{3.1875}{504} = -6.32 \times 10^{-3} \text{ rad} \qquad \theta_C = 6.32 \times 10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

Deflection at *C*.

$$y_C = -\frac{2.796875}{504} = -5.55 \times 10^{-3} \text{ m} \qquad y_C = 5.55 \text{ mm} \downarrow \blacktriangleleft$$

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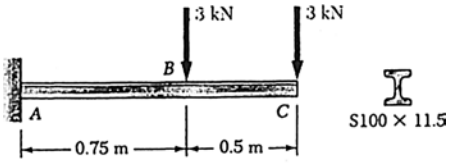


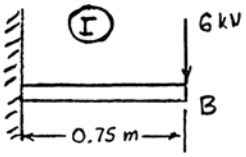


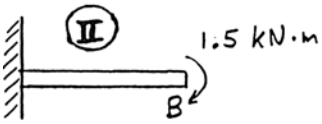
PROBLEM 9.74

For the cantilever beam and loading shown, determine the slope and deflection at point *B*. Use $E = 200$ GPa.

SOLUTION







Units: Forces in kN; lengths in m.

The slope and deflection at *B* depend only on the deformation of portion *AB*.

Reduce the force at *C* to an equivalent force-couple system at *B* and add the force already at *B* to obtain the loadings I and II shown.

Loading I: Case 1 of Appendix *D*.

$$\theta'_B = -\frac{PL^2}{2EI} = -\frac{(6)(0.75)^2}{2EI} = -\frac{1.6875}{EI}$$

$$y'_B = -\frac{PL^3}{3EI} = -\frac{(6)(0.75)^3}{3EI} = -\frac{0.84375}{EI}$$

Loading II: Case 3 of Appendix *D*.

$$\theta''_B = -\frac{ML}{EI} = -\frac{(1.5)(0.75)}{EI} = -\frac{1.125}{EI}$$

$$y''_B = -\frac{ML^2}{2EI} = -\frac{(1.5)(0.75)^2}{EI} = -\frac{0.421875}{EI}$$

By superposition,

$$\theta_B = \theta'_B + \theta''_B = -\frac{2.8125}{EI}$$

$$y_B = y'_B + y''_B = -\frac{1.265625}{EI}$$

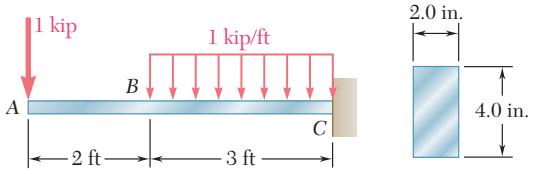
Data: $E = 200 \times 10^9$ Pa, $I = 2.52 \times 10^6$ mm^4 $= 2.52 \times 10^{-6}$ m^4

$$EI = (200 \times 10^9)(2.52 \times 10^{-6}) = 504 \times 10^3 \text{ N} \cdot \text{m}^2 = 504 \text{ kN} \cdot \text{m}^2$$

Slope at *B*. $\theta_B = -\dfrac{2.8125}{504} = -5.58 \times 10^{-3}$ rad $\qquad \theta_B = 5.58 \times 10^{-3}$ rad ↻

Deflection at *B*. $y_B = -\dfrac{1.265625}{504} = -2.51 \times 10^{-3}$ m $\qquad y_B = 2.51$ mm ↓

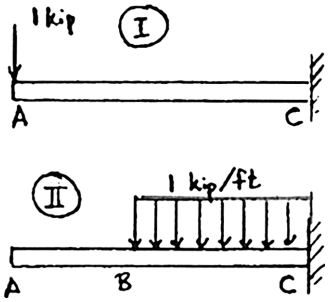
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PROBLEM 9.75

For the cantilever beam and loading shown, determine the slope and deflection at end *A*. Use $E = 29 \times 10^6$ psi.

SOLUTION



Units: Forces in kips; lengths in ft.

Loading I: Concentrated load at *A*.

Case 1 of Appendix *D*.

$$\theta'_A = \frac{PL^2}{2EI} = \frac{(1)(5)^2}{2EI} = \frac{12.5}{EI}$$

$$y'_A = -\frac{PL^3}{3EI} = -\frac{(1)(5)^3}{3EI} = -\frac{41.667}{EI}$$

Loading II: Uniformly distributed load over portion *BC*.

Case 2 of Appendix *D* applied to portion *BC*.

$$\theta''_B = \frac{wL^3}{6EI} = \frac{(1)(3)^3}{6EI} = \frac{4.5}{EI}$$

$$y''_B = -\frac{wL^4}{8EI} = -\frac{(1)(3)^4}{8EI} = -\frac{10.125}{EI}$$

Portion *AB* remains straight. $\theta''_A = \theta''_B = \dfrac{4.5}{EI}$

$$y''_A = y''_B - a\theta''_B = -\frac{10.125}{EI} - (2)\left(\frac{4.5}{EI}\right) = -\frac{19.125}{EI}$$

By superposition,

$$\theta_A = \theta'_A + \theta''_A = \frac{12.5}{EI} + \frac{4.5}{EI} = \frac{17}{EI}$$

$$y_A = y'_A + y''_A = -\frac{41.667}{EI} - \frac{19.125}{EI} = -\frac{60.792}{EI}$$

Data:

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = \frac{1}{12}(2.0)(4.0)^3 = 10.667 \text{ in}^4$$

$$EI = (29 \times 10^3)(10.667) = 309.33 \times 10^3 \text{ kip} \cdot \text{in}^2 = 2148 \text{ kip} \cdot \text{ft}^2$$

Slope at *A*.

$$\theta_A = \frac{17}{2148}$$

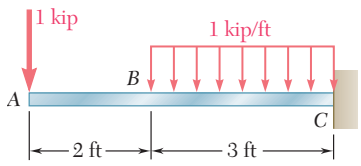
$$\theta_A = 7.91 \times 10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

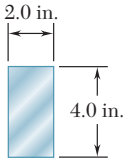
Deflection at *A*.

$$y_A = -\frac{60.792}{2148} = -28.30 \times 10^{-3} \text{ ft}$$

$$y_A = 0.340 \text{ in.} \downarrow \blacktriangleleft$$

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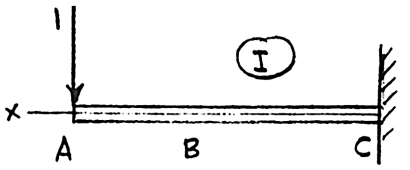


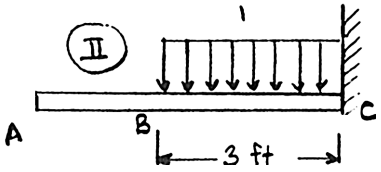


PROBLEM 9.76

For the cantilever beam and loading shown, determine the slope and deflection at point *B*. Use $E = 29 \times 10^6$ psi.

SOLUTION





Units: Forces in kips; lengths in ft.

Loading I: Concentrated load at *A*.

Case 1 of Appendix *D*.

$$y = \frac{P}{6EI}[x^3 - 3Lx^2]$$

$$\frac{dy}{dx} = \frac{P}{6EI}[3x^2 - 6Lx]$$

with $P = 1$ kip, $L = 5$ ft, $x = 3$ ft

$$y'_B = \frac{1}{6EI}[(3)^3 - (3)(5)(3)^2] = -\frac{18}{EI}$$

$$\left.\frac{dy}{dx}\right|_B = \frac{1}{6EI}[(3)(3)^2 - (6)(5)(3)] = -\frac{10.5}{EI}$$

Adjusting the sign, $\theta'_B = \dfrac{10.5}{EI}$

Loading II: Uniformly distributed load over portion *BC*.

Case 2 of Appendix *D* applied to portion *BC*.

$$y''_B = -\frac{wL^4}{8EI} = -\frac{(1)(3)^4}{8EI} = -\frac{10.125}{EI} \qquad \theta''_B = \frac{wL^3}{6EI} = \frac{(1)(3)^3}{6EI} = \frac{4.5}{EI}$$

By superposition,

$$\theta_B = \theta'_B + \theta''_B = \frac{10.5}{EI} + \frac{4.5}{EI} = \frac{15}{EI} \qquad y_B = y'_B + y''_B = -\frac{18}{EI} - \frac{10.125}{EI} = -\frac{28.125}{EI}$$

Data:

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi} \qquad I = \frac{1}{12}(2.0)(4.0)^3 = 10.667 \text{ in}^4$$

$$EI = (29 \times 10^3)(10.667) = 309.33 \times 10^3 \text{ kip} \cdot \text{in}^2 = 2148 \text{ kip} \cdot \text{ft}^2$$

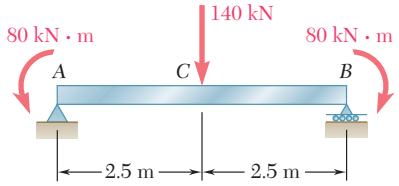
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PROBLEM 9.76 *(Continued)*

Slope at *B*. $\theta_B = \dfrac{15}{2148} = 6.98 \times 10^{-3}$ $\theta_B = 6.98 \times 10^{-3}\,\text{rad}$ ◀

Deflection at *B*. $y_B = -\dfrac{28.125}{2148} = -13.09 \times 10^{-3}\,\text{ft}$ $y_B = 0.1571\,\text{in.}\downarrow$ ◀

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PROBLEM 9.77

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in m.

Loading I: Moment at *B*.

Case 7 of Appendix *D*. $M = 80\ \text{kN}\cdot\text{m}, \quad L = 5.0\ \text{m}, \quad x = 2.5\ \text{m}$

$$\theta_A = \frac{ML}{6EI} = \frac{(80)(5.0)}{6EI} = \frac{66.667}{EI}$$

$$y_C = -\frac{M}{6EIL}(x^3 - L^2x) = -\frac{80}{6EI(5.0)}[2.5^3 - (5.0)^2(2.5)] = \frac{125}{EI}$$

Loading II: Moment at *A*. Case 7 of Appendix *D*.

$$M = 80\ \text{kN}\cdot\text{m}, \quad L = 5.0\ \text{m}, \quad x = 2.5\ \text{m}$$

$$\theta_A = \frac{ML}{3EI} = \frac{(80)(5.0)}{3EI} = \frac{133.333}{EI}$$

$$y_C = \frac{125}{EI} \qquad \text{(Same as loading I.)}$$

Loading III: 140 kN concentrated load at *C*. $P = 140$ kN

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{(140)(5.0)^2}{16EI} = -\frac{218.75}{EI}$$

$$y_C = -\frac{PL^3}{48EI} = -\frac{(140)(5.0)^3}{48EI} = -\frac{364.583}{EI}$$

Data: $E = 200\times10^9\ \text{Pa}, \quad I = 156\times10^6\ \text{mm}^4 = 156\times10^{-6}\ \text{m}^4$

$$EI = (200\times10^9)(156\times10^{-6}) = 31.2\times10^6\ \text{N}\cdot\text{m}^2 = 31{,}200\ \text{kN}\cdot\text{m}^2$$

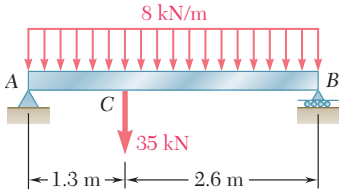
(*a*) Slope at *A*. $$\theta_A = \frac{67.667 + 133.333 - 218.75}{31{,}200} = -0.601\times10^{-3}\ \text{rad}$$

$$\theta_A = 0.601\times10^{-3}\ \text{rad} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at *C*. $$y_C = \frac{125 + 125 - 364.583}{31{,}200} = -3.67\times10^{-3}\ \text{m}$$

$$y_C = 3.67\ \text{mm}\downarrow \blacktriangleleft$$

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W360 × 39

PROBLEM 9.78

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in m.

Loading I: 8 kN/m uniformly distributed.

Case 6: $w = 8$ kN/m, $L = 3.9$ m, $x = 1.3$ m

$$\theta_A = -\frac{WL^3}{24EI} = -\frac{(8)(3.9)^3}{24EI} = -\frac{19.773}{EI}$$

$$y_C = -\frac{w}{24EI}[x^4 - 2Lx^3 + L^3x] = -\frac{8}{24EI}[(1.3)^4 - (2)(3.9)(1.3)^3 + (3.9)^3(1.3)]$$

$$= -\frac{20.945}{EI}$$

Loading II: 35 kN concentrated load at *C*. Case 5 of Appendix *D*.

$P = 35$ kN, $L = 3.9$ m, $a = 1.3$ m, $b = 2.6$ m, $x = a = 1.3$ m

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(35)(2.6)(3.9^2 - 2.6)^2}{6EI\,(3.9)} = -\frac{32.861}{EI}$$

$$y_C = -\frac{Pa^2b^2}{3EIL} = -\frac{(35)(1.3)^2(2.6)^2}{3EI(3.9)} = -\frac{34.176}{EI}$$

Data: $E = 200 \times 10^9$, $I = 102 \times 10^6\ \text{mm}^4 = 102 \times 10^{-6}\ \text{m}^4$

$$EI = (200 \times 10^9)(102 \times 10^{-6}) = 20.4 \times 10^6\ \text{N} \cdot \text{m}^2 = 20{,}400\ \text{kN} \cdot \text{m}^2$$

(*a*) Slope at *A*.

$$\theta_A = -\frac{19.773 + 32.861}{20{,}400} = -2.58 \times 10^{-3}\ \text{rad}$$

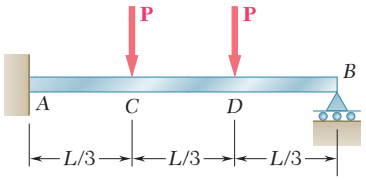
$$\theta_A = 2.58 \times 10^{-3}\ \text{rad} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at *C*.

$$y_C = -\frac{20.934 + 34.176}{20{,}400} = -2.70 \times 10^{-3}\ \text{m}$$

$$y_C = 2.70\ \text{mm} \downarrow \blacktriangleleft$$

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PROBLEM 9.79

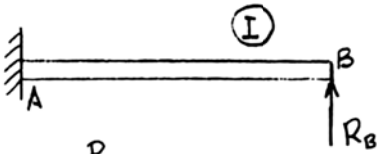
For the uniform beam shown, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.

SOLUTION

Consider R_B as redundant and replace loading system by I, II and III.

Loading I: Case 1 of Appendix *D* applied to *AB*.

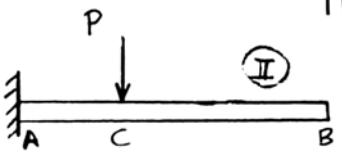
$$(y_B)_I = \frac{R_B L^3}{3EI}$$



Loading II: Case 1 applied to portion *AC*.

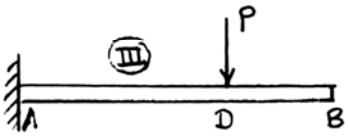
$$(\theta_C)_{II} = -\frac{P(L/3)^2}{2EI} = -\frac{1}{18}\frac{PL^2}{EI}$$

$$(y_C)_{II} = -\frac{P(L/3)^3}{3EI} = -\frac{1}{81}\frac{PL^3}{EI}$$



Portion *CB* remains straight.

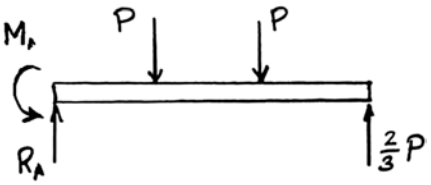
$$(y_B)_{II} = (Y_C)_{II} + \frac{2L}{3}(\theta_C)_{II} = -\frac{4}{81}\frac{PL^3}{EI}$$



Loading III: Case 1 applied to portion *AD*.

$$(\theta_D)_{III} = \frac{P(2L/3)^2}{2EI} = -\frac{2}{9}\frac{PL^2}{EI}$$

$$(y_D)_{III} = \frac{P(2L/3)^3}{3EI} = -\frac{8}{81}\frac{PL^3}{EI}$$



Portion *DB* remains straight.

$$(y_C)_{III} = (y_D)_{III} + \frac{L}{3}(\theta_D)_{III} = -\frac{14}{81}\frac{PL^3}{EI}$$

Superposition and constraint:

$$y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$$

$$\frac{1}{3}R_B L^3 - \frac{4}{81}\frac{PL^3}{EI} - \frac{14}{81}\frac{PL^3}{EI} = \frac{1}{3}\frac{R_B L^3}{EI} - \frac{2}{9}\frac{PL^3}{EI} = 0$$

(*b*) $\mathbf{R_B} = \frac{2}{3}P \uparrow$ ◀

Statics:

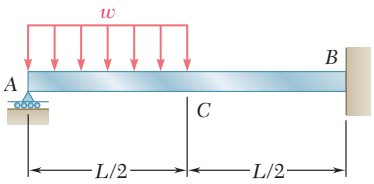
$$+\uparrow \Sigma F_y = 0: \quad R_A - P - P + \frac{2}{3}P = 0$$

(*a*) $\mathbf{R_A} = \frac{4}{3}P \uparrow$ ◀

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - P\left(\frac{L}{3}\right) - P\left(\frac{2L}{3}\right) + \left(\frac{2}{3}P\right)(L) = 0$$

$\mathbf{M_A} = \frac{1}{3}PL \circlearrowleft$ ◀

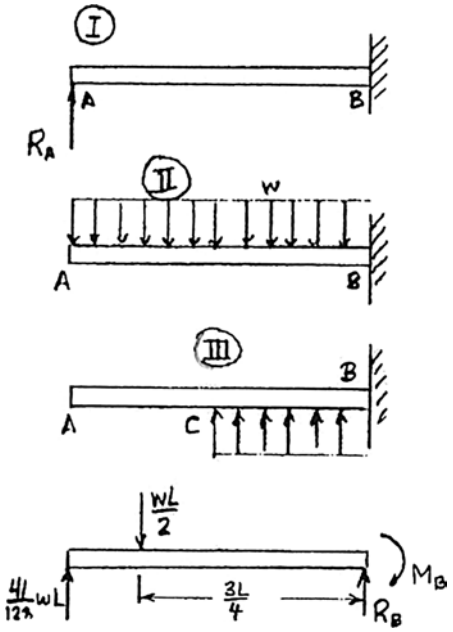
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PROBLEM 9.80

For the uniform beam shown, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.

SOLUTION



Beam is indeterminate to first degree. Consider R_A as redundant and replace the given loading by loadings I, II, and III.

Loading I: Case 1 of Appendix *D*.

$$(y_A)_I = \frac{R_A L^3}{3EI}$$

Loading II: Case 2 of Appendix *D*.

$$(y_A)_{II} = -\frac{wL^4}{8EI}$$

Loading III: Case 2 of Appendix *D* (portion *CB*).

$$(\theta_C)_{III} = -\frac{w(L/2)^3}{6EI} = -\frac{1}{48}\frac{wL^3}{EI}$$

$$(y_C)_{III} = \frac{w(L/2)^4}{8EI} = \frac{1}{128}\frac{wL^4}{EI}$$

Portion *AC* remains straight.

$$(y_A)_{III} = (y_C)_{III} + \frac{L}{2}(\theta_C)_{III} = \frac{7}{384}\frac{wL^4}{EI}$$

Superposition and constraint: $y_A = (y_A)_I + (y_A)_{II} + (y_A)_{III} = 0$

(*a*) $$\frac{1}{3}\frac{R_A L^3}{3EI} - \frac{1}{8}\frac{wL^4}{EI} + \frac{7}{384}\frac{wL^4}{EI} = \frac{1}{3}\frac{R_A L^3}{EI} - \frac{41}{384}\frac{wL^4}{EI} = 0$$

$$R_A = \frac{41}{128}wL \uparrow \blacktriangleleft$$

Statics:

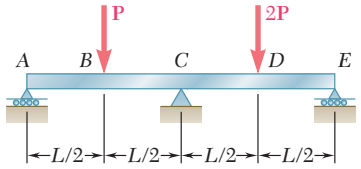
(*b*) $$+\uparrow \Sigma F_y = 0: \quad \frac{41}{128}wL - \frac{1}{2}wL + R_B = 0$$

$$R_B = \frac{23}{128}wL \uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -\left(\frac{41}{128}wL\right)L - \left(\frac{1}{2}wL\right)\left(\frac{3L}{4}\right) - M_B = 0$$

$$M_B = \frac{7}{128}wL^2 \circlearrowleft \blacktriangleleft$$

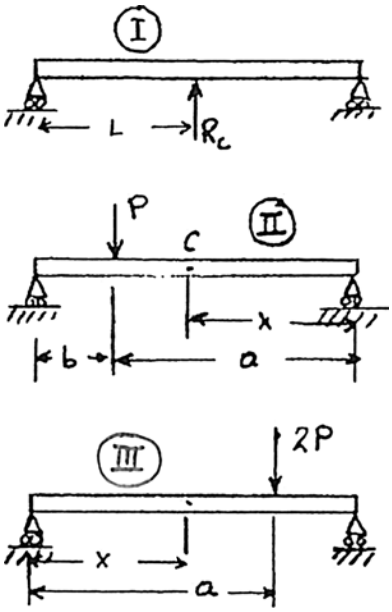
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PROBLEM 9.81

For the uniform beam shown, determine the reaction at each of the three supports.

SOLUTION



Beam is indeterminate to first degree. Consider R_C to be the redundant reaction, and replace the loading by loadings I, II, and III.

Loading I: Case 4 of Appendix *D*.

$$(y_C)_I = \frac{R_C(2L)^3}{48EI} = \frac{1}{6}\frac{R_C L^2}{EI}$$

Loading II: Case 5 of Appendix *D*.

$$(y_C)_{II} = \frac{Pb}{6EI(2L)}[x^3 - \{(2L)^2 - b^2\}x]$$

$$= \frac{P(L/2)}{12EIL}\left[L^3 - \left\{4L^2 - \left(\frac{L}{2}\right)^2\right\}L\right] = -\frac{11}{48}\frac{PL^3}{EI}$$

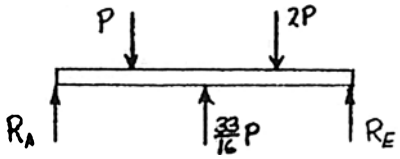
Loading III: Case 5 of Appendix *D*.

$$(y_C)_{III} = 2(y_C)_{II} = -\frac{11}{24}\frac{PL^3}{EI}$$

Superposition and constraint: $y_C = (y_C)_I + (y_C)_{II} + (y_C)_{III} = 0$

$$\frac{1}{6}\frac{R_C L^3}{EI} - \frac{11}{48}\frac{PL^3}{EI} - \frac{11}{24}\frac{PL^3}{EI} = \frac{1}{6}\frac{R_C L^3}{EI} - \frac{11}{16}\frac{PL^3}{EI} = 0$$

$$R_C = \frac{33}{16}P \uparrow \blacktriangleleft$$



Statics: $+\circlearrowleft \sum M_E = 0$:

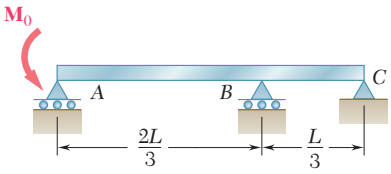
$$-R_A(2L) + P\left(\frac{3L}{2}\right) - \left(\frac{33}{16}P\right)L + (2P)\left(\frac{L}{2}\right) = 0$$

$$R_A = \frac{7}{32}P \uparrow \blacktriangleleft$$

$$+\uparrow \sum F_y = 0: \quad \frac{7}{32}P - P + \frac{33}{16}P - 2P + R_E = 0$$

$$R_E = \frac{23}{32}P \uparrow \blacktriangleleft$$

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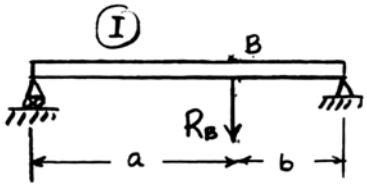


PROBLEM 9.82

For the uniform beam shown, determine the reaction at each of the three supports.

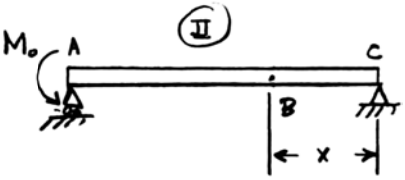
SOLUTION

Beam is statically indeterminate to first degree. Consider R_B to be the redundant reaction, and replace the loading by loading I and II.



Loading I: Case 5 of Appendix *D*.

$$(y_B)_I = -\frac{R_B a^2 b^2}{3EIL} = -\frac{R_B (2L/3)^2 (L/3)^2}{3EIL} = -\frac{4}{243}\frac{R_B L^3}{EI}$$



Loading II: Case 7 of Appendix *D*.

$$(y_B)_{II} = -\frac{M_0}{6EIL}(x^3 - L^2 x) = -\frac{M_0}{6EIL}\left[\left(\frac{L}{3}\right)^3 - L^2\left(\frac{L}{3}\right)\right]$$

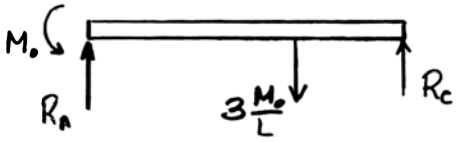
$$= \frac{4}{81}\frac{M_0 L^2}{EI}$$

Superposition and constraint:

$$y_B = (y_B)_I + (y_B)_{II} = 0$$

$$-\frac{4}{243}\frac{R_B L^3}{EI} + \frac{4}{81}\frac{M_0 L^2}{EI} = 0 \qquad R_B = 3\frac{M_0}{L} \downarrow \blacktriangleleft$$

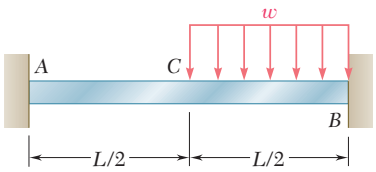
Statics:



$$+\circlearrowleft \Sigma M_C = 0: \quad -R_A L + M_0 + 3\frac{M_0}{L}\cdot\frac{L}{3} = 0 \qquad R_A = 2\frac{M_0}{L} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2\frac{M_0}{L} - 3\frac{M_0}{L} + R_C = 0 \qquad R_C = \frac{M_0}{L} \uparrow \blacktriangleleft$$

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PROBLEM 9.83

For the beam shown, determine the reaction at *B*.

SOLUTION



Portion *AC*: Superposition of Cases 3 and 1 of Appendix *D*.

$$y_C = \frac{M_C(L/2)^2}{2EI} - \frac{V_C(L/2)^3}{3EI} = \frac{M_C L^2}{8EI} - \frac{V_C L^3}{24EI}$$

$$\theta_C = \frac{M_C(L/2)}{EI} - \frac{V_C(L/2)^2}{2EI} = \frac{M_C L}{2EI} - \frac{V_C L^3}{8EI}$$

Portion *CB*: Superposition of Cases 3, 1, and 2 of Appendix *D*.

$$y_C = \frac{M_C(L/2)^2}{2EI} + \frac{V_C(L/2)^3}{3EI} - \frac{w(L/2)^4}{8EI}$$

$$= \frac{M_C L^2}{8EI} + \frac{V_C L^3}{24EI} - \frac{wL^4}{128EI}$$

$$\theta_C = -\frac{M_C(L/2)}{EI} - \frac{V_C(L/2)^3}{2EI} + \frac{w(L/2)^3}{6EI}$$

$$= -\frac{M_C L}{2EI} - \frac{V_C L^3}{8EI} + \frac{wL^3}{48EI}$$

Matching expressions for y_C,

$$\frac{M_C L^2}{8EI} - \frac{V_C L^3}{24EI} = \frac{M_C L^2}{8EI} + \frac{V_C L^4}{24EI} - \frac{wL^4}{128EI} \qquad V_C = \frac{3}{32}wL$$

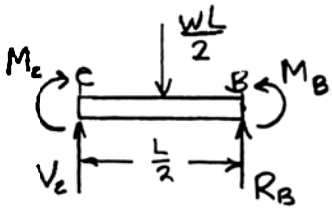
Matching expressions for θ_C,

$$\frac{M_C L}{2EI} - \frac{V_C L^3}{8EI} = -\frac{M_C L}{2EI} - \frac{V_C L^3}{8EI} + \frac{wL^3}{48EI} \qquad M_C = \frac{1}{48}wL^2$$

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PROBLEM 9.83 ***(Continued)***

Using portion *CB* as a free body,



$$+\uparrow \Sigma F_y = 0: \quad R_B + V_C - \frac{wL}{2} = 0$$

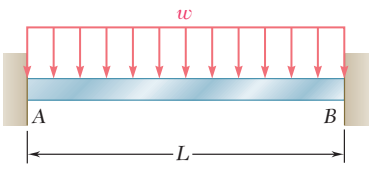
$$R_B = \frac{wL}{2} - \frac{3}{32}wL \qquad R_B = \frac{13}{32}wL \uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_B = 0: \quad M_B - M_C - V_C\frac{L}{2} + \frac{wL}{2}\cdot\frac{L}{4} = 0$$

$$M_B = \frac{1}{48}wL^2 + \left(\frac{3}{32}wL\right)\left(\frac{L}{2}\right) - \frac{wL^2}{8}$$

$$M_B = -\frac{11}{192}wL^2 \qquad M_B = \frac{11}{192}wL^2 \circlearrowleft \blacktriangleleft$$

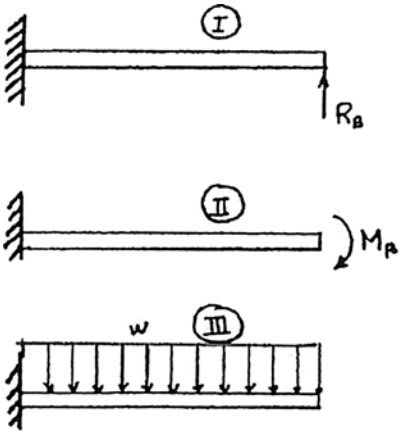
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PROBLEM 9.84

For the beam shown, determine the reaction at *B*.

SOLUTION



Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.

Loading I: Case 1 of Appendix *D*.

$$(y_B)_I = \frac{R_B L^3}{3EI} \qquad (\theta_B)_I = \frac{R_B L^2}{2EI}$$

Loading II: Case 3 of Appendix *D*.

$$(y_B)_{II} = -\frac{M_B L^2}{2EI} \qquad (\theta_B)_{II} = -\frac{M_B L}{EI}$$

Loading III: Case 2 of Appendix *D*.

$$(y_B)_{III} = -\frac{wL^4}{8EI} \qquad (\theta_B)_{III} = -\frac{wL^2}{6EI}$$

Superposition and constraint:

$$y_B = (y_B)_{\mathrm{I}} + (y_B)_{\mathrm{II}} + (y_B)_{\mathrm{III}} = 0$$

$$\frac{L^3}{3EI} R_B - \frac{L^2}{2EI} M_B - \frac{wL^4}{8EI} = 0 \qquad (1)$$

$$\theta_B = (\theta_B)_{\mathrm{I}} + (\theta_B)_{\mathrm{II}} + (\theta_B)_{\mathrm{III}} = 0$$

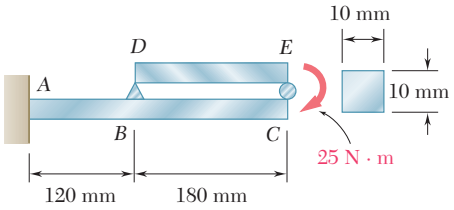
$$\frac{L^2}{2EI} R_B - \frac{L}{EI} M_B - \frac{wL^3}{6EI} = 0 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$R_B = \frac{1}{2} wL \uparrow \blacktriangleleft$$

$$M_B = \frac{1}{12} wL^2 \circlearrowright \blacktriangleleft$$

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PROBLEM 9.85

Beam *DE* rests on the cantilever beam *AC* as shown. Knowing that a square rod of side 10 mm is used for each beam, determine the deflection at end *C* if the 25-N · m couple is applied (*a*) to end *E* of the beam *DE*, (*b*) to end *C* of the beam *AC*. Use $E = 200$ GPa.

SOLUTION

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = \frac{1}{12}(10)(10)^3 = 833.33 \text{ mm}^4 = 833.33 \times 10^{-12} \text{ m}^4$$

$$EI = 166.667 \text{ N} \cdot \text{m}^2$$

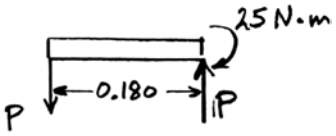
(*a*) Couple applied to beam *DE*.

Free body *DE*. $+\circlearrowleft \Sigma M = 0$: $0.180P - 25 = 0$ $\quad P = 138.889$ N

Loads on cantilever beam *ABC* are $P\uparrow$ at point *B* and $P\downarrow$ at point *C* as shown.

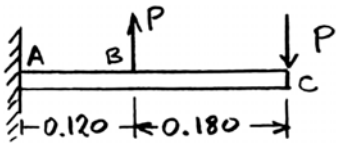
Due to $P\uparrow$ at point *B*.

Using portion *AB* and applying case 1 of Appendix *D*,



$$(y_B)_1 = \frac{PL^3}{3EI} = \frac{(138.889)(0.120)^3}{(3)(166.667)} = 0.480 \times 10^{-3} \text{ m}$$

$$(\theta_B)_1 = \frac{PL^2}{2EI} = \frac{(138.889)(0.120)^2}{(2)(166.667)} = 6.00 \times 10^{-3}$$



$$(y_C)_1 = (y_B)_1 + L_{BC}(\theta_B)_1$$
$$= 0.480 \times 10^{-3} + (0.180)(6.00 \times 10^{-3})$$
$$= 1.56 \times 10^{-3} \text{ m}$$

Due to load $P\downarrow$ at point *C*. Case 1 of App. *D* applied to *ABC*.

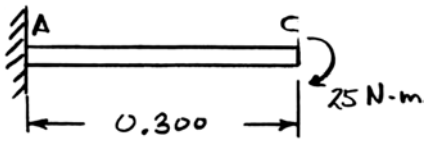
$$(y_C)_2 = -\frac{PL^3}{3EI} = -\frac{(138.889)(0.120 + 0.180)^3}{(3)(166.667)} = -7.50 \times 10^{-3} \text{ m}$$

Total deflection at point *C*. $\quad y_C = (y_C)_1 + (y_C)_2 = -5.94 \times 10^{-3}$ m

$$y_C = 5.94 \text{ mm} \downarrow \blacktriangleleft$$

(*b*) Couple applied to beam *AC*.

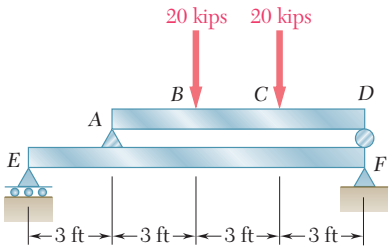
Case 3 of Appendix *D*.



$$y_C = -\frac{ML^2}{2EI} = -\frac{(25)(0.300)^2}{(2)(166.667)} = -6.75 \times 10^{-3} \text{ m}$$

$$y_C = 6.75 \text{ mm} \downarrow \blacktriangleleft$$

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PROBLEM 9.86

Beam *AD* rests on beam *EF* as shown. Knowing that a W12 × 26 rolled-steel shape is used for each beam, determine for the loading shown the deflection at points *B* and *C*. Use $E = 29 \times 10^6$ psi.

SOLUTION

$$E = 29 \times 10^3 \text{ ksi} \quad I = 204 \text{ in}^4$$

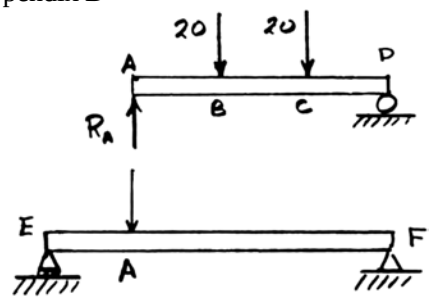
$$EI = (29 - 10^3)(204) = 5.916 \times 10^6 \text{ kip} \cdot \text{in}^2$$

$$= 41{,}083 \text{ kip} \cdot \text{ft}^2$$

For equilibrium of beam *ABCD*, $R_A = 20$ kips

Deflection at point *A* is due to bending of beam *EAF*. Using Case 5 of Appendix *D*

$$y_A = -\frac{Pa^2b^2}{3EIL} = -\frac{(20)(3)^2(9)^2}{(3)(EI)(12)} = -\frac{405}{EI} \text{ ft}$$



Assuming that beam *ABCD* is rigid,

$$y'_B = \frac{6}{9} y_A = -\frac{270}{EI} \text{ ft}$$

$$y'_C = \frac{3}{9} y_A = -\frac{135}{EI} \text{ ft}$$

Additional deflection at *B* due to bending of beam *ABCD*. Using case 5,

$$y''_B = -\frac{P_B a^2 b^2}{3EIL} + \frac{P_C b}{6EIL}[x^3 - (L^2 - b^2)x]$$

$$= -\frac{(20)(3)^2(6)^2}{(3)(EI)(9)} + \frac{(20)(3)[(3)^3 - (9^2 - 3^2)(3)]}{(6)(EI)(9)}$$

$$= -\frac{240}{EI} - \frac{210}{EI} = -\frac{450}{EI} \text{ ft}$$

Additional deflection at *C* due to bending of beam *ABCD*.

By symmetry, $y''_C = y''_B = -\frac{450}{EI}$ ft

Total deflection at *B*.

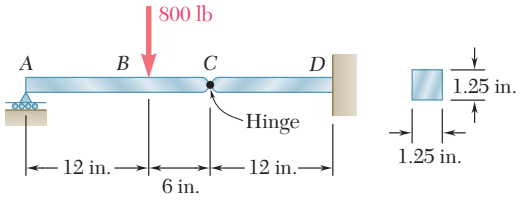
$$y_B = y'_B + y''_B = -\frac{270}{EI} - \frac{450}{EI} = -\frac{720}{EI} = -\frac{720}{41{,}083} = -17.525 \times 10^{-3} \text{ ft}$$

$$y_B = 0.210 \text{ in.} \downarrow \blacktriangleleft$$

$$y_C = y'_C + y''_C = -\frac{135}{EI} - \frac{450}{EI} = -\frac{585}{EI} = -\frac{585}{41{,}083} = -14.239 \times 10^{-3} \text{ ft}$$

$$y_C = 0.1709 \text{ in.} \downarrow \blacktriangleleft$$

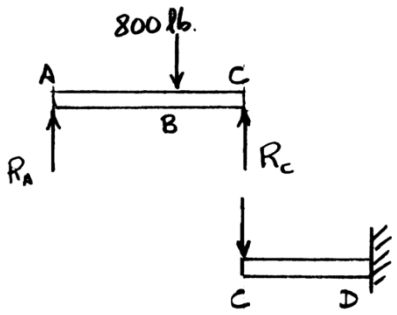
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PROBLEM 9.87

The two beams shown have the same cross section and are joined by a hinge at *C*. For the loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *B*. Use $E = 29\times10^6$ psi.

SOLUTION



Using free body *ABC*,

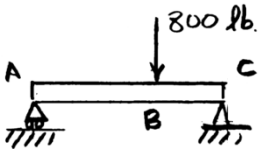
$$+\circlearrowleft \Sigma M_A = 0: \quad 18R_C - (12)(800) = 0 \qquad R_C = 533.33 \text{ lb}$$

$$E = 29\times10^6 \text{ psi}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(1.25)(1.25)^3 = 0.20345 \text{ in}^4$$

$$EI = (29\times10^6)(0.20345) = 5.900\times10^6 \text{ lb}\cdot\text{in}^2$$

Using cantilever beam *CD* with load R_C,



Case 1 of Appendix *D*.

$$y_C = -\frac{R_C L_{CD}^3}{3EI} = -\frac{(533.33)(12)^3}{(3)(5.900\times10^6)} = -52.067\times10^{-3} \text{ in.}$$

Calculation of θ'_A and y'_B assuming that point *C* does not move.

Case 5 of Appendix *D*.

$$P = 800 \text{ lb}, \quad L = 18 \text{ in.}, \quad a = 12 \text{ in.}, \quad b = 6 \text{ in.}$$

$$\theta'_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(800)(6)(18^2 - 6^2)}{(6)(5.900\times10^6)(18)} = -2.1695\times10^{-3} \text{ rad}$$

$$y'_B = -\frac{Pb^2a^2}{3EIL} = -\frac{(800)(6)^2(12)^2}{(3)(5.900\times10^6)(18)} = -13.017\times10^{-3} \text{ in.}$$

Additional slope and deflection due to movement of point *C*.

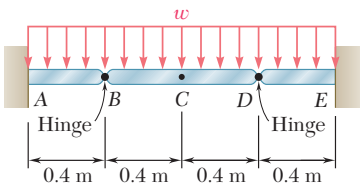
$$\theta''_A = \frac{y_C}{L_{AC}} = -\frac{52.067\times10^{-3}}{18} = -2.8926\times10^{-3} \text{ rad}$$

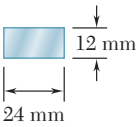
$$y''_B = \frac{a}{L}y_C = -\frac{(12)(52.067\times10^{-3})}{18} = -34.711\times10^{-3} \text{ in.}$$

(*a*) Slope at *A*. $\theta_A = \theta'_A + \theta''_A = -2.1695\times10^{-3} - 2.8926\times10^{-3}$ $\qquad \theta_A = 5.06\times10^{-3}$ rad ◀

(*b*) Deflection at *B*. $y_B = y'_B + y''_B = -13.017\times10^{-3} - 34.711\times10^{-3}$ $\qquad y_B = 0.0477$ in. ↓ ◀

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PROBLEM 9.88

A central beam *BD* is joined at hinges to two cantilever beams *AB* and *DE*. All beams have the cross section shown. For the loading shown, determine the largest *w* so that the deflection at *C* does not exceed 3 mm. Use $E = 200$ GPa.

SOLUTION

Let $a = 0.4$ m.

Cantilever beams *AB* and *CD*.

Cases 1 and 2 of Appendix *D*.

$$y_B = y_D = -\frac{(wa)a^3}{3EI} - \frac{wa^4}{8EI} = -\frac{11}{24}\frac{wa^4}{EI}$$

Beam *BCD*, with $L = 0.8$ m, assuming that points *B* and *D* do not move.

Case 6 of Appendix *D*.

$$y'_C = -\frac{5wL^4}{384EI}$$

Additional deflection due to movement of points *B* and *D*.

$$y''_C = y_B = y_D = -\frac{11}{24}\frac{wa^4}{EI}$$

Total deflection at *C*.

$$y_C = y'_C + y''_C$$

$$y_C = -\frac{w}{EI}\left\{\frac{5L^4}{384} + \frac{11a^4}{24}\right\}$$

Data:

$$E = 200\times10^9 \text{ Pa},$$

$$I = \frac{1}{12}(24)(12)^3 = 3.456\times10^{-3}\text{ mm}^4 = 3.456\times10^{-9}\text{ m}^4$$

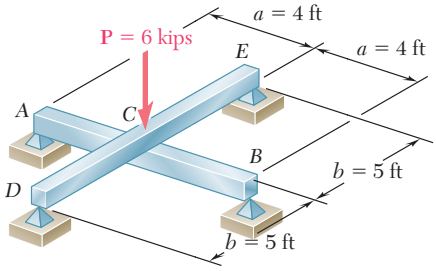
$$EI = (200\times10^9)(3.456\times10^{-9}) = 691.2\text{ N}\cdot\text{m}^2$$

$$y_C = -3\times10^{-3}\text{ m}$$

$$-3\times10^{-3} = -\frac{w}{691.2}\left\{\frac{(5)(0.8)^4}{384} + \frac{(11)(0.4)^4}{24}\right\} = -24.69\times10^{-6}w$$

$$w = 121.5\text{ N/m} \blacktriangleleft$$

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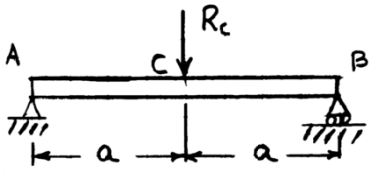
PROBLEM 9.89

For the loading shown, and knowing that beams *AB* and *DE* have the same flexural rigidity, determine the reaction (*a*) at *B*, (*b*) at *E*.

SOLUTION

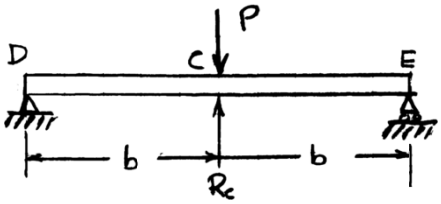
Units: Forces in kips; lengths in ft.

For beam *ACB*, using Case 4 of Appendix *D*:



$$(y_C)_1 = -\frac{R_C(2a)^3}{48EI}$$

For beam *DCE*, using Case 4 of Appendix *D*:



$$(y_C)_2 = \frac{(R_C - P)(2b)^3}{48EI}$$

Matching deflections at *C*,

$$-\frac{R_C(2a)^3}{48EI} = \frac{(R_C - P)(2b)^3}{48EI}$$

$$R_C = \frac{Pb^3}{a^3 + b^3} = \frac{(6)(5)^3}{4^3 + 5^3} = 3.968 \text{ kips}$$

$$P - R_C = 6 - 3.968 = 2.032 \text{ kips}$$

Using free body *ACB*,

$$+\circlearrowleft M_A = 0: \quad 2aR_B - aR_C = 0$$

(*a*) Reaction at *B*.

$$R_B = \frac{1}{2}R_C \qquad R_B = 1.984 \text{ kips} \uparrow \blacktriangleleft$$

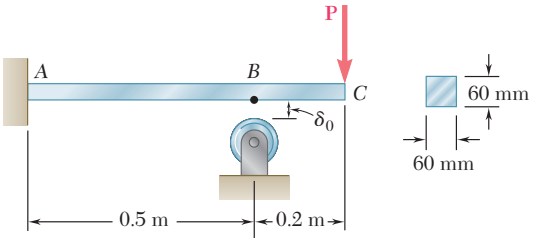
Using free body *DCE*,

$$+\circlearrowleft M_D = 0: \quad 2bR_E - b(P - R_C) = 0$$

(*b*) Reaction at *E*.

$$R_E = \frac{1}{2}(P - R_C) \qquad R_E = 1.016 \text{ kips} \uparrow \blacktriangleleft$$

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PROBLEM 9.90

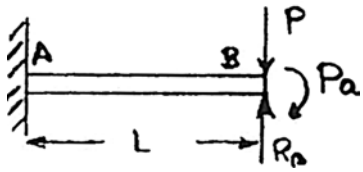
Before the load **P** was applied, a gap, $\delta_0 = 0.5\,\text{mm}$, existed between the cantilever beam *AC* and the support at *B*. Knowing that $E = 200$ GPa, determine the magnitude of **P** for which the deflection at *C* is 1 mm.

SOLUTION

Let length $AB = L = 0.5$ m

length $BC = a = 0.2$ m

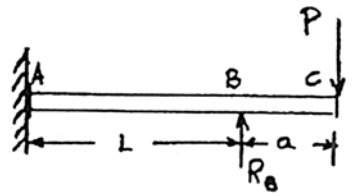
Consider portion *AB* of beam *ABC*.



The loading becomes forces *P* and R_B at *B* plus the couple *Pa*. The deflection at *B* is δ_0. Using Cases 1 and 3 of Appendix *D*,

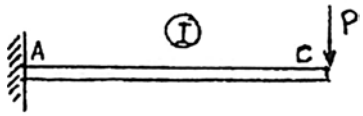
$$\delta_0 = \frac{(P - R_B)L^3}{3EI} + \frac{PaL^2}{2EI}$$

$$\left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)P - \frac{L^3}{3}R_B = EI\delta_0 \qquad (1)$$



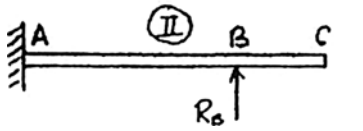
The deflection at *C* depends on the deformation of beam *ABC* subjected to loads *P* and R_B. For loading I, using Case 1 of Appendix *D*,

$$(\delta_C)_1 = \frac{P(L + a)^3}{3EI}$$



For loading II, using Case 1 of Appendix *D*,

$$y_B = \frac{R_B L^3}{3EI} \quad \theta_B = \frac{R_B L^2}{2EI}$$



Portion *BC* remains straight.

$$y_C = y_B + a\theta_B = \left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)\frac{R_B}{EI}$$

By superposition, the downward deflection at *C* is

$$\delta_C = \frac{P(L + a)^3}{3EI} - \left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)\frac{R_B}{EI}$$

$$\frac{(L + a)^3}{3}P - \left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)R_B = EI\delta_C \qquad (2)$$

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PROBLEM 9.90 *(Continued)*

Data: $E = 200 \times 10^9 \text{ Pa}$ $\quad I = \frac{1}{12}(60)(60)^3 = 1.08 \times 10^6 \text{ mm}^4 = 1.08 \times 10^{-6} \text{ m}^4$

$$EI = 216 \times 10^3 \text{ N} \cdot \text{m}^2$$

$$\delta_0 = 0.5 \times 10^{-3} \text{ m} \quad \delta_C = 1.0 \times 10^{-3} \text{ m}$$

Using the data, Eqs. (1) and (2) become

$$0.06667P - 0.04167R_B = 108 \tag{1}'$$

$$0.11433P - 0.06667R_B = 216 \tag{2}'$$

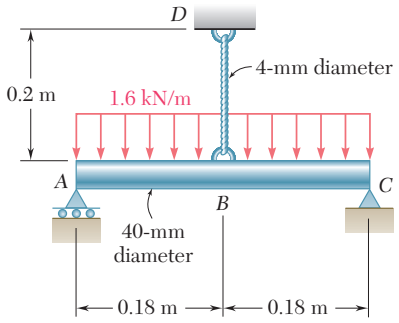
Solving simultaneously,

$$P = 5.63 \times 10^3 \text{ N}$$

$$R_B = 6.42 \times 10^3 \text{ N}$$

$P = 5.63 \text{ kN} \downarrow$ ◀

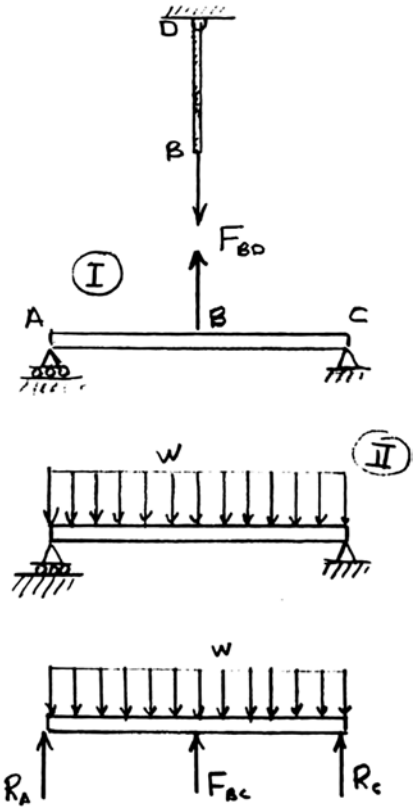
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PROBLEM 9.91

Knowing that the rod *ABC* and the wire *BD* are both made of steel, determine (*a*) the deflection at *B*, (*b*) the reaction at *A*. Use $E = 200$ GPa.

SOLUTION



Let F_{BD} be the tension in wire *BD*. The elongation of the wire is

$$\delta_{BD} = \frac{F_{BD}l}{EA}$$

Beam *ABC* is subjected to loads F_{BD} (I) and w (II).

Loading I: Case 4 of Appendix *D*.

$$(y_B)_I = \frac{F_{BD}L^3}{48EI}$$

Loading II: Case 6 of Appendix *D*.

$$(y_B)_{II} = -\frac{5}{384}\frac{wL^4}{EI}$$

Deflection at *B*.

$$-\delta_{BD} = y_B = (y_B)_I + (y_B)_{II} - \frac{F_{BD}l}{EA}$$

$$= \frac{F_{BD}L^3}{48EI} - \frac{5}{384}\frac{wL^4}{EI}\left(\frac{l}{EA} + \frac{L^3}{48EI}\right)F_{BD}$$

$$= \frac{5}{384}\frac{wL^4}{EI}$$

Data:

$$l = 0.2\text{ m}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(4)^2 = 12.566\text{ mm}^2$$

$$= 12.566 \times 10^{-6}\text{ m}^2$$

$$E = 200 \times 10^9\text{ Pa}$$

$$\frac{l}{EA} = 79.58 \times 10^{-9}\text{ m/N}$$

$$L = 0.36\text{ m}$$

$$w = 1.6 \times 10^3\text{ N/m}$$

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PROBLEM 9.91 ***(Continued)***

$$I = \frac{\pi}{4}C^4 = \frac{\pi}{4}\left(\frac{40}{2}\right)^4 = 125.66 \times 10^3 \text{ mm}^4 = 125.66 \times 10^{-9} \text{ m}^4$$

$$EI = (200 \times 10^9)(125.66 \times 10^{-9}) = 25.132 \times 10^3 \text{ N} \cdot \text{m}^2$$

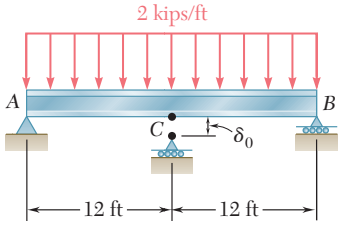
$$\left[79.58 \times 10^{-9} + \frac{(0.36)^3}{(48)(25.132 \times 10^3)}\right] F_{BD} = \frac{(5)(1.6 \times 10^3)(0.36)^4}{(384)(25.132 \times 10^3)}$$

$$118.256 \times 10^{-9} F_{BD} = 13.923 \times 10^{-6} \quad F_{BD} = 117.74 \text{ N}$$

(*a*) Deflection at *B*. $\delta_B = \dfrac{F_{BD}l}{EA} = (117.74)(79.58 \times 10^{-9}) = 9.37 \times 10^{-6} \text{ m} = 0.00937 \text{ mm} \downarrow$ ◀

(*b*) $R_A = R_C = \dfrac{1}{2}[wL - F_{BD}] = \dfrac{1}{2}\left[(1600)(0.36) - 117.74\right] = 229 \text{ N} \uparrow$ ◀

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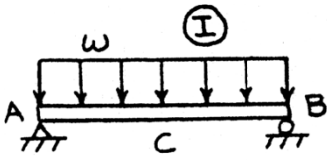
PROBLEM 9.92

Before the 2-kip/ft load is applied, a gap, $\delta_0 = 0.8$ in., exists between the W16×40 beam and the support at *C*. Knowing that $E = 29\times10^6$ psi, determine the reaction at each support after the uniformly distributed load is applied.

SOLUTION

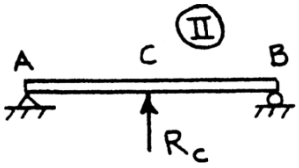
Data:

$$\delta_0 = 0.8 \text{ in.} = 66.667\times10^{-3} \text{ ft}$$
$$E = 29\times10^6 \text{ psi} = 29\times10^3 \text{ ksi}$$
$$I = 518 \text{ in}^4$$
$$EI = 15.022\times10^6 \text{ kip}\cdot\text{in}^2$$
$$= 104.319\times10^3 \text{ kip}\cdot\text{ft}^2$$



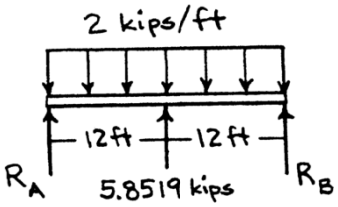
Loading I: Case 6 of Appendix *D*.

$$y_C' = -\frac{5wL^4}{384EI}$$
$$= -\frac{5(2)(24)^4}{384(104.319\times10^3)}$$
$$= -82.823\times10^{-3} \text{ ft}$$



Loading II: Case 4 of Appendix *D*.

$$y_C'' = \frac{R_C L^3}{48EI} = \frac{R_C(24)^3}{48(104.319\times10^3)}$$
$$= 2.7608\times10^{-3} R_C$$



Deflection at *C*.

$$y_C = y_C' + y_C'' = -\delta_0$$
$$-82.823\times10^{-3} + 2.7608\times10^{-3} R_C = -66.667\times10^{-3}$$
$$R_C = 5.8519 \text{ kips}$$

$R_C = 5.85$ kips ↑ ◀

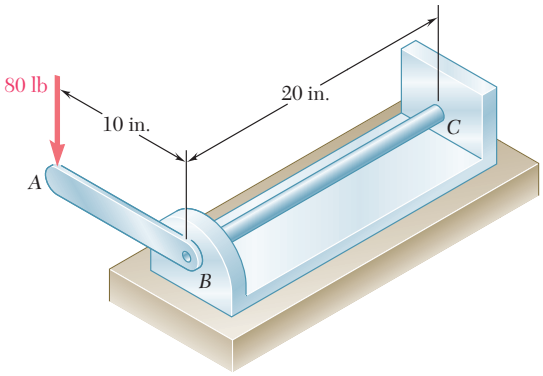
$$+\circlearrowleft \Sigma M_B = 0: \quad (2)(24)(12) - R_A(24) - (5.8519)(12) = 0$$
$$R_A = 21.074 \text{ kips}$$

$R_A = 21.1$ kips ↑ ◀

$$+\uparrow \Sigma F_y = 0: \quad 21.074 - 2(24) + 5.8519 + R_B = 0$$

$R_B = 21.1$ kips ↑ ◀

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PROBLEM 9.93

A $\frac{7}{8}$-in.-diameter rod *BC* is attached to the lever *AB* and to the fixed support at *C*. Lever *AB* has a uniform cross section $\frac{3}{8}$ in. thick and 1 in. deep. For the loading shown, determine the deflection of point *A*. Use $E = 29\times10^{6}$ psi and $G = 11.2\times10^{6}$ psi.

SOLUTION

Deformation of rod *BC*. (Torsion)

$$c = \frac{1}{2}d = \frac{1}{2}\left(\frac{7}{8}\right) = 0.4375 \text{ in.}$$

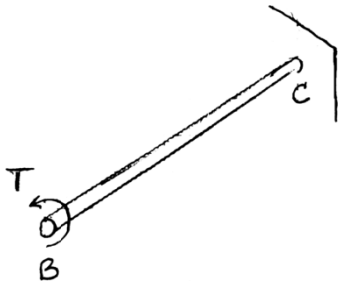
$$J = \frac{\pi}{2}C^4 = 57.548\times10^{-3} \text{ in}^4$$

$$T = Pa = (80)(10) = 800 \text{ lb}\cdot\text{in.}$$

$$L = 20 \text{ in.}$$

$$\varphi_B = \frac{TL}{GJ} = \frac{(800)(20)}{(11.2\times10^{6})(57.548\times10^{-3})}$$

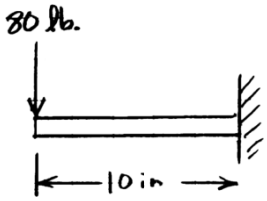
$$= 24.824\times10^{-3} \text{ rad}$$



Deflection of point *A* assuming lever *AB* to be rigid.

$$(y_A)_1 = a\varphi_B = (10)(24.824\times10^{-3})$$

$$= 0.24824 \text{ in.}\downarrow$$



Additional deflection due to bending of lever *AB*.

Refer to Case 1 of Appendix *D*.

$$I = \frac{1}{12}\left(\frac{3}{8}\right)(1)^3 = 31.25\times10^{-3} \text{ in}^4$$

$$(y_A) = \frac{PL^3}{3EI} = \frac{(80)(10)^3}{(3)(29\times10^{6})(31.25\times10^{-3})}$$

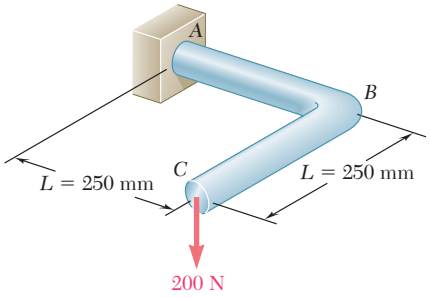
$$= 0.02943 \text{ in.}\downarrow$$

Total deflection at point *A*.

$$y_A = (y_A)_1 + (y_A)_2$$

$$y_A = 0.278 \text{ in.}\downarrow \blacktriangleleft$$

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PROBLEM 9.94

A 16-mm-diameter rod has been bent into the shape shown. Determine the deflection of end C after the 200-N force is applied. Use $E = 200$ GPa and $G = 80$ GPa.

SOLUTION

Let 200 N = P.

Consider torsion of rod AB.

$$\phi_B = \frac{TL}{JG} = \frac{(PL)L}{JG} = \frac{PL^2}{JG}$$

$$y'_C = -L\phi_B = -\frac{PL^3}{JG}$$

Consider bending of AB. (Case 1 of Appendix D.)

$$y''_C = y_B = -\frac{PL^3}{3EI}$$

Consider bending of BC. (Case 1 of Appendix D.)

$$y'''_C = -\frac{PL^3}{3EI}$$

Superposition:

$$y_C = y'_C + y''_C + y'''_C$$

$$= -\frac{PL^3}{JG} - \frac{PL^3}{3EI} - \frac{PL^3}{3EI} = -\frac{PL^3}{EI}\left(\frac{EI}{JG} + \frac{2}{3}\right)$$

Data:

$$G = 80(10^9)\ \text{Pa} \qquad J = \frac{1}{2}\pi(0.008)^4 = 6.4340(10^{-9})\ \text{m}^4$$

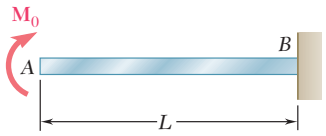
$$E = 200(10^9)\ \text{Pa} \qquad I = \frac{1}{2}J = 3.2170(10^{-9})\ \text{m}^4$$

$$EI = 643.40\ \text{N}\cdot\text{m}^2 \qquad JG = 514.72\ \text{N}\cdot\text{m}^2$$

$$y_C = -\frac{(200)(0.25)^3}{643.40}\left(\frac{643.40}{514.72} + \frac{2}{3}\right) = -9.3093(10^{-3})\ \text{m}$$

$$y_C = 9.31\ \text{mm} \downarrow \blacktriangleleft$$

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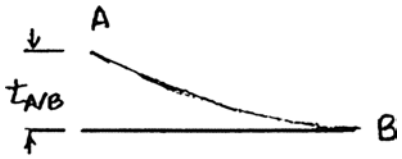
PROBLEM 9.95

For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

SOLUTION

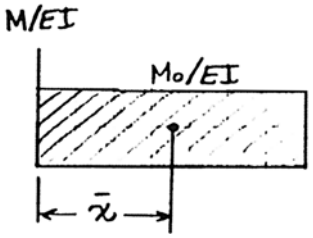
Place reference tangent at *B*.

Draw *M*/*EI* diagram.



$$A = \left(\frac{M_0}{EI}\right)L = \frac{M_0 L}{EI}$$

$$\bar{x} = \frac{1}{2}L$$



$$\theta_{B/A} = A = \frac{M_0 L}{EI}$$

$$t_{B/A} = A\bar{x} = \left(\frac{M_0 L}{EI}\right)\left(\frac{1}{2}L\right)$$

$$= \frac{M_0 L^2}{2EI}$$

(*a*) Slope at end *A*.

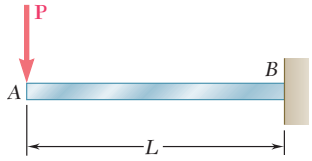
$$\theta_B = \theta_A + A \qquad 0 = \theta_A + \frac{M_0 L}{EI}$$

$$\theta_A = -\frac{M_0 L}{EI} \qquad \theta_A = \frac{M_0 L}{EI} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at *A*.

$$y_A = t_{A/B} = \frac{M_0 L^2}{2EI} \qquad y_A = \frac{M_0 L^2}{2EI} \uparrow \blacktriangleleft$$

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PROBLEM 9.96

For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

SOLUTION

Place reference tangent at *B*.

Draw *M*/*EI* diagram.

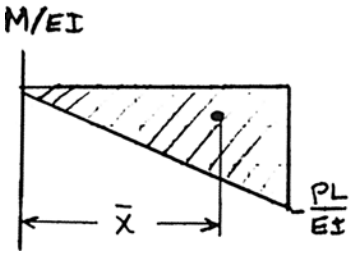


$$A = \frac{1}{2}\left(-\frac{PL}{EI}\right)L = -\frac{PL^2}{2EI}$$

$$\bar{x} = \frac{2}{3}L$$

$$\theta_{B/A} = A = -\frac{PL^2}{2EI}$$

$$t_{A/B} = A\bar{x} = \left(-\frac{PL^3}{2EI}\right)\left(\frac{2}{3}L\right) = -\frac{PL^3}{3EI}$$



(*a*) Slope at end *A*.

$$\theta_B = \theta_A + A$$

$$0 = \theta_A - \frac{PL^2}{2EI}$$

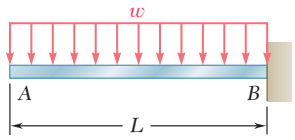
$$\theta_A = \frac{PL^2}{2EI}$$ ◀

(*b*) Deflection at *A*.

$$y_A = t_{A/B} = -\frac{PL^3}{3EI}$$

$$y_A = \frac{PL^3}{3EI}\downarrow$$ ◀

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PROBLEM 9.97

For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

SOLUTION

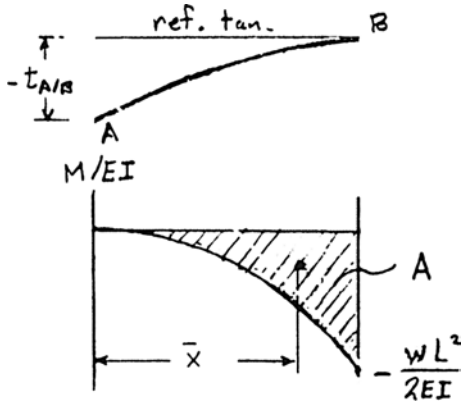
Place reference tangent at *B*.

$$\theta_B = 0$$

Draw *M*/*EI* curve as parabola.

$$A = -\frac{1}{3}\left(\frac{wL^2}{2EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

$$\bar{x} = L - \frac{1}{4}L = \frac{3}{4}L$$



By first moment-area theorem,

$$\theta_{B/A} = A = -\frac{1}{6}\frac{wL^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{6}\frac{wL^3}{EI} = \frac{1}{6}\frac{wL^3}{EI}$$

By second moment-area theorem,

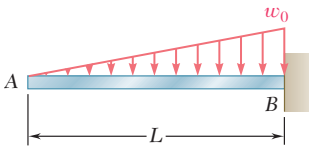
$$t_{A/B} = \bar{x}A = \left(\frac{3}{4}L\right)\left(-\frac{1}{6}\frac{wL^3}{EI}\right) = -\frac{1}{8}\frac{wL^4}{EI}$$

$$y_A = t_{A/B} = -\frac{1}{8}\frac{wL^4}{EI}$$

(*a*) $\theta_A = \dfrac{wL^3}{6EI}$ ◀

(*b*) $y_A = \dfrac{wL^4}{8EI}\downarrow$ ◀

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PROBLEM 9.98

For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

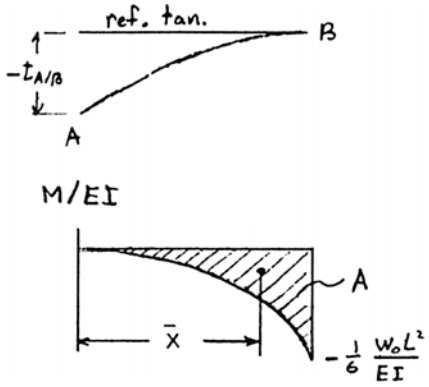
SOLUTION

Place reference tangent at *B*.

$$\theta_B = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad \left(\frac{1}{2}w_0 L\right)\frac{L}{3} + M_B = 0$$

$$M_B = -\frac{1}{6}w_0 L^2$$



Draw M/EI curve as cubic parabola.

$$A = -\frac{1}{4}\left(\frac{1}{6}\frac{w_0 L^2}{EI}\right)L = -\frac{1}{24}\frac{w_0 L^3}{EI}$$

$$\bar{x} = L - \frac{1}{5}L = \frac{4}{5}L$$

By first moment-area theorem,

$$\theta_{B/A} = A = -\frac{1}{24}\frac{w_0 L^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{24}\frac{w_0 L^3}{EI} = \frac{1}{24}\frac{w_0 L^3}{EI}$$

By second moment-area theorem,

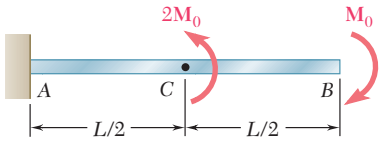
$$t_{A/B} = \bar{x}A = \left(\frac{4}{5}L\right)\left(-\frac{1}{24}\frac{w_0 L^3}{EI}\right) = -\frac{1}{30}\frac{w_0 L^4}{EI}$$

$$y_A = t_{A/B} = -\frac{1}{30}\frac{w_0 L^4}{EI}$$

(*a*) $\theta_A = \dfrac{w_0 L^3}{24EI}$ ◀

(*b*) $y_A = \dfrac{w_0 L^4}{30EI} \downarrow$ ◀

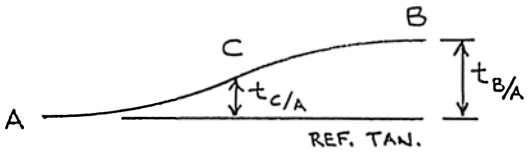
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PROBLEM 9.99

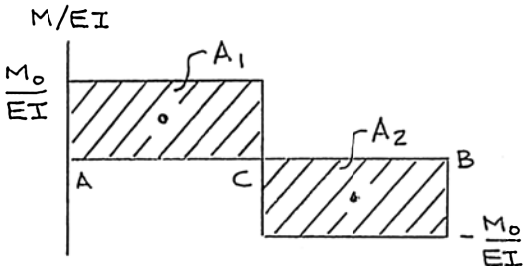
For the uniform cantilever beam and loading shown, determine the slope and deflection at (*a*) point *B*, (*b*) point *C*.

SOLUTION



Place reference tangent at *A*. $\theta_A = 0$

Draw $\dfrac{M}{EI}$ diagram.



$$A_1 = \left(\frac{M_0}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{2}\frac{M_0 L}{EI}$$

$$A_2 = \left(-\frac{M_0}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{2}\frac{M_0 L}{EI}$$

(*a*) Slope at *B*.

$$\theta_{B/A} = A_1 + A_2 = \frac{1}{2}\frac{M_0 L}{EI} - \frac{1}{2}\frac{M_0 L}{EI} = 0$$

$$\theta_B = \theta_A + \theta_{B/A} = 0 \qquad \theta_B = 0 \blacktriangleleft$$

Deflection at *B*.

$$y_B = t_{B/A} = A_1\left(\frac{L}{2} + \frac{1}{2}\cdot\frac{L}{2}\right) + A_2\left(\frac{1}{2}\cdot\frac{L}{2}\right)$$

$$= \frac{3}{8}\frac{M_0 L^2}{EI} - \frac{1}{8}\frac{M_0 L^2}{EI} = \frac{1}{4}\frac{M_0 L^2}{EI} \qquad y_B = \frac{1}{4}\frac{M_0 L^2}{EI} \uparrow \blacktriangleleft$$

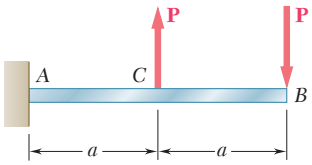
(*b*) Slope at *C*.

$$\theta_{C/A} = A_1 = \frac{1}{2}\frac{M_0 L}{EI} \qquad \theta_C = \theta_A + \theta_{C/A} \qquad \theta_C = \frac{1}{2}\frac{M_0 L}{EI} \blacktriangleleft$$

Deflection at *C*.

$$y_C = t_{C/A} = A_1\left(\frac{1}{2}\cdot\frac{L}{2}\right) = \frac{1}{8}\frac{M_0 L^2}{EI} \qquad y_C = \frac{1}{8}\frac{M_0 L^2}{EI} \uparrow \blacktriangleleft$$

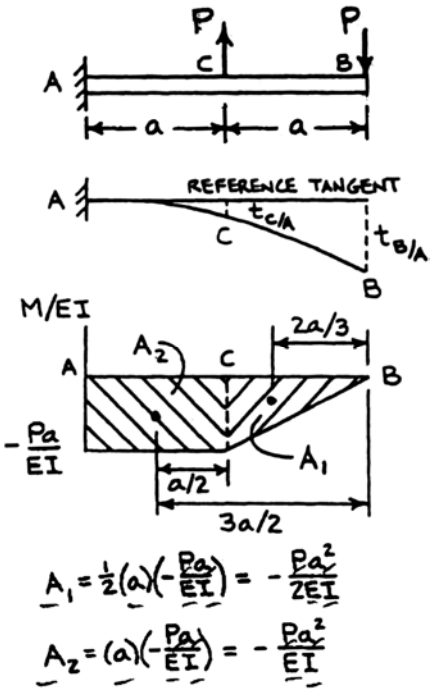
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PROBLEM 9.100

For the uniform cantilever beam and loading shown, determine the slope and deflection at (*a*) point *B*, (*b*) point *C*.

SOLUTION



(*a*) At point *B*:

$$\theta_B = \theta_{B/A} = A_1 + A_2 = -\frac{Pa^2}{2EI} - \frac{Pa^2}{EI} = -\frac{3Pa^2}{2EI}$$

$$\theta_B = \frac{3Pa^2}{2EI} \curvearrowright \blacktriangleleft$$

$$y_B = t_{B/A} = A_1\left(\frac{2a}{3}\right) + A_2\left(\frac{3a}{2}\right)$$

$$= \left(-\frac{Pa^2}{2EI}\right)\left(\frac{2a}{3}\right) + \left(-\frac{Pa^2}{EI}\right)\left(\frac{3a}{2}\right)$$

$$= -\frac{11Pa^3}{6EI}$$

$$y_B = \frac{11Pa^3}{6EI} \downarrow \blacktriangleleft$$

(*b*) At point *C*:

$$\theta_C = \theta_{C/A} = A_2 = -\frac{Pa^2}{EI}$$

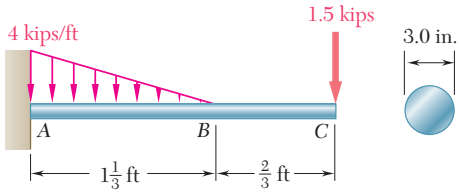
$$\theta_C = \frac{Pa^2}{EI} \curvearrowright \blacktriangleleft$$

$$y_C = t_{C/A} = A_2\left(\frac{a}{2}\right)$$

$$= \left(-\frac{Pa^2}{EI}\right)\left(\frac{a}{2}\right) = -\frac{Pa^3}{2EI}$$

$$y_C = \frac{Pa^3}{2EI} \downarrow \blacktriangleleft$$

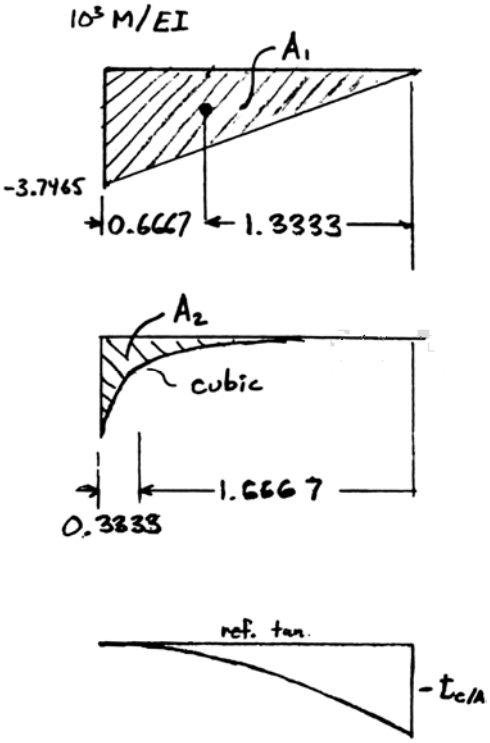
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PROBLEM 9.101

For the cantilever beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *C*. Use $E = 29 \times 10^6$ psi.

SOLUTION



Units: Forces in kips, lengths in ft.

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}(1.5)^4 = 3.9761 \text{ in}^4$$

$$EI = (29 - 10^3)(3.9761)$$

$$= 115.307 \times 10^3 \text{ kip} \cdot \text{in}^2 = 800.74 \text{ kip} \cdot \text{ft}^2$$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = -\frac{(1.5)(2)}{800.74} = -3.7465 \times 10^{-3} \text{ ft}^{-1}$$

$$A_1 = \frac{1}{2}(-3.7465 \times 10^{-3})(2) = -3.7465 \times 10^{-3}$$

$$\bar{x}_1 = \frac{1}{3}(2) = 0.66667 \text{ ft}$$

$$\frac{M_2}{EI} = \frac{\frac{1}{2}(4)\left(\frac{4}{3}\right)\left(\frac{1}{3} \times \frac{4}{3}\right)}{800.74} = -1.48011 \times 10^{-3} \text{ ft}^{-1}$$

$$A_1 = \frac{1}{4}(-1.48011 \times 10^{-3})\left(\frac{4}{3}\right)$$

$$= -0.49337 \times 10^{-3}$$

$$\bar{x} = \frac{1}{4} \cdot \frac{4}{3} = 0.33333 \text{ ft}$$

Place reference tangent at *A*. $\theta_A = 0$

$$\theta_{C/A} = A_1 + A_2 = -4.24 \times 10^{-3} \text{ rad}$$

$$\theta_C = \theta_A + \theta_{C/A} = -4.24 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

$$t_{C/A} = (1.3333)(-3.7465 \times 10^{-3})$$

$$+ (1.6667)(-0.49337 \times 10^{-3})$$

$$= -6.71 \times 10^{-3} \text{ ft}$$

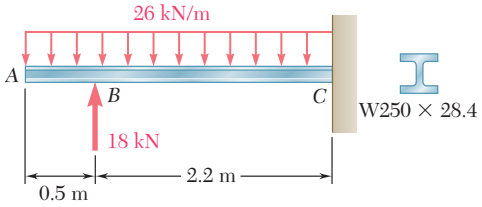
$$y_C = y_A + (2)(\theta_A) + t_{C/A}$$

$$= 0 + 0 - 5.82 \times 10^{-3}$$

$$= -5.82 \times 10^{-3} \text{ ft}$$

$$y_C = 0.0698 \text{ in.} \downarrow \quad \blacktriangleleft$$

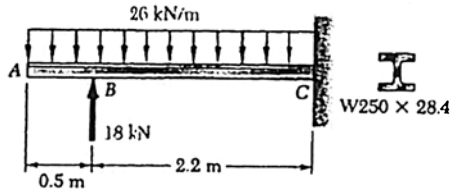
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PROBLEM 9.102

For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*. Use $E = 200$ GPa.

SOLUTION



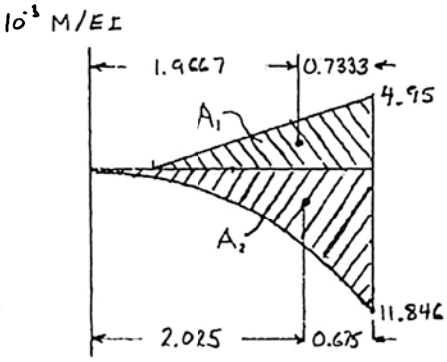
Units: Forces in kN; lengths in m.

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 40.1 \times 10^6 \text{ mm}^4 = 40.1 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(40.1 \times 10^{-6}) = 8.02 \times 10^6 \text{ N} \cdot \text{m}^2$$

$$= 8020 \text{ kN} \cdot \text{m}^2$$



Draw M/EI diagram by parts.

$$\frac{M_1}{EI} = \frac{(18)(2.2)}{8020} = 4.9377 \times 10^{-3} \text{ m}^{-1}$$

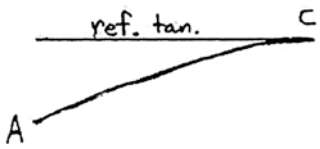
$$A_1 = \frac{1}{2}(4.9377 \times 10^{-3})(2.2) = 5.4315 \times 10^{-3}$$

$$\bar{x}_1 = \frac{1}{3}(2.2) = 0.7333 \text{ m}$$

$$\frac{M_2}{EI} = -\frac{(26)(2.7)^2}{(2)(8020)} = -11.8167 \times 10^{-3} \text{ m}^{-1}$$

$$A_2 = \frac{1}{3}(-11.8167 \times 10^{-3})(2.7) = -10.6350 \times 10^{-3}$$

$$\bar{x}_2 = \frac{1}{4}(2.7) = 0.675 \text{ m}$$



Draw reference tangent at *C*.

$$\theta_C = \theta_A + \theta_{C/A} = \theta_A + A_1 + A_2 = 0$$

(*a*) Slope at *A*.

$$\theta_A = -A_1 - A_2 = -5.4315 \times 10^{-3} + 10.6350 \times 10^{-3}$$

$$= 5.20 \times 10^{-3} \text{ rad}$$

$$\theta_A = 5.20 \times 10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

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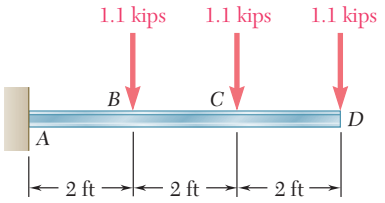
PROBLEM 9.102 *(Continued)*

(*b*) Deflection at *A*.

$$\begin{aligned} y_A &= y_C - \theta_C L + t_{A/C} \\ &= 0 - 0 + A_1\bar{x}_1 + A_2\bar{x}_2 \\ &= 0 - 0 + (5.4315 \times 10^{-3})(1.9667) - (10.6350 \times 10^{-3})(2.025) \\ &= -10.85 \times 10^{-3}\ \text{m} \end{aligned}$$

$$y_A = 10.85\ \text{mm} \downarrow \blacktriangleleft$$

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PROBLEM 9.103

Two C6×8.2 channels are welded back to back and loaded as shown. Knowing that $E = 29\times10^6$ psi, determine (*a*) the slope at point *D*, (*b*) the deflection at point *D*.

SOLUTION

Units: Forces in kips; lengths in ft.

$$E = 29\times10^6 \text{ psi} = 29\times10^3 \text{ ksi} \qquad I = (2)(13.1) = 26.2 \text{ in}^4$$

$$EI = (29.10^3)(26.2) = 759.8\times10^3 \text{ kip}\cdot\text{in}^2 = 5276 \text{ kip}\cdot\text{ft}^2$$

Draw *M*/*EI* diagram by parts.

$$\frac{M_1}{EI} = \frac{(1.1)(6)}{EI} = -\frac{6.6}{EI} \text{ ft}^{-1}$$

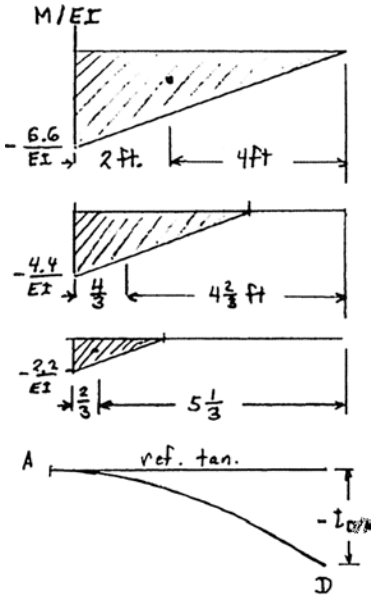
$$A_1 = \frac{1}{2}\left(\frac{6.6}{EI}\right)(6) = -\frac{19.8}{EI} \qquad \bar{x}_1 = \frac{1}{3}(6) = 2 \text{ ft}$$

$$\frac{M_2}{EI} = -\frac{(1.1)(4)}{EI} = -\frac{4.4}{EI} \text{ ft}^{-1}$$

$$A_2 = \frac{1}{2}\left(-\frac{4.4}{EI}\right)(4) = -\frac{8.8}{EI} \qquad \bar{x}_2 = \frac{1}{3}(4) = \frac{4}{3} \text{ ft}$$

$$\frac{M_3}{EI} = -\frac{(1.1)(2)}{EI} = -\frac{2.2}{EI} \text{ ft}^{-1}$$

$$A_3 = \frac{1}{2}\left(-\frac{2.2}{EI}\right)(2) = -\frac{2.2}{EI} \qquad \bar{x}_3 = \frac{1}{3}(2) = \frac{2}{3} \text{ ft}$$



Place reference tangent at *A*. $\theta_A = 0$

(*a*) Slope at *D*. $\theta_{D/A} = A_1 + A_2 + A_3 = -\frac{30.8}{EI} = -\frac{30.8}{5276} = -5.84\times10^{-3} \text{ rad}$

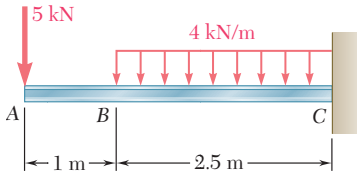
$$\theta_D = \theta_A + \theta_{D/A} \qquad \theta_D = 5.89\times10^{-3} \text{ rad.} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at *D*. $t_{D/A} = \left(-\frac{19.8}{EI}\right)(4) + \left(-\frac{8.8}{EI}\right)\left(4\frac{2}{3}\right) + \left(-\frac{2.2}{EI}\right)\left(5\frac{1}{3}\right)$

$$= -\frac{132.0}{EI} = -\frac{132.0}{5276} = -25.02\times10^{-3} \text{ ft}$$

$$y_D = t_{D/A} = -25.02\times10^{-3} \text{ ft} \qquad y_D = 0.300 \text{ in.} \downarrow \blacktriangleleft$$

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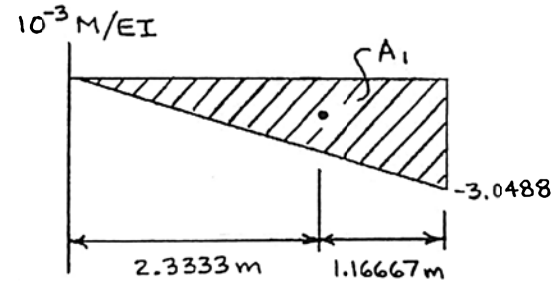


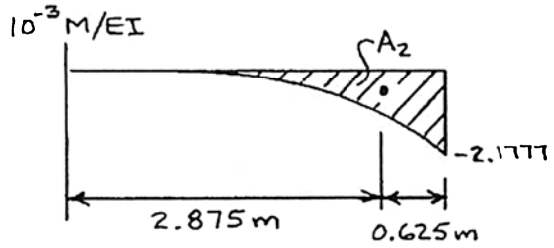


PROBLEM 9.104

For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*. Use $E = 250$ GPa.

SOLUTION







Units: Forces in kN; lengths in m.

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 28.7 \times 10^6 \text{ mm}^4 = 28.7 \times 10^{-6} \text{m}^4$$

$$EI = (200 \times 10^9)(28.7 \times 10^6)$$

$$= 5.74 \times 10^6 \text{ N} \cdot \text{m}^2$$

$$= 5740 \text{ kN} \cdot \text{m}^2$$

Draw M/EI diagram by parts.

$$\frac{M_1}{EI} = -\frac{(5)(3.5)}{5740} = -3.0488 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}(-3.0488 \times 10^{-3})(3.5) = -5.3354 \times 10^{-3}$$

$$\bar{x}_1 = \frac{1}{3}(3.5) = 1.16667 \text{ m}$$

$$\frac{M_2}{EI} = -\frac{(4)(2.5)^2}{(2)(5740)} = -2.1777 \times 10^{-3} \text{ m}^{-1}$$

$$A_2 = \frac{1}{3}(-2.1777 \times 10^{-3})(2.5) = -1.81475 \times 10^{-3}$$

$$\bar{x}_2 = \frac{1}{4}(2.5) = 0.625 \text{ m}$$

Place reference tangent at *C*. $\theta_C = 0$

$$\theta_{C/A} = A_1 + A_2 = -7.1502 \times 10^{-3}$$

(*a*) Slope at *A*. $\theta_A = \theta_C - \theta_{C/A} = 7.1502 \times 10^{-3}$

$$\theta_A = 7.15 \times 10^{-3} \text{ rad} \measuredangle \blacktriangleleft$$

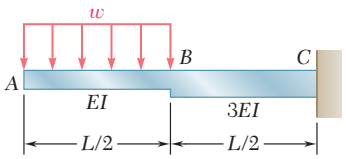
$$t_{A/C} = (2.3333)(-5.3354 \times 10^{-3}) + (2.875)(-1.81475 \times 10^{-3})$$

$$= -17.6665 \times 10^{-3} \text{ m}$$

(*b*) Deflection at *A*.

$$y_A = t_{AC} = -17.67 \times 10^{-3} \text{ m} \qquad y_A = 17.67 \text{ mm} \downarrow \blacktriangleleft$$

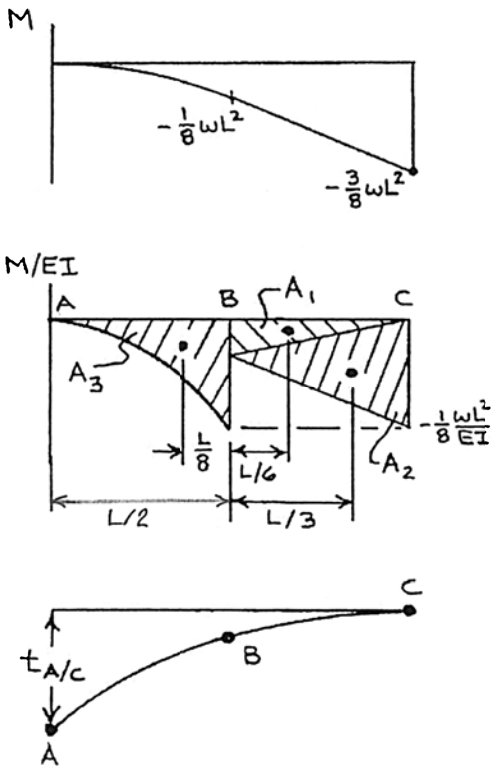
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PROBLEM 9.105

For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*.

SOLUTION



Draw the $\frac{M}{EI}$ diagram using the M diagram.

$$A_1 = \frac{1}{2}\left(-\frac{1}{24}\frac{wL^2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{96}\frac{wL^3}{EI}$$

$$A_2 = \frac{1}{2}\left(-\frac{1}{8}\frac{wL^2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{32}\frac{wL^3}{EI}$$

$$A_3 = \frac{1}{3}\left(-\frac{1}{8}\frac{wL^2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$

Place reference tangent at *C*.

(*a*) Slope at *A*. $\theta_A = -\theta_{C/A}$

$$\theta_A = -A_1 - A_2 - A_3 = \frac{1}{16}\frac{wL^3}{EI}$$

$$\theta_A = \frac{1}{16}\frac{wL^3}{EI} \measuredangle \blacktriangleleft$$

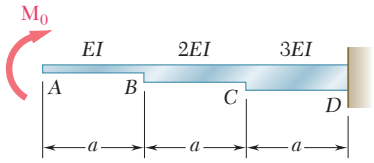
(*b*) Deflection at *A*. $y_A = t_{A/C}$

$$y_A = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{5}{6}L\right) + A_3\left(\frac{3}{8}L\right)$$

$$= -\frac{47}{1152}\frac{wL^4}{EI}$$

$$y_A = \frac{47}{1152}\frac{wL^4}{EI} \downarrow \blacktriangleleft$$

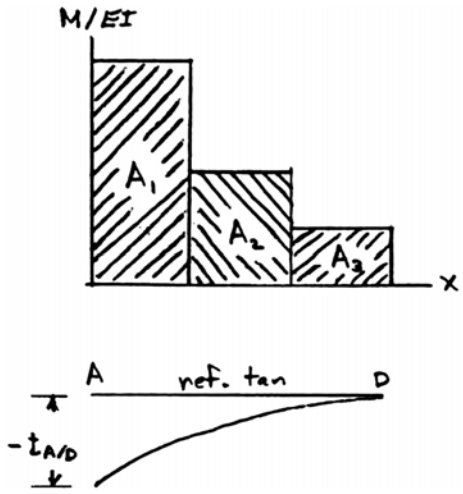
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PROBLEM 9.106

For the cantilever beam and loading shown, determine the deflection and slope at end A caused by the moment $\mathbf{M}_0$.

SOLUTION



Draw $\dfrac{M}{EI}$ diagram. $A_1 = +\dfrac{M_0 a}{EI}$

$$A_2 = +\frac{M_0 a}{2EI}$$

$$A_3 = +\frac{M_0 a}{3EI}$$

$$\theta_D = 0, \quad y_D = 0$$

Place reference tangent at D.

Deflection at A. $y_A = t_{A/D}$

$$y_A = A_1\left(\frac{1}{2}a\right) + A_2\left(\frac{3}{2}a\right) + A_3\left(\frac{5}{2}a\right)$$

$$= \frac{25M_0a^2}{12EI} \uparrow$$ ◀

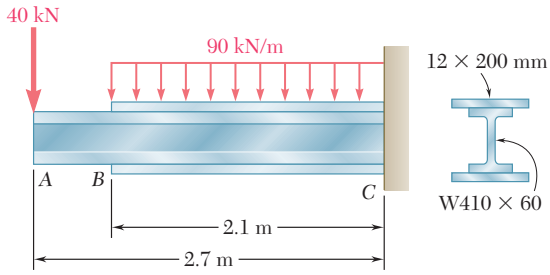
Slope at A. $\theta_A = -\theta_{C/A}$

$$\theta_A = -A_1 - A_2 - A_3$$

$$= -\frac{11M_0a}{6EI}$$

$$\theta_A = \frac{11M_0a}{6EI}$$ ◀

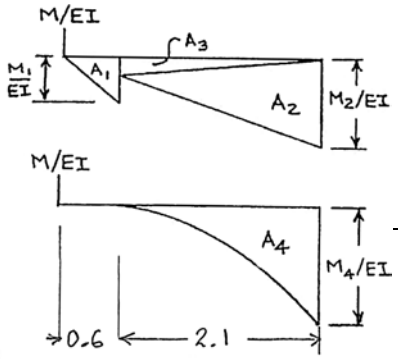
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PROBLEM 9.107

Two cover plates are welded to the rolled-steel beam as shown. Using $E = 200$ GPa, determine (*a*) the slope at end *A*, (*b*) the deflection at end *A*.

SOLUTION



Portion *AB*: $I = 216 \times 10^6 \text{ mm}^4$

$EI = (200 \times 10^6 \text{ kPa})(216 \times 10^{-6} \text{ m}^4) = 43{,}200 \text{ kN} \cdot \text{m}^2$

Portion *BC*:

	$A(\text{mm}^2)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
Top plate	2400	209	104.834×10^6	28,800
W410 × 60				216×10^6
Bot. plate	2400	209	104.834×10^6	28,800
Σ			209.67×10^6	216.06×10^6

$$I = 209.67 \times 10^6 + 216.06 \times 10^6 = 425.73 \times 10^6 \text{ mm}^4$$

$$EI = (200 \times 10^6 \text{ kPa})(425.73 \times 10^{-6} \text{ m}^4) = 85{,}146 \text{ kN} \cdot \text{m}^2$$

Draw $\dfrac{M}{EI}$ diagram.

$$\frac{M_1}{EI} = -\frac{(40)(0.6)}{43{,}200} = -0.55556 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(40)(2.7)}{85{,}146} = -1.26841 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_4}{EI} = -\frac{(90)(2.1)(1.05)}{85{,}146} = -2.3307 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}(0.6)(-0.55556 \times 10^{-3}) = -0.166668 \times 10^{-3}$$

$$A_2 = \frac{1}{2}(2.1)(-1.26841 \times 10^{-3}) = -1.33183 \times 10^{-3}$$

$$A_3 = \frac{0.6}{2.7} A_2 = -0.29596 \times 10^{-3}$$

$$A_4 = \frac{1}{3}(2.1)(-2.3307 \times 10^{-3}) = -1.63149 \times 10^{-3}$$

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PROBLEM 9.107 *(Continued)*

Place reference tangent at C. $\theta_C = 0$

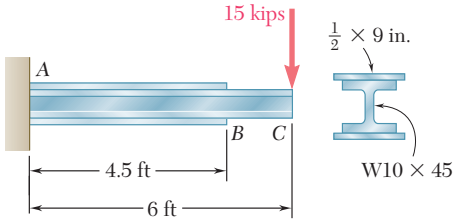
(*a*) Slope at A. $\theta_A = \theta_C - \theta_{A/C} = 0 - (A_1 + A_2 + A_3 + A_4)$ $\theta_A = 3.43 \times 10^{-3}$ rad ◀

(*b*) Deflection at A. $y_A = t_{A/C}$

$$y_A = (0.4)(A_1) + (2)(A_2) + (1.3)(A_3) + (2.175)(A_4) = -6.66 \times 10^{-3}\,\text{m}$$

$y_A = 6.66$ mm ↓ ◀

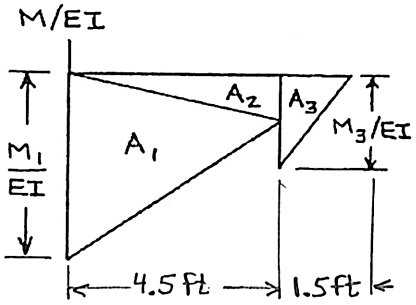
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PROBLEM 9.108

Two cover plates are welded to the rolled-steel beam as shown. Using $E = 29 \times 10^6$ psi, determine (*a*) the slope at end *C*, (*b*) the deflection at end *C*.

SOLUTION



Portion *BC*: $I = 248 \text{ in}^4$

$EI = (29 \times 10^6)(248) = 7.192 \times 10^9 \text{ lb} \cdot \text{in}^2 = 49{,}944 \text{ kip} \cdot \text{ft}^2$

Portion *AB*:

	$A(\text{in}^2)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
Top plate	4.5	5.3	126.405	0.09375
W12 × 45				248
Bot. plate	4.5	5.3	126.405	0.09375
Σ			252.81	248.19

$I = 252.81 + 248.19 = 501.00 \text{ in}^4 \quad EI = (29 \times 10^6)(501) = 14.529 \times 10^9 \text{ lb} \cdot \text{in}^2 = 100{,}896 \text{ kip} \cdot \text{ft}^2$

Draw $\frac{M}{EI}$ diagram.

$$\frac{M_1}{EI} = -\frac{(15)(6)}{100{,}896} = -892.01 \times 10^{-6} \text{ ft}^{-1}$$

$$\frac{M_3}{EI} = -\frac{(15)(1.5)}{49{,}944} = -450.50 \times 10^{-6} \text{ ft}^{-1}$$

$$A_1 = \frac{1}{2}(4.5)(-892.01 \times 10^{-6}) = -2.0070 \times 10^{-3}$$

$$A_2 = \frac{1.5}{6} A_1 = -0.50175 \times 10^{-3}$$

$$A_3 = \frac{1}{2}(1.5)(-450.50 \times 10^{-6}) = -0.33788 \times 10^{-3}$$

Place reference tangent at *A*.

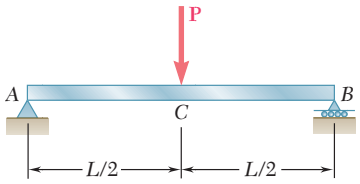
(*a*) Slope at *C*. $\theta_C = \theta_{C/A} = A_1 + A_2 + A_3$ $\theta_C = 2.85 \times 10^{-3} \text{ rad}$ ◀

(*b*) Deflection at *C*. $y_C = t_{C/A}$

$$y_C = (4.5)(A_1) + (3)(A_2) + (1)(A_3) = -10.8746 \times 10^{-3} \text{ ft}$$

$y_C = 0.1305 \text{ in.} \downarrow$ ◀

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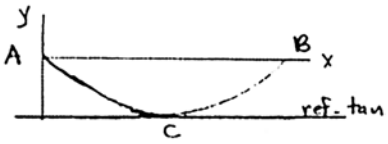
PROBLEM 9.109

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

Symmetric beam and loading.

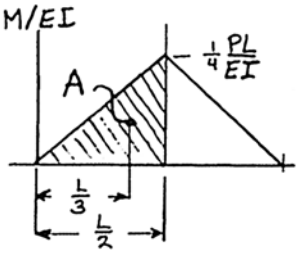
Place reference tangent at *C*.



$$\theta_C = 0, \quad y_C = -t_{A/C}$$

Reactions: $R_A = R_B = \frac{1}{2}P$

Bending moment at *C*. $M_C = \frac{1}{4}PL$



$$A = \frac{1}{2}\left(\frac{1}{4}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{16}\frac{PL^2}{EI}$$

(*a*) Slope at *A*. $\theta_A = \theta_C - \theta_{C/A}$

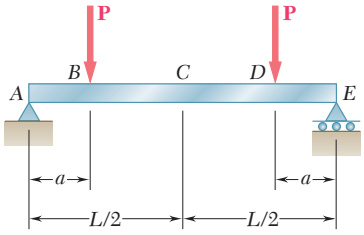
$$\theta_A = 0 - \frac{1}{16}\frac{PL^2}{EI}$$

$$\theta_A = -\frac{1}{16}\frac{PL^2}{EI} \curvearrowleft \blacktriangleleft$$

(*b*) Deflection at *C*. $y_C = -t_{A/C} = -A\left(\frac{L}{3}\right) = -\left(\frac{1}{16}\frac{PL^2}{EI}\right)\left(\frac{L}{3}\right)$

$$y_C = \frac{1}{48}\frac{PL^3}{EI} \downarrow \blacktriangleleft$$

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PROBLEM 9.110

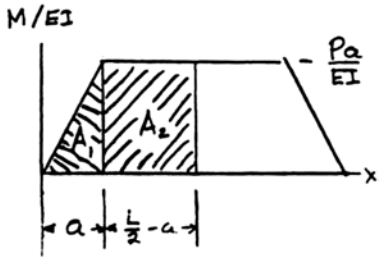
For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

Symmetric beam with symmetric loading.

$\theta_c = 0$ Place reference tangent at *C*.

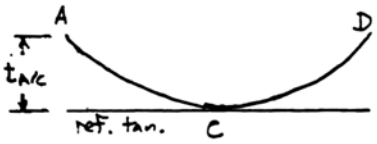
Draw $\dfrac{M}{EI}$ diagram.



$$A_1 = \frac{1}{2}a\left(\frac{Pa}{EI}\right) = \frac{Pa^2}{2EI}$$

$$A_2 = \left(\frac{L}{2} - a\right)\left(\frac{Pa}{EI}\right) = \frac{Pa(L-2a)}{2EI}$$

Slope at end *A*. $\theta_A = -\theta_{C/A}$



$$\theta_A = A_1 + A_2 = -\frac{Pa(L-a)}{2EI}$$ ◀

Deflection at *C*. $y_C = -t_{A/C}$

$$t_{A/C} = \left(\frac{2}{3}a\right)A_1 + \left[\frac{1}{2}\left(\frac{L}{2} + a\right)\right]A_2$$

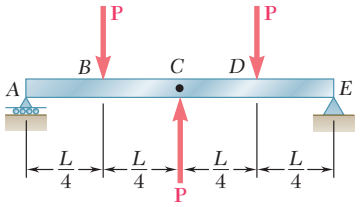
$$= \frac{2}{3}\frac{Pa^3}{2EI} + \left(\frac{L+2a}{4}\right)\left(\frac{Pa(L-2a)}{2EI}\right)$$

$$= \frac{Pa}{EI}\left\{\frac{1}{3}a^2 + \frac{1}{8}(L^2 - 4a^2)\right\}$$

$$= \frac{Pa}{24EI}(3L^2 - 4a^2)$$

$$y_C = -\frac{Pa}{24EI}(3L^2 - 4a^2)$$ ◀

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PROBLEM 9.111

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

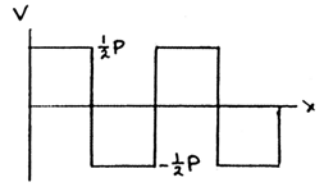
SOLUTION

Symmetric beam and loading.

Place reference tangent at *C*.

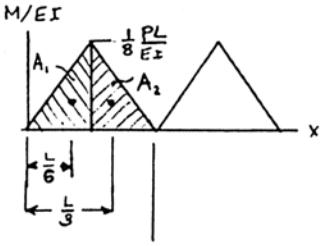
$$\theta_C = 0$$

Reactions: $$R_A = R_E = \frac{1}{2}P$$



Draw *V* (shear) and *M/EI* diagrams.

$$A_1 = A_2 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\frac{L}{4} = \frac{1}{64}\frac{PL^2}{EI}$$

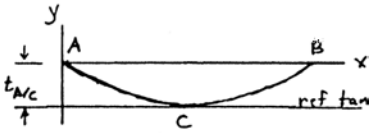


(*a*) Slope at *A*.

$$\theta_A = \theta_C - \theta_{A/C} = 0 - A_1 - A_2$$

$$= -\frac{1}{32}\frac{PL^2}{EI}$$

$$\theta_A = \frac{PL^2}{32EI} \curvearrowright \blacktriangleleft$$



(*b*) Deflection at *C*.

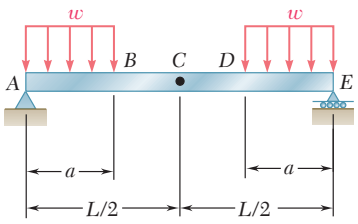
$$y_C = -t_{A/C} = -\left(A_1\frac{L}{6} + A_2\frac{L}{3}\right)$$

$$= -\left(\frac{1}{64}\frac{PL^3}{EI}\cdot\frac{L}{6} + \frac{1}{64}\frac{PL^3}{EI}\cdot\frac{L}{3}\right)$$

$$= -\frac{1}{128}\frac{PL^3}{EI}$$

$$y_C = \frac{PL^3}{128EI} \downarrow \blacktriangleleft$$

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PROBLEM 9.112

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

Symmetric beam and loading.

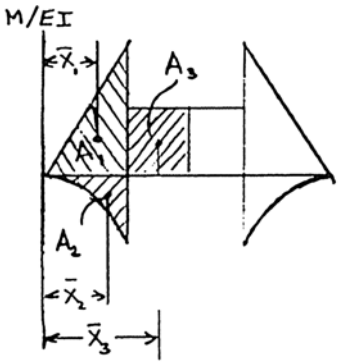
Place reference tangent at *C*. $\theta_C = 0$

Reactions: $R_A = R_E = wa$

Bending moment.

Over *AB*: $M = wax - \frac{1}{2}wa^2$

Over *BD*: $M = \frac{1}{2}wa^2$

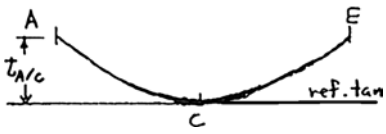


Draw *M*/*EI* diagram by parts.

$$\frac{M_1}{EI} = \frac{wa^2}{EI}$$

$$\frac{M_2}{EI} = -\frac{1}{2}\frac{wa^2}{EI}$$

$$\frac{M_3}{EI} = \frac{1}{2}\frac{wa^2}{EI}$$



$$A_1 = \frac{1}{2}\frac{M_1}{EI}a = \frac{1}{2}\frac{wa^3}{EI}$$

$$A_2 = -\frac{1}{3}\frac{M_2}{EI}a = -\frac{1}{6}\frac{wa^3}{EI}$$

$$A_3 = \frac{M_3}{EI}\left(\frac{L}{2} - a\right) = \frac{1}{4}\frac{wa^2}{EI}(L - 2a)$$

(*a*) Slope at *A*.

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{1}{2}\frac{wa^3}{EI} + \frac{1}{6}\frac{wa^3}{EI} - \frac{1}{4}\frac{wa^2}{EI}(L - 2a)$$

$$= -\frac{wa^2}{EI}\left(\frac{1}{4}L - \frac{1}{6}a\right)$$

$$= -\frac{1}{12}\frac{wa^2}{EI}(3L - 2a)$$

$$\theta_A = \frac{wa^2}{12EI}(3L - 2a) \quad \curvearrowright \blacktriangleleft$$

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PROBLEM 9.112 *(Continued)*

(*b*) Deflection at *C*.

$$y_C = -t_{C/A}$$

$$\bar{x}_1 = \frac{2}{3}a,$$

$$\bar{x}_2 = \frac{3}{4}a,$$

$$\bar{x}_3 = a + \frac{1}{2}\left(\frac{L}{2} - a\right) = \frac{1}{4}(L + 2a)$$

$$y_C = -t_{C/A} = -A_1\bar{x}_1 - A_2\bar{x}_2 - A_3\bar{x}_3$$

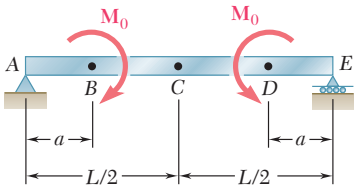
$$= -\left(\frac{1}{2}\frac{wa^3}{EI}\right)\left(\frac{2}{3}a\right) + \left(\frac{1}{6}\frac{wa^3}{EI}\right)\left(\frac{3}{4}a\right) - \frac{1}{4}\left(\frac{wa^2}{EI}\right)(L - 2a)\frac{1}{4}(L + 2a)$$

$$= -\frac{1}{3}\frac{wa^3}{EI} + \frac{1}{8}\frac{wa^3}{EI} - \frac{1}{16}\frac{wa^2}{EI}(L^2 - 4a^2)$$

$$= -\frac{wa^2}{EI}\left(\frac{1}{16}L^2 - \frac{1}{24}a^2\right) = -\frac{1}{48}\frac{wa^2}{EI}(3L^2 - 2a^2)$$

$$y_A = \frac{wa^2}{48EI}(3L^2 - 2a^2) \downarrow \blacktriangleleft$$

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PROBLEM 9.113

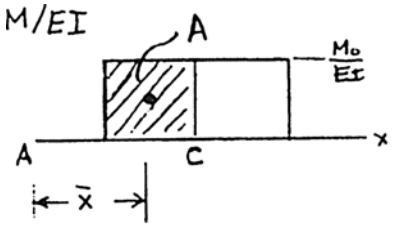
For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

Symmetric beam and loading.

Place reference tangent at *C*. $\theta_C = 0$.

Draw $\dfrac{M}{EI}$ diagram.



(*a*) Slope at *A*. $\theta_A = 0$

$$A = \frac{M_0}{EI}\left(\frac{L}{2} - a\right) = \frac{1}{2}\frac{M_0}{EI}(L - 2a)$$

$$\theta_A = \theta_C - \theta_{C/A} = 0 - A = -\frac{1}{2}\frac{M}{EI}(L - 2a)$$

$$\theta_A = \frac{1}{2}\frac{M_0}{EI}(L - 2a) \measuredangle \blacktriangleleft$$

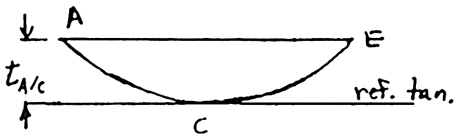
(*b*) Deflection at *C*.

$$\bar{x} = a + \frac{1}{2}\left(\frac{L}{2} - a\right) = \frac{1}{4}(L + 2a)$$

$$y_C = -t_{C/A} = A\bar{x}$$

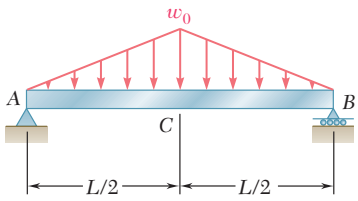
$$= -\frac{1}{2}\frac{M_0}{EI}(L - 2a)\frac{1}{4}(L + 2a)$$

$$= -\frac{1}{8}\frac{M_0}{EI}(L^2 - 4a^2)$$



$$y_C = \frac{1}{8}\frac{M_0}{EI}(L^2 - 4a^2) \downarrow \blacktriangleleft$$

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PROBLEM 9.114

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

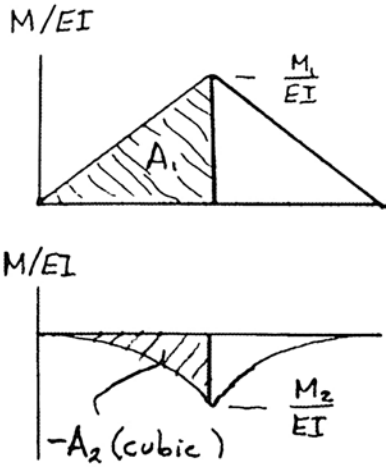
SOLUTION

Symmetric beam and loading.

Place reference tangent at *C*. $\theta_C = 0$

Reactions: $R_A = R_B = \dfrac{w_0 L}{4}$

Draw $\dfrac{M}{EI}$ diagram by parts.



$$\frac{M_1}{EI} = \frac{R_A L}{2} = \frac{w_0 L^2}{8EI}$$

$$A_1 = \frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{M_1}{EI}\right) = \frac{w_0 L^3}{32EI}$$

$$\frac{M_2}{EI} = \frac{1}{EI}\cdot\frac{1}{2}\left(-\frac{w_0 L}{2}\right)\left(\frac{1}{3}\cdot\frac{L}{2}\right) = -\frac{w_0 L^2}{24EI}$$

$$A_2 = \frac{1}{4}\left(\frac{L}{2}\right)\left(-\frac{w_0 L^2}{24EI}\right) = -\frac{w_0 L^3}{192EI}$$

(*a*) Slope at *A*. $\theta_A = -\theta_{C/A}$

$$\theta_A = -A_1 - A_2 = \left(-\frac{1}{32} + \frac{1}{192}\right)\frac{w_0 L^3}{EI}$$

$$\theta_A = \frac{5 w_0 L^3}{192EI} \curvearrowright \blacktriangleleft$$

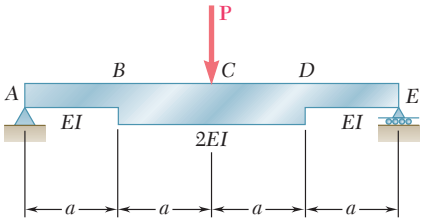
(*b*) Deflection at *C*. $y_C = t_{A/C}$

$$t_{A/C} = A_1\left[\left(\frac{2}{3}\right)\left(\frac{L}{2}\right)\right] + A_2\left[\left(\frac{4}{5}\right)\left(\frac{L}{2}\right)\right] = \left[\left(\frac{1}{3}\right)\left(\frac{1}{32}\right) - \left(\frac{2}{5}\right)\left(\frac{1}{192}\right)\right]\frac{w_0 L^4}{EI}$$

$$= \frac{w_0 L^4}{120EI}$$

$$y_C = \frac{w_0 L^4}{120EI} \downarrow \blacktriangleleft$$

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PROBLEM 9.115

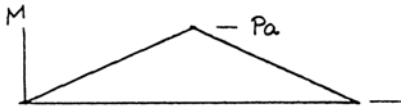
For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

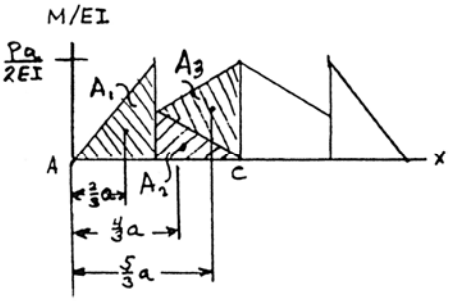
Symmetric beam and loading.

$$R_A = R_E = \frac{1}{2}P$$

$$M_{\max} = \left(\frac{1}{2}P\right)(2a) = Pa$$



Draw *M* and *M*/*EI* diagrams.



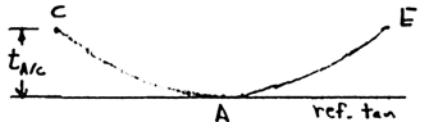
$$A_1 = \frac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{Pa}{4EI}\right)a = \frac{1}{8}\frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$

Place reference tangent at *C*.

$$\theta_C = 0$$



(*a*) Slope at *A*.

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{5}{8}\frac{Pa^2}{EI}$$

$$\theta_A = \frac{5Pa^2}{8EI} \curvearrowright \blacktriangleleft$$

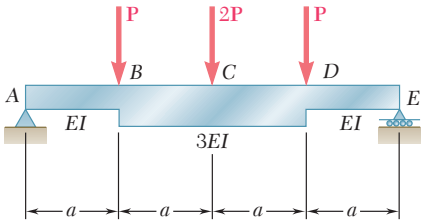
(*b*) Deflection at *C*.

$$|y_C| = t_{A/C} = A_1\left(\frac{2}{3}a\right) + A_2\left(\frac{4}{3}a\right) + A_3\left(\frac{5}{3}a\right)$$

$$= \frac{1}{6}\frac{Pa^3}{EI} + \frac{1}{6}\frac{Pa^3}{EI} + \frac{5}{12}\frac{Pa^3}{EI} = \frac{3}{4}\frac{Pa^3}{EI}$$

$$y_C = \frac{3Pa^3}{4EI} \downarrow \blacktriangleleft$$

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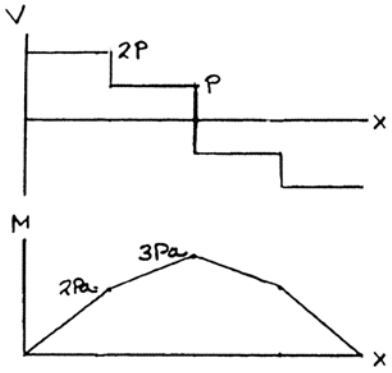
PROBLEM 9.116

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

Symmetric beam and loading.

$$R_A = R_E = 2P.$$



Draw *V*, *M*, and *M*/*EI* diagrams.

$$A_1 = \frac{1}{2}\left(\frac{2Pa}{EI}\right)a = \frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{2}{3}\frac{Pa}{EI}\right)a = \frac{1}{3}\frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{Pa}{EI}\right)a = \frac{1}{2}\frac{Pa^2}{EI}$$

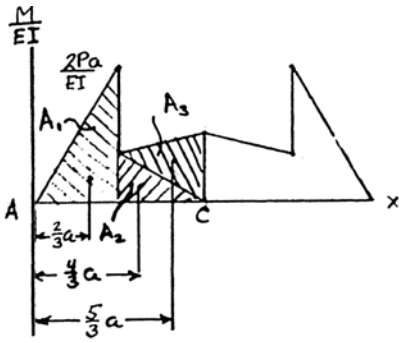
Place reference tangent at *C*.

$$\theta_C = 0$$

(*a*) Slope at *A*.

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{11}{6}\frac{Pa^2}{EI} \qquad \theta_A = \frac{11Pa^2}{6EI} \text{ ↻} \blacktriangleleft$$

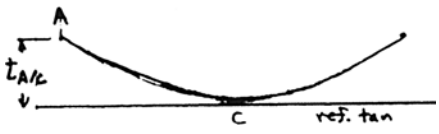


(*b*) Deflection at *C*.

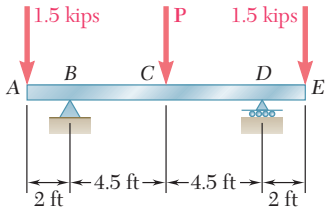
$$|y_C| = t_{A/C}$$

$$= A_1\left(\frac{2}{3}a\right) + A_2\left(\frac{4}{3}a\right) + A_3\left(\frac{5}{3}a\right)$$

$$= \frac{35}{18}\frac{Pa^3}{EI} \qquad y_C = \frac{35Pa^3}{18EI} \downarrow \blacktriangleleft$$



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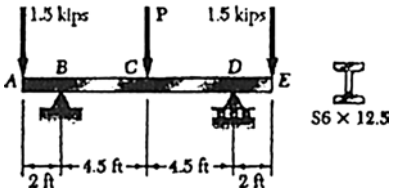




PROBLEM 9.117

Knowing that the magnitude of the load **P** is 7 kips, determine (*a*) the slope at end *A*, (*b*) the deflection at end *A*, (*c*) the deflection at midpoint *C* of the beam. Use $E = 29 \times 10^6$ psi.

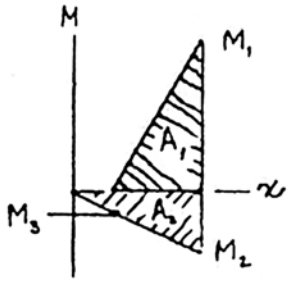
SOLUTION



Use units of kips and ft. $P = 7$ kips

For S6 × 12.5, $I = 22.0 \text{ in}^4$

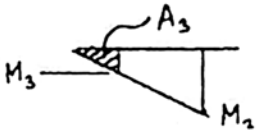
$$EI = (29 \times 10^6)(22.0) = 638 \times 10^6 \text{ lb} \cdot \text{in}^2$$
$$= 4430.6 \text{ kip} \cdot \text{ft}^2$$



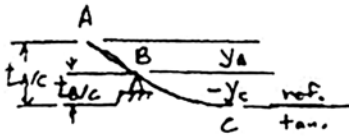
Symmetric beam with symmetric loading. Place reference tangent at midpoint *C* where $\theta_C = 0$.

$$R_B = R_D = \frac{1}{2}(1.5 + 7 + 1.5) = 5 \text{ kips} \uparrow$$

Draw the bending moment diagram by parts for the left half of the beam.



$$M_1 = (4.5)(5) = 22.5 \text{ kip} \cdot \text{ft}$$
$$A_1 = \frac{1}{2}(4.5)(22.5) = 50.625 \text{ kip} \cdot \text{ft}^2$$
$$M_2 = -(2 + 4.5)(1.5) = -9.75 \text{ kip} \cdot \text{ft}$$
$$A_2 = \frac{1}{2}(6.5)(-9.75) = -31.6875 \text{ kip} \cdot \text{ft}^2$$
$$M_3 = -(2)(1.5) = -3 \text{ kip} \cdot \text{ft}$$
$$A_3 = \frac{1}{2}(2)(-3) = -3 \text{ kip} \cdot \text{ft}^2$$



Formulas: $\theta_A = -\theta_{C/A}$, $y_A - y_C = t_{A/C}$ $y_C = y_A - t_{A/C}$

$$y_B = y_A - 2\theta_A + t_{B/A} = 0, \qquad y_A = -2\theta_A - t_{B/A}$$

$$\theta_{C/A} = \frac{1}{EI}(A_1 + A_2) = \frac{50.625 - 31.6875}{4430.6} = 4.27425 \times 10^{-3}$$

$$t_{A/C} = \frac{1}{EI}\left\{(2 + 3)A_1 + \frac{2}{3}(6.5)A_2\right\} = \frac{115.8125}{4430.6} = 26.1392 \times 10^{-3} \text{ ft}$$

$$t_{B/A} = \frac{1}{EI}\left\{\frac{1}{3}(2)A_3\right\} = \frac{-2}{4430.6} = -0.45141 \times 10^{-3} \text{ ft}$$

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PROBLEM 9.117 *(Continued)*

(*a*) Slope at end *A*. $\theta_A = -4.27 \times 10^{-3}\,\text{rad}$ $\theta_A = 4.27 \times 10^{-3}\,\text{rad}$ ◀

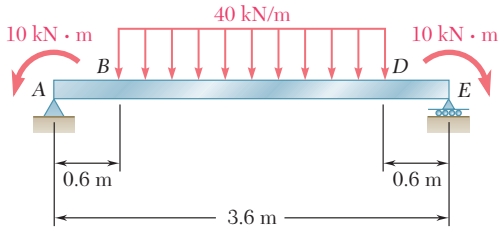
(*b*) Deflection at *A*. $y_A = -(2)(-4.27425 \times 10^{-3}) - (-0.45141 \times 10^{-3})$

$= 8.9999 \times 10^{-3}\,\text{ft}$ $y_A = 0.1080\,\text{in.} \uparrow$ ◀

(*c*) Deflection at *C*. $y_C = 8.9999 \times 10^{-3} - 26.1392 \times 10^{-3} = -17.1393 \times 10^{-3}\,\text{ft}$

$y_C = -0.206\,\text{in.}$ $y_C = 0.206\,\text{in.} \downarrow$ ◀

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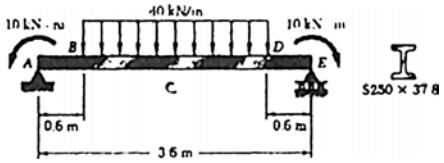




PROBLEM 9.118

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint of the beam. Use $E = 200$ GPa.

SOLUTION



Use units of kN and m.

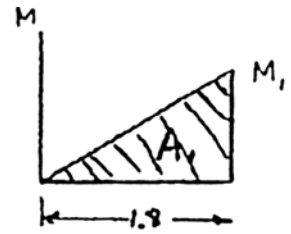
For S250 × 37.8,

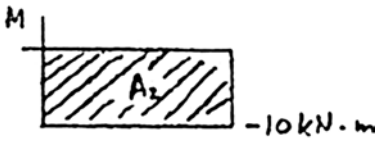
$$I = 51.2 \times 10^6 \text{ mm}^4 = 51.2 \times 10^{-6} \text{ m}^4$$

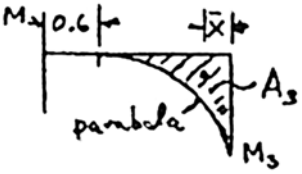
$$EI = (200 \times 10^9)(51.2 \times 10^{-6})$$

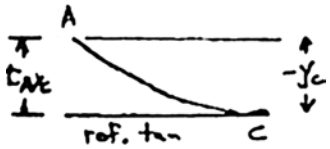
$$= 10.24 \times 10^6 \text{ N} \cdot \text{m}^2 = 10,240 \text{ kN} \cdot \text{m}^2$$

Place reference tangent at midpoint *C*.









Reactions: $R_A = R_E = \frac{1}{2}(40)(3.6 - 1.2) = 48 \text{ kN} \uparrow$

Draw bending moment diagram of left half of beam by parts.

$$M_1 = (48)(1.8) = 86.4 \text{ kN} \cdot \text{m}$$

$$A_1 = \frac{1}{2}(1.8)(86.4) = 77.76 \text{ kN} \cdot \text{m}^2$$

$$A_2 = (1.8)(-10) = -18 \text{ kN} \cdot \text{m}^2$$

$$M_3 = \frac{1}{2}(40)(1.8 - 0.6)^2 = -28.8 \text{ kN} \cdot \text{m}$$

$$A_3 = \frac{1}{3}(1.2)(-28.8) = -11.52 \text{ kN} \cdot \text{m}^2$$

$$\bar{x} = \frac{1}{4}(1.2) = 0.30 \text{ m}$$

(*a*) Slope at end *A*. $\theta_A = -\theta_{A/C}$

$$\theta_A = \frac{1}{EI}\{-A_1 - A_2 - A_3\} = \frac{-77.76 + 18 + 11.52}{10,240}$$

$$= -4.71 \times 10^{-3} \text{ rad}$$

$$\theta_A = 4.71 \times 10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

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PROBLEM 9.118 ***(Continued)***

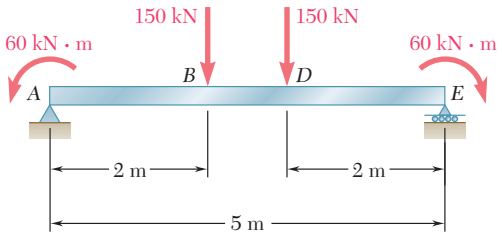
(*b*) Deflection at midpoint *C*. $y_C = -t_{A/C}$

$$t_{A/C} = \frac{1}{EI}\{1.2A_1 + 0.9A_2 + (1.8 - 0.3)A_3\}$$

$$= \frac{(1.2)(77.76) - (0.9)(18) - (1.5)(11.52)}{10{,}240} = 5.84 \times 10^{-3} \text{ m}$$

$$y_C = -5.84 \times 10^{-3} \text{ m} \qquad y_C = 5.84 \text{ mm} \downarrow \blacktriangleleft$$

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PROBLEM 9.119

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint of the beam. Use $E = 200$ GPa.

SOLUTION

Use units of kN and m.

For W460×74,

$$I = 333\times10^{6}\ \text{mm}^4 = 333\times10^{-6}\ \text{m}^4$$

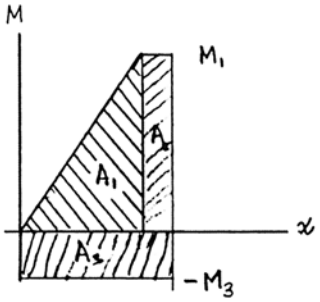
$$EI = (200\times10^{9})(333\times10^{-6})$$

$$= 66.6\times10^{6}\ \text{N}\cdot\text{m}^2 = 66{,}600\ \text{kN}\cdot\text{m}^2$$

Symmetric beam and loading. Place reference tangent at midpoint *C* where $\theta_C = 0$.

Reactions: $R_A = R_E = 150\ \text{kN}\uparrow$

Draw bending moment diagram of left half of beam by parts.



$$M_1 = (2)(150) = 300\ \text{kN}\cdot\text{m}$$

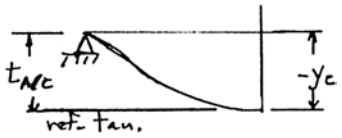
$$A_1 = \left(\frac{1}{2}\right)(2)(300) = 300\ \text{kN}\cdot\text{m}^2$$

$$A_2 = (0.5)(300) = 150\ \text{kN}\cdot\text{m}^2$$

$$M_3 = -60\ \text{kN}\cdot\text{m}$$

$$A_3 = (2.5)(-60) = -150\ \text{kN}\cdot\text{m}^2$$

(*a*) Slope at end *A*. $\theta_A = -\theta_{C/A}$



$$\theta_A = \frac{1}{EI}\{-A_1 - A_2 - A_3\}$$

$$= \frac{-300-150+150}{66{,}600}$$

$$= -4.50\times10^{-3}\ \text{rad}$$

$\theta_A = 4.50\times10^{-3}\ \text{rad}$ ◀

(*b*) Deflection at midpoint *C*. $y_C = -t_{A/C}$

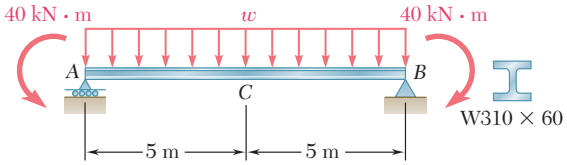
$$t_{A/C} = \frac{1}{EI}\left\{\left(\frac{2}{3}\cdot 2\right)A_1 + \left(2+\frac{0.5}{2}\right)A_2 + \left(\frac{2.5}{2}\right)A_3\right\}$$

$$= \frac{400+337.5-187.5}{66{,}600} = 8.26\times10^{-3}\ \text{m}$$

$$y_C = -8.26\times10^{-3}\ \text{m}$$

$y_C = 8.26\ \text{mm}\downarrow$ ◀

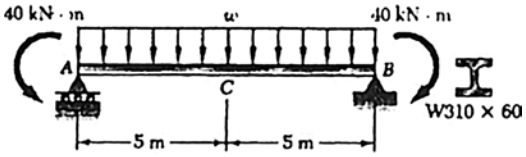
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PROBLEM 9.120

For the beam and loading shown and knowing that $w = 8$ kN/m, determine (*a*) the slope at end *A*, (*b*) the deflection at midpoint *C*. Use $E = 200$ GPa.

SOLUTION



$$E = 200\times10^{9}\ \text{Pa}$$

$$I = 128\times10^{6}\ \text{mm}^4 = 128\times10^{-6}\ \text{m}^4$$

$$EI = (200\times10^{9})(128\times10^{-6}) = 25.6\times10^{6}\ \text{N}\cdot\text{m}^2$$

$$= 25{,}600\ \text{kN}\cdot\text{m}^2$$

Symmetrical beam and loading.

$$R_A = R_B = \frac{1}{2}(8)(10) = 40\ \text{kN}$$

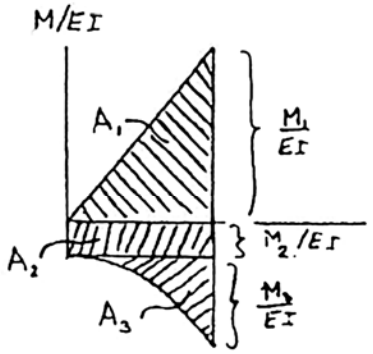
Bending moment:

$$M = 40x - 40 - \frac{1}{2}(8)x^2$$

At $x = 5$,

$$M = 200 - 40 - 100$$

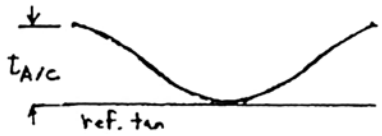
Draw $\dfrac{M}{EI}$ diagram by parts.



$$\frac{M_1}{EI} = \frac{200}{25{,}600} = 7.8125\times10^{-3}\ \text{m}^{-1}$$

$$\frac{M_2}{EI} = \frac{-40}{25{,}600} = -1.5625\times10^{-3}\ \text{m}^{-1}$$

$$\frac{M_3}{EI} = \frac{-100}{25{,}600} = -3.9063\times10^{-3}\ \text{m}^{-1}$$



$$A_1 = \frac{1}{2}(7.8125\times10^{-3})(5) = 19.5313\times10^{-3} \qquad \bar{x}_1 = \left(\frac{2}{3}\right)(5) = 3.3333\ \text{m}$$

$$A_2 = -(1.5625)(5) = -7.8125\times10^{-3} \qquad \bar{x}_2 = \left(\frac{1}{2}\right)(5) = 2.5\ \text{m}$$

$$A_3 = -\frac{1}{3}(3.9063)(5) = -6.5105\times10^{-3} \qquad \bar{x}_3 = \left(\frac{3}{4}\right)(5) = 3.75\ \text{m}$$

Place reference tangent at *C*. $\theta_C = 0$

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PROBLEM 9.120 *(Continued)*

(*a*) Slope at *A*.

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$\theta_A = -(19.5313\times10^{-3} - 7.8125\times10^{-3} - 6.5105\times10^{-3}) = -5.21\times10^{-3}$$

$$\theta_A = 5.21\times10^{-3}\ \text{rad} \curvearrowright \blacktriangleleft$$

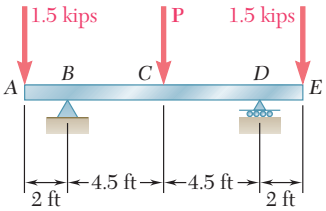
(*b*) Deflection at *C*.

$$|y_C| = t_{A/C}$$

$$= (19.5313\times10^{-3})(3.3333) - (7.8125\times10^{-3})(2.5) - (6.5105\times10^{-3})(3.75)$$

$$= 21.2\times10^{-3}\ \text{m} \qquad y_C = 21.2\ \text{mm}\downarrow \blacktriangleleft$$

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PROBLEM 9.121

For the beam and loading of Prob. 9.117, determine (*a*) the load **P** for which the deflection is zero at the midpoint *C* of the beam, (*b*) the corresponding deflection at end *A*. Use $E = 29 \times 10^6$ psi.

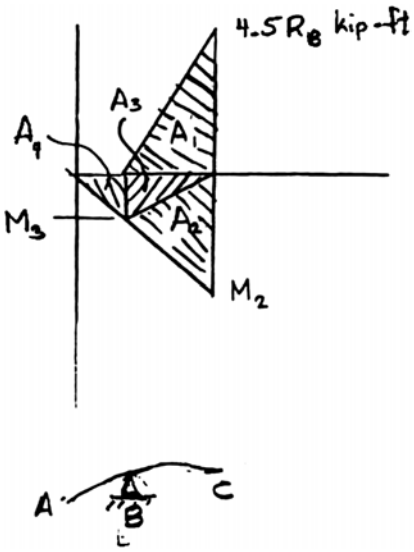
SOLUTION

Use units of kips and ft.

For S6 × 12.5, $I = 22.1\,\text{in}^4$

$$EI = (29 \times 10^6)(22.1) = 640.9 \times 10^6\,\text{lb} \cdot \text{in}^2$$

$$= 4451\,\text{kip} \cdot \text{ft}^2$$



Symmetric beam with symmetric loading. Place reference tangent at midpoint *C* where $\theta_C = 0$.

Draw the bending moment diagram by parts for the left half of the beam.

$$A_1 = \frac{1}{2}(4.5)(4.5)R_B = 10.1250R_B\ \text{kip} \cdot \text{ft}^2$$

$$M_2 = -(2 + 4.5)(1.5) = -9.75\ \text{kip} \cdot \text{ft}$$

$$A_2 = \frac{1}{2}(4.5)(-9.75) = -21.938\ \text{kip} \cdot \text{ft}^2$$

$$M_3 = -(2)(1.5) = -3\ \text{kip} \cdot \text{ft}$$

$$A_3 = \frac{1}{2}(4.5)(-3) = -6.75\ \text{kip} \cdot \text{ft}^2$$

$$A_4 = \frac{1}{2}(2)(-3) = -3\ \text{kip} \cdot \text{ft}^2$$

(*a*) $t_{B/C} = 0$: $\dfrac{1}{EI}\left\{\dfrac{2}{3}(4.5)A_1 + \dfrac{2}{3}(4.5)A_2 + \dfrac{1}{3}(4.5)A_3\right\}$

$$= \frac{30.375R_B - 75.938}{EI} = 0 \quad R_B = 2.5\ \text{kips}$$

By statics, $-(2)(1.5) + 2R_B - P = 0$ $\qquad P = 2.00\ \text{kips} \downarrow$ ◀

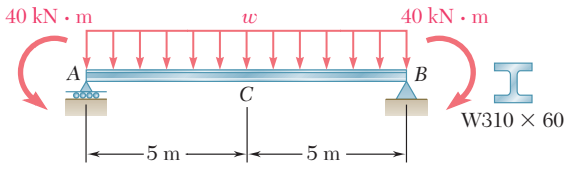
$$A_1 = (10.125)(2.5) = 25.313\,\text{kip} \cdot \text{ft}^2$$

(*b*) $y_A = t_{A/C} = \dfrac{1}{EI}\left\{(2 + 3)A_1 + (2 + 3)A_2 + (2 + 1.5)A_3 + \left(\dfrac{2}{3} \cdot 2\right)A_4\right\}$

$$= \frac{(5)(25.313) + (5)(-21.938) + (3.5)(-6.75) + \left(\dfrac{4}{3}\right)(-3)}{4451}$$

$$= -2.4152 \times 10^{-3}\,\text{ft} = -0.0290\,\text{in.} \qquad y_A = 0.0290\,\text{in.} \downarrow$$ ◀

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PROBLEM 9.122

For the beam and loading of Prob. 9.120, determine the value of w for which the deflection is zero at the midpoint C of the beam. Use $E = 200$ GPa.

SOLUTION

Symmetric beam and loading:

$$R_A = R_B = 5w \qquad (w \text{ in kN/m})$$

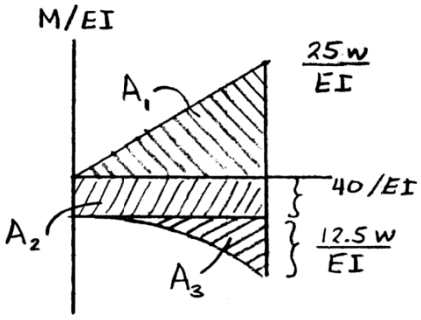
Bending moment in kN · m:

$$M = 5wx - 40 - \frac{1}{2}wx^2$$

At $x = 5$ m,

$$M = 25w - 40 - 12.5w$$

Draw M/EI diagram by parts.



$$A_1 = \frac{1}{2}\left(\frac{25w}{EI}\right)(5) = \frac{62.5w}{EI}$$

$$A_2 = -\frac{(40)(5)}{EI} = -\frac{200}{EI}$$

$$A_3 = -\frac{1}{3}\left(\frac{12.5w}{EI}\right)(5) = -\frac{20.833w}{EI}$$

$$\bar{x}_1 = \frac{2}{3}(5) = 3.3333 \text{ m}$$

$$\bar{x}_2 = \frac{1}{2}(5) = 2.5 \text{ m}$$

$$\bar{x}_3 = \frac{3}{4}(5) = 3.75 \text{ m}$$



Place reference tangent at C.

Deflection at C is zero.

$$t_{A/C} = y_A - y_C = 0$$

$$A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 = 0$$

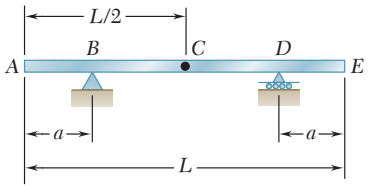
$$\left(\frac{62.5w}{EI}\right)(3.3333) - \left(\frac{200}{EI}\right)(2.5) - \left(\frac{20.833w}{EI}\right)(3.75) = 0$$

$$\frac{130.21w}{EI} - \frac{500}{EI} = 0$$

$$w = \frac{500}{130.21} = 3.84 \text{ kN/m}$$

$$w = 3.84 \text{ kN/m} \blacktriangleleft$$

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PROBLEM 9.123*

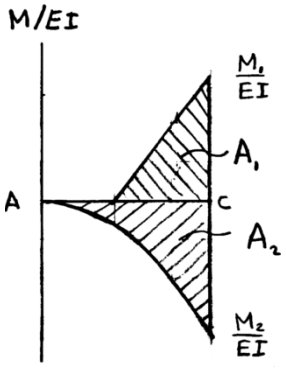
A uniform rod *AE* is to be supported at two points *B* and *D*. Determine the distance *a* for which the slope at ends *A* and *E* is zero.

SOLUTION

Let w = weight per unit length of rod.

Symmetric beam and loading:

$$R_B = R_D = \frac{1}{2}wL$$



Bending moment:

Over *AB*: $M = -\frac{1}{2}wx^2$

Over *BCD*: $M = -\frac{1}{2}wx^2 + \frac{1}{2}wL(x-a)$

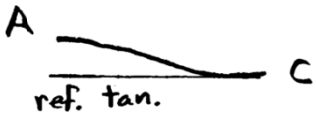
Draw M/EI diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2}\frac{wL(\frac{L}{2}-a)}{EI} = \frac{1}{4}\frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = \frac{1}{2}\frac{w(\frac{L}{2})^2}{EI} = -\frac{1}{8}\frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2}\frac{M_1}{EI}\left(\frac{L}{2}-a\right) = \frac{1}{16}\frac{wL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3}\left(\frac{M_2}{EI}\right)\frac{L}{2} = -\frac{1}{48}\frac{wL^3}{EI}$$



Place reference tangent at *C*. $\theta_C = 0$

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2) = 0$$

$$-\frac{1}{16}\frac{wL(L-2a)^2}{EI} + \frac{1}{48}\frac{wL^3}{EI} = 0$$

Let $u = \frac{a}{L}$ and divide by $\frac{wL^3}{48EI}$.

$$1 - 3(1-2u)^2 = 0$$

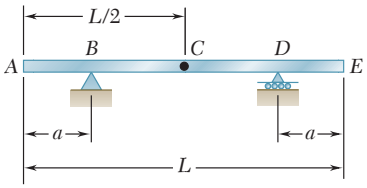
$$1 - 2u = \frac{\sqrt{3}}{3}$$

$$u = \frac{1}{2}\left(1 - \frac{\sqrt{3}}{3}\right) = 0.21132$$

$$\frac{a}{L} = 0.211$$

$$a = 0.211L \blacktriangleleft$$

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PROBLEM 9.124*

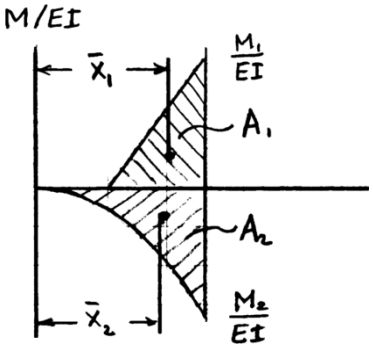
A uniform rod *AE* is to be supported at two points *B* and *D*. Determine the distance *a* from the ends of the rod to the points of support, if the downward deflections of points *A*, *C*, and *E* are to be equal.

SOLUTION

Let w = weight per unit length of rod.

Symmetric beam and loading:

$$R_B = R_D = \frac{1}{2}wL$$



Bending moment:

Over *AB*: $M = -\frac{1}{2}wx^2$

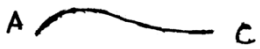
Over *BCD*: $M = -\frac{1}{2}wx^2 + \frac{1}{2}wL(x-a)$

Draw M/EI diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2}\frac{wL(\frac{L}{2}-a)}{EI} = \frac{1}{4}\frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = -\frac{1}{2}\frac{w(\frac{L}{2})^2}{EI} = -\frac{1}{8}\frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2}\frac{M_1}{EI}\left(\frac{L}{2}-a\right) = \frac{1}{16}\frac{wL(L-2a)^2}{EI}$$



$$A_2 = \frac{1}{3}\left(\frac{M_2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$

$$\bar{x}_1 = a + \frac{2}{3}\left(\frac{L}{2}-a\right) = \frac{1}{3}(L+a)$$

$$\bar{x}_2 = \frac{L}{2} - \frac{1}{4}\left(\frac{L}{2}\right) = \frac{3}{8}L$$

Place reference tangent at *C*.

$$y_A - y_C = t_{A/C} = 0$$

$$A_1\bar{x}_1 + A_2\bar{x}_2 = 0$$

$$\frac{1}{16}\frac{wL(L-2a)^2}{EI}\frac{1}{3}(L+a) - \frac{1}{48}\frac{wL^3}{EI}\frac{3}{8}L = 0$$

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PROBLEM 9.124* *(Continued)*

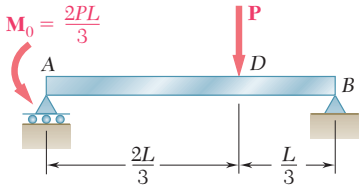
Let $u = \frac{a}{L}$. Divide by $\frac{wL^4}{48EI}$.

$$(1-2u)^2(1+u) - \frac{3}{8} = 0$$

$$4u^3 - 3u + \frac{5}{8} = 0$$

Solving for u, $u = 0.22315$ $\frac{a}{L} = 0.223$ $a = 0.223L$ ◀

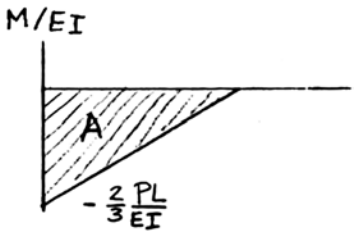
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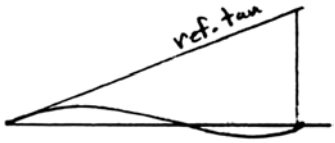


PROBLEM 9.125

For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

SOLUTION





$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{2PL}{3} - R_A L + P\frac{L}{3} = 0 \qquad R_A = P$$

$$+\circlearrowleft M_A = 0: \quad \frac{2PL}{3} - P\frac{2L}{3} + R_B L = 0 \qquad R_B = 0$$

Draw $\frac{M}{EI}$ diagram. Reference tangent at *A*.

$$A = -\frac{1}{2}\cdot\left(\frac{2}{3}\frac{PL}{EI}\right)\left(\frac{2L}{3}\right) = -\frac{2}{9}\frac{PL^2}{EI}$$

$$t_{B/A} = \left(-\frac{2}{9}\frac{PL^2}{EI}\right)\left(\frac{2}{3}\frac{2L}{3} + \frac{L}{3}\right) = -\frac{14}{81}\frac{PL^3}{EI}$$

$$t_{D/A} = \left(-\frac{2}{9}\frac{Pl^2}{EI}\right)\left(\frac{2}{3}\cdot\frac{2L}{3}\right) = -\frac{8}{81}\frac{PL^3}{EI}$$

(*a*) Deflection at *D*.

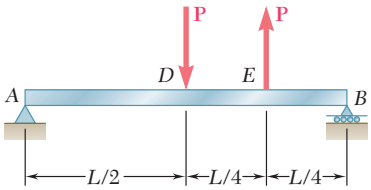
$$y_D = t_{D/A} - \frac{x_D}{L}t_{B/A}$$

$$= -\frac{8}{81}\frac{PL^3}{EI} + \frac{2}{3}\times\frac{14}{81}\frac{PL^3}{EI} = \frac{4}{243}\frac{PL^3}{EI}\uparrow$$ ◀

(*b*) Slope at *A*.

$$\theta_A = -\frac{t_{B/A}}{L} = \frac{14}{81}\frac{PL^2}{EI}$$ ◀

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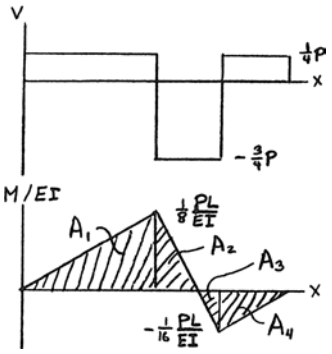
PROBLEM 9.126

For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \frac{PL}{2} - \frac{PL}{4} = 0 \qquad R_A = \frac{1}{4}P \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -\frac{PL}{2} + P\frac{3L}{4} + R_B L = 0 \qquad R_B = \frac{1}{4}P \downarrow$$



Draw *V* (shear) diagram and $\frac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{32}\frac{PL^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\left(\frac{L}{6}\right) = \frac{1}{96}\frac{PL^2}{EI}$$

$$A_3 = \frac{1}{2}\left(-\frac{1}{16}\frac{PL}{EI}\right)\left(\frac{L}{12}\right) = -\frac{1}{384}\frac{PL^2}{EI}$$

$$A_4 = \frac{1}{2}\left(-\frac{1}{16}\frac{PL}{EI}\right)\left(\frac{L}{4}\right) = -\frac{1}{128}\frac{PL^3}{EI}$$

Place reference tangent at *A*.

$$t_{B/A} = \left(\frac{1}{32}\frac{PL^2}{EI}\right)\left(\frac{2L}{3}\right) + \left(\frac{1}{96}\frac{PL^2}{EI}\right)\left(\frac{L}{2} - \frac{1}{3}\cdot\frac{L}{6}\right)$$
$$+\left(-\frac{1}{384}\frac{PL^2}{EI}\right)\left(\frac{L}{4} + \frac{1}{3}\cdot\frac{L}{12}\right) + \left(-\frac{1}{128}\frac{PL^2}{EI}\right)\left(\frac{2}{3}\cdot\frac{L}{4}\right)$$
$$= \frac{1}{48}\frac{PL^3}{EI} + \frac{1}{216}\frac{PL^3}{EI} - \frac{5}{6912}\frac{PL^3}{EI} - \frac{1}{768}\frac{PL^3}{EI} = \frac{3}{128}\frac{PL^3}{EI}$$

$$t_{D/A} = \left(\frac{1}{32}\frac{PL^2}{EI}\right)\left(\frac{1}{3}\cdot\frac{L}{2}\right) = \frac{1}{192}\frac{PL^3}{EI}$$

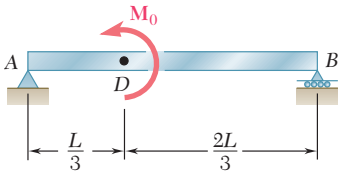
(*a*) Deflection at *D*.

$$y_D = t_{D/A} - \frac{x_D}{L}t_{B/A} = \frac{1}{192}\frac{PL^3}{EI} - \frac{1}{2}\left(\frac{3}{128}\frac{PL^3}{EI}\right) \qquad y_D = -\frac{5}{768}\frac{PL^3}{EI} \blacktriangleleft$$

(*b*) Slope at *A*.

$$\theta_A = -\frac{t_{B/A}}{L} \qquad \theta_A = -\frac{3}{128}\frac{PL^2}{EI} \blacktriangleleft$$

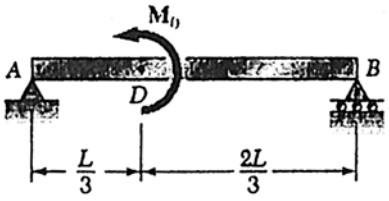
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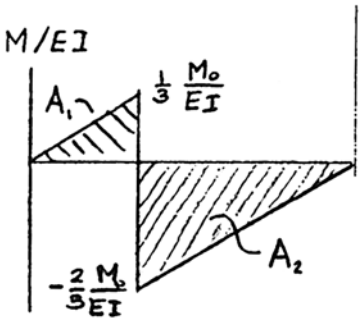


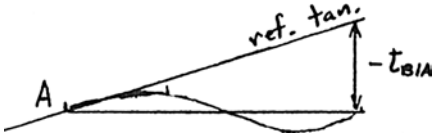
PROBLEM 9.127

For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

SOLUTION







Reactions: $R_A = \dfrac{M_0}{L} \uparrow, \qquad R_B = \dfrac{M_0}{L} \downarrow$

Draw $\dfrac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2}\left(\frac{1}{3}\frac{M_0}{EI}\right)\frac{L}{3} = \frac{1}{18}\frac{M_0 L}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{2}{3}\frac{M_0}{EI}\right)\frac{2L}{3} = -\frac{2}{9}\frac{M_0 L}{EI}$$

Place reference tangent at *A*.

$$t_{B/A} = A_1\left(\frac{L}{9} + \frac{2L}{3}\right) + A_2\left(\frac{2}{3}\cdot\frac{2L}{3}\right)$$

$$= \frac{7}{162}\frac{M_0 L^2}{EI} - \frac{8}{81}\frac{M_0 L^2}{EI} = -\frac{1}{18}\frac{M_0 L^2}{EI}$$

$$t_{D/A} = A_1\frac{L}{9} = \frac{1}{162}\frac{M_0 L^2}{EI}$$

(*a*) Deflection at *D*.

$$y_D = t_{D/A} - \frac{x_D}{L}t_{B/A}$$

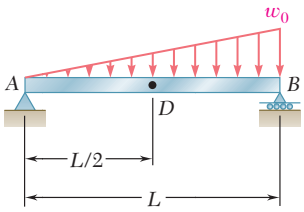
$$= \frac{1}{162}\frac{M_0 L^2}{EI} - \frac{1}{3}\left(-\frac{1}{18}\frac{M_0 L^2}{EI}\right) = \frac{2}{81}\frac{M_0 L^2}{EI}$$

$$y_D = \frac{2}{81}\frac{M_0 L^2}{EI} \uparrow \blacktriangleleft$$

(*b*) Slope at end *A*.

$$\theta_A = -\frac{t_{B/A}}{L} = \frac{1}{18}\frac{M_0 L}{EI} \qquad \theta_A = \frac{1}{18}\frac{M_0 L}{EI} \blacktriangleleft$$

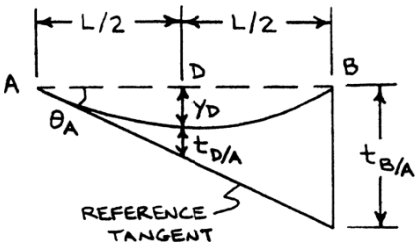
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PROBLEM 9.128

For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

SOLUTION

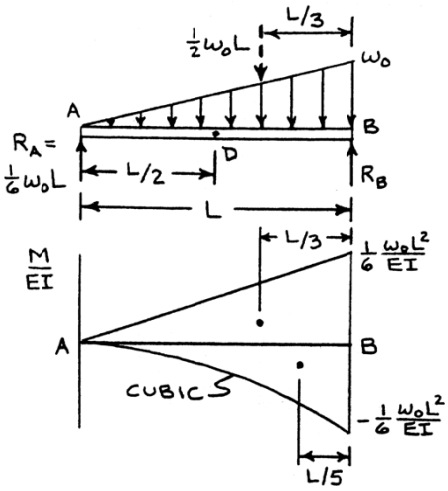


$$t_{B/A} = \frac{1}{2}\left(\frac{1}{6}\frac{w_0L^2}{EI}\right)(L)\left(\frac{L}{3}\right) + \frac{1}{4}\left(-\frac{1}{6}\frac{w_0L^2}{EI}\right)(L)\left(\frac{L}{5}\right)$$

$$= \frac{7w_0L^4}{360EI}$$

$$t_{D/A} = \frac{1}{2}\left(\frac{1}{12}\frac{w_0L^2}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) + \frac{1}{4}\left(-\frac{1}{48}\frac{w_0L^2}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{10}\right)$$

$$= \frac{37w_0L^4}{11{,}520EI}$$



(*a*) Deflection at *D*.

$$y_D = \frac{1}{2}t_{B/A} - t_{D/A}$$

$$= \frac{1}{2}\left(\frac{7w_0L^4}{360EI}\right) - \frac{37w_0L^4}{11{,}520EI}$$

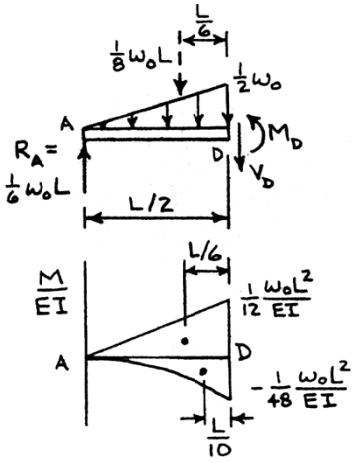
$$= \frac{75w_0L^4}{11{,}520EI}$$

$$y_D = \frac{5w_0L^4}{768EI} \downarrow \blacktriangleleft$$

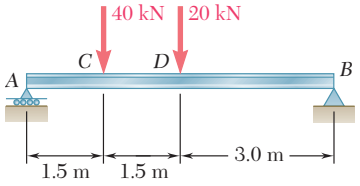
(*b*) Slope at *A*.

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{7w_0L^3}{360EI}$$

$$\theta_A = \frac{7w_0L^3}{360EI} \blacktriangleleft$$



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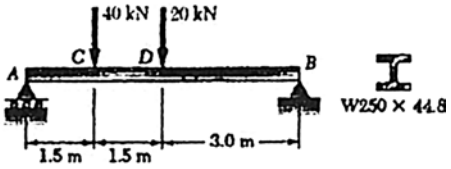




PROBLEM 9.129

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.

SOLUTION



$$E = 200 \times 10^9 \text{ Pa}$$

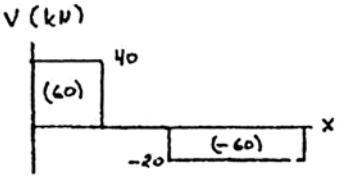
$$I = 70.8 \times 10^6 \text{ mm}^4 = 70.8 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(70.8 \times 10^{-6}) = 19.16 \times 10^6 \text{ N} \cdot \text{m}^2$$

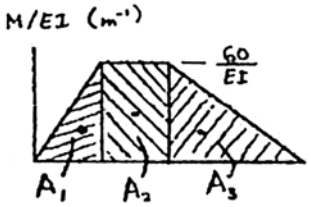
$$= 14{,}160 \text{ kN} \cdot \text{m}^2$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -6R_A + (4.5)(40) + (3)(20) = 0$$

$$R_A = 40 \text{ kN}$$



Draw shear and $\dfrac{M}{EI}$ diagrams.



$$A_1 = \frac{1}{2}\left(\frac{60}{EI}\right)(1.5) = \frac{45}{EI}$$

$$A_2 = \left(\frac{60}{EI}\right)(1.5) = \frac{90}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{60}{EI}\right)(3) = \frac{90}{EI}$$

Place reference tangent at *A*.



$$t_{B/A} = A_1(4.5 + 0.5) + A_2(3 + 0.75) + A_3(2.0)$$

$$= \frac{742.5}{EI} \text{ m}$$

$$t_{D/A} = A_1(1.5 + 0.5) + A_2(0.75)$$

$$= \frac{157.5}{EI} \text{ m}$$

(*a*) Slope at *A*.

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{742.5}{6EI} = -\frac{123.75}{EI} = -\frac{123.75}{14{,}160}$$

$$= -8.74 \times 10^{-3}$$

$$\theta_A = 8.74 \times 10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

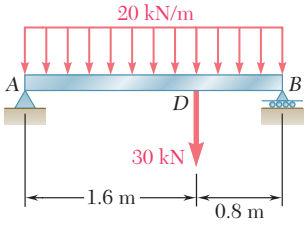
(*b*) Deflection at *D*.

$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{157.5}{EI} - \left(\frac{3}{6}\right)\left(\frac{742.5}{EI}\right) = -\frac{213.75}{EI}$$

$$= -\frac{213.75}{14{,}160} = -15.10 \times 10^{-3} \text{ m}$$

$$y_D = 15.10 \text{ mm} \downarrow \blacktriangleleft$$

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PROBLEM 9.130

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in meters.

For W150 × 24,

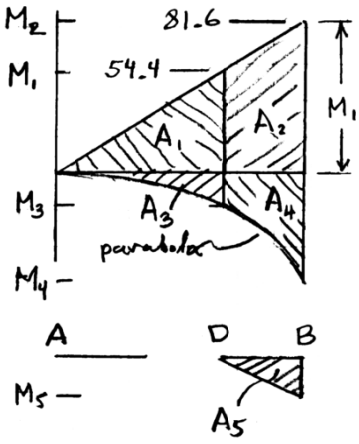
$$I = 13.4 \times 10^6 \text{ mm}^4$$
$$= 13.4 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(13.4 \times 10^{-6}) = 2.68 \times 10^6 \text{ N} \cdot \text{m}^2$$
$$= 2680 \text{ kN} \cdot \text{m}^2$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -2.4R_A + (0.8)(30) + (1.2)(2.4)(20) = 0$$
$$R_A = 34 \text{ kN} \uparrow$$

Draw bending moment diagram by parts.

$$M_1 = (1.6)(34) = 54.4 \text{ kN} \cdot \text{m}$$
$$M_2 = (2.4)(34) = 81.6 \text{ kN} \cdot \text{m}$$
$$M_3 = -\frac{1}{2}(20)(1.6)^2 = -25.6 \text{ kN} \cdot \text{m}$$
$$M_4 = -\frac{1}{2}(20)(2.4)^2 = -57.6 \text{ kN} \cdot \text{m}$$
$$M_5 = -(0.8)(30) = -24 \text{ kN} \cdot \text{m}$$
$$A_1 = \frac{1}{2}(1.6)(54.4) = 43.52 \text{ kN} \cdot \text{m}^2$$
$$A_1 + A_2 = \frac{1}{2}(2.4)(81.6) = 97.92 \text{ kN} \cdot \text{m}^2$$
$$A_3 = \frac{1}{3}(1.6)(-25.6) = -13.6533 \text{ kN} \cdot \text{m}^2$$
$$A_3 + A_4 = \frac{1}{3}(2.4)(-57.6) = -46.08 \text{ kN} \cdot \text{m}^2$$
$$A_5 = \frac{1}{2}(0.8)(-24) = -9.6 \text{ kN} \cdot \text{m}^2$$



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PROBLEM 9.130 *(Continued)*

(*a*) Slope at *A*. Place reference tangent at *A*.

$$\theta_A = -\frac{1}{L} t_{B/A}$$

$$t_{B/A} = \frac{1}{EI}\left\{(A_1 + A_2)\left(\frac{1}{3}\right)(2.4) + (A_3 + A_4)\left(\frac{1}{4}\right)(2.4) + A_5\left(\frac{1}{3}\right)(0.8)\right\}$$

$$= \frac{48.128}{2680} = 17.9582 \times 10^{-3}\,\text{m}$$

$$\theta_A = -\frac{17.9582 \times 10^{-3}}{2.4} = -7.48258 \times 10^{-3}$$

$$\theta_A = 7.48 \times 10^{-3}\,\text{rad.} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at point *D*.

$$y_D = t_{D/A} + \theta_A x_D$$

$$t_{D/A} = \frac{1}{EI}\left\{A_1\left(\frac{1}{3}\right)(1.6) + A_2\left(\frac{1}{4}\right)(1.6)\right\}$$

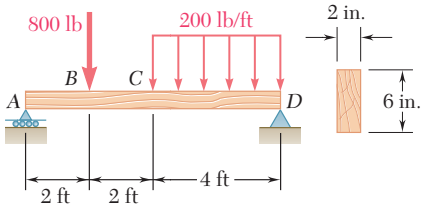
$$= \frac{17.7493}{2680} = 6.62289 \times 10^{-3}\,\text{m}$$

$$y_D = 6.62289 \times 10^{-3} + (-7.48258 \times 10^{-3})(1.6)$$

$$= -5.3492 \times 10^{-3}\,\text{m}$$

$$y_D = 5.35\ \text{mm} \downarrow \blacktriangleleft$$

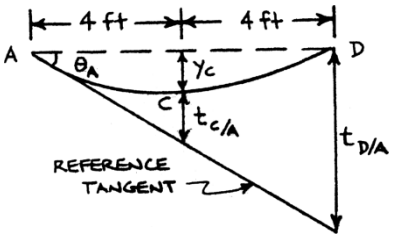
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PROBLEM 9.131

For the timber beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *C*. Use $E = 1.7 \times 10^6$ psi.

SOLUTION



$$I = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$

$$EI = (1.7 \times 10^3 \text{ ksi})(36 \text{ in}^4) = 61.2 \times 10^3 \text{ kip} \cdot \text{in}^2$$

$$A_1 = \frac{1}{2}(6.4)(8) = 25.6 \text{ kip} \cdot \text{ft}^2$$

$$A_2 = \frac{1}{2}(-4.8)(6) = -14.4 \text{ kip} \cdot \text{ft}^2$$

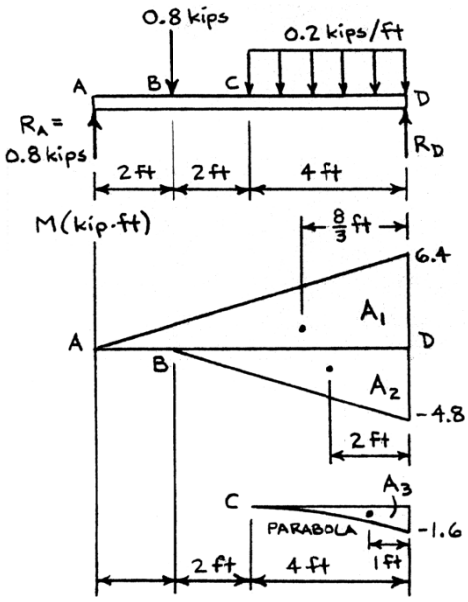
$$A_3 = \frac{1}{3}(-1.6)(4) = -2.1333 \text{ kip} \cdot \text{ft}^2$$

$$EIt_{D/A} = A_1\left(\frac{8}{3} \text{ ft}\right) + A_2(2 \text{ ft}) + A_3(1 \text{ ft})$$

$$= (25.6)\left(\frac{8}{3}\right) + (-14.4)(2) + (-2.1333)(1)$$

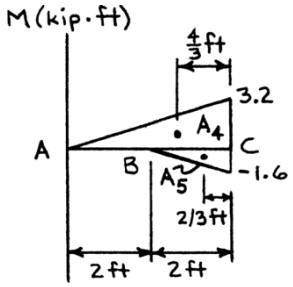
$$= 37.333 \text{ kip} \cdot \text{ft}^3 = 64{,}512 \text{ kip} \cdot \text{in}^3$$

$$t_{D/A} = \frac{64{,}512}{61.2 \times 10^3} = 1.05412 \text{ in.}$$



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PROBLEM 9.131 *(Continued)*



$$A_4 = \frac{1}{2}(3.2)(4) = 6.4 \text{ kip} \cdot \text{ft}^2$$

$$A_5 = \frac{1}{2}(-1.6)(2) = -1.6 \text{ kip} \cdot \text{ft}^2$$

$$EIt_{C/A} = A_4\left(\frac{4}{3}\text{ft}\right) + A_5\left(\frac{2}{3}\text{ft}\right) = (6.4)\left(\frac{4}{3}\right) + (-1.6)\left(\frac{2}{3}\right)$$

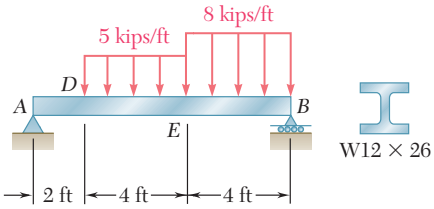
$$= 7.4667 \text{ kip} \cdot \text{ft}^3 = 12{,}902.4 \text{ kip} \cdot \text{in}^3$$

$$t_{C/A} = \frac{12{,}902.4}{61.2 \times 10^3} = 0.21082 \text{ in.}$$

(*a*) $\theta_A = -\dfrac{t_{D/A}}{L} = -\dfrac{1.05412}{96} = -0.0109804 \text{ rad}$ $\theta_A = 10.98 \times 10^{-3} \text{ rad}$ ◀

(*b*) $y_C = t_{C/A} - \dfrac{1}{2}t_{D/A} = 0.21082 \text{ in.} - \dfrac{1}{2}(1.05412 \text{ in.})$ $y_C = 0.316 \text{ in.} \downarrow$ ◀

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PROBLEM 9.132

For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *E*. Use $E = 29\times10^6$ psi.

SOLUTION

Units: Forces in kips; lengths in ft.

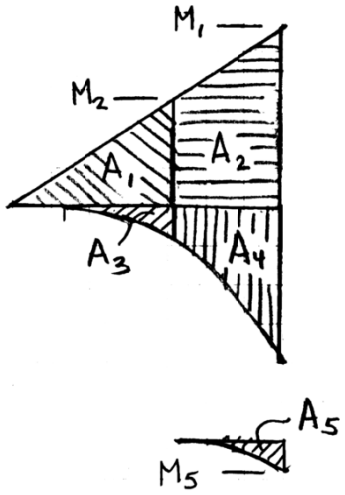
For W12 × 26, $I = 204 \text{ in}^4$

$$EI = (29\times10^6)(204) = 5.916\times10^9 \text{ lb}\cdot\text{in}^2$$
$$= 41{,}083 \text{ kip}\cdot\text{ft}^2$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -10R_A + (6)(4)(5) + (2)(4)(8) = 0 \qquad R_A = 18.4 \text{ kips} \uparrow$$

Consider loading as 5 kips/ft from *D* to *B* plus 3 kips/ft from *E* to *B*. Draw bending moment diagram by parts.

$$M_1 = 10R_A = 184 \text{ kip}\cdot\text{ft}$$
$$M_2 = 6R_A = 110.4 \text{ kip}\cdot\text{ft}$$
$$M_3 = -\frac{1}{2}(5)(8)^2 = -160 \text{ kip}\cdot\text{ft}$$
$$M_4 = -\frac{1}{2}(5)(4)^2 = -40 \text{ kip}\cdot\text{ft}$$
$$M_5 = -\frac{1}{2}(3)(4)^2 = -24 \text{ kip}\cdot\text{ft}$$
$$A_1 + A_2 = \frac{1}{2}(10)(184) = 920 \text{ kip}\cdot\text{ft}^2$$
$$A_1 = \frac{1}{2}(6)(110.4) = 331.2 \text{ kip}\cdot\text{ft}^2$$
$$A_3 + A_4 = \frac{1}{3}(8)(-160) = -426.667 \text{ kip}\cdot\text{ft}^2$$
$$A_3 = \frac{1}{3}(4)(-40) = -53.333 \text{ kip}\cdot\text{ft}^2$$
$$A_5 = \frac{1}{3}(4)(-24) = -32 \text{ kip}\cdot\text{ft}^2$$



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PROBLEM 9.132 *(Continued)*

(*a*) Slope at *A*. $y_B = y_A + \theta_A L + t_{B/A} \qquad y_A = y_B = 0$

$$\theta_A = -t_{B/A}/L$$

$$t_{B/A} = \frac{1}{EI}\left\{(A_1 + A_2)\left(\frac{1}{3}\right)(10) + (A_3 + A_4)\left(\frac{1}{4}\right)(8) + (A_5)\left(\frac{1}{4}\right)(4)\right\}$$

$$= \frac{2181.33}{41{,}083} = 53.096 \times 10^{-3}\,\text{ft}$$

$$\theta_A = -\frac{53.096 \times 10^{-3}}{10} = -5.3096 \times 10^{-3}$$

$$\theta_A = 5.31 \times 10^{-3}\ \text{rad} \curvearrowright \blacktriangleleft$$

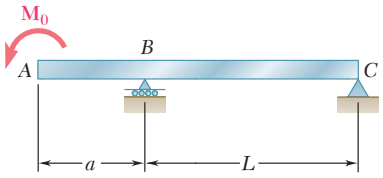
(*b*) Deflection at *E*. $y_E = x_E \theta_A + t_{E/A}$

$$t_{E/A} = \frac{1}{EI}\left\{(A_1)\left(\frac{1}{3}\right)(6) + (A_3)\left(\frac{1}{4}\right)(4)\right\} = \frac{609.067}{41{,}083} = 14.8253 \times 10^{-3}\,\text{ft}$$

$$y_E = (6)(-5.3096 \times 10^{-3}) + 14.8253 \times 10^{-3} = -17.0323 \times 10^{-3}\,\text{ft}$$

$$y_E = 0.204\ \text{in.} \downarrow \blacktriangleleft$$

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PROBLEM 9.133

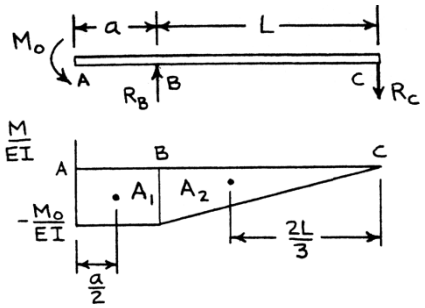
For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*.

SOLUTION

$$A_1 = -\frac{M_0 a}{EI}$$

$$A_2 = -\frac{M_0 L}{2EI}$$

$$t_{C/B} = A_2\left(\frac{2L}{3}\right) = \left(-\frac{M_0 L}{2EI}\right)\left(\frac{2L}{3}\right) = -\frac{M_0 L^2}{3EI}$$

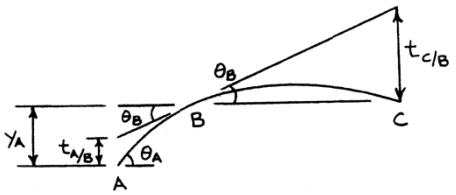


(*a*) Slope at *A*.

$$\theta_B = \frac{t_{C/B}}{L} = \frac{M_0 L}{3EI}$$

$$\theta_B = \theta_A + \theta_{B/A} = \theta_A + A_1$$

$$\frac{M_0 L}{3EI} = \theta_A - \frac{M_0 a}{EI}$$



$$\theta_A = \frac{M_0}{3EI}(L + 3a) \curvearrowright \blacktriangleleft$$

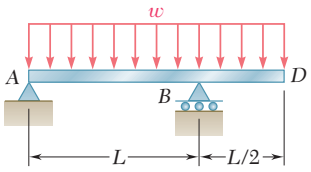
(*b*) Deflection at *A*.

$$t_{A/B} = A_1\left(\frac{a}{2}\right) = -\frac{M_0 a^2}{2EI}$$

$$y_A = \frac{a}{L}t_{C/B} + t_{A/B} = \frac{a}{L}\left(-\frac{M_0 L^2}{3EI}\right) - \frac{M_0 a^2}{2EI}$$

$$y_A = \frac{M_0 a}{6EI}(2L + 3a) \downarrow \blacktriangleleft$$

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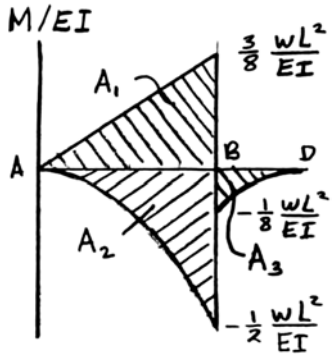


PROBLEM 9.134

For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *D*.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \left(\frac{3}{2}wL\right)\left(\frac{1}{4}L\right) = 0 \quad R_A = \frac{3}{8}wL$$

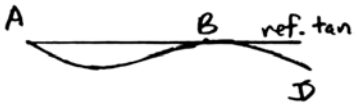


Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{3}{8}\frac{wL^2}{EI}\right)L = \frac{3}{16}\frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3}\left(\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

$$A_3 = -\frac{1}{3}\left(\frac{1}{8}\frac{wL^2}{EI}\right)\frac{L}{2} = -\frac{1}{48}\frac{wL^3}{EI}$$



Place reference tangent at *A*.

$$t_{B/A} = A_1\frac{L}{3} + A_2\frac{L}{4}$$

$$= \frac{1}{16}\frac{wL^4}{EI} - \frac{1}{24}\frac{wL^4}{EI} = \frac{1}{48}\frac{wL^4}{EI}$$

(*a*) Slope at *A*.

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{1}{48}\frac{wL^3}{EI}$$ ◀

$$t_{D/A} = A_1\left(\frac{L}{3} + \frac{L}{2}\right) + A_2\left(\frac{L}{4} + \frac{L}{2}\right) + A_3\left(\frac{3}{4}\cdot\frac{L}{2}\right)$$

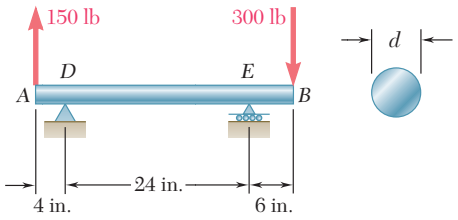
$$= \frac{5}{32}\frac{wL^4}{EI} - \frac{1}{8}\frac{wL^4}{EI} - \frac{1}{128}\frac{wL^4}{EI} = \frac{3}{128}\frac{wL^4}{EI}$$

(*b*) Deflection at *D*.

$$y_D = t_{D/A} - \frac{x_D}{L}t_{B/A} = \frac{3}{128}\frac{wL^4}{EI} - \frac{3}{2}\cdot\frac{1}{48}\frac{wL^4}{EI} = -\frac{1}{128}\frac{wL^4}{EI}$$

$$y_D = \frac{1}{128}\frac{wL^4}{EI}\downarrow$$ ◀

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PROBLEM 9.135

Knowing that the beam AB is made of a solid steel rod of diameter $d = 0.75$ in., determine for the loading shown (*a*) the slope at point D, (*b*) the deflection at point A. Use $E = 29 \times 10^6$ psi.

SOLUTION

Units: Forces in lb; lengths in inches.

$$c = \frac{1}{2} = \frac{1}{2}(0.75) = 0.375 \text{ in.}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.375)^4 = 0.0155316 \text{ in}^4$$

$$EI = (29 \times 10^6)(0.0155316) = 450.4 \times 10^3 \text{ lb} \cdot \text{in}^2$$

Draw $\frac{M}{EI}$ diagram by parts by considering the bending moment diagram due to each of the applied loads.

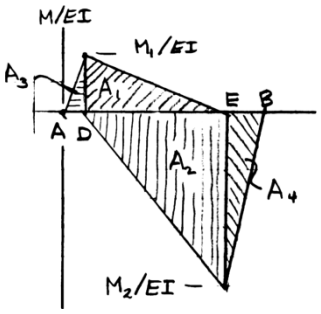
$$\frac{M_1}{EI} = \frac{(150)(4)}{450.4 \times 10^3} = 1.33215 \times 10^{-3} \text{ in.}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(300)(6)}{450.4 \times 10^3} = -3.99645 \times 10^{-3} \text{ in.}^{-1}$$

$$A_1 = \frac{1}{2}(24)(1.33215 \times 10^{-3}) = 15.9858 \times 10^{-3}$$

$$A_2 = \frac{1}{2}(24)(-3.99645 \times 10^{-3}) = -47.9574 \times 10^{-3}$$

$$A_3 = \frac{1}{2}(4)(1.33215 \times 10^{-3}) = 2.6643 \times 10^{-3}$$



Place reference tangent at D.

(*a*) Slope at point D.

$$y_E = y_D + L\theta_D + t_{E/D} \qquad \theta_D = -t_{E/D}/L$$

$$t_{E/A} = 16A_1 + 8A_2 = -127.8864 \times 10^{-3} \text{ in.}$$

$$\theta_D = \frac{-127.8864 \times 10^3}{24} = 5.3286 \times 10^{-3}$$

$$\theta_D = 5.33 \times 10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

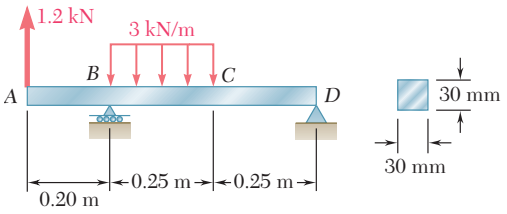
(*b*) Deflection at A.

$$y_A = y_D - a\theta_D + t_{A/D} = t_{A/D} - a\theta_D$$

$$y_A = A_3\left(\frac{2}{3}\right)(4) - (4)(5.3286 \times 10^{-3}) = -14.21 \times 10^{-3} \text{ in.}$$

$$y_A = 0.01421 \text{ in.} \downarrow \blacktriangleleft$$

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PROBLEM 9.136

Knowing that the beam *AD* is made of a solid steel bar, determine (*a*) the slope at point *B*, (*b*) the deflection at point *A*. Use $E = 200$ GPa.

SOLUTION

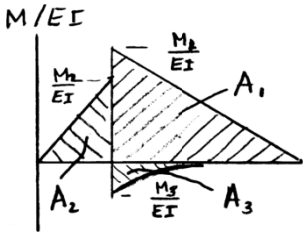
$$E = 200\times10^{9}\text{ Pa} \quad I = \frac{1}{12}(30)(30)^3 = 67.5\times10^{3}\text{ mm}^4 = 67.5\times10^{-9}\text{ m}^4$$

$$EI = (200\times10^{9})(67.5\times10^{-9}) = 13{,}500\text{ N}\cdot\text{m}^2 = 13.5\text{ kN}\cdot\text{m}^2$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -(0.2)(1.2) - (3)(0.25)(0.125) + 5R_D = 0 \quad R_D = 0.6675\text{ kN}$$

Draw $\frac{M}{EI}$ diagram by parts.

$$M_1 = (0.6675)(0.5) = 0.33375\text{ kN}\cdot\text{m}$$
$$M_2 = (1.2)(0.2) = 0.240\text{ kN}\cdot\text{m}$$
$$M_3 = -\frac{1}{2}(3)(0.25)^2 = -0.09375\text{ kN}\cdot\text{m}$$



$$A_1 = \frac{1}{2}(0.33375)(0.5)/EI = 0.0834375/EI$$
$$A_2 = \frac{1}{2}(0.240)(0.2)/EI = 0.024/EI$$
$$A_3 = \frac{1}{3}(-0.09375)(0.25)/EI = -0.0078125/EI$$

Place reference tangent at *B*.

$$t_{D/B} = A_1\left(\frac{2}{3}\cdot 0.5\right) + A_3\left(\frac{3}{4}\cdot(0.25) + 0.25\right) = 0.024395/EI$$

(*a*) Slope at *B*. $\theta_B = -\frac{t_{D/B}}{L} = -\frac{0.024395}{0.5EI} = -\frac{0.048789}{EI}$

$$= -3.6140\times10^{-3} \qquad \theta_B = 3.61\times10^{-3}\text{ rad} \circlearrowright \blacktriangleleft$$

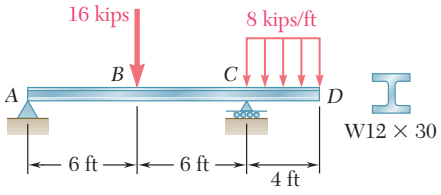
$$t_{A/B} = A_2\left(\frac{2}{3}(0.20)\right) = 0.0032/EI = 0.23704\times10^{-3}\text{ m}$$

(*b*) Deflection at *A*. $y_A = t_{A/B} - L_{AB}\theta_B$

$$= 0.23704\times10^{-3} - (0.2)(-3.6140\times10^{-3}) = 0.960\times10^{-3}\text{ m}$$

$$y_A = 0.960\text{ mm}\uparrow \blacktriangleleft$$

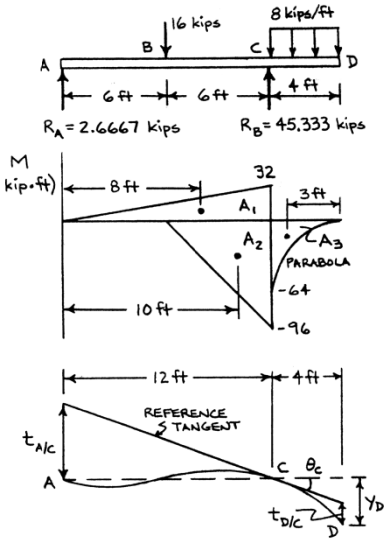
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PROBLEM 9.137

For the beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *D*. Use $E = 29 \times 10^6$ psi.

SOLUTION



Free body *AD*: $+\circlearrowleft \Sigma M_C = 0: \quad (16)(6) - (32)(2) - 12R_A = 0$

$$R_A = 2.6667 \text{ kips} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 2.6667 - 16 + R_B - 32 = 0$$

$$R_B = 45.333 \text{ kips} \uparrow$$

For W12×30, $I = 238 \text{ in}^4$

$$EI = (29 \times 10^3 \text{ ksi})(238 \text{ in}^4)$$

$$= 6.902 \times 10^6 \text{ kip} \cdot \text{in}^2 = 47{,}931 \text{ kip} \cdot \text{ft}^2$$

(*a*) Slope at *C*.

$$A_1 = \frac{1}{2}(32)(12) = 192 \text{ kip} \cdot \text{ft}^2$$

$$A_2 = \frac{1}{2}(-96)(6) = -288 \text{ kip} \cdot \text{ft}^2$$

$$EIt_{A/C} = A_1(8 \text{ ft}) + A_2(10 \text{ ft})$$

$$= (192)(8) + (-288)(10) = -1344 \text{ kip} \cdot \text{ft}^3$$

$$t_{A/C} = -\frac{1344}{47{,}931} = -28.040 \times 10^{-3} \text{ ft} = -0.33648 \text{ in.}$$

$$\theta_C = \frac{t_{A/C}}{L} = -\frac{28.040 \times 10^{-3} \text{ ft}}{12 \text{ ft}}$$

$$\theta_C = 2.34 \times 10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at *D*.

$$EIt_{D/C} = A_1(3 \text{ ft}) = \frac{1}{3}(-64)(4)(3) = -256 \text{ kip} \cdot \text{ft}^3$$

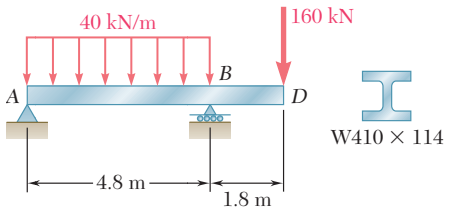
$$t_{D/C} = -\frac{256}{47{,}931} = -5.3410 \times 10^{-3} \text{ ft} = -0.064092 \text{ in.}$$

$$y_D = t_{D/C} + \frac{4}{12} t_{A/C} = -0.064092 + \frac{1}{3}(-0.33648)$$

$$= -0.176252 \text{ in.}$$

$$y_D = 0.1763 \text{ in.} \downarrow \blacktriangleleft$$

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PROBLEM 9.138

For the beam and loading shown, determine (*a*) the slope at point *B*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in meters.

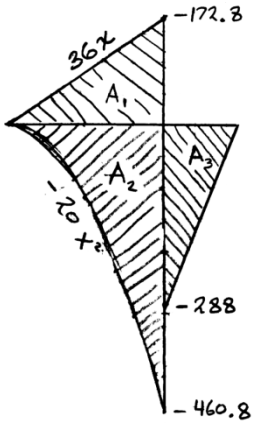
$$I = 462\times10^{6}\,\text{mm}^4 = 462\times10^{-6}\,\text{m}^4$$

$$EI = (200\times10^{9})(462\times10^{-6})$$

$$= 92.4\times10^{6}\,\text{N}\cdot\text{m}^2 = 92{,}400\ \text{kN}\cdot\text{m}^2$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -4.8R_A + (40)(4.8)(2.4) - (160)(1.8) = 0$$

$$R_A = 36\ \text{kN}$$



Draw bending moment diagram by parts.

$$A_1 = \frac{1}{2}(4.8)(172.8) = 414.72\ \text{kN}\cdot\text{m}^2$$

$$A_2 = \frac{1}{3}(4.8)(-460.8) = -737.28\ \text{kN}\cdot\text{m}^2$$

$$A_3 = \frac{1}{2}(1.8)(-288) = -259.2\ \text{kN}\cdot\text{m}^2$$

Place reference tangent at *B*.

(*a*) Slope at *B*.

$$y_A = y_B - L\theta_B + t_{A/B}$$



$$\theta_B = \frac{t_{B/A}}{L} = \frac{1}{EIL}\left\{A_1\left(\frac{2}{3}\right)(4.8) + A_2\left(\frac{3}{4}\right)(4.8)\right\}$$

$$= \frac{-1327.104}{(92{,}400)(4.8)} = -2.9922\times10^{-3}$$

$$\theta_B = 2.99\times10^{-3}\ \text{rad} \circlearrowright \blacktriangleleft$$

(*b*) Deflection at *D*.

$$y_D = y_B + a\theta_B + t_{D/B}$$

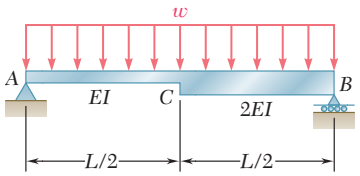
$$= 0 + (1.8)(-2.9922\times10^{-3}) - \frac{1}{EI}\left\{A_3\left(\frac{2}{3}\right)(1.8)\right\}$$

$$= -5.3860\times10^{-3} - \frac{311.04}{92{,}400}$$

$$= -8.75\times10^{-3}\ \text{m}$$

$$y_D = 8.75\ \text{mm}\downarrow \blacktriangleleft$$

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PROBLEM 9.139

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the slope at end *B*, (*c*) the deflection at the midpoint *C*.

SOLUTION

Reactions: $R_A = R_B = \frac{1}{2}wL$

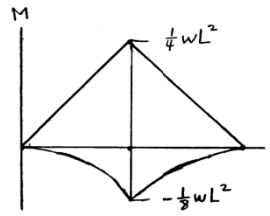
Draw bending moment and *M/EI* diagrams by parts as shown.

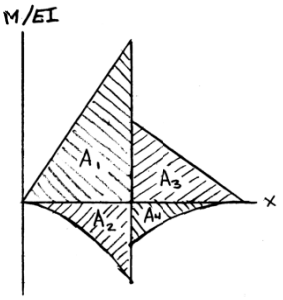
$$A_1 = \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{wL^2}{4EI} = \frac{wL^3}{16EI}$$

$$A_2 = -\frac{1}{3}\cdot\frac{L}{2}\cdot\frac{wL^2}{8EI} = -\frac{wL^3}{48EI}$$

$$A_3 = \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{wL^2}{8EI} = -\frac{wL^3}{32EI}$$

$$A_4 = -\frac{1}{3}\cdot\frac{L}{2}\cdot\frac{wL^2}{16EI} = -\frac{wL^3}{96EI}$$





Place reference tangent at *A*.

(*a*) Slope at end *A*.

$$y_B = y_A + L\theta_A + t_{B/A}$$

$$\theta_A = -t_{B/A}/L$$

$$t_{B/A} = \left(\frac{L}{2}+\frac{L}{6}\right)A_1 + \left(\frac{L}{2}+\frac{L}{8}\right)A_2 + \frac{L}{3}A_3 + \frac{3L}{8}A_4$$

$$= \frac{wL^4}{EI}\left(\frac{1}{24} - \frac{5}{384} + \frac{1}{96} - \frac{1}{256}\right) = \frac{9wL^4}{256EI}$$

$$\theta_A = -\frac{9wL^4}{256EI}\cdot\frac{1}{L} = -\frac{9wL^3}{256EI}$$

$$\theta_A = \frac{9wL^3}{256EI}$$ ◀

(*b*) Slope at end *B*.

$$\theta_B = \theta_A + \theta_{B/A} = -\frac{9wL^3}{256EI} + A_1 + A_2 + A_3 + A_4$$

$$\theta_B = \frac{7wL^3}{256EI}$$

$$\theta_B = \frac{7wL^3}{256EI}$$ ◀

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PROBLEM 9.139 *(Continued)*

(*c*) Deflection at midpoint *C*.

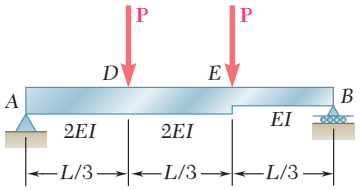
$$y_A = y_C + \frac{L}{2}\theta_A + t_{C/A}$$

$$t_{C/A} = \left(\frac{L}{6}\right)A_1 + \left(\frac{L}{8}\right)A_2 = \frac{wL^4}{128EI}$$

$$y_C = 0 + \left(\frac{L}{2}\right)\left(-\frac{9wL^3}{256EI}\right) + \frac{wL^4}{128EI} = -\frac{5wL^4}{512EI}$$

$$y_C = \frac{5wL^4}{512EI} \downarrow \blacktriangleleft$$

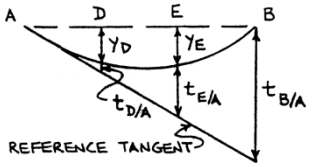
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PROBLEM 9.140

For the beam and loading shown, determine the deflection (*a*) at point *D*, (*b*) at point *E*.

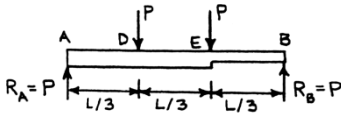
SOLUTION



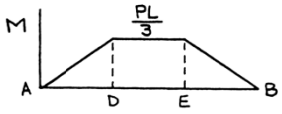
$$A_1 = \frac{1}{2}\left(\frac{PL}{6EI}\right)\left(\frac{L}{3}\right) = \frac{PL^2}{36EI}$$

$$A_2 = \left(\frac{PL}{6EI}\right)\left(\frac{L}{3}\right) = \frac{PL^2}{18EI}$$

$$A_3 = \frac{1}{2}\left(\frac{PL}{3EI}\right)\left(\frac{L}{3}\right) = \frac{PL^2}{18EI}$$



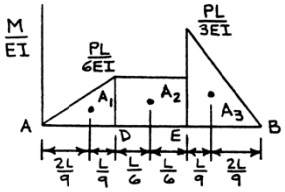
$$t_{D/A} = A_1\left(\frac{L}{9}\right) = \left(\frac{PL^2}{36EI}\right)\left(\frac{L}{9}\right) = \frac{PL^3}{324EI}$$



$$t_{E/A} = A_1\left(\frac{L}{9} + \frac{L}{3}\right) + A_2\left(\frac{L}{6}\right)$$

$$= \left(\frac{PL^3}{36EI}\right)\left(\frac{4L}{9}\right) + \left(\frac{PL^2}{18EI}\right)\left(\frac{L}{6}\right)$$

$$= \frac{7PL^3}{324EI}$$



$$t_{B/A} = A_1\left(\frac{7L}{9}\right) + A_2\left(\frac{L}{2}\right) + A_3\left(\frac{2L}{9}\right)$$

$$= \left(\frac{PL^3}{36EI}\right)\left(\frac{7L}{9}\right) + \left(\frac{PL^2}{18EI}\right)\left(\frac{L}{2}\right) + \left(\frac{PL^2}{18EI}\right)\left(\frac{2L}{9}\right)$$

$$= \frac{5PL^3}{81EI}$$

(*a*) Deflection at *D*.

$$y_D = \frac{1}{3}t_{B/A} - t_{D/A} = \frac{1}{3}\left(\frac{5PL^3}{81EI}\right) - \frac{PL^3}{324EI} = \frac{17PL^3}{972EI}$$

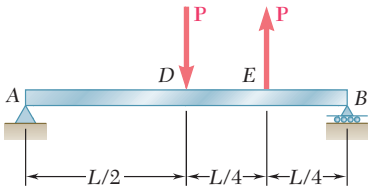
$$y_D = \frac{17PL^3}{972EI} \downarrow \blacktriangleleft$$

(*b*) Deflection at *E*.

$$y_E = \frac{2}{3}t_{B/A} - t_{E/A} = \frac{2}{3}\left(\frac{5PL^3}{81EI}\right) - \frac{7PL^3}{324EI} = \frac{19PL^3}{972EI}$$

$$y_E = \frac{19PL^3}{972EI} \downarrow \blacktriangleleft$$

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PROBLEM 9.141

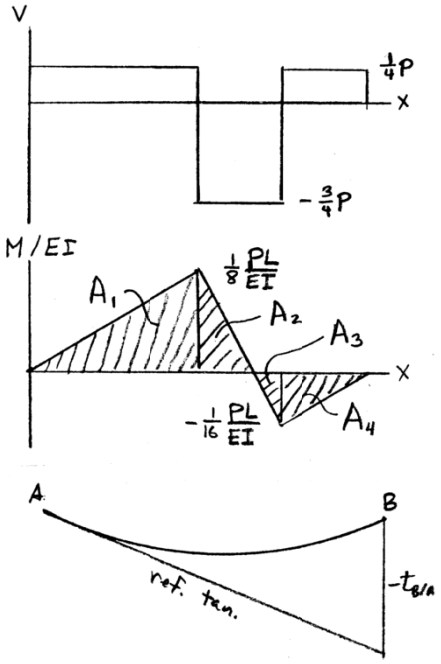
For the beam and loading of Prob. 9.126, determine the magnitude and location of the largest downward deflection.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \frac{PL}{2} - \frac{PL}{4} = 0 \qquad R_A = \frac{1}{4}P \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -\frac{PL}{2} + P\frac{3L}{4} + R_B L = 0 \qquad R_B = \frac{1}{4}P \downarrow$$

Draw V (shear) diagram and M/EI diagram.



$$A_1 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{32}\frac{PL^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\left(\frac{L}{6}\right) = \frac{1}{96}\frac{PL^2}{EI}$$

$$A_3 = \frac{1}{2}\left(-\frac{1}{16}\frac{PL}{EI}\right)\left(\frac{L}{12}\right) = -\frac{1}{384}\frac{PL^2}{EI}$$

$$A_4 = \frac{1}{2}\left(-\frac{1}{16}\frac{PL}{EI}\right)\left(\frac{L}{4}\right) = -\frac{1}{128}\frac{PL^3}{EI}$$

Place reference tangent at A.

$$t_{B/A} = \left(\frac{1}{32}\frac{PL^2}{EI}\right)\left(\frac{2L}{3}\right) + \left(\frac{1}{96}\frac{PL^2}{EI}\right)\left(\frac{L}{2} - \frac{1}{3}\frac{L}{6}\right)$$
$$+ \left(-\frac{1}{384}\frac{PL^2}{EI}\right)\left(\frac{L}{4} + \frac{1}{3}\frac{L}{12}\right) + \left(-\frac{1}{128}\frac{PL^2}{EI}\right)\left(\frac{2}{3}\cdot\frac{L}{4}\right)$$
$$= \left(\frac{1}{48} + \frac{1}{216} - \frac{5}{6912} - \frac{1}{768}\right)\frac{PL^3}{EI} = \frac{3}{128}\frac{PL^3}{EI}$$

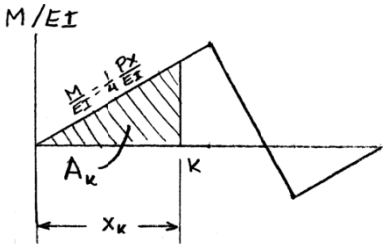
$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{3}{128}\frac{PL^2}{EI}$$

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PROBLEM 9.141 *(Continued)*

Let point K be the location of $|y_m|$.

$$\begin{aligned}\theta_K &= \theta_A + \theta_{K/A} \\ &= -\frac{3}{128}\frac{PL^2}{EI} + A_K \\ &= -\frac{3}{128}\frac{PL^2}{EI} + \frac{1}{2}\left(\frac{1}{4}\frac{Px_K}{EI}\right)x_K \\ &= \frac{P}{EI}\left(-\frac{3}{128}L^2 + \frac{1}{8}x_K^2\right) = 0\end{aligned}$$



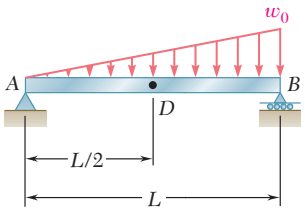
$$x_K = \sqrt{\frac{3}{16}}\,L = \frac{1}{4}\sqrt{3}\,L \qquad x_K = 0.433L \blacktriangleleft$$

$$t_{K/A} = A_K\left(\frac{1}{3}x_K\right) = \frac{1}{2}\left(\frac{1}{4}\frac{Px_K^2}{EI}\right)\frac{x_K}{3} = \frac{1}{24}\frac{Px_K^3}{EI} = \frac{\sqrt{3}}{512}\frac{PL^3}{EI}$$

$$y_K = t_{K/A} - \frac{x_K}{L}t_{B/A} = \frac{\sqrt{3}}{512}\frac{PL^3}{EI} - \left(\frac{1}{4}\sqrt{3}\right)\frac{3}{128}\frac{PL^3}{EI} = -\frac{\sqrt{3}}{256}\frac{PL^3}{EI}$$

$$y_K = 0.00677\frac{PL^3}{EI}\downarrow \blacktriangleleft$$

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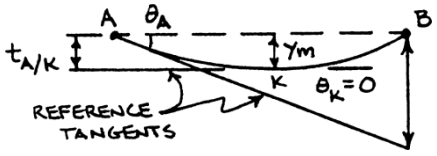


PROBLEM 9.142

For the beam and loading of Prob. 9.128, determine the magnitude and location of the largest downward deflection.

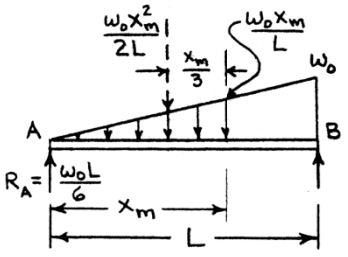
SOLUTION

From Prob. 9.128: $\theta_A = -\dfrac{7w_0L^3}{360EI}$



$$A_1 = \frac{1}{2}\left(\frac{w_0L}{6EI}x_m\right)(x_m) = \frac{w_0Lx_m^2}{12EI}$$

$$A_2 = \frac{1}{4}\left(-\frac{w_0x_m^3}{6EIL}\right)(x_m) = -\frac{w_0x_m^4}{24EIL}$$

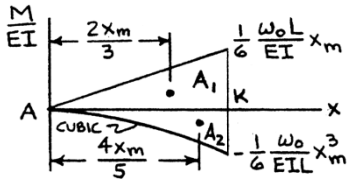


Maximum deflection occurs at K, where $\theta_K = 0$.

$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + A_1 + A_2$$

$$0 = -\frac{7w_0L^3}{360EI} + \frac{w_0Lx_m^2}{12EI} - \frac{w_0x_m^4}{24EIL}$$

Rearranging: $$0 = \frac{w_0L^2}{360EI}\left[-7 + 30\left(\frac{x_m}{L}\right)^2 - 15\left(\frac{x_m}{L}\right)^4\right]$$



Solving biquadratic: $\left(\dfrac{x_m}{L}\right)^2 = 0.26970 \qquad x_m = 0.51933L$

y_m is $0.519L$ from A. ◀

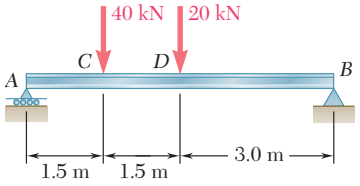
$$t_{A/K} = A_1\frac{2x_m}{3} + A_2\frac{4x_m}{5} = \left(\frac{w_0Lx_m^2}{12EI}\right)\frac{2x_m}{3} + \left(-\frac{w_0x_m^4}{24EIL}\right)\frac{4x_m}{5}$$

$$= \frac{w_0L^4}{90EI}\left[5\left(\frac{x_m}{L}\right)^3 - 3\left(\frac{x_m}{L}\right)^5\right] = \frac{w_0L^4}{90EI}\left[5(0.51933)^3 - 3(0.51933)^5\right]$$

$$= 0.0065222\frac{w_0L^4}{EI} \qquad y_m = |t_{A/K}|$$

$$y_m = 6.52\times10^{-3}\frac{w_0L^4}{EI}$$ ◀

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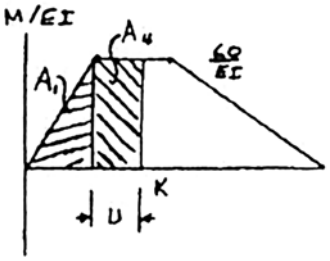


W250 × 44.8

PROBLEM 9.143

For the beam and loading of Prob. 9.129, determine the magnitude and location of the largest downward deflection.

SOLUTION





Referring to the solution to Prob. 9.129,

$$EI = 14{,}160 \text{ kN} \cdot \text{m}^2$$

$$R_A = 40 \text{ kN}, \quad A_1 = \frac{45}{EI}$$

$$t_{B/A} = \frac{742.5}{EI} \text{ m}$$

$$\theta_A = -\frac{123.75}{EI}$$

Let K be the location of the maximum deflection. Assume that K lies between C and D.

$$\theta_K = \theta_A + \theta_{K/A}$$

$$= -\frac{123.75}{EI} + A_1 + A_4$$

$$= -\frac{123.75}{EI} + \frac{45}{EI} + \frac{60u}{EI} = 0$$

$$u = \frac{123.75 - 45}{60} = 1.3125 \text{ m}$$

$$x_K = 1.5 + u = 2.8125 \text{ m}$$

$$t_{K/A} = A_1(u + 0.5) + A_4\left(\frac{1}{2}u\right)$$

$$= \frac{45}{EI}(1.8125) + \frac{(60)(1.3125)\left(\frac{1}{2}\right)(1.3125)}{EI} = \frac{133.242}{EI}$$

$$y_K = t_{K/A} - \frac{x_K}{L} t_{B/A}$$

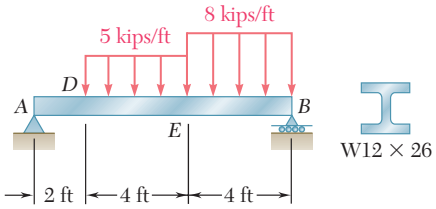
$$= \frac{133.242}{EI} - \frac{2.8125}{6}\left(\frac{742.5}{EI}\right) = -\frac{214.80}{EI} = -\frac{214.80}{14{,}160}$$

$$= -15.17 \times 10^{-3} \text{ m}$$

$$y_K = 15.17 \text{ mm} \downarrow \blacktriangleleft$$

$$x_K = 2.81 \text{ m} \blacktriangleleft$$

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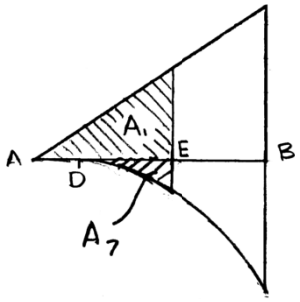


PROBLEM 9.144

For the beam and loading of Prob. 9.132, determine the magnitude and location of the largest downward deflection.

SOLUTION

From the solution to Prob. 9.132,



$$EI = 41{,}083 \text{ kip} \cdot \text{ft}^2$$
$$R_A = 18.4 \text{ kips}$$
$$A_1 = 331.2 \text{ kip} \cdot \text{ft}^2$$
$$A_3 = -53.333 \text{ kip} \cdot \text{ft}^3$$
$$\theta_A = -5.3096 \times 10^{-3}$$

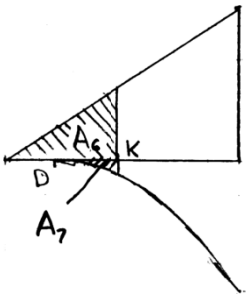
Slope at *E*.

$$\theta_E = \theta_A + \theta_{E/A}$$

$$\theta_{E/A} = \frac{1}{EI}\{A_1 + A_3\} = \frac{278.767}{41{,}083} = 6.7855 \times 10^{-3}$$

$$\theta_E = 1.4759 \times 10^{-3}$$

Since $\theta_E > 0$, the point *K* of zero slope lies to the left of point *E*. Let x_K be the coordinate of point *K*.



$$A_6 = \frac{1}{2} R_A x_K^2 = 9.2\, x_K^2$$

$$A_7 = -\frac{1}{6}(5)(x_K - 2)^3$$

$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + \frac{1}{EI}\{A_6 + A_7\} = 0$$

$$A_6 + A_7 + EI\theta_A = 0$$

$$f(x_K) = 9.2 x_K^2 - \frac{5}{6}(x_K - 2)^3 - 218.134 = 0$$

$$\frac{df}{dx_K} = 18.4\, x_K - 2.5(x_K - 2)^2$$

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PROBLEM 9.144 *(Continued)*

Solve for x_K by iteration.

$$x_K = (x_K)_0 - \frac{f}{df/dx_K}$$

x_K	5	5.1473	5.1525	$x_K = 5.1525$ ft
f	−10.634	−0.362	0.001	
df/dx_K	72.2	70.131		

$$A_6 = 244.244 \text{ kip} \cdot \text{ft}^2, \quad A_7 = -26.108 \text{ kip} \cdot \text{ft}^2$$

Maximum deflection. $y_A = y_K + t_{A/K} = 0 \qquad y_K = -t_{A/K}$

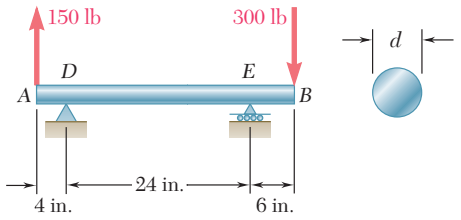
$$\bar{x}_6 = \frac{2}{3}x_K \quad \bar{x}_7 = 2 + \frac{3}{4}(x_K - 2) = \frac{3x_K + 2}{4}$$

$$y_7 = -\frac{1}{EI}\{A_6\bar{x}_6 + A_7\bar{x}_7\} = -\frac{725.033}{41{,}083} = -17.648 \times 10^{-3} \text{ ft}$$

$y_K = 0.212$ in.↓ ◀

$x_K = 5.15$ ft ◀

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PROBLEM 9.145

For the beam and loading of Prob. 9.135, determine the largest upward deflection in span *DE*.

SOLUTION

Units: Forces in lbs; lengths in inches.

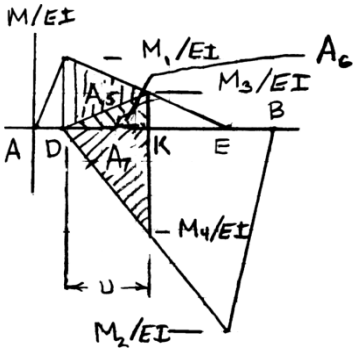
From the solution to Prob. 9.135,

$$EI = 450.4\times10^{3}\ \text{lb}\cdot\text{in.}$$

$$\frac{M_1}{EI} = 1.33215\times10^{-3}\ \text{in.}^{-1}$$

$$\frac{M_2}{EI} = -3.99645\times10^{-3}\ \text{in.}^{-1}$$

$$\theta_D = 5.3286\times10^{-3}$$



Location of maximum deflection.

$$\frac{M_3}{EI} = \frac{M_1}{EI}\left(1-\frac{u}{24}\right)$$

$$\frac{M_4}{EI} = \frac{M_2}{EI}\frac{u}{24}$$

$$A_5 = \frac{1}{2}\frac{M_1}{EI}\cdot u = 0.666075\times10^{-3}u$$

$$A_6 = \frac{1}{2}\frac{M_3}{EI}\left(\frac{u}{24}\right) = 0.666075\times10^{-3}\left(1-\frac{u}{24}\right)u$$

$$A_7 = \frac{1}{2}\frac{M_4}{EI}\frac{u}{24} = -1.998225\times10^{-3}\left(\frac{u}{24}\right)u$$

$$\theta_K = \theta_D + A_5 + A_6 + A_7 = 0$$

Multiply by 10^3.

$$5.3286 + 0.666075u + 0.666075\left(1-\frac{u}{24}\right)u - (1.998225)\frac{u}{24}u = 0$$

$$5.3286 + 1.33215u - 0.1110125u^2 = 0$$

$$u = 15.16515\ \text{in.}$$

$$A_5 = 10.10113\times10^{-3},\quad A_6 = 3.71842\times10^{-3},\quad A_7 = -19.14814\times10^{-3}$$

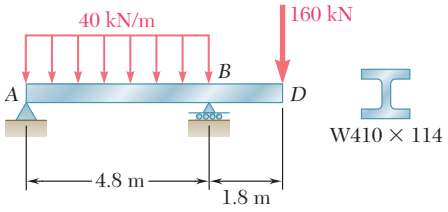
Maximum deflection in portion *DE*. $y_D = y_K + t_{D/K} = 0$

$$y_K = -t_{D/K} = -\left\{A_5\left(\frac{u}{3}\right) + A_6\left(\frac{2u}{3}\right) + A_7\left(\frac{2u}{3}\right)\right\}$$

$$= -\{-0.1049\}$$

$$y_K = 0.1049\ \text{in.} \blacktriangleleft$$

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PROBLEM 9.146

For the beam and loading of Prob. 9.138, determine the largest upward deflection in span *AB*.

SOLUTION

Units: Forces in kN; lengths in meters.

$$I = 462\times10^{6}\ \text{mm}^4 = 462\times10^{-6}\ \text{m}^4$$

$$EI = (200\times10^{9})(462\times10^{-6})$$

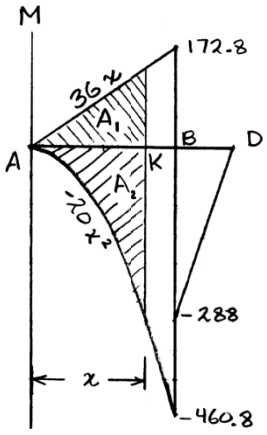
$$= 92.4\times10^{6}\ \text{N}\cdot\text{m}^2 = 92{,}400\ \text{kN}\cdot\text{m}$$

$$+\circlearrowleft M_B = 0: \quad -4.8R_A + (40)(4.8)(2.4) - (160)(1.8) = 0$$

$$R_A = 36\ \text{kN}$$

$$A_1 = \frac{1}{2}x(36x) = 18x^2$$

$$A_2 = \frac{1}{3}x(-20x^2) = -\frac{20}{3}x^3$$



Place reference tangent at *A*.

$$y_B = y_A + L\theta_A + t_{B/A} = 0$$

$$\theta_A = -\frac{t_{B/A}}{L}$$

$$(A_1)_B = (18)(4.8)^2 = 414.72\ \text{kN}\cdot\text{m}^2$$

$$(A_2)_B = \left(\frac{20}{3}\right)(4.8)^3 = -737.28\ \text{kN}\cdot\text{m}^2$$

$$\theta_A = -\frac{1}{EIL}\left\{(A_1)_B\left(\frac{1}{3}\right)(4.8) + (A_2)_B\left(\frac{1}{4}\right)(4.8)\right\}$$

$$= -\frac{-221.184}{(92{,}400)(4.8)} = 0.49870\times10^{-3}$$

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PROBLEM 9.146 *(Continued)*

Locate Point K of maximum deflection.

$$\theta_K = \theta_A + \theta_{K/A} = 0$$

$$EI\theta_A + A_1 + A_2 = 0$$

$$f = 46.08 + 18x_K^2 - \frac{20}{3}x_K^3 = 0 \qquad \frac{df}{dx} = 36x_K - 20x_K^2$$

Solve by iteration.

$$x_K = (x_K)_0 - \frac{f}{df/dx}$$

x_K	3	3.39	3.327	3.3251	3.32514 ←
df/dx	−72	−107.8	−101.6	−101.42	
f	28.08	−6.78	−0.188	0.005	

Place reference tangent at K.

$$y_A = y_K + t_{A/K}$$

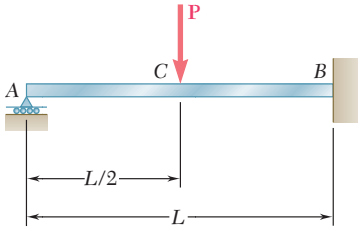
$$y_A - y_K = -t_{A/K}$$

$$= -\frac{1}{EI}\left\{(A_1)\left(\frac{2}{3}x_K\right) + A_2\left(\frac{3}{4}x_K\right)\right\} = -\frac{1}{EI}\left\{12x_K^3 + 5x_K^4\right\}$$

$$= -\frac{170.064}{92{,}400} = -1.841\times10^{-3}\ \text{m}$$

$$y_K = 1.841\ \text{mm} \blacktriangleleft$$

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PROBLEM 9.147

For the beam and loading shown, determine the reaction at the roller support.

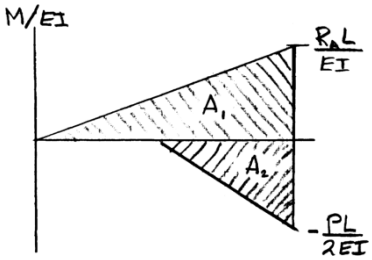
SOLUTION

Remove support *A* and treat R_A as redundant.

Draw the *M*/*EI* diagram by parts.

$$A_1 = \frac{1}{2} L \frac{R_A L}{EI} = \frac{R_A L^2}{2EI}$$

$$A_2 = -\frac{1}{2}\frac{L}{2}\frac{PL}{2} = -\frac{PL^2}{8EI}$$



Place reference tangent at *B*.

$$y_A = y_B - \theta_B L + t_{A/B} = 0$$

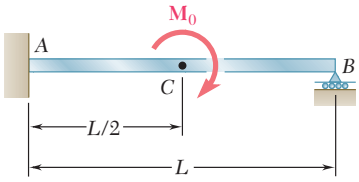
$$t_{A/B} = 0$$

$$A_1\left(\frac{2L}{3}\right) + A_2\left(\frac{L}{2} + \frac{L}{3}\right) = 0$$

$$\frac{R_A L^3}{3EI} - \frac{5PL^3}{48EI} = 0$$

$$R_A = \frac{5}{16} P \uparrow \blacktriangleleft$$

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PROBLEM 9.148

For the beam and loading shown, determine the reaction at the roller support.

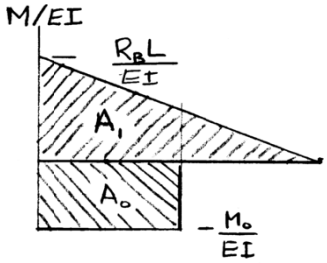
SOLUTION

Remove support B and treat R_B as redundant.

Draw M/EI diagram.

$$A_1 = \frac{1}{2} L \frac{R_A L}{EI} = \frac{R_A L^2}{2EI}$$

$$A_2 = \frac{L}{2} \cdot \frac{M_0 L}{EI} = \frac{M_0 L^2}{2EI}$$



Place reference tangent at A.

$$y_B = y_A + L\theta_A + t_{B/A} = 0$$

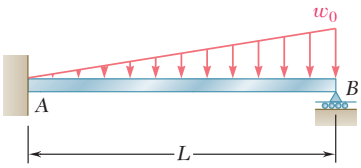
$$t_{B/A} = 0$$

$$A_1\left(\frac{2L}{3}\right) + A_2\left(\frac{L}{2} + \frac{L}{4}\right) = 0$$

$$\frac{R_A L^3}{3EI} - \frac{3M_0 L^2}{8EI} = 0$$

$$R_A = \frac{9}{8}\frac{M_0}{L} \uparrow \blacktriangleleft$$

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PROBLEM 9.149

For the beam and loading shown, determine the reaction at the roller support.

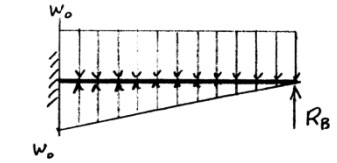
SOLUTION

Remove support B and treat R_B as redundant.

Replace loading by equivalent shown at left.

Draw M/EI diagram for load w_0 and R_B.

Use parts as shown.



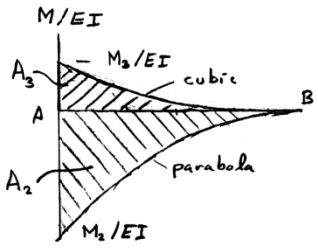
$$A_1 = \frac{1}{2}\left(\frac{R_B L}{EI}\right)(L) = \frac{1}{2}\frac{R_B L^2}{EI}$$

$$M_2 = -\frac{1}{2}w_0 L^2$$

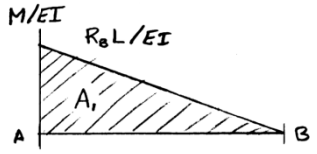
$$A_2 = \frac{1}{3}\left(-\frac{1}{2}\frac{w_0 L^2}{EI}\right)L = -\frac{1}{6}\frac{w_0 L^3}{EI}$$

$$M_3 = \frac{1}{6}\frac{w_0}{L}L^3 = \frac{1}{6}w_0 L^2$$

$$A_3 = \frac{1}{4}\left(\frac{1}{6}\frac{w_0 L^2}{EI}\right)L = \frac{1}{24}\frac{w_0 L^3}{EI}$$

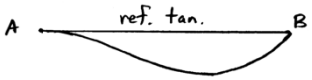


Place reference tangent at A.



$$t_{B/A} = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{3}{4}L\right) + A_3\left(\frac{4}{5}L\right)$$

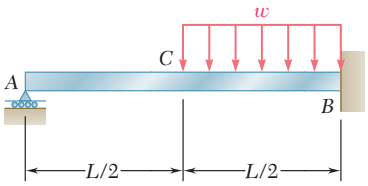
$$= \frac{1}{3}\frac{R_B L^3}{EI} - \frac{1}{8}\frac{w_0 L^4}{EI} + \frac{1}{30}\frac{w_0 L^4}{EI} = 0$$



$$R_B = \frac{11}{40}w_0 L \uparrow$$

$$R_B = 0.275 w_0 L \uparrow \blacktriangleleft$$

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PROBLEM 9.150

For the beam and loading shown, determine the reaction at the roller support.

SOLUTION

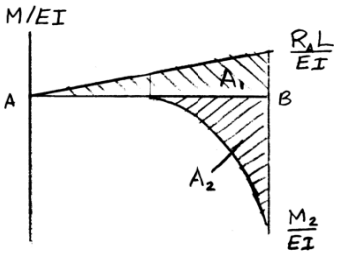
Remove support *A* and treat R_A as redundant.

Draw *M*/*EI* diagram for loads R_A and *w*.

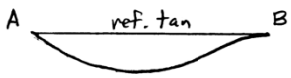
$$M_2 = -\frac{1}{2}w\left(\frac{L}{2}\right)^2 = -\frac{1}{8}wL^2$$

$$A_1 = \frac{1}{2}\left(\frac{R_A L}{EI}\right)L = \frac{1}{2}\frac{R_A L^2}{EI}$$

$$A_2 = \frac{1}{3}\left(-\frac{1}{8}\frac{wL^2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$



Place reference tangent at *B*.



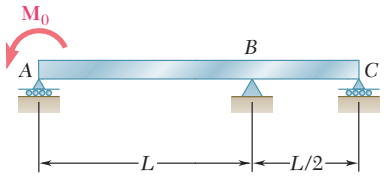
$$t_{A/B} = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{L}{2} + \frac{3}{4}\frac{L}{2}\right)$$

$$= \frac{1}{3}\frac{R_A L^3}{EI} - \frac{7}{384}\frac{wL^4}{EI} = 0$$

$$R_A = \frac{7}{128}wL$$

$$R_A = \frac{7}{128}wL \uparrow \blacktriangleleft$$

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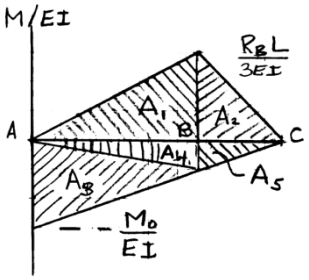
PROBLEM 9.151

For the beam and loading shown, determine the reaction at each support.

SOLUTION

Choose $R_B \downarrow$ as the redundant reaction.

Draw M/EI diagram for the loads R_B and M_0.



$$A_1 = \frac{1}{2}(L)\left(\frac{R_B L}{3EI}\right) = \frac{R_B L^2}{6EI}$$

$$A_2 = \frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{R_B L}{3EI}\right) = \frac{R_B L^2}{12EI}$$

$$A_3 = \frac{1}{2}(L)\left(-\frac{M_0}{EI}\right) = -\frac{M_0 L}{2EI}$$

$$A_4 = \frac{1}{2}(L)\left(\frac{1}{3}\right)\left(-\frac{M_0}{EI}\right) = -\frac{M_0 L}{6EI}$$

$$A_3 + A_4 + A_5 = \frac{1}{2}\left(\frac{3L}{2}\right)\left(-\frac{M_0}{EI}\right) = -\frac{3M_0 L}{4EI}$$

$$y_B = y_A + L\theta_A + t_{B/A} \qquad \theta_A = -t_{B/A}/L$$

$$y_C = y_A + \frac{3L}{2}\theta_A + t_{C/A} = 0 \qquad -\frac{3}{2}t_{B/A} + t_{C/A} = 0$$

$$t_{B/A} = (A_1)\left(\frac{L}{3}\right) + A_3\left(\frac{2L}{3}\right) + A_4\left(\frac{L}{3}\right) = \frac{R_B L^3}{18EI} - \frac{7M_0 L^2}{18EI}$$

$$t_{C/A} = (A_1)\left(\frac{L}{2} + \frac{L}{3}\right) + A_2\left(\frac{L}{3}\right) + (A_3 + A_4 + A_5)(L) = \frac{R_B L^3}{6EI} - \frac{3M_0 L^2}{4EI}$$

$$-\frac{3}{2}t_{B/A} + t_{C/A} = \frac{R_B L^3}{12EI} - \frac{M_0 L^2}{6EI} = 0 \qquad R_B = \frac{2M_0}{L} \downarrow \blacktriangleleft$$

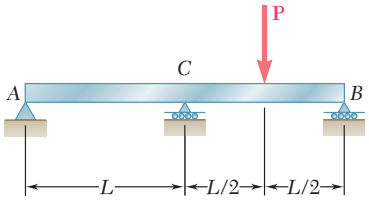
$$+\circlearrowleft \Sigma M_C = 0: \quad M_0 + \frac{L}{2}R_B - \frac{3L}{2}R_A = 0$$

$$R_A = \frac{2}{3L}[M_0 + M_0] \qquad R_A = \frac{4M_0}{3L} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B + R_C = 0 \quad \frac{4}{3}\frac{M_0}{L} - \frac{2M_0}{L} + R_C = 0$$

$$R_C = \frac{2M_0}{3L} \uparrow \blacktriangleleft$$

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PROBLEM 9.152

For the beam and loading shown, determine the reaction at each support.

SOLUTION

Remove support C and add reaction R_C.

Draw M/EI due to each of the loads P and R_C.

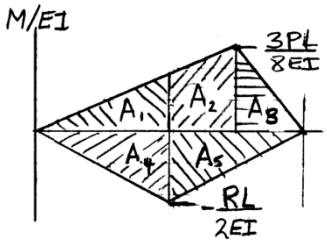
$$A_1 = \frac{1}{2}\cdot L\cdot\frac{2}{3}\cdot\frac{3PL}{8EI} = \frac{PL^2}{8EI}$$

$$A_1 + A_2 = \frac{1}{2}\cdot\frac{3L}{2}\cdot\frac{3PL}{8EI} = \frac{9PL^2}{32EI}$$

$$A_3 = \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{3PL}{8EI} = \frac{3PL^2}{32EI}$$

$$A_4 = \frac{1}{2}\cdot L\left(-\frac{R_C L}{2EI}\right) = -\frac{R_C L^2}{4EI}$$

$$A_4 + A_5 = \frac{1}{2}(2L)\left(-\frac{R_C L}{2EI}\right) = -\frac{R_C L^2}{2EI}$$



Place reference tangent at A. $y_A = 0$

$$y_C = L\theta_A + t_{C/A} = 0 \qquad \theta_A = -\frac{t_{C/A}}{L}$$

$$y_B = 2L\theta_A + t_{B/A} = 0 \qquad -2t_{C/A} + t_{B/A} = 0$$

$$-2\left[A_1\frac{L}{3} + A_4\cdot\frac{L}{3}\right] + \left[(A_1 + A_2)\left(\frac{L}{2} + \frac{2}{3}\cdot\frac{3L}{2}\right) + A_3\cdot\frac{2}{3}\cdot\frac{L}{2} + (A_4 + A_5)\cdot L\right] = 0$$

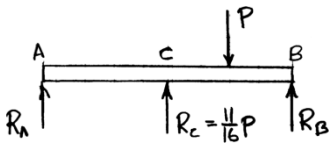
$$-2\left[\frac{PL^3}{24EI} - \frac{R_C L^3}{12EI}\right] + \left[\frac{9PL^3}{32EI} + \frac{PL^3}{32EI} - \frac{R_C L^3}{2EI}\right] = 0$$

$$-\frac{R_C L^3}{3EI} + \frac{11PL^3}{48EI} = 0$$

$$R_C = \frac{11}{16}P\uparrow \blacktriangleleft$$

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PROBLEM 9.152 ***(Continued)***



$$+\circlearrowleft \Sigma M_B = 0: \quad -2LR_A - LR_C + \frac{L}{2}P = 0$$

$$R_A = \frac{P}{4} - \frac{1}{2}R_C = -\frac{3}{32}P$$

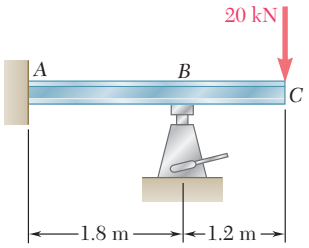
$$R_A = \frac{3}{32}P \downarrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 2LR_B + LR_C - \frac{3L}{2}P = 0$$

$$R_B = \frac{3P}{4} - \frac{1}{2}R_C = \frac{13}{32}P$$

$$R_B = \frac{13}{32}P \uparrow \blacktriangleleft$$

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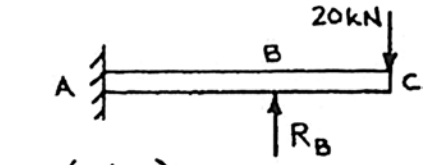


W130 × 23.8

PROBLEM 9.153

A hydraulic jack can be used to raise point *B* of the cantilever beam *ABC*. The beam was originally straight, horizontal, and unloaded. A 20-kN load was then applied at point *C*, causing this point to move down. Determine (*a*) how much point *B* should be raised to return point *C* to its original position, (*b*) the final value of the reaction at *B*. Use $E = 200$ GPa.

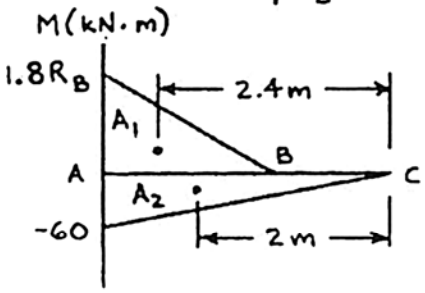
SOLUTION



For W130 × 23.8, $I_x = 8.91 \times 10^6\ \text{mm}^4$

$$EI = (200 \times 10^6\ \text{kPa})(8.91 \times 10^{-6}\ \text{m}^4) = 1782\ \text{kN} \cdot \text{m}^2$$

Let R_B be the jack force in kN.



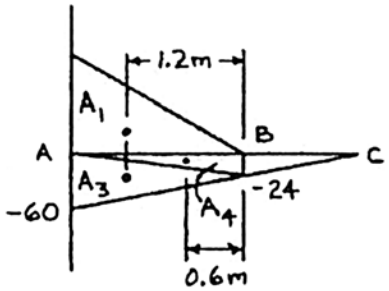
$$A_1 = \frac{1}{2}(1.8R_B)(1.8) = 1.62R_B$$

$$A_2 = \frac{1}{2}(-60)(3) = -90\ \text{kN} \cdot \text{m}^2$$

$$EIt_{C/A} = (2.4)A_1 + (2)A_2$$

$$0 = (2.4)(1.62R_B) + (2)(-90)$$

$$R_B = 46.296\ \text{kN}$$



$$A_1 = 75\ \text{kN} \cdot \text{m}^2$$

$$A_3 = \frac{1}{2}(-60)(1.8) = -54\ \text{kN} \cdot \text{m}^2$$

$$A_4 = \frac{1}{2}(-24)(1.8) = -21.6\ \text{kN} \cdot \text{m}^2$$

$$EIt_{B/A} = (1.2)A_1 + (1.2)A_3 + (0.6)A_4$$

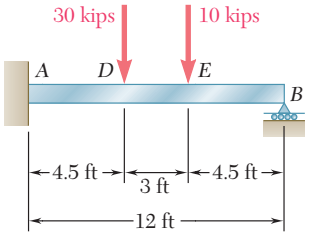
$$= (1.2)(75) + (1.2)(-54) + (0.6)(-21.6)$$

$$= 12.24\ \text{kN} \cdot \text{m}^2$$

(*a*) Deflection at *B*. $y_B = t_{B/A} = \dfrac{EI\,t_{B/A}}{EI} = \dfrac{12.24}{1782} = 6.8687 \times 10^{-3}\ \text{m}$ $\quad y_B = 6.87\ \text{mm} \uparrow$ ◀

(*b*) Reaction at *B*. $\quad R_B = 46.3\ \text{kN} \uparrow$ ◀

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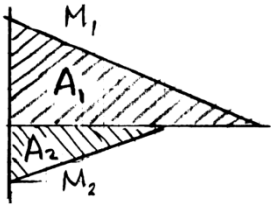


PROBLEM 9.154

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION

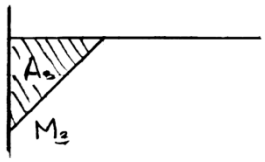
Units: Forces in kips; lengths in feet.



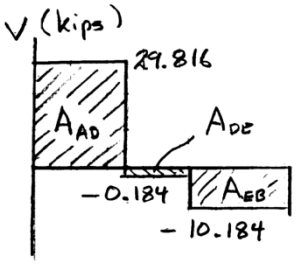
Let R_B be the redundant reaction.

Remove support B and add load R_B.

Draw bending moment diagram by parts.

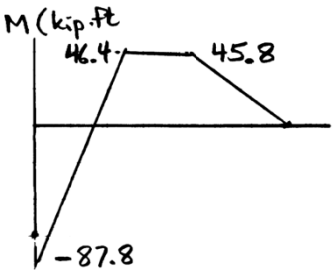


$$M_1 = 12R_B \text{ kip}\cdot\text{ft}$$
$$M_2 = -(4.5+3)(10) = -75 \text{ kip}\cdot\text{ft}$$
$$M_3 = -(4.5)(30) = -135 \text{ kip}\cdot\text{ft}$$
$$A_1 = \frac{1}{2}(12)(12R_B) = 72R_B \text{ kip}\cdot\text{ft}^2$$
$$A_2 = \frac{1}{2}(7.5)(-75) = -281.25 \text{ kip}\cdot\text{ft}^2$$
$$A_3 = \frac{1}{2}(4.5)(-135) = -303.75 \text{ kip}\cdot\text{ft}^2$$
$$y_B = y_A + 12\theta_A + t_{B/A} = 0$$
$$t_{B/A} = 0$$



$$t_{B/A} = \frac{1}{EI}\{(72R_B)(8) + (-281.25)(4.5+5) + (-303.75)(7.5+3)\} = 0$$

$$576R_B - 5861.25 = 0 \qquad R_B = 10.18 \text{ kips} \uparrow \blacktriangleleft$$



Draw shear diagram working from right to left.

B to E: $V = -R_B = -10.176$ kips

E to D: $V = -10.184 + 10 = -0.176$ kips

D to A: $V = -0.176 + 30 = 29.824$ kips

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PROBLEM 9.154 *(Continued)*

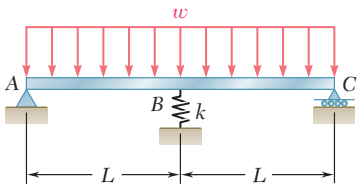
Areas of shear diagram.

$$A_{AD} = (4.5)(29.824) = 134.21 \text{ kip} \cdot \text{ft}$$
$$A_{DE} = (3)(-0.176) = -0.53 \text{ kip} \cdot \text{ft}$$
$$A_{EB} = (4.5)(-10.176) = 45.79 \text{ kip} \cdot \text{ft}$$

Bending moments.

$$M_A = M_1 + M_2 + M_3 = -87.888 \text{ kip} \cdot \text{ft} \qquad M_A = -87.9 \text{ kip} \cdot \text{ft} \blacktriangleleft$$
$$M_D = M_A + A_{AD} = 46.322 \text{ kip} \cdot \text{ft} \qquad M_D = 46.3 \text{ kip} \cdot \text{ft} \blacktriangleleft$$
$$M_E = M_D + A_{DE} = 45.792 \text{ kip} \cdot \text{ft} \qquad M_E = 45.8 \text{ kip} \cdot \text{ft} \blacktriangleleft$$
$$M_B = M_E + A_{EB} = 0 \qquad M_B = 0 \blacktriangleleft$$

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PROBLEM 9.155

For the beam and loading shown, determine the spring constant k for which the force in the spring is equal to one-third of the total load on the beam.

SOLUTION

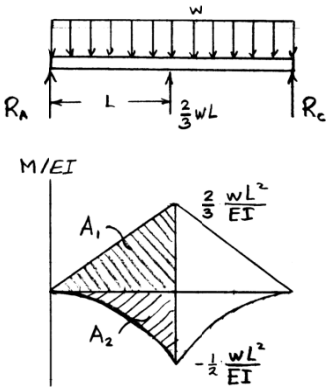
Symmetric beam and loading: $R_C = R_A$

Spring force: $F = \frac{1}{3}(2wL) = \frac{2}{3}wL$

$$+\uparrow \Sigma F_y = 0: \quad R_A + F - 2wL + R_C = 0$$

$$R_A = R_C = \frac{2}{3}wL$$

Draw M/EI diagram by parts.



$$A_1 = \frac{1}{2}\left(\frac{2}{3}\frac{wL^2}{EI}\right)L = \frac{1}{3}\frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3}\left(\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

Place reference tangent at B. $\theta_B = 0$

$$y_B = -t_{A/B}$$

$$= -\left(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L\right)$$

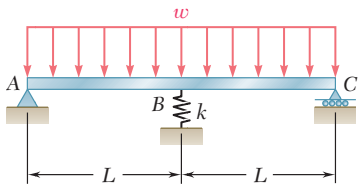
$$= -\frac{7}{72}\frac{wL^4}{EI}$$

$$F = -ky_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{2}{3}wL}{\frac{7}{72}\frac{wL^4}{EI}}$$

$$k = \frac{48}{7}\frac{EI}{L^3} \blacktriangleleft$$

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PROBLEM 9.156

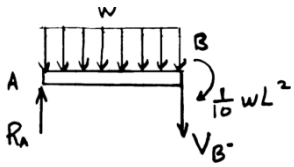
For the beam and loading shown, determine the spring constant k for which the bending moment at B is $M_B = -wL^2/10$.

SOLUTION

Using free body *AB*,

$$+\circlearrowleft M_B = 0: \quad -R_A L + (wL)\left(\frac{L}{2}\right) - \frac{1}{10}wL^2 = 0$$

$$R_A = \frac{2}{5}wL \uparrow$$

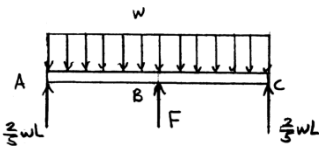


Symmetric beam and loading: $R_C = R_A$

Using free body *ABC*,

$$+\uparrow \Sigma F_y = 0: \quad \frac{2}{5}wL + F + \frac{2}{5}wL - 2wL = 0$$

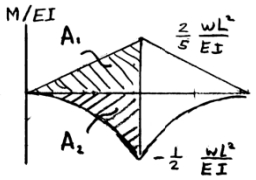
$$F = \frac{6}{5}wL$$



Draw M/EI diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{2}{5}\frac{wL^2}{EI}\right)L = \frac{1}{5}\frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3}\left(\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$



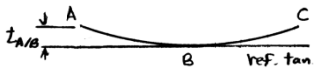
Place reference tangent at *B*.

$$\theta_B = 0$$

$$y_B = -t_{A/B}$$

$$= -\left(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L\right)$$

$$= -\frac{1}{120}\frac{wL^4}{EI}$$

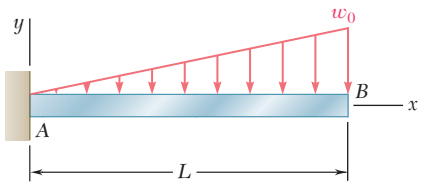


$$F = -ky_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{6}{5}wL}{\frac{1}{120}\frac{wL^4}{EI}}$$

$$k = 144\frac{EI}{L^3} \blacktriangleleft$$

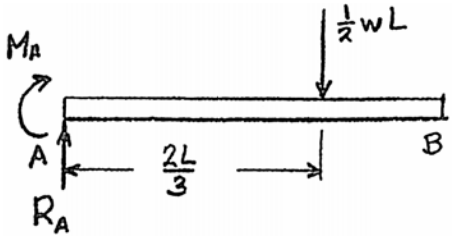
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PROBLEM 9.157

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

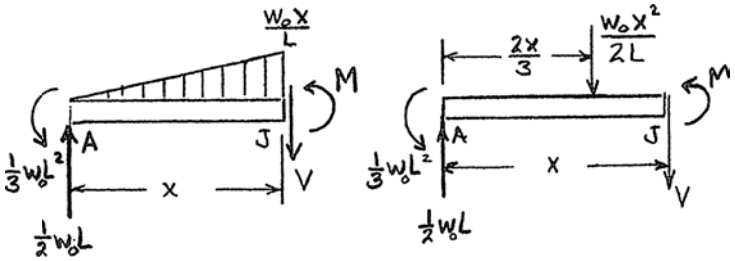
SOLUTION



$$+\uparrow \Sigma F_y = 0:$$

$$R_A = \frac{1}{2}wL = 0$$

$$R_A = \frac{1}{2}w_0 L$$



$$+\circlearrowleft \Sigma M_A = 0:$$

$$-M_A = \frac{2L}{3}\cdot\frac{wL}{2} = 0$$

$$M_A = -\frac{1}{3}w_0L^2$$

$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{1}{3}w_0L^2 - \frac{1}{2}w_0Lx + \frac{w_0x^2}{2L}\cdot\frac{x}{3} + M = 0$$

$$M = -\frac{1}{3}w_0L^2 + \frac{1}{2}w_0Lx - \frac{w_0x^3}{6L}$$

$$EI\frac{d^2y}{dx^2} = -\frac{1}{3}w_0L^2 + \frac{1}{2}w_0Lx - \frac{w_0x^3}{6L}$$

$$EI\frac{dy}{dx} = -\frac{1}{3}w_0L^2x + \frac{1}{4}w_0Lx^2 - \frac{w_0x^4}{24L} + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right]: \quad 0 = -0 + 0 - 0 + C_1 \qquad C_1 = 0$$

$$EIy = -\frac{1}{6}w_0L^2x^2 + \frac{1}{12}w_0Lx^3 - \frac{w_0x^5}{120L} + C_2$$

$$[x = 0, y = 0]: \quad 0 = -0 + 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

(*a*) Elastic curve:

$$y = -\frac{w_0}{EIL}\left(\frac{1}{6}L^3x^2 - \frac{1}{12}Lx^4 + \frac{1}{120}x^5\right) \blacktriangleleft$$

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PROBLEM 9.157 *(Continued)*

(*b*) y at $x = L$

$$y_B = -\frac{w_0 L^4}{EI}\left(\frac{1}{6} - \frac{1}{12} + \frac{1}{120}\right) = -\frac{11}{120}\frac{w_0 L^4}{EI}$$

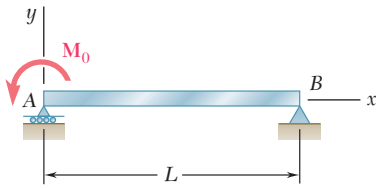
$$y_B = \frac{11}{120}\frac{w_0 L^4}{EI} \downarrow \blacktriangleleft$$

(*c*) $\dfrac{dy}{dx}$ at $x = L$

$$\left.\frac{dy}{dx}\right|_B = -\frac{w_0 L^3}{EI}\left(\frac{1}{3} - \frac{1}{4} + \frac{1}{24}\right) = -\frac{1}{8}\frac{w_0 L^4}{EI}$$

$$\theta_B = \frac{1}{8}\frac{w_0 L^3}{EI} \blacktriangleleft$$

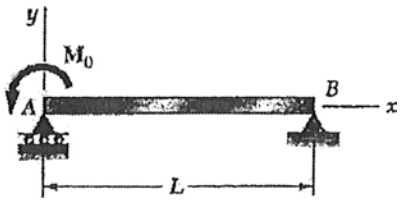
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PROBLEM 9.158

(*a*) Determine the location and magnitude of the maximum deflection of beam *AB*. (*b*) Assuming that beam *AB* is a W360 × 64, $L = 3.5$ m, and $E = 200$ GPa, calculate the maximum allowable value of the applied moment $\mathbf{M}_0$ if the maximum deflection is not to exceed 1 mm.

SOLUTION

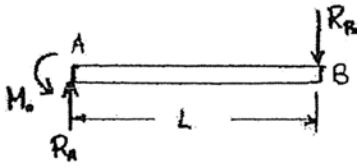


Using entire beam as a free body,

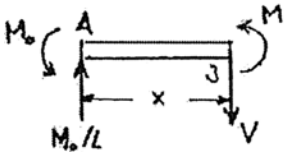
$$+\circlearrowleft \Sigma M_B = 0: \quad M_0 - R_A L = 0 \qquad R_A = \frac{M_0}{L}$$

Using portion *AJ*,

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$ $+\circlearrowleft \Sigma M_J = 0: \quad M_0 - \dfrac{M_0}{L}x + M = 0$



$$M = \frac{M_0}{L}(x - L)$$



$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}(x - L)$$

$$EI\frac{dy}{dx} = \frac{M_0}{L}\left(\frac{1}{2}x^2 - Lx\right) + C_1$$

$$EIy = \frac{M_0}{L}\left(\frac{1}{6}x^3 - \frac{1}{2}Lx^2\right) + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L,\ y = 0]$ $\quad 0 = \dfrac{M_0}{L}\left(\dfrac{1}{6}L^3 - \dfrac{1}{2}L^3\right) + C_1 L + 0 \qquad C_1 = \dfrac{1}{3}M_0 L$

$$y = \frac{M_0}{EIL}\left(\frac{1}{6}x^3 - \frac{1}{2}Lx^2 + \frac{1}{3}L^2 x\right) \qquad \frac{dy}{dx} = \frac{M_0}{EIL}\left(\frac{1}{2}x^2 - Lx + \frac{1}{3}L^2\right)$$

(*a*) To find location of maximum deflection, set $\dfrac{dy}{dx} = 0$.

$$\frac{1}{2}x_m^2 - Lx_m + \frac{1}{3}L^2 = 0 \qquad x_m = L - \sqrt{L^2 - (4)\left(\frac{1}{2}\right)\left(\frac{1}{3}L^2\right)} = \left(1 - \sqrt{\frac{1}{3}}\right)L$$

$$= 0.42265L \qquad x_m = 0.423L \blacktriangleleft$$

$$y_m = \frac{M_0 L^2}{EI}\left\{\left(\frac{1}{6}\right)(0.42265)^3 - \left(\frac{1}{2}\right)(0.42265)^2 + \left(\frac{1}{3}\right)(0.42265)\right\} \qquad y_m = 0.06415\frac{M_0 L^2}{EI} \blacktriangleleft$$

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PROBLEM 9.158 ***(Continued)***

Solving for M_0, $$M_0 = \frac{EIy_m}{0.06415L^2}$$

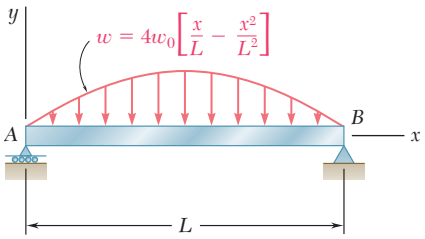
(*b*) Data: $$E = 200 \times 10^9 \text{ Pa}, \quad I = 178 \times 10^6 \text{ mm}^4 = 178 \times 10^{-6} \text{ m}^4$$

$$L = 3.5 \text{ m} \quad y_m = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$M_0 = \frac{(200 \times 10^9)(178 \times 10^{-6})(10^{-3})}{(0.06415)(3.5)^2} = 45.3 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_0 = 45.3 \text{ kN} \cdot \text{m} \blacktriangleleft$$

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PROBLEM 9.159

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection at the midpoint of the span.

$[x = 0,\ M = 0]$ $[x = L,\ M = 0]$

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

SOLUTION

Boundary conditions at *A* and *B* are noted.

$$w = \frac{w_0}{L^2}(4Lx - 4x^2)$$

$$\frac{dV}{dx} = -w = \frac{w_0}{L^2}(4x^2 - 4Lx)$$

$$\frac{dM}{dx} = V = \frac{w_0}{L^2}\left(\frac{4}{3}x^3 - 2Lx^2\right) + C_1$$

$$M = \frac{w_0}{L^2}\left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3\right) + C_1x + C_2$$

$[x = 0,\ M = 0]$ $\quad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L,\ M = 0]$ $\quad 0 = \frac{w_0}{L^2}\left(\frac{1}{3}L^4 - \frac{2}{3}L^4\right) + C_1L + 0 \qquad C_1 = \frac{1}{3}w_0L$

$$EI\frac{d^2y}{dx^2} = M = \frac{w_0}{L^2}\left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3 + \frac{1}{3}L^3x\right)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L^2}\left(\frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^3x^2\right) + C_3$$

$$EIy = \frac{w_0}{L^2}\left(\frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^3x^3\right) + C_3x + C_4$$

$[x = 0,\ y = 0]$ $\quad 0 = 0 + 0 + 0 + 0 + C_4 \qquad C_4 = 0$

$[x = L,\ y = 0]$ $\quad 0 = \frac{w_0}{L^2}\left(\frac{1}{90}L^6 - \frac{1}{30}L^6 + \frac{1}{18}L^6\right) + C_3L + 0 \qquad C_3 = -\frac{1}{30}w_0L^3$

(*a*) Elastic curve.

$$y = \frac{w_0}{EIL^2}\left(\frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^3x^3 - \frac{1}{30}L^5x\right) \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{w_0}{EIL^2}\left(\frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^3x^2 - \frac{1}{30}L^5\right)$$

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PROBLEM 9.159 *(Continued)*

(*b*) Slope at end *A*. Set $x = 0$ in $\dfrac{dy}{dx}$. $\left.\dfrac{dy}{dx}\right|_A = -\dfrac{1}{30}\dfrac{w_0 L^3}{EI}$

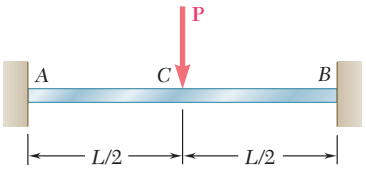
$$\theta_A = \frac{1}{30}\frac{w_0 L^3}{EI} \curvearrowright \blacktriangleleft$$

(*c*) Deflection at midpoint. Set $x = \dfrac{L}{2}$ in y.

$$y_C = \frac{w_0 L^4}{EI}\left\{\left(\frac{1}{90}\right)\left(\frac{1}{2}\right)^6 - \left(\frac{1}{30}\right)\left(\frac{1}{2}\right)^5 + \frac{1}{18}\left(\frac{1}{2}\right)^3 - \frac{1}{30}\left(\frac{1}{2}\right)\right\}$$

$$= \frac{w_0 L^4}{EI}\left\{\frac{1}{5760} - \frac{1}{960} + \frac{1}{144} - \frac{1}{60}\right\} = -\frac{61}{5760}\frac{w_0 L^4}{EI} \qquad y_C = \frac{61}{5760}\frac{w_0 L^4}{EI} \downarrow \blacktriangleleft$$

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PROBLEM 9.160

Determine the reaction at *A* and draw the bending moment diagram for the beam and loading shown.

$[x = 0,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

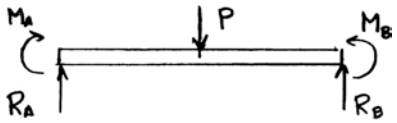
$\left[x = \dfrac{L}{2},\ \dfrac{dy}{dx} = 0\right]$

SOLUTION

By symmetry, $R_A = R_B$ and $\dfrac{dy}{dx} = 0$ at $x = \dfrac{L}{2}$.

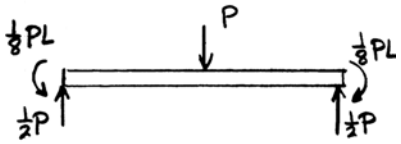
$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \qquad R_A = R_B = \frac{1}{2}P \blacktriangleleft$$

Moment reaction is statically indeterminate.



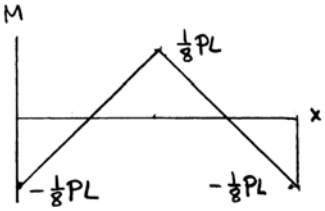
$$0 < x < \frac{L}{2}:$$

$$M = M_A + R_A x = M_A + \frac{1}{2}Px$$



$$EI\frac{d^2y}{dx^2} = M_A + \frac{1}{2}Px$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{4}Px^2 + C_1$$



$$y_B = \frac{P}{EI}\left\{-0 + 0 + \frac{1}{12}a^3\right\} = \frac{Pa^3}{12EI}$$

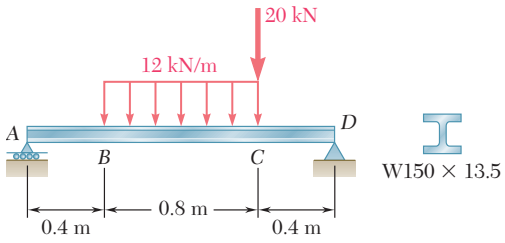
$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right] \quad M_A\frac{L}{2} + \frac{1}{4}P\left(\frac{L}{2}\right)^2 + 0 = 0$$

$$M_A = -\frac{1}{8}PL \qquad M_A = \frac{1}{8}PL \blacktriangleleft$$

By symmetry, $M_B = M_A$ $\qquad M_B = \dfrac{1}{8}PL \blacktriangleleft$

$$M_C = M_A + \frac{1}{2}P\frac{L}{2} = -\frac{1}{8}PL + \frac{1}{4}PL \qquad M_C = \frac{1}{8}PL \blacktriangleleft$$

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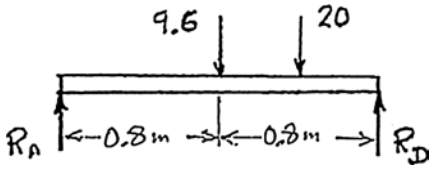


PROBLEM 9.161

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN, lengths in m



$$+\circlearrowleft M_D = 0:$$

$$-1.6R_A + (9.6)(0.8) + (20)(0.4) = 0$$

$$R_A = 9.8 \text{ kN}$$

$$w(x) = 12\langle x - 0.4\rangle^0 - 12\langle x - 1.2\rangle^0 \text{ kN/m}$$

$$\frac{dV}{dx} = -w(x) = -12\langle x - 0.4\rangle^0 + 12\langle x - 1.2\rangle^0 \text{ kN/m}$$

$$\frac{dM}{dx} = V = 9.8 - 12\langle x - 0.4\rangle^1 + 12\langle x - 1.2\rangle^1 - 20\langle x - 1.2\rangle^0 \quad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = 9.8x - 6\langle x - 0.4\rangle^2 + 6\langle x - 1.2\rangle^2 - 20\langle x - 1.2\rangle^1 \quad \text{kN} \cdot \text{m}$$

$$EI\frac{dy}{dx} = 4.9x^2 - 2\langle x - 0.4\rangle^3 + 2\langle x - 1.2\rangle^3 - 10\langle x - 1.2\rangle^2 + C_1 \quad \text{kN} \cdot \text{m}^2$$

$$EIy = 1.63333x^3 - \frac{1}{2}\langle x - 0.4\rangle^4 + \frac{1}{2}\langle x - 1.2\rangle^4 - \frac{10}{3}\langle x - 1.2\rangle^3 + C_1x + C_2 \quad \text{kN} \cdot \text{m}^3$$

$$[x = 0,\ y = 0]: 0 - 0 + 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = 1.6,\ y = 0]: (1.63333)(1.6)^3 - \frac{1}{2}(1.2)^4 + \frac{1}{2}(0.4)^4 - \frac{10}{3}(0.4)^3 + C_1(1.6) + 0 = 0$$

$$C_1 = -3.4080 \text{ kN} \cdot \text{m}^2$$

Data: $E = 200 \times 10^9$ Pa, $I = 6.83 \times 10^6 \text{ mm}^4 = 6.83 \times 10^{-6} \text{ mm}^4$

$$EI = (200 \times 10^4)(6.83 \times 10^{-6}) = 1.366 \times 10^6 \text{ N} \cdot \text{m}^2 = 1366 \text{ kN} \cdot \text{m}^2$$

(*a*) Slope at *A*. $\left(\frac{dy}{dx} \text{ at } x = 0\right)$

$$EI\frac{dy}{dx} = 0 - 0 + 0 - 0 - 3.4080 \quad \text{kN} \cdot \text{m}^2$$

$$\theta_A = -\frac{3.4080}{1366} = -2.49 \times 10^{-3} \text{ rad} \qquad \theta_A = 2.49 \times 10^{-3} \text{ rad} \ \searrow \blacktriangleleft$$

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PROBLEM 9.161 *(Continued)*

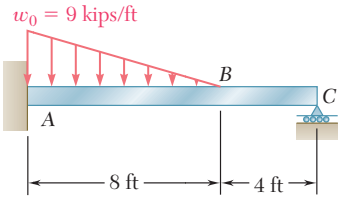
(*b*) Deflection at *C*. (y at $x = 1.2$ m)

$$EIy_C = (1.63333)(1.2)^3 - \frac{1}{2}(0.8)^4 + 0 - 0 - (3.4080)(1.2) + 0$$

$$= -1.4720 \text{ kN} \cdot \text{m}^3$$

$$y_C = -\frac{1.4720}{1366} = -1.078 \times 10^{-3} \text{ m} \qquad y_C = 1.078 \text{ mm} \downarrow \blacktriangleleft$$

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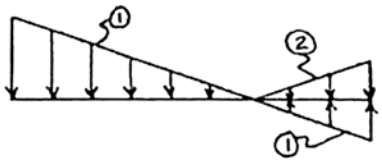




PROBLEM 9.162

For the beam and loading shown, determine (*a*) the reaction at point *C*, (*b*) the deflection at point *B*. Use $E = 29 \times 10^6$ psi.

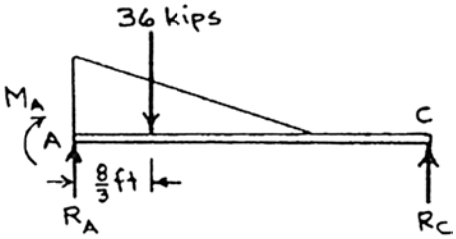
SOLUTION



Distributed loads: $k = \frac{9}{8}$

(1) $w_1(x) = w_0 - kx$ (2) $w_2(x) = k\langle x - 8\rangle^1$

$w_0 = 9$ kips/ft, $k = \frac{9}{8}$ kips/ft^2



$+\uparrow \sum F_y = 0$: $R_A - 36 + R_C = 0$ $R_A = (36 - R_C)$ kips

$+\circlearrowleft \sum M_A = 0$: $12R_C - M_A - \left(\frac{8}{3}\right)(36) = 0$

$M_A = (12R_C - 96)$ kip · ft

$w(x) = w_0 - kx + k\langle x - 8\rangle^1 = 9 - \frac{9}{8}x + \frac{9}{8}\langle x - 8\rangle^1$ kips/ft

$\frac{dV}{dx} = -w = -9 + \frac{9}{8}x - \frac{9}{8}\langle x - 8\rangle^1$ kips/ft

$\frac{dM}{dx} = V = R_A - 9x + \frac{9}{16}x^2 - \frac{9}{16}\langle x - 8\rangle^2$ kips

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - \frac{9}{2}x^2 + \frac{3}{16}x^3 - \frac{3}{16}\langle x - 8\rangle^3 \quad \text{kip} \cdot \text{ft}$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{3}{2}x^3 + \frac{3}{64}x^4 - \frac{3}{64}\langle x - 8\rangle^4 + C_1 \quad \text{kip} \cdot \text{ft}^2$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{3}{8}x^4 + \frac{3}{320}x^5 - \frac{3}{320}\langle x - 8\rangle^5 + C_1 x + C_2 \quad \text{kip} \cdot \text{ft}^3$$

$\left[x = 0, \frac{dy}{dx} = 0\right]$: $C_1 = 0$ $[x = 0, y = 0]$: $C_2 = 0$

$$[x = 12, y = 0]: \frac{1}{2}M_A(12)^2 + \frac{1}{6}R_A(12)^3 - \frac{3}{8}(12)^4 + \frac{3}{320}(12)^5 - \frac{3}{320}(4)^5 + 0 + 0 = 0$$

$$72(12R_C - 96) + 288(36 - R_C) - 5452.8 = 0$$

$R_C = 3.4667$ $R_C = 3.47$ kips ↑ ◀

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PROBLEM 9.162 *(Continued)*

(*a*) Reaction at *C*. $R_C = 3.47 \text{ kips} \uparrow$ ◀

$$M_A = 12(3.4667) - 96 = -54.400 \text{ kip} \cdot \text{ft} \qquad R_A = 36 - 3.4667 = 32.533 \text{ kips}$$

Data: $E = 29 \times 10^3 \text{ ksi} \quad I = 307 \text{ in}^4 \quad EI = (29 \times 10^3)(307) = 8.903 \times 10^6 \text{ kip} \cdot \text{in}^2$

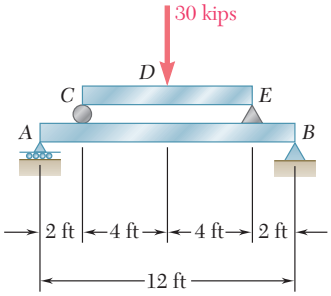
$$= 61{,}826 \text{ kip} \cdot \text{ft}^2$$

(*b*) Deflection at *B*. (y at $x = 8$ ft)

$$EIy_B = \frac{1}{2}(-54.400)(8)^2 + \frac{1}{6}(32.533)(8)^3 - \frac{3}{8}(8)^4 + \frac{3}{320}(8)^5 - 0 = -193.451 \quad \text{kip} \cdot \text{ft}^3$$

$$y_B = \frac{-193.451}{61{,}826} = -3.1290 \times 10^{-3} \text{ ft} \qquad y_B = 0.0376 \text{ in.} \downarrow$$ ◀

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PROBLEM 9.163

Beam *CE* rests on beam *AB* as shown. Knowing that a W10 × 30 rolled-steel shape is used for each beam, determine for the loading shown the deflection at point *D*. Use $E = 29 \times 10^6$ psi.

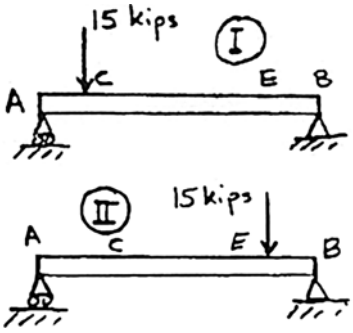
SOLUTION

For W10 × 30, $I = 170 \text{ in}^4$

$$EI = (29 \times 10^6)(170) = 4.93 \times 10^9 \text{ lb} \cdot \text{in}^2 = 34{,}236 \text{ kip} \cdot \text{ft}^2$$

Beam *AB*: 15 kip downward loads at *C* and *E*.

Refer to Case 5 of Appendix *D*.



Loading I: $(y_C)_1 = -\dfrac{Pa^2b^2}{3EIL}$

with $a = 2$ ft, $b = 10$ ft, $L = 12$ ft

$$(y_C)_1 = -\frac{(15)(2)^2(10)^2}{(3)(34{,}236)(12)} = -4.8682 \times 10^{-3} \text{ ft}$$

Loading II: $(y_C)_2 = \dfrac{Pb\left[x^2 - (L^2 - b^2)x\right]}{6EIL}$

with $b = 2$ ft, $x = 2$ ft, $L = 12$ ft

$$(y_C)_2 = \frac{(15)(2)[2^3 - (12^2 - 2^2)(2)]}{(6)(34{,}236)(12)} = -3.3104 \times 10^{-3} \text{ ft}$$

$$y_C = (y_C)_1 + (y_C)_2 = -8.1786 \times 10^{-3} \text{ ft}$$

By symmetry, $y_E = y_C$

Beam *CDE*: 30 kip downward loads at *D*.

Refer to Case 4 of Appendix *D*.

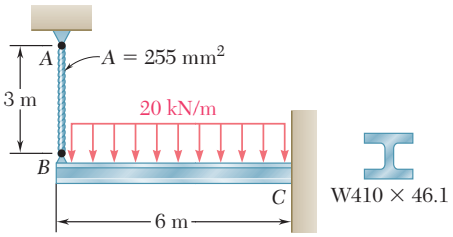
$$y_{D/C} = -\frac{PL^3}{48EI}$$

with $P = 30$ kips and $L = 8$ ft

$$y_{D/C} = -\frac{(30)(8)^3}{(48)(34{,}236)} = -9.3469 \times 10^{-3} \text{ ft}$$

Total deflection at *D*. $y_D = y_C + y_{D/C} = 17.5255 \times 10^{-3}$ ft $\qquad y_D = 0.210 \text{ in.} \downarrow$ ◀

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PROBLEM 9.164

The cantilever beam *BC* is attached to the steel cable *AB* as shown. Knowing that the cable is initially taut, determine the tension in the cable caused by the distributed load shown. Use $E = 200$ GPa.

SOLUTION

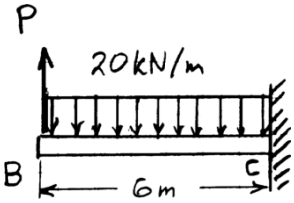
Let P be the tension developed in member AB and δ_B be the elongation of that member.

Cable *AB*:

$$A = 255 \text{ mm}^2 = 255 \times 10^{-6} \text{ m}^2$$

$$\delta_B = \frac{PL}{EA} = \frac{(P)(3)}{(200 \times 10^9)(255 \times 10^{-6})}$$

$$= 58.82 \times 10^{-9} P$$



Beam *BC*:

$$I = 156 \times 10^6 \text{ mm}^4 = 156 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(156 \times 10^{-6})$$

$$= 31.2 \times 10^6 \text{ N} \cdot \text{m}^2$$

Loading I: 20 kN/m downward.

Refer to Case 2 of Appendix *D*.

$$(y_B)_1 = -\frac{wL^4}{8EI} = -\frac{(20 \times 10^3)(6)^4}{(8)(31.2 \times 10^6)}$$

$$= -103.846 \times 10^{-3} \text{ m}$$

Loading II: Upward force P at point *B*.

Refer to Case 1 of Appendix *D*.

$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(6)^3}{(3)(31.2 \times 10^6)} = 2.3077 \times 10^{-6} P$$

By superposition,

$$y_B = (y_B)_1 + (y_B)_2$$

Also, matching the deflection at *B*,

$$y_B = -\delta_B$$

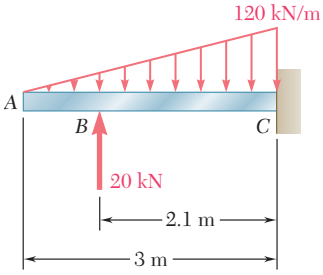
$$-103.846 \times 10^{-3} + 2.3077 \times 10^{-6} P = -58.82 \times 10^{-9} P$$

$$2.3666 \times 10^{-6} P = 103.846 \times 10^{-3}$$

$$P = 43.9 \times 10^3 \text{ N}$$

$$P = 43.9 \text{ kN} \blacktriangleleft$$

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PROBLEM 9.165

For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in meters.

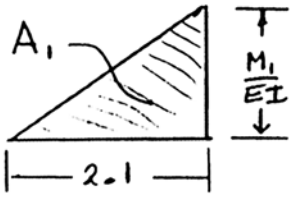
$$I = 178\times10^{6}\ \text{mm}^4 = 178\times10^{-6}$$

$$EI = (200\times10^{9})(178\times10^{-6}) = 35{,}600\ \text{kN}\cdot\text{m}^2$$

Draw $\dfrac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{(20)(2.1)}{35{,}600} = 1.17978\times10^{-3}\,\text{m}^{-1}$$

$$A_1 = \left(\frac{1}{2}\right)(2.1)(1.17978\times10^{-3}) = 1.23876\times10^{-3}$$



$$M_2 = -\frac{\left(\frac{1}{2}\right)(120)(3)(1)}{35{,}600} = -5.0562\times10^{-3}\,\text{m}^{-1}$$

$$A_2 = \left(\frac{1}{4}\right)(3)(-5.0562\times10^{-3}) = -3.7921\times10^{-3}$$



Place reference tangent at *C*.

$$\theta_C = 0$$

(*a*) Slope at *A*.

$$\theta_A = -\theta_{C/A} = -A_1 - A_2$$

$$\theta_A = 2.55\times10^{-3}\ \text{rad}\ \curvearrowright \blacktriangleleft$$

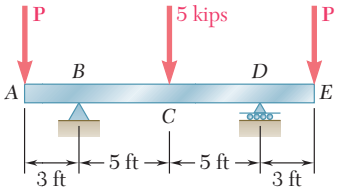
(*b*) Deflection at *A*.

$$y_A = t_{A/C}$$

$$y_C = A_1(3-0.7) + A_2\left(3-\frac{3}{5}\right) = -6.25\times10^{-3}\ \text{m}$$

$$y_C = 6.25\ \text{mm}\downarrow \blacktriangleleft$$

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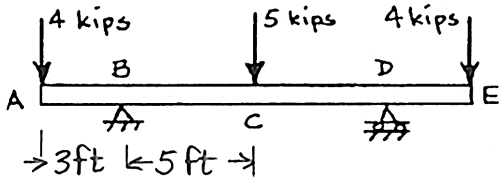


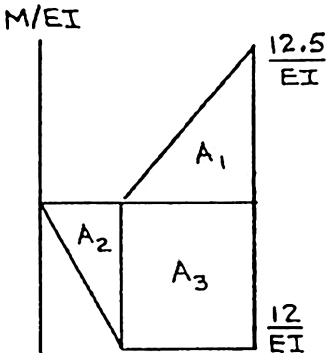


PROBLEM 9.166

Knowing that $P = 4$ kips, determine (*a*) the slope at end *A*, (*b*) the deflection at midpoint *C* of the beam. Use $E = 29 \times 10^6$ psi.

SOLUTION





$$E = 29 \times 10^3 \text{ ksi} \qquad I = 39.6 \text{ in}^4$$

$$EI = (29 \times 10^3)(39.6) = 1.1484 \times 10^6 \text{ kip} \cdot \text{in}^2$$

$$= 7975 \text{ kip} \cdot \text{ft}^2$$

Symmetric beam and loading:

$$R_A = R_B = P + 2.5 = 4 + 2.5 = 6.5 \text{ kips}$$

Bending moment:

Over *AB*: $M = -Px = -4x$

Over *BC*: $M = -4x + 6.5(x - 3)$

$$= 2.5(x - 3) - 12$$

Draw $\dfrac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{12.5}{EI}\right)(5) = \frac{31.25}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{12}{EI}\right)(3) = -\frac{18}{EI}$$

$$A_3 = -\left(\frac{12}{EI}\right)(5) = -\frac{60}{EI}$$

Place reference tangent at *C*. $\theta_C = 0$

(*a*) Slope at *A*. $\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$

$$\theta_A = -\left(\frac{31.25}{EI} - \frac{18}{EI} - \frac{60}{EI}\right) = \frac{46.75}{EI} = \frac{46.75}{7975}$$

$$\theta_A = 5.86 \times 10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

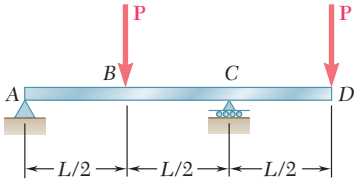
(*b*) Deflection at *C*. $y_C = -t_{B/C}$

$$= -(A_1\bar{x}_1 + A_3\bar{x}_3)$$

$$= -\left[\left(\frac{31.25}{EI}\right)\left(\frac{2}{3}(5)\right) - \left(\frac{60}{EI}\right)\left(\frac{1}{2}(5)\right)\right] = \frac{45.833}{EI} = \frac{45.833}{7975} = 5.7471 \times 10^{-3} \text{ ft}$$

$$y_C = 0.0690 \text{ in.} \uparrow \blacktriangleleft$$

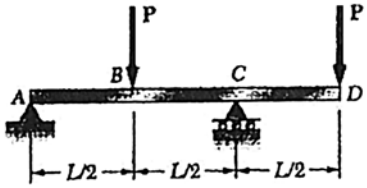
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PROBLEM 9.167

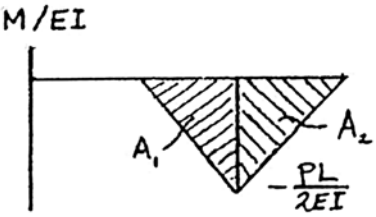
For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *D*.

SOLUTION



$$+\circlearrowleft \Sigma M_C = 0: \quad -R_A L + P\frac{L}{2} - P\frac{L}{2} = 0 \qquad R_A = 0$$

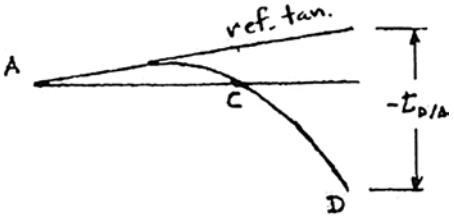
Draw $\dfrac{M}{EI}$ diagram.



$$A_1 = -\frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{8}\frac{PL^2}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{8}\frac{PL^2}{EI}$$

Place reference tangent at *A*.



$$t_{C/A} = A_1 \cdot \left(\frac{1}{3}\cdot\frac{L}{2}\right) = -\frac{1}{48}\frac{PL^3}{EI}$$

(*a*) Slope at *A*.

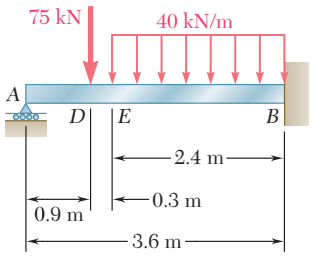
$$\theta_A = -\frac{t_{C/A}}{L} \qquad \theta_A = \frac{1}{48}\frac{PL^2}{EI} \blacktriangleleft$$

(*b*) Deflection at *D*.

$$t_{D/A} = A_1\left(\frac{L}{2}+\frac{L}{6}\right) + A_2\left(\frac{2}{3}\cdot\frac{L}{2}\right) = -\frac{1}{8}\frac{PL^3}{EI}$$

$$y_D = t_{D/A} - \frac{x_D}{L}t_{C/A} = -\frac{1}{8}\frac{PL^3}{EI} - \left(\frac{3}{2}\right)\left(-\frac{1}{48}\frac{PL^3}{EI}\right) \qquad y_D = -\frac{3}{32}\frac{PL^3}{EI} \blacktriangleleft$$

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PROBLEM 9.168

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION

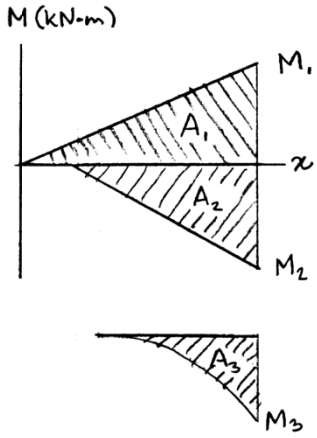
Units: Forces in kN; lengths in meters.

Let R_A be the redundant reaction.

Remove support at *A* and add reaction $R_A \uparrow$.

Draw bending moment diagram by parts.

$$M_1 = 3.6\ R_A \text{ kN} \cdot \text{m}$$
$$M_2 = -(75)(0.3 + 2.4) = -202.5 \text{ kN} \cdot \text{m}$$
$$M_3 = -\frac{1}{2}(40)(2.4)^2 = -115.2 \text{ kN} \cdot \text{m}$$



$$A_1 = \frac{1}{2}(3.6)(3.6R_A) = 6.48 \text{ kN} \cdot \text{m}^2$$
$$A_2 = \frac{1}{2}(2.7)(-202.5) = -273.375 \text{ kN} \cdot \text{m}^2$$
$$A_3 = \frac{1}{3}(2.4)(-115.2) = -92.16 \text{ kN} \cdot \text{m}^2$$

Place reference tangent at *B*, where

$$\theta_B = 0 \quad \text{and} \quad y_B = 0.$$

Then

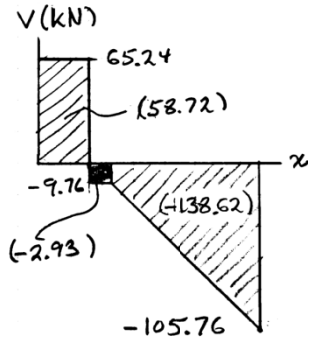
$$y_A = t_{A/B} = 0$$
$$t_{A/B} = \frac{1}{EI}\left\{\left(\frac{2}{3}\cdot 3.6\right)A_1 + \left(0.9 + \frac{2}{3}\cdot 2.7\right)A_2 + \left(0.9 + 0.3 + \frac{3}{4}\cdot 2.4\right)A_3\right\}$$
$$= \frac{1}{EI}\{15.552R_A - 1014.5925\} = 0$$
$$R_A = 65.24 \text{ kN}$$

$$R_A = 65.2 \text{ kN} \uparrow \blacktriangleleft$$

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PROBLEM 9.168 *(Continued)*

Draw shear diagram.



A to D: $V = R_A = 65.24$ kN

D to E: $V = 65.24 - 75 = -9.76$ kN

E to B: $V = -9.76 - 40(x - 1.2)$ kN

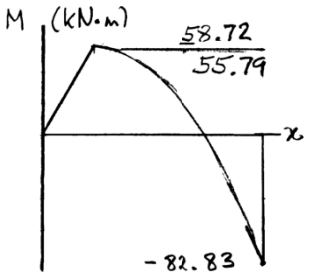
At B, $V_B = -105.76$ kN

Bending moment diagram. $M_A = 0$

$M_D = M_A + 58.72 = 58.72$ kN · m

$M_E = 58.72 - 2.93 = 55.79$ kN · m

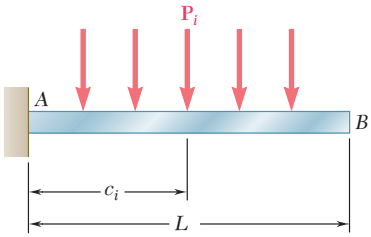
$M_B = 55.79 - 138.62 = -82.83$ kN · m



$M_D = 58.7$ kN · m ◀

$M_B = -82.8$ kN · m ◀

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PROBLEM 9.C1

Several concentrated loads can be applied to the cantilever beam *AB*. Write a computer program to calculate the slope and deflection of beam *AB* from $x = 0$ to $x = L$, using given increments Δx. Apply this program with increments $\Delta x = 50$ mm to the beam and loading of Prob 9.73 and Prob. 9.74.

SOLUTION

For each load, enter

$$P_i, \quad c_i$$

Compute reaction at *A*.

For $i = 1$ to number loads

$$R_A = R_A + P_i$$

$$M_A = M_A - P_i c_i$$

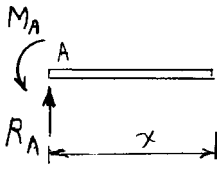
Compute slope and deflection.

Using method of integration starting with $x = 0$ and updating through increments, superpose:

(1) Due to reaction at *A*:

$$\theta = (1/EI)(R_A x^2/2.0 + M_A x)$$

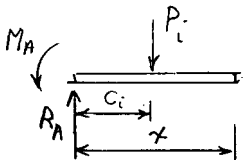
$$y = (1/EI)(R_A x^3/6.0 + M_A x^2/2.0)$$



(2) Due to each load with $c_i < x$:

$$\theta = -(1/EI)(P_i/2.0)(x - c_i)^2$$

$$y = -(1/EI)(P_i/6.0)(x - c_i)^3$$



At $x = 0, \quad y = \dfrac{dy}{dx} = 0$

$\therefore$ The constants of integration equal zero

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PROBLEM 9.C1 *(Continued)*

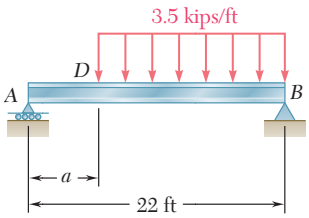
Program Output

Problems 9.73 and 9.74

At *A*: Force = 6.0 kN Couple = –6.0 kN · m

x m	Slope Radians	Deflection m	
0.00	0.000000	0.000000	
0.05	–0.000578	–0.000015	
0.10	–0.001126	–0.000057	
0.15	–0.001645	–0.000127	
0.20	–0.002134	–0.000221	
0.25	–0.002594	–0.000340	
0.30	–0.003024	–0.000480	
0.35	–0.003424	–0.000642	
0.40	–0.003794	–0.000822	
0.45	–0.004135	–0.001021	
0.50	–0.004447	–0.001235	
0.55	–0.004728	–0.001465	
0.60	–0.004980	–0.001708	
0.65	–0.005203	–0.001962	
0.70	–0.005395	–0.002227	
0.75	–0.005558	–0.002501	◀
0.80	–0.005699	–0.002783	
0.85	–0.005825	–0.003071	
0.90	–0.005936	–0.003365	
0.95	–0.006033	–0.003664	
1.00	–0.006114	–0.003968	
1.05	–0.006181	–0.004275	
1.10	–0.006233	–0.004586	
1.15	–0.006270	–0.004898	
1.20	–0.006292	–0.005213	
1.25	–0.006299	–0.005527	◀

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PROBLEM 9.C2

The 22-ft beam *AB* consists of a W21×62 rolled-steel shape and supports a 3.5-kip/ft distributed load as shown. Write a computer program and use it to calculate for values of *a* from 0 to 22 ft, using 1-ft increments, (*a*) the slope and deflection at *D*, (*b*) the location and magnitude of the maximum deflection. Use $E = 29\times10^6$ psi.

SOLUTION

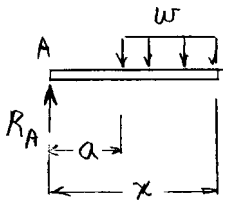
Enter load *w*, length *L*, *a*.

Compute reaction at *A*.

$$R_A = w(L-a)^2/(2.0L)$$

Compute slope and deflection at *D*.

Using singularity functions,



$$C_1 = -\frac{w}{24L}(L-a)^4 - \frac{1}{6}R_A L^2$$

$$\theta = (1/EI)(R_A a^2/2.0 + C_1)$$

$$y = (1/EI)(R_A a^3/6.0 + C_1 a)$$

$$EI\frac{d^2y}{dx^2} = R_A x - \frac{w}{2}\langle x-a\rangle^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{w}{6}\langle x-a\rangle^3 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{w}{24}\langle x-a\rangle^4 + C_1 x + C_2$$

From boundary conditions,

$$C_2 = 0$$

$$C_1 = -\frac{w}{24L}(L-a)^4 - \frac{1}{6}R_A L^2$$

Maximum *y* at $\theta = 0$:

$$0 = \frac{1}{2}R_A x^2 - \frac{w}{6}\langle x-a\rangle^3 + C_1$$

If $x_{\max} \leq a$,

$$\frac{1}{2}R_A x^2 + C_1 = 0$$

$$x_{\max} = \sqrt{\frac{-2.0\,C_1}{R_A}}$$

$$y_{\max} = \frac{1}{6}R_A x_{\max}^3 + C_1 x_{\max}$$

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PROBLEM 9.C2 *(Continued)*

Compute location and magnitude of maximum deflection.

Assume $x < A$: $x_{\max} = (-2.0\ C_1/R_A)^{1/2}$

If $x_{\max} < a$, then $y_{\max} = (1/EI)\left(\frac{1}{6}R_A x_{\max}^3 + C_1 x_{\max}\right)$

If $x_{\max} > a$, then

Begin with $x = a$

$$\theta = (1/EI)\left(\frac{1}{2}R_A x - \frac{1}{6}(x-a)^3 + C_1\right)$$

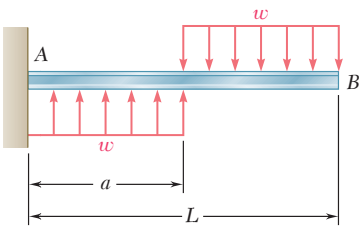
Increase x by small amount until θ is approximately 0

$$y_{\max} = (1/EI)\left(\frac{1}{6}R_A x^3 - \frac{w}{24}(x-a)^4 + C_1 x\right)$$

Program Output

a ft	θ_D Radians	$y_D\phi$ in.	x_m ft	y_m in.
0.	–0.00580	0.000000	11.000	–0.478290
1.	–0.00569	–0.068758	11.008	–0.475922
2.	–0.00539	–0.133047	11.030	–0.468860
3.	–0.00494	–0.189440	11.068	–0.457231
4.	–0.00439	–0.235551	11.121	–0.441245
5.	–0.00378	–0.269927	11.189	–0.421192
6.	–0.00314	–0.291944	11.272	–0.397443
7.	–0.00250	–0.301695	11.370	–0.370441
8.	–0.00188	–0.299889	11.481	–0.340699
9.	–0.00131	–0.287738	11.606	–0.308795
10.	–0.00080	–0.266855	11.742	–0.275364
11.	–0.00036	–0.239145	11.885	–0.241090
12.	–0.00001	–0.206699	12.028	–0.206700
13.	0.00025	–0.171684	12.159	–0.172954
14.	0.00043	–0.136240	12.275	–0.140603
15.	0.00052	–0.102374	12.376	–0.110339
16.	0.00054	–0.071846	12.463	–0.082792
17.	0.00049	–0.046069	12.537	–0.058515
18.	0.00039	–0.026001	12.596	–0.037987
19.	0.00027	–0.012036	12.643	–0.021604
20.	0.00014	–0.003896	12.675	–0.009677
21.	0.00004	–0.000530	12.695	–0.002431
22.	0.00000	0.000000	12.702	0.000000

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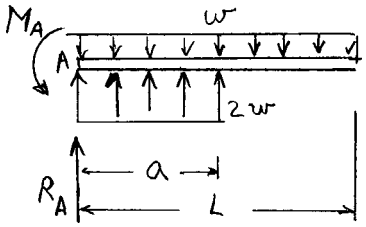
PROBLEM 9.C3

The cantilever beam *AB* carries the distributed loads shown. Write a computer program to calculate the slope and deflection of beam *AB* from $x = 0$ to $x = L$ using given increments Δx. Apply this program with increments $\Delta x = 100$ mm, assuming that $L = 2.4$ m, $w = 36$ kN/m, and (*a*) $a = 0.6$ m, (*b*) $a = 1.2$ m, (*c*) $a = 1.8$ m. Use $E = 200$ GPa.

SOLUTION

Enter *w*, *a*, *L*.

Compute reaction at *A*.



$$R_A = wL - 2.0wa$$

$$M_A = \frac{1}{2}wL^2 - \frac{1}{2}wa^2$$

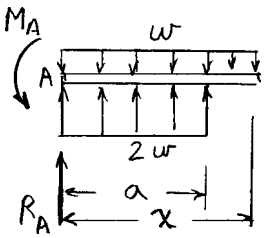
Compute slope and deflection.

Using equation of elastic curve starting with $x = 0$ and updating through increments, superpose:

(1) Due to reactions at *A*:

$$\theta = (1/EI)\left(\frac{1}{2}R_A x^2 + M_A x\right)$$

$$y = (1/EI)\left(\frac{1}{6}R_A x^3 + \frac{1}{2}M_A x\right)$$



(2) Due to load *w*:

$$\theta = -(1/EI)\left(\frac{1}{6}wx^3\right)$$

$$y = -(1/EI)\left(\frac{1}{24}wx^4\right)$$

At $x = 0$, $y = \frac{dy}{dx} = 0$

$\therefore$ The constants of integration are zero.

(3) Due to load 2*w*:

If $x \leq a$,

$$\theta = (1/EI)\left(\frac{1}{3}wx^3\right)$$

$$y = (1/EI)\left(\frac{1}{12}wx^4\right)$$

If $x > a$,

$$\theta = (1/EI)\left(\frac{1}{3}wx^3 - \frac{1}{3}w(x-a)^3\right)$$

$$y = (1/EI)\left(\frac{1}{12}wx^4 - \frac{1}{12}w(x-a)^4\right)$$

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PROBLEM 9.C3 *(Continued)*

Program Output

Problem 9.C3 (*a*) $a = 0.6$ m

At *A*: Force = 43.2 kN Couple = –90.7 kN · m

x m	Slope Radians	Deflection m
0.00	0.000000	0.000000
0.10	–0.000905	–0.000046
0.20	–0.001762	–0.000179
0.30	–0.002567	–0.000396
0.40	–0.003318	–0.000691
0.50	–0.004009	–0.001058
0.60	–0.004638	–0.001491
0.70	–0.005202	–0.001983
0.80	–0.005703	–0.002529
0.90	–0.006145	–0.003122
1.00	–0.006533	–0.003756
1.10	–0.006868	–0.004427
1.20	–0.007156	–0.005128
1.30	–0.007399	–0.005856
1.40	–0.007602	–0.006607
1.50	–0.007769	–0.007376
1.60	–0.007902	–0.008160
1.70	–0.008006	–0.008955
1.80	–0.008083	–0.009760
1.90	–0.008139	–0.010571
2.00	–0.008177	–0.011387
2.10	–0.008199	–0.012206
2.20	–0.008211	–0.013027
2.30	–0.008215	–0.013848
2.40	–0.008216	–0.014669

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PROBLEM 9.C3 *(Continued)*

Program Output (*Continued*)

Problem 9.C3 (*b*) $a = 1.2$ m

At A: Force $= 0.0$ kN Couple $= -51.8$ kN · m

x m	Slope Radians	Deflection m
0.00	0.000000	0.000000
0.10	–0.000529	–0.000026
0.20	–0.001055	–0.000106
0.30	–0.001574	–0.000237
0.40	–0.002081	–0.000420
0.50	–0.002574	–0.000653
0.60	–0.003048	–0.000934
0.70	–0.003500	–0.001262
0.80	–0.003926	–0.001633
0.90	–0.004323	–0.002046
1.00	–0.004687	–0.002497
1.10	–0.005014	–0.002982
1.20	–0.005301	–0.003498
1.30	–0.005544	–0.004041
1.40	–0.005747	–0.004606
1.50	–0.005913	–0.005189
1.60	–0.006047	–0.005787
1.70	–0.006150	–0.006398
1.80	–0.006228	–0.007017
1.90	–0.006284	–0.007642
2.00	–0.006321	–0.008273
2.10	–0.006344	–0.008906
2.20	–0.006356	–0.009541
2.30	–0.006360	–0.010177
2.40	–0.006361	–0.010813

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PROBLEM 9.C3 *(Continued)*

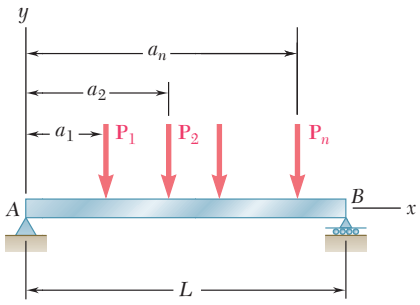
Program Output (*Continued*)

Problem 9.C3 (*c*) $a = 1.8$ m

At A: Force $= -43.2$ kN Couple $= 13.0$ kN · m

x m	Slope Radians	Deflection m
0.00	0.000000	0.000000
0.10	0.000111	0.000006
0.20	0.000182	0.000021
0.30	0.000215	0.000041
0.40	0.000216	0.000063
0.50	0.000187	0.000083
0.60	0.000133	0.000099
0.70	0.000056	0.000109
0.80	–0.000039	0.000110
0.90	–0.000149	0.000101
1.00	–0.000270	0.000080
1.10	–0.000398	0.000046
1.20	–0.000530	0.000000
1.30	–0.000662	–0.000060
1.40	–0.000790	–0.000132
1.50	–0.000911	–0.000217
1.60	–0.001021	–0.000314
1.70	–0.001116	–0.000421
1.80	–0.001193	–0.000537
1.90	–0.001248	–0.000659
2.00	–0.001286	–0.000786
2.10	–0.001309	–0.000916
2.20	–0.001320	–0.001047
2.30	–0.001325	–0.001179
2.40	–0.001325	–0.001312

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PROBLEM 9.C4

The simple beam *AB* is of constant flexural rigidity *EI* and carries several concentrated loads as shown. Using the *Method of Integration*, write a computer program that can be used to calculate the slope and deflection at points along the beam from $x = 0$ to $x = L$ using given increments Δx. Apply this program to the beam and loading of (*a*) Prob. 9.13 with $\Delta x = 1$ ft, (*b*) Prob. 9.16 with $\Delta x = 0.05$ m, (*c*) Prob. 9.129 with $\Delta x = 0.25$ m.

SOLUTION

For each load, enter P_i, a_i

Compute reaction at *A*.

For $i = 1$ to number loads,

$$M_A = M_A + P_i a_i$$

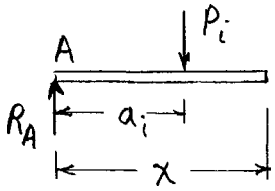
$$\text{Load} = \text{Load} + P_i$$

Then

$$R_B = M_A / L$$

$$R_A = \text{Load} - R_B$$

For Load P_i:



For $x < a_i$,

$$EI\frac{d^2y}{dx^2} = R_A x$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 + C_1 x + C_2$$

For $x > a_i$,

$$EI\frac{d^2y}{dx^2} = R_A x - P_i(x - a_i)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{2}P_i(x - a_i)^2 + C_3$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{6}(x - a_i)^3 + C_3 x + C_4$$

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PROBLEM 9.C4 *(Continued)*

From boundary conditions,

$$C_2 = C_4 = 0$$

$$C_1 = C_3 = \frac{P_i}{6L}(L - a_i)^3 - \frac{1}{6}R_A L^2$$

Note: R_A for load P_i

Compute slope and deflection.

Starting with $x = 0$ and updating through increments, superpose:

(1) Due to reaction at *A*:

$$\theta = (1/EI)\left(\frac{1}{2}R_A x^2\right)$$

$$y = (1/EI)\left(\frac{1}{6}R_A x^3\right)$$

(2) Due to loads—constant part:

$$\text{Const}_1 = -\frac{1}{6}R_A L^2$$

For 1 to number loads,

$$\text{Const}_2 = \frac{1}{6L}P_i(L - a_i)^3 + \text{Const}_2$$

Then total contribution for constant:

$$\text{Const} = (1/EI)(\text{Const}_1 + \text{Const}_2)$$

(3) Due to loads—remaining part:

If $x \le a_i$,

$$\theta = (1/EI)\left(\frac{1}{2.0}R_A x^2\right)$$

$$y = (1/EI)\left(\frac{1}{6.0}R_A x^3\right)$$

If $x > a_i$,

$$\theta = (1/EI)\left(\frac{1}{2.0}R_A x^2 - \frac{1}{2.0}P_i(x - a_i)^2\right)$$

$$y = (1/EI)\left(\frac{1}{6.0}R_A x^3 - \frac{1}{6.0}P_i(x - a_i)^3\right)$$

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PROBLEM 9.C4 *(Continued)*

Program Outputs:

Problem 9.13

x in.	θ rad $\times 10^3$	y in.
0.00	–8.2948	0.000000
12.00	–8.0957	–0.098742
24.00	–7.4985	–0.192705
36.00	–6.5031	–0.277113
48.00	–5.1096	–0.347188
60.00	–3.3179	–0.398151
72.00	–1.4267	–0.426420
84.00	0.2654	–0.433189
96.00	1.7585	–0.420846
108.00	3.0525	–0.391781
120.00	4.1474	–0.348383
132.00	5.0433	–0.293039
144.00	5.7400	–0.228141
156.00	6.2377	–0.156075
168.00	6.5363	–0.079232
180.00	6.6359	0.000000

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PROBLEM 9.C4 *(Continued)*

Program Outputs (*Continued*)

Problem 9.129

x m	θ rad $\times 10^3$	y mm	
0.000	–8.703	0.000	(a) ◀
0.250	–8.615	–2.168	
0.500	–8.351	–4.293	
0.750	–7.911	–6.329	
1.000	–7.296	–8.234	
1.250	–6.505	–9.962	
1.500	–5.538	–11.472	
1.750	–4.483	–12.724	
2.000	–3.428	–13.713	
2.250	–2.373	–14.438	
2.500	–1.319	–14.900	
2.750	–0.264	–15.098	
3.000	0.791	–15.032	(b) ◀
3.250	1.802	–14.706	
3.500	2.725	–14.138	
3.750	3.560	–13.350	
4.000	4.307	–12.365	
4.250	4.967	–11.204	
4.500	5.538	–9.889	
4.750	6.021	–8.442	
5.000	6.417	–6.886	
5.250	6.725	–5.241	
5.500	6.944	–3.531	
5.750	7.076	–1.776	
6.000	7.120	0.000	

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PROBLEM 9.C4 *(Continued)*

Program Outputs (*Continued*)

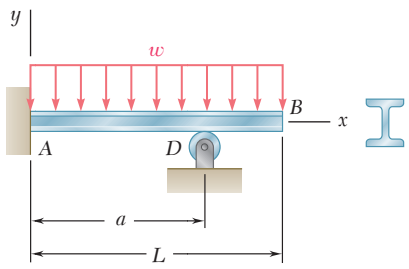
Problem 9.16

x m	θ rad $\times 10^3$	y mm	x m	θ rad $\times 10^3$	y mm
0.000	–2.490	0.000	1.300	0.146	–1.972
0.050	–2.485	–0.124	1.350	0.293	–1.961
0.100	–2.471	–0.248	1.400	0.439	–1.943
0.150	–2.448	–0.371	1.450	0.586	–1.917
0.200	–2.416	–0.493	1.500	0.732	–1.884
0.250	–2.375	–0.613	1.550	0.879	–1.844
0.300	–2.325	–0.730	1.600	1.025	–1.796
0.350	–2.265	–0.845	1.650	1.172	–1.741
0.400	–2.197	–0.957	1.700	1.318	–1.679
0.450	–2.119	–1.065	1.750	1.460	–1.610
0.500	–2.032	–1.168	1.800	1.593	–1.533
0.550	–1.936	–1.268	1.850	1.716	–1.451
0.600	–1.831	–1.362	1.900	1.831	–1.362
0.650	–1.716	–1.451	1.950	1.936	–1.268
0.700	–1.593	–1.533	2.000	2.032	–1.168
0.750	–1.460	–1.610	2.050	2.119	–1.065
0.800	–1.318	–1.679	2.100	2.197	–0.957
0.850	–1.172	–1.741	2.150	2.265	–0.845
0.900	–1.025	–1.796	2.200	2.325	–0.730
0.950	–0.879	–1.844	2.250	2.375	–0.613
1.000	–0.732	–1.884	2.300	2.416	–0.493
1.050	–0.586	–1.917	2.350	2.448	–0.371
1.100	–0.439	–1.943	2.400	2.471	–0.248
1.150	–0.293	–1.961	2.450	2.485	–0.124
1.200	–0.146	–1.972	2.500	2.490	0.000
1.250	0.000	–1.976			

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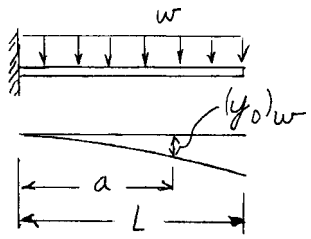
PROBLEM 9.C5



The supports of beam *AB* consist of a fixed support at end *A* and a roller support located at point *D*. Write a computer program that can be used to calculate the slope and deflection at the free end of the beam for values of *a* from 0 to *L* using given increments Δa. Apply this program to calculate the slope and deflection at point *B* for each of the following cases:

	L	ΔL	w	E	Shape
(*a*)	12 ft	0.5 ft	1.6 k/ft	29×10^6 psi	W16×57
(*b*)	3 m	0.2 m	18 kN/m	200 GPa	W460×113

SOLUTION



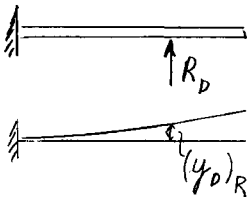
Beam is indeterminate.

Use Appendix *D* and superposition.

Determine reaction at *D*.

Due to distributed load:

$$(y_D)_W = -\frac{w}{24EI}(a^4 - 4La^3 + 6L^2a^2)$$



Due to redundant load:

$$(y_D)_R = \frac{R_D L^3}{3EI}$$

Redundant reaction:

Since $(y_D)_w + (y_D)_R = 0$: $\quad R_D = \dfrac{3EI}{L^3}(y_D)_w$

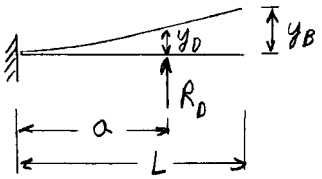
Compute slope and deflection at *B*.

Superpose:

Due to distributed load:

$$\theta_B = -\frac{wL^3}{6EI}$$

$$y_B = -\frac{wL^4}{8EI}$$



$\theta_B = \theta_D$

$y_B = y_D + (L-a)\theta_D$

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PROBLEM 9.C5 *(Continued)*

Due to R_D:

$$\theta_B = \frac{Pa^2}{2EI}$$

$$y_B = \frac{Pa^3}{3EI} + (L-a)\frac{Pa^2}{2EI}$$

Program Output

Problem 9.C5(*a*)

a ft	θ_B rad $\times 10^{-3}$	*y* at *B* in.
0.0	–3.019	–0.3260
0.5	–2.743	–0.2869
1.0	–2.483	–0.2511
1.5	–2.238	–0.2183
2.0	–2.007	–0.1885
2.5	–1.790	–0.1614
3.0	–1.586	–0.1369
3.5	–1.395	–0.1149
4.0	–1.216	–0.0953
4.5	–1.049	–0.0778
5.0	–0.893	–0.0624
5.5	–0.748	–0.0490
6.0	–0.613	–0.0374
6.5	–0.488	–0.0274
7.0	–0.373	–0.0191
7.5	–0.266	–0.0122
8.0	–0.168	–0.0067
8.5	–0.077	–0.0025
9.0	0.006	0.0006
9.5	0.082	0.0027
10.0	0.152	0.0037
10.5	0.216	0.0039
11.0	0.274	0.0033
11.5	0.328	0.0020
12.0	0.377	0.0000

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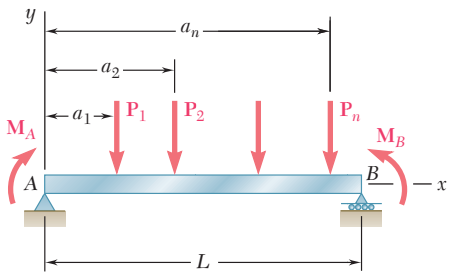
PROBLEM 9.C5 *(Continued)*

Program Output ***(Continued)***

Problem 9.C5(*b*)

a m	θ_B rad $\times 10^{-3}$	y at B mm
0.0	–0.728	–1.6389
0.2	–0.624	–1.3324
0.4	–0.529	–1.0663
0.6	–0.442	–0.8374
0.8	–0.364	–0.6426
1.0	–0.293	–0.4789
1.2	–0.230	–0.3435
1.4	–0.174	–0.2338
1.6	–0.124	–0.1472
1.8	–0.079	–0.0813
2.0	–0.040	–0.0337
2.2	–0.006	–0.0024
2.4	0.023	0.0149
2.6	0.049	0.0198
2.8	0.072	0.0143

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PROBLEM 9.C6

For the beam and loading shown, use the *Moment-Area Method* to write a computer program to calculate the slope and deflection at points along the beam from $x = 0$ to $x = L$ using given increments Δx. Apply this program to calculate the slope and deflection at each concentrated load for the beam of (*a*) Prob. 9.77 with $\Delta x = 0.5$ m, (*b*) Prob. 9.119 with $\Delta x = 0.5$ m.

SOLUTION

Enter M_A and M_B

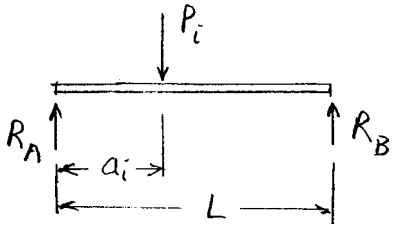
For each load, enter P_i and a_i

Determine reaction at *A*.

Due to moments at ends:

$$(R_A)_1 = -(M_A - M_B)/L$$

Due to loads P_i:



For $i = 1$ to number of loads,

$$R_B = R_B + P_i\, a_i/L$$

Load = Load + P_i

$$(R_A)_2 = \text{Load} - R_B$$

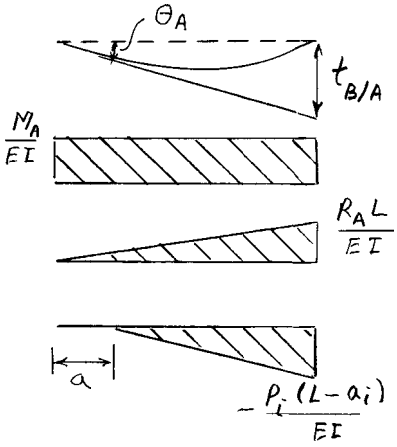
$$R_A = (R_A)_1 + (R_A)_2$$

Determine slope at *A*.

Use second moment-area theorem to get tangential deviation at *B*.

Due to M_A:

$$t_{B/A} = M_A L^2/(2.0EI)$$



Due to R_A:

$$t_{B/A} = R_A L^3/6.0EI)$$

Due to loads P_i:

For $i = 1$ to number of loads,

$$t_{B/A} = -P_i(L - a_i)^3/(6.0EI)$$

Sum $t_{B/A}$:

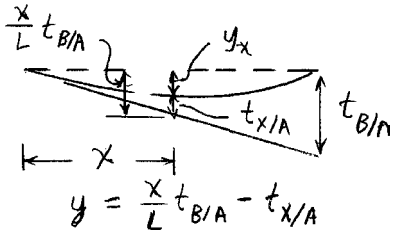
$$\theta_A = -t_{B/A}/L$$

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PROBLEM 9.C6 *(Continued)*

Determine slope and deflections.

For $x = 0$ to L, superpose:



Due to M_A and R_A:

$$\theta_x = \theta_A + (M_A x + R_A x_2^2/2.0)/EI$$

$$y_x = \frac{x}{L} t_{B/A} - M_A x^2/(2.0EI) - R_A x^3/(6.0EI)$$

Due to loads P_i:

Do for all loads with $a_i < x$

$$\theta_x = P_i (x - a_i)^2/(2.0EI)$$

$$y_x = P_i (x - a_i)^3/(6.0EI)$$

Program Outputs

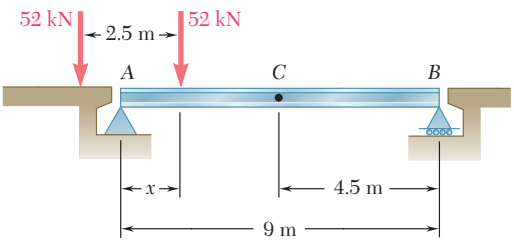
Problem 9.77

x m	θ rad × 1000	y at x mm		
0.000	–0.600962	0.000000	(*a*)	◀
0.500	–1.602564	0.574252		
1.000	–2.043269	1.509081		
1.500	–1.923077	2.524039		
2.000	–1.241987	3.338675		
2.500	0.000000	3.672543	(*b*)	◀
3.000	1.241987	3.338676		
3.500	1.923077	2.524039		
4.000	2.043269	1.509082		
4.500	1.602564	0.574253		
5.000	0.600962	0.000000		

Problem 9.119

x m	θ rad × 1000	y at x mm		
0.000	–4.504505	0.000000	(*a*)	◀
0.500	–4.673423	2.317943		
1.000	–4.279279	4.579579		
1.500	–3.322072	6.503378		
2.000	–1.801802	7.807808		
2.500	0.000000	8.258258	(*b*)	◀
3.000	1.801802	7.807808		
3.500	3.322072	6.503378		
4.000	4.279279	4.579579		
4.500	4.673423	2.317943		
5.000	4.504505	0.000000		

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PROBLEM 9.C7

Two 52-kN loads are maintained 2.5 m apart as they are moved slowly across beam *AB*. Write a computer program to calculate the deflection at the midpoint *C* of the beam for values of *x* from 0 to 9 m, using 0.5-m increments. Use $E = 200$ GPa.

SOLUTION

Enter load *P*, beam length *L* and space between loads *D*.

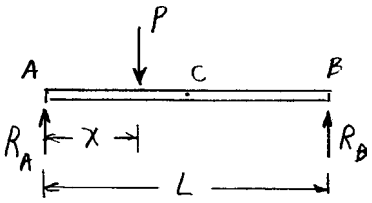
We will solve with moment-area method.

Determine deflection at *C*.

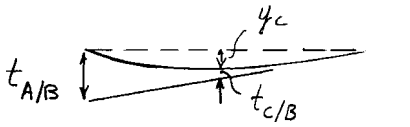
For $x = 0$ to *L*,

If $0 \leq x \leq D$:

We have one load to left of *C*.

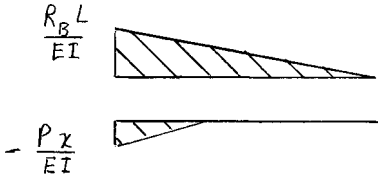


$$R_B = P_x/L$$
$$t_{A/B} = (R_B L^3 - Px^3)/(6.0EI)$$
$$t_{C/B} = R_B L^3/(48.0EI)$$
$$y_C = \frac{1}{2}t_{A/B} - t_{C/B}$$



If $D < x \leq L/2$,

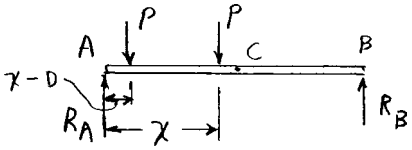
We have two loads to left of *C*.



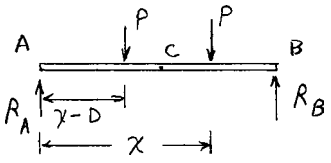
$$R_B = Px/L + P(x - D)/L$$
$$t_{A/B} = (R_B L^3 - Px^3 - P(x - D)^3)/(6.0EI)$$
$$t_{C/B} = R_B L^3/(48.0EI)$$
$$y_C = \frac{1}{2}t_{A/B} - t_{C/B}$$

If $L/2 < x \leq (L/2 + D)$,

We have one load to left of *C* and one to right of *C* or at *C*.



$$R_B = Px/L + P(x - D)/L$$
$$t_{A/B} = (R_B L^3 - Px^3 - P(x - D)^3)/(6.0EI)$$
$$t_{C/B} = \left(R_B L^3/48.0 - P\left(x - \frac{L}{2} \right)^3 / 6.0 \right)/EI$$
$$y_C = \frac{1}{2}t_{A/B} - t_{C/B}$$



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PROBLEM 9.C7 *(Continued)*

If $(L/2 + D) < x < L$,

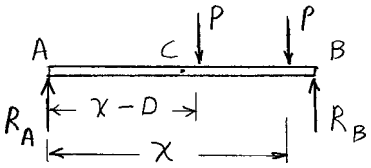
We have both loads to right of *C*.

$$R_B = Px/L + P(x - D)/L$$

$$t_{A/B} = (R_B L^3 - Px^3 - P(x - b)^3)/(6.0EI)$$

$$t_{C/B} = \left\{ R_B L^3 \Big/ 48.0 - P\left(x - \frac{L}{2} \right)^3 \Big/ 6.0 - P\left(x - D - \frac{L}{2} \right)^3 \Big/ 6.0 \right\} \Big/ EI$$

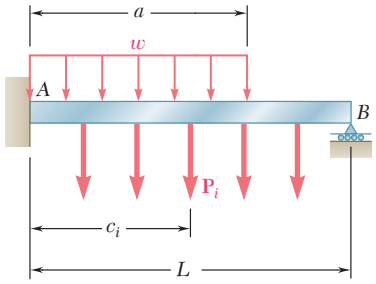
$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$



Program Output

x m	R_B kN	θ_B rad	y_C mm
0.0	0.000	0.00000	0.00000
0.5	2.889	0.00315	1.17881
1.0	5.778	0.00624	2.32839
1.5	8.667	0.00921	3.41951
2.0	11.556	0.01200	4.42296
2.5	14.444	0.01456	5.30950
3.0	20.222	0.01998	7.22872
3.5	26.000	0.02499	8.94335
4.0	31.778	0.02947	10.39493
4.5	37.556	0.03331	11.52503
5.0	43.333	0.03639	12.28492
5.5	49.111	0.03859	12.66487
6.0	54.889	0.03980	12.66487
6.5	60.667	0.03989	12.28492
7.0	66.444	0.03876	11.52503
7.5	72.222	0.03629	10.39493
8.0	78.000	0.03235	8.94335
8.5	83.778	0.02684	7.22872
9.0	89.556	0.01963	5.30950

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PROBLEM 9.C8

A uniformly distributed load w and several distributed loads P_i may be applied to beam AB. Write a computer program to determine the reaction at the roller support and apply this program to the beam and loading of (*a*) Prob. 9.53*a*, (*b*) Prob. 9.154.

SOLUTION

The beam is indeterminate.

Use equation of elastic curve.

Enter w and for each load P_i and C_i

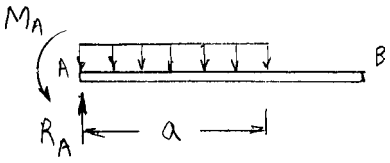
Compute displacement at B due to loads.

Reaction at A:

Due to w:

$$R_A = wa$$

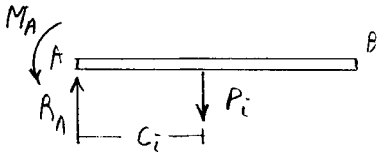
$$M_A = \frac{1}{2}wa^2$$



For $i = 1$ to number loads P_i,

$$R_A = R_A - P_i$$

$$M_A = M_A - P_i c_i$$



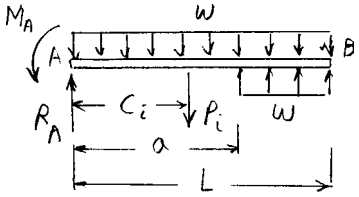
For displacement at B, superpose:

Due to reaction at A:

$$EIy_B = \frac{1}{6}R_A L^3 + \frac{1}{2}M_A L^2$$

Due to distributed loads:

$$EIy_B = \frac{1}{24}(-wL^4 + w(L-a)^4)$$



Due to P_i:

For $i = 1$ to number loads,

$$EIy_B = \frac{1}{6}P_i(L - c_i)^3$$

At $x = 0,\ y = \frac{dy}{dx} = 0$

$\therefore$ The constants of integration are zero.

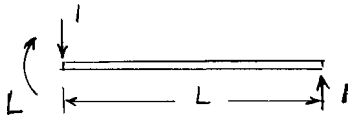
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PROBLEM 9.C8 *(Continued)*

Compute displacement at B due to unit R_B.

$$EI\,(y_B)_{\text{unit}} = \frac{1}{3}L^3$$

Compute reaction at B.



From $$EIy_B + R_B EI\,(y_B)_{\text{unit}} = 0$$

$$R_B = -y_B/(y_B)_{\text{unit}}$$

$$EI\frac{d^2y}{dx^2} = -x + L$$

$$EI\frac{dy}{dx} = -\frac{1}{2}x^2 + Lx + C_1$$

$$EI\,y = -\frac{1}{6}x^3 + \frac{1}{2}Lx^2 + C_1x + C_2$$

Boundary conditions give $C_1 = C_2 = 0$

Program Outputs

Problem 9.53(*a*)

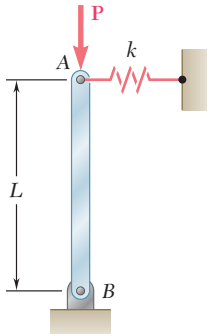
Reaction at roller support = 11.5356 kN

Problem 9.154

Reaction at roller support = 10.1758 kN

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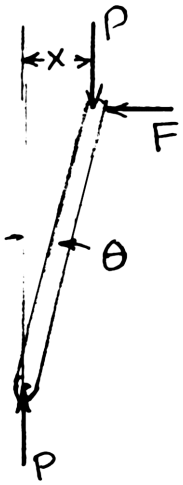
CHAPTER 10



PROBLEM 10.1

Knowing that the spring at A is of constant k and that the bar AB is rigid, determine the critical load P_{cr}.

SOLUTION



Let θ be the angle change of bar AB.

$$F = kx = kL\sin\theta$$

$$+\circlearrowleft \Sigma M_B = 0: \quad FL\cos\theta - Px = 0$$

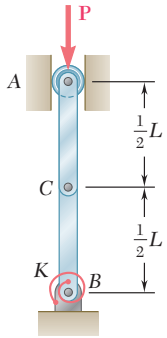
$$kL^2\sin\theta\cos\theta - PL\sin\theta = 0$$

Using

$$\sin\theta \approx \theta \quad \text{and} \quad \cos\theta \approx 1, \quad kL^2\theta - PL\theta = 0$$

$$(kL^2 - PL)\theta = 0 \qquad P_{cr} = kL \blacktriangleleft$$

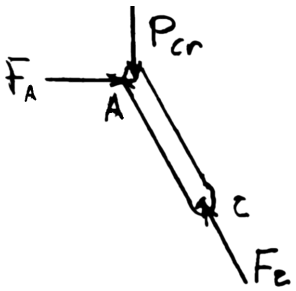
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PROBLEM 10.2

Two rigid bars *AC* and *BC* are connected by a pin at *C* as shown. Knowing that the torsional spring at *B* is of constant *K*, determine the critical load P_{cr} for the system.

SOLUTION

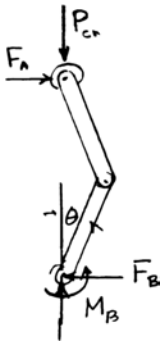


Let θ be the angle change of each bar.

$$M_B = K\theta$$

$$+\circlearrowleft \Sigma M_B = 0: \quad K\theta - F_A L = 0$$

$$F_A = \frac{K\theta}{L}$$

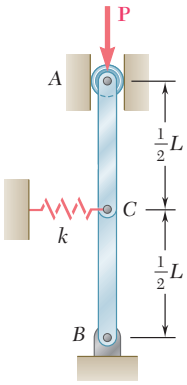


Bar *AC*. $$+\circlearrowleft \Sigma M_C = 0: \quad P_{cr}\frac{1}{2}L\theta - \frac{1}{2}LF_A = 0$$

$$P_{cr} = \frac{F_A}{\theta}$$

$$P_{cr} = \frac{K}{L} \blacktriangleleft$$

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PROBLEM 10.3

Two rigid bars *AC* and *BC* are connected as shown to a spring of constant *k*. Knowing that the spring can act in either tension or compression, determine the critical load P_{cr} for the system.

SOLUTION

Let *x* be the lateral deflection of point *C*.

$$x = \frac{1}{2}L\ \sin\theta \qquad F_C = kx = \frac{1}{2}\ kL\ \sin\theta$$

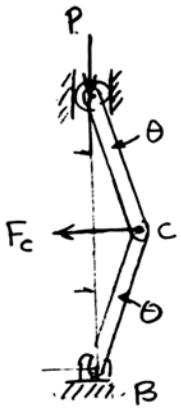
Joint *C*: $+F_y = 0$: $F_{AC}\cos\ \theta - F_{CB}\cos\ \theta = 0$

$$F_{AC} = F_{CB}$$

$$\xrightarrow{+}\Sigma F_x = 0: \qquad F_{AB}\sin\theta + F_{CB}\sin\theta - F_C = 0$$

$$-2\,F_{AB}\sin\theta - \frac{1}{2}kL\sin\theta = 0$$

$$-\left(F_{AB} + \frac{1}{4}kL\right)\sin\theta = 0 \qquad F_{AB} = -\frac{1}{4}kL$$

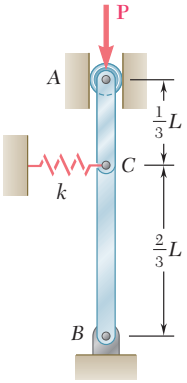


Joint *A*: $\Sigma F_y = 0$: $-P - F_{AB}\cos\ \theta = 0$ $\qquad P = -F_{AB}\cos\ \theta = \frac{1}{4}kL\cos\theta$

With $\theta \to 0$ $\qquad P_{cr} = \frac{1}{4}kL$

◀

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PROBLEM 10.4

Two rigid bars *AC* and *BC* are connected as shown to a spring of constant *k*. Knowing that the spring can act in either tension or compression, determine the critical load P_{cr} for the system.

SOLUTION

Let δ be the deflection of point *C*.

Using free body *AC* and

$$+\circlearrowleft \sum M_C = 0: \quad -\frac{1}{3}LR_A + P\delta = 0 \qquad R_A = \frac{3P\delta}{L}$$

Using free body *BC* and

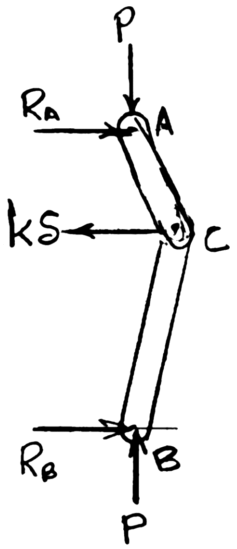
$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{2}{3}LR_B - P\delta = 0 \qquad R_B = \frac{3P\delta}{2L}$$

Using both free bodies together,

$$\pm\rightarrow \Sigma F_x = 0: \quad R_A + R_B - k\delta = 0$$

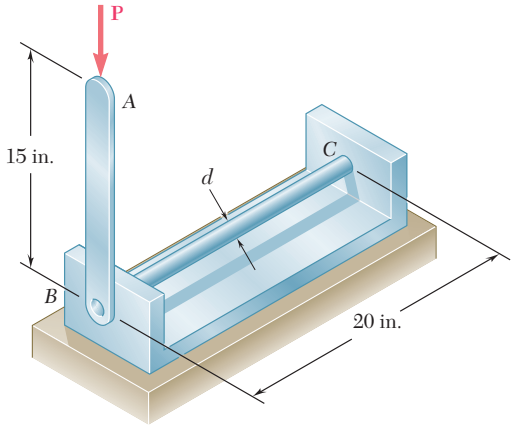
$$\frac{3P\delta}{L} + \frac{3P\delta}{2L} - k\delta = 0$$

$$\left(\frac{9}{2}\frac{P}{L} - k\right)\delta = 0$$



$$P_{cr} = \frac{2kL}{9} \blacktriangleleft$$

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PROBLEM 10.5

The steel rod *BC* is attached to the rigid bar *AB* and to the fixed support at *C*. Knowing that $G = 11.2 \times 10^6$ psi, determine the diameter of rod *BC* for which the critical load P_{cr} of the system is 80 lb.

SOLUTION

Look at torsion spring *BC*.

$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ}{L}\varphi = K\varphi$$

$$G = 11.2 \times 10^6 \text{ psi}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$$

$$L = 20 \text{ in.}$$

$$K = \frac{(11.2 \times 10^6)\pi d^4}{(20)(32)} = 54{,}978d^4$$



$$+\circlearrowleft \Sigma M_B = 0: \quad T - Pl\sin\varphi = 0$$

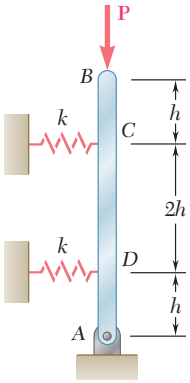
$$K\varphi - Pl\sin\varphi = 0$$

$$P = \frac{K\varphi}{l\sin\varphi} \qquad P_{cr} = \frac{K}{l}$$

$$K = 54{,}978d^4 = P_{cr}l = (80)(15) = 1200$$

$$d = \sqrt[4]{\frac{1200}{54{,}978}} \qquad d = 0.384 \text{ in.} \blacktriangleleft$$

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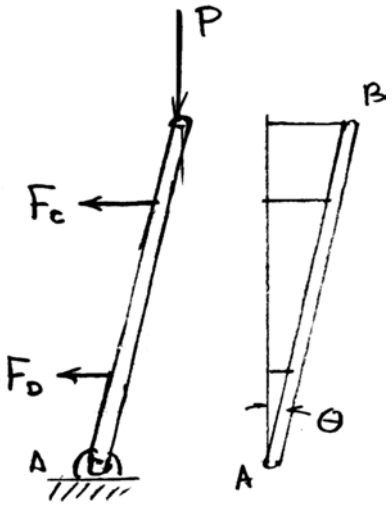


PROBLEM 10.6

The rigid rod AB is attached to a hinge at A and to two springs, each of constant $k = 2$ kip/in., that can act in either tension or compression. Knowing that $h = 2$ ft, determine the critical load.

SOLUTION

Let θ be the small rotation angle.



$$x_D \approx h\theta$$
$$x_C \approx 3h\theta$$
$$x_B \approx 4h\theta$$
$$F_C = kx_C \approx 3kh\theta$$
$$F_D = kx_D \approx kh\theta$$

$$+\circlearrowleft \Sigma M_A = 0: \quad hF_D + 3hF_C - Px_B = 0$$

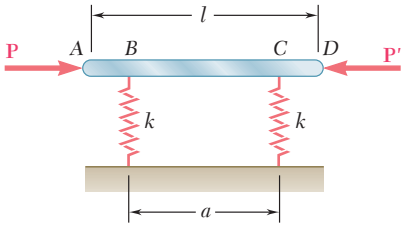
$$kh^2\theta + 9kh^2\theta - 4hP = 0, \quad P = \frac{5}{2}kh$$

Data: $k = 2.0$ kip/in. $\quad h = 2\,\text{ft} = 24$ in.

$$P = \frac{5}{2}(2.0)(24)$$

$$P = 120.0 \text{ kips} \blacktriangleleft$$

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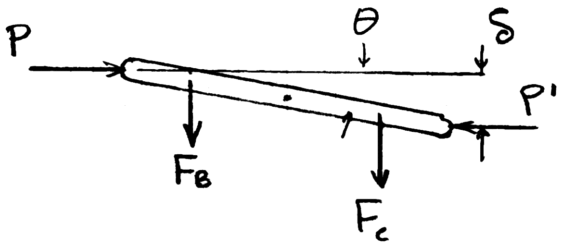


PROBLEM 10.7

The rigid bar *AD* is attached to two springs of constant *k* and is in equilibrium in the position shown. Knowing that the equal and opposite loads **P** and **P′** *remain horizontal*, determine the magnitude P_{cr} of the critical load for the system.

SOLUTION

Let y_B and y_C be the deflections of points *B* and *C*, positive upward.



$$F_B = -ky_B \qquad F_C = -ky_C$$

$$+\uparrow \Sigma F_y = 0: \quad F_B + F_C = 0 \qquad F_C = -F_B$$

$y_C = -y_B$ F_B and F_C form a counter clockwise couple.

Let θ be the angle change.

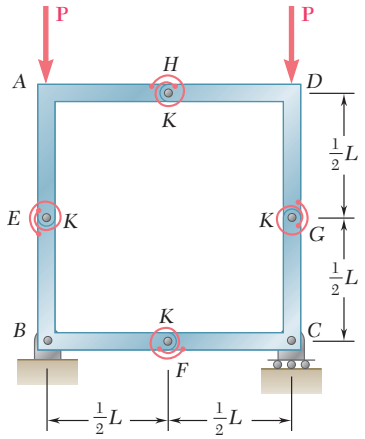
$$y_B = -y_C = \frac{1}{2}a\sin\theta, \quad \delta = l\sin\theta$$

P and P' form a clockwise couple of magnitude $P\delta$.

$$+\circlearrowleft \Sigma M = 0: \quad k\left(\frac{1}{2}a\sin\theta\right)a\cos\theta - Pl\sin\theta = 0 \qquad P = \frac{ka^2}{2l}\cos\theta$$

Let $\theta \to 0$. $$P_{cr} = \frac{ka^2}{2l}$$ ◀

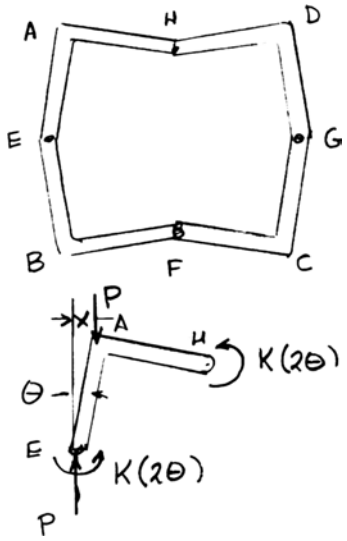
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PROBLEM 10.8

A frame consists of four L-shaped members connected by four torsional springs, each of constant *K*. Knowing that equal loads **P** are applied at points *A* and *D* as shown, determine the critical value P_{cr} of the loads applied to the frame.

SOLUTION



Let θ be the rotation of each L-shaped member.

Angle change across each torsional spring is 2θ.

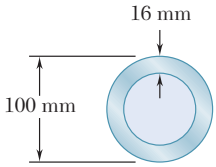
$$x = \frac{1}{2}L\sin\theta \approx \frac{1}{2}L\theta$$

$$\Sigma M_E = 0:$$

$$K(2\theta) + K(2\theta) - Px = 0$$

$$P_{cr} = \frac{4K\theta}{x} \qquad P_{cr} = \frac{8K}{L} \blacktriangleleft$$

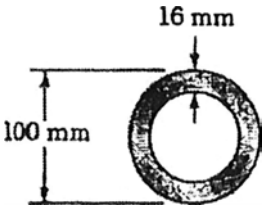
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PROBLEM 10.9

Determine the critical load of a pin-ended steel tube that is 5 m long and has a 100-mm outer diameter and a 16-mm wall thickness. Use $E = 200$ GPa.

SOLUTION



$$c_o = \frac{1}{2}d_o = 50 \text{ mm} \quad c_i = c_o - t = 50 - 16 = 34 \text{ mm}$$

$$I = \frac{\pi}{4}\left(c_o^4 - c_i^4\right) = 3.859 \times 10^6 \text{ mm}^4 = 3.859 \times 10^{-6} \text{ m}^4$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9)(3.859 \times 10^{-6})}{(5.0)^2} = 305 \times 10^3 \text{ N}$$

$$P_{\text{cr}} = 305 \text{ kN} \blacktriangleleft$$

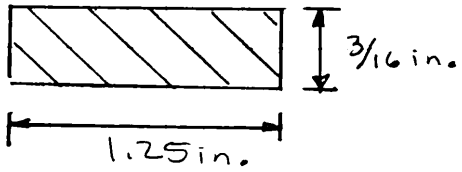
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PROBLEM 10.10

Determine the critical load of a pin-ended wooden stick that is 3 ft long and has a $\frac{3}{16} \times 1\frac{1}{4}$-in. rectangular cross section. Use $E = 1.6 \times 10^6$ psi.

SOLUTION

$$I = \frac{1}{12}(1.25 \text{ in.})\left(\frac{3}{16} \text{ in.}\right)^3 = 686.65 \times 10^{-6} \text{ in}^4$$



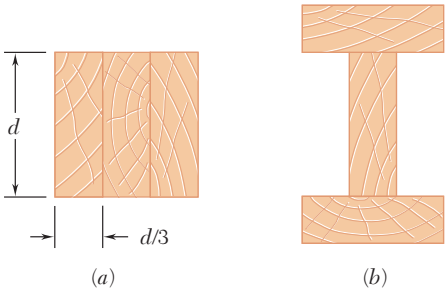
$$L_e = 36 \text{ in.}$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{L_e^2}$$

$$= \frac{\pi^2 (1.6 \times 10^6 \text{ psi})(686.65 \times 10^{-6} \text{ in}^4)}{(36 \text{ in.})^2}$$

$$P_{\text{cr}} = 8.37 \text{ lb} \blacktriangleleft$$

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PROBLEM 10.11

A column of effective length L can be made by gluing together identical planks in either of the arrangements shown. Determine the ratio of the critical load using the arrangement a to the critical load using the arrangement b.

SOLUTION

Arrangement (a).

$$I_a = \frac{1}{12} d^4$$

$$P_{\text{cr},a} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E d^4}{12 L_e^2}$$

Arrangement (b).

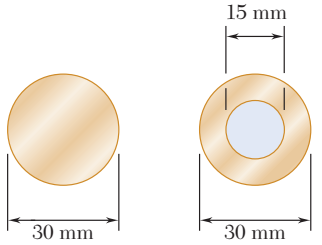
$$I_{\min} = I_y = \frac{1}{12}\left(\frac{d}{3}\right)(d^3) + \frac{1}{12}(d)\left(\frac{d}{3}\right)^3 + \frac{1}{12}\left(\frac{d}{3}\right)(d)^3 = \frac{19}{324} d^4$$

$$P_{\text{cr},b} = \frac{\pi^2 EI}{L_e^2} = \frac{19\pi^2 E d^4}{324 L_e^2}$$

$$\frac{P_{\text{cr},a}}{P_{\text{cr},b}} = \frac{1}{12} \cdot \frac{324}{19} = \frac{27}{19}$$

$$\frac{P_{\text{cr},a}}{P_{\text{cr},b}} = 1.421 \blacktriangleleft$$

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PROBLEM 10.12

A compression member of 1.5-m effective length consists of a solid 30-mm-diameter brass rod. In order to reduce the weight of the member by 25%, the solid rod is replaced by a hollow rod of the cross section shown. Determine (*a*) the percent reduction in the critical load, (*b*) the value of the critical load for the hollow rod. Use $E = 200$ GPa.

SOLUTION

(*a*) $P_{cr} = \dfrac{\pi^2 EI}{L^2}$

P_{cr} is proportional to I.

For solid rod, $c = \dfrac{1}{2}d, \qquad I_s = \dfrac{\pi}{4}c^4$

$$c = \frac{1}{2}(30) = 15\text{ mm} \qquad I_s = \frac{\pi}{4}(15)^4 = 39.761\times10^3\text{ mm}^4 = 39.761\times10^{-9}\text{ m}^4$$

For hollow rod, $c_i = \dfrac{1}{2}d_i \qquad I_h = \dfrac{\pi}{4}(c^4 - c_i^4)$

$$\frac{(P_{cr})_h}{(P_{cr})_s} = \frac{I_h}{I_s} = \frac{c^4 - c_i^4}{c^4} = 1 - \left(\frac{c_i}{c}\right)^4 = 1 - \left(\frac{d_i}{d}\right)^4$$

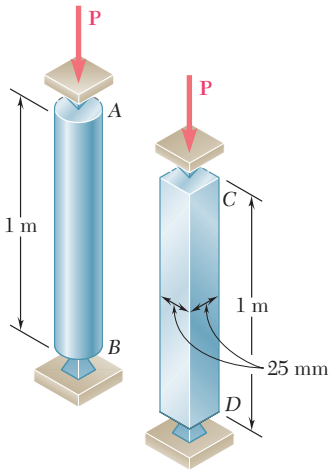
$$= 1 - \left(\frac{15}{30}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16}$$

Percent reduction in P_{cr}: $\dfrac{1}{16}\times 100\,\%$ 6.25% ◀

(*b*) $P_{cr} = \dfrac{15}{16}\dfrac{\pi^2 EI_s}{L^2} = \dfrac{15}{16}\cdot\dfrac{\pi^2(105\times10^9)(39.761\times10^{-9})}{(1.5)^2} = 17.17\times10^3\text{ N}$

$P_{cr} = 17.17$ kN ◀

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PROBLEM 10.13

Determine the radius of the round strut so that the round and square struts have the same cross-sectional area and compute the critical load for each. Use $E = 200$ GPa.

SOLUTION

For square strut, $A = 25^2 = 625 \text{ mm}^2$

$$I = \frac{1}{12}(25)^4 = 32.552 \times 10^3 \text{ mm}^4 = 32.552 \times 10^{-9} \text{ m}^4$$

For round strut, $\frac{1}{4}\pi d^2 = A$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(625)}{\pi}} = 28.2 \text{ mm}$$

$$c = \frac{1}{2}d = 14.10 \text{ mm} \qquad c = 14.10 \text{ mm} \blacktriangleleft$$

$$I = \frac{\pi}{4}c^4 = 31.085 \times 10^3 \text{ mm}^3 = 31.085 \times 10^{-9} \text{ m}^4$$

Critical loads: $P_{cr} = \dfrac{\pi^2 EI}{L^2}$

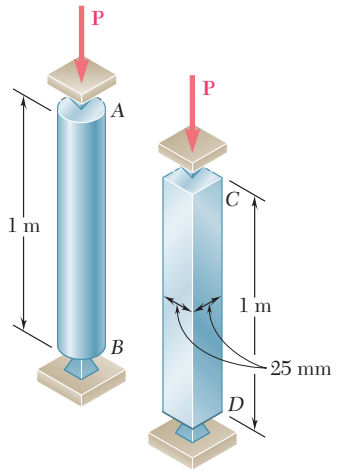
Round strut. $P_{cr} = \dfrac{\pi^2(200 \times 10^9)(31.085 \times 10^{-9})}{(1)^2} = 61.4 \times 10^3 \text{ N}$

$P_{cr} = 61.4 \text{ kN} \blacktriangleleft$

Square strut. $P_{cr} = \dfrac{\pi^2(200 \times 10^9)(32.552 \times 10^{-9})}{(1)^2} = 64.3 \times 10^3 \text{ N}$

$P_{cr} = 64.3 \text{ kN} \blacktriangleleft$

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PROBLEM 10.14

Determine (*a*) the critical load for the square strut, (*b*) the radius of the round strut for which both struts have the same critical load. (*c*) Express the cross-sectional area of the square strut as a percentage of the cross-sectional area of the round strut. Use $E = 200$ GPa.

SOLUTION

(*a*) Square strut. $A = (25)^2 = 625\ \text{mm}^2$

$$I = \frac{1}{12}(25)^4 = 32.552 \times 10^3\ \text{mm}^4 = 32.552 \times 10^{-9}\ \text{m}^4$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(200 \times 10^9)(32.552 \times 10^{-9})}{(1)^2}$$

$$= 64.3 \times 10^3\ \text{N} \qquad = 64.3\ \text{kN} \blacktriangleleft$$

(*b*) Round strut. For the same critical load, modulus of elasticity, and length, the moments of inertia must be equal.

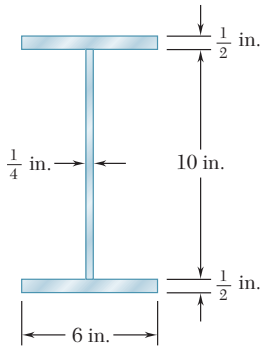
$$\frac{\pi}{4}c^4 = I$$

$$c = \sqrt[4]{\frac{4I}{\pi}} = \sqrt[4]{\frac{(4)(32.552 \times 10^3)}{\pi}} = 14.27\ \text{mm}$$

$$d = 2c = 28.5\ \text{mm} \blacktriangleleft$$

(*c*) Area. $A = \pi c^2 = 639.58\ \text{mm}$ Ratio $= \dfrac{625}{639.58} = 0.977 = 97.7\%$ ◀

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PROBLEM 10.15

A column with the cross section shown has a 13.5-ft effective length. Using a factor of safety equal to 2.8, determine the allowable centric load that can be applied to the column. Use $E = 29 \times 10^6$ psi.

SOLUTION

$$I_{\min} = 2I_1 + I_2$$

$$= 2\left(\frac{1}{12}\right)\left(\frac{1}{2}\text{ in.}\right)(6\text{ in.})^3 + \frac{1}{12}(10\text{ in.})\left(\frac{1}{4}\text{ in.}\right)^3$$

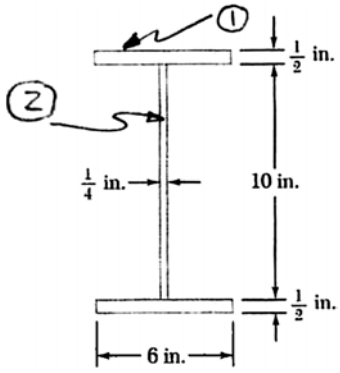
$$= 18.0130\text{ in}^4$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{L_e^2}$$

$$= \frac{\pi^2(29 \times 10^6\text{ psi})(18.0130\text{ in}^4)}{[(13.5\text{ ft})(12\text{ in./ft})]^2}$$

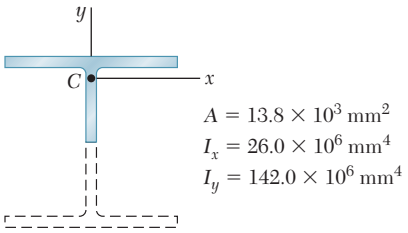
$$= 196.451\text{ kips}$$

$$P_{\text{all}} = \frac{196.451}{2.8}\text{ kips}$$



$P_{\text{all}} = 70.2\text{ kips}$ ◀

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PROBLEM 10.16

A column is made from half of a W360 × 216 rolled-steel shape, with the geometric properties as shown. Using a factor of safety equal to 2.6, determine the allowable centric load if the effective length of the column is 6.5 m. Use $E = 200$ GPa.

SOLUTION

$$P_{\text{cr}} = \frac{\pi^2 EI}{L_e^2}$$

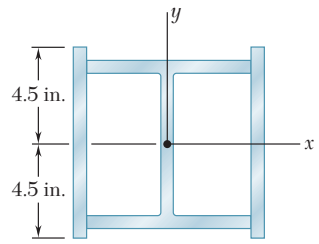
$$= \frac{\pi^2 (200 \times 10^9\ \text{Pa})(26.0 \times 10^{-6}\ \text{m}^4)}{(6.5\ \text{m})^2}$$

$$= 1214.72\ \text{kN}$$

$$P_{\text{all}} = \frac{1214.72}{2.6}\ \text{kN}$$

$$P_{\text{all}} = 467\ \text{kN} \blacktriangleleft$$

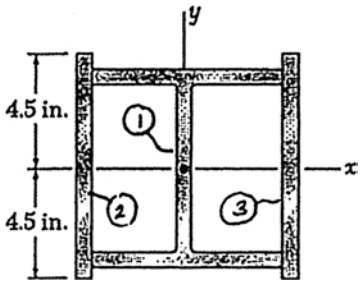
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PROBLEM 10.17

A column of 22-ft effective length is to be made by welding two 9×0.5-in. plates to a W8 $\times$ 35 as shown. Determine the allowable centric load if a factor of safety 2.3 is required. Use 29×10^6 psi.

SOLUTION



① W8 × 35: $I_x = 127 \text{ in}^4$ $I_y = 42.6 \text{ in}^4$

$b_f = 8.02$ in.

② and ③: For each plate, $A = (0.5)(9.0) = 4.5 \text{ in}^2$

$$I_x = \frac{1}{12}(0.5)(9)^3 = 30.375 \text{ in}^4$$

$$I_y = \frac{1}{12}(9)(0.5)^3 + (4.5)\left(\frac{8.02}{2} + \frac{0.5}{2}\right)^2 = 81.758 \text{ in}^4$$

Total: $I_x = 127 + (2)(30.375) = 187.75 \text{ in}^4 = I_{\min}$

$I_y = 42.6 + (2)(81.758) = 206.12 \text{ in}^4$

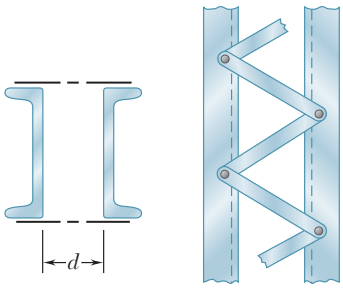
$L = 22 \text{ ft} = 264 \text{ in.}$

$$P_{\text{cr}} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(29 \times 10^6)(187.75)}{264^2} = 771.0 \times 10^3 \text{ lb} = 771 \text{ kips}$$

$$P_{\text{all}} = \frac{P_{\text{cr}}}{F.S.} = \frac{771}{2.3}$$

$P_{\text{all}} = 335$ kips ◀

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PROBLEM 10.18

A single compression member of 8.2-m effective length is obtained by connecting two C200 × 17.1 steel channels with lacing bars as shown. Knowing that the factor of safety is 1.85, determine the allowable centric load for the member. Use $E = 200$ GPa and $d = 100$ mm.

SOLUTION

For C200 × 17.1 steel channel, $A = 2170\ \text{mm}^2$

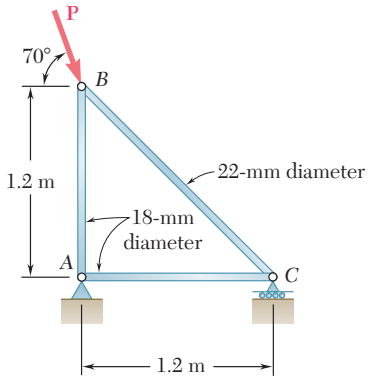
$$\bar{I}_x = 13.5 \times 10^6\ \text{mm}^4, \quad \bar{I}_y = 0.545 \times 10^6\ \text{mm}^2$$
$$\bar{x} = 14.5\ \text{mm}$$

For the fabricated column,

$$I_x = 2\bar{I}_x = (2)(13.5 \times 10^6) = 27.0 \times 10^6\ \text{mm}^4$$
$$I_y = 2\left[\bar{I}_y + A\left(\frac{d}{2} + \bar{x}\right)^2\right]$$
$$= 2\left[0.545 \times 10^6 + 2170\left(\frac{100}{2} + 14.5\right)^2\right]$$
$$= 19.1455 \times 10^6\ \text{mm}^4$$
$$I_{\min} = I_y = 19.1455 \times 10^6\ \text{mm}^4 = 19.1455 \times 10^{-6}\ \text{m}^4 \qquad E = 200 \times 10^9\ \text{Pa}$$
$$P_{\text{cr}} = \frac{\pi^2 E I_{\min}}{L^2} = \frac{\pi^2 (200 \times 10^9)\,(19.1455 \times 10^{-6})}{(8.2)^2} = 562.04 \times 10^3\ \text{N}$$
$$P_{\text{all}} = \frac{P_{\text{cr}}}{F.S.} = \frac{562.04 \times 10^3}{1.85} = 303 \times 10^3\ \text{N}$$

$$P_{\text{all}} = 304\ \text{kN} \blacktriangleleft$$

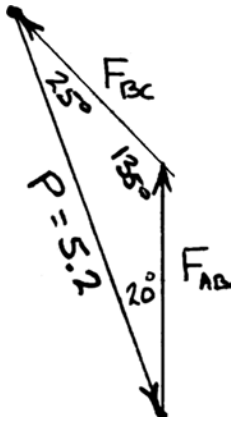
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PROBLEM 10.19

Knowing that $P = 5.2$ kN, determine the factor of safety for the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.

SOLUTION



Joint *B*: From force triangle,

$$\frac{F_{AB}}{\sin 25^\circ} = \frac{F_{BC}}{\sin 20^\circ} = \frac{5.2}{\sin 135^\circ}$$

$$F_{AB} = 3.1079 \text{ kN} \quad \text{(Comp)}$$

$$F_{BC} = 2.5152 \text{ kN} \quad \text{(Comp)}$$

Member *AB*:

$$I_{AB} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{18}{2}\right)^4 = 5.153\times10^{3}\,\text{mm}^4$$

$$= 5.153\times10^{-9}\,\text{m}^4$$

$$F_{AB,\text{cr}} = \frac{\pi^2 EI_{AB}}{L_{AB}^2} = \frac{\pi^2(200\times10^9)(5.153\times10^{-9})}{(1.2)^2}$$

$$= 7.0636\times10^{3}\,\text{N} = 7.0636 \text{ kN}$$

$$F.S. = \frac{F_{AB,\text{cr}}}{F_{AB}} = \frac{7.0636}{3.1079} = 2.27$$

Member *BC*:

$$I_{BC} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{22}{2}\right)^4$$

$$= 11.499\times10^{3}\,\text{mm}^4 = 11.499\times10^{-9}\,\text{m}^4$$

$$L_{BC}^2 = 1.2^2 + 1.2^2 = 2.88\ \text{m}^2$$

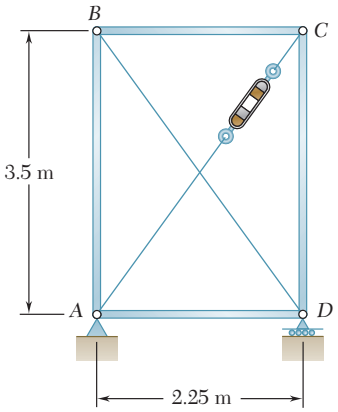
$$F_{BC,\text{cr}} = \frac{\pi^2 EI_{BC}}{L_{BC}^2} = \frac{\pi^2(200\times10^9)(11.499\times10^{-9})}{2.88}$$

$$= 7.8813\times10^{3}\,\text{N} = 7.8813 \text{ kN}$$

$$F.S. = \frac{F_{BC,\text{cr}}}{F_{BC}} = \frac{7.8813}{2.5152} = 3.13$$

Smallest *F.S.* governs. $F.S. = 2.27$ ◀

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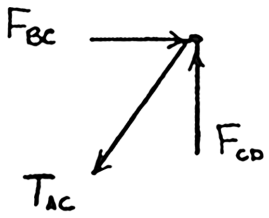


PROBLEM 10.20

Members *AB* and *CD* are 30-mm-diameter steel rods, and members *BC* and *AD* are 22-mm-diameter steel rods. When the turnbuckle is tightened, the diagonal member *AC* is put in tension. Knowing that a factor of safety with respect to buckling of 2.75 is required, determine the largest allowable tension in *AC*. Use $E = 200$ GPa and consider only buckling in the plane of the structure.

SOLUTION

$$L_{AC} = \sqrt{(3.5)^2 + (2.25)^2} = 4.1608 \text{ m}$$



Joint *C*:

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \quad F_{BC} - \frac{2.25}{4.1608} T_{AC} = 0$$

$$T_{AC} = 1.84926 F_{BC}$$

$$+\uparrow \Sigma F_y = 0: \quad F_{CD} - \frac{3.5}{4.1608} T_{AC} = 0$$

$$T_{AC} = 1.1888 F_{CD}$$

Members *BC* and *AD*:

$$I_{BC} = \frac{\pi}{4}\left(\frac{d_{BC}}{2}\right)^4 = \frac{\pi}{4}\left(\frac{22}{2}\right)^4 = 11.499 \times 10^3 \text{ mm}^4 = 11.499 \times 10^{-9} \text{ m}^4$$

$$L_{BC} = 2.25 \text{ m}$$

$$F_{BC,\text{cr}} = \frac{\pi^2 E I_{BC}}{L_{BC}^2} = \frac{\pi^2 (200 \times 10^9)(11.499 \times 10^{-9})}{(2.25)^2} = 4.4836 \times 10^3 \text{ N}$$

$$F_{BC,\text{all}} = \frac{F_{BC,\text{cr}}}{F.S.} = 1.6304 \times 10^3 \text{ N} \quad T_{AC,\text{all}} = 3.02 \times 10^3 \text{ N}$$

Members *AB* and *CD*:

$$I_{CD} = \frac{\pi}{4}\left(\frac{d_{CD}}{2}\right)^4 = \frac{\pi}{4}\left(\frac{30}{2}\right)^4 = 39.761 \times 10^3 \text{ mm}^4 = 39.761 \times 10^{-9} \text{ m}^4$$

$$L_{CD} = 3.5 \text{ m}$$

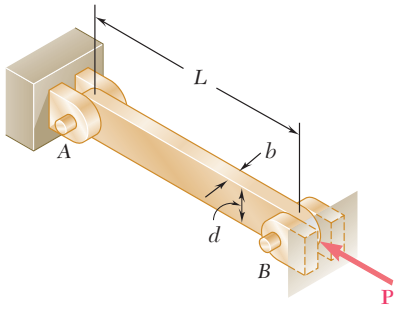
$$F_{CD,\text{cr}} = \frac{\pi^2 E I_{CD}}{L_{CD^2}} = \frac{\pi^2 (200 \times 10^9)(39.761 \times 10^{-9})}{(3.5)^2} = 6.4069 \times 10^3 \text{ N}$$

$$F_{CD,\text{all}} = \frac{F_{CD,\text{cr}}}{F.S.} = 2.3298 \times 10^3 \text{ N} \quad T_{AC,\text{all}} = 2.77 \times 10^3 \text{ N}$$

Smaller value for $T_{AC,\text{all}}$ governs.

$$T_{AC,\text{all}} = 2.77 \text{ kN} \blacktriangleleft$$

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PROBLEM 10.21

The uniform brass bar *AB* has a rectangular cross section and is supported by pins and brackets as shown. Each end of the bar can rotate freely about a horizontal axis through the pin, but rotation about a vertical axis is prevented by the brackets. (*a*) Determine the ratio *b*/*d* for which the factor of safety is the same about the horizontal and vertical axes. (*b*) Determine the factor of safety if $P = 1.8$ kips, $L = 7$ ft, $d = 1.5$ in., and $E = 29 \times 10^6$ psi.

SOLUTION

Buckling in horizontal plane: $L_e = \frac{1}{2}L, \quad I = \frac{1}{12}db^3$

$$P_{cr1} = \frac{\pi^2 EI}{L_e^2} = \frac{4\pi^2 Edb^3}{12L^2} \qquad (1)$$

Buckling in vertical plane: $L_e = L, \quad I = \frac{1}{12}bd^3$

$$P_{cr2} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 Ebd^3}{12L^2} \qquad (2)$$

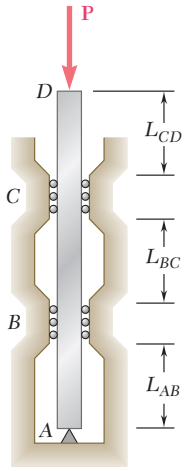
(*a*) Equating, $P_{cr1} = P_{cr2}$ $\quad \frac{4\pi^2 Edb^3}{12L^2} = \frac{\pi^2 Ebd^3}{12L^2} \quad 4b^2 = d^2$ $\quad b = \frac{1}{2}d$ ◀

(*b*) $b = \frac{1}{2}d = 0.75$ in. $\quad L = 7$ ft $= 84$ in.

Using (2), $P_{cr} = \frac{\pi^2 (15 \times 10^6)(0.75)(1.5)^3}{(12)(84)^2} = 4.4257 \times 10^3$ lb $= 4.426$ kips.

$$F.S. = \frac{P_{cr}}{P} = \frac{4.4257}{1.8} = 2.46 \blacktriangleleft$$

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PROBLEM 10.22

A 1-in.-square aluminum strut is maintained in the position shown by a pin support at *A* and by sets of rollers at *B* and *C* that prevent rotation of the strut in the plane of the figure. Knowing that $L_{AB} = 3$ ft, determine (*a*) the largest values of L_{BC} and L_{CD} that can be used if the allowable load **P** is to be as large as possible, (*b*) the magnitude of the corresponding allowable load. Consider only buckling in the plane of the figure and use $E = 10.4 \times 10^6$ psi.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(1)(1)^3 = 0.083333 \text{ in}^4$$

(*a*) Equivalent lengths:

AB: $L_e = 0.7L_{AB} = 2.1 \text{ ft} = 25.2 \text{ in.}$

BC: $L_e = 0.5L_{BC}$

$$L_{BC} = \frac{2.1}{0.5} \qquad L_{BC} = 4.20 \text{ ft} \blacktriangleleft$$

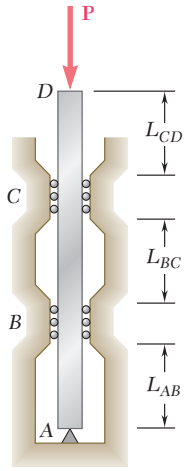
CD: $L_e = 2L_{CD}$

$$L_{CD} = \frac{2.1}{2} \qquad L_{CD} = 1.050 \text{ ft} \blacktriangleleft$$

(*b*)
$$P_{\text{all}} = \frac{P_{\text{cr}}}{F.S.} = \frac{\pi^2 EI}{(F.S.)L_e^2} = \frac{\pi^2(10.4 \times 10^6)(0.083333)}{(3.2)(25.2)^2}$$
$$= 4.21 \times 10^3 \text{ lb}$$

$$P_{\text{all}} = 4.21 \text{ kips} \blacktriangleleft$$

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PROBLEM 10.23

A 1-in.-square aluminum strut is maintained in the position shown by a pin support at *A* and by sets of rollers at *B* and *C* that prevent rotation of the strut in the plane of the figure. Knowing that $L_{AB} = 3$ ft, $L_{BC} = 4$ ft, and $L_{CD} = 1$ ft, determine the allowable load **P** using a factor of safety with respect to buckling of 3.2. Consider only buckling in the plane of the figure and use $E = 10.4 \times 10^6$ psi.

SOLUTION

$$I = \frac{1}{12} bh^3 = \frac{1}{12}(1)(1)^3 = 0.083333 \text{ in}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$P_{all} = \frac{(P_{cr})_{min}}{F.S.} = \frac{\pi^2 EI}{(F.S.)(L_e)^2_{max}}$$

Portion *AB*: $L_e = 0.7 L_{AB} = (0.7)(3) = 2.1$ ft

Portion *BC*: $L_e = 0.5 L_{BC} = (0.5)(4) = 2.0$ ft

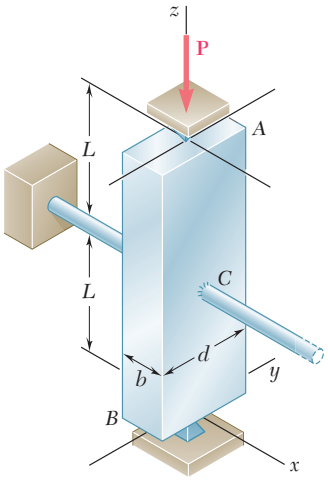
Portion *CD*: $L_e = 2L_e = (2)(1) = 2.0$ ft

$$(L_e)_{max} = 2.1 \text{ ft} = 25.2 \text{ in.}$$

$$P_{all} = \frac{\pi^2 (10.4 \times 10^6)(0.083333)}{(3.2)(25.2^2)} = 4.21 \times 10^3 \text{ lb}$$

$P_{all} = 4.21$ kips ◀

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PROBLEM 10.24

Column *ABC* has a uniform rectangular cross section with $b = 12$ mm and $d = 22$ mm. The column is braced in the *xz* plane at its midpoint *C* and carries a centric load **P** of magnitude 3.8 kN. Knowing that a factor of safety of 3.2 is required, determine the largest allowable length *L*. Use $E = 200$ GPa.

SOLUTION

$$P_{cr} = (F.S.)P = (3.2)(3.8\times10^{3}) = 12.16\times10^{3}\ \text{N}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \qquad L_e = \pi\sqrt{\frac{EI}{P_{cr}}}$$

Buckling in *xz*-plane.

$$L = L_e = \pi\sqrt{\frac{EI}{P_{cr}}}$$



$$I = \frac{1}{12}db^3 = \frac{1}{12}(22)(12)^3 = 3.168\times10^{3}\ \text{mm}^4$$
$$= 3.168\times10^{-9}\ \text{m}^4$$

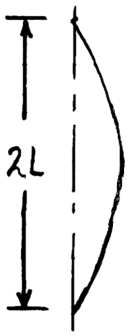
$$L = \pi\sqrt{\frac{(200\times10^{9})(3.168\times10^{-9})}{12.16\times10^{3}}} = 0.717\ \text{m}$$

Buckling in *yz*-plane.

$$L_e = 2L \qquad L = \frac{L_e}{2} = \frac{\pi}{2}\sqrt{\frac{EI}{P_{cr}}}$$

$$I = \frac{1}{12}bd^3 = \frac{1}{12}(12)(22)^3 = 10.648\times10^{3}\ \text{mm}^4$$
$$= 10.648\times10^{-9}\ \text{m}^4$$

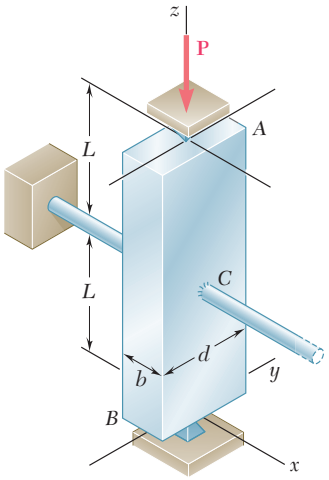
$$L = \frac{\pi}{2}\sqrt{\frac{(200\times10^{9})(10.648\times10^{-9})}{12.16\times10^{3}}} = 0.657\ \text{m}$$



The smaller length governs. $L = 0.657$ m

$L = 657$ mm ◀

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PROBLEM 10.25

Column *ABC* has a uniform rectangular cross section and is braced in the *xz* plane at its midpoint *C*. (*a*) Determine the ratio *b*/*d* for which the factor of safety is the same with respect to buckling in the *xz* and *yz* planes. (*b*) Using the ratio found in part *a*, design the cross section of the column so that the factor of safety will be 3.0 when $P = 4.4$ kN, $L = 1$ m, and $E = 200$ GPa.

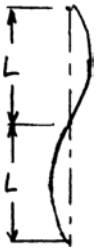
SOLUTION

Buckling in *xz*-plane.

$$L_e = L, \quad I = \frac{db^3}{12}$$

$$(P_{cr})_1 = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 Edb^3}{12L^2}$$

$$(F.S.)_1 = \frac{(P_{cr})_1}{P} = \frac{\pi^2 Edb^3}{12PL^2}$$

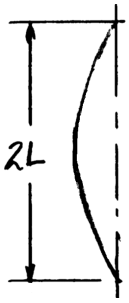


Buckling in *yz*-plane.

$$L_e = 2L, \quad I = \frac{bd^3}{12}$$

$$(P_{cr})_2 = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 Ebd^3}{12(2L)^2}$$

$$(F.S.)_2 = \frac{(P_{cr})_2}{P} = \frac{\pi^2 Ebd^3}{48PL^2}$$



(*a*) Equating the two factors of safety,

$$\frac{\pi^2 Edb^3}{12PL^2} = \frac{\pi^2 Ebd^3}{48PL^2}$$

$$b^2 = \frac{1}{4}d^2$$

$$b/d = 1/2 \blacktriangleleft$$

Then

$$(F.S.) = \frac{\pi^2 Ed^4}{96PL^2}$$

$$d^4 = \frac{96(F.S.)PL^2}{\pi^2 E} = \frac{(96)(3.0)(4.4\times10^3)(1)^2}{\pi^2(200\times10^9)}$$

$$= 641.97\times10^{-9}\,\text{m}^4$$

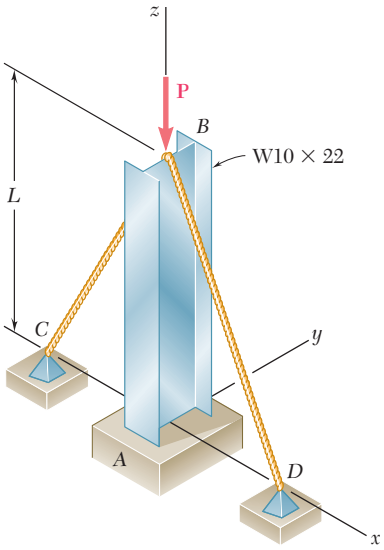
(*b*)

$$d = 28.3\times10^{-3}\,\text{m}$$

$$d = 28.3\,\text{mm} \blacktriangleleft$$

$$b = 14.15\,\text{mm} \blacktriangleleft$$

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PROBLEM 10.26

Column *AB* carries a centric load **P** of magnitude 15 kips. Cables *BC* and *BD* are taut and prevent motion of point *B* in the *xz* plane. Using Euler's formula and a factor of safety of 2.2, and neglecting the tension in the cables, determine the maximum allowable length *L*. Use $E = 29\times10^6$ psi.

SOLUTION

W10×22: $I_x = 118 \text{ in}^4$

$I_y = 11.4 \text{ in}^4$

$P = 15\times10^3 \text{ lb}$

$P_{cr} = (F.S.)P = (2.2)(15\times10^3) = 33\times10^3 \text{ lb}$

Buckling in *xz*-plane. $L_e = 0.7L$

$$P_{cr} = \frac{\pi^2 EI_y}{(0.7L)^2} \qquad L = \frac{\pi}{0.7}\sqrt{\frac{EI_y}{P_{cr}}}$$

$$L = \frac{\pi}{0.7}\sqrt{\frac{(29\times10^6)(11.4)}{33\times10^3}} = 449.21 \text{ in.}$$

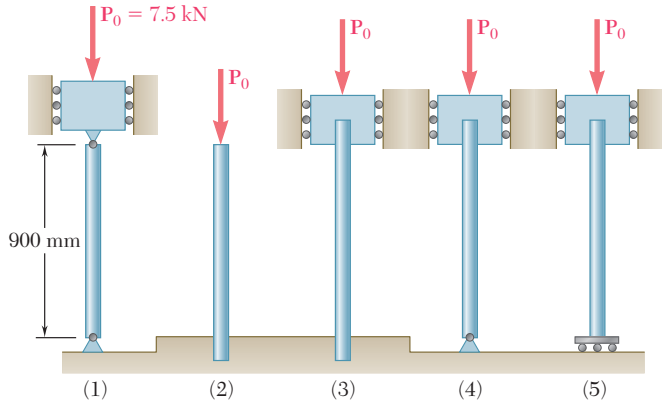
Buckling in *yz*-plane. $L_e = 2L$

$$P_{cr} = \frac{\pi^2 EI_x}{(2L)^2}$$

$$L = \frac{\pi}{2}\sqrt{\frac{EI_x}{P_{cr}}} = \frac{\pi}{2}\sqrt{\frac{(29\times10^6)(118)}{33\times10^3}} = 505.83 \text{ in.}$$

Smaller value for *L* governs. $L = 449.21$ in. $L = 37.4$ ft ◀

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PROBLEM 10.27

Each of the five struts shown consists of a solid steel rod. (*a*) Knowing that the strut of Fig. (1) is of a 20-mm diameter, determine the factor of safety with respect to buckling for the loading shown. (*b*) Determine the diameter of each of the other struts for which the factor of safety is the same as the factor of safety obtained in part *a*. Use $E = 200$ GPa.

SOLUTION

Solid circular cross section: $c = \frac{1}{2}d = 10$ mm

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(10)^4 = 7.854 \times 10^3 \text{ mm}^4 = 7.854 \times 10^{-9} \text{ m}^4$$

$$E = 200 \text{ GPa} = 200 \times 10^9 \text{ Pa}$$

$$EI = (200 \times 10^9)(7.854 \times 10^{-9}) = 1570.8 \text{ N} \cdot \text{m}^2$$

For strut (1), $L_e = L = 900 \text{ mm} = 0.9 \text{ m}$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(1570.8)}{(0.9)^2} = 19{,}140 \text{ N} = 19.140 \text{ kN}$$

(*a*) $$F.S. = \frac{P_{cr}}{P_0} = \frac{19.140 \text{ kN}}{7.5 \text{ kN}}$$ $F.S. = 2.55$ ◀

(*b*) For the same factor of safety, the struts must have the same critical load.

$$P_{cr} = \frac{\pi^2 EI_i}{L_i^2} \text{ where } i = 1, 2, 3, 4, \text{ and } 5$$

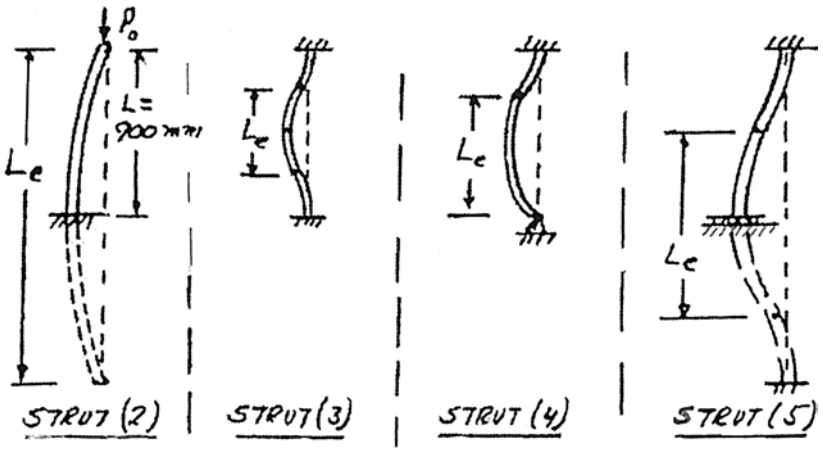
For $i = 2, 3, 4$, and 5, $\frac{I_i}{L_i^2} = \frac{I_1}{L_1^2}$ or $\frac{I_i}{I_1} = \frac{L_i^2}{L_1^2}$

Since I is proportional to d^4, $\frac{d_i^4}{d_1^4} = \frac{L_i^2}{L_1^2}$

or $\frac{d_i}{d_1} = \sqrt{\frac{L_i}{L_1}}$, where L_i is the effective length.

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PROBLEM 10.27 *(Continued)*



Strut (1): $L_1 = 900$ mm $\quad d_1 = 20$ mm

Strut (2): $L_2 = 2L_1 = 1800$ mm

$$\frac{d_2}{20} = \sqrt{\frac{1800}{900}} \qquad d_2 = 28.3 \text{ mm} \blacktriangleleft$$

Strut (3): $L_3 = \frac{1}{2}L_1 = 450$ mm

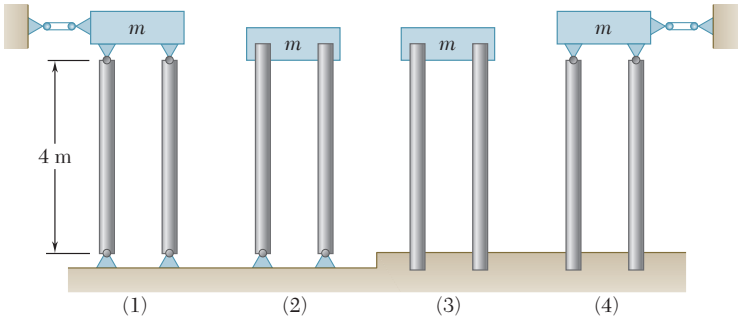
$$\frac{d_3}{20} = \sqrt{\frac{450}{900}} \qquad d_3 = 14.14 \text{ mm} \blacktriangleleft$$

Strut (4): $L_4 = 0.669 \quad L_1 = 629.1$ mm

$$\frac{d_4}{20} = \sqrt{\frac{629.1}{900}} \qquad d_4 = 16.72 \text{ mm} \blacktriangleleft$$

Strut (5): $L_5 = L_1 = 900$ mm $\qquad d_5 = 20.0$ mm ◀

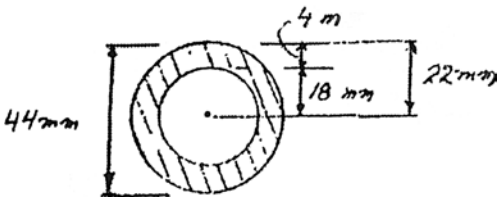
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PROBLEM 10.28

A rigid block of mass m can be supported in each of the four ways shown. Each column consists of an aluminum tube that has a 44-mm outer diameter and a 4-mm wall thickness. Using $E = 70$ GPa and a factor of safety of 2.8, determine the allowable load for each support condition.

SOLUTION



$$I = \frac{\pi}{4}[(22\text{ mm})^4 - (18\text{ mm})^4]$$

$$I = 101.54 \times 10^3\text{ mm}^4 = 101.54 \times 10^{-9}\text{ m}^4$$

$$E = 70\text{ GPa} = 70 \times 10^9\text{ Pa}$$

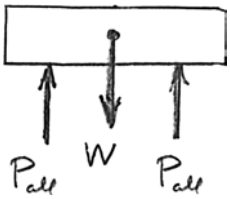
$$EI = (70\text{ GPa})(101.54 \times 10^{-9}\text{ m}^4) = 7108\text{ N} \cdot \text{m}^2$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(7108\text{ N} \cdot \text{m}^2)}{L_e^2}$$

Allowable value of P:

$$P_{all} = \frac{P_{cr}}{F.S.}$$

$$P_{all} = \frac{\pi^2(7108\text{ N} \cdot \text{m}^2)}{2.8L_e^2} = \frac{25{,}055}{L_e^2}$$

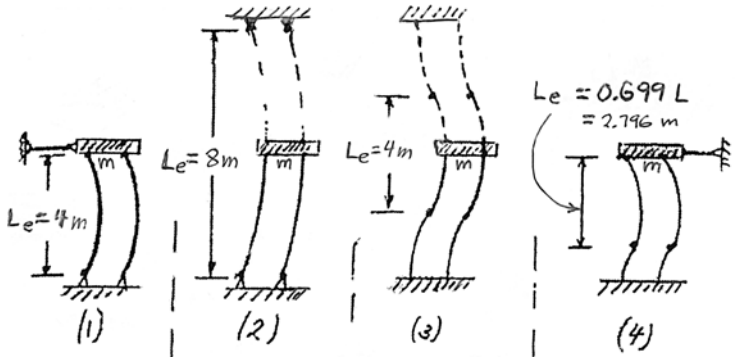


Equilibrium. $\sum F_y = 0$: $2P_{all} - W = 0$

$$m = \frac{W}{g} = \frac{2P_{all}}{g} = \frac{(2)(25{,}055)}{9.81L_e^2} = \frac{5108}{L_e^2}$$

Column length: $L_1 = 4$ m

Effective lengths for columns.



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PROBLEM 10.28 *(Continued)*

Allowable capacity:

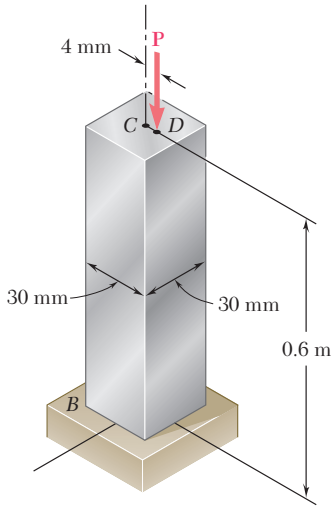
Case (1): $L_e = L = 4$ m $\quad m = \dfrac{5108}{(4)^2}$ $\quad m = 319$ kg ◀

Case (2): $L_e = 2L = 8$ m $\quad m = \dfrac{5108}{(8)^2}$ $\quad m = 79.8$ kg ◀

Case (3): $L_e = L = 4$ m $\quad m = \dfrac{5108}{(4)^2}$ $\quad m = 319$ kg ◀

Case (4): $L_e = 0.699L = 2.796$ m $\quad m = \dfrac{5108}{(2.796)^2}$ $\quad m = 653$ kg ◀

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PROBLEM 10.29

An axial load **P** = 15 kN is applied at point *D* that is 4 mm from the geometric axis of the square aluminum bar *BC*. Using $E = 70$ GPa, determine (*a*) the horizontal deflection of end *C*, (*b*) the maximum stress in the column.

SOLUTION

$$A = (30)^2 = 900 \text{ mm}^2$$
$$= 900 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$$
$$= 67.5 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}(30) = 15 \text{ mm} = 0.015 \text{ m} \qquad e = 4 \times 10^{-3} \text{ m}$$

$$L_e = 2L = (2)(0.6) = 1.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (70 \times 10^9)(67.5 \times 10^{-9})}{(1.2)^2}$$
$$= 32.385 \times 10^3 \text{ N} = 38.385 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{15}{32.385} = 0.46318$$

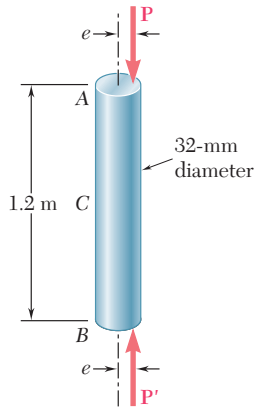
(*a*) $$y_m = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = (4 \times 10^3)\left[\sec\left(\frac{\pi}{2}\sqrt{0.46318}\right) - 1\right]$$

$$= (4 \times 10^{-3})\left[\sec(1.06904) - 1\right] = 4.3166 \times 10^{-3} \text{ m} \qquad y_m = 4.32 \text{ mm} \blacktriangleleft$$

(*b*) $$M_{max} = P(e + y_m) = (15 \times 10^3)(4 \times 10^{-3} + 4.3166 \times 10^{-3})$$
$$= 124.75 \text{ N} \cdot \text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{15 \times 10^3}{900 \times 10^{-6}} + \frac{(124.75)(0.015)}{67.5 \times 10^{-9}}$$
$$= 44.4 \times 10^6 \text{ Pa} \qquad \sigma_{max} = 44.4 \text{ MPa} \blacktriangleleft$$

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PROBLEM 10.30

An axial load **P** is applied to the 32-mm-diameter steel rod *AB* as shown. For $P = 37$ kN and $e = 1.2$ mm, determine (*a*) the deflection at the midpoint *C* of the rod, (*b*) the maximum stress in the rod. Use $E = 200$ GPa.

SOLUTION

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{32}{2}\right)^4 = 51.47 \times 10^3 \text{ mm}^4 = 51.47 \times 10^{-9} \text{ m}^4$$

$$L_e = L = 1.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^{\,2}} = \frac{\pi^2 (200 \times 10^9)(51.47 \times 10^{-9})}{(1.2)^2} = 70.556 \times 10^3 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{37 \times 10^3}{70.556 \times 10^3} = 0.52440$$

(*a*) Deflection at *C*.

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = 1.3817e = (1.3817)(1.2)$$

$$y_{max} = 1.658 \text{ mm} \blacktriangleleft$$

(*b*) Maximum normal stress.

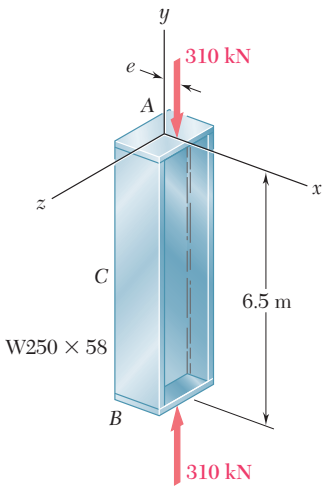
$$M_{max} = P(e + y_{max}) = (37 \times 10^3)(1.2 + 1.658)(10^{-3}) = 105.75 \text{ N} \cdot \text{m}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2, \quad c = 16 \times 10^{-3} \text{ m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{37 \times 10^3}{804.25 \times 10^{-6}} + \frac{(105.75)(16 \times 10^{-3})}{51.47 \times 10^{-9}} = 78.9 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 78.9 \text{ MPa} \blacktriangleleft$$

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PROBLEM 10.31

The line of action of the 310-kN axial load is parallel to the geometric axis of the column *AB* and intersects the *x* axis at $x = e$. Using $E = 200$ GPa, determine (*a*) the eccentricity *e* when the deflection of the midpoint *C* of the column is 9 mm, (*b*) the maximum stress in the column.

SOLUTION

For W250 × 58, $A = 7420 \text{ mm}^2 = 7420 \times 10^{-6} \text{ m}^2$

$$I_y = 18.7 \times 10^6 \text{ mm}^3 = 18.7 \times 10^{-6} \text{ m}^4$$

$$S_y = 185 \times 10^3 \text{ mm}^3 = 185 \times 10^{-6} \text{ m}^3$$

$$L = 6.5 \text{ m} \qquad L_e = 6.5 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(18.7 \times 10^{-6})}{(6.5)^2} = 873.7 \times 10^3 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{310 \times 10^3}{873.7 \times 10^3} = 0.35483$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = 0.68558\, e$$

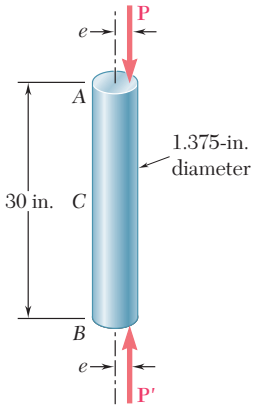
(*a*) $$e = \frac{y_{max}}{0.67990} = \frac{9 \times 10^{-3}}{0.67990} = 13.13 \times 10^{-3} \text{ m} = 13.13 \text{ mm}$$ ◀

(*b*) $$M_{max} = P(e + y_{max}) = (310 \times 10^3)(9 + 13.13)(10^{-3}) = 6859.6 \text{ N} \cdot \text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{310 \times 10^3}{7420 \times 10^{-6}} + \frac{6859.6}{185 \times 10^{-6}}$$

$$= 41.78 \times 10^6 + 37.08 \times 10^6 = 78.86 \times 10^6 \text{ Pa} = 78.9 \text{ MPa}$$ ◀

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PROBLEM 10.32

An axial load **P** is applied to the 1.375-in.-diameter steel rod *AB* as shown. When $P = 21$ kips, it is observed that the horizontal deflection of the midpoint *C* is 0.03 in. Using $E = 29 \times 10^6$ psi, determine (*a*) the eccentricity *e* of the load, (*b*) the maximum stress in the rod.

SOLUTION

$$c = \frac{1}{2}d = 0.6875 \text{ in.} \quad A = \pi c^2 = 1.4849 \text{ in}^2$$

$$I = \frac{\pi}{4}c^4 = 0.175461 \text{ in}^4$$

$$L_e = L = 30 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29 \times 10^3)(0.175461)}{(30)^2} = 55.800 \times 10^3 \text{ lb}$$

$$\frac{P}{P_{cr}} = \frac{21 \times 10^3}{55.8 \times 10^3} = 0.37634$$

(*a*) Eccentricity of the load.

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = e[\sec 0.96363 - 1] = 0.75272e$$

$$e = \frac{y_{max}}{0.75272} = \frac{0.03}{0.75272} \qquad e = 0.0399 \text{ in.} \blacktriangleleft$$

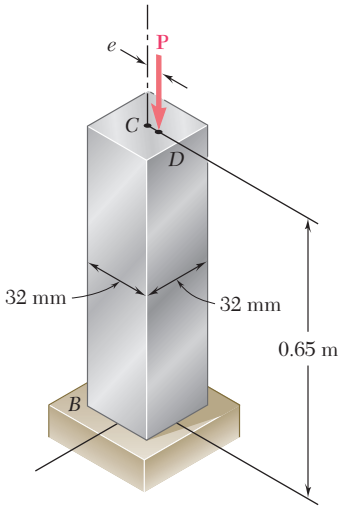
(*b*) Maximum normal stress.

$$M_{max} = P(e + y_{max}) = (21 \times 10^3)(0.0399 + 0.03) = 1.467 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}c}{I} = \frac{21 \times 10^3}{1.4849} + \frac{(1.467 \times 10^3)(0.6875)}{0.175461} = 19.89 \times 10^3 \text{ psi}$$

$$\sigma_{max} = 19.89 \text{ ksi} \blacktriangleleft$$

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PROBLEM 10.33

An axial load **P** is applied to the 32-mm-square aluminum bar *BC* as shown. When $P = 24$ kN, the horizontal deflection at end *C* is 4 mm. Using $E = 70$ GPa, determine (*a*) the eccentricity *e* of the load, (*b*) the maximum stress in the rod.

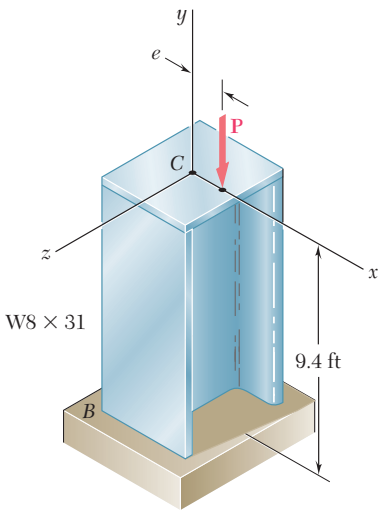
SOLUTION

$$I = \frac{1}{12}(32)^4 = 87.3813 \times 10^3 \text{ mm}^4$$
$$= 87.3813 \times 10^{-9} \text{ m}^4$$
$$A = (32)^2 = 1.024 \times 10^3 \text{ mm}^2$$
$$= 1.024 \times 10^{-3} \text{ m}^2$$
$$L_e = 2L = (2)(0.65) = 1.30 \text{ m}$$
$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(70 \times 10^9)(87.3813 \times 10^{-9})}{(1.30)^2}$$
$$= 35.7215 \times 10^3 \text{ N}$$
$$= 35.7215 \text{ kN}$$
$$\frac{P}{P_{cr}} = \frac{24}{35.7215} = 0.67186$$

(*a*) $$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = e\left[\sec\left(\frac{\pi}{2}\sqrt{0.67186}\right) - 1\right]$$
$$= e[\sec 1.28754 - 1] = 2.5780e$$
$$e = \frac{y_{max}}{2.5780} = \frac{4}{2.5780}$$ $e = 1.552$ mm ◀

(*b*) $$M_{max} = P(e + y_{max}) = (24 \times 10^3)[1.552 \times 10^{-3} + 4 \times 10^{-3}]$$
$$= 133.24 \text{ N} \cdot \text{m}$$
$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}c}{I} = \frac{24 \times 10^3}{1.024 \times 10^{-3}} + \frac{(133.24)(0.016)}{87.3813 \times 10^{-9}}$$
$$= 47.8 \times 10^6 \text{ Pa}$$ $\sigma_{max} = 47.8$ MPa ◀

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PROBLEM 10.34

The axial load **P** is applied at a point located on the *x* axis at a distance *e* from the geometric axis of the rolled-steel column *BC*. When $P = 82$ kips, the horizontal deflection of the top of the column is 0.20 in. Using $E = 29 \times 10^6$ psi, determine (*a*) the eccentricity *e* of the load, (*b*) the maximum stress in the column.

SOLUTION

W8 × 31: $A = 9.12\ \text{in}^2$, $I_y = 37.1\ \text{in}^4$, $S_y = 9.27\ \text{in}^3$

$$L = 9.4\ \text{ft} = 112.8\ \text{in.} \qquad L_e = 2L = 225.6\ \text{in.}$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(29 \times 10^6)(37.1)}{(225.6)^2} = 208.63 \times 10^3$$

$$\frac{P}{P_{\text{cr}}} = \frac{82 \times 10^3}{208.63 \times 10^3} = 0.39304$$

(a) $$y_{\max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) - 1\right] = 0.80811e$$

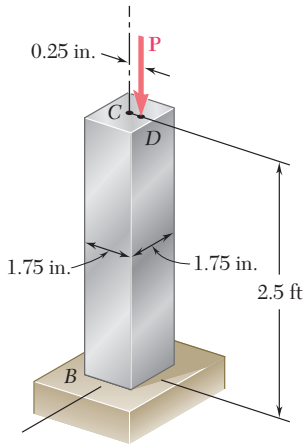
$$e = \frac{y_{\max}}{0.80811} = \frac{0.20}{0.80816} = 0.247\ \text{in.}$$ ◀

(*b*) $$M_{\max} = P(e + y_{\max}) = (82 \times 10^3)(0.247 + 0.20) = 36.693 \times 10^3\ \text{lb} \cdot \text{in.}$$

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{82 \times 10^3}{9.12} + \frac{36.693 \times 10^3}{9.27} = 12.95 \times 10^3\ \text{psi}$$

$\sigma_{\max} = 12.95\ \text{ksi}$ ◀

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PROBLEM 10.35

An axial load **P** is applied at a point *D* that is 0.25 in. from the geometric axis of the square aluminum bar *BC*. Using $E = 10.1 \times 10^6$ psi, determine (*a*) the load **P** for which the horizontal deflection of end *C* is 0.50 in., (*b*) the corresponding maximum stress in the column.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(1.75)(1.75)^3 = 0.78157 \text{ in}^4$$

$$A = (1.75)^2 = 3.0625 \text{ in}^2 \quad c = \frac{1}{2}(1.75) = 0.875 \text{ in.}$$

$$L = 2.5 \text{ ft} = 30 \text{ in.} \quad L_e = 2L = 60 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(10.1 \times 10^3)(0.78157)}{(60)^2} = 21.641 \text{ kips}$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_c}}\right) - 1\right], \quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}, \quad \cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

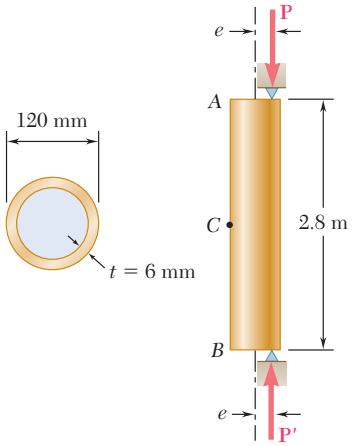
(*a*)
$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\frac{e}{e + y_{max}}\right]^2 = \left[\frac{2}{\pi}\arccos\frac{0.25}{0.25 + 0.50}\right]^2$$

$$= 0.61411 \qquad P = 0.61411P_{cr} \qquad P = 13.29 \text{ kips} \blacktriangleleft$$

(*b*)
$$M_{max} = P(e + y_{max}) = (13.29)(0.25 + 0.50) = 9.9675 \text{ kip} \cdot \text{in.}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{13.29}{3.0625} + \frac{(9.9675)(0.875)}{0.78157} \qquad \sigma_{max} = 15.50 \text{ ksi} \blacktriangleleft$$

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PROBLEM 10.36

A brass pipe having the cross section shown has an axial load **P** applied 5 mm from its geometric axis. Using $E = 120$ GPa, determine (*a*) the load **P** for which the horizontal deflection at the midpoint *C* is 5 mm, (*b*) the corresponding maximum stress in the column.

SOLUTION

$$c_o = \frac{1}{2}d_o = 60 \text{ mm} \qquad c_i = c_o - t = 54 \text{ mm}$$

$$I = \frac{\pi}{4}\left(c_o^4 - c_i^4\right) = 3.5005\times10^{6} \text{ mm}^4 = 3.5005\times10^{-6} \text{ m}^4$$

$$L = 2.8 \text{ m} \qquad L_e = 2.8 \text{ m}$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(120\times10^9)(3.5005\times10^{-6})}{(2.8)^2}$$

$$= 528.8\times10^3 \text{ N} = 528.8 \text{ kN}$$

(*a*) $$y_{\max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{y_{\max}+e}{e}$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{e}{y_{\max}+e} \qquad \frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi}\arccos\frac{e}{y_{\max}+e}\right]^2$$

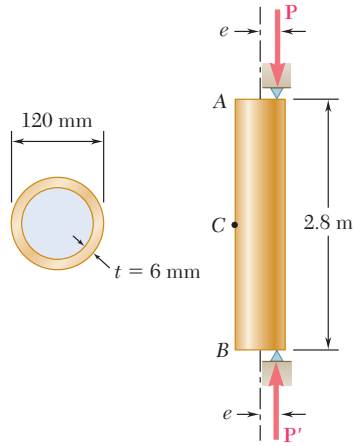
$$\frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi}\arccos\frac{5}{5+5}\right]^2 = 0.44444 \qquad P = 0.44444P_{\text{cr}} \qquad P = 235 \text{ kN} \blacktriangleleft$$

(*b*) $$M_{\max} = P(e + y_{\max}) = (235\times10^3)(5+5)(10^{-3}) = 2350 \text{ N}\cdot\text{m}$$

$$A = \pi\left(c_o^2 - c_i^2\right) = \pi(60^2 - 54^2) = 2.1488\times10^3 \text{ mm}^2 = 2.1488\times10^{-3} \text{ m}^2$$

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I} = \frac{235\times10^3}{2.1488\times10^{-3}} + \frac{(2350)(60\times10^{-3})}{3.5005\times10^{-6}} = 149.6\times10^6 \text{ Pa} \qquad \sigma_{\max} = 149.6 \text{ MPa} \blacktriangleleft$$

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PROBLEM 10.37

Solve Prob. 10.36, assuming that the axial load **P** is applied 10 mm from the geometric axis of the column.

PROBLEM 10.36 A brass pipe having the cross section shown has an axial load **P** applied 5 mm from its geometric axis. Using $E = 120$ GPa, determine (*a*) the load **P** for which the horizontal deflection at the midpoint *C* is 5 mm, (*b*) the corresponding maximum stress in the column.

SOLUTION

$$c_o = \frac{1}{2}d_o = 60 \text{ mm} \qquad c_i = c_o - t = 54 \text{ mm}$$

$$I = \frac{\pi}{4}\left(c_o^4 - c_i^4\right) = 3.5005 \times 10^6 \text{ mm}^4 = 3.5005 \times 10^{-6} \text{ m}^4$$

$$L = 2.8 \text{ m} \qquad L_e = 2.8 \text{ m}$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(120 \times 10^9)(3.5005 \times 10^{-6})}{(2.8)^2}$$

$$= 528.8 \times 10^3 \text{ N} = 528.8 \text{ kN}$$

(*a*) $$y_{\max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{y_{\max} + e}{e}$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{e}{y_{\max} + e} \qquad \frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi}\arccos\frac{e}{y_{\max} + e}\right]^2$$

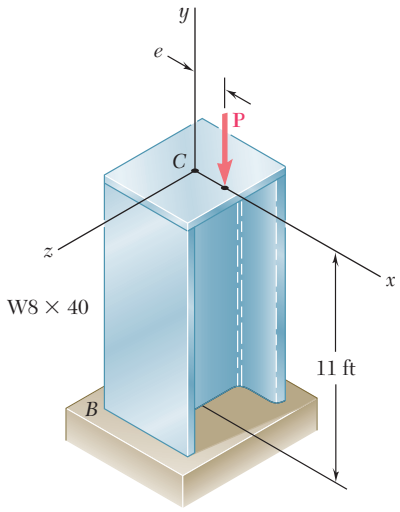
$$\frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi}\arccos\frac{10}{5+10}\right]^2 = 0.28670 \qquad P = 0.28670 P_{\text{cr}} \qquad P = 151.6 \text{ kN} \blacktriangleleft$$

(*b*) $$M_{\max} = P(e + y_{\max}) = (151.6 \times 10^3)(10 + 5)(10^{-3}) = 2274 \text{ N} \cdot \text{m}$$

$$A = \pi\left(c_o^2 - c_i^2\right) = \pi(60^2 - 54^2) = 2.1488 \times 10^3 \text{ mm}^2 = 2.1488 \times 10^{-3} \text{ m}^2$$

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I} = \frac{151.6 \times 10^3}{2.1488 \times 10^{-3}} + \frac{(2274)(60 \times 10^{-3})}{3.5005 \times 10^{-6}} = 109.5 \times 10^6 \text{ Pa} \qquad \sigma_{\max} = 109.5 \text{ MPa} \blacktriangleleft$$

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PROBLEM 10.38

The line of action of the axial load **P** is parallel to the geometric axis of the column *AB* and intersects the *x* axis at $x = 0.8$ in. Using $E = 29 \times 10^6$ psi, determine (*a*) the load **P** for which the horizontal deflection at the end *C* is 0.5 in., (*b*) the corresponding maximum stress in the column.

SOLUTION

$$L = 11 \text{ ft}$$

$$L_e = 2L = 22 \text{ ft} = 264 \text{ in.} \qquad e = 0.8 \text{ in.}$$

W8 × 40:

$$A = 11.7 \text{ in}^2 \qquad I_y = 49.1 \text{ in}^4$$

$$S_y = 12.2 \text{ in}^3$$

$$E = 29{,}000 \text{ ksi} \qquad y_m = 0.8 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29{,}000)(49.1)}{(264)^2} = 201.64 \text{ kips}$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right]$$

$$\sec\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = \frac{y_m + e}{e}$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_m + e} = \frac{0.8}{0.5 + 0.8} = 0.61538$$

$$\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 0.90792$$

$$\frac{P}{P_{cr}} = \left[\frac{(2)(0.90792)}{\pi}\right]^2 = 0.33409$$

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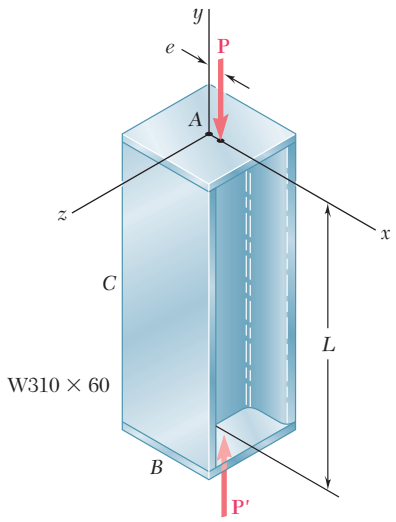
PROBLEM 10.38 *(Continued)*

(*a*) $$P = (0.33409)(201.64) = 67.366$$ $P = 67.4$ kips ◀

(*b*) $$M_{\max} = P(e + y_m) = (67.4)(0.8 + 0.5) = 87.575 \text{ kip} \cdot \text{in.}$$

$$\sigma_{\max} = \frac{P}{A} + \frac{M_{\max}}{S_y} = \frac{67.366}{11.7} + \frac{87.575}{12.2}$$ $\sigma_{\max} = 12.94$ ksi ◀

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PROBLEM 10.39

The line of action of the axial load **P** is parallel to the geometric axis of the column and applied at a point located on the *x* axis at a distance $e = 12$ mm from the geometric axis of the W310 × 60 rolled-steel column *BC*. Assuming that $L = 7.0$ m and using $E = 200$ GPa, determine (*a*) the load **P** for which the horizontal deflection of the midpoint *C* of the column is 15 mm, (*b*) the corresponding maximum stress in the column.

SOLUTION

W310 × 60: $A = 7550 \text{ mm}^2 = 7550 \times 10^{-6} \text{ m}^2$ $I_y = 18.4 \times 10^6 \text{ mm}^4 = 18.4 \times 10^{-6} \text{ m}^4$

$S_y = 180 \times 10^3 \text{ mm}^3 = 180 \times 10^{-6} \text{ m}^3$

$L_e = 7.0 \text{ m}$

Critical load:

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(18.3 \times 10^{-6})}{(7.0)^2} = 741.2 \times 10^3 \text{ N} = 741.2 \text{ kN}$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\left(\frac{e}{y_{max} + e}\right)\right]^2 = \left[\frac{2}{\pi}\arccos\left(\frac{12}{15 + 12}\right)\right]^2 = 0.49957$$

(*a*) Load *P*:

$$P = 0.49957 P_{cr} = 370.3 \text{ kN}$$

$P = 370$ kN ◀

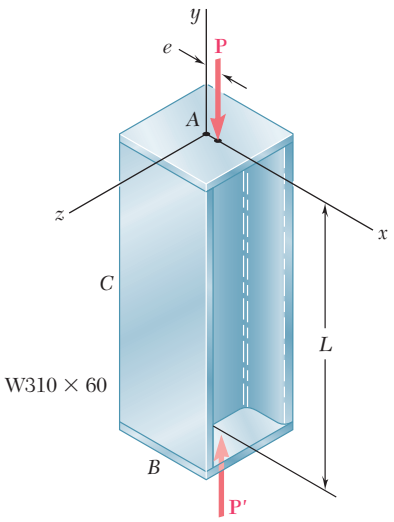
(*b*) Maximum normal stress.

$$M_{max} = P(e + y_{max}) = (370.3 \times 10^3)(12 + 15)(10^{-3}) = 9998 \text{ N} \cdot \text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{370.3 \times 10^3}{7550 \times 10^{-6}} + \frac{9998}{180 \times 10^{-6}} = 104.6 \times 10^6 \text{ Pa}$$

$\sigma_{max} = 104.6$ MPa ◀

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PROBLEM 10.40

Solve Prob. 10.39, assuming that L is 9.0 m.

PROBLEM 10.39 The line of action of the axial load **P** is parallel to the geometric axis of the column and applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W310 × 60 rolled-steel column BC. Assuming that $L = 7.0$ m and using $E = 200$ GPa, determine (*a*) the load **P** for which the horizontal deflection of the midpoint C of the column is 15 mm, (*b*) the corresponding maximum stress in the column.

SOLUTION

W310 × 60: $A = 7550\ \text{mm}^2 = 7550\times10^{-6}\,\text{m}^2$ $I_y = 18.4\times10^{6}\,\text{mm}^4 = 18.4\times10^{-6}\,\text{m}^4$

$S_y = 180\times10^{3}\,\text{mm}^3 = 180\times10^{-6}\,\text{m}^3$ $L_e = 9.0$ m

Critical load:

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(200\times10^9)(18.4\times10^{-6})}{(9.0)^2} = 448.4\times10^3\ \text{N} = 448.4\ \text{kN}$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max}+e}{e}$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max}+e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\left(\frac{e}{y_{max}+e}\right)\right]^2 = \left[\frac{2}{\pi}\arccos\left(\frac{12}{15+12}\right)\right]^2 = 0.49957$$

(*a*) Load P:

$$P = 0.49957P_{cr} = 224.01\ \text{kN}$$

$P = 224$ kN ◀

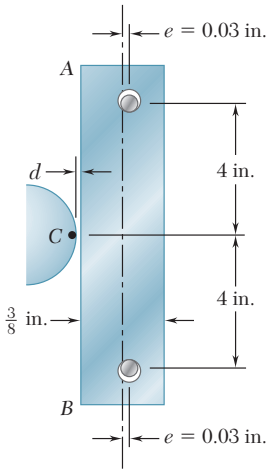
$$M_{max} = P(e + y_{max}) = (224.01\times10^3)(12+15)(10^{-3}) = 6048\ \text{N}\cdot\text{m}$$

(*b*) Maximum normal stress.

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{224.01\times10^3}{7550\times10^{-6}} + \frac{6048}{180\times10^{-6}} = 63.3\times10^6\ \text{Pa}$$

$\sigma_{max} = 63.3$ MPa ◀

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PROBLEM 10.41

The steel bar AB has a $\frac{3}{8} \times \frac{3}{8}$-in. square cross section and is held by pins that are a fixed distance apart and are located at a distance $e = 0.03$ in. from the geometric axis of the bar. Knowing that at temperature T_0 the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point C if $d = 0.01$ in. Use $E = 29 \times 10^6$ psi and a coefficient of thermal expansion $\alpha = 6.5 \times 10^{-6}/°\text{F}$.

SOLUTION

$$A = \left(\frac{3}{8}\right)\left(\frac{3}{8}\right) = 0.140625 \text{ in}^2$$

$$I = \frac{1}{12}\left(\frac{3}{8}\right)^4 = 1.64795 \times 10^{-3} \text{ in}^4$$

$$EI = (29 \times 10^6)(1.64795 \times 10^{-3}) = 47{,}791 \text{ lb} \cdot \text{in}^2$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (47{,}791)}{(8)^2} = 7370 \text{ lb}$$

Calculate P using the secant formula.

$$y_{\max} = d = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) - 1\right] \qquad \sec\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}} = 1 + \frac{d}{e}$$

$$\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}} = \cos^{-1}\left(1 + \frac{d}{e}\right)^{-1} = \cos^{-1}\left(1 + \frac{0.01}{0.03}\right)^{-1} = \cos^{-1}(0.75) = 0.72273$$

$$\frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi}(0.72273)\right]^2 = 0.21170 \qquad P = 0.21170 P_{\text{cr}} = 1560.2 \text{ lb}$$

Thermal analysis:

(1) Simple approximation by ignoring eccentricity.

$$\text{Total elongation} = \alpha L(\Delta T) - \frac{PL}{EA} = 0$$

$$\Delta T = \frac{PL}{EA}\frac{1}{\alpha L} = \frac{P}{EA\alpha} = \frac{1560.2}{(29 \times 10^6)(0.140625)(6.5 \times 10^{-6})} \qquad \Delta T = 58.9°\text{F} \blacktriangleleft$$

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PROBLEM 10.41 *(Continued)*

(2) Analysis with inclusion of eccentricity.

Total elongation of centroidal axis $= \alpha L(\Delta T) - \dfrac{PL}{EA} = 2e\left.\dfrac{dy}{dx}\right|_{x=0}$

To calculate $\dfrac{dy}{dx}$, differentiate Eq. (10.26).

$$\frac{dy}{dx} = e\left(p\tan\frac{pL}{2}\cos px - p\sin px \right)$$

At $x = 0$, $\left.\dfrac{dy}{dx}\right|_{x=0} = ep\tan\dfrac{pL}{2} = e\sqrt{\dfrac{P}{EI}}\tan\dfrac{\pi}{2}\sqrt{\dfrac{P}{P_{\text{cr}}}}$

The elongation of the centroidal axis is $2e^2\sqrt{\dfrac{P}{EI}}\tan\left(\dfrac{\pi}{2}\sqrt{\dfrac{P}{P_{\text{cr}}}}\right)$

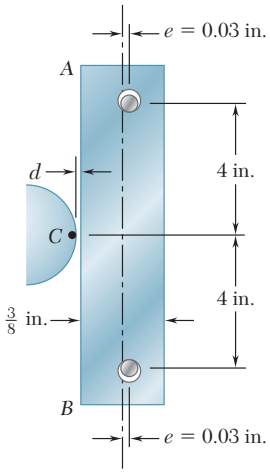
$$= (2)(0.03)^2\sqrt{\frac{1560.2}{47{,}791}}\tan(0.72273) = 286.8\times10^{-6}\text{ in.}$$

$$\alpha L(\Delta T) = \frac{PL}{EA} + 2e\left.\frac{dy}{dx}\right|_{x=0}$$

$$\Delta T = \frac{P}{EA\alpha} + \frac{286.8\times10^{-6}}{\alpha L} = 58.9 + \frac{286.8\times10^{-6}}{(6.5\times10^{-6})(8)} = 58.9 + 5.5°\text{F}$$

$$= 64.4°\text{F}$$

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PROBLEM 10.42

For the bar of Prob. 10.41, determine the required distance d for which the bar will just make contact with point C when the temperature increases by 120°F.

PROBLEM 10.41 The steel bar AB has a $\frac{3}{8} \times \frac{3}{8}$-in. square cross section and is held by pins that are a fixed distance apart and are located at a distance $e = 0.03$ in. from the geometric axis of the bar. Knowing that at temperature T_0 the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point C if $d = 0.01$ in. Use $E = 29 \times 10^6$ psi and a coefficient of thermal expansion $\alpha = 6.5 \times 10^{-6}/°\text{F}$.

SOLUTION

$$A = \left(\frac{3}{8}\right)\left(\frac{3}{8}\right) = 0.140625 \text{ in}^2$$

$$I = \frac{1}{12}\left(\frac{3}{8}\right)^4 = 1.64795 \times 10^{-3} \text{ in}^4$$

$$EI = (29 \times 10^6)(1.64795 \times 10^{-3}) = 47{,}791 \text{ lb} \cdot \text{in}^2$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (47{,}791)}{(8)^2} = 7370 \text{ lb}$$

Calculate P from thermal analysis. To obtain an approximate value, neglect the effect of eccentricity in the thermal analysis.

$$\text{Total elongation} = \alpha L(\Delta T) - \frac{PL}{EA} = 0$$

$$P = EA\alpha(\Delta T) = (29 \times 10^6)(0.140625)(6.5 \times 10^{-6})(120) = 3181 \text{ lb}$$

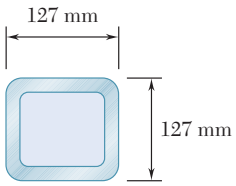
Calculate the deflection using the secant formula.

$$d = y_{\max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) - 1\right] = (0.03)\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{3181}{7370}}\right) - 1\right]$$

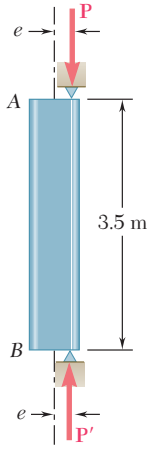
$$= (0.03)[\sec(1.03197) - 1] = (0.03)(0.94883) \qquad d = 0.0285 \text{ in.} \blacktriangleleft$$

For an improved thermal analysis including eccentricity, see solution of Prob. 10.41.

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$A = 3400 \text{ mm}^2$
$I = 7.93 \times 10^{-6} \text{ m}^4$
$r = 48.3 \text{ mm}$



PROBLEM 10.43

A 3.5-m-long steel tube having the cross section and properties shown is used as a column. For the grade of steel used $\sigma_Y = 250$ MPa and $E = 200$ GPa. Knowing that a factor of safety of 2.6 with respect to permanent deformation is required, determine the allowable load **P** when the eccentricity e is (*a*) 15 mm, (*b*) 7.5 mm. (*Hint*: Since the factor of safety must be applied to the load **P** not to the stress, use Fig. 10.24 to determine P_Y).

SOLUTION

$$A = 3400 \times 10^{-6} \text{ m}^2 \qquad r = 48.3 \times 10^{-3} \text{ m}$$

$$L_e = 3.5 \text{ m} \qquad \frac{L_e}{r} = \frac{3.5}{48.3 \times 10^{-3}} = 72.46$$

$$c = \frac{127}{2} = 63.5 \text{ mm}$$

(*a*) $e = 15$ mm. $\qquad \dfrac{ec}{r^2} = \dfrac{(15)(63.5)}{(48.3)^2} = 0.40829$

Using Fig. 10.23 with $L_e/r = 72.46$ and $ec/r^2 = 0.40829$,

$$P/A = 144.75 \text{ MPa} = 144.75 \times 10^6 \text{ Pa}$$

$$P = (144.75 \times 10^6)(3400 \times 10^{-6}) = 492 \times 10^3 \text{ N}$$

Using factor of safety, $\quad P_{\text{all}} = \dfrac{492 \times 10^3}{2.6} = 189.0 \times 10^3 \text{ N}$ $\qquad P_{\text{all}} = 189.0 \text{ kN}$ ◀

(*b*) $e = 7.5$ mm. $\qquad \dfrac{ec}{r^2} = \dfrac{(7.5)(63.5)}{(48.3)^2} = 0.20415$

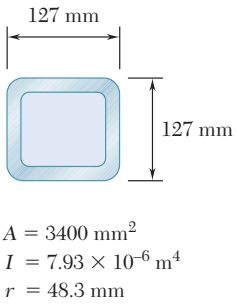
Using Fig. 10.23 with $L_e/r = 72.46$ and $ec/r^2 = 0.20415$,

$$P/A = 175.2 \text{ MPa} = 175.2 \times 10^6 \text{ Pa}$$

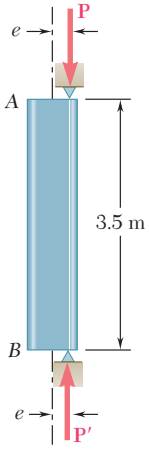
$$P = (175.2 \times 10^6)(3400 \times 10^{-6}) = 596 \times 10^3 \text{ N}$$

Using factor of safety, $\quad P_{\text{all}} = \dfrac{596 \times 10^3}{2.6} = 229 \times 10^3 \text{ N}$ $\qquad P_{\text{all}} = 229 \text{ kN}$ ◀

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$A = 3400 \text{ mm}^2$
$I = 7.93 \times 10^{-6} \text{ m}^4$
$r = 48.3 \text{ mm}$



PROBLEM 10.44

Solve Prob. 10.43, assuming that the length of the steel tube is increased to 5 m.

PROBLEM 10.43 A 3.5-m-long steel tube having the cross section and properties shown is used as a column. For the grade of steel used, $\sigma_Y = 250$ MPa and $E = 200$ GPa. Knowing that a factor of safety of 2.6 with respect to permanent deformation is required, determine the allowable load **P** when the eccentricity e is (*a*) 15 mm, (*b*) 7.5 mm. (*Hint*: Since the factor of safety must be applied to the load **P** not to the stress, use Fig. 10.24 to determine P_Y).

SOLUTION

$$A = 3400 \times 10^{-6} \text{ m}^2 \qquad r = 48.3 \times 10^{-3} \text{ m}$$

$$L_e = 5 \text{ m} \qquad \frac{L_e}{r} = \frac{5}{48.3 \times 10^{-3}} = 103.52$$

$$c = \frac{127}{2} = 63.5 \text{ mm}$$

(*a*) $e = 15$ mm. $\qquad \dfrac{ec}{r^2} = \dfrac{(15)(63.5)}{(48.3)^2} = 0.40829$

Using Fig. 10.23 with $\dfrac{L_e}{r} = 103.52$ and $\dfrac{ec}{r^2} = 0.40829$ gives $\dfrac{P}{A} = 112.75 \text{ MPa} = 112.75 \times 10^{-6} \text{ Pa}$

$$P = (112.75 \times 10^6)(3400 \times 10^{-6}) = 383 \times 10^3 \text{ N}$$

Using factor of safety, $\quad P_{all} = \dfrac{383 \times 10^3}{2.6} = 147.0 \times 10^3 \text{ N}$ $\qquad P_{all} = 147.0 \text{ kN} \blacktriangleleft$

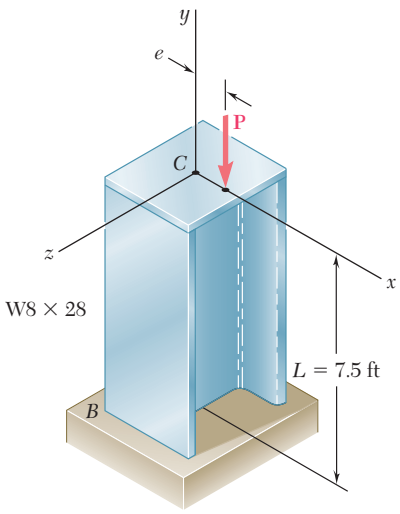
(*b*) $e = 7.5$ mm. $\qquad \dfrac{ec}{r^2} = \dfrac{(7.5)(63.5)}{(48.3)^2} = 0.20415$

Using Fig. 10.23 gives $\dfrac{P}{A} = 133.2 \text{ MPa} = 133.2 \times 10^6 \text{ Pa}$.

$$P = (133.2 \times 10^6)(3400 \times 10^{-6}) = 453 \times 10^3 \text{ N}$$

Using factor of safety, $\quad P_{all} = \dfrac{453 \times 10^3}{2.6} = 174.0 \times 10^3 \text{ N}$ $\qquad P_{all} = 174.0 \text{ kN} \blacktriangleleft$

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PROBLEM 10.45

An axial load **P** is applied to the W8 × 28 rolled-steel column *BC* that is free at its top *C* and fixed at its base *B*. Knowing that the eccentricity of the load is $e = 0.6$ in. and that for the grade of steel used $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi, determine (*a*) the magnitude of *P* of the allowable load when a factor of safety of 2.5 with respect to permanent deformation is required, (*b*) the ratio of the load found in part *a* to the magnitude of the allowable centric load for the column. (See hint of Prob. 10.43.)

SOLUTION

(*a*) $\sigma_Y = 36$ ksi

$E = 29 \times 10^3$ ksi

$e = 0.60$ in.

W8 × 28: $A = 8.24\ \text{in}^2$, $I_y = 21.7\ \text{in}^4$, $r_y = 1.62$ in.

$$e = \frac{1}{2}(6.54\ \text{in.}) = 3.27\ \text{in.}$$

$$L_e = 2L = 2(6\ \text{ft}) = 12\ \text{ft} = 144\ \text{in.}$$

$$\frac{L_e}{r} = \frac{144\ \text{in.}}{1.62\ \text{in.}} = 88.9$$

$$\frac{ec}{r^2} = \frac{(0.60\ \text{in.})(3.27\ \text{in.})}{(1.62\ \text{in.})^2} = 0.748$$

From Fig. 10.23, we read $P/A = 15$ ksi

$$\therefore \quad P = (15\ \text{ksi})(8.24\ \text{in}^2)$$

$$P = 123.6\ \text{kips}$$

with *F.S.* = 2.5: $P_{\text{all}} = \dfrac{123.6}{2.5}$

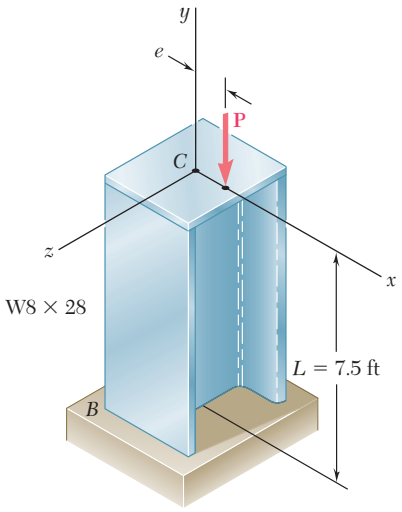
$$P_{\text{all}} = 49.4\ \text{kips} \blacktriangleleft$$

(*b*) $$P_{\text{cr}} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi(29 \times 10^3\ \text{ksi})(21.7\ \text{in}^4)}{(144\ \text{in.})^2} = 299.52\ \text{kips}$$

$$P_{\text{all}} = \frac{299.52}{2.5}\ \text{kips} = 119.808\ \text{kips}$$

Ratio: $\dfrac{49.6\ \text{kips}}{119.808\ \text{kips}} = 0.413$ $\qquad$ 0.413 ◀

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PROBLEM 10.46

An axial load **P** of magnitude 50 kips is applied at a point located on the x axis at a distance $e = 0.25$ in. from the geometric axis of the W8 × 28 rolled-steel column BC. Knowing that the column is free at its top C and fixed at its base B and that $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi, determine the factor of safety with respect to yield. (See hint of Prob. 10.43.)

SOLUTION

(*a*) $\sigma_Y = 36$ ksi W8 × 28: $A = 8.24\ \text{in}^2$, $I_y = 21.7\ \text{in}^4$, $r_y = 1.62$ in.

$E = 26 \times 10^3$ ksi $C = \frac{1}{2}(6.54\ \text{in.}) = 3.27$ in.

$e = 0.60$ in. $L_e = 2L = 2(6\ \text{ft}) = 12\ \text{ft} = 144$ in.

$$\frac{L_e}{r} = \frac{144\ \text{in.}}{1.62\ \text{in.}} = 88.9$$

$$\frac{ec}{r^2} = \frac{(0.25\ \text{in.})(3.27\ \text{in.})}{(1.62\ \text{in.})^2} = 0.312$$

From Fig. 10.23, we read $P/A = 20$ ksi

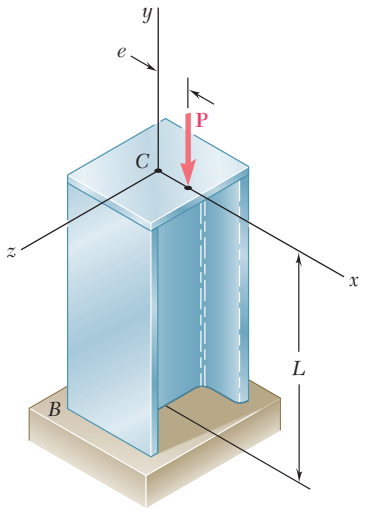
$$\therefore\ P_{\text{all}} = (20\ \text{ksi})(8.24\ \text{in}^2)$$
$$= 164.8\ \text{kips}$$

For $P = 50$ kips,

$$F.S. = \frac{164.8\ \text{kips}}{50\ \text{kips}}$$

$F.S. = 3.30$ ◀

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PROBLEM 10.47

A 100-kN axial load **P** is applied to the W150×18 rolled-steel column *BC* that is free at its top *C* and fixed at its base *B*. Knowing that the eccentricity of the load is $e = 6\text{mm}$, determine the largest permissible length *L* if the allowable stress in the column is 80 MPa. Use $E = 200$ GPa.

SOLUTION

W150×18:

$$A = 2290\ \text{mm}^2 = 2290\times10^{-6}\,\text{m}^2$$

$$b_f = 102\ \text{mm} \qquad c = \frac{b_f}{2} = 51\ \text{mm}$$

$$I_y = 1.24\times10^{6}\ \text{mm}^4 = 10^{-6}\,\text{m}^4 \qquad r_y = 23.3\ \text{mm}$$

$$\sigma_{\max} = 80\times10^{6}\,\text{Pa} \qquad P = 100\times10^{3}\,\text{N}$$

$$\sigma_{\max} = \frac{P}{A}\left[1+\frac{ec}{r_y^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right)\right] \qquad \frac{A\sigma_{\max}}{P}-1 = \frac{ec}{r_y^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r_y^2}{ec}\left(\frac{A\sigma_{\max}}{P}-1\right) = \frac{(23.3)^2}{(6)(51)}\left[\frac{(2290\times10^{-6})(80\times10^{6})}{100\times10^{3}}-1\right] = 1.47609$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) = 0.67746 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}} = 0.82649$$

$$\frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi}(0.82649)\right]^2 = 0.27684$$

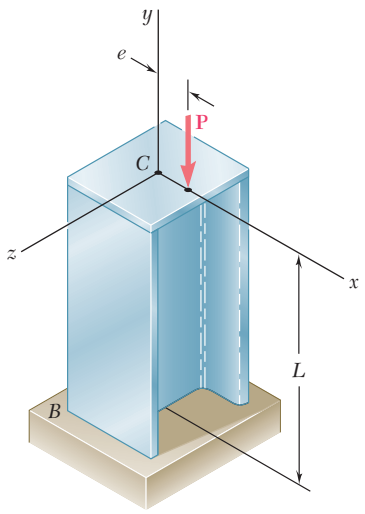
$$P_{\text{cr}} = \frac{P}{0.27684} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.27684\pi^2 EI}{P} = \frac{0.27684\pi^2(200\times10^{9})(1.29\times10^{-6})}{100\times10^{3}} = 6.7762\ \text{m}^2$$

$$L_e = 2.6031\ \text{m} = 2L$$

$$L = 1.302\ \text{m} \blacktriangleleft$$

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PROBLEM 10.48

A 26-kip axial load **P** is applied to a W6 × 12 rolled-steel column *BC* that is free at its top *C* and fixed at its base *B*. Knowing that the eccentricity of the load is $e = 0.25$ in., determine the largest permissible length *L* if the allowable stress in the column is 14 ksi. Use $E = 29 \times 10^6$ psi.

SOLUTION

Data: $P = 26$ kips, $e = 0.25$ in.

$E = 29 \times 10^6$ psi $= 29{,}000$ ksi

W6 × 12: $A = 3.55\ \text{in}^2$ $b_f = 4.000$ in.

$$c = \frac{b_f}{2} = 2.000 \text{ in.}, \qquad I_y = 2.99\ \text{in}^4, \qquad r_y = 0.918 \text{ in.}$$

$$\sigma_{\max} = 14 \text{ ksi}$$

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right)\right]$$

$$\frac{A\sigma_{\max}}{P} - 1 = \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{r^2}{ec}\left(\frac{A\sigma_{\max}}{P} - 1\right) = \frac{(0.918)^2}{(0.25)(2.000)}\left[\frac{(3.55)(14)}{26} - 1\right] = 1.53635$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) = 0.65089 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}} = 0.86204$$

$$\frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi}(0.86204)\right]^2 = 0.30117$$

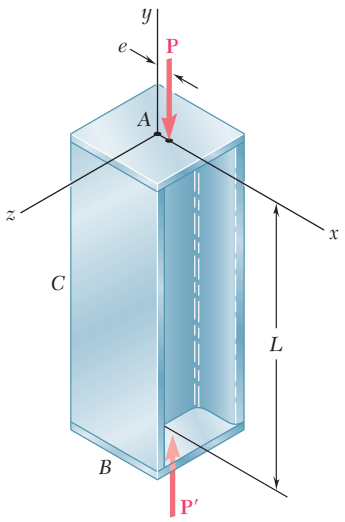
$$P_{\text{cr}} = \frac{P}{0.30117} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.30117\pi^2 EI}{P} = \frac{0.30117\pi^2(29{,}000)(2.99)}{26} = 9.913 \times 10^3\ \text{in}^2$$

$$L_e = 99.56 \text{ in.} = 2L$$

$$L = 49.78 \text{ in.} = 4.15 \text{ ft} \blacktriangleleft$$

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PROBLEM 10.49

Axial loads of magnitude $P = 135$ kips are applied parallel to the geometric axis of the W10 × 54 rolled-steel column *AB* and intersect the *x* axis at a distance *e* from the geometric axis. Knowing that $\sigma_{all} = 12$ ksi and $E = 29 \times 10^6$ psi, determine the largest permissible length *L* when (*a*) $e = 0.25$ in., (*b*) $e = 0.5$ in.

SOLUTION

Data: $P = 135$ kips $\quad E = 29 \times 10^3$ ksi

W10 × 54: $A = 15.8 \text{ in}^2, \quad c = \dfrac{b_f}{2} = 5.00$ in.

$$I_y = 103 \text{ in}^4 \qquad r_y = 2.56 \text{ in.}$$

$$\sigma_{all} = \sigma_{max} = 12 \text{ ksi}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec}\left[\frac{A\sigma_{max}}{P} - 1\right]$$

(*a*) $e = 0.25$ in.

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(2.56)^2}{(0.25)(5.00)}\left[\frac{(15.8)(12)}{135} - 1\right] = 2.1205$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.4716 \quad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 1.0797$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(1.0737)\right]^2 = 0.47246$$

$$P_{cr} = \frac{P}{0.47246} = \frac{\pi^2 EI}{L_e^2}$$

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PROBLEM 10.49 *(Continued)*

$$L_e^2 = \frac{0.47246\pi^2 EI}{P} = \frac{0.47246\pi^2(29\times10^3)(103)}{135} = 103.172\times10^3 \text{ in}^2$$

$L_e = 321.2$ in. $\qquad L = L_e = 321.2$ in. $\qquad L = 26.8$ ft ◀

(*b*) $\underline{e = 0.5 \text{ in.}}$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(2.56)^2}{(0.5)(5.00)}\left[\frac{(15.8)(12)}{135} - 1\right] = 1.06023$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.94319 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 0.33868$$

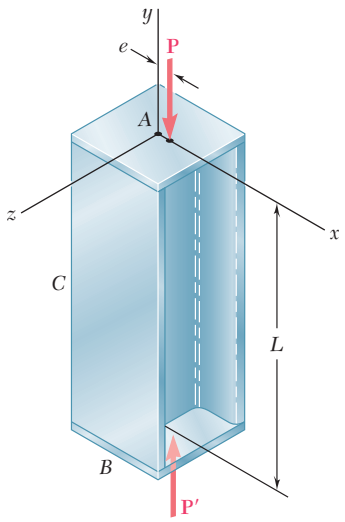
$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(0.33868)\right]^2 = 0.046488$$

$$P_{cr} = \frac{P}{0.046488} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.046488\pi^2 EI}{P} = \frac{0.046488\pi^2(29\times10^3)(103)}{135} = 10.152\times10^3 \text{ in}^2$$

$L_e = 100.8$ in. $\qquad L = L_e = 100.8$ in. $\qquad L = 8.40$ ft ◀

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PROBLEM 10.50

Axial loads of magnitude $P = 84$ kN are applied parallel to the geometric axis of the W200 × 22.5 rolled-steel column AB and intersect the x axis at a distance e from the geometric axis. Knowing that $\sigma_{all} = 75$ MPa and $E = 200$ GPa, determine the largest permissible length L when (*a*) $e = 5$ mm, (*b*) $e = 12$ mm.

SOLUTION

Data: $P = 84 \times 10^3$ N $\quad E = 200 \times 10^9$ Pa

W200 × 22.5: $A = 2860\ \text{mm}^2 = 2860 \times 10^{-6}\ \text{m}^2$

$$b_f = 102\ \text{mm} \quad c = \frac{b_f}{2} = 51\ \text{mm} \quad r_y = 22.3\ \text{mm}$$

$$I_y = 1.42 \times 10^6\ \text{mm}^4 = 1.42 \times 10^{-6}\ \text{m}^4$$

$$\sigma_{all} = \sigma_{max} = 75\ \text{MPa} = 75 \times 10^6\ \text{Pa}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec}\left(\frac{A\sigma_{max}}{P} - 1\right)$$

(*a*) $e = 5$ mm:

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(22.3)^2}{(5)(51)}\left[\frac{(2860 \times 10^{-6})(75 \times 10^6)}{84 \times 10^3} - 1\right] = 3.0297$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.33006 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 1.2344$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(1.2344)\right]^2 = 0.61757$$

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PROBLEM 10.50 *(Continued)*

$$P_{\text{cr}} = \frac{P}{0.61757} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.61757\pi^2 EI}{P} = \frac{0.61757\pi^2(200\times10^9)(1.42\times10^{-6})}{84\times10^3} = 20.61\,\text{m}^2$$

$$L_e = 4.54\,\text{m} \qquad L = L_e = 4.54\,\text{m} \blacktriangleleft$$

(*b*) $e = 12\,\text{mm}$:

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{(22.3)^2}{(12)(51)}\left[\frac{(2860\times10^{-6})(75\times10^6)}{84\times10^3} - 1\right] = 1.26238$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) = 0.79216 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}} = 0.65646$$

$$\frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi}(0.65646)\right]^2 = 0.17466$$

$$P_{\text{cr}} = \frac{P}{0.17466} = \frac{\pi^2 EI}{L_e^2}$$

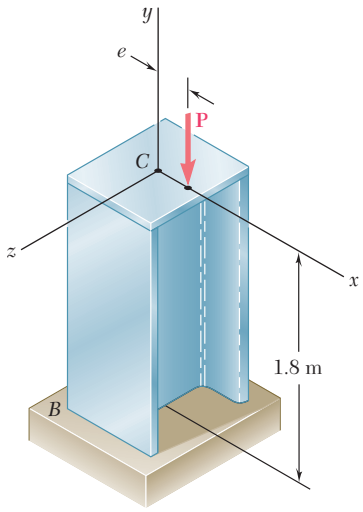
$$L_e^2 = \frac{0.17466\pi^2 EI}{P}$$

$$= \frac{0.17466\pi^2(200\times10^9)(1.42\times10^{-6})}{84\times10^3}$$

$$= 5.828\,\text{m}^2$$

$$L_e = 2.41\,\text{m} \qquad L = L_e = 2.41\,\text{m} \blacktriangleleft$$

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PROBLEM 10.51

An axial load of magnitude $P = 220$ kN is applied at a point located on the x axis at a distance $e = 6$ mm from the geometric axis of the wide-flange column BC. Knowing that $E = 200$ GPa, choose the lightest W200 shape that can be used if $\sigma_{all} = 120$ MPa.

SOLUTION

$$P = 220 \times 10^3 \text{ N} \qquad L = 1.8 \text{ m} \qquad L_e = 2L = 3.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI_y}{L_e^2} = \frac{\pi^2 (200 \times 10^9) I_y}{3.6^2} = 152.3 \times 10^9 I_y \text{ N}$$

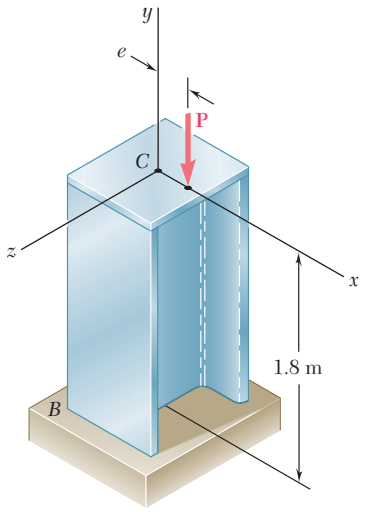
$$e = 6 \text{ mm} \qquad c = \frac{b_f}{2} \qquad \frac{ec}{r^2} = \frac{eb_f}{2r_y^2}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

Shape	$A(10^{-6}\text{m}^2)$	b_f (mm)	I_y (10^{-6}m^4)	r_y (mm)	P_{cr} (kN)	$\frac{ec}{r^2}$	σ_{max} (MPa)	
W200 × 41.7	5320	166	9.03	41.1	137.5	0.2948	56.4	
W200 × 26.6	3390	133	3.32	31.2	505.7	0.4099	117.1	←
W200 × 22.5	2860	102	1.42	22.3	216.3			

$\sigma_{max} = 117.1 \text{ MPa} < 120 \text{ MPa}$ Use W200 × 26.6. ◀

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PROBLEM 10.52

Solve Prob. 10.51, assuming that the magnitude of the axial load is $P = 345$ kN.

PROBLEM 10.51 An axial load of magnitude $P = 220$ kN is applied at a point located on the x axis at a distance $e = 6$ mm from the geometric axis of the wide-flange column BC. Knowing that $E = 200$ GPa, choose the lightest W200 shape that can be used if $\sigma_{all} = 120$ MPa.

SOLUTION

$$P = 345 \times 10^3 \text{ N} \qquad L = 1.8 \text{ m} \qquad L_e = 2L = 3.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 E I_y}{L_e^2} = \frac{\pi^2 (200 \times 10^9) I_y}{(3.6)^2} = 152.3 \times 10^9 I_y \text{ N}$$

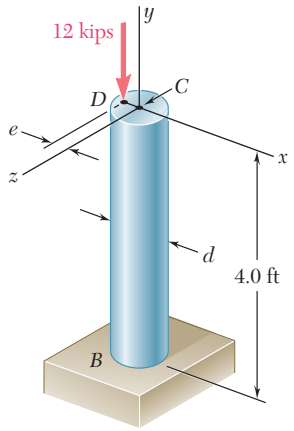
$$e = 6 \text{ mm} \qquad c = \frac{b_f}{2} \qquad \frac{ec}{r^2} = \frac{eb_f}{2r_y^2}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

Shape	$A(10^{-6}\text{m}^2)$	b_f (mm)	$I_y(10^{-6}\text{m}^4)$	r_y (mm)	P_{cr} (kN)	$\frac{ec}{r^2}$	σ_{max} (MPa)	
W200 × 41.7	5320	166	9.03	41.1	1375	0.2948	91.9	
W200 × 26.6	3390	133	3.32	31.2	505.7	0.4099	256	
W200 × 35.9	4570	165	7.62	40.9	1161	0.2959	109.6	←
W200 × 31.3	3970	134	4.07	32.0	619.3	0.3926	174.7	

$\sigma_{max} = 109.6 \text{ MPa} < 120 \text{ MPa}$ Use W200 × 35.9. ◀

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PROBLEM 10.53

A 12-kip axial load is applied with an eccentricity $e = 0.375$ in. to the circular steel rod BC that is free at its top C and fixed at its base B. Knowing that the stock of rods available for use have diameters in increments of $\frac{1}{8}$ in. from 1.5 in. to 3.0 in., determine the lightest rod that can be used if $\sigma_{all} = 15$ ksi. Use $E = 29 \times 10^6$ psi.

SOLUTION

$$E = 29 \times 10^6 \text{ psi} = 29{,}000 \text{ ksi} \qquad d = \text{diameter (in.)}$$

$$A = \frac{\pi}{4} d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

$$c = \frac{1}{2} d \qquad e = 0.375 \text{ in.}$$

$$L = 4.0 \text{ ft} = 48 \text{ in.}$$

$$L_e = 2L = 96 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29{,}000)\pi d^4}{(64)(96)^2} = 1.52449 d^4 \text{ kips}$$

$$r^2 = \frac{I}{A} = \frac{\pi d^4}{64} \cdot \frac{4}{\pi d^2} = \frac{d^2}{16} \qquad P = 12 \text{ kips}$$

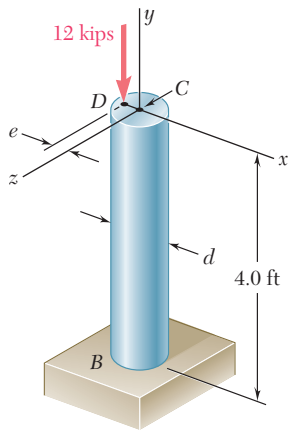
$$\frac{ec}{r^2} = \frac{(0.375)\left(\frac{1}{2} d\right)}{\frac{1}{16} d^2} = \frac{3}{d}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

d (in.)	A (in^2)	P_{cr} (kips)	ec/r^2	σ_{max} (ksi)	
2.25	3.976	39.07	1.3333	9.26	
2.0	3.1416	24.39	1.5	16.49	
2.125	3.546	31.09	1.4118	11.90	←

$\sigma_{max} = 11.90 \text{ ksi} < 15 \text{ ksi}$ Use $d = 2.125$ in. ◀

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PROBLEM 10.54

Solve Prob. 10.53, assuming that the 12-kip axial load will be applied to the rod with an eccentricity $e = \frac{1}{2}d$.

PROBLEM 10.53 A 12-kip axial load is applied with an eccentricity $e = 0.375$ in. to the circular steel rod *BC* that is free at its top *C* and fixed at its base *B*. Knowing that the stock of rods available for use have diameters in increments of $\frac{1}{8}$ in. from 1.5 in. to 3.0 in., determine the lightest rod that can be used if $\sigma_{all} = 15$ ksi. Use $E = 29 \times 10^6$ psi.

SOLUTION

$$E = 29 \times 10^6 \text{ psi} = 29{,}000 \text{ ksi} \qquad d = \text{diameter (in.)}$$

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^2 = \frac{\pi}{64}d^4$$

$$c = \frac{1}{2}d \qquad e = \frac{1}{2}d$$

$$L = 4 \text{ ft} = 48 \text{ in.}$$

$$L_e = 2L = 96 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29{,}000)(\pi d^4)}{(64)(96)^2} = 1.52449 d^4$$

$$r^2 = \frac{I}{A} = \frac{\pi d^4}{64} \cdot \frac{4}{\pi d^2} = \frac{1}{16}d^2 \qquad P = 12 \text{ kips}$$

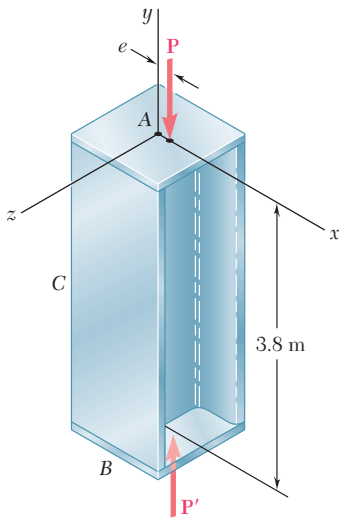
$$\frac{ec}{r^2} = \frac{\left(\frac{1}{2}d\right)\left(\frac{1}{2}d\right)}{\frac{1}{16}d^2} = 4.0$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right] = \frac{P}{A}\left[1 + 4.0\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

d (in.)	A (in^2)	P_{cr} (kips)	σ_{max} (ksi)
2.25	3.976	39.07	21.75
3.0	7.068	123.48	9.39
2.5	4.909	59.55	15.28
2.625	5.412	72.38	13.27

$\sigma_{max} = 13.27 \text{ ksi} < 15 \text{ ksi}$ Use $d = 2.625$ in. ◀

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PROBLEM 10.55

Axial loads of magnitude $P = 175$ kN are applied parallel to the geometric axis of a W250 × 44.8 rolled-steel column AB and intersect the axis at a distance $e = 12$ mm from its geometric axis. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine the factor of safety with respect to yield. (*Hint:* Since the factor of safety must be applied to the load **P,** not to the stresses, use Fig. 10.24 to determine P_Y.)

SOLUTION

For W250 × 44.8,

$$A = 5700 \text{ mm}^2 \qquad r_y = 34.8 \text{ mm}$$

$$L_e = 3800 \text{ mm} \qquad L_e/r = 108.26$$

$$c = \frac{b_f}{2} = \frac{148}{2} = 74 \text{ mm} \qquad e = 12 \text{ mm}$$

$$\frac{ec}{r^2} = \frac{(12)(74)}{(34.8)^2} = 0.73325$$

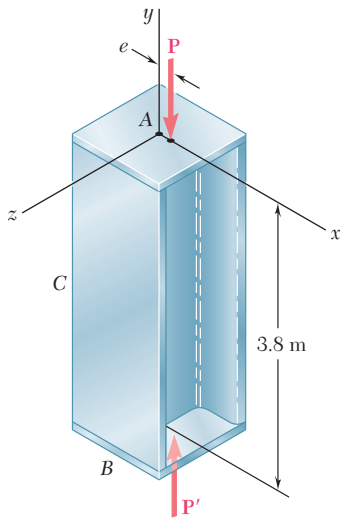
Using Fig 10.24 with $L_e/r = 108.26$ and $\dfrac{ec}{r^2} = 0.73325$,

$$P_Y/A = 93 \text{ MPa} = 93 \times 10^6 \text{ N/m}^2$$

$$P_Y = AP_Y/A = (5700 \times 10^{-6})(93 \times 10^6) = 530 \times 10^3 \text{ N} = 530 \text{ kN}$$

$$F.S. = \frac{P_Y}{P} = \frac{530}{175} \qquad F.S. = 3.02 \blacktriangleleft$$

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PROBLEM 10.56

Solve Prob. 10.55, assuming that $e = 0.16$ mm and $P = 155$ kN.

PROBLEM 10.55 Axial loads of magnitude $P = 175$ kN are applied parallel to the geometric axis of a $W250 \times 44.8$ rolled-steel column AB and intersect the axis at a distance $e = 12$ mm from its geometric axis. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine the factor of safety with respect to yield. (*Hint:* Since the factor of safety must be applied to the load **P,** not to the stresses, use Fig. 10.24 to determine P_Y.)

SOLUTION

For W250 × 44.8,

$$A = 5700 \text{ mm}^2 \qquad r_y = 34.8 \text{ mm}$$

$$L_e = 3800 \text{ mm} \qquad L_e/r = 108.26$$

$$c = \frac{b_f}{2} = \frac{148}{2} = 74 \text{ mm} \qquad e = 16 \text{ mm}$$

$$\frac{ec}{r^2} = \frac{(16)(74)}{(34.8)^2} = 0.97767$$

Using Fig 10.24 with $L_e/r = 108.26$ and $\dfrac{ec}{r^2} = 0.97767$,

$$P_Y/A = 84 \text{ MPa} = 84 \times 10^6 \text{ N/m}^2$$

$$P_Y = A(P_Y/A) = (5700 \times 10^{-6})(84 \times 10^6) = 479 \times 10^3 \text{ N} = 479 \text{ kN}$$

$$F.S. = \frac{P_Y}{P} = \frac{479}{155} \qquad F.S. = 3.09 \blacktriangleleft$$

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PROBLEM 10.57

Using allowable stress design determine the allowable centric load for a column of 6-m effective length that is made from the following rolled-steel shape: (*a*) W200 × 35.9, (*b*) W200 × 86. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa..

SOLUTION

Steel: $\sigma_Y = 250$ MPa $E = 200 \times 10^3$ MPa

Transition L/r: $4.71\sqrt{\dfrac{E}{\sigma_Y}} = 133.22$

(*a*) W200 × 35.9: $A = 4570 \times 10^{-6}\ \text{m}^2$ $r_y = 40.9 \times 10^{-3}\ \text{m}$

$$\frac{L_e}{r_y} = \frac{6}{40.9 \times 10^{-3}} = 146.70 > 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r_y)^2} = \frac{\pi^2(200 \times 10^3)}{(146.70)^2} = 91.72 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{F.S.} = \frac{(0.877)(91.72)}{1.67} = 48.17 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (48.17 \times 10^6)(4570 \times 10^{-6}) \qquad P_{\text{all}} = 220 \text{ kN} \blacktriangleleft$$

(*b*) W200 × 86: $A = 11{,}000 \times 10^{-6}\ \text{m}^2$ $r_y = 53.3 \times 10^{-3}\ \text{m}$

$$\frac{L_e}{r_y} = \frac{6}{53.3 \times 10^{-3}} = 112.57 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(200 \times 10^3)}{(112.57)^2} = 155.77 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{F.S.} = \frac{1}{1.67}[0.658^{250/155.77}](250) = 76.47 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (76.47 \times 10^6)(11{,}000 \times 10^{-6}) \qquad P_{\text{all}} = 841 \text{ kN} \blacktriangleleft$$

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PROBLEM 10.58

A $W8 \times 31$ rolled-steel shape is used for a column of 21-ft effective length. Using allowable stress design, determine the allowable centric load if the yield strength of the grade of steel used is (*a*) $\sigma_Y = 36$ ksi, (*b*) $\sigma_Y = 50$ ksi. Use $E = 29 \times 10^6$ psi.

SOLUTION

Steel: $E = 29{,}000$ ksi W8 × 31: $A = 9.12 \text{ in}^2$ $r_{\min} = 2.02$ in.

$$L_e = 21 \text{ ft} = 252 \text{ in.} \qquad L_e/r = 124.75$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(29{,}000)}{(124.75)^2} = 18.391 \text{ ksi}$$

(*a*) $\underline{\sigma_Y = 36 \text{ ksi.}}$ Transition (L/r): $4.71\sqrt{\dfrac{E}{\sigma_Y}} = 133.68 > 124.75$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{F.S.} = \frac{1}{1.67}[0.658^{36/18.391}](36) = 9.501 \text{ ksi}$$

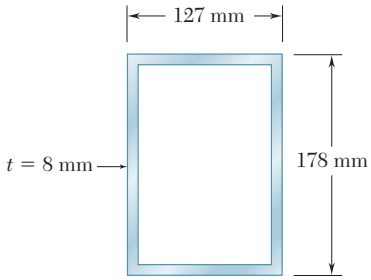
$$P_{\text{all}} = \sigma_{\text{all}} A = (9.501)(9.12) \qquad P_{\text{all}} = 86.6 \text{ kips} \blacktriangleleft$$

(*b*) $\underline{\sigma_Y = 50 \text{ ksi.}}$ Transition (L/r): $4.71\sqrt{\dfrac{E}{\sigma_Y}} = 113.43 < 124.75$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{F.S.} = \frac{(0.877)(18.391)}{1.67} = 9.658 \text{ ksi}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (9.658)(9.12) \qquad P_{\text{all}} = 88.1 \text{ kips} \blacktriangleleft$$

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PROBLEM 10.59

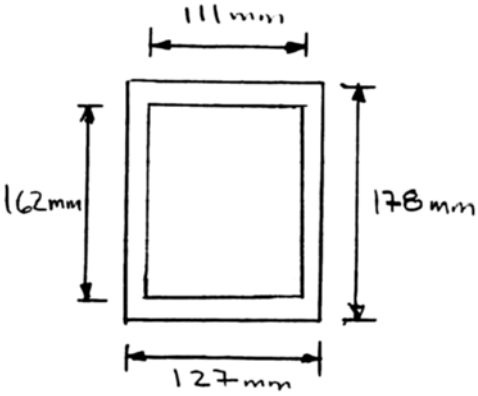
A rectangular structural tube having the cross section shown is used as a column of 5-m effective length. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, use allowable stress design to determine the largest centric load that can be applied to the steel column.

SOLUTION

$$A = (0.178)(0.127) - (0.162)(0.111)$$
$$= 4.624 \times 10^{-3} \text{ m}^2$$

$$I_y = \frac{1}{12}\left[(0.178)(0.127)^3 - (0.162)(0.111)^3\right]$$
$$= 11.9213 \times 10^{-6} \text{ m}^4$$

$$r = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{11.9213 \times 10^{-6}}{4.624 \times 10^{-3}}} = 50.775 \times 10^{-3} \text{ m}$$



$L_e = 5$ m

$$\frac{L_e}{r} = \frac{5}{50.775 \times 10^{-3}} = 98.473$$

$$\frac{L}{r} = 4.71\sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.219$$ Eq. (10.38)

$$\sigma_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2(200 \times 10^9)}{(98.473)^2} = 203.56 \text{ MPa}$$

$$\sigma_{\text{cr}} = \left[0.658^{(\sigma_y/\sigma_e)}\right]\sigma_y$$
$$= \left[0.658^{(250/203.56)}\right]250 \times 10^6 = 149.519 \text{ MPa}$$

$$P_{\text{all}} = A\frac{\sigma_{\text{cr}}}{1.67} = (4.624 \times 10^{-3})\frac{149.519 \times 10^6}{1.67} = 414 \text{ kN}$$ ◀

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PROBLEM 10.60

A column having a 3.5-m effective length is made of sawn lumber with a 114×140-mm cross section. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_C = 7.6$ MPa and the adjusted modulus is $E = 2.8$ GPa, determine the maximum allowable centric load for the column.

SOLUTION

Sawn lumber: $c = 0.8 \qquad \sigma_C = 7.6 \text{ MPa} \qquad E = 2800 \text{ MPa}$

$$A = (114)(140) = 15{,}960 \text{ mm}^2$$
$$= 15{,}960 \times 10^{-6} \text{ m}^2$$
$$d = 114 \text{ mm} = 114 \times 10^{-3} \text{ m}$$
$$L/d = 3.5/114 \times 10^{-3} = 30.70$$
$$\sigma_{CE} = \frac{0.822E}{(L/d)^2} = \frac{(0.822)(2800)}{(30.70)^2} = 2.442 \text{ MPa}$$
$$\frac{\sigma_{CE}}{\sigma_C} = 0.32132$$
$$u = \frac{1 + \sigma_{CE}/\sigma_C}{2c} = \frac{1 + 0.32132}{(2)(0.8)} = 0.82583$$
$$v = \frac{\sigma_{CE}/\sigma_C}{c} = 0.40165$$
$$C_P = u - \sqrt{u^2 - v} = 0.29635$$
$$\sigma_{\text{all}} = C_P \sigma_C = (0.29635)(7.6) = 2.252 \text{ MPa}$$
$$P_{\text{all}} = \sigma_{\text{all}} A = (2.252 \times 10^6)(15{,}960 \times 10^{-6}) \qquad P_{\text{all}} = 35.9 \text{ kN} \blacktriangleleft$$

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PROBLEM 10.61

A sawn lumber column with a 7.5×5.5-in. cross section has an 18-ft effective length. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_C = 1200$ psi and that the adjusted modulus is $E = 470 \times 10^3$ psi, determine the maximum allowable centric load for the column.

SOLUTION

Sawn lumber:

$$c = 0.8 \qquad \sigma_C = 1200 \text{ psi} \qquad E = 470 \times 10^3 \text{ psi}$$

$$A = (7.5)(5.5) = 41.25 \text{ in}^2 \qquad d = 5.5 \text{ in.} \qquad L = 18 \text{ ft} = 216 \text{ in.}$$

$$L/d = 216/5.5 = 39.273$$

$$\sigma_{CE} = \frac{0.822E}{(L/d)^2} = \frac{(0.822)(470 \times 10^3)}{(39.273)^2} = 250.49 \text{ psi}$$

$$\sigma_{CE}/\sigma_C = 0.20874$$

$$u = \frac{1 + \sigma_{CE}/\sigma_C}{2c} = \frac{1.20874}{(2)(0.8)} = 0.75546$$

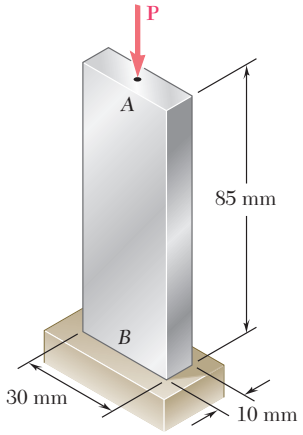
$$v = \frac{\sigma_{CE}/\sigma_C}{c} = 0.26093$$

$$C_P = u - \sqrt{u^2 - v} = 0.19887$$

$$\sigma_{\text{all}} = C_P \sigma_C = (0.19877)(1200) = 238.6 \text{ psi}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (238.6)(41.25) = 9.84 \times 10^3 \text{ lb} \qquad P_{\text{all}} = 9.84 \text{ kips} \blacktriangleleft$$

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PROBLEM 10.62

Bar *AB* is free at its end *A* and fixed at its base *B*. Determine the allowable centric load **P** if the aluminum alloy is (*a*) 6061-T6, (*b*) 2014-T6.

SOLUTION

$$A = (30)(10) = 300 \text{ mm}^2 = 300 \times 10^{-6} \text{ m}^2$$

$$I_{\min} = \frac{1}{12}(30)(10)^3 = 2.50 \times 10^3 \text{ mm}^4$$

$$r_{\min} = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.50 \times 10^3}{300}} = 2.887 \text{ mm}$$

$$L_e = 2L = (2)(85) = 170 \text{ mm} \qquad \frac{L_e}{r_{\min}} = 58.88$$

(*a*) 6061-T6: $L/r < 66$

$$\sigma_{\text{all}} = 140 - 0.874(L/r) = 140 - (0.874)(58.88)$$
$$= 88.53 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (88.53 \times 10^6)(300 \times 10^{-6}) = 26.6 \times 10^3 \text{ N}$$

Allowable centric load. $P_{\text{all}} = 26.6 \text{ kN}$ ◀

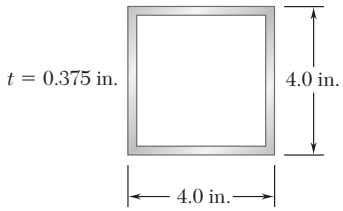
(*b*) 2014-T6: $L/r > 55$

$$\sigma_{\text{all}} = \frac{382 \times 10^3}{(L/r)^2} = \frac{382 \times 10^3}{(58.88)^2} = 110.15 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (110.15 \times 10^6)(300 \times 10^{-6}) = 33.0 \times 10^3 \text{ N}$$

Allowable centric load. $P_{\text{all}} = 33.0 \text{ kN}$ ◀

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PROBLEM 10.63

A compression member has the cross section shown and an effective length of 5 ft. Knowing that the aluminum alloy used is 2014-T6, determine the allowable centric load.

SOLUTION

$$b_o = 4.0 \text{ in.} \qquad b_i = b_o - 2t = 3.25 \text{ in.}$$

$$A = (4.0)^2 - (3.25)^2 = 5.4375 \text{ in}^2$$

$$I = \frac{1}{12}[(4.0)^4 - (3.25)^4] = 12.036 \text{ in}^4$$

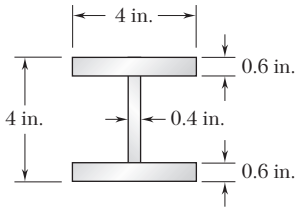
$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{12.036}{5.4375}} = 1.488 \text{ in.} \qquad L_e = 5 \text{ ft} = 60 \text{ in.}$$

$$\frac{L}{r} = \frac{60}{1.488} = 40.33 < 55 \qquad \text{for 2014-T6 aluminum alloy.}$$

$$\sigma_{all} = 30.9 - 0.229(L/r) = 30.9 - (0.229)(40.33) = 21.66 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (21.66)(5.4375) \qquad P_{all} = 117.8 \text{ kips} \blacktriangleleft$$

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PROBLEM 10.64

A compression member has the cross section shown and an effective length of 5 ft. Knowing that the aluminum alloy used is 6061-T6, determine the allowable centric load.

SOLUTION

$$I_x = \frac{1}{12}(0.4)(2.8)^3 + 2\left[\frac{1}{12}(4)(0.6)^3 + (4)(0.6)(1.7)^2\right]$$

$$= 14.7477 \text{ in}^4$$

$$I_y = \frac{1}{12}(2.8)(0.4)^3 + 2\left[\frac{1}{12}(0.6) + (4)^3\right]$$

$$= 6.4149 \text{ in}^4 = I_{\min}$$

$$A = (0.4)(2.8) + (2)(4)(0.6) = 5.92 \text{ in}^2$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{6.4149}{5.92}} = 1.04096 \text{ in.}$$

$$L = 5 \text{ ft} = 60 \text{ in.} \qquad \frac{L}{r} = \frac{60}{1.04096} = 57.639$$

6061-T6 aluminum alloy with $L/r < 66$:

Using Eq. 10.43, we have

$$\sigma_{\text{all}} = 20.3 - (0.127)(57.634) = 12.9799 \text{ ksi}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (12.9799)(5.92) \qquad P_{\text{all}} = 76.8 \text{ kips} \blacktriangleleft$$

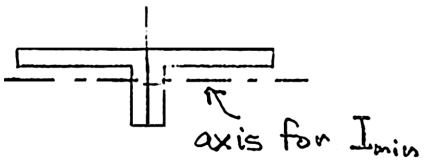
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PROBLEM 10.65



A compression member of 8.2-ft effective length is obtained by bolting together two $L5 \times 3 \times \frac{1}{2}$-in. steel angles as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION



For one $5 \times 3 \times \frac{1}{2}$-in. steel angle, Appendix *C* gives

$$A = 3.75 \text{ in}^2$$

$$I_x = 9.43 \text{ in}^4, \quad r_x = 1.58 \text{ in.}, \quad y = 1.74 \text{ in.}$$

$$I_y = 2.55 \text{ in}^4, \quad r_y = 0.824 \text{ in.}, \quad x = 0.746 \text{ in.}$$

For the column made using the two angles as shown,

$$A = (2)(3.75) = 7.50 \text{ in}^2, \quad I_{\min} = (2)(2.55) = 5.10 \text{ in}^4$$

$$r = 0.824 \text{ in.} \quad L = 8.2 \text{ ft} = 98.4 \text{ in.} \quad \frac{L}{r} = \frac{98.4}{0.824} = 119.42$$

Steel: $E = 29{,}000$ ksi, $\sigma_Y = 36$ ksi

Transition: $$\frac{L}{r} = 4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{29{,}000}{36}} = 133.68$$

$$\sigma_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2(29{,}000)}{(119.42)^2} = 20.070 \text{ ksi}$$

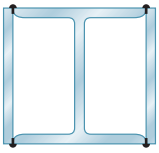
$$\sigma_{\text{cr}} = [0.658^{\sigma_Y/\sigma_e}]\sigma_Y = [0.658^{(36/20.070)}](36) = 16.992 \text{ ksi}$$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{1.67} = \frac{16.992}{1.67} = 10.175 \text{ ksi}$$

Allowable centric load: $P_{\text{all}} = \sigma_{\text{all}} A$

$$P_{\text{all}} = (10.175)(7.50) \qquad P_{\text{all}} = 76.3 \text{ kips} \blacktriangleleft$$

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PROBLEM 10.66

A compression member of 9-m effective length is obtained by welding two 10-mm-thick steel plates to a $W250 \times 80$ rolled-steel shape as shown. Knowing that $\sigma_Y = 345$ MPa and $E = 200$ GPa and using allowable stress design, determine the allowable centric load for the compression member.

SOLUTION

For $W250 \times 80$, $A = 10{,}200 \text{ mm}^2$, $d = 257$ mm, $b_f = 254$ mm

$I_x = 126 \times 10^6 \text{ mm}^4$, $I_y = 42.9 \times 10^6 \text{ mm}^4$

For one plate, $A = (257)(10) = 2570 \text{ mm}^4$

$$I_x = \frac{1}{12}(10)(257)^3 = 14.145 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{1}{12}(257)(10)^3 + (2570)\left(\frac{254}{2} + \frac{10}{2}\right)^2 = 44.801 \times 10^6 \text{ mm}^4$$

For column, $A = 10{,}200 + (2)(2570) = 15.34 \times 10^3 \text{ mm}^2 = 15.34 \times 10^{-3} \text{ m}^2$

$I_x = 126 \times 10^6 + (2)(14.145 \times 10^6) = 154.29 \times 10^6 \text{ mm}^4$

$I_y = 42.9 \times 10^6 + (2)(44.801 \times 10^6) = 132.50 \times 10^6 \text{ mm}^4 = I_{\min}$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{132.50 \times 10^6}{15.34 \times 10^3}} = 92.938 \text{ mm} = 92.938 \times 10^{-3} \text{m}$$

$$\frac{L_e}{r} = \frac{9}{92.938 \times 10^{-3}} = 96.838$$

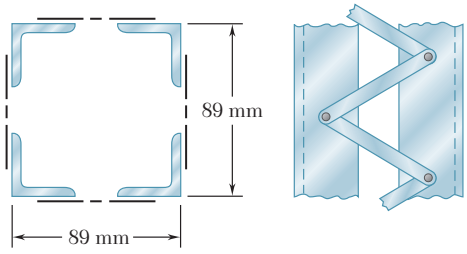
Steel: Transition L/r: $4.71\sqrt{\dfrac{E}{\sigma_Y}} = 4.71\sqrt{\dfrac{200 \times 10^9}{345 \times 10^6}} = 113.40 > 96.838$

$$\sigma_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(96.838)^2} = 210.49 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{F.S.} = \frac{1}{1.67}[0.658^{345/210.49}](345) = 104.03 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (104.03 \times 10^6)(15.34 \times 10^{-3}) \qquad P_{\text{all}} = 1596 \text{ kN} \blacktriangleleft$$

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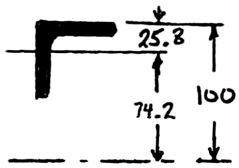


PROBLEM 10.67

A column of 6.4-m effective length is obtained by connecting four L89 × 89 × 9.5-mm steel angles with lacing bars as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_Y = 345$ MPa and $E = 200$ GPa.

SOLUTION

89 × 89 × 9.5 mm angle:



$$A_L = 1600 \text{ mm}^2$$

$$x = 25.4 \text{ mm}$$

$$I_x = 1.19 \times 10^6 \text{ mm}^2$$

$$d = 100 - x = 74.2 \text{ mm}$$

$$I = 4(Ad^2 + I_x) = 4[(1600)(74.2)^2 + 1.19 \times 10^6]$$

$$= 39.996 \times 10^6 \text{ mm}^4$$

$$A = 4A_L = 6400 \text{ mm}^2 = 6400 \times 10^{-6} \text{ m}^2$$

$$r = \sqrt{\frac{I}{A}} = 79.053 \text{ mm} = 79.053 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{6.4}{79.053 \times 10^{-3}} = 80.958$$

$$\frac{L}{r} = 4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.4 > 80.958$$

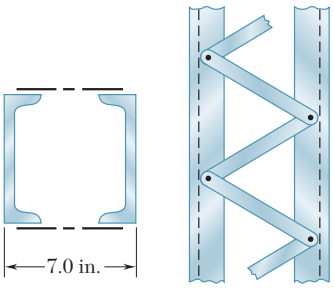
$$\sigma_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2(200 \times 10^9)}{(80.958)^2} = 301.17 \text{ MPa}$$

$$\sigma_{cr} = [0.658^{(\sigma_Y/\sigma_e)}]\sigma_Y = [0.658^{(345/301.17)}](345 \times 10^6) = 213.59$$

$$\sigma_{all} = \frac{\sigma_{cr}}{1.67} = \frac{213.59}{1.67} = 127.9 \text{ MPa}$$

$$P_{all} = A\,\sigma_{all} = (6.4 \times 10^{-3})(127.9 \times 10^6) = 819 \text{ MPa}$$ ◀

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PROBLEM 10.68

A column of 21-ft effective length is obtained by connecting C10 × 20 steel channels with lacing bars as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION



C10 × 20: $A = 5.87 \text{ in}^2$ $x = 0.606$ in.

$I_x = 78.9 \text{ in}^4$ $I_y = 2.80 \text{ in}^4$

$d = 3.5 - x = 2.894$ in.

For the column, $A = (2)(5.87) = 11.74 \text{ in}^2$

$I_x = (2)(78.9) = 157.8 \text{ in}^4$

$I_y = 2\,[2.81 + (5.87)(2.894)^2] = 103.945 \text{ in}^4$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{103.95}{11.74}} = 2.976 \text{ in.} \qquad L_e = 21 \text{ ft} = 252 \text{ in.}$$

$$\frac{L_e}{r} = 84.69$$

$$\frac{L}{r} = 4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{29 \times 10^6}{36 \times 10^3}} = 133.68 > 84.69$$

$$\sigma_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 (29 \times 10^3)}{(84.69)^2} = 39.906$$

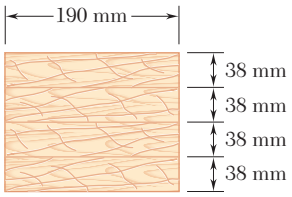
$$\sigma_{\text{cr}} = [0.658^{(\sigma_Y/\sigma_e)}]\sigma_Y = [0.658^{(36/39.906)}](36) = 24.678 \text{ ksi}$$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{1.67} = \frac{24.678}{1.67} = 14.7775$$

$$P_{\text{all}} = A\sigma_{\text{all}} = (11.74)(14.7775) = 173.5 \text{ kips}$$ ◀

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PROBLEM 10.69



The glued laminated column shown is made from four planks, each of 38 × 190-mm cross section. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_C = 10$ MPa and $E = 12$ GPa, determine the maximum allowable centric load if the effective length of the column is (*a*) 7 m, (*b*) 3 m.

SOLUTION

Glued laminated column. $c = 0.9, \qquad K_{CE} = 0.418$

$$\sigma_C = 10 \text{ MPa} \qquad E = 12{,}000 \text{ MPa}$$

$$4 \times 38 = 152 \text{ mm} = d \qquad b = 190 \text{ mm}$$

$$A = (152)(190) = 28.88 \times 10^3 \text{ mm}^2 = 28.88 \times 10^{-3} \text{ m}^2$$

(*a*) $L = 7 \text{ m} = 7000 \text{ mm} \qquad \dfrac{L}{d} = 46.053$

$$\sigma_{CE} = \frac{K_{CE}E}{(L/d)^2} = \frac{(0.418)(12{,}000)}{(46.053)^2} = 2.36510 \text{ MPa}$$

$$\frac{\sigma_{CE}}{\sigma_C} = 0.236510$$

$$u = \frac{1 + \sigma_{CE}/\sigma_C}{2c} = \frac{1.19709}{(2)(0.9)} = 0.68695$$

$$\frac{\sigma_{CE}/\sigma_C}{c} = 0.262788$$

$$C_P = u - \sqrt{u^2 - v} = 0.22966$$

$$\sigma_{\text{all}} = \sigma_C C_P = (10)(0.22966) = 2.2966 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (2.2966 \times 10^6)(28.88 \times 10^{-3}) = 66.3 \times 10^3 \text{ N} = 66.3 \text{ kN} \quad \blacktriangleleft$$

(*b*) $L = 3 \text{ m} = 3000 \text{ mm} \qquad \dfrac{L}{d} = 19.7368$

$$\sigma_{CE} = \frac{K_{CE}E}{(L/d)^2} = \frac{(0.418)(12{,}000)}{(19.7368)^2} = 12.8766 \text{ MPa} \qquad \frac{\sigma_{CE}}{\sigma_C} = 1.28766$$

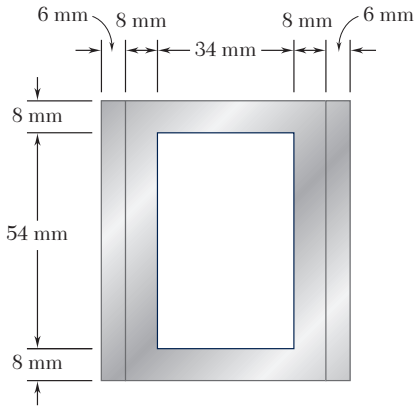
$$u = \frac{1 + \sigma_{CE}/\sigma_C}{2c} = \frac{2.28766}{(2)(0.9)} = 1.27092 \qquad v = \frac{\sigma_{CE}/\sigma_C}{c} = 1.43074$$

$$C_p = u\sqrt{u^2 - v} = 0.84138$$

$$\sigma_{\text{all}} = \sigma_C C_P (10)(0.84138) = 8.4138 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (8.4138 \times 10^6)(28.88 \times 10^{-3}) = 243 \times 10^{-3} \text{ N} = 243 \text{ kN} \quad \blacktriangleleft$$

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PROBLEM 10.70

An aluminum structural tube is reinforced by bolting two plates to it as shown for use as a column of 1.7-m effective length. Knowing that all material is aluminum alloy 2014-T6, determine the maximum allowable centric load.

SOLUTION

$$b_o = 6 + 8 + 34 + 8 + 6 = 62 \text{ mm}$$

$$b_i = 34 \text{ mm}$$

$$h_o = 8 + 54 + 8 = 70 \text{ mm}$$

$$h_i = 54 \text{ mm}$$

$$A = b_o h_o - b_i h_i = (62)(70) - (34)(54)$$

$$= 2.504 \times 10^3 \text{ mm}^2 = 2.504 \times 10^{-3} \text{ m}^2$$

$$I_x = \frac{1}{12}\left[b_o h_o^3 - b_i h_i^3\right] = \frac{1}{12}[(62)(70)^3 - (34)(54)^3]$$

$$= 1.32602 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{1}{12}\left[h_o b_o^3 - h_i b_i^3\right] = \frac{1}{12}[(70)(62)^3 - (54)(34)^3] = 1.21337 \times 10^6 \text{ mm}^4 = I_{\min}$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{1.21337 \times 10^6}{2.504 \times 10^3}} = 22.013 \text{ mm} = 22.013 \times 10^{-3} \text{ m} \qquad L = 1.7 \text{ m}$$

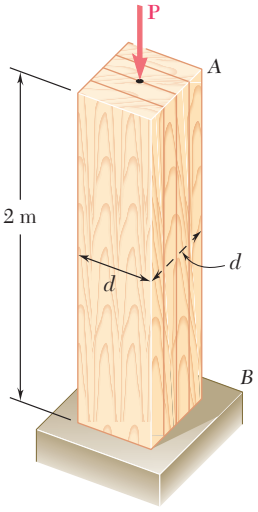
$$\frac{L}{r} = \frac{1.7}{22.013 \times 10^{-3}} = 77.23 > 55 \quad \text{(aluminum alloy 2014-T6)}$$

$$\sigma_{\text{all}} = \frac{382 \times 10^3}{(L/r)^2} = \frac{382 \times 10^3}{77.23^2} = 64.05 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (64.05 \times 10^6)(2.504 \times 10^{-3}) = 160.4 \times 10^3 \text{ N}$$

$$P_{\text{all}} = 160.4 \text{ kN} \blacktriangleleft$$

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PROBLEM 10.71

The glued laminated column shown is free at its top *A* and fixed at its base *B*. Using wood that has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 9.2$ MPa and an adjusted modulus of elasticity $E = 5.7$ GPa, determine the smallest cross section that can support a centric load of 62 kN.

SOLUTION

Glued laminated column: $c = 0.9 \qquad E = 5700 \text{ MPa}$

$$L_e = 2L = (2)(2) = 4 \text{ m} = 4000 \text{ mm}$$

$$\sigma_C = 9.2 \text{ MPa}$$

$$A = d^2 \qquad P_{\text{all}} = 62{,}000 \text{ N}$$

$$L_e/d = 4000/d \quad \text{with } d \text{ in mm}$$

$$\sigma_{\text{all}} = \sigma_C C_P$$

Assume C_P. $\sigma_{\text{all}} = 9.2\, C_P \text{ (MPa)}$

$$d = \sqrt{\frac{P_{\text{all}}}{\sigma_{\text{all}}}} = \sqrt{\frac{62{,}000}{\sigma_{\text{all}}}} = \frac{249}{\sqrt{\sigma_{\text{all}}}}$$

$$\sigma_{CE} = \frac{0.822E}{(L/d)^2} = \frac{(0.822)(5700)}{(L/d)^2} = \frac{4685}{(L/d)^2} \text{ (MPa)}$$

Checking:
$$C_P = \frac{1 + \sigma_{CE}/\sigma_C}{2c} - \sqrt{\left(\frac{1 + \sigma_{CE}/\sigma_C}{2c}\right)^2 - \frac{\sigma_{CE}/\sigma_C}{c}}$$

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PROBLEM 10.71 ***(Continued)***

Calculations are carried out in the following table:

C_P (assumed)	σ_{all} (MPa)	d (mm)	L/d	σ_{CE} (MPa)	σ_{CE}/σ_C	C_P (calc.)	ΔC_P
0.5	4.6	116.1	34.45	3.948	0.4291	0.4021	–0.0979
0.4	3.68	129.8	30.82	4.932	0.5361	0.4892	+0.0892
0.448	4.122	121.7	32.87	4.336	0.4713	0.4373	–0.0107
0.443	4.076	123.3	32.44	4.452	0.4839	0.4476	+0.0046

Using interpolation,

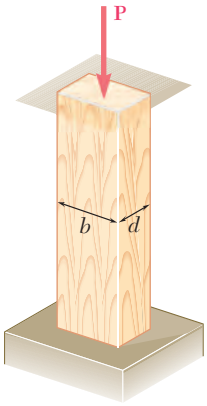
$$C_P = 0.443 + \frac{0.0046}{0.0153}(0.005) = 0.4445$$

$$\sigma_{\text{all}} = (0.4445)(9.2) = 4.0894 \text{ MPa}$$

$$d = \frac{249}{\sqrt{4.0894}} = 123.1 \text{ mm}$$

$d = 123.1$ mm ◀

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PROBLEM 10.72

An 18-kip centric load is applied to a rectangular sawn lumber column of 22-ft effective length. Using sawn lumber for which the adjusted allowable stress for compression parallel to the grain is $\sigma_C = 1050$ psi and the adjusted modulus is $E = 440 \times 10^3$ psi, determine the smallest cross section that can be used. Use $b = 2d$.

SOLUTION

Sawn lumber: $c = 0.8 \qquad L = 22 \text{ ft} = 264 \text{ in.}$

$$\frac{\sigma_{CE}}{\sigma_C} = \frac{(0.822)E}{\sigma_C (L/d)^2}$$

$$= \frac{(0.822)(440 \times 10^3)d^2}{(1050)(264)^2}$$

$$= 4.9423 \times 10^{-3} d^2$$

Let $$x = \frac{d}{6 \text{ in.}}$$

where 6 in. is a reference value for d.

$$\frac{\sigma_{CE}}{\sigma_C} = 0.17792x^2$$

$$u = \frac{1 + \sigma_{CE}/\sigma_C}{2c} = \frac{1 + 0.17792x^2}{1.6}$$

$$v = \frac{\sigma_{CE}/\sigma_C}{c} = 0.2224x^2$$

$$C_P = u - \sqrt{u^2 - v} = \frac{1 + 0.17792x^2}{1.6} - \sqrt{\left(\frac{1 + 0.17792x^2}{1.6}\right)^2 - 0.2224x^2}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (C_P \sigma_C)(bd) = C_P \sigma_C (2d^2)$$

$$= C_P (1050)(72x^2) = 75{,}600 C_P x^2$$

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PROBLEM 10.72 *(Continued)*

Determine P_{all} for various values of x.

x	u	v	C_P	P_{all} (lb)
1.0	0.73620	0.22240	0.17087	12,920
1.2	0.78513	0.32026	0.24092	26,227
1.1	0.75955	0.26910	0.20473	18,729
1.09	0.75712	0.26423	0.20124	18,075
1.089	0.75687	0.26374	0.20089	18,011

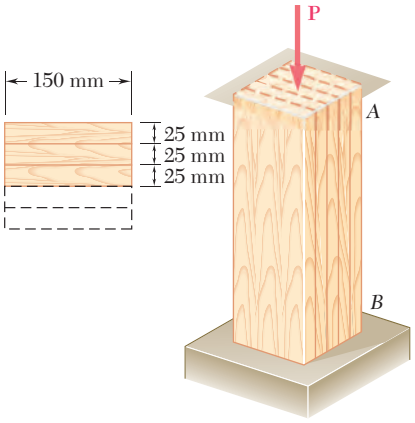
$$P_{all} = 18 \text{ kips}$$
$$= 18{,}000 \text{ lb}$$

For $P_{all} = 18{,}000$ lb $x = 1.089$

$$d = (1.089)(6 \text{ in.})$$

$$d = 6.53 \text{ in.} \blacktriangleleft$$

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PROBLEM 10.73

A laminated column of 2.1-m effective length is to be made by gluing together wood pieces of 25 × 150-mm cross section. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_C = 7.7$ MPa and the adjusted modulus is $E = 5.4$ GPa, determine the number of wood pieces that must be used to support the concentric load shown when (*a*) $P = 52$ kN, (*b*) $P = 108$ kN.

SOLUTION

Glued laminated column: $c = 0.90$

Let n = number of pieces.

$$A = (150)(25n) = 3750n \text{ mm}^2$$

$$= 3750 \times 10^{-6} n \text{ m}^2$$

$$d = 25 \times 10^{-3} n \text{ m} \quad \text{if} \quad n \le 6$$

$$d = 150 \times 10^{-3} \text{ m} \quad \text{if} \quad n > 6$$

$$L_e = 2.1 \text{ m}$$

$$\frac{L_e}{d} = \frac{2.1}{25 \times 10^{-3} n} = \frac{84}{n} \quad \text{if} \quad n \le 6$$

$$= \frac{2.1}{0.15} = 14 \quad \text{if} \quad n \ge 6$$

$$\sigma_C = 7.7 \text{ MPa} \qquad E = 5400 \text{ MPa} \qquad \sigma_{CE} = \frac{(0.822)(5400)}{(L/d)^2} = \frac{4439}{(L/d)^2} \text{ MPa}$$

$$C_P = \frac{1 + \sigma_{CE}/\sigma_C}{2c} - \left(\frac{1 + \sigma_C/\sigma_{CE}}{2c} \right)^2 - \frac{\sigma_{CE}/\sigma_C}{c}$$

$$\sigma_{\text{all}} = C_P \sigma_C \qquad P_{\text{all}} = \sigma_{\text{all}} A$$

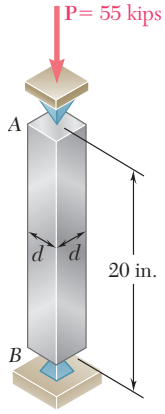
Calculations are carried out in the table below.

n	d(m1)	$A(10^{-6}$ m$^2)$	$\frac{L}{d}$	σ_{CE} (MPa)	$\frac{\sigma_{CE}}{\sigma_C}$	C_P	σ_{all} (MPa)	P_{all} kN
3	0.075	11,250	28	5.662	0.7353	0.6288	4.842	54.5
4	0.100	15,000	21	10.066	1.3073	0.8453	6.509	97.6
5	0.125	18,750	16.8	15.727	2.0421	0.9237	7.112	133.6

(*a*) For $P = 52$ kN, use $n = 3$. ◀

(*b*) For $P = 108$ kN, use $n = 5$. ◀

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PROBLEM 10.74

For a rod made of the aluminum alloy 2014-T6, select the smallest square cross section that may be used if the rod is to carry a 55-kip centric load.

SOLUTION

Square cross section: $A = d^2, \quad I = \frac{1}{12}d^4, \quad L = 20 \text{ in.}$

$$r\sqrt{\frac{I}{A}} = \frac{d}{\sqrt{12}} \qquad \frac{L}{r} = \frac{20\sqrt{12}}{d} = \frac{69.282}{d}$$

2014-T6 aluminum alloy: Assume $\frac{L}{r} < 55.$

$$\sigma_{\text{all}} = 30.9 - 0.229\frac{L}{r} \text{ ksi} = 30.9 - \frac{15.866}{d} \text{ ksi}$$

$$P_{\text{all}} = \sigma_{\text{all}}A = 30.9d^2 - 15.866d \text{ kips}$$

But $P = 55$ kips: $30.9d^2 - 15.866d = 55$

$$d^2 - 0.51346d - 1.7799 = 0$$

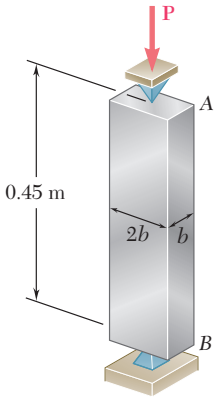
$$d = \frac{0.51346 \pm \sqrt{(0.51346)^2 - (4)(-1.7799)}}{2}$$

$$= 0.25673 \pm 1.35861 \qquad d = 1.6153 \text{ in.}$$

$$\frac{L}{d} = \frac{69.282}{1.6153} = 43.89 < 55 \qquad \text{as required.}$$

$d = 1.615$ in. ◀

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PROBLEM 10.75

A 72-kN centric load must be supported by an aluminum column as shown. Using the aluminum alloy 6061-T6, determine the minimum dimension b that can be used.

SOLUTION

Rectangular cross section $2b \times b$: $A = 2b^2$

$$I_{\min} = \frac{(2b)b^3}{12} = \frac{b^4}{6}, \quad r = \sqrt{\frac{I_{\min}}{A}} = \frac{b}{\sqrt{12}}, \quad L = 0.45\,\text{m}$$

$$\frac{L}{b} = \frac{0.45\sqrt{12}}{b} = \frac{1.5588}{b}$$

6061-T6 aluminum alloy: Assume $\dfrac{L}{r} < 66$

$$\sigma_{\text{all}} = \left(140 - 0.874\frac{L}{r}\right)\text{MPa} = \left[140 - \frac{(0.874)(1.5588)}{b}\right](10^6)\ \text{Pa}$$

$$= 140 \times 10^6 - \frac{1.3624 \times 10^6}{b}\ \text{Pa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = 280 \times 10^6 b^2 - 2.7248 \times 10^6 b\ \text{N}$$

But $P = 72\,\text{kN} = 72 \times 10^3\,\text{N}$

$$280 \times 10^6 b^2 - 2.7248 \times 10^6 b = 72 \times 10^3$$

$$b^2 - 0.0097314b - 0.00025714 = 0$$

$$b = \frac{0.0097314}{2} \pm \frac{\sqrt{(0.0097314)^2 - (4)(-0.00025714)}}{2}$$

$$= 0.0048658 \pm 0.0167575 \qquad b = 0.02162\ \text{m}$$

$$\frac{L}{b} = \frac{1.5588}{0.02162} = 72.09 > 66 \qquad \text{(Assumption is false.)}$$

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PROBLEM 10.75 *(Continued)*

Use $$\sigma_{\text{all}} = \frac{354 \times 10^3}{(L/r)^2} \text{ MPa} = \frac{354 \times 10^3}{(1.5588)^2} b^2 \text{ MPa}$$

$$= 145.688 \times 10^3 \text{ MPa} = 145.688 \times 10^9 \text{ Pa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (145.688 \times 10^9)(2b^2) = 291.375 \times 10^9 b^4 \text{ N}$$

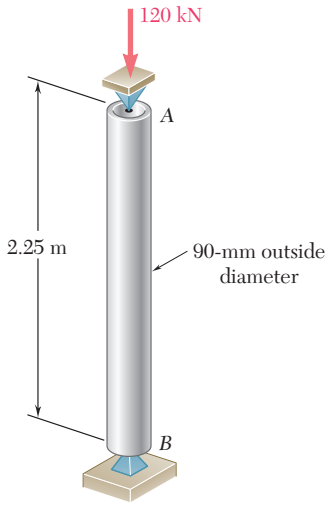
But $$P = 72 \times 10^3 \text{ N}$$

$$291.375 \times 10^9 b^4 = 72 \times 10^3 \qquad b = 0.0223 \text{ m}$$

$$\frac{L}{r} = \frac{1.5588}{0.0223} = 69.9 > 66$$

$b = 22.3$ mm ◀

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PROBLEM 10.76

An aluminum tube of 90-mm outer diameter is to carry a centric load of 120 kN. Knowing that the stock of tubes available for use are made of alloy 2014-T6 and with wall thicknesses in increments of 3 mm from 6 mm to 15 mm, determine the lightest tube that can be used.

SOLUTION

$$L = 2250 \text{ mm}, \quad P = 120 \times 10^3 \text{ N} \quad r_o = 45 \text{ mm}$$

$$r_i = r_o - t \qquad A = \pi\left(r_o^2 - r_i^2\right) \qquad I = \frac{\pi}{4}\left(r_o^4 - r_i^4\right)$$

$$r = \sqrt{I/A}$$

For 2014-T6 aluminum alloy,

$$\sigma_{\text{all}} = 213 - 1.577(L/r) \text{ MPa} \qquad \text{if} \qquad L/r < 55$$

$$\sigma_{\text{all}} = \frac{382 \times 10^3}{(L/r)^2} \text{ MPa} \qquad \text{if} \qquad L/r > 55$$

$$P_{\text{all}} = \sigma_{\text{all}} A$$

Calculate P_{all} for each thickness.

	T mm	r_i mm	A mm^2	I 10^6 mm^4	R mm	L/r	σ_{all} MPa	P_{all} kN	
	6	39	1583	1.404	29.78	75.56	66.91	105.9	
→	9	36	2290	1.901	28.82	78.08	62.66	143.5	←
	12	33	2941	2.289	27.90	80.65	58.73	172.7	
	15	30	3534	2.584	27.04	83.20	55.18	195.0	

Since P_{all} must be greater than 120 kN, use $t = 9$ mm. ◀

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PROBLEM 10.77

A column of 4.6-m effective length must carry a centric load of 525 kN. Knowing that $\sigma_Y = 345$ MPa and $E = 200$ GPa, use allowable stress design to select the wide-flange shape of 200-mm nominal depth that should be used.

SOLUTION

Transition $\dfrac{L}{r}$:

$$4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200\times10^9}{345\times10^6}} = 113.40$$

$$P < \frac{\sigma_Y A}{1.67} \qquad A > \frac{1.67P}{\sigma_Y} = \frac{(1.67)(525\times10^3)}{345\times10^6} = 2.541\times10^{-3}\,\text{m}^2 = 2541\,\text{mm}^2$$

$$P < \frac{0.877\pi^2 E I_{\min}}{1.67L_e^2}$$

$$I_{\min} > \frac{1.67PL_e^2}{0.877\pi^2 E} = \frac{(1.67)(525\times10^3)(4.6)^2}{0.877\pi^2(200\times10^9)} = 10.72\times10^6\,\text{m}^4 = 10.72\times10^{-6}\,\text{mm}^4$$

Try W200 × 46.1. $A = 5880\,\text{mm}^2$, $I_{\min} = 15.4\times10^6\,\text{mm}^4$, $r = 51.3$ mm

$$\frac{L_e}{r} = \frac{4.6}{51.1\times10^{-3}} = 89.67 < 113.40$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = 245.50\ \text{MPa}$$

$$\sigma_{\text{cr}} = (0.658^{\sigma_Y/\sigma_e})\sigma_Y = [0.658^{345/245.50}](345) = 191.589\ \text{MPa}$$

$$P_{\text{all}} = \frac{\sigma_{\text{cr}} A}{1.67} = \frac{(191.589\times10^6)(5880\times10^{-6})}{1.67} = 675\ \text{kN} > 525\ \text{kN}$$

All lighter sections fail the minimum moment of inertia criterion. Use W200 × 46.1. ◀

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PROBLEM 10.78

A column of 22.5-ft effective length must carry a centric load of 288 kips. Using allowable stress design, select the wide-flange shape of 14-in. nominal depth that should be used. Use $\sigma_Y = 50$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

Preliminary calculations.

$$P < \frac{\sigma_Y A}{1.67} \qquad A > \frac{1.67P}{\sigma_Y} = \frac{(1.67)(288)}{50} = 9.62 \text{ in}^2$$

$$L_e = 22.5 \text{ ft} = 270 \text{ in.} \quad E = 29{,}000 \text{ ksi}$$

$$P < \frac{0.877\pi^2 E I_{\min}}{1.67L^2} \qquad I_{\min} > \frac{1.67PL_e^2}{0.877\pi^2 E} = \frac{(1.67)(288)(270)^2}{0.877\pi^2(29{,}000)} = 139.7 \text{ in}^4$$

Transition $\frac{L_e}{r}$: $\quad 4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.43$

Try W14 × 82. $A = 24.0$ in^2, $I_{\min} = 148$ in^4, $r = 2.48$ in.

$$\frac{L_e}{r} = \frac{270}{2.48} = 108.87 < 113.43$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(29{,}000)}{(108.87)^2} = 24.148 \text{ ksi}$$

$$\sigma_{\text{cr}} = (0.658^{\sigma_Y/\sigma_e})\sigma_Y = [0.658^{50/24.148}](50) = 21.018 \text{ ksi}$$

$$P_{\text{all}} = \frac{\sigma_{\text{cr}} A}{1.67} = \frac{(21.018)(24.1)}{1.67} = 302 \text{ kips} > 288 \text{ kips}$$

Use W14 × 82. ◀

All lighter 14-in. wide-flange shapes fail the minimum moment of inertia criterion.

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PROBLEM 10.79

A column of 17-ft effective length must carry a centric load of 235 kips. Using allowable stress design, select the wide-flange shape of 10-in. nominal depth that should be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

Preliminary calculations.

$$P < \frac{\sigma_Y A}{F.S.} \qquad A > \frac{(F.S.)P}{\sigma_Y} = \frac{(1.67)(235)}{36} = 10.90 \text{ in}^2$$

$$L_e = 17 \text{ ft} = 204 \text{ in.}$$

$$E = 29 \times 10^6 \text{ psi} = 29{,}000 \text{ ksi}$$

$$P < \frac{0.877\pi^2 EI}{(F.S.)L_e^2} \qquad I > \frac{(F.S.)(PL_e)^2}{0.877\pi^2 E} = \frac{(1.67)(235)(204)^2}{0.877\pi^2(29{,}000)} = 65.1 \text{ in}^4$$

Transition L_e/r: $\quad 4.71\sqrt{\dfrac{E}{\sigma_Y}} = 4.71\sqrt{\dfrac{29{,}000}{36}} = 133.68$

Try W10 × 54. $\quad A = 15.8 \text{ in}^2 \quad I_y = 103 \text{ in}^4 \quad r_y = 2.56 \text{ in.}$

$$\frac{L_e}{r} = \frac{204}{2.56} = 79.69 < 133.68$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(29{,}000)}{(79.69)^2} = 45.07 \text{ ksi}$$

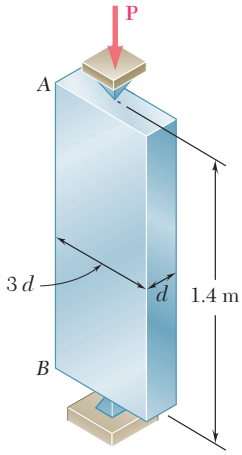
$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{F.S.} = \frac{1}{1.67}[0.658^{\sigma_Y/\sigma_e}]\sigma_Y = \frac{1}{1.67}[0.658^{36/45.07}](36) = 15.431 \text{ ksi}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (15.431)(15.8)$$

$$P_{\text{all}} = 244 \text{ kips}$$

$P_{\text{all}} > P = 235$ kips $\qquad$ Use W10 × 54. ◀

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PROBLEM 10.80

A centric load **P** must be supported by the steel bar *AB*. Using allowable stress design, determine the smallest dimension d of the cross section that can be used when (*a*) $P = 108$ kN, (*b*) $P = 166$ kN. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

SOLUTION

Transition $\dfrac{L_e}{r}$: $\quad 4.71\sqrt{\dfrac{E}{\sigma_Y}} = 4.71\sqrt{\dfrac{200\times10^9}{250\times10^6}} = 133.22$

$$L_e = L = 1.4\text{ m} \qquad A = (3d)(d) = 3d^2 \qquad I = \frac{1}{12}(3d)(d)^3 = \frac{1}{4}d^4$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{\sqrt{12}} = 0.288675d$$

(*a*) $\underline{P = 108\times10^3\text{ N}}$. $\qquad$ Assume $\dfrac{L_e}{r} > 133.22$

$$P_{\text{all}} = \frac{0.877\pi^2 EI}{1.67} \qquad I = \frac{(1.67)P_{\text{all}}L_e^2}{0.877\pi^2 E} = \frac{1}{4}d^4$$

$$d^4 = \frac{(4)(1.67)PL_e^2}{0.877\pi^2 E} = \frac{(4)(1.67)(108\times10^3)(1.4)^2}{(0.877\pi^2)(200\times10^9)} = 816\times10^{-9}\text{ m}^4$$

$$d = 30.063\times10^{-3}\text{ m} \qquad r = 8.678\times10^{-3}\text{ m}$$

$$\frac{L_e}{r} = \frac{1.4}{8.678\times10^{-3}} = 161.32 > 133.22 \qquad d = 30.1\text{ mm} \blacktriangleleft$$

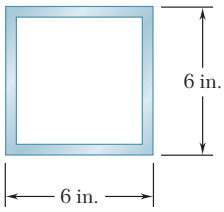
(*b*) $\underline{P = 166\times10^3\text{ N}}$. $\qquad$ Assume $\dfrac{L_e}{r} > 133.22$

$$d^4 = \frac{(4)(1.67)(166\times10^3)(1.4)^2}{(0.877\pi^2)(200\times10^9)} = 1.25548\times10^{-9}\text{ m}^4$$

$$d = 33.474\times10^{-3}\text{ m} \qquad r = 9.663\times10^{-3}\text{ m}$$

$$\frac{L_e}{r} = \frac{1.4}{9.633\times10^{-3}} = 144.88 > 133.22 \qquad d = 33.5\text{ mm} \blacktriangleleft$$

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PROBLEM 10.81

A square steel tube having the cross section shown is used as a column of 26-ft effective length to carry a centric load of 65 kips. Knowing that the tubes available for use are made with wall thicknesses ranging from $\frac{1}{4}$ in. to $\frac{3}{4}$ in. in increments of $\frac{1}{16}$ in., use allowable stress design to determine the lightest tube that can be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

$$b_o = 6 \text{ in.} \qquad b_i = b_o - 2t \qquad A = b_o^2 - b_i^2$$

$$I = \frac{1}{12}(b_o^4 - b_i^4)$$

$$L_e = 26 \text{ ft} = 312 \text{ in.} \qquad P = 65 \text{ kips}$$

$$\sigma_Y = 36 \text{ ksi} \qquad E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

Transition L_e/r:

$$4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{29{,}000}{36}} = 133.68$$

Try $t = \frac{1}{2}$ in. $= 0.5$ in. $\quad b_i = 5.0$ in. $\quad A = 6^2 - 5.0^2 = 11 \text{ in}^2$

$$I = \frac{1}{12}[(6)^4 - (5.0)^4] = 55.9167 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 2.2546 \text{ in.}$$

$$\frac{L_e}{r} = \frac{312}{2.2546} = 138.382 > 133.68$$

$$\sigma_{\text{cr}} = \frac{0.877\pi^2 E}{(L_e/r)^2} = \frac{(0.877)\pi^2(29{,}000)}{(138.382)^2} = 13.108 \text{ ksi}$$

$$P_{\text{all}} = \frac{\sigma_{\text{cr}} A}{1.67} = \frac{(13.108)(11)}{1.67} = 86.34 \text{ kips}$$

P_{all} is approximately proportional to t.

$$\frac{t}{0.5} = \frac{65}{86.34} \qquad t \approx 0.3765 \text{ in.}$$

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PROBLEM 10.81 *(Continued)*

Try $t = \frac{3}{8}$ in. $= 0.375$ in. $\quad b_i = 5.25$ in. $\quad A = 8.4375$ in^2

$$I = \frac{1}{12}[(6)^4 - (5.25)^4] = 44.6924 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 2.3015 \text{ in.}$$

$$\frac{L_e}{r} = \frac{312}{2.3015} = 135.564 > 133.68$$

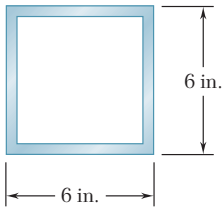
$$\sigma_{\text{cr}} = \frac{0.877\pi^2 E}{(L_e/r)^2} = 13.659 \text{ ksi}$$

$$P_{\text{all}} = \frac{\sigma_{\text{cr}} A}{1.67} = \frac{(13.659)(8.4375)}{1.67} = 69.0 \text{ kips} > 65 \text{ kips}$$

Use $t = 3/8$ in. ◀

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PROBLEM 10.82



Solve Prob. 10.81, assuming that the effective length of the column is decreased to 20 ft.

PROBLEM 10.81 A square structural tube having the cross section shown is used as a column of 26-ft effective length to carry a centric load of 65 kips. Knowing that the tubes available for use are made with wall thicknesses ranging from $\frac{1}{4}$ in. to $\frac{3}{4}$ in. in increments of $\frac{1}{16}$ in., use allowable stress design to determine the lightest tube that can be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

$$b_o = 6 \text{ in.} \qquad b_i = b_o - 2t \qquad A = b_o^2 - b_i^2$$

$$I = \frac{1}{12}\left(b_o^4 - b_i^4\right)$$

$$L_e = 20 \text{ ft} = 240 \text{ in.} \qquad P = 65 \text{ kips}$$

$$\sigma_Y = 36 \text{ ksi} \qquad E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

Steel: Transition $\frac{L}{r}$: $4.71\sqrt{\frac{E}{\sigma_Y}} = 133.68$

Try $t = \frac{1}{2}$ in. $= 0.5$ in. $\quad b_i = 5.0$ in. $\quad A = 6^2 - 5.0^2 = 11 \text{ in}^2$

$$I = \frac{1}{12}[(6)^4 - (5.0)^4] = 55.9167 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 2.2546 \text{ in.}$$

$$\frac{L_e}{r} = \frac{240}{2.2546} = 106.449 < 133.68$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(29 \times 10^3)}{(106.442)^2} = 25.262 \text{ ksi}$$

$$\sigma_{\text{cr}} = \left(0.658^{\sigma_Y/\sigma_e}\right)\sigma_Y = [0.658^{36/25.262}](36) = 19.827 \text{ ksi}$$

$$P_{\text{all}} = \frac{\sigma_{\text{cr}} A}{1.67} = \frac{(19.827)(11)}{1.67} = 130.60 \text{ kips}$$

P_{all} is approximately proportional to t.

$$\frac{t}{0.5} \approx \frac{65}{130.6} \qquad t \approx 0.249 \text{ in.}$$

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PROBLEM 10.82 *(Continued)*

Try $t = \frac{1}{4}$ in. $= 0.25$ in. $\quad b_i = 5.5$ in. $\quad A = 5.75$ in^2

$$I = \frac{1}{12}[(6)^4 - (5.5)^4] = 31.745 \text{ in}^4 \qquad r = \sqrt{\frac{I}{A}} = 2.3496 \text{ in.}$$

$$\frac{L_e}{r} = \frac{240}{2.3496} = 102.143 < 133.68$$

$$\sigma_e = \frac{\pi^2(29 \times 10^3)}{(102.143)^2} = 27.433 \text{ ksi}$$

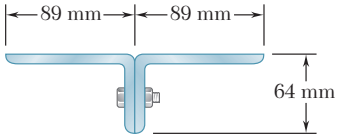
$$\sigma_{\text{cr}} = [0.658^{36/27.433}](36) = 20.786 \text{ ksi}$$

$$P_{\text{all}} = \frac{(20.786)(5.75)}{1.67} = 71.5 \text{ kips} > 65 \text{ kips}$$

Use $t = 1/4$ in. ◀

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PROBLEM 10.83



Two 89 × 64-mm angles are bolted together as shown for use as a column of 2.4-m effective length to carry a centric load of 180 kN. Knowing that the angles available have thickness of 6.4 mm, 9.5 mm, and 12.7 mm, use allowable stress design to determine the lightest angles that can be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

SOLUTION

Transition $\dfrac{L}{r}$: $4.71\sqrt{\dfrac{E}{\sigma_Y}}$

$$4.71\sqrt{\frac{200\times10^9}{250\times10^6}} = 133.22$$

Try L89 × 64 × 9.5. $A = (2)(1360) = 2720\ \text{mm}^2 = 2720\times10^{-6}\ \text{mm}$

$$r = 18.2\ \text{mm} = 18.2\times10^{-3}\ \text{m} \qquad (r_y \text{ in Appendix } C)$$

$$L_e/r = 2.4/18.2\times10^{-3} = 131.9 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(200\times10^9)}{(131.9)^2} = 113.51\ \text{MPa}$$

$$\sigma_{\text{cr}} = (0.658^{\sigma_Y/\sigma_e})\sigma_Y = [0.658^{250/113.51}](250) = 99.45\ \text{MPa}$$

$$P_{\text{all}} = \frac{\sigma_{\text{cr}}A}{1.67} = \frac{(99.45\times10^6)(2720\times10^{-6})}{1.67} = 162.0\ \text{kN} < 180\ \text{kN}$$

Do not use.

Try L89 × 64 × 12.7. $A = (2)(1770) = 3540\ \text{mm}^2 = 3540\times10^{-6}\ \text{m}^2$

$$r_x = 17.8\ \text{mm} = 17.8\times10^{-3}\ \text{m}$$

$$L_e/r = 2.4/17.8\times10^{-3} = 134.83 > 133.22$$

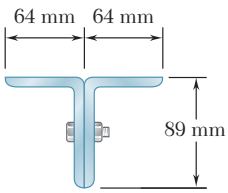
$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(200\times10^9)}{(134.83)^2} = 108.58\ \text{MPa}$$

$$\sigma_{\text{cr}} = 0.877\sigma_e = (0.877)(108.58) = 95.224\ \text{MPa}$$

$$P_{\text{all}} = \frac{\sigma_{\text{cr}}A}{1.67} = \frac{(95.224\times10^6)(3540\times10^{-6})}{1.67} = 202\ \text{kN} > 180\ \text{kN}$$

Use L89 × 64 × 12.7. ◀

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PROBLEM 10.84

Two 89 × 64-mm angles are bolted together as shown for use as a column of 2.4-m effective length to carry a centric load of 325 kN. Knowing that the angles available have thicknesses of 6.4 mm, 9.5 mm, and 12.7 mm, use allowable stress design to determine the lightest angles that can be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

SOLUTION

Transition $\dfrac{L}{r}$: $4.71\sqrt{\dfrac{E}{\sigma_Y}}$

$$4.71\sqrt{\frac{200\times10^9}{250\times10^6}} = 133.22$$

Try L89 × 64 × 9.5. $A = (2)(1360) = 2720\ \text{mm}^2 = 2720\times10^{-6}\ \text{m}^2$

$$I_x = (2)(1.07\times10^6) = 2.14\times10^6\ \text{mm}^4$$

$$I_y = (2)[0.463\times10^6 + (1360)(16.9)^2] = 1.70285\times10^6\ \text{mm}^4 = I_{\min}$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{1.70285\times10^6}{2720}} = 25.021\,\text{mm} \times 25.021\times10^{-3}\,\text{m}$$

$$\frac{L_e}{r} = \frac{2.4}{25.021\times10^{-3}} = 95.919 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(200\times10^9)}{(95.919)^2} = 214.55\,\text{MPa}$$

$$\sigma_{\text{cr}} = (0.658^{\sigma_Y/\sigma_e})\sigma_Y = [0.658^{250/214.55}](250) = 153.51\,\text{MPa}$$

$$P_{\text{all}} = \frac{\sigma_{\text{cr}}A}{1.67} = \frac{(153.51\times10^6)(2720\times10^{-6})}{1.67} = 250\ \text{kN} < 325\ \text{kN}$$

Do not use.

Try L89 × 64 × 12.7. $A = (2)(1780) = 3560\ \text{mm}^2 = 3560\times10^{-6}\ \text{m}^2$

$$I_x = (2)(1.36\times10^6) = 2.72\times10^6\ \text{mm}^4$$

$$I_y = (2)[0.581\times10^6 + (1780)(18.1)^2] = 2.3283\times10^6\ \text{mm}^4 = I_{\min}$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{2.3283\times10^6}{3560}} = 25.574\,\text{mm} = 25.574\times10^{-3}\,\text{m}$$

$$\frac{L_e}{r} = \frac{2.4}{25.574\times10^{-3}} = 93.846 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(200\times10^9)}{(93.846)^2} = 224.13\,\text{MPa}$$

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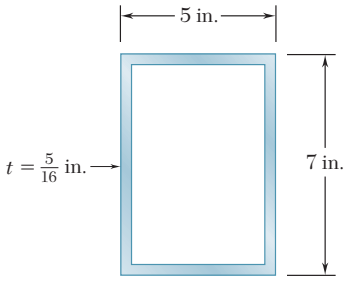
PROBLEM 10.84 *(Continued)*

$$\sigma_{cr} = [0.658^{250/224.13}](250) = 156.74 \text{ MPa}$$

$$P_{all} = \frac{\sigma_{cr} A}{1.67} = \frac{(156.74 \times 10^{6})(3560 \times 10^{-6})}{1.67} = 334 \text{ kN} > 325 \text{ kN}$$

Use L89 × 64 × 12.7. ◀

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PROBLEM 10.85*

A rectangular steel tube having the cross section shown is used as a column of 14.5-ft effective length. Knowing that $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi, use load and resistance factor design to determine the largest centric live load that can be applied if the centric dead load is 54 kips. Use a dead load factor $\gamma_D = 1.2$, a live load factor $\gamma_L = 1.6$, and the resistance factor $\varphi = 0.90$.

SOLUTION

$$L_e = 14.5 \text{ ft} = 174 \text{ in.}$$

$$b_o = 7 \text{ in.} \qquad b_i = 7 - (2)\left(\frac{5}{16}\right) = 6\frac{3}{8} \text{ in.}$$

$$h_o = 5 \text{ in.} \qquad h_i = 5 - (2)\left(\frac{5}{16}\right) = 4\frac{3}{8} \text{ in.}$$

$$A = (7)(5) - \left(6\frac{3}{8}\right)\left(4\frac{3}{8}\right) = 7.109375 \text{ in}^2$$

$$I = \frac{1}{12}\left[(7)(5)^3 - \left(6\frac{3}{8}\right)\left(4\frac{3}{8}\right)^3\right] = 28.42967 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.99972 \text{ in.}$$

Transition L/r:

$$4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{29{,}000}{36}} = 133.68$$

$$\frac{L_e}{r} = \frac{174}{1.99972} = 87.012 < 133.68$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = 37.804 \text{ ksi}$$

$$\sigma_{\text{cr}} = [0.658^{\sigma_Y/\sigma_e}]\sigma_Y = [0.658^{36/37.804}](36) = 24.166 \text{ ksi}$$

$$P_U = A\sigma_{\text{cr}} = (7.109375)(24.166) = 171.804 \text{ kips}$$

$$\gamma_D P_D + \gamma_L P_L = \varphi P_U$$

$$(1.2)(54) + (1.6)P_L = (0.90)(17.804) \qquad P_L = 56.1 \text{ kips} \blacktriangleleft$$

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PROBLEM 10.86*

A column with a 5.8-m effective length supports a centric load, with a ratio of dead to live load equal to 1.35. The dead load factor is $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$, and the resistance factor $\varphi = 0.90$. Use load and resistance factor design to determine the allowable centric dead and live loads if the column is made of the following rolled-steel shapes: (*a*) W250 × 67, (*b*) W360 × 101. Use $\sigma_Y = 345$ MPa and $E = 200$ GPa.

SOLUTION

Transition L/r:

$$4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200\times10^9}{345\times10^6}} = 113.40$$

(*a*) W250 × 67.

$$A = 8580\ \text{mm}^2 = 8580\times10^{-6}\text{m}^2,$$
$$r_y = 51.1\ \text{mm} = 51.1\times10^{-3}\text{m}$$
$$L_e/r_y = \frac{5.8}{51.1\times10^{-3}} = 113.725 > 113.40$$
$$\sigma_{\text{cr}} = \frac{0.877\pi^2E}{(L_e/r_y)^2} = \frac{0.877\pi^2(200\times10^9)}{(113.725)^2} = 133.85\ \text{MPa}$$
$$P_U = A\sigma_{\text{cr}} = [8580\times10^{-6}](133.85\times10^6) = 1.1484\times10^6\ \text{N}$$
$$\gamma_D P_D + \gamma_L P_L = \varphi P_U \qquad \frac{P_D}{P_L} = 1.35 \qquad P_L = \frac{P_D}{1.35}$$
$$1.2P_D + 1.6\frac{P_D}{1.35} = (0.9)(1.1484\times10^6) \qquad P_D = 433\ \text{kN} \blacktriangleleft$$
$$P_L = 321\ \text{kN} \blacktriangleleft$$

(*b*) W360 × 101.

$$A = 12{,}900\ \text{mm}^2 = 12{,}900\times10^{-6}\ \text{m}^2$$
$$r_y = 62.5\ \text{mm} = 62.5\times10^{-3}\ \text{m}$$
$$L_e/r_y = \frac{5.8}{62.5\times10^{-3}} = 92.652 < 113.40$$
$$\sigma_e = \frac{\pi^2E}{(L_e/r_y)^2} = \frac{\pi^2(200\times10^9)}{(92.652)^2} = 229.94\ \text{MPa}$$
$$\sigma_{\text{cr}} = [0.658^{\sigma_Y/\sigma_e}]\sigma_y = [0.658^{345/229.94}](345) = 184.12\ \text{MPa}$$
$$P_U = A\sigma_{\text{cr}} = (12{,}900\times10^{-6})(184.12\times10^6) = 2.3751\times10^6\ \text{N}$$
$$\gamma_D P_D + \gamma_L P_L = \varphi P_U$$
$$1.2P_D + 1.6\frac{P_D}{1.35} = (0.90)(2.3751\times10^6) \qquad P_D = 896\ \text{kN} \blacktriangleleft$$
$$P_L = 664\ \text{kN} \blacktriangleleft$$

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PROBLEM 10.87

A steel column of 5.5-m effective length must carry a centric dead load of 310 kN and a centric live load of 375 kN. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, use load and resistance factor design to select the wide-flange shape of 310-mm nominal depth that should be used. The dead load factor is $\gamma_D = 1.2$, the live load factor is $\gamma_L = 1.6$, and the resistance factor is $\varphi = 0.90$.

SOLUTION

Transition $\frac{L}{r}$: $$4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

Design criterion: $$\gamma_D P_D + \gamma_L P_L \le \varphi P_U$$

Required minimum P_U: $$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\varphi} = \frac{(1.2)(310) + (1.6)(375)}{0.90} = 1080 \text{ kN}$$

Preliminary calculations:

$$P_U < \sigma_Y A \qquad A > \frac{P_U}{\sigma_Y} = \frac{1080 \times 10^3}{250 \times 10^6} = 4.32 \times 10^{-3} \text{ m}^2 = 4320 \text{ mm}^2$$

$$P_U < \frac{0.877\pi^2 E I_y}{L_e^2}$$

$$I_y > \frac{P_U L_e^2}{0.877\pi^2 E} = \frac{(1080 \times 10^3)(5.5)^2}{0.877\pi^2(200 \times 10^9)} = 18.87 \times 10^{-6} \text{ m}^4 = 18.87 \times 10^6 \text{ mm}^4$$

Try W310 × 74. $A = 9420 \text{ mm}^2$ $I_y = 23.4 \times 10^6 \text{ mm}^4$ $r_y = 49.8 \text{ mm}$

$$\frac{L_e}{r_y} = \frac{5.5}{49.8 \times 10^{-3}} = 110.44 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r_y)^2} = \frac{\pi^2(200 \times 10^9)}{(110.44)^2} = 161.83 \text{ MPa}$$

$$\sigma_{cr} = [0.658^{\sigma_Y/\sigma_e}]\sigma_Y = [0.658^{250/161.83}](250) = 130.96 \text{ MPa}$$

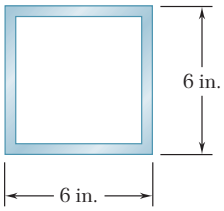
$$P_U = A\sigma_{cr} = (9420 \times 10^{-6} \text{ m}^2)(130.96 \times 10^6 \text{ Pa}) = 1234 \text{ kN} > 1080 \text{ kN}$$

(Acceptable)

The next lighter shape, W310 × 60, with $I_y = 18.3 \times 10^6 \text{ mm}^4$, fails the moment of inertia criterion given above.

Use W310 × 74. ◀

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PROBLEM 10.88*

The steel tube having the cross section shown is used as a column of 15-ft effective length to carry a centric dead load of 51 kips and a centric live load of 58 kips. Knowing that the tubes available for use are made with wall thicknesses in increments of $\frac{1}{16}$ in. from $\frac{3}{16}$ in. to $\frac{3}{8}$ in., use load and resistance factor design to determine the lightest tube that can be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. The dead load factor is $\gamma_D = 1.2$, the live load factor is $\gamma_L = 1.6$, and the resistance factor is $\varphi = 0.90$.

SOLUTION

Transition L/r:

$$4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{29{,}000}{36}} = 133.68$$

$$L_e = 15 \text{ ft} = 180 \text{ in.}$$

$$\gamma_D P_D + \gamma_L P_L = \varphi P_U$$

Required:

$$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\varphi} = \frac{(1.2)(51) + (1.6)(58)}{0.90}$$

$$= 171.11 \text{ kips}$$

Try

$$t = \frac{1}{4} \text{ in.} = 0.25 \text{ in.} \qquad b_o = 6.0 \text{ in.} \qquad b_i = b_o - 2t = 5.5 \text{ in.}$$

$$A = b_o^2 - b_i^2 = (6)^2 - (5.5)^2 = 5.75 \text{ in}^2$$

$$I = \frac{1}{12}(b_o^4 - b_i^4) = \frac{1}{12}[(6)^4 - (5.5)^4] = 31.74 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{31.74}{5.75}} = 2.3496 \text{ in.}$$

$$\frac{L_e}{r} = \frac{180}{2.3496} = 76.61 < 133.68$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(29{,}000)}{(76.61)^2} = 48.767 \text{ ksi}$$

$$\sigma_{\text{cr}} = [0.658^{\sigma_Y/\sigma_e}]\sigma_Y = [0.658^{36/48.767}](36) = 26.431 \text{ ksi}$$

$$P_U = A\sigma_{\text{cr}} = (5.75)(26.431) = 151.98 \text{ kips} < 171.11 \text{ kips}$$

Thickness is too small.

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PROBLEM 10.88* ***(Continued)***

Since P_U is approximately proportional to thickness, the required thickness is approximately

$$\frac{t_{\text{reg}}}{0.25} = \frac{P_{U(\text{reg})}}{151.98} = \frac{171.11}{151.98} \qquad t_{\text{reg}} = 0.2815 \text{ in.}$$

Try

$$t = \frac{5}{16} \text{ in.} = 0.3125 \text{ in.} \qquad b_i = 5.375 \text{ in.}$$

$$A = 7.1094 \text{ in}^2$$

$$I = 38.44 \text{ in}^4 \qquad r = 2.3254 \text{ in.}$$

$$\frac{L_e}{r} = 77.41 < 133.68$$

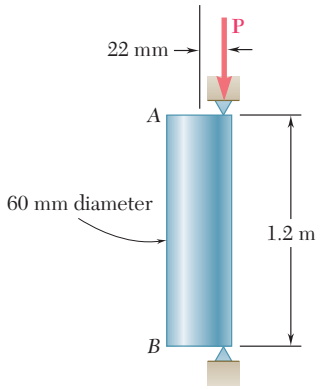
$$\sigma_e = \frac{\pi^2(29{,}000)}{(77.41)^2} = 47.764 \text{ ksi}$$

$$\sigma_{\text{cr}} = [0.658^{36/47.764}](36) = 26.26 \text{ ksi}$$

$$P_U = (7.1094)(26.26) = 186.69 \text{ kips} > 171.11 \text{ kips}$$

Use $t = \frac{5}{16}$ in. ◀

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PROBLEM 10.89

An eccentric load is applied at a point 22 mm from the geometric axis of a 60-mm-diameter rod made of a steel for which $\sigma_Y = 250$ MPa and $E = 200$ GPa. Using the allowable-stress method, determine the allowable load **P**.

SOLUTION

For the solid circular cross section, $c = \frac{1}{2}d = 30 \text{ mm} = 0.030 \text{ m}$

$$A = \pi c^2 = \pi(0.030)^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

$$I = \frac{\pi}{4}c^4 \qquad r = \sqrt{\frac{I}{A}} = \frac{1}{2}c = 0.015 \text{ m} = 15 \text{ mm}$$

$$L = 1.2 \text{ m} \qquad \frac{L}{r} = \frac{1.2}{0.015} = 80$$

Steel: $E = 200 \times 10^3$ MPa, $\sigma_Y = 250$ MPa

transition $\frac{L}{r} = 4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200 \times 10^3}{250}} = 133.22$

$$\sigma_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2(200 \times 10^3)}{(80)^2} = 308.43 \text{ MPa}$$

$$\sigma_{\text{cr}} = (0.658^{\sigma_Y/\sigma_e}) = [0.658^{250/308.43}](250) = 178.07 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{1.67} = 106.63 \text{ MPa}$$

Eccentric loading: $M = Pe \qquad e = 22$ mm

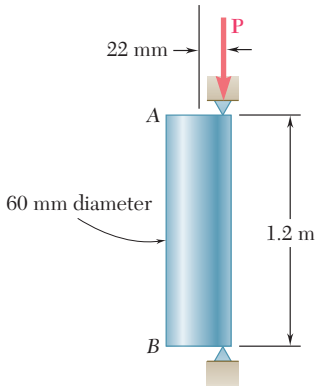
$$\sigma_{\text{all}} = \frac{P}{A} + \frac{Mc}{I} = P\left(\frac{1}{A} + \frac{ec}{I}\right) = \frac{P}{A}\left(1 + \frac{ec}{r^2}\right)$$

$$= \frac{P}{A}\left(1 + \frac{(22)(30)}{(15)^2}\right) = 3.9333\frac{P}{A}$$

$$P = \frac{\sigma_{\text{all}}A}{3.9333} = \frac{(106.63 \times 10^6)(2.8274 \times 10^{-3})}{3.9333} = 76.7 \times 10^3 \text{ N}$$

Allowable load. $P = 76.7$ kN ◀

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PROBLEM 10.90

Solve Prob. 10.89, assuming that the load is applied at a point 40 mm from the geometric axis and that the effective length is 0.9 m.

PROBLEM 10.89 An eccentric load is applied at a point 22 mm from the geometric axis of a 60-mm-diameter rod made of a steel for which $\sigma_Y = 250$ MPa and $E = 200$ GPa. Using the allowable-stress method, determine the allowable load **P**.

SOLUTION

For the solid circular cross section, $c = \frac{1}{2}d = 30 \text{ mm} = 0.030 \text{ m}$

$$A = \pi c^2 = \pi(0.030)^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

$$I = \frac{\pi}{4}c^4 \qquad r = \sqrt{\frac{I}{A}} = \frac{1}{2}c = 0.015 \text{ m} = 15 \text{ mm}$$

$$L = 0.9 \text{ m} \qquad \frac{L}{r} = \frac{0.9}{0.015} = 60$$

Steel: $E = 200 \times 10^3$ MPa, $\sigma_Y = 250$ MPa

$$\text{transition } \frac{L}{r} = 4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200 \times 10^3}{250}} = 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2(200 \times 10^3)}{(60)^2} = 548.31 \text{ MPa}$$

$$\sigma_{\text{cr}} = [0.658^{\sigma_Y/\sigma_e}] = [0.658^{(250/548.30)}](250) = 206.57 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{1.67} = 123.69 \text{ MPa} = 123.69 \times 10^6 \text{ Pa}$$

Eccentric loading: $M = Pe \qquad e = 40$ mm

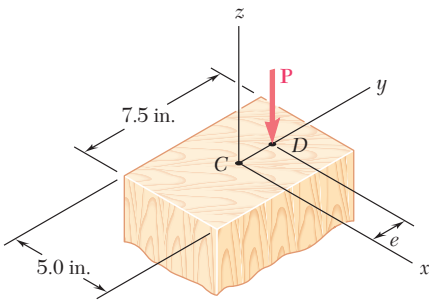
$$\sigma_{\text{all}} = \frac{P}{A} + \frac{Mc}{I} = P\left(\frac{1}{A} + \frac{ec}{I}\right) = \frac{P}{A}\left(1 + \frac{ec}{r^2}\right)$$

$$= \frac{P}{A}\left(1 + \frac{(40)(30)}{(15)^2}\right) = 6.3333\frac{P}{A}$$

$$P = \frac{\sigma_{\text{all}}A}{6.3333} = \frac{(123.69 \times 10^6)(2.8274 \times 10^{-3})}{6.3333} = 55.2 \times 10^3 \text{ N}$$

Allowable load. $P = 55.2$ kN ◀

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PROBLEM 10.91

A sawn-lumber column of 5.0 × 7.5-in. cross section has an effective length of 8.5 ft. The grade of wood used has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 1180$ psi and an adjusted modulus $E = 440 \times 10^3$ psi. Using the allowable-stress method, determine the largest eccentric load **P** that can be applied when (*a*) $e = 0.5$ in., (*b*) $e = 1.0$ in.

SOLUTION

Sawn lumber:

$$c = 0.8$$

$$L_e = 8.5 \text{ ft} = 102 \text{ in.}$$

$$b = 7.5 \text{ in.} \quad d = 5.0 \text{ in.} \qquad c = \frac{b}{2} = 3.75 \text{ in.}$$

$$A = bd = (7.5)(5.0) = 37.5 \text{ in}^2 \qquad I_x = \frac{1}{12}(5.0)(7.5)^3 = 175.78 \text{ in}^4$$

$$\frac{L}{d} = \frac{102}{5.0} = 20.4 \qquad \sigma_{CE} = \frac{0.822E}{(L/d)^2} = \frac{(0.822)(440 \times 10^3)}{(20.4)^2} = 869.1 \text{ psi}$$

$$\sigma_{CE}/\sigma_C = 869.1/1180 = 0.73652$$

$$C_P = \frac{1 + \sigma_{CE}/\sigma_C}{2c} - \sqrt{\left(\frac{1 + \sigma_{CE}/\sigma_C}{2c}\right)^2 - \frac{\sigma_{CE}/\sigma_C}{c}} = 0.5777$$

$$\sigma_{\text{all}} = \sigma_C C_P = (1180)(0.5777) = 681.7 \text{ psi}$$

$$\frac{P_{\text{all}}}{A} + \frac{P_{\text{all}}\, ec}{I_x} = \sigma_{\text{all}} = BP_{\text{all}} \quad \text{where} \quad B = \frac{1}{A} + \frac{ec}{I_x}.$$

$$P_{\text{all}} = \frac{\sigma_{\text{all}}}{B}$$

(*a*) $e = 0.5$ in.

$$B = \frac{1}{37.5} + \frac{(0.5)(3.75)}{175.78} = 0.037333 \text{ in}^{-2}$$

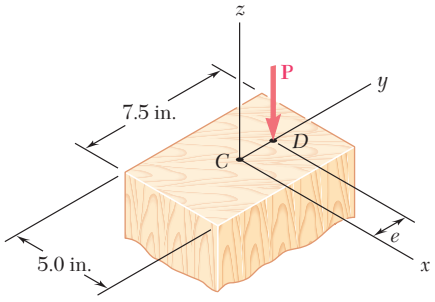
$$P_{\text{all}} = \frac{681.7}{0.037333} = 18{,}260 \text{ lb} \qquad P_{\text{all}} = 18.26 \text{ kips} \blacktriangleleft$$

(*b*) $e = 1.0$ in.

$$B = \frac{1}{37.5} + \frac{(1.0)(3.75)}{175.78} = 0.048000 \text{ in}^{-1}$$

$$P_{\text{all}} = \frac{681.7}{0.048000} = 14{,}202 \text{ lb} \qquad P_{\text{all}} = 14.20 \text{ kips} \blacktriangleleft$$

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PROBLEM 10.92

Solve Prob. 10.91 using the interaction method and an allowable stress in bending of 1300 psi.

PROBLEM 10.91 A sawn-lumber column of 5.0×7.5-in. cross section has an effective length of 8.5 ft. The grade of wood used has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 1180$ psi and an adjusted modulus $E = 440 \times 10^3$ psi. Using the allowable-stress method, determine the largest eccentric load **P** that can be applied when (*a*) $e = 0.5$ in., (*b*) $e = 1.0$ in.

SOLUTION

Sawn lumber:

$$c = 0.8$$

$$L_e = 8.5 \text{ ft} = 102 \text{ in.}$$

$$b = 7.5 \text{ in.} \qquad d = 5.0 \text{ in.} \qquad c = \frac{b}{2} = 3.75 \text{ in.}$$

$$A = bd = (7.5)(5.0) = 37.5 \text{ in}^2$$

$$I_x = \frac{1}{12}(5.0)(7.5)^3 = 175.78 \text{ in}^4$$

$$\frac{L}{d} = \frac{102}{5.0} = 20.4$$

$$\sigma_{CE} = \frac{0.822E}{(L/d)^2} = \frac{(0.822)(440 \times 10^3)}{(20.4)^2} = 869.1 \text{ psi}$$

$$\sigma_{CE}/\sigma_C = 869.1/1180 = 0.73652$$

$$C_P = \frac{1 + \sigma_{CE}/\sigma_C}{2c} + \sqrt{\left(\frac{1 + \sigma_{CE}/\sigma_C}{2c}\right)^2 - \frac{\sigma_{CE}/\sigma_C}{c}} = 0.5777$$

$$\sigma_{\text{all, centric}} = \sigma_C C_P = (1180)(0.5777) = 681.7 \text{ psi}$$

$$\frac{P_{\text{all}}}{A\sigma_{\text{all, centric}}} + \frac{P_{\text{all}}\, ec}{I_x\, \sigma_{\text{all, bending}}} = 1 \qquad P_{\text{all}} = B^{-1} \text{ (lb)}$$

where

$$B = \frac{1}{A\sigma_{\text{all, centric}}} + \frac{ec}{I_x \sigma_{\text{all, bending}}} \text{ (lb}^{-1}\text{)}$$

(*a*) $\underline{e = 0.5 \text{ in.}}$

$$B = \frac{1}{(37.5)(681.7)} + \frac{(0.5)(3.75)}{(175.78)(1300)} = 47.323 \times 10^{-6} \text{ lb}^{-1}$$

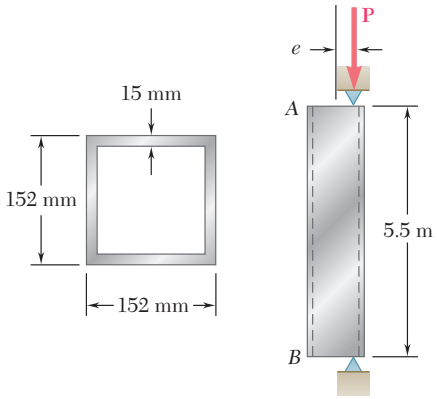
$$P_{\text{all}} = (47.323 \times 10^{-6})^{-1} = 21{,}131 \text{ lb} \qquad P_{\text{all}} = 21.1 \text{ kips} \blacktriangleleft$$

(*b*) $\underline{e = 1.0 \text{ in.}}$

$$B = \frac{1}{(37.5)(681.7)} + \frac{(1.0)(3.75)}{(175.78)(1300)} = 55.528 \times 10^{-6} \text{ lb}^{-1}$$

$$P_{\text{all}} = (55.528 \times 10^{-6})^{-1} = 18{,}009 \text{ lb} \qquad P_{\text{all}} = 18.01 \text{ kips} \blacktriangleleft$$

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PROBLEM 10.93

A column of 5.5-m effective length is made of the aluminum alloy 2014-T6, for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load **P**, knowing that the eccentricity is (*a*) $e = 0$, (*b*) $e = 40$ mm.

SOLUTION

$$b_o = 152 \text{ mm} \qquad b_i = b_o - 2t = 122 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 8220 \text{ mm}^2 = 8220 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}\left(b_o^4 - b_i^4\right) = 26.02 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 56.26 \text{ mm} = 56.26 \times 10^{-3} \text{ m}$$

$$\frac{L}{r} = \frac{5.5}{56.26 \times 10^{-3}} = 97.76 > 55$$

$$\sigma_{\text{all},c} = \frac{382 \times 10^3}{(L/r)^2} = \frac{382 \times 10^3}{(97.76)^2} = 39.98 \text{ MPa} \quad \text{for centric loading}$$

$$\frac{P}{A\sigma_{\text{all},c}} + \frac{Pec}{I\sigma_{\text{all},b}} = 1$$

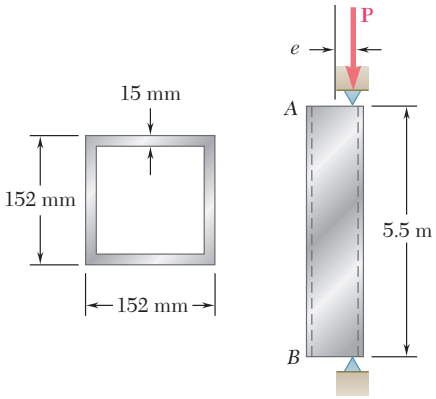
(*a*) $e = 0$: $P = A\sigma_{\text{all},c} = (8220 \times 10^{-6})(39.97 \times 10^6) = 329 \times 10^3$ N $\qquad P = 329$ kN ◀

(*b*) $e = 40 \times 10^{-3}$ m: $\qquad c = \frac{1}{2}(152) = 76 \text{ mm} = 76 \times 10^{-3}$ m

$$\frac{P}{(8220 \times 10^{-6})(39.98 \times 10^6)} + \frac{P(40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^{-6})(220 \times 10^6)} = 3.574 \times 10^{-6} P = 1$$

$P = 279.8 \times 10^3$ N $\qquad P = 280$ kN ◀

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PROBLEM 10.94

Solve Prob. 10.93, assuming that the effective length of the column is 3.0 m.

PROBLEM 10.93 A column of 5.5-m effective length is made of the aluminum alloy 2014-T6, for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load **P**, knowing that the eccentricity is (*a*) $e = 0$, (*b*) $e = 40$ mm.

SOLUTION

$$b_o = 152 \text{ mm} \qquad b_i = b_o - 2t = 122 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 8220 \text{ mm}^2 = 8220 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}\left(b_o^4 - b_i^4\right) = 26.02 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 56.26 \text{ mm} = 56.26 \times 10^{-3} \text{ m}$$

$$\frac{L}{r} = \frac{3.0}{56.26 \times 10^{-3}} = 53.32 < 55 \qquad \text{(2014-T6 aluminum alloy)}$$

$$\sigma_{\text{all},c} = 213 - 1.577(L/r) = 213 - (1.577)(53.32) = 128.91 \text{ MPa}$$

$$\frac{P}{A\sigma_{\text{all},c}} + \frac{Pec}{I\sigma_{\text{all},b}} = 1$$

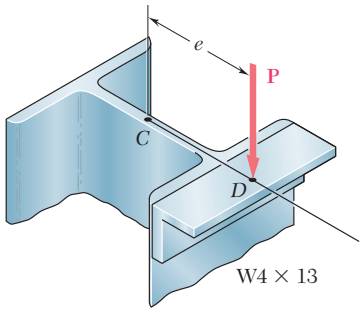
(*a*) $e = 0$: $P = A\sigma_{\text{all},c} = (8220 \times 10^{-6})(128.91 \times 10^6) = 1060 \times 10^3$ N $\qquad P = 1060$ kN ◀

(*b*) $e = 40 \text{ mm} = 40 \times 10^{-3}$ m: $\qquad c = \frac{1}{2}(152) = 76 \text{ mm} = 76 \times 10^{-3}$ m

$$\frac{P}{(8220 \times 10^{-6})(128.91 \times 10^6)} + \frac{P(40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^{-6})(220 \times 10^6)} = 1.4747 \times 10^{-6} P = 1$$

$P = 678 \times 10^3$ N $\qquad P = 678$ kN ◀

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PROBLEM 10.95

A steel compression member of 9-ft effective length supports an eccentric load as shown. Using the allowable-stress method, determine the maximum allowable eccentricity e if (a) $P = 30$ kips, (b) $P = 18$ kips. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

W4 × 13: $A = 3.83 \text{ in}^2$, $S_x = 5.46 \text{ in}^3$

$r_y = 1.00$ in. $L_e = 9$ ft $= 108$ in. $L_e/r_y = 108$

$$\frac{L}{r} = 4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{29 \times 10^3}{36}} = 133.7 > 108$$

$$\sigma_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 (29 \times 10^3)}{(108)^2} = 24.539$$

$$\sigma_{cr} = [0.658^{\sigma_Y/\sigma_e}]\,\sigma_Y = [0.658^{(36/24.539)}](36) = 19.4819 \text{ ksi}$$

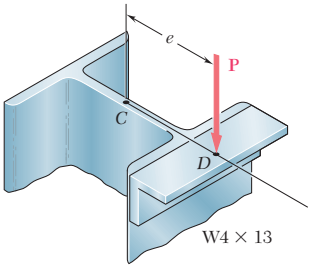
$$\sigma_{all} = \frac{\sigma_{cr}}{1.67} = \frac{19.4819}{1.67} = 11.6658 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pec}{I_x} = \frac{P}{A} + \frac{Pe}{S_x} = \sigma_{all} \qquad e = \frac{S_x}{P}\left(\sigma_{all} - \frac{P}{A}\right)$$

(a) $P = 30$ kips: $e = \dfrac{5.46}{30}\left(11.6658 - \dfrac{30}{3.83}\right) = 0.698$ in. ◀

(b) $P = 18$ kips: $e = \dfrac{5.46}{18}\left(11.6658 - \dfrac{18}{3.83}\right) = 2.11$ in. ◀

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PROBLEM 10.96

Solve Prob. 10.95, assuming that the effective length of the column is increased to 12 ft and that (*a*) $P = 20$ kips, (*b*) $P = 15$ kips.

PROBLEM 10.95 A steel compression member of 9-ft effective length supports an eccentric load as shown. Using the allowable-stress method, determine the maximum allowable eccentricity e if (*a*) $P = 30$ kips, (*b*) $P = 18$ kips. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

W4 × 13: $A = 3.83 \text{ in}^2$, $S_x = 5.46 \text{ in}^2$

$r_y = 1.00$ in. $L_e = 12$ ft $= 144$ in. $L_e/r = 144$

$$\frac{L}{r} = 4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{29 \times 10^3}{36}} = 133.7 < 144$$

$$\sigma_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 (29 \times 10^3)}{(144)^2} = 13.803 \text{ ksi}$$

$$\sigma_{\text{cr}} = 0.877\sigma_e = 0.877(13.803) = 12.105 \text{ ksi}$$

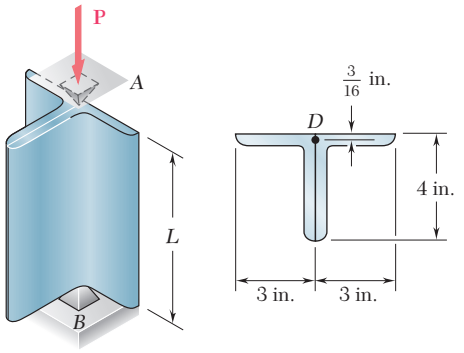
$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{1.67} = \frac{12.105}{1.67} = 7.2486 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pec}{I_x} = \frac{P}{A} + \frac{Pe}{S_x} = \sigma_{\text{all}} \qquad e = \frac{S_x}{P}\left(\sigma_{\text{all}} - \frac{P}{A}\right)$$

(*a*) $P = 20$ kips: $e = \dfrac{5.46}{20}\left(7.2486 - \dfrac{20}{3.83}\right) = 0.553$ in. ◀

(*b*) $P = 15$ kips: $e = \dfrac{5.46}{18}\left(7.2486 - \dfrac{15}{3.83}\right) = 1.213$ in. ◀

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PROBLEM 10.97

Two L4 × 3 × $\frac{3}{8}$-in. steel angles are welded together to form the column *AB*. An axial load **P** of magnitude 14 kips is applied at point *D*. Using the allowable-stress method, determine the largest allowable length *L*. Assume $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

One angle: L4 × 3 × $\frac{3}{8}$ $\quad A = 2.48 \text{ in}^2$

$$I_x = 3.94 \text{ in}^4, \quad S_x = 1.44 \text{ in}^3, \quad r_x = 1.26 \text{ in.}, \quad y = 1.27 \text{ in.}$$

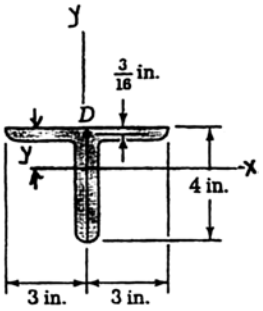
$$I_y = 1.89 \text{ in}^4, \quad r_y = 0.873 \text{ in.}, \quad x = 0.775 \text{ in.}$$

Two angles: $\quad A = 2(2.48) = 4.96 \text{ in}^2, \quad I_x = 2(3.94) = 7.88 \text{ in}^4$

$$r_x = 1.26 \text{ in.}, \quad y = 1.27 \text{ in.}$$

$$I_y = 2[I_y + Ax^2] = 2[1.89 + (2.48)(0.775)^2]$$

$$= 6.7591 \text{ in}^4$$



$$r_{\min} = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{6.7591}{4.96}} = 1.1674 \text{ in.}$$

$$e = y - \frac{3}{16} = 1.27 - \frac{3}{16} = 1.0825 \text{ in.}$$

$$P = 14 \text{ kips}$$

$$\sigma_{\text{all}} = \frac{P}{A} + \frac{Pe_y}{I_x} = \frac{14}{4.96} + \frac{(14)(1.0825)(1.27)}{7.88} = 5.265 \text{ ksi}$$

$$\frac{L}{r} = 4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{29{,}000}{36}} = 133.7$$

Assume $\frac{L_e}{r} > 133.7$

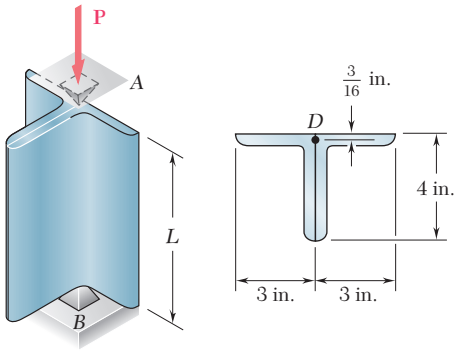
$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{1.67} = 0.877\frac{\pi^2 E}{1.67(L/r)^2} = 5.265$$

$$\frac{L}{r} = \sqrt{\frac{(0.877)\pi^2(29{,}000)}{1.67(5.265)}} = 168.96 > 133.7$$

$$L = (168.96)(1.1674) = 197.2 \text{ in.}$$

$$L = 16.44 \text{ ft} \blacktriangleleft$$

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PROBLEM 10.98

Solve Prob. 10.97 using the interaction method with $P = 18$ kips and an allowable stress in bending of 22 ksi.

PROBLEM 10.97 Two L4 × 3 × $\frac{3}{8}$-in. steel angles are welded together to form the column *AB*. An axial load **P** of magnitude 14 kips is applied at point *D*. Using the allowable-stress method, determine the largest allowable length *L*. Assume $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

One angle: L4 × 3 × $\frac{3}{8}$ $\quad A = 2.48 \text{ in}^2$

$$I_x = 3.94 \text{ in}^4, \quad S_x = 1.44 \text{ in}^3, \quad r_x = 1.26 \text{ in.}, \quad y = 1.27 \text{ in.}$$

$$I_y = 1.89 \text{ in}^4, \quad r_y = 0.873 \text{ in.}, \quad x = 0.775 \text{ in.}$$

Two angles: $A = 2(2.48) = 4.96 \text{ in}^2, \quad I_x = 2(3.94) = 7.88 \text{ in}^4$

$$r_x = 1.26 \text{ in.}, \quad y = 1.27 \text{ in.}$$

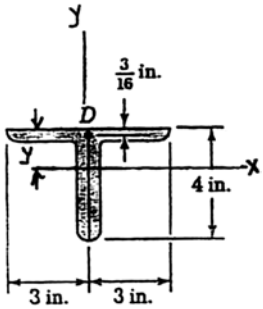
$$I_y = 2[I_y + Ax^2] = 2[1.89 + (2.48)(0.775)^2]$$

$$= 6.7591 \text{ in}^4$$

$$r_{\min} = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{6.7591}{4.96}} = 1.16736 \text{ in.}$$

$$e = y - \frac{3}{16} = 1.27 - \frac{3}{16} = 1.0825 \text{ in.}$$

$$P = 18 \text{ kips}$$



$$\frac{P}{A\sigma_{\text{all},c}} + \frac{P_{ey}}{I_x\sigma_{\text{all},b}} = 1 \qquad \frac{1}{\sigma_{\text{all},c}} = \frac{A}{P}\left(1 - \frac{P_{ey}}{I_x\sigma_{\text{all},b}}\right)$$

$$\frac{1}{\sigma_{\text{all},c}} = \frac{4.96}{18}\left[1 - \frac{(18)(1.0825)(1.27)}{(7.88)(22)}\right]$$

$$\sigma_{\text{all},c} = 4.2333 \text{ ksi}$$

$$\frac{L}{r} = 4.71\sqrt{\frac{E}{\sigma_y}} = 4.71\sqrt{\frac{29{,}000}{36}} = 133.7$$

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PROBLEM 10.98 *(Continued)*

Assume $\frac{L_e}{r} > 133.7$

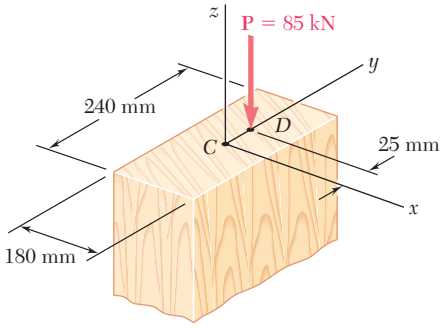
$$\sigma_{\text{all}} = \frac{\sigma_{cr}}{1.67} = 0.877\frac{\pi^2 E}{1.67(L/r)^2} = 4.2333$$

$$\frac{L}{r} = \sqrt{\frac{(0.877)\pi^2(29{,}000)}{1.67(4.2333)}} = 188.43 > 133.7$$

$$L = (188.43)(1.16736) = 219.97 \text{ in.}$$

$L = 18.33$ ft ◀

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PROBLEM 10.99

A rectangular column is made of a grade of sawn wood that has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 8.3$ MPa and a modulus of elasticity $E = 11.1$ GPa. Using the allowable-stress method, determine the largest allowable effective length L that can be used.

SOLUTION

$E = 11{,}100$ MPa $\quad d = 180$ mm $= 0.180$ m $\quad b = 240$ mm $= 0.240$ m

$$A = bd = 43.2 \times 10^{-3} \text{ m}^2 \qquad I_x = \frac{1}{12}db^3 = \frac{1}{12}(0.180)(0.240)^3 = 207.36 \times 10^{-6} \text{ m}^4$$

$$e = 25 \text{ mm} = 0.025 \text{ m} \qquad c = \frac{b}{2} = 0.120 \text{ m}$$

$$\frac{P}{A} + \frac{Pec}{I} \le \sigma_{\text{all}} \qquad \sigma_{\text{all}} \ge \frac{85 \times 10^3}{43.2 \times 10^{-3}} + \frac{(85 \times 10^3)(0.025)(0.120)}{207.36 \times 10^{-6}} = 3.1973 \times 10^6 \text{ Pa}$$

$$= 3.1973 \text{ MPa}$$

Define y and x as $\qquad C_p = \frac{\sigma_{\text{all}}}{\sigma_C} = \frac{3.1973}{8.3} = 0.38522 = y \qquad x = \sigma_{CE}/\sigma_C$

$$y = \frac{1+x}{2c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}} \qquad \text{where } c = 0.8 \text{ for sawn lumber.}$$

$$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$$

$$\left(\frac{1+x}{2c}\right)^2 - y\left(\frac{1+x}{c}\right) + y^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

$$x = y\frac{(1-cy)}{1-y} = (0.38522)\frac{1-(0.8)(0.38522)}{1-0.38522} = 0.43350$$

$$\sigma_{CE} = \sigma_C(0.43350) = (8.3)(0.43350) = 3.598 \text{ MPa}$$

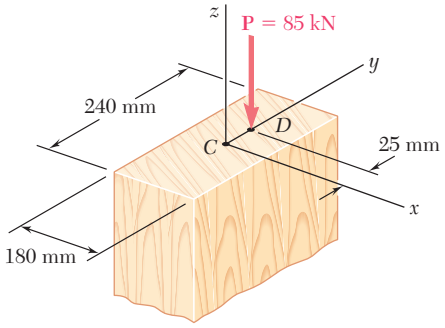
$$\sigma_{CE} = \frac{K_{CE}E}{(L/d)^2} \qquad L^2 = \frac{K_{CE}Ed^2}{\sigma_{CE}} \qquad \text{where } K_{CE} = 0.300$$

Largest allowable effective length.

$$L = d\sqrt{\frac{K_{CE}E}{\sigma_{CE}}} = (0.180)\sqrt{\frac{(0.300)(11{,}100)}{3.598}}$$

$$L = 5.48 \text{ m} \blacktriangleleft$$

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PROBLEM 10.100

Solve Prob. 10.99, summing that $P = 105$ kN.

PROBLEM 10.99 A rectangular column is made of a grade of sawn wood that has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 8.3$ MPa and a modulus of elasticity $E = 11.1$ GPa. Using the allowable-stress method, determine the largest allowable effective length L that can be used.

SOLUTION

$E = 11{,}100$ MPa $\quad d = 180$ mm $= 0.180$ m $\quad b = 240$ mm $= 0.240$ m

$$A = bd = 43.2 \times 10^{-3}\ \text{m}^2 \qquad I_x = \frac{1}{12}db^3 = \frac{1}{12}(0.180)(0.240)^3 = 207.36 \times 10^{-6}\ \text{m}^4$$

$$e = 25\ \text{mm} = 0.025\ \text{m} \qquad c = \frac{b}{2} = 0.120\ \text{m}$$

$$\frac{P}{A} + \frac{Pec}{I_x} \le \sigma_{\text{all}} \qquad \sigma_{\text{all}} \ge \frac{105 \times 10^3}{43.2 \times 10^{-3}} + \frac{(105 \times 10^3)(0.025)(0.120)}{207.36 \times 10^{-6}} = 3.9496 \times 10^6\ \text{Pa}$$

$$= 3.9496\ \text{MPa}$$

Define y and x as $\quad C_p = \dfrac{\sigma_{\text{all}}}{\sigma_C} = \dfrac{3.9496}{8.3} = 0.47586 = y \qquad x = \sigma_{CE}/\sigma_C$

$$y = \frac{1+x}{c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}} \qquad \text{where } c = 0.8 \text{ for sawn lumber.}$$

$$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$$

$$\left(\frac{1+x}{2c} - y\right)^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

$$\left(\frac{1+x}{2c}\right)^2 - \frac{1+x}{c}y + y^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

$$x = y\frac{1-cy}{1-y} = (0.47586)\frac{1-(0.8)(0.47586)}{1-0.47586} = 0.56227$$

$$\sigma_{CE} = \sigma_C(0.56227) = (8.3)(0.56227) = 4.6668\ \text{MPa}$$

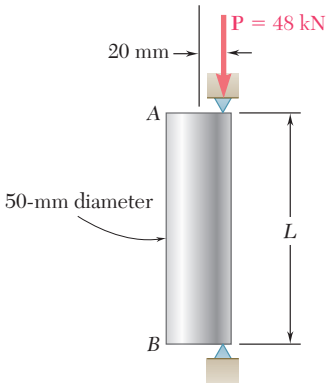
$$\sigma_{CE} = \frac{K_{CE}E}{(L/d)^2} \qquad L^2 = \frac{K_{CE}Ed^2}{\sigma_{CE}} \qquad \text{where } K_{CE} = 0.300.$$

Largest allowable effective length.

$$L = d\sqrt{\frac{K_{CE}E}{\sigma_{CE}}} = 0.180\sqrt{\frac{(0.300)(11{,}100)}{4.6668}}$$

$$L = 4.81\ \text{m} \blacktriangleleft$$

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PROBLEM 10.101

An eccentric load $P = 48$ kN is applied at a point 20 mm from the geometric axis of a 50-mm-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 145 MPa, determine the largest allowable effective length L that can be used.

SOLUTION

$$c = \frac{1}{2}d = 25\text{ mm} \qquad A = \pi c^2 = 1.9635 \times 10^3\text{ mm}^2$$

$$I = \frac{\pi}{4}c^4 = 306.8 \times 10^3\text{ mm}^4 \qquad r = \sqrt{\frac{I}{A}} = 12.5\text{ mm}$$

$$e = 20\text{ mm} \qquad \sigma_{\text{all},b} = 145 \times 10^6\text{ Pa}$$

$$\frac{P}{A\sigma_{\text{all},c}} + \frac{Pec}{I\sigma_{\text{all},b}} = 1$$

$$\frac{P}{A\sigma_{\text{all},c}} = 1 - \frac{Pec}{I\sigma_{\text{all},b}}$$

$$\frac{1}{\sigma_{\text{all},c}} = \frac{A}{P}\left[1 - \frac{P_{ec}}{I\sigma_{\text{all},b}}\right]$$

$$= \frac{1.9635 \times 10^{-3}}{48 \times 10^3}\left[1 - \frac{(48 \times 10^3)(20 \times 10^{-3})(25 \times 10^{-3})}{(306.8 \times 10^{-9})(145 \times 10^6)}\right]$$

$$= 18.838 \times 10^{-9}\text{ Pa}^{-1}$$

$$\sigma_{\text{all},c} = 53.086 \times 10^6\text{ Pa} = 53.086\text{ MPa}$$

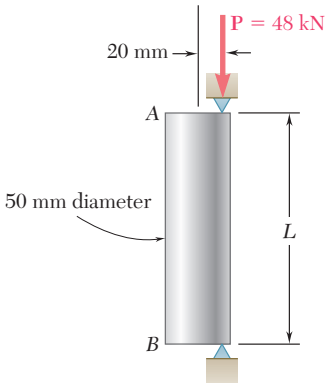
Assume $\dfrac{L}{r} > 66.$ $\qquad \sigma_{\text{all},c} = \dfrac{354 \times 10^3}{(L/r)^2}$

$$\frac{L}{r} = \sqrt{\frac{354 \times 10^3}{\sigma_{\text{all},c}}} = \sqrt{\frac{354 \times 10^3}{53.086}} = 81.660 > 66$$

$$L = 81.660r = (81.660)(12.5 \times 10^{-3})$$

$$L = 1.021\text{ m} \blacktriangleleft$$

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PROBLEM 10.102

Solve Prob. 10.101, assuming that the aluminum alloy used is 2014-T6 and that the allowable stress in bending is 180 MPa.

PROBLEM 10.101 An eccentric load $P = 48$ kN is applied at a point 20 mm from the geometric axis of a 50-mm-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 145 MPa, determine the largest allowable effective length L that can be used.

SOLUTION

$$c = \frac{1}{2}d = 25\text{ mm} \qquad A = \pi c^2 = 1.9635 \times 10^3\text{ mm}^2$$

$$I = \frac{\pi}{4}c^4 = 306.8 \times 10^3\text{ mm}^4 \qquad r = \sqrt{\frac{I}{A}} = 12.5\text{ mm}$$

$$e = 20\text{ mm} \qquad \sigma_{\text{all},b} = 180 \times 10^6\text{ Pa}$$

$$\frac{P}{A\sigma_{\text{all},c}} + \frac{Pec}{I\sigma_{\text{all},b}} = 1$$

$$\frac{P}{A\sigma_{\text{all},c}} = 1 - \frac{Pec}{I\sigma_{\text{all},b}}$$

$$\frac{1}{\sigma_{\text{all},c}} = \frac{A}{P}\left[1 - \frac{P_{ec}}{I\sigma_{\text{all},b}}\right]$$

$$= \frac{1.9635 \times 10^{-3}}{48 \times 10^3}\left[1 - \frac{(48 \times 10^3)(20 \times 10^{-3})(25 \times 10^{-3})}{(306.8 \times 10^{-9})(180 \times 10^6)}\right]$$

$$= 23.129 \times 10^{-9}\text{ Pa}^{-1} \qquad \sigma_{\text{all},c} = 43.237 \times 10^6\text{ Pa} = 43.237\text{ MPa}$$

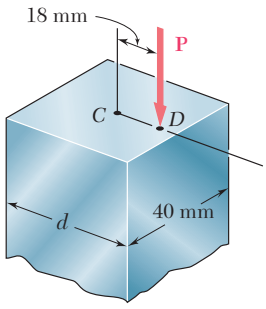
Assume $\frac{L}{r} > 55$. $\qquad \sigma_{\text{all},c} = \frac{382 \times 10^3}{(L/r)^2}$

$$\frac{L}{r} = \sqrt{\frac{382 \times 10^3}{\sigma_{\text{all},c}}} = \sqrt{\frac{382 \times 10^3}{43.236}} = 93.995 > 55$$

$$L = 93.996r = (93.996)(12.5 \times 10^{-3})$$

$$L = 1.175\text{ m} \blacktriangleleft$$

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PROBLEM 10.103

A compression member made of steel has a 720-mm effective length and must support the 198-kN load **P** as shown. For the material used, $\sigma_Y = 250$ MPa and $E = 200$ GPa. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension d of the cross section that can be used.

SOLUTION

Using dimensions in meters,

$$A = 40 \times 10^{-3} d \qquad L_e = 720 \text{ mm} = 0.720 \text{ m}$$

$$I_x = \frac{1}{12}(40 \times 10^{-3}) d = 5.3333 \times 10^{-6} d$$

$$I_y = \frac{1}{12}(40 \times 10^{-3}) d^3 = 3.3333 \times 10^{-3} d^3$$

$$|x| = \frac{d}{2}, \quad |y| = 20 \text{ mm} = 0.020 \text{ m} \quad |e_x| = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

Transition $\frac{L}{r}$: $4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$

Assume $d > 40 \text{ mm} = 40 \times 10^{-3}$ m. Then $I_{\min} = I_x$

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333 \times 10^{-6} d}{40 \times 10^{-3} d}} = 11.547 \times 10^{-3} \text{ m}, \quad \frac{L_e}{r} = 62.35 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(62.35)^2} = 507.76 \text{ MPa}$$

$$\sigma_{\text{all,centric}} = \frac{\sigma_{\text{cr}}}{1.67} = \frac{1}{1.67}(0.658^{250/507.76})(250) = 121.82 \text{ MPa}$$

$$\sigma_{\text{all,bending}} = 150 \text{ MPa}$$

$$\frac{P}{A\sigma_{\text{all, centric}}} + \frac{Pe_x x}{I_y \sigma_{\text{all,bending}}} = 1$$

$$\frac{198 \times 10^3}{(40 \times 10^{-3} d)(127.82 \times 10^6)} + \frac{(198 \times 10^3)(18 \times 10^{-3})\left(\frac{1}{2} d\right)}{(3.3333 \times 10^{-3} d^3)(150 \times 10^6)} = 1$$

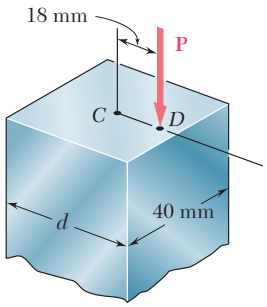
$$\frac{40.634 \times 10^{-3}}{d} + \frac{3.5640 \times 10^{-3}}{d^2} = 1$$

$$d^2 - 40.634 \times 10^{-3} d - 3.5640 \times 10^{-3} = 0$$

$$d = \frac{1}{2}\left\{40.634 \times 10^{-3} + \sqrt{(40.034 \times 10^{-3})^2 + (4)(3.5640 \times 10^{-3})}\right\}$$

$$d = 83.4 \times 10^{-3} \text{ m} > 40 \times 10^{-3} \text{ m} \qquad d = 83.4 \text{ mm} \blacktriangleleft$$

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PROBLEM 10.104

Solve Prob. 10.103, assuming that the effective length is 1.62 m and that the magnitude of P of the eccentric load is 128 kN.

PROBLEM 10.103 A compression member made of steel has a 720-mm effective length and must support the 198-kN load **P** as shown. For the material used, $\sigma_Y = 250$ MPa and $E = 200$ GPa. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension d of the cross section that can be used.

SOLUTION

Using dimensions in meters,

$$A = 40 \times 10^{-3} d \qquad L_e = 1.62 \text{ m}$$

$$I_x = \frac{1}{12}(40 \times 10^{-3})^3 d = 5.3333 \times 10^{-6} d$$

$$I_y = \frac{1}{12}(40 \times 10^{-3}) d^3 = 3.3333 \times 10^{-3} d^3$$

$$|x| = \frac{1}{2} d, \quad |y| = 20 \text{ mm} = 20 \times 10^{-3} \text{ m} \quad |e_x| = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

Transition $\frac{L}{r}$: $4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$

Assume $d > 40 \text{ mm} = 40 \times 10^{-3}$ m. Then $I_{\min} = I_x$

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333 \times 10^{-6} d}{3.3333 \times 10^{-3} d}} = 11.547 \times 10^{-3} \text{ m}, \quad \frac{L_e}{r} = 140.29 > 133.22$$

$$\sigma_{\text{cr}} = \frac{0.877\pi^2 E}{(L_e/r)^2} = \frac{0.877\pi^2 (200 \times 10^9)}{(140.29)^2} = 87.958 \text{ MPa}$$

$$\sigma_{\text{all,centric}} = \frac{\sigma_{cr}}{1.67} = 52.67 \text{ MPa}$$

$$\sigma_{\text{all,bending}} = 150 \text{ MPa} \qquad \frac{P}{A\,\sigma_{\text{all,centric}}} + \frac{P e_x x}{I_y\,\sigma_{\text{all,bending}}} = 1$$

$$\frac{128 \times 10^3}{(40 \times 10^{-3} d)(52.67 \times 10^6)} + \frac{(128 \times 10^3)(18 \times 10^{-3})\left(\frac{1}{2} d\right)}{(3.3333 \times 10^{-3} d^3)(150 \times 10^6)} = 1$$

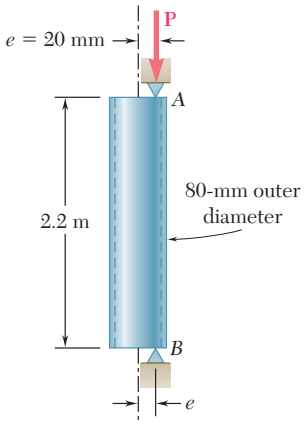
$$\frac{60.756 \times 10^{-3}}{d} + \frac{2.304 \times 10^{-3}}{d^2} = 1$$

$$d^2 - 60.756 \times 10^{-3} d - 2.304 \times 10^{-3} = 0$$

$$d = \frac{1}{2}\left\{60.756 \times 10^{-3} + \sqrt{(60.756 \times 10^{-3})^2 + (4)(2.304 \times 10^{-3})}\right\}$$

$$d = 87.2 \times 10^{-3} \text{ m} > 40 \times 10^{-3} \text{ m} \qquad d = 87.2 \text{ mm} \blacktriangleleft$$

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PROBLEM 10.105

A steel tube of 80-mm outer diameter is to carry a 93-kN load **P** with an eccentricity of 20 mm. The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm. Using the allowable-stress method, determine the lightest tube that can be used. Assume $E = 200$ GPa and $\sigma_Y = 250$ MPa.

SOLUTION

$$r_o = \frac{1}{2}d_o = 40 \text{ mm},$$

$$r_i = r_o - t$$

$$A = \pi(r_o^2 - r_i^2),$$

$$I = \frac{\pi}{4}(r_o^4 - r_i^4)$$

$$r = \sqrt{\frac{I}{A}}$$

t (mm)	r_i (mm)	A (mm^2)	I (10^6 mm^4)	r (mm)
3	37	726	0.539	27.24
6	34	1395	0.961	26.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$L_e = 2.2$ m

$P = 93 \times 10^3$ N

Transition L/r:
$$4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

Try $t = 9$ mm.
$$\frac{L_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(200 \times 10^9)}{(86.92)^2} = 261.27 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{1.67} = \frac{1}{1.67}[0.658^{250/261.27}](250) = 100.30 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2007 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.285 \times 10^{-6}}$$

$$= 104.2 \text{ MPa} > 100.3 \text{ MPa}$$

(not allowed)

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PROBLEM 10.105 ***(Continued)***

Approximate required area: $\left(\frac{104.2}{100.3}\right)(2007) = 2085 \text{ mm}^2$

Try $t = 12$ mm.

$$\frac{L_e}{r} = \frac{2.2}{24.41 \times 10^{-3}} = 90.12 < 133.22$$

$$\sigma_e = 243.05 \text{ MPa}$$

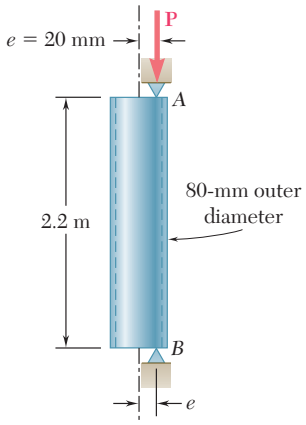
$$\sigma_{\text{all}} = 97.33 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2564 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.528 \times 10^{-6}}$$

$$= 85.0 \text{ MPa} < 97.3 \text{ MPa}$$

Use $t = 12.00$ mm. ◀

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PROBLEM 10.106

Solve Prob. 10.105, using the interaction method with $P = 165$ kN, $e = 15$ mm, and an allowable stress in bending of 150 MPa.

PROBLEM 10.105 A steel tube of 80-mm outer diameter is to carry a 93-kN load **P** with an eccentricity of 20 mm. The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm. Using the allowable-stress method, determine the lightest tube that can be used. Assume $E = 200$ GPa and $\sigma_Y = 250$ MPa.

SOLUTION

$$r_o = \frac{1}{2}d_o = 40 \text{ mm}$$

$$r_i = r_o - t$$

$$A = \pi(r_o^2 - r_i^2)$$

$$I = \frac{\pi}{4}(r_o^4 - r_i^4)$$

$$r = \sqrt{\frac{I}{A}}$$

t (mm)	r_i (mm)	A (mm^2)	I (10^6 mm^4)	r (mm)
3	37	726	0.539	27.24
6	34	1395	0.961	26.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$L_e = 2.2$ m
$P = 165 \times 10^3$ N
$\sigma_{\text{all, bending}} = 150$ MPa

Transition L/r:

$$4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

Try $t = 9$ mm.

$$\frac{L_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(200 \times 10^9)}{(86.92)^2} = 261.27 \text{ MPa}$$

$$\sigma_{\text{all, centric}} = \frac{\sigma_{\text{cr}}}{1.67} = \frac{1}{1.67}[0.658^{250/261.27}](250) = 100.30 \text{ MPa}$$

$$\frac{P}{A\sigma_{\text{all, centric}}} + \frac{Pec}{I\sigma_{\text{all, bending}}} = \frac{165 \times 10^3}{(2007 \times 10^{-6})(100.3 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.285 \times 10^{-6})(150 \times 10^6)}$$

$$= 0.820 + 0.514 = 1.334 > 1 \quad \text{(not allowed)}$$

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PROBLEM 10.106 *(Continued)*

Approximate required area: $A = (1.334)(2007) = 2677 \text{ mm}^2$

Try $t = 12$ mm. $\dfrac{L_e}{r} = \dfrac{2.2}{24.41 \times 10^{-3}} = 90.12 < 133.22$

$$\sigma_e = 243.05 \text{ MPa}$$

$$\sigma_{\text{all, centric}} = 97.33 \text{ MPa}$$

$$\frac{P}{A\sigma_{\text{all, centric}}} + \frac{Pec}{I\sigma_{\text{all, bending}}} = \frac{165 \times 10^3}{(2564 \times 10^{-6})(97.33 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.528 \times 10^{-6})(150 \times 10^6)}$$

$0.661 + 0.432 = 1.093 > 1$ (not allowed)

Try $t = 15$ mm. $\dfrac{L_e}{r} = 93.26$

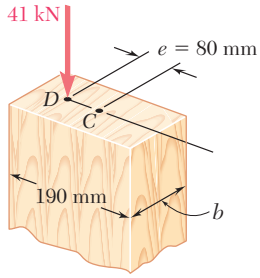
$$\sigma_e = 226.95 \text{ MPa}$$

$$\sigma_{\text{all, centric}} = 94.40 \text{ MPa}$$

$$\frac{165 \times 10^3}{(3063 \times 10^{-6})(94.40 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.704 \times 10^{-6})(150 \times 10^6)} = 0.958 < 1$$

Use $t = 15.00$ mm. ◀

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PROBLEM 10.107

A sawn lumber column of rectangular cross section has a 2.2-m effective length and supports a 41-kN load as shown. The sizes available for use have b equal to 90 mm, 140 mm, 190 mm, and 240 mm. The grade of wood has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 8.1\,\text{MPa}$ and $E = 8.3\,\text{GPa}$. Use the allowable-stress method to determine the lightest section that can be used.

SOLUTION

Sawn lumber: $\sigma_C = 8.1\,\text{MPa}$ $E = 8.3\,\text{GPa}$

$c = 0.8$ $K_{CE} = 0.300$

$L_e = 2.2\ \text{m}$

$$\frac{P_{\text{all}}}{A} + \frac{P_{\text{all}}ec}{I_x} = \sigma_{\text{all}}$$

$e = 80 \times 10^{-3}\ \text{m}$, $c = \frac{1}{2}(190) = 95\ \text{mm} = 95 \times 10^{-3}\ \text{m}$

$A = 0.190b\ \text{m}^2$ $I_x = \frac{1}{12}b(0.190)^3 = 571.58 \times 10^{-6}b\ \text{m}^4$

$$\frac{P_{\text{all}}}{0.190b} + \frac{P_{\text{all}}(80 \times 10^{-3})(95 \times 10^{-3})}{571.58 \times 10^{-6}b} = \frac{18.56P_{\text{all}}}{b} = \sigma_{\text{all}}$$

$$P_{\text{all}} = 0.05388\sigma_{\text{all}}b$$

$d = 0.190\ \text{m}$ or b, whichever is smaller.

$$\sigma_{CE} = \frac{K_{CE}E}{(L/d)^2} = \frac{(0.300)(8300)}{(2.2/d)^2} = 514.5d^2\ \text{MPa}$$

$$\sigma_{CE}/\sigma_C = \frac{514.5d^2}{8.1} = 63.51d^2$$

$$C_p = \frac{1 + \sigma_{CE}/\sigma_C}{2c} - \sqrt{\left(\frac{1 + \sigma_{CE}/\sigma_C}{2c}\right)^2 - \frac{\sigma_{CE}/\sigma_c}{c}}$$

$$= \frac{1 + \sigma_{CE}/\sigma_C}{1.6} - \sqrt{\left(\frac{1 + \sigma_{CE}/\sigma_C}{1.6}\right)^2 - \frac{\sigma_{CE}/\sigma_C}{0.8}}$$

$$\sigma_{\text{all}} = \sigma_C C_p = (8.1 \times 10^6)C_p$$

$$P_{\text{all}} = (0.05388b)(8.1 \times 10^6)C_p = 472.47 \times 10^3 bC_p$$

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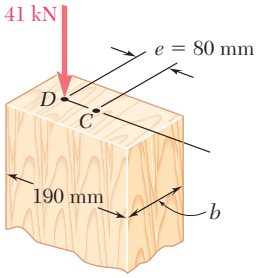
PROBLEM 10.107 *(Continued)*

Calculate P_{all} for all four values of b. See table below.

	b (m)	d (m)	σ_{CE}/σ_C	C_p	P_{all} (kN)	
	0.090	0.090	0.51443	0.44367	18.87	
→	0.140	0.140	1.24480	0.76081	50.3	← $P = 41$ kN
	0.190	0.190	2.2927	0.8878	79.7	
	0.240	0.190	2.2927	0.8878	100.7	Use $b = 0.140$ m.

$b = 140.0$ mm ◀

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PROBLEM 10.108

Solve Prob. 10.107, assuming that $e = 40$ mm.

PROBLEM 10.107 A sawn lumber column of rectangular cross section has a 2.2-m effective length and supports a 41-kN load as shown. The sizes available for use have b equal to 90 mm, 140 mm, 190 mm, and 240 mm. The grade of wood has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 8.1$ MPa and $E = 8.3$ GPa. Use the allowable-stress method to determine the lightest section that can be used.

SOLUTION

Sawn lumber: $\sigma_C = 8.1$ MPa $\quad E = 8.3$ GPa

$c = 0.8 \quad K_{CE} = 0.300$

$L_e = 2.2$ m

$$\frac{P_{\text{all}}}{A} + \frac{P_{\text{all}}ec}{I_x} = \sigma_{\text{all}}$$

$e = 40 \times 10^{-3}$ m, $\quad c = \frac{1}{2}(190) = 95$ mm $= 95 \times 10^{-3}$ m

$A = 0.190b$ m^2 $\quad I_x = \frac{1}{12}b(0.190)^3 = 571.58 \times 10^{-6}b$ m^4

$$\frac{P_{\text{all}}}{0.190b} + \frac{P_{\text{all}}(40 \times 10^{-3})(95 \times 10^{-3})}{571.58 \times 10^{-6}b} = \frac{11.911P_{\text{all}}}{b} = \sigma_{\text{all}}$$

$P_{\text{all}} = 0.083953\sigma_{\text{all}}b$

$d = 0.190$ m or b, whichever is smaller.

$$\sigma_{CE} = \frac{K_{CE}E}{(L/d)^2} = \frac{(0.300)(8300)}{(2.2/d)^2} = 514.5d^2 \text{ MPa}$$

$$\sigma_{CE}/\sigma_C = \frac{514.5d^2}{8.1} = 63.51d^2$$

$$C_p = \frac{1 + \sigma_{CE}/\sigma_C}{2c} - \sqrt{\left(\frac{1 + \sigma_{CE}/\sigma_C}{2c}\right)^2 - \frac{\sigma_{CE}/\sigma_C}{c}}$$

$$= \frac{1 + \sigma_{CE}/\sigma_C}{1.6} - \sqrt{\left(\frac{1 + \sigma_{CE}/\sigma_C}{1.6}\right)^2 - \frac{\sigma_{CE}/\sigma_C}{0.8}}$$

$\sigma_{\text{all}} = \sigma_C C_p = (8.1 \times 10^6)C_p$

$P_{\text{all}} = (0.083953b)(8.1 \times 10^6)C_p = 680.02 \times 10^3 bC_p$

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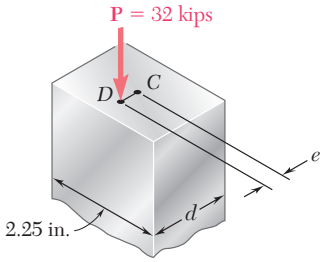
PROBLEM 10.108 *(Continued)*

Calculate P_{all} for all four values of b. See table below.

	b (m)	d (m)	σ_{CE}/σ_C	C_p	P_{all} (kN)	
	0.090	0.090	0.51443	0.44367	27.2	
→	0.140	0.140	1.24480	0.76081	72.4	← $P = 41$ kN
	0.190	0.190	2.2927	0.8878	114.7	
	0.240	0.190	2.2927	0.8878	144.9	Use $b = 0.140$ m.

$b = 140.0$ mm ◀

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PROBLEM 10.109

A compression member of rectangular cross section has an effective length of 36 in. and is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 24 ksi. Using the interaction method, determine the smallest dimension d of the cross section that can be used when $e = 0.4$ in.

SOLUTION

$$A = 2.25d \qquad c = \frac{1}{2}d \qquad e = 0.4 \text{ in.} \qquad L_e = 36 \text{ in.}$$

$$\sigma_{\text{all},b} = 24 \text{ ksi} \qquad P = 32 \text{ kips}$$

$$I_x = \frac{1}{12}(2.25)d^3 \qquad r_x = \frac{d}{\sqrt{12}}$$

Assume $r_x = r_{\min}$, i.e. $d < 2.25$ in.

$$L_e/r_{\min} = \sqrt{12}L_e/d$$

Assume $L_e/r_{\min} > 55$. $\sigma_{\text{all},c} = \dfrac{55{,}400}{(L_e/r_x)^2} = \dfrac{55{,}400d^2}{12L_e^2} = \dfrac{55{,}400}{(12)(36)^2}d^2 = 3.56224d^2$

$$\frac{P}{\sigma_{\text{all},c}} + \frac{Pec}{I\sigma_{\text{all},b}} = \frac{32}{(2.25d)(3.56224d^2)} + \frac{(12)(32)(0.4)\left(\frac{1}{2}d\right)}{(2.25d^3)(24)} = 1$$

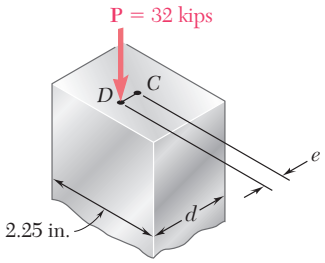
$$\frac{3.9925}{d^3} + \frac{1.42222}{d^2} = 1 \qquad \text{Let } x = \frac{1}{d} \qquad 3.9925x^3 + 1.42222x^2 = 1$$

Solving for x, $x = 0.53126$ $d = \dfrac{1}{x} = 1.8823$ in. < 2.25 in.

$$L/r_x = \left(\sqrt{12}\right)(36)/1.894 = 65.8 > 55$$

$d = 1.882$ in. ◀

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PROBLEM 10.110

Solve Prob. 10.109, assuming that $e = 0.2$ in.

PROBLEM 10.109 A compression member of rectangular cross section has an effective length of 36 in. and is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 24 ksi. Using the interaction method, determine the smallest dimension d of the cross section that can be used when $e = 0.4$ in.

SOLUTION

$$A = 2.25d \qquad c = \frac{1}{2}d \qquad e = 0.2 \text{ in.} \qquad L_e = 36 \text{ in.}$$

$$\sigma_{\text{all},b} = 24 \text{ ksi} \qquad P = 32 \text{ kips}$$

$$I_x = \frac{1}{12}(2.25)d^3 \qquad r_x = \frac{d}{\sqrt{12}}$$

Assume $r_x = r_{\min}$, i.e. $d < 2.25$ in.

$$L_e/r_{\min} = \sqrt{12}L_e/d$$

Assume $L_e/r_{\min} > 55$. $\sigma_{\text{all},c} = \dfrac{55{,}400}{(L_e/r_x)^2} = \dfrac{55{,}400d^2}{12L_e^2} = \dfrac{55{,}400d^2}{(12)(36)^2} = 3.56224d^2$

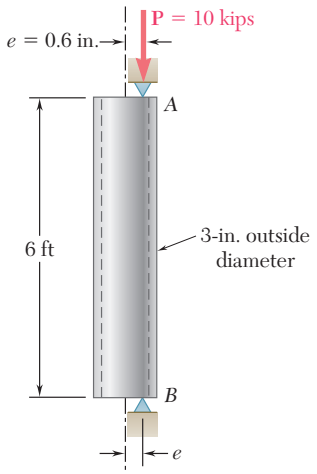
$$\frac{P}{A\sigma_{\text{all},c}} + \frac{Pec}{I\sigma_{\text{all},b}} = \frac{32}{(2.25d)(3.56224d^2)} + \frac{(12)(32)(0.2)\left(\frac{1}{2}d\right)}{(2.25d^3)(24)} = 1$$

$$\frac{3.9925}{d^3} + \frac{0.71111}{d^2} = 1 \qquad \text{Let } x = \frac{1}{d} \qquad 3.9925x^3 + 0.71111x^2 = 1$$

Solving for x, $x = 0.57623 \text{ in}^{-1}$ $d = \dfrac{1}{x} = 1.735 \text{ in.} < 2.25 \text{ in.}$

$L_e/r_x = \left(\sqrt{12}\right)(36)/1.748 = 71.4 > 55$ $d = 1.735$ in. ◀

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PROBLEM 10.111

An aluminum tube of 3-in. outside diameter is to carry a load of 10 kips having an eccentricity $e = 0.6$ in. Knowing that the stock of tubes available for use are made of aluminum alloy 2014-T6 and have wall thicknesses in increments of $\frac{1}{16}$ in. up to $\frac{1}{2}$ in., determine the lightest tube that can be used. Use the allowable-stress method.

SOLUTION

$$L_e = 6 \text{ ft} = 72 \text{ in.} \qquad c_o = \frac{1}{2}d_o = 1.500 \text{ in.} = c$$

$$c_i = c_o - t = 1.500 - t$$

$$A = \pi\left(c_o^2 - c_i^2\right) \qquad I = \frac{\pi}{4}\left(c_o^2 - c_i^2\right)$$

Try $t = \frac{1}{4}$ in. $= 0.25$ in. $\qquad c_i = 1.250$ in.

$$A = 2.1598 \text{ in}^2 \qquad I = 2.0586 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 0.97629 \text{ in.}$$

$$\frac{L}{r} = \frac{72}{0.97629} = 73.748 > 55 \qquad \text{(2014-T6 aluminum alloy)}$$

$$\sigma_{\text{all}} = \frac{55{,}400}{(L/r)^2} = \frac{55{,}400}{(73.748)^2} = 10.186 \text{ ksi}$$

$$\frac{P_{\text{all}}}{A} + \frac{Mc}{I} = \frac{P_{\text{all}}}{A} + \frac{P_{\text{all}}ec}{I} = \left[\frac{1}{2.1598} + \frac{(0.6)(1.500)}{2.0586}\right]P_{\text{all}} = 0.9002P_{\text{all}}$$

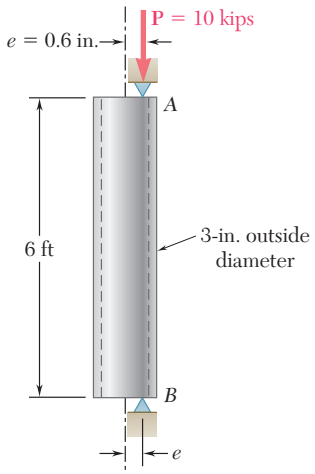
$$P_{\text{all}} = \frac{\sigma_{\text{all}}}{0.9002} = \frac{10.186}{0.9002} = 11.315 \text{ kips} > 10 \text{ kips}$$

Since P_{all} is nearly proportional to the thickness t,

$$\frac{t}{0.25} \approx \frac{10}{11.315} \qquad t \approx 0.221 \text{ in.} > \frac{3}{16} \text{ in.} = 0.1875 \text{ in.}$$

$\frac{3}{16}$-in. thickness would be too small. Use $t = \frac{1}{4}$ in. ◀

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PROBLEM 10.112

Solve Prob. 10.111, using the interaction method of design with an allowable stress in bending of 25 ksi.

PROBLEM 10.111 An aluminum tube of 3-in. outside diameter is to carry a load of 10 kips having an eccentricity $e = 0.6$ in. Knowing that the stock of tubes available for use are made of aluminum alloy 2014-T6 and have wall thicknesses in increments of $\frac{1}{16}$ in. up to $\frac{1}{2}$ in., determine the lightest tube that can be used. Use the allowable-stress method.

SOLUTION

$$L_e = 6 \text{ ft} = 72 \text{ in.} \qquad c_o = \frac{1}{2} d_o = 1.500 \text{ in.} = c$$

$$c_i = c_o - t = 1.500 - t$$

$$A = \pi\left(c_o^2 - c_i^2\right) \qquad I = \frac{\pi}{4}\left(c_o^2 - c_i^2\right)$$

Try $t = \frac{1}{4}$ in. $= 0.25$ in. $\qquad c_i = 1.250$ in.

$$A = 2.1598 \text{ in}^2 \qquad I = 2.0586 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 0.97629 \text{ in.}$$

$$\frac{L}{r} = \frac{72}{0.97629} = 73.748 > 55. \qquad \text{(2014-T6 aluminum alloy)}$$

$$\sigma_{\text{all},c} = \frac{55{,}400}{(L/r)^2} = \frac{55{,}400}{(73.748)^2} = 10.186 \text{ ksi}$$

$$\frac{P_{\text{all}}}{A\sigma_{\text{all},c}} + \frac{Mc}{I\sigma_{\text{all},b}} = \frac{P_{\text{all}}}{A\sigma_{\text{all}}} + \frac{P_{\text{all}} ec}{I\sigma_{\text{all},b}}$$

$$= \left[\frac{1}{(2.1598)(10.186)} + \frac{(0.6)(1.500)}{(2.0586)(25)}\right] P_{\text{all}} = 0.062943 P_{\text{all}} = 1$$

$$P_{\text{all}} = \frac{1}{0.062943} = 15.89 \text{ kips} > 10 \text{ kips}$$

Since P_{all} is nearly proportional to the thickness t,

$$\frac{t}{0.25} \approx \frac{10}{15.89} \qquad t \approx 0.1574 \text{ in.} < \frac{3}{16} \text{ in.} = 0.1875 \text{ in.}$$

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PROBLEM 10.112 *(Continued)*

Try $t = \frac{3}{16} = 0.1875$ in. $\qquad c_i = 1.3125$ in.

$$A = 1.6567 \text{ in}^2, \quad I = 1.64537 \text{ in}^4, \quad r = 0.99658 \text{ in.}$$

$$\frac{L}{r} = 72.247 > 55 \qquad \sigma_{\text{all},c} = 10.614 \text{ ksi}$$

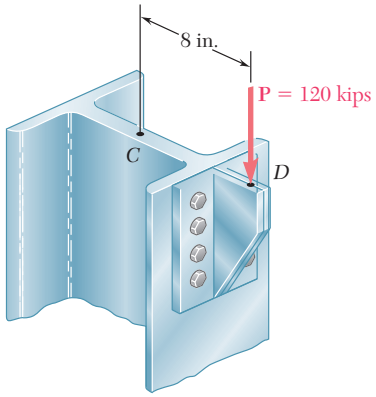
$$\left[\frac{1}{(1.6567)(10.614)} + \frac{(0.6)(1.500)}{(1.64537)(25)}\right] P_{\text{all}} = 0.07875 P_{\text{all}} = 1$$

$$P_{\text{all}} = \frac{1}{0.07875} = 12.70 \text{ kips} > 10 \text{ kips}$$

$$\frac{t}{0.1875} \approx \frac{10}{12.70} \qquad t \approx 0.1477 \text{ in.} < \frac{3}{16} \text{ in.}$$

Use $t = \frac{3}{16}$ in. ◀

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PROBLEM 10.113

A steel column having a 24-ft effective length is loaded eccentrically as shown. Using the allowable-stress method, select the wide-flange shape of 14-in. nominal depth that should be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

$$E = 29{,}000 \text{ ksi} \qquad L_e = 24 \text{ ft} = 288 \text{ in.}$$

Transition $\dfrac{L}{r}$: $\qquad 4.71\sqrt{\dfrac{E}{\sigma_Y}} = 133.68$

$$e = 8 \text{ in.}$$

$$\frac{P}{A} + \frac{Pec}{I_x} < \sigma_{\text{all}} \quad \text{or} \quad \frac{P}{A}\left(1 + \frac{ec}{r_x^2}\right) < \sigma_{\text{all}}$$

For 14-in. nominal depth, $c \approx 7$ in., $r_x \approx 6$ in.

$$1 + \frac{ec}{r_x^2} \approx 1 + \frac{(8)(7)}{(6)^2} = 2.33$$

If the column is short, $\qquad \sigma_{\text{all}} = \dfrac{\sigma_Y}{1.67} = 21.56 \text{ ksi}$

$$A > \frac{2.33P}{\sigma_{\text{all}}} = \frac{(2.33)(120)}{21.56} = 12.97 \text{ in}^2$$

If the column is long, $\qquad 2.33P < \dfrac{0.877\pi^2 EI}{1.67L_e^2}$

$$I > \frac{(1.67)(2.33)PL_e^2}{0.877\pi^2 E} = \frac{(1.67)(2.33)(120)(288)^2}{0.877\pi^2(29{,}000)} = 154 \text{ in}^4$$

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PROBLEM 10.113 ***(Continued)***

Try W14 × 82. $A = 24.0 \text{ in}^2$, $S_x = 123 \text{ in}^3$, $r_y = 2.48$ in.

$$\frac{L_e}{r_y} = \frac{288}{2.48} = 116.13 < 133.66 \qquad \sigma_e = \frac{\pi^2(29{,}000)}{(116.13)^2} = 21.221 \text{ ksi}$$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{1.67} = \frac{1}{1.67}\left[0.658^{36/21.221}\right](36) = 10.60 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pe}{S_x} = \frac{120}{24.0} + \frac{(120)(8)}{123} = 12.80 \text{ ksi} > 10.60 \text{ ksi}$$

(not allowed)

Try W14 × 145. $A = 42.7 \text{ in}^2$, $S_x = 232 \text{ in}^3$, $r_y = 3.98$ in.

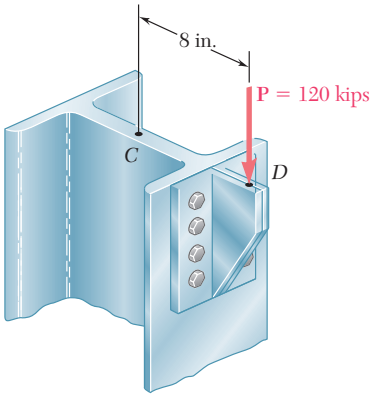
$$\frac{L_e}{r_y} = \frac{288}{3.98} = 72.36 < 133.68 \qquad \sigma_e = \frac{\pi^2(29{,}000)}{(72.36)^2} = 54.664 \text{ ksi}$$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{1.67} = \frac{1}{1.67}[0.658^{36/54.664}](36) = 16.36 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pe}{S_x} = \frac{120}{42.7} + \frac{(120)(8)}{232} = 6.95 \text{ ksi} < 16.36 \text{ ksi}$$

Use W14 × 145. ◀

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PROBLEM 10.114

Solve Prob. 10.113 using the interaction method, assuming that $\sigma_Y = 50$ ksi and the allowable stress in bending is 30 ksi.

PROBLEM 10.113 A steel column having a 24-ft effective length is loaded eccentrically as shown. Using the allowable-stress method, select the wide-flange shape of 14-in. nominal depth that should be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

$$E = 29{,}000 \text{ ksi} \qquad L_e = 24 \text{ ft} = 288 \text{ in.}$$

Transition $\dfrac{L}{r}$:

$$4.71\sqrt{\frac{E}{\sigma_Y}} = 113.43$$

$$\frac{288}{r_y} = 113.43 \qquad r_y = 2.54 \text{ in.}$$

For $r_y \le 2.54$ in.,

$$\sigma_{\text{all,centric}} = \frac{0.877\pi^2 E}{1.67(L/r)^2} = \frac{5.183Er^2}{L_e^2}$$

Interaction formula:

$$\frac{P}{A\sigma_{\text{all,centric}}} + \frac{Pe}{S_x\sigma_{\text{all,bending}}} < 1$$

$$\frac{PL_e^2}{5.183Er_y^2A} + \frac{Pe}{S_x\sigma_{\text{all,bending}}} = \beta < 1$$

$$\frac{(120)(288)^2}{(5.183)(29{,}000)I_y} + \frac{(120)(8)}{S_x(30)} = \beta < 1$$

$$\frac{66.22}{I_y} + \frac{32}{S_x} = \beta < 1$$

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PROBLEM 10.114 *(Continued)*

Obviously, $I_y > 66.22 \text{ in}^4$ and $S_x > 32 \text{ in}^3$

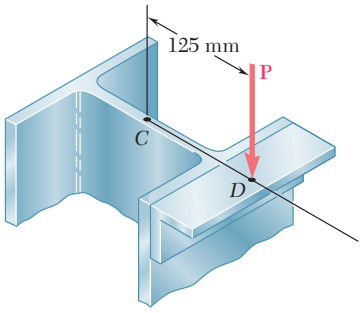
Shape	r_y (in.)	I_y (in^4)	S_x (in^3)	β
W14 × 82	2.48	148	123	0.708
W14 × 68	2.46	121	103	0.858
W14 × 53	1.92	57.7*		

*too small

W14 × 68 is the lightest shape with $\beta < 1$.

← Use W14 × 68. ◀

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PROBLEM 10.115

A steel compression member of 5.8-m effective length is to support a 296-kN eccentric load **P**. Using the interaction method, select the wide-flange shape of 200-mm nominal depth that should be used. Use $E = 200$ GPa, $\sigma_Y = 250$ MPa, and $\sigma_{all} = 150$ MPa in bending.

SOLUTION

Transition $\frac{L}{r}$: $\quad 4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200\times10^9}{250\times10^6}} = 133.22 \qquad L_e = 5.8 \text{ m}$

At transition, $\quad \frac{5.8}{r_y} = 133.22$

$$r_y = 45.4\times10^{-3} \text{ m} = 45.4 \text{ mm}$$

For 200-mm nominal depth wide-flange sections,

$$c = \frac{1}{2}d \approx 104 \text{ mm} \qquad r_x \approx 88 \text{ mm} \qquad \frac{ec}{r_x^2} \approx \frac{(125)(104)}{(88)^2} = 1.68$$

For $r_y = 45.4$ mm, $\quad \sigma_{all,centric} = \frac{0.877\pi^2 E}{(1.67)(133.22)^2} = 58.4\times10^6 \text{ Pa}$

Interaction formula: $\quad \frac{P}{A\sigma_{all,centric}} + \frac{Pec}{Ar_x^2\sigma_{all,bending}} < 1$

$$A > P\left[\frac{1.67}{\sigma_Y} + \frac{ec/r_x^2}{\sigma_{all,bending}}\right] = 296\times10^3\left(\frac{1.67}{250\times10^6} + \frac{1.68}{150\times10^6}\right)$$

$$= 5.29\times10^{-3} \text{ m}^2 = 8380 \text{ mm}^2$$

Try W200 × 59. $\quad A = 7550 \text{ mm}^2, \quad S_x = 582\times10^3 \text{ mm}^3, \quad r_y = 51.8 \text{ mm}$

$$\frac{L_e}{r_y} = \frac{5.8}{51.8\times10^{-3}} = 111.97 < 133.22 \qquad \sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = 157.45 \text{ MPa}$$

$$\sigma_{all,centric} = \frac{\sigma_{cr}}{1.67} = \frac{1}{1.67}[0.658^{250/157.45}](250) = 77.02 \text{ MPa}$$

$$\frac{P}{A\sigma_{all,centric}} + \frac{Pe}{S_x\sigma_{all,bending}} = \frac{296\times10^3}{(7550\times10^{-6})(77.02\times10^6)} + \frac{(296\times10^3)(125\times10^{-3})}{(582\times10^{-6})(150\times10^6)}$$

$$= 0.5090 + 0.4238 = 0.9329 < 1 \qquad \text{(allowed)}$$

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PROBLEM 10.115 *(Continued)*

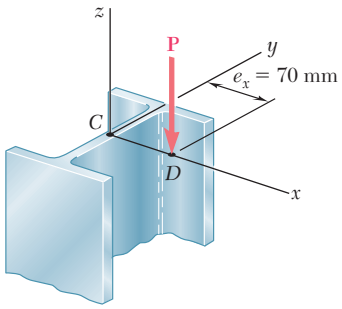
Try W200 × 52. $A = 6650 \text{ mm}^2, \quad S_x = 511 \times 10^3 \text{ mm}^3, \quad r_y = 51.6 \text{ mm}$

$$\frac{L_e}{r_y} = 112.40 \qquad \sigma_e = 156.23 \text{ MPa} \qquad \sigma_{\text{all,centric}} = 76.62 \text{ MPa}$$

$$\frac{296 \times 10^3}{(6550 \times 10^{-6})(76.62 \times 10^6)} + \frac{(296 \times 10^3)(125 \times 10^{-3})}{(512 \times 10^{-6})(150 \times 10^6)} = 1.064 > 1 \quad \text{(not allowed)}$$

Use W200 × 59. ◀

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PROBLEM 10.116

A steel column of 7.2-m effective length is to support as 83-kN eccentric load **P** at a point *D*, located on the *x* axis as shown. Using the allowable-stress method, select the wide-flange shape of 250-mm nominal depth that should be used. Use $E = 200$ GPa and $\sigma_Y = 250$ MPa.

SOLUTION

Transition $\frac{L}{r}$: $$4.71\sqrt{\frac{E}{\sigma_Y}} = 133.22$$

At the transition, $$\frac{7.2}{r_y} = 133.22$$

$$r_y = 54.05 \times 10^{-3} \text{ m} = 54.05 \text{ mm}$$

All sections meet $$\frac{L}{r_y} > 133.22.$$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{1.67} = \frac{0.877\pi^2 E}{1.67(L/r_y)^2} = \frac{0.877\pi^2(200 \times 10^9)}{(1.67)(7.2/r_y)^2} = 19.9962 \times 10^9 r_y^2$$

Try W250 × 49.1. $$A = 6260 \text{ mm}^2, \qquad I_y = 15.2 \times 10^6 \text{ mm}^4$$

$$S_y = 151 \times 10^3 \text{ mm}^3 \qquad r_y = 49.3 \text{ mm}$$

$$\sigma_{\text{all}} = (19.9962 \times 10^9)(49.3 \times 10^{-3})^2 = 48.601 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pe}{S_y} = \frac{83 \times 10^3}{6260 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})}{151 \times 10^{-6}} = 51.736 \text{ MPa} > \sigma_{\text{all}}$$

(not allowed)

Approximate required area: $$A \approx \frac{51.736}{48.601}(6260) = 6664 \text{ mm}^2$$

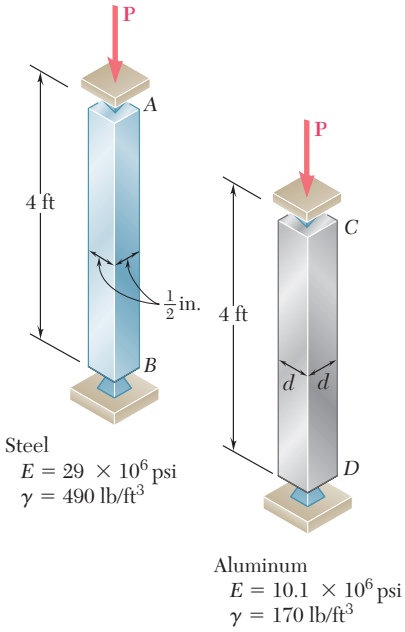
Try W250 × 58. $$A = 7420 \text{ mm}^2, \quad S_y = 185 \times 10^3 \text{ mm}^3, \quad r_y = 50.3 \text{ mm}$$

$$\sigma_{\text{all}} = (19.9962 \times 10^9)(50.3 \times 10^{-3})^2 = 50.592 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pe}{S_y} = \frac{83 \times 10^3}{7420 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})}{185 \times 10^{-6}} = 42.591 \text{ MPa} < \sigma_{\text{all}}$$

Use W250 × 58. ◀

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PROBLEM 10.117

Determine (*a*) the critical load for the steel strut, (*b*) the dimension *d* for which the aluminum strut will have the same critical load. (*c*) Express the weight of the aluminum strut as a percent of the weight of the steel strut.

SOLUTION

Steel: $E = 29\times10^6$ psi $\quad \gamma = 490$ lb/ft^3 $= 0.28356$ lb/in^3

Aluminum: $E = 10.1\times10^6$ psi $\quad \gamma = 170$ lb/ft^3 $= 0.09838$ lb/in^3

Length: $L = 4$ ft $= 48$ in.

(*a*) Steel strut:

$$I = \frac{1}{12}d_s^4 = \frac{1}{12}\left(\frac{1}{2}\right)^4 = 5.2083\times10^{-3}\ \text{in}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(29\times10^6)(5.2083\times10^{-3})}{(48)^2} \qquad P_{cr} = 647\ \text{lb} \blacktriangleleft$$

Weight: $W_s = \gamma_s L d_s^2 = (0.28356)(48)\left(\frac{1}{2}\right)^2 = 3.4028$ lb

(*b*) Aluminum strut:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad I = \frac{P_{cr}L^2}{\pi^2 E} = \frac{(647)(48)^2}{\pi^2(10.1\times10^6)} = 14.9546\times10^{-3}\ \text{in}^4$$

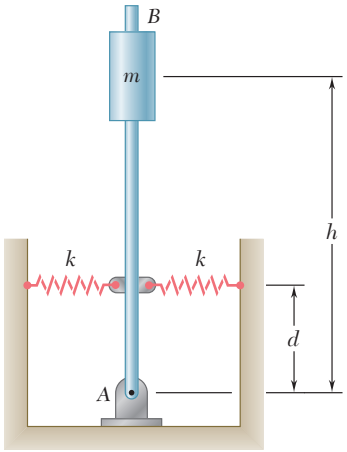
$$I = \frac{1}{12}d^4 \qquad d = \sqrt[4]{12I} = \sqrt[4]{(12)(14.9546\times10^{-3})} \qquad d = 0.651\ \text{in.} \blacktriangleleft$$

Weight: $W_a = \gamma_a L d^2 = (0.09838)(48)(0.651)^2 = 2.0004$ lb

(*c*) Weight ratio as a percent:

$$\frac{w_a}{w_s}\times100\% = \frac{2.0004}{3.4028}\times100\% \qquad 58.8\% \blacktriangleleft$$

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PROBLEM 10.118

The rigid rod *AB* is attached to a hinge at *A* and to two springs, each of constant *k*. If $h = 450$ mm, $d = 300$ mm, and $m = 200$ kg, determine the range of values of *k* for which the equilibrium of rod *AB* is stable in the position shown. Each spring can act in either tension or compression.

SOLUTION

Let θ be the angle change of bar *AB*.

$$\delta_1 = h \sin\theta$$

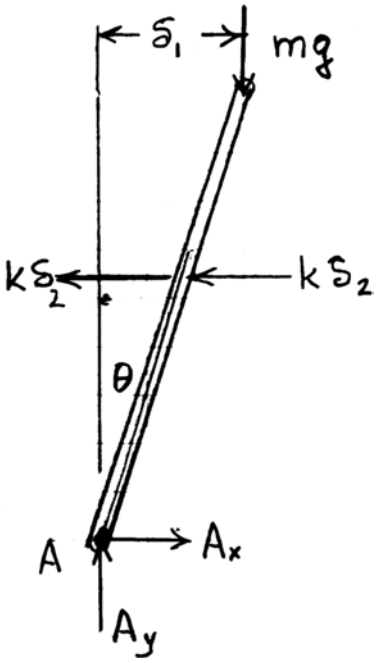
$$\delta_2 = d \sin\theta$$

$$+\circlearrowleft \Sigma M_A = 0:$$

$$(2k\,\delta_2)(d\cos\theta) - mg\;\delta_1 = 0$$

$$(2kd^2\cos\theta - mgh)\sin\theta = 0$$

$$k = \frac{mgh}{2d^2\cos\theta}$$



Let θ be small.

$$\cos\theta \approx 1$$

For stability,

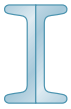
$$k > \frac{mgh}{2d^2}$$

Data: $m = 200$ kg, $g = 9.81$ m/s^2, $h = 0.450$ m, $d = 0.300$ m

$$k > \frac{(200)(9.81)(0.450)}{(2)(0.300)^2} = 4.905\times10^3 \text{ kg/s}^2$$

$k > 4.91$ kN/m ◀

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(*a*)



(*b*)

PROBLEM 10.119

A column of 3-m effective length is to be made by welding together two C130 × 13 rolled-steel channels. Using $E = 200$ GPa, determine for each arrangement shown the allowable centric load if a factor of safety of 2.4 is required.

SOLUTION

For channel C130 × 13, $A = 1700 \text{ mm}^2$ $b_f = 48.0 \text{ mm}$

$I_x = 3.70 \times 10^6 \text{ mm}^4$ $I_y = 0.260 \times 10^6 \text{ mm}^4$ $\bar{x} = 12.1 \text{ mm}$

Arrangement (*a*):

$$I_x = (2)(3.70 \times 10^6) = 7.40 \times 10^6 \text{ mm}^4$$

$$I_y = 2[0.260 \times 10^6 + (1700)(12.1)^2] = 1.0178 \times 10^6 \text{ mm}^2$$

$$I_{\min} = I_y = 1.0178 \times 10^6 \text{ mm} = 1.0178 \times 10^{-6} \text{ m}^4$$

$$P_{\text{cr}} = \frac{\pi^2 E I_{\min}}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(1.0178 \times 10^{-6})}{(3.0)^2} = 223 \times 10^3 \text{ N} = 223 \text{ kN}$$

$$P_{\text{all}} = \frac{P_{\text{cr}}}{F.S.} = \frac{223}{2.4} \qquad P_{\text{all}} = 93.0 \text{ kN} \blacktriangleleft$$

Arrangement (*b*):

$$I_x = (2)(3.70 \times 10^6) \text{ mm}^4 = 7.40 \times 10^6 \text{ mm}^4$$

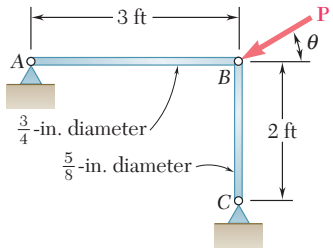
$$I_y = 2[0.260 \times 10^6 + (1700)(48 - 12.1)^2] = 4.902 \times 10^6 \text{ mm}^4$$

$$I_{\min} = I_y = 4.902 \times 10^6 \text{ mm}^4 = 4.902 \times 10^{-6} \text{ m}^4$$

$$P_{\text{cr}} = \frac{\pi^2 E I_{\min}}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(4.902 \times 10^{-6})}{(3.0)^2} = 1075 \times 10^3 \text{ N} = 1075 \text{ kN}$$

$$P_{\text{all}} = \frac{P_{\text{cr}}}{F.S.} = \frac{1075}{2.4} \qquad P_{\text{all}} = 448 \text{ kN} \blacktriangleleft$$

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PROBLEM 10.120

(*a*) Considering only buckling in the plane of the structure shown and using Euler's formula, determine the value of θ between 0° and 90° for which the allowable magnitude of the load **P** is maximum. (*b*) Determine the corresponding maximum value of P knowing that a factor of safety of 3.2 is required. Use $E = 29 \times 10^6$ psi.

SOLUTION

Strut *AB*. $L = 3 \text{ ft} = 36 \text{ in.}$

$$c = \frac{1}{2}d = \frac{1}{2}\left(\frac{3}{4}\right) = 0.375 \text{ in.}$$

$$I = \frac{\pi}{4}c^4 = 15.5316 \times 10^{-3} \text{ in}^4$$

$$(P_{AB})_{\text{cr}} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(29 \times 10^6)(15.5316 \times 10^{-3})}{(36)^2}$$

$$= 3.4301 \times 10^3 \text{ lb}$$

Strut *BC*: $L = 2 \text{ ft} = 24 \text{ in.}$, $c = \frac{1}{2}\left(\frac{5}{8}\right) = 0.3125 \text{ in.}$, $I = 7.4901 \times 10^{-3} \text{ in}^4$

$$(P_{BC})_{\text{cr}} = \frac{\pi^2(29 \times 10^6)(7.4901 \times 10^{-3})}{(24)^2} = 3.7219 \times 10^3 \text{ lb}$$

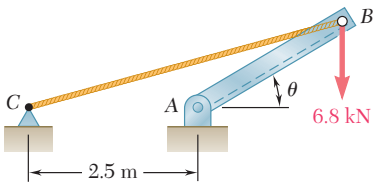
For structure, P is maximum if both struts buckle simultaneously.

(*a*) $\tan\theta = \dfrac{(P_{BC})_{\text{cr}}}{(P_{AB})_{\text{cr}}} = 1.08507$ $\qquad \theta = 47.2°$ ◀

(*b*) $P_{\text{cr}} = \sqrt{(P_{AB})_{\text{cr}}^2 + (P_{BC})_{\text{cr}}^2} = 5.0614 \times 10^3 \text{ lb} = 5.0614 \text{ kips}$

With *F.S.* = 3.2, $\qquad P = \dfrac{5.0614}{3.2}$ $\qquad P = 1.582 \text{ kips}$ ◀

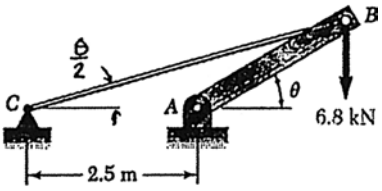
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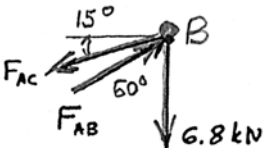


PROBLEM 10.121

Member AB consists of a single $C130 \times 10.4$ steel channel of length 2.5 m. Knowing that the pins at A and B pass through the centroid of the cross section of the channel, determine the factor of safety for the load shown with respect to buckling in the plane of the figure when $\theta = 30°$. Use Euler's formula with $E = 200$ GPa.

SOLUTION





Geometry:

Since $AB = 2.5$ m, triangle ABC is isosceles.

Equilibrium of connection at B.

$$\sum F_x = 0: \quad -F_{AC}\cos 15° + F_{AB}\cos 30° = 0$$

$$F_{AC} = \frac{F_{AB}\cos 30°}{\cos 15°}$$

$$+\uparrow \sum F_y = 0: \quad F_{AB}\sin 30° - F_{AC}\sin 15° - 6.8 = 0$$

$$F_{AB}\left(\sin 30° - \frac{\sin 15°\cos 30°}{\cos 15°}\right) = 0.26795F_{AB} = 6.8$$

$$F_{AB} = 25.378 \text{ kN}$$

Rolled-steel shape $C130 \times 10.4$:

$$I_{\min} = 0.196 \times 10^6 \text{ mm}^4 = 0.196 \times 10^6 \text{ m}^4$$

Critical load:

$$P_{cr} = \frac{\pi^2 E I_{\min}}{L_{AB}^2}$$

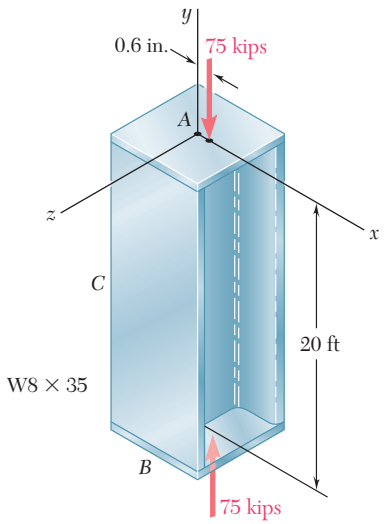
$$P_{cr} = \frac{\pi^2 (200 \times 10^9)(0.196 \times 10^{-6})}{(2.5)^2} = 61.902 \times 10^3 \text{ N} = 61.902 \text{ kN}$$

Factor of safety:

$$F.S. = \frac{P_{cr}}{F_{AB}} = \frac{61.902}{25.378}$$

$$F.S. = 2.44 \blacktriangleleft$$

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PROBLEM 10.122

The line of action of the 75-kip axial load is parallel to the geometric axis of the column AB and intersects the x axis at $x = 0.6$ in. Using $E = 29 \times 10^6$ psi, determine (*a*) the horizontal deflection of the midpoint C of the column, (*b*) the maximum stress in the column.

SOLUTION

$L_e = L = 20 \text{ ft} = 240 \text{ in.} \qquad e = 0.6 \text{ in.}$

W8 × 35: $A = 10.3 \text{ in}^2 \quad I_y = 42.6 \text{ in}^4$

$S_y = 10.6 \text{ in}^3 \quad E = 29{,}000 \text{ ksi}$

$$P_{cr} = \frac{\pi^2 E I_y}{L^2} = \frac{\pi^2 (29{,}000)(42.6)}{(240)^2} = 211.68 \text{ kips}$$

$$\frac{P}{P_{cr}} = \frac{75}{211.68} = 0.35404$$

(*a*) $$y_m = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right]$$

$$= (0.6)\left[\sec\left(\frac{\pi}{2}\sqrt{0.35404}\right) - 1\right]$$

$$= (0.6)\left[\sec(0.93499) - 1\right]$$

$$= 0.41040 \text{ in.}$$

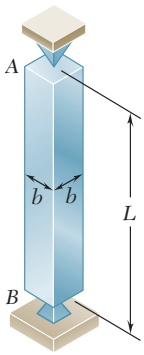
$y_m = 0.410$ in. ◀

(*b*) $$M_{\max} = P(y_m + e) = (75)(0.6 + 0.41040) = 75.780 \text{ kip} \cdot \text{in.}$$

$$\sigma_{\max} = \frac{P}{A} + \frac{M_{\max}}{S_y} = \frac{75}{10.3} + \frac{75.780}{10.6}$$

$\sigma_{\max} = 14.43$ ksi ◀

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PROBLEM 10.123

Supports A and B of the pin-ended column shown are at a fixed distance L from each other. Knowing that at a temperature T_0 the force in the column is zero and that buckling occurs when the temperature is $T_1 = T_0 + \Delta T$, express ΔT in terms of b, L, and the coefficient of thermal temperature α.

SOLUTION

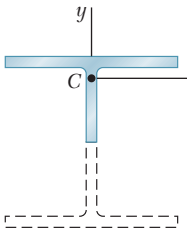
Let P be the compressive force in the column.

$$L\alpha(\Delta T) - \frac{PL}{EA} = 0 \qquad P = EA\alpha(\Delta T)$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = P = EA\alpha(\Delta T)$$

$$\Delta T = \frac{\pi^2 EI}{L^2 EA\alpha} = \frac{\pi^2 Eb^4/12}{L^2 Eb^2} \qquad \Delta T = \frac{\pi^2 b^2}{12L^2\alpha} \blacktriangleleft$$

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$A = 13.75 \times 10^3 \text{ mm}^2$
$I_x = 26.0 \times 10^6 \text{ mm}^4$
$I_y = 141.0 \times 10^6 \text{ mm}^4$

PROBLEM 10.124

A column is made from half of a W360 × 216 rolled-steel shape, with the geometric properties as shown. Using allowable stress design, determine the allowable centric load if the effective length of the column is (*a*) 4.0 m, (*b*) 6.5 m. Use $\sigma_Y = 345$ MPa and $E = 200$ GPa.

SOLUTION

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{26.0 \times 10^6}{13.75 \times 10^3}} = 43.485 \text{ mm}$$

$$= 43.485 \times 10^{-3} \text{ m}$$

$$A = 13.75 \times 10^{-3} \text{ m}^2$$

Transition L/r: $\quad 4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.4$

(*a*) $\underline{L_e = 4.0 \text{ m}}$.

$$\frac{L_e}{r} = \frac{4.0}{43.485 \times 10^{-3}} = 91.987 < 113.4$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(91.987)^2} = 233.28 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{1}{F.S.}\sigma_{\text{cr}} = \frac{1}{1.67}[0.658^{345/233.28}](345) = 111.2 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (111.2 \times 10^6)(13.75 \times 10^{-3}) \qquad P_{\text{all}} = 1530 \text{ kN} \blacktriangleleft$$

(*b*) $\underline{L_e = 6.5 \text{ m}}$.

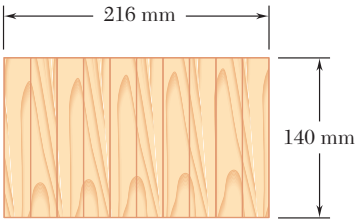
$$\frac{L_e}{r} = \frac{6.5}{43.485 \times 10^{-3}} = 149.48 > 113.4$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(149.48)^2} = 88.3 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{1}{F.S.}\sigma_{\text{cr}} = \frac{1}{1.67}[(0.877)(88.3)] = 46.39 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (46.39 \times 10^6)(13.75 \times 10^{-3}) \qquad P_{\text{all}} = 638 \text{ kN} \blacktriangleleft$$

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PROBLEM 10.125

A rectangular column with a 4.4-m effective length is made of glued laminated wood. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_C = 8.3$ MPa and the adjusted modulus is $E = 4.6$ GPa, determine the maximum allowable centric load for the column.

SOLUTION

Glued laminated column: $c = 0.9 \qquad E = 4600$ MPa

$$A = (216)(140) = 30{,}240 \text{ mm}^2$$
$$= 30{,}240 \times 10^{-6} \text{ m}^2$$
$$d = 140 \text{ mm} = 140 \times 10^{-3} \text{ m}$$
$$L = 4.4 \text{ m}$$
$$\frac{L}{d} = \frac{4.4}{140 \times 10^{-3}} = 31.429$$
$$\sigma_{CE} = \frac{0.822E}{(L/d)^2} = \frac{(0.822)(4600)}{(31.429)^2} = 3.8281 \text{ MPa}$$
$$\frac{\sigma_{CE}}{\sigma_C} = \frac{3.8281}{8.3} = 0.46121$$
$$u = \frac{1 + \sigma_{CE}/\sigma_C}{2c} = \frac{1.46121}{(2)(0.9)} = 0.81178$$
$$v = \frac{\sigma_{CE}/\sigma_C}{c} = 0.51246$$
$$C_p = u - \sqrt{u^2 - v} = 0.42908$$
$$\sigma_{\text{all}} = C_p \sigma_C = (0.42908)(8.3) = 3.5614 \text{ MPa}$$
$$P_{\text{all}} = \sigma_{\text{all}} A = (3.5614 \times 10^6)(30{,}240 \times 10^{-6}) \qquad P_{\text{all}} = 107.7 \text{ kN} \blacktriangleleft$$

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PROBLEM 10.126

A column of 4.5-m effective length must carry a centric load of 900 kN. Knowing that $\sigma_Y = 345$ MPa and $E = 200$ GPa, use allowable-stress design to select the wide-flange shape of 250-mm nominal depth that should be used.

SOLUTION

Transition $\dfrac{L}{r}$:

$$4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200\times10^9}{345\times10^6}} = 113.40$$

$$P < \frac{\sigma_Y A}{1.67} \qquad A > \frac{1.67P}{\sigma_Y} = \frac{(1.67)(900\times10^3)}{345\times10^6} = 4357\times10^{-6}\,\text{m}^2 = 4357\,\text{mm}^2$$

$$P < \frac{\sigma_{\text{cr}} A}{1.67} = \frac{0.877\pi^2 E I_{\min}}{1.67 L_e^2}$$

$$I_{\min} > \frac{1.67PL_e^2}{0.877\pi^2 E} = \frac{(1.67)(900\times10^3)(4.5)^2}{0.877\pi^2(200\times10^9)} = 17.58\times10^{-6}\,\text{m}^4 = 17.58\times10^6\,\text{mm}^4$$

Try W250 × 58. $A = 7420\,\text{mm}^2$, $r_y = 50.3\,\text{mm}$

$$\frac{L_e}{r} = \frac{4.5}{50.3\times10^{-3}} = 89.46 < 113.40$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(200\times10^9)}{(89.46)^2} = 246.64\,\text{MPa}$$

$$\sigma_{\text{cr}} = (0.658^{\sigma_Y/\sigma_e})\sigma_Y = [0.658^{345/246.64}](345) = 192.11\,\text{MPa}$$

$$P_{\text{all}} = \frac{\sigma_{\text{cr}} A}{1.67} = \frac{(192.11\times10^6)(7420\times10^{-6})}{1.67} = 854\,\text{kN} < 900\,\text{kN}$$

(not acceptable)

Try W250 × 67. $A = 8580\,\text{mm}^2$, $r_y = 51.1\,\text{mm}$

$$\frac{L_e}{r} = \frac{4.5}{51.1\times10^{-3}} = 88.063 < 113.40$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(200\times10^9)}{(88.063)^2} = 254.53\,\text{MPa}$$

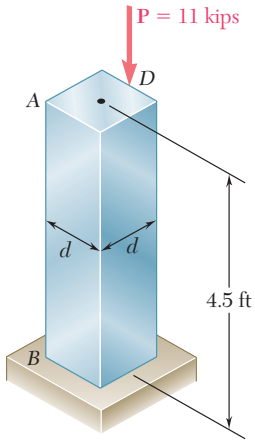
$$\sigma_{\text{cr}} = (0.658^{\sigma_Y/\sigma_e})\sigma_Y = [0.658^{345/254.53}](345) = 195.63\,\text{MPa}$$

$$P_{\text{all}} = \frac{\sigma_{\text{cr}} A}{1.67} = \frac{(195.63\times10^6)(8580\times10^{-6})}{1.67} = 1005\,\text{kN} > 900\,\text{kN}$$

(acceptable)

Use W250 × 67. ◀

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PROBLEM 10.127

An 11-kip vertical load **P** is applied at the midpoint of one edge of the square cross section of the steel compression member *AB*, which is free at its top *A* and fixed at its base *B*. Knowing that for the grade of steel used $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi, and using the allowable-stress method, determine the smallest allowable dimension *d*.

SOLUTION

Steel: $\sigma_Y = 36$ ksi $E = 29{,}000$ ksi

$$\text{transition } \frac{L}{r} = 4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{29{,}000}{36}} = 133.68$$

Assume $\dfrac{L}{r} > 133.68$ so that $\sigma_{cr} = 0.877\sigma_e$ and $\sigma_{\text{all}} = \dfrac{\sigma_{\text{cr}}}{1.67} = \dfrac{0.877\pi^2 E}{1.67(L/r)^2} = \dfrac{5.1830E}{(L/r)^2}$

Square cross section: $A = d^2$

$I = \dfrac{1}{12}d^4$ for all axes through the center of the square.

$$r = \sqrt{\frac{I}{A}} = \frac{d}{\sqrt{12}} \qquad L_e = 2L = (2)(4.5 \text{ ft}) = 9 \text{ ft} = 108 \text{ in.}$$

Allowable stress for centric loading.

$$\sigma_{\text{all}} = \frac{(5.1830)E}{(108)^2/(d^2/12)} = \frac{(5.1830)(29{,}000)}{(108)^2(12)}d^2 = 1.07387d^2 \text{ ksi}$$

Eccentric loading: $M = Pe$ $P = 11$ kips

$e = c = \dfrac{1}{2}d$ when the loading is at point *D*.

Allowable stress method.

$$\sigma_{\text{all}} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\right)$$

$$= \frac{P}{d^2}\left[1 + \frac{(d/2)(d/2)}{d^2/12}\right] = \frac{4P}{d^2} = \frac{44}{d^2} \text{ ksi}$$

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PROBLEM 10.127 *(Continued)*

Equating the two expressions for σ_{all},

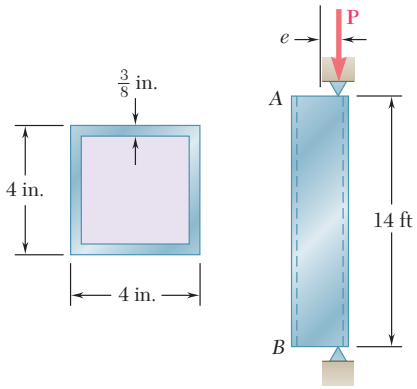
$$1.07387d^2 = \frac{44}{d^2} \qquad d = \sqrt[4]{\frac{44}{1.07387}} = 2.53 \text{ in.}$$

$$r = \frac{2.53}{\sqrt{12}} = 0.730 \text{ in.} \qquad \frac{L_e}{r} = \frac{108}{0.730} = 147.9 > 133.68$$

Assumption is verified.

Smallest allowable dimension. $d = 2.53$ in. ◀

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PROBLEM 10.128

A column of 14-ft effective length consists of a section of steel tubing having the cross section shown. Using the allowable-stress method, determine the maximum allowable eccentricity e if (a) $P = 55$ kips, (b) $P = 35$ kips. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

$$E = 29{,}000 \text{ ksi}$$

Transition L/r:
$$4.71\sqrt{\frac{E}{\sigma_Y}} = 133.68$$

$$b_o = 4.0 \text{ in.} \quad b_i = b_o - 2t = 3.25 \text{ in.} \quad c = 2.0 \text{ in.}$$

$$A = b_o^2 - b_i^2 = 5.4375 \text{ in}^2 \qquad I = \frac{1}{12}\left(b_o^4 - b_i^4\right) = 12.036 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.4878 \text{ in.}$$

$$L_e = 14 \text{ ft} = 168 \text{ in.}$$

$$\frac{L_e}{r} = \frac{168}{1.4878} = 112.92 < 133.68$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2(29{,}000)}{(112.92)^2} = 22.447 \text{ ksi}$$

$$\sigma_{\text{all}} = \frac{\sigma_{\text{cr}}}{1.67} = \frac{1}{1.67}[0.658^{36/22.447}](36) = 11.017 \text{ ksi}$$

$$\frac{P_{\text{all}}}{A} + \frac{P_{\text{all}}ec}{I} = \sigma_{\text{all}} \qquad \frac{P_{\text{all}}ec}{I} = \sigma_{\text{all}} - \frac{P_{\text{all}}}{A} \qquad e = \frac{I}{cP_{\text{all}}}\left(\sigma_{\text{all}} - \frac{P_{\text{all}}}{A}\right)$$

(a) $P_{\text{all}} = 55$ kips.

$$e = \frac{12.036}{(2.0)(55)}\left[11.017 - \frac{55}{5.4375}\right] \qquad e = 0.0987 \text{ in.} \blacktriangleleft$$

(b) $P_{\text{all}} = 35$ kips.

$$e = \frac{12.036}{(2.0)(35)}\left[11.017 - \frac{35}{5.4375}\right] \qquad e = 0.787 \text{ in.} \blacktriangleleft$$

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PROBLEM 10.C1

A solid steel rod having an effective length of 500 mm is to be used as a compression strut to carry a centric load **P.** For the grade of steel used, $E = 200$ GPa and $\sigma_Y = 245$ MPa. Knowing that a factor of safety of 2.8 is required and using Euler's formula, write a computer program and use it to calculate the allowable centric load P_{all} for values of the radius of the rod from 6 mm to 24 mm, using 2-mm increments.

SOLUTION

Enter radius rad, effective length L_e and factor of safety *FS*.

Compute radius of gyration.

$$A = \pi \text{ rad}^2$$

$$I = \frac{1}{4}\pi \text{ rad}^4$$

$$r = \sqrt{\frac{I}{A}}$$

Determine allowable centric load.

Critical stress:

$$\sigma_{\text{cr}} = \frac{\pi^2 E}{(L_e/r)^2}$$

Let σ equal smaller of σ_{cr} and σ_Y.

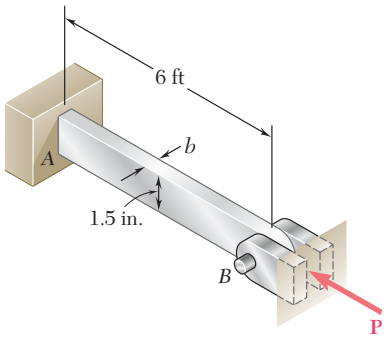
$$P_{\text{all}} = \frac{\sigma A}{FS}$$

Program Output

Radius of Rod m	Critical Stress MPa	Allowable Load kN
0.006	71.1	2.87
0.008	126.3	9.07
0.010	197.4	22.15
0.012	284.2	39.58
0.014	386.9	53.88
0.016	505.3	70.37
0.018	639.6	89.06
0.020	789.6	109.96
0.022	955.4	133.05
0.024	1137.0	158.34

Below the dashed line we have: Critical Stress > Yield Strength.

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PROBLEM 10.C2

An aluminum bar is fixed at end *A* and supported at end *B* so that it is free to rotate about a horizontal axis through the pin. Rotation about a vertical axis at end *B* is prevented by the brackets. Knowing that $E = 10.1\times10^6$ psi, use Euler's formula with a factor of safety of 2.5 to determine the allowable centric load **P** for values of *b* from 0.75 in. to 1.5 in., using 0.125-in. increments.

SOLUTION

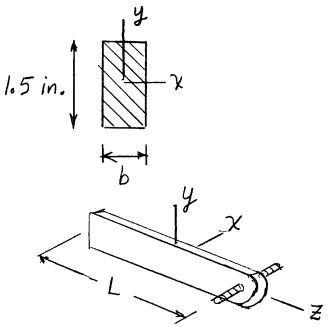
Enter *E*, length *L* and factor of safety *FS* for $b = 0.75$ to 1.5 with 0.125 increments.

Compute radius of gyration.

$$A = 1.5b$$

$$I_x = \frac{1}{12}b1.5^3 \qquad r_x = \sqrt{\frac{I_x}{A}}$$

$$I_y = \frac{1}{8}b^3 \qquad r_y = \sqrt{\frac{I_y}{A}}$$

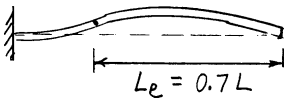


Buckling in *yz* Plane

Compute critical stresses.

$$(\sigma_{cr})_x = \frac{\pi^2 E}{(0.7L/r_x)^2}$$

$$(\sigma_{cr})_y = \frac{\pi^2 E}{(0.5L/r_y)^2}$$

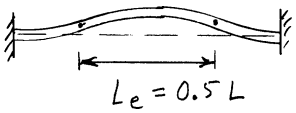


Buckling in *xz* Plane

Let σ_{cr} equal smaller stress.

Compute allowable centric load.

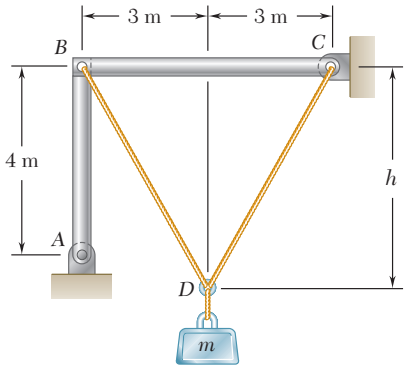
$$P_{all} = \frac{\sigma_{cr} A}{FS}$$



Program Output

b in.	Critical Stress *x* axis ksi	Critical Stress *y* axis ksi	Allowable Load kips
0.750	7.358	3.6	1.62
0.875	7.358	4.9	2.58
1.000	7.358	6.4	3.85
1.125	7.358	8.1	4.97
1.250	7.358	10.0	5.52
1.375	7.358	12.1	6.07
1.500	7.358	14.4	6.62

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PROBLEM 10.C3

The pin-ended members *AB* and *BC* consist of sections of aluminum pipe of 120-mm outer diameter and 10-mm wall thickness. Knowing that a factor of safety of 3.5 is required, determine the mass *m* of the largest block that can be supported by the cable arrangement shown for values of *h* from 4 m to 8 m, using 0.25-m increments. Use $E = 70$ GPa and consider only buckling in the plane of the structure.

SOLUTION

Compute moment of inertia:

$$I = \frac{\pi}{4}(0.06^4 - 0.05^4)$$

for $h = 4$ to 8 using 0.25 increments.

Compute allowable loads for members.

$$(F_{AB})_{cr} = \frac{\pi^2 EI}{3.5(4)^2};$$

$$(F_{BC})_{cr} = \frac{\pi^2 EI}{3.5(6)^2}$$

Determine allowable *W*.

$$(W_{all})_1 = 2(F_{AB})_{cr}$$

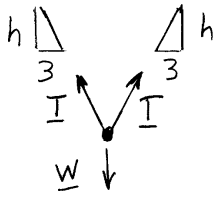
$$(W_{all})_2 = \frac{h}{1.5}(F_{BC})_{cr}$$

W_{all} equals smaller value.

Compute mass *m*.

$$m = \frac{W_{all}}{9.81}$$

Joint *D*:



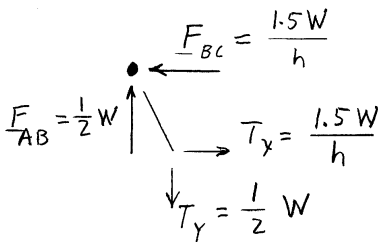
$\Sigma F_y = 0$ yields

$$T_y = \frac{1}{2}W$$

$$\frac{T_x}{T_y} = \frac{3}{h} \text{ yields}$$

$$T_x = \frac{1.5W}{h}$$

Joint *B*:



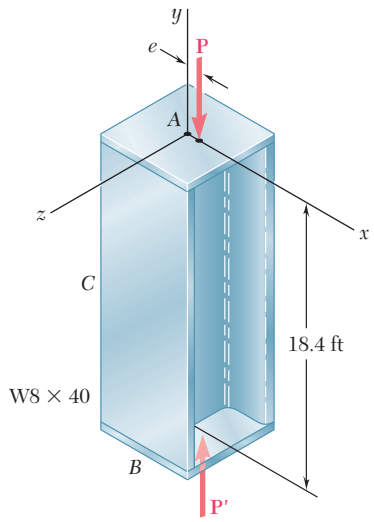
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PROBLEM 10.C3 *(Continued)*

Program Output

h m	Weight Critical Stress *AB* kN	Weight Critical Stress *BC* kN	Mass kg
4.00	455.11	269.7	7854.88
4.25	455.11	286.6	8345.80
4.50	455.11	303.4	8836.74
4.75	455.11	320.3	9327.66
5.00	455.11	337.1	9818.59
5.25	455.11	354.0	10,309.52
5.50	455.11	370.8	10,800.45
5.75	455.11	387.7	11,291.38
6.00	455.11	404.5	11,782.31
6.25	455.11	421.4	12,273.24
6.50	455.11	438.3	12,764.17
6.75	455.11	455.1	13,255.10
7.00	455.11	472.0	13,255.10
7.25	455.11	488.8	13,255.10
7.50	455.11	505.7	13,255.10
7.75	455.11	522.5	13,255.10
8.00	455.11	539.4	13,255.10

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PROBLEM 10.C4

An axial load **P** is applied at a point located on the *x* axis at a distance $e = 0.5$ in. from the geometric axis of the W8×40 rolled-steel column *AB*. Using $E = 29\times10^6$ psi, write a computer program and use it to calculate for values of *P* from 25 to 75 kips, using 5-kip increments, (*a*) the horizontal deflection at the midpoint *C*, (*b*) the maximum stress in the column.

SOLUTION

Enter length *L*, eccentricity *e*.

Enter properties *A*, I_y, r_y, b_f.

Compute critical load:

$$P_{\text{cr}} = \frac{\pi^2 E I_y}{L^2}$$

for $P = 25$ to 75 kips in increments of 5.

Compute horizontal deflection at *C*.

$$y_C = e\left(\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right) - 1.0\right)$$

Compute maximum stress.

$$\sigma_{\max} = \frac{P}{A}\left(1 + \frac{e b_f}{2 r_y^2}\sec\frac{\pi}{2}\sqrt{\frac{P}{P_{\text{cr}}}}\right)$$

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PROBLEM 10.C4 *(Continued)*

Program Output

Load kip	Maximum Deflection in.	Maximum Stress kips
25.0	0.059	3.29
30.0	0.072	3.99
35.0	0.086	4.69
40.0	0.100	5.41
45.0	0.115	6.14
50.0	0.130	6.88
55.0	0.146	7.65
60.0	0.163	8.43
65.0	0.181	9.22
70.0	0.199	10.04
75.0	0.219	10.88

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PROBLEM 10.C5

A column of effective length L is made from a rolled-steel shape and carries a centric axial load **P**. The yield strength for the grade of steel used is denoted by σ_Y, the modulus of elasticity by E, the cross-sectional area of the selected shape by A, and its smallest radius of gyration by r. Using the AISC design formulas for allowable stress design, write a computer program that can be used with either SI or U.S. customary units to determine the allowable load **P**. Use this program to solve (*a*) Prob. 10.57, (*b*) Prob. 10.58, (*c*) Prob. 10.124.

SOLUTION

Enter L, E, σ_Y.

Enter properties A, r_y.

Determine allowable stress.

$$C = 4.71\sqrt{\frac{E}{\sigma_Y}}$$

If $L/r_y \geqslant C$,

$$\sigma_{\text{all}} = \frac{0.877\pi^2 E}{1.67(L/r_y)^2}$$

If $L/r_y < C$,

$$\sigma_e = \frac{\pi^2 E}{(L/r_y)^2}$$

$$\sigma_{\text{all}} = \frac{1}{1.67}\left[0.658^{\sigma_Y/\sigma_e}\right]\sigma_y$$

Calculate allowable load.

$$P_{\text{all}} = \sigma_{\text{all}}\, A$$

Program Outputs

Problem 10.57*a*

Effective length = 6.00 m

A = 4580.0 mm^2

r_y = 40.8 mm

Yield strength = 250.0 MPa

E = 200 GPa

Allowable centroid load: P = 219.5 kN

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PROBLEM 10.C5 *(Continued)*

Program Outputs (*Continued*)

Problem 10.57*b*

Effective length = 6.00 m
$A = 11{,}000.0 \text{ mm}^2$
$r_y = 53.2$ mm
Yield strength = 250.0 MPa
$E = 200$ GPa

Allowable centroid load: $P = 839.0$ kN

Problem 10.58*a*

Effective length = 21.00 ft
$A = 9.130 \text{ in}^2$
$r_y = 2.020$ in.
Yield strength = 36.0 ksi
$E = 29{,}000$ ksi

Allowable centroid load: $P = 86.7$ kips

Problem 10.58*b*

Effective length = 21.00 ft
$A = 9.130 \text{ in}^2$
$r_y = 2.020$ in.
Yield strength = 50.0 ksi
$E = 29{,}000$ ksi

Allowable centroid load: $P = 88.2$ kips

Problem 10.124*a*

Effective length = 4.00 m
$A = 13.8 \text{ mm}^2$
$r_y = 43.4$ mm
Yield strength = 345.0 MPa
$E = 200$ GPa

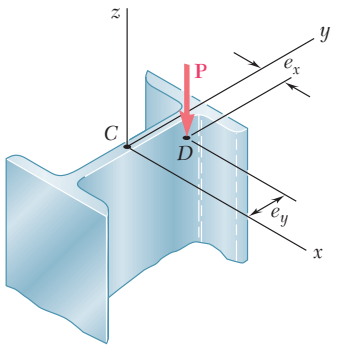
Allowable centroid load: $P = 1532$ kN

Problem 10.124*b*

Effective length = 6.50 m
$A = 13{,}800.0 \text{ mm}^2$
$r_y = 43.4$ mm
Yield strength = 345.0 MPa
$E = 200$ GPa

Allowable centroid load: $P = 638.0$ kN

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PROBLEM 10.C6

A column of effective length L is made from a rolled-steel shape and is loaded eccentrically as shown. The yield strength of the grade of steel used is denoted by σ_Y, the allowable stress in bending by σ_{all}, the modulus of elasticity by E, the cross-sectional area of the selected shape by A, and its smallest radius of gyration by r. Write a computer program that can be used with either SI or U.S. customary units to determine the allowable load **P**, using either the allowable-stress method or the interaction method. Use this program to check the given answer for (*a*) Prob. 10.113, (*b*) Prob. 10.114.

SOLUTION

Enter L, E, σ_Y, $(\sigma_{\text{all}})_{\text{bending}}$, e_x, e_y.

Enter properties A, S_x, S_y, r_y.

Determine allowable stress.

$$C = 4.71\sqrt{\frac{E}{\sigma_Y}}$$

If $L/r_y \geqslant C$,

$$\sigma_{\text{all}} = \frac{0.877\pi^2 E}{1.67(L/r_y)^2}$$

If $L/r_y < C$,

$$\sigma_e = \frac{\pi^2 E}{(L/r_y)^2}$$

$$\sigma_{\text{all}} = \frac{1}{1.67}[0.658^{\sigma_Y/\sigma_e}]\sigma_Y$$

For allowable-stress method:

$$\text{coeff.} = \frac{1}{A} + \frac{e_x}{S_x} + \frac{e_y}{S_y}$$

$$P_{\text{all}} = \frac{\sigma_{\text{all}}}{\text{coeff.}}$$

For interaction method:

$$\text{coeff.} = \frac{1}{A\sigma_{\text{all}}} + \frac{(e_x/S_x) + (e_y/S_y)}{(\sigma_{\text{all}})_{\text{bending}}}$$

$$P_{\text{all}} = \frac{1.0}{\text{coeff.}}$$

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PROBLEM 10.C6 *(Continued)*

Program Outputs

Problem 10.113

Effective length = 24.00 ft

$A = 42.700$ in^2

$r_y = 3.980$ in.

$S_x = 232.00$ in^3

Yield strength = 36.0 ksi

$E = 29 \times 10^3$ ksi

Using allowable-stress method,

Allowable load: $P = 282.6$ kips

Problem 10.114

Effective length = 24.00 ft

$A = 20.000$ in^2

$r_y = 2.460$ in.

$S_x = 103.00$ in^3

Yield strength = 50.0 ksi

$E = 29 \times 10^3$ ksi

Using interaction method,

Allowable load: $P = 139.9$ kips

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CHAPTER 11

PROBLEM 11.1

Determine the modulus of resilience for each of the following grades of structural steel:

(*a*) ASTM A709 Grade 50: $\sigma_Y = 50$ ksi
(*b*) ASTM A913 Grade 65: $\sigma_Y = 65$ ksi
(*c*) ASTM A709 Grade 100: $\sigma_Y = 100$ ksi

SOLUTION

Structural steel: $E = 29 \times 10^6$ psi for all three steels given.

(*a*) $\sigma_Y = 50 \text{ ksi} = 50 \times 10^3 \text{ psi}$

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(50 \times 10^3)^2}{(2)(29 \times 10^6)} \qquad u_Y = 43.1 \text{ in.} \cdot \text{lb/in}^3 \blacktriangleleft$$

(*b*) $\sigma_Y = 65 \text{ ksi} = 65 \times 10^3 \text{ psi}$

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(65 \times 10^6)^2}{(2)(29 \times 10^6)} \qquad u_Y = 72.8 \text{ in.} \cdot \text{lb/in}^3 \blacktriangleleft$$

(*c*) $\sigma_Y = 100 \text{ ksi} = 100 \times 10^3 \text{ psi}$

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(100 \times 10^3)^2}{(2)(29 \times 10^6)} \qquad u_Y = 172.4 \text{ in.} \cdot \text{lb/in}^3 \blacktriangleleft$$

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PROBLEM 11.2

Determine the modulus of resilience for each of the following aluminum alloys:

(*a*) 1100-H14: $E = 70\text{ GPa}$ $\sigma_Y = 55\text{ MPa}$
(*b*) 2014-T6: $E = 72\text{ GPa}$ $\sigma_Y = 220\text{ MPa}$
(*c*) 6061-T6: $E = 69\text{ GPa}$ $\sigma_Y = 150\text{ MPa}$

SOLUTION

Aluminum alloys:

(*a*) $E = 70\times10^9\text{ Pa}$ $\sigma_Y = 55\times10^6\text{ Pa}$

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(55\times10^6)^2}{(2)(70\times10^9)} = 21.6\times10^3\text{ N}\cdot\text{m/m}^3$$

$u_Y = 21.6\text{ kJ/m}^3$ ◀

(*b*) $E = 72\times10^9\text{ Pa}$ $\sigma_Y = 220\times10^6\text{ Pa}$

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(220\times10^6)^2}{(2)(72\times10^9)} = 336\times10^3\text{ N}\cdot\text{m/m}^3$$

$u_Y = 336\text{ kJ/m}^3$ ◀

(*c*) $E = 69\times10^9\text{ Pa}$ $\sigma_Y = 150\times10^6\text{ Pa}$

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(150\times10^6)^2}{(2)(69\times10^9)} = 163.0\times10^3\text{ N}\cdot\text{m/m}^3$$

$u_Y = 163.0\text{ kJ/m}^3$ ◀

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PROBLEM 11.3

Determine the modulus of resilience for each of the following metals:

(*a*) Stainless steel
AISI 302 (annealed): $E = 190$ GPa $\sigma_Y = 260$ MPa

(*b*) Stainless steel
AISI 302 (cold-rolled): $E = 190$ GPa $\sigma_Y = 520$ MPa

(*c*) Malleable cast iron: $E = 165$ GPa $\sigma_Y = 230$ MPa

SOLUTION

(*a*) $E = 190 \times 10^9$ Pa, $\sigma_Y = 260 \times 10^6$ Pa

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(260 \times 10^6)^2}{(2)(190 \times 10^9)} = 177.9 \times 10^3 \text{ N} \cdot \text{m/m}^3$$

$$u_Y = 177.9 \text{ kJ/m}^3 \blacktriangleleft$$

(*b*) $E = 190 \times 10^9$ Pa, $\sigma_Y = 520 \times 10^6$ Pa

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(520 \times 10^6)^2}{(2)(190 \times 10^9)} = 712 \times 10^3 \text{ N} \cdot \text{m/m}^3$$

$$u_Y = 712 \text{ kJ/m}^3 \blacktriangleleft$$

(*c*) $E = 165 \times 10^9$ Pa, $\sigma_Y = 230 \times 10^6$ Pa

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(230 \times 10^6)^2}{(2)(165 \times 10^9)} = 160.3 \times 10^3 \text{ N} \cdot \text{m/m}^3$$

$$u_Y = 160.3 \text{ kJ/m}^3 \blacktriangleleft$$

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PROBLEM 11.4

Determine the modulus of resilience for each of the following alloys:

(*a*) Titanium: $E = 16.5 \times 10^6$ psi $\sigma_Y = 120$ ksi

(*b*) Magnesium: $E = 6.5 \times 10^6$ psi $\sigma_Y = 29$ ksi

(*c*) Cupronickel (annealed): $E = 20 \times 10^6$ psi $\sigma_Y = 16$ ksi

SOLUTION

(*a*) $E = 16.5 \times 10^6$ psi, $\sigma_Y = 120 \times 10^3$ psi

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(120 \times 10^3)^2}{(2)(16.5 \times 10^6)} \qquad u_Y = 436 \text{ in.} \cdot \text{lb/in}^3 \blacktriangleleft$$

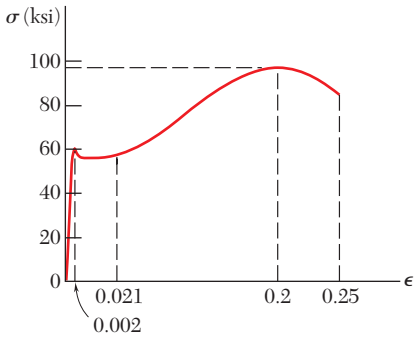
(*b*) $E = 6.5 \times 10^6$ psi, $\sigma_Y = 29 \times 10^3$ psi

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(29 \times 10^3)^2}{(2)(6.5 \times 10^6)} \qquad u_Y = 64.7 \text{ in.} \cdot \text{lb/in}^3 \blacktriangleleft$$

(*c*) $E = 20 \times 10^6$ psi, $\sigma_Y = 16 \times 10^3$ psi

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(16 \times 10^3)^2}{(2)(20 \times 10^6)} \qquad u_Y = 6.40 \text{ in.} \cdot \text{lb/in}^3 \blacktriangleleft$$

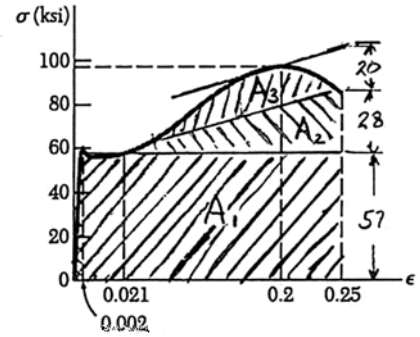
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PROBLEM 11.5

The stress-strain diagram shown has been drawn from data obtained during a tensile test of a specimen of structural steel. Using $E = 29 \times 10^6$ psi, determine (*a*) the modulus of resilience of the steel, (*b*) the modulus of toughness of the steel.

SOLUTION



(*a*) $\sigma_Y = E\varepsilon_Y$

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{1}{2}E\varepsilon_Y^2 = \frac{1}{2}(29 \times 10^6)(0.002)^2$$

$$u_Y = 58.0 \text{ in.} \cdot \text{lb/in}^3 \blacktriangleleft$$

(*b*) Modulus of toughness = total area under the stress-strain curve

$$A_1 = (57)(0.25 - 0.002) = 14.14 \text{ kips/in}^2$$
$$= 14.14 \text{ in.} \cdot \text{kip/in}^3$$

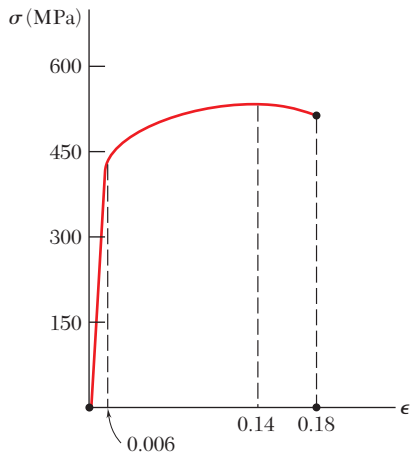
$$A_2 = \frac{1}{2}(28)(0.25 - 0.021) = 3.21 \text{ kips/in}^2$$
$$= 3.21 \text{ in.} \cdot \text{kip/in}^3$$

$$A_3 = \frac{2}{3}(20)(0.25 - 0.075) = 2.33 \text{ kips/in}^2$$
$$= 2.33 \text{ in.} \cdot \text{kip/in}^3$$

$$\underline{\text{modulus of toughness}} = u_Y + A_1 + A_2 + A_3$$

modulus of toughness $\approx 20.0 \text{ in.} \cdot \text{kip/in}^3 \blacktriangleleft$

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PROBLEM 11.6

The stress-strain diagram shown has been drawn from data obtained during a tensile test of an aluminum alloy. Using $E = 72$ GPa, determine (*a*) the modulus of resilience of the alloy, (*b*) the modulus of toughness of the alloy.

SOLUTION

(*a*) $\sigma_Y = E\varepsilon_Y$

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{1}{2}E\varepsilon_Y^2 = \frac{1}{2}(72\times10^9)(0.006)^2$$

$$= 1296\times10^3\ \text{N}\cdot\text{m/m}^3 \qquad u_Y = 1296\ \text{kJ/m}^3 \blacktriangleleft$$

(*b*) Modulus of toughness = total area under the stress-strain curve

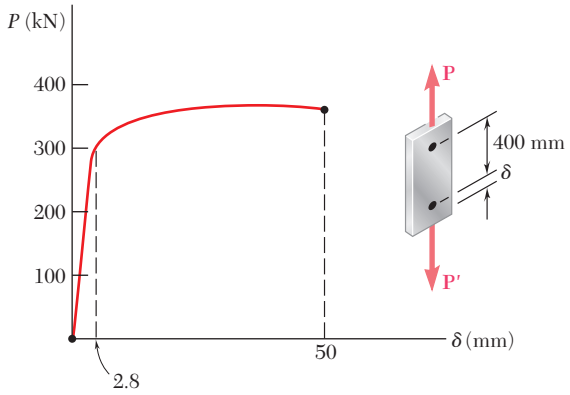
The average ordinate of the stress-strain curve is

$$500\ \text{MPa} = 500\times10^6\ \text{N/m}^2.$$

The area under the curve is $A = (500\times10^6)(0.18) = 90.0\times10^6\ \text{N/m}^2.$

$$\underline{\text{modulus of toughness}} = 90.0\times10^6\ \text{J/m}^3 = 90.0\ \text{MJ/m}^3 \blacktriangleleft$$

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PROBLEM 11.7

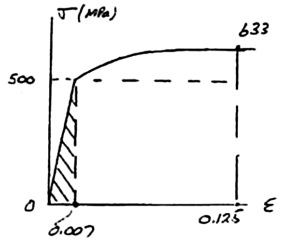
The load-deformation diagram shown has been drawn from data obtained during a tensile test of a specimen of an aluminum alloy. Knowing that the cross-sectional area of the specimen was 600 mm^2 and that the deformation was measured using a 400-mm gage length, determine by approximate means (*a*) the modulus of resilience of the alloy, (*b*) the modulus of toughness of the alloy.

SOLUTION

$$\sigma = \frac{P}{A} = \frac{P}{600 \times 10^{-6}\ \text{m}^2}$$

$$\varepsilon = \frac{\delta}{L} = \frac{\delta}{400\ \text{mm}}$$

Draw $\sigma - \varepsilon$ curve:



(*a*) Modules of resilience:

(shaded area)

$$u_Y = \frac{1}{2}(500\ \text{MPa})(0.007)$$

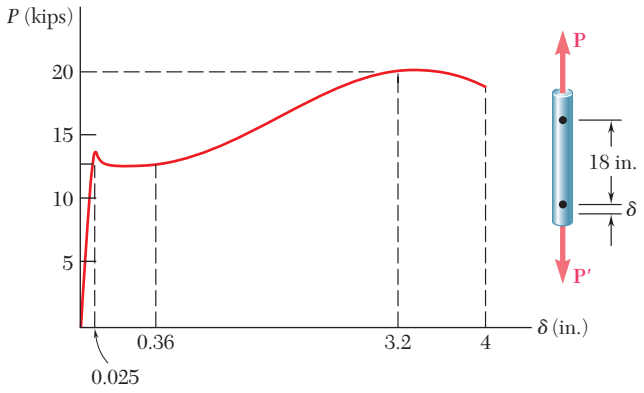
$$u_Y = 1.750\ \text{MJ/m}^3 \blacktriangleleft$$

(*b*) Modules of toughness: (total area under $\sigma - \varepsilon$ curve)

$$= 1.750\ \text{MJ/m}^3 + (500\ \text{MPa})(0.125 - 0.007) + \frac{1}{2}(6.33 - 500)(0.125 - 0.007)$$

$$= 1.750\ \text{MJ/m}^3 + 59\ \text{MJ/m}^3 + 10.46\ \text{MJ/m}^3 = 71.2\ \text{MJ/m}^3 \blacktriangleleft$$

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PROBLEM 11.8

The load-deformation diagram shown has been drawn from data obtained during a tensile test of a $\frac{5}{8}$-in.-diameter rod of structural steel. Knowing that the deformation was measured using an 18-in. gage length, determine by approximate means (*a*) the modulus of resilience of the steel, (*b*) the modulus of toughness of the steel.

SOLUTION

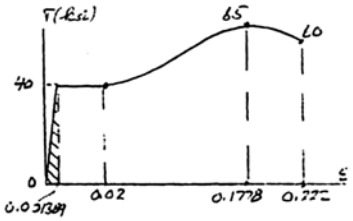
$\frac{5}{3}$-in.-diameter rod:

$$A = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.3068\ \text{in}^2$$

$$\sigma = \frac{P}{A} = \frac{12.5\ \text{kips}}{0.3068\ \text{in}^2} = 40\ \text{ksi}$$

$$\varepsilon = \frac{\delta}{L} = \frac{0.025\ \text{in.}}{18\ \text{in.}} = 1.389 \times 10^{-3}$$

$\sigma_Y \approx 40$ ksi



Draw σ-ε curve:

(*a*) Mod. of resilience:

(shaded area)

$$u_Y = \frac{1}{2}\sigma_Y\varepsilon_Y = \frac{1}{2}(40 \times 10^3\ \text{psi})(1.389 \times 10^{-3})$$

$$u_Y \approx 28.0\ \text{in.} \cdot \text{lb/in}^3 \blacktriangleleft$$

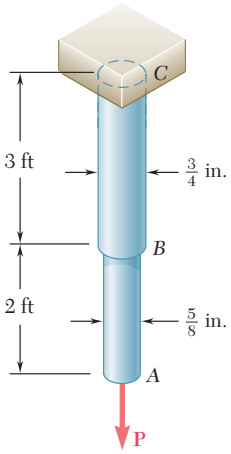
(*b*) Mod. of toughness: (total area under σ-ε curve)

$$= 28\ \text{in.} \cdot \text{lb/in}^2 + (40\ \text{ksi})(0.02 - 0.0014) + \frac{1}{2}(40 + 65)(0.1778 - 0.02) + (62.5)(0.222 - 0.1778)$$

$$= 28 + 744 + 9860 + 2760 = 13{,}390\ \text{in.} \cdot \text{lb/in}^2$$

Modulus of toughness $\approx 13.40\ \text{in.} \cdot \text{kips/in}^3 \blacktriangleleft$

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PROBLEM 11.9

Using $E = 29\times10^{6}$ psi, determine (*a*) the strain energy of the steel rod *ABC* when $P = 8$ kips, (*b*) the corresponding strain energy density in portions *AB* and *BC* of the rod.

SOLUTION

$$P = 8 \text{ kips}, \quad E = 29\times10^{3} \text{ ksi}$$

$$A = \frac{\pi}{4}d^{2}, \quad V = AL, \quad \sigma = \frac{P}{A}, \quad u = \frac{\sigma^{2}}{2E}$$

$$U = uV$$

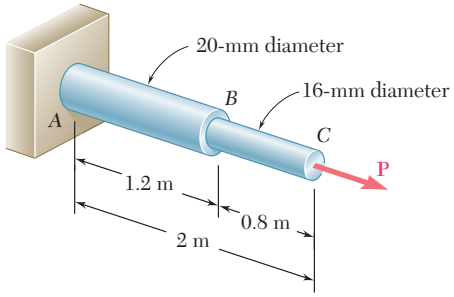
Portion	d(in.)	L(in.)	A(in^2)	V(in^3)	σ(ksi)	u(in.·kip/in^3)	U(in.·kip)
AB	0.625	24	0.3608	7.363	26.08	11.72×10^{-3}	86.32×10^{-3}
BC	0.75	36	0.4418	15.904	18.11	5.65×10^{-3}	89.92×10^{-3}
Σ							176.24×10^{-3}

(*a*) $U = 176.2\times10^{-3}$ in.·kip $\qquad U = 176.2$ in.·lb ◀

(*b*) In *AB*: $u = 11.72\times10^{-3}$ in.·kip/in^3 $\qquad u_{AB} = 11.72$ in.·lb/in^3 ◀

In *BC*: $u = 5.65\times10^{-3}$ in.·kip/in^3 $\qquad u_{BC} = 5.65$ in.·lb/in^3 ◀

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PROBLEM 11.10

Using $E = 200$ GPa, determine (*a*) the strain energy of the steel rod *ABC* when $P = 25$ kN, (*b*) the corresponding strain-energy density in portions *AB* and *BC* of the rod.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2$$

$$= 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2$$

$$= 201.06 \times 10^{-6} \text{ m}^2$$

$$P = 25 \times 10^3 \text{ N}$$

$$U = \sum \frac{P^2 L}{2EA}$$

$$= \frac{(25\times10^3)^2(1.2)}{(2)(200\times10^9)(314.16\times10^{-6})} + \frac{(25\times10^3)^2(0.8)}{(2)(200\times10^9)(201.06\times10^{-6})}$$

(*a*) $U = 5.968 + 6.213 = 12.18 \text{ N}\cdot\text{m}$ $\qquad U = 12.18 \text{ J}$ ◀

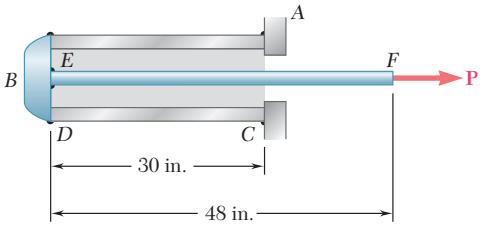
(*b*) $$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{25\times10^3}{314.16\times10^{-6}} = 79.58\times10^6 \text{ Pa}$$

$$u_{AB} = \frac{\sigma_{AB}^2}{2E} = \frac{(79.58\times10^6)^2}{(2)(200\times10^9)} = 15.83\times10^3 \qquad u_{AB} = 15.83 \text{ kJ/m}^3$$ ◀

$$\sigma_{BC} = \frac{P}{A_{AB}} = \frac{25\times10^3}{201.16\times10^{-6}} = 124.28\times10^6 \text{ Pa}$$

$$u_{BC} = \frac{\sigma_{BC}^2}{2E} = \frac{(124.28\times10^6)^2}{(2)(200\times10^9)} = 38.6\times10^3 \qquad u_{BC} = 38.6 \text{ kJ/m}^3$$ ◀

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PROBLEM 11.11

A 30-in. length of aluminum pipe of cross-sectional area 1.85 in^2 is welded to a fixed support *A* and to a rigid cap *B*. The steel rod *EF*, of 0.75-in. diameter, is welded to cap *B*. Knowing that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the total strain energy of the system when $P = 8$ kips, (*b*) the corresponding strain-energy density of the pipe *CD* and in the rod *EF*.

SOLUTION

Member *EF* carries a force $P = 8000$ lb in tension while member *CD* carries 8000 lb in compression.

Area of member *EF*: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.75)^2 = 0.4418 \text{ in}^2$

(*a*) Strain energy.

CD: $U_{CD} = \frac{P^2L}{2EA} = \frac{(-8000)^2(30)}{(2)(10.6\times10^6)(1.85)} = 48.95 \text{ in.}\cdot\text{lb}$

EF: $U_{EF} = \frac{P^2L}{2EA} = \frac{(8000)^2(48)}{(2)(29\times10^6)(0.4418)} = 119.89 \text{ in.}\cdot\text{lb}$

Total: $U = U_{CD} + U_{EF} = 168.8 \text{ in.}\cdot\text{lb}$ $\quad U = 168.8 \text{ in.}\cdot\text{lb.}$ ◀

(*b*) Strain energy density.

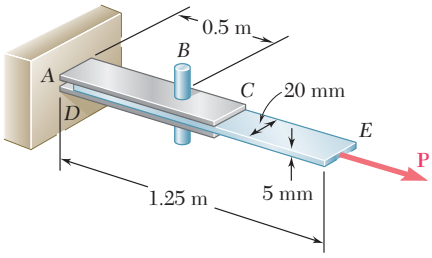
CD: $\sigma = -\frac{8000}{1.85} = -4324 \text{ psi}, \qquad u = \frac{\sigma^2}{2E} = \frac{(-4324)^2}{(2)(10.6\times10^6)}$

$u = 0.882 \text{ in.}\cdot\text{lb/in}^3$ ◀

EF: $\sigma = \frac{8000}{0.4418} = 18{,}108 \text{ psi}, \qquad u = \frac{\sigma^2}{2E} = \frac{(18{,}108)^2}{(2)(29\times10^6)}$

$u = 5.65 \text{ in.}\cdot\text{lb/in}^3$ ◀

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PROBLEM 11.12

A single 6-mm-diameter steel pin *B* is used to connect the steel strip *DE* to two aluminum strips, each of 20-mm width and 5-mm thickness. The modulus of elasticity is 200 GPa for the steel and 70 GPa for the aluminum. Knowing that for the pin at *B* the allowable shearing stress is $\tau_{all} = 85$ MPa, determine, for the loading shown, the maximum strain energy that can be acquired by the assembled strips.

SOLUTION

$$A_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(6)^2 = 28.274 \text{ mm}^2$$
$$= 28.274 \times 10^{-6} \text{ m}^2$$

$$\tau_{all} = 85 \times 10^6 \text{ Pa}$$

Double shear:

$$P = 2A\tau = (2)(28.274 \times 10^{-6})(85 \times 10^6)$$
$$= 4.8066 \times 10^3 \text{ N}$$

For strips *AB*, *DB*, *BE*,

$$A = (20)(5) = 100 \text{ mm}^2 = 100 \times 10^{-6} \text{ m}^2$$

$$F_{AB} = F_{DB} = \frac{1}{2} P = 2.4033 \times 10^3 \text{ N}$$

$$U_{AB} = U_{DB} = \frac{F_{AB}^2 L_{AB}}{2E_a A_{AB}} = \frac{(2.4033 \times 10^3)(0.5)}{(2)(70 \times 10^9)(100 \times 10^{-6})} = 0.2063 \text{ J}$$

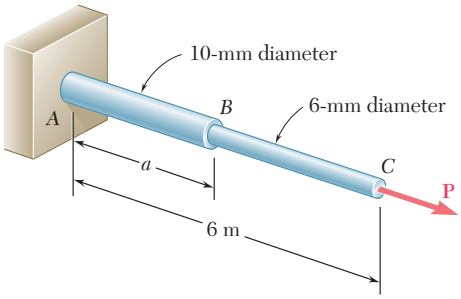
$$U_{BE} = \frac{F_{BE}^2 L_{BE}}{2E_s A_{BE}} = \frac{(4.8066 \times 10^3)^2 (1.25 - 0.5)}{(2)(200 \times 10^9)(100 \times 10^{-6})} = 0.4332 \text{ J}$$

Total:

$$U = U_{AB} + U_{DB} + U_{BE} = 0.846 \text{ J}$$

$U = 0.846$ J ◀

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PROBLEM 11.13

Rods *AB* and *BC* are made of a steel for which the yield strength is $\sigma_Y = 300$ MPa and the modulus of elasticity is $E = 200$ GPa. Determine the maximum strain energy that can be acquired by the assembly without causing any permanent deformation when the length *a* of rod *AB* is (*a*) 2 m, (*b*) 4 m.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(10)^2 = 78.54\ \text{mm}^2 = 78.54 \times 10^{-6}\ \text{m}^2$$

$$A_{BC} = \frac{\pi}{4}(6)^2 = 28.274\ \text{mm}^2 = 28.274 \times 10^{-6}\ \text{m}^2$$

$$P = \sigma_Y A_{\min} = (300 \times 10^6)(28.274 \times 10^{-6})$$

$$= 8.4822 \times 10^3\ \text{N}$$

$$U = \sum \frac{P^2 L}{2EA}$$

(*a*) $a = 2$ m $\quad L - a = 6 - 2 = 4$ m

$$U = \frac{(8.4822 \times 10^3)^2(2)}{(2)(200 \times 10^9)(78.54 \times 10^{-6})} + \frac{(8.4822 \times 10^3)^2(4)}{(2)(200 \times 10^9)(28.274 \times 10^{-6})}$$

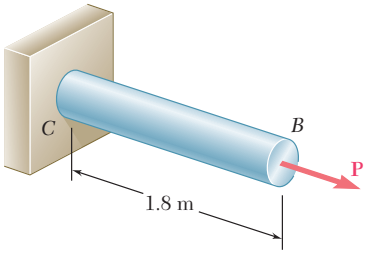
$$= 4.5803 + 25.4466 = 30.0\ \text{N} \cdot \text{m} = 30.0\ \text{J}$$ ◀

(*b*) $a = 4$ m $\quad L - a = 6 - 4 = 2$ m

$$U = \frac{(8.4822 \times 10^3)^2(4)}{(2)(200 \times 10^9)(78.54 \times 10^{-6})} + \frac{(8.4822 \times 10^3)^2(2)}{(2)(200 \times 10^9)(28.274 \times 10^{-6})}$$

$$= 9.1606 + 12.7233 = 21.9\ \text{N} \cdot \text{m} = 21.9\ \text{J}$$ ◀

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PROBLEM 11.14

Rod *BC* is made of a steel for which the yield strength is $\sigma_Y = 300$ MPa and the modulus of elasticity is $E = 200$ GPa. Knowing that a strain energy of 10 J must be acquired by the rod when the axial load **P** is applied, determine the diameter of the rod for which the factor of safety with respect to permanent deformation is six.

SOLUTION

For factor of safety of six on the energy,

$$U_Y = (6)(10) = 60 \text{ J}$$

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(300\times10^6)^2}{(2)(200\times10^9)}$$

$$= 225\times10^3 \text{ J/m}^3$$

$$U_Y = ALu_Y$$

$$A = \frac{U_Y}{Lu_Y} = \frac{60}{(1.8)(225\times10^3)}$$

$$= 148.148\times10^{-6} \text{ m}^2$$

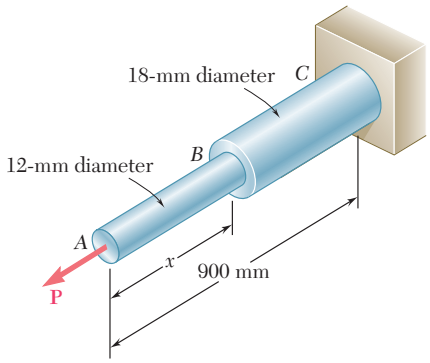
$$A = \frac{\pi}{4}d^2$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(148.148\times10^{-6})}{\pi}}$$

$$= 13.73\times10^{-3} \text{ m}$$

$$d = 13.73 \text{ mm} \blacktriangleleft$$

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PROBLEM 11.15

The assembly *ABC* is made of a steel for which $E = 200$ GPa and $\sigma_Y = 320$ MPa. Knowing that a strain energy of 5 J must be acquired by the assembly as the axial load **P** is applied, determine the factor of safety with respect to permanent deformation when (*a*) $x = 300$ mm, (*b*) $x = 600$ mm.

SOLUTION

$$\sigma_Y = 320 \text{ MPa} = 320 \times 10^6 \text{ Pa}, \qquad E = 200 \text{ GPa} = 200 \times 10^9 \text{ Pa}$$

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(12)^2 = 113.097 \text{ mm}^2 = 113.097 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(18)^2 = 254.47 \text{ mm}^2 = 254.47 \times 10^{-6} \text{ m}^2$$

$$A_{\min} = A_{AB}$$

Force at yielding or allowable axial force.

$$P = P_Y = \sigma_Y A_{\min} = (320 \times 10^6)(113.097 \times 10^{-6}) = 36.191 \times 10^3 \text{ N}$$

(*a*) $x = 300$ mm: $L_{AB} = 0.300$ m, $L_{BC} = 0.600$ m

$$U_Y = U_{AB} + U_{BC} = \frac{P^2 L_{AB}}{2EA_{AB}} + \frac{P^2 L_{BC}}{2EA_{BC}} = \frac{P^2}{2E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$= \frac{(36.191 \times 10^3)^2}{(2)(200 \times 10^9)}\left[\frac{0.300}{113.097 \times 10^{-6}} + \frac{0.600}{254.97 \times 10^{-6}}\right]$$

$$= (3.2745 \times 10^{-3})(2652.6 + 2353.2) = 16.392 \text{ J}$$

Applied energy: $U = 5$ J

Factor of safety: $\dfrac{U_Y}{U} = \dfrac{16.392}{5}$ $F.S. = 3.28$ ◀

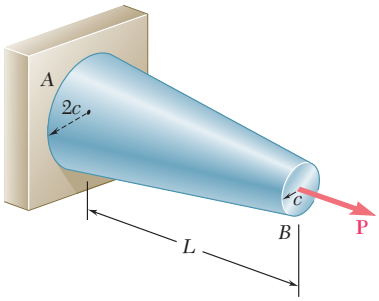
(*b*) $x = 600$ mm: $L_{AB} = 0.600$ m, $L_{BC} = 0.300$ m

$$U_Y = \frac{36.191 \times 10^3}{(2)(200 \times 10^9)}\left[\frac{0.600}{113.097 \times 10^{-6}} + \frac{0.300}{254.97 \times 10^{-6}}\right]$$

$$= (3.2745 \times 10^{-3})(5305.2 + 1176.6) = 21.225 \text{ J}$$

Factor of safety: $\dfrac{U_Y}{U} = \dfrac{21.225}{5}$ $F.S. = 4.25$ ◀

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PROBLEM 11.16

Show by integration that the strain energy of the tapered rod *AB* is

$$U = \frac{1}{4}\frac{P^2 L}{EA_{\min}}$$

where $A_{\min}$ is the cross-sectional area at end *B*.

SOLUTION

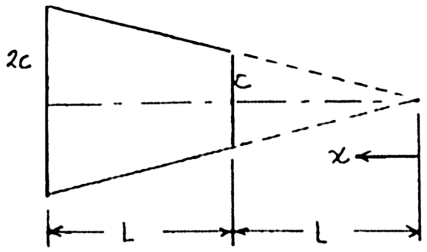
Radius: $r = \dfrac{cx}{L}$ $\qquad A_{\min} = \pi c^2$

$$A = \pi r^2 = \frac{\pi c^2}{L^2}x^2$$

$$U = \int_L^{2L} \frac{P^2 dx}{2EA} = \frac{P^2}{2E}\int_L^{2L} \frac{L^2}{\pi c^2}\frac{dx}{x^2}$$

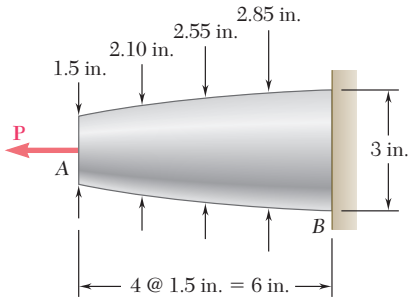
$$= \frac{P^2 L^2}{2E\pi c^2}\left(-\frac{1}{x}\right)\Bigg|_L^{2L}$$

$$= \frac{P^2 L^2}{2EA_{\min}}\left(-\frac{1}{2L}+\frac{1}{L}\right)$$



$$U = \frac{P^2 L}{4EA_{\min}} \blacktriangleleft$$

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PROBLEM 11.17

Using $E = 10.6 \times 10^6$ psi, determine by approximate means the maximum strain energy that can be acquired by the aluminum rod shown if the allowable normal stress is $\sigma_{all} = 22$ ksi.

SOLUTION

$$A_{min} = \frac{\pi}{4}(1.5)^2 = 1.7671 \text{ in}^2$$

$$\sigma_{all} = 22{,}000 \text{ psi}$$

$$P_{all} = \sigma_{all} A_{min} = 38{,}877 \text{ lb}$$

$$U = \int \frac{P^2 dx}{2EA} = \frac{P^2}{2E} \int \frac{dx}{\frac{\pi}{4} d^2} = \frac{2P^2}{\pi E} \int \frac{dx}{d^2}$$

Use Simpson's rule to compute the integral. $h = 1.5$ in.

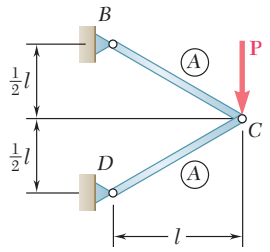
Section	d (in.)	$1/d^2$ (in^{-2})	multiplier	$m(1/d^2)$ (in^{-2})
1	1.50	0.4444	1	0.4444
2	2.10	0.22675	4	0.9070
3	2.55	0.15379	2	0.3076
4	2.85	0.12311	4	0.4924
5	3.00	0.11111	1	0.1111
Σ				2.2625

$$\int_A^B \frac{dx}{d^2} = \frac{h}{3} \Sigma m \left(\frac{1}{d^2} \right) = \frac{1.5}{3}(2.2625) = 1.13125 \text{ in}^{-1}$$

$$U = \frac{(2)(38877)^2 (1.13125)}{\pi (10.6 \times 10^6)}$$

$$U = 102.7 \text{ in.} \cdot \text{lb} \blacktriangleleft$$

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PROBLEM 11.18

In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load **P** is applied.

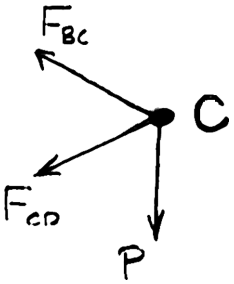
SOLUTION

$$L_{BC} = L_{CD} = \sqrt{l^2 + \left(\frac{1}{2}l\right)^2} = \frac{\sqrt{5}}{2}l$$

Joint *C*. (equilibrium)

$$\pm\rightarrow \Sigma F_x = 0: \qquad -\frac{2}{\sqrt{5}}F_{BC} - \frac{2}{\sqrt{5}}F_{CD} = 0$$

$$F_{CD} = -F_{BC}$$



$$+\uparrow \Sigma F_y = 0: \qquad \frac{1}{\sqrt{5}}F_{BC} - \frac{1}{\sqrt{5}}F_{CD} - P = 0$$

$$F_{BC} = \frac{\sqrt{5}}{2}P \qquad F_{CD} = -\frac{\sqrt{5}}{2}P$$

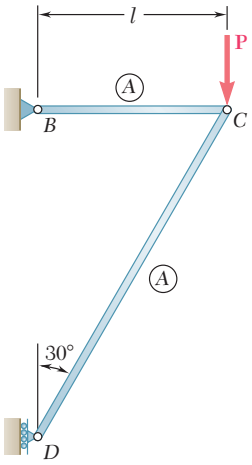
Strain energy.

$$U = \Sigma\frac{F^2L}{2EA} = \frac{1}{2EA}\left[F_{BC}^2L_{BC} + F_{CD}^2L_{CD}\right]$$

$$= \frac{1}{2EA}\left[\left(\frac{\sqrt{5}}{2}P\right)^2\left(\frac{\sqrt{5}}{2}l\right) + \left(\frac{\sqrt{5}}{2}P\right)^2\left(\frac{\sqrt{5}}{2}l\right)\right]$$

$$U = 1.398\frac{P^2l}{EA} \blacktriangleleft$$

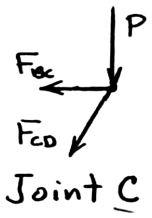
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PROBLEM 11.19

In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load **P** is applied.

SOLUTION



$$+\uparrow \Sigma F_y = 0: \quad -\frac{\sqrt{3}}{2}F_{CD} - P = 0 \qquad F_{CD} = -\frac{2}{\sqrt{3}}P$$

$$+\rightarrow \Sigma F_x = 0: \quad -F_{BC} - \frac{1}{2}F_{CD} = 0 \qquad F_{BC} = \frac{1}{\sqrt{3}}P$$

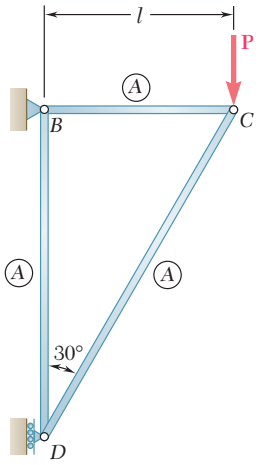
$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E}\sum \frac{F^2 L}{A}$$

Member	F	L	A	F^2L/A
BC	$\frac{1}{\sqrt{3}}P$	l	A	$\frac{1}{3}P^2l/A$
CD	$-\frac{2}{\sqrt{3}}P$	$2l$	A	$\frac{8}{3}P^2l/A$
Σ				$3P^2l/A$

$$U = \frac{1}{2E}\left(3\frac{P^2 l}{A}\right)$$

$$U = 1.5\frac{P^2 l}{EA} \blacktriangleleft$$

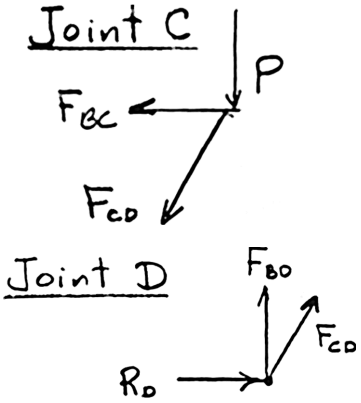
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PROBLEM 11.20

In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load **P** is applied.

SOLUTION



Equilibrium of joint *C*.

$$+\uparrow \sum F_y = 0: \quad -\frac{\sqrt{3}}{2}F_{CD} - P = 0 \qquad F_{CD} = -\frac{2}{\sqrt{3}}P$$

$$+\rightarrow \sum F_x = 0: \quad -F_{BC} - \frac{1}{2}F_{CD} = 0 \qquad F_{BC} = \frac{1}{\sqrt{3}}P$$

Equilibrium of joint *D*.

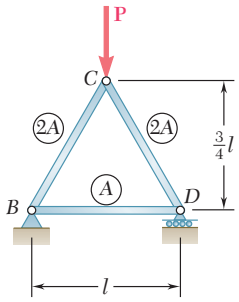
$$+\uparrow \sum F_y = 0: \quad F_{BD} + \frac{\sqrt{3}}{2}F_{CD} = 0 \qquad F_{BD} = P$$

Strain energy. $U = \sum \frac{1}{2}\frac{F^2L}{EA} = \frac{1}{2E}\sum \frac{F^2L}{A}$

Member	F	L	A	F^2L/A
BC	$\frac{1}{\sqrt{3}}P$	l	A	$\frac{1}{3}P^2l/A$
CD	$-\frac{2}{\sqrt{3}}P$	$2l$	A	$\frac{8}{3}P^2l/A$
BD	P	$\sqrt{3}l$	A	$\sqrt{3}P^2l/A$
Σ				$4.732P^2l/A$

$$U = \frac{1}{2E}\left(4.732\frac{P^2l}{A}\right) \qquad U = 2.37\frac{P^2l}{EA} \blacktriangleleft$$

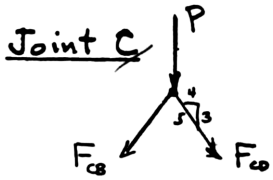
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PROBLEM 11.21

In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load **P** is applied.

SOLUTION

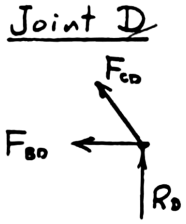


$$+\rightarrow \Sigma F_x = 0: \quad \frac{3}{5}F_{CD} - \frac{3}{5}F_{CB} = 0$$

$$F_{CB} = F_{CD}$$

$$+\uparrow \Sigma F_y = 0: \quad -P - 2\cdot\frac{4}{5}F_{CD} = 0$$

$$F_{CB} = F_{CD} = -\frac{5}{8}P$$



$$+\rightarrow \Sigma F_x = 0: \quad -F_{BD} - \frac{3}{5}F_{CD} = 0$$

$$-F_{BD} - \frac{3}{5}F_{CD} = 0$$

$$F_{BD} = -\frac{3}{5}\cdot\frac{5}{8}P = \frac{3}{8}P$$

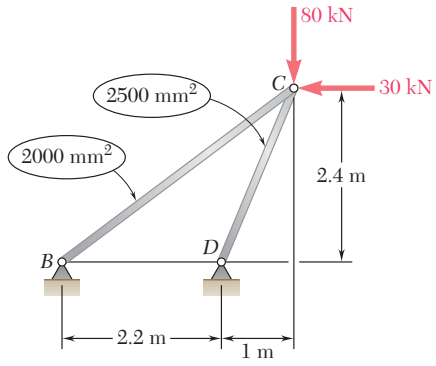
$$U = \Sigma\frac{F^2L}{2EA} = \frac{1}{2E}\Sigma\frac{F^2L}{A}$$

$$= \frac{1}{2E}\left(\frac{179}{384}\frac{P^2l}{A}\right)$$

$$= \frac{179}{768}\frac{P^2l}{EA} \qquad U = 0.233\frac{P^2l}{EA} \blacktriangleleft$$

Member	F	L	A	F^2L/A
CB	$-\frac{5}{8}P$	$\frac{5}{6}l$	$2A$	$\frac{125}{768}P^2l/A$
CD	$-\frac{5}{8}P$	$\frac{5}{6}l$	$2A$	$\frac{125}{768}P^2l/A$
BD	$\frac{3}{8}P$	l	A	$\frac{9}{64}P^2l/A$
Σ				$\frac{179}{384}P^2l/A$

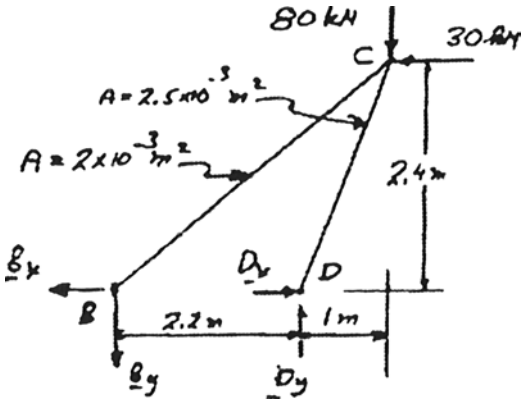
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PROBLEM 11.22

Each member of the truss shown is made of aluminum and has the cross-sectional area shown. Using $E = 72$ GPa, determine the strain energy of the truss for loading shown.

SOLUTION



Lengths of members:

$$L_{BC} = (3.2^2 + 2.4^2)^{1/2} = 4 \text{ m}$$

$$L_{CD} = (1^2 + 2.4^2)^{1/2} = 2.6 \text{ m}$$

$$E = 72 \text{ GPa} = 72 \times 10^9 \text{ Pa}$$

Forces in kN.

Equilibrium of truss.

$$+\circlearrowleft \sum M_B = 0: \quad (30)(2.4) - (80)(3.2) + D_y(2.2) = 0$$

$$D_y = 83.636 \text{ kN} \uparrow$$

$$+\uparrow \sum F_y = 0: \quad D_y - B_y - 80 = 0$$

$$83.636 - B_y - 80 = 0 \qquad B_y = 3.636 \text{ kN} \downarrow$$

Member forces.

$$F_{BC} = B_y \frac{4 \text{ m}}{2.4 \text{ m}} = (3.636 \text{ kN})\left(\frac{4}{2.4}\right) = 6.061 \text{ kN}$$

$$F_{CD} = -D_y \frac{2.6 \text{ m}}{2.4 \text{ m}} = -(83.636 \text{ kN})\left(\frac{2.6}{2.4}\right) = -90.606 \text{ kN}$$

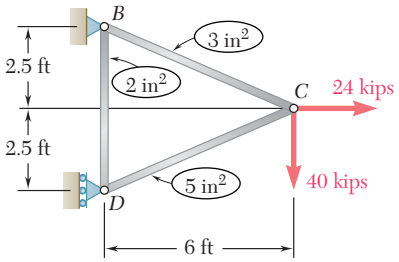
Strain energy. $U = U_{BC} + U_{CD} = \sum \dfrac{F_i L_i}{2AE}$

$$U = \frac{F_{BC}^2 L_{BC}}{2EA_{BC}} + \frac{F_{CD}^2 L_{CD}}{2EA_{CD}} = \frac{(6.061 \times 10^3)^2 (4)}{(2)(72 \times 10^9)(2 \times 10^{-3})} + \frac{(90.606 \times 10^3)^2 (2.6)}{(2)(72 \times 10^9)(2.5 \times 10^{-3})}$$

$$= 0.510 \text{ J} + 59.290 \text{ J}$$

$$U = 59.8 \text{ J} \blacktriangleleft$$

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PROBLEM 11.23

Each member of the truss shown is made of aluminum and has the cross-sectional area shown. Using $E = 10.5 \times 10^6$ psi, determine the strain energy of the truss for the loading shown.

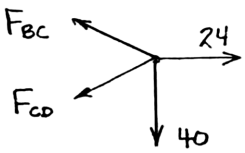
SOLUTION

$$L_{BC} = L_{CD} = \sqrt{6^2 + 2.5^2} = 6.5 \text{ ft} = 78 \text{ in.}$$

Joint *C*:

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \quad -\frac{6}{6.5} F_{BC} - \frac{6}{6.5} F_{CD} + 24 = 0 \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{2.5}{6.5} F_{BC} - \frac{2.5}{6.5} F_{CD} - 40 = 0 \tag{2}$$



Solving (1) and (2) simultaneously,

$$F_{BC} = 65 \text{ kips} \quad F_{CD} = -39 \text{ kips}$$

Joint *D*



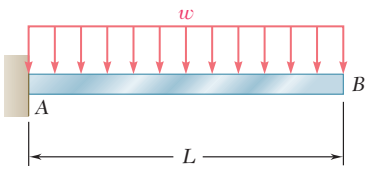
$$+\uparrow \Sigma F_y = 0: \quad F_{BD} + \frac{2.5}{6.5} F_{CD} = 0 \qquad F_{BD} = 15 \text{ kips}$$

$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2E} \Sigma \frac{F^2 L}{A}$$

Member	$F(10^3 \text{ lb})$	L(in.)	$A(\text{in}^2)$	$F^2L/A\ (10^9 \text{ lb}^2/\text{in.})$
BC	65	78	3	109.85
BD	15	60	2	6.75
CD	–39	78	5	23.73
Σ				140.33

$$U = \frac{140.33 \times 10^9}{(2)(10.5 \times 10^6)} = 6682 \text{ lb} \cdot \text{in.} = 6.68 \text{ kip} \cdot \text{in.} \blacktriangleleft$$

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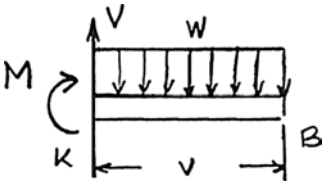
PROBLEM 11.24

Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam *AB* for the loading shown.

SOLUTION

$$+\circlearrowleft \sum M_K = 0: \quad -M - (wv)\left(\frac{v}{2}\right) = 0$$

$$M = -\frac{1}{2}wv^2$$



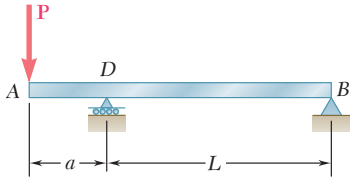
$$U = \int_0^L \frac{M^2}{2EI}dv = \frac{1}{2EI}\int_0^L \left(\frac{1}{2}wv^2\right)^2 dv$$

$$= \frac{w^2}{8EI}\int_0^L v^4 dv = \frac{w^2}{8EI}\frac{v^5}{5}\Big|_0^L$$

$$= \frac{w^2L^5}{40EI}$$

$$U = \frac{w^2L^5}{40EI} \blacktriangleleft$$

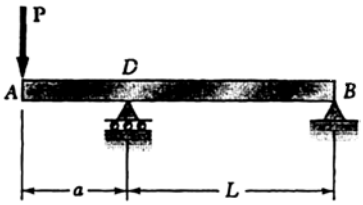
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PROBLEM 11.25

Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam *AB* for the loading shown.

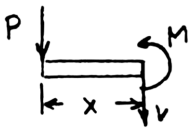
SOLUTION



$$+\circlearrowleft \Sigma M_D = 0: \quad aP + LR_B = 0$$

$$R_B = -\frac{aP}{L} = \frac{aP}{L} \downarrow$$

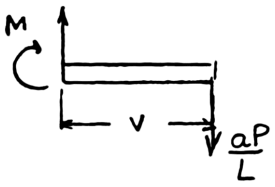
Over portion *AD*:



$$M = -Px$$

$$U_{AD} = \int_0^a \frac{M^2}{2EI}dx = \frac{1}{2EI}\int_0^a P^2x^2dx = \frac{P^2}{2EI}\frac{x^3}{3}\Big|_0^a = \frac{P^2a^3}{6EI}$$

Over portion *DB*:



$$M = -\frac{aP}{L}v$$

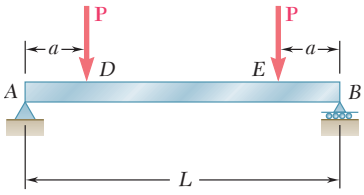
$$U_{DB} = \int_0^L \frac{M^2}{2EI}dv = \frac{1}{2EI}\int_0^L \frac{a^2P^2}{L^2}v^2\,dv$$

$$= \frac{P^2a^2}{2EIL^2}\int_0^L v^2dv = \frac{P^2a^2}{2EIL^2}\frac{v^3}{3}\Big|_0^L = \frac{P^2a^2L}{6EI}$$

Total: $U = U_{AD} + U_{DB}$

$$U = \frac{P^2a^2}{6EI}(a + L) \blacktriangleleft$$

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PROBLEM 11.26

Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam *AB* for the loading shown.

SOLUTION

Symmetric beam and loading: $R_A = R_B$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - 2P = 0 \qquad R_A = R_B = P$$

Over portion *AD*, $M = R_A x = Px$

$$U_{AD} = \int_0^a \frac{M^2}{2EI}dx = \frac{P^2}{2EI}\int_0^a x^2 dx = \frac{P^2}{2EI}\frac{x^3}{3}\bigg|_0^a = \frac{P^2a^3}{6EI}$$

Over portion *DE*, $M = Pa \qquad U_{DE} = \dfrac{P^2a^2(L-2a)}{2EI}$

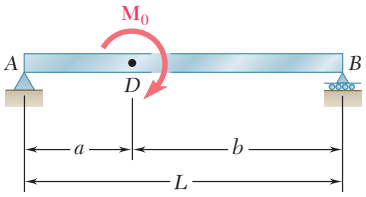
Over portion *EB*,

By symmetry,

$$U_{EB} = U_{AD} = \frac{P^2a^3}{6EI}$$

Total: $U = U_{AD} + U_{DE} + U_{EB}$ $\qquad U = \dfrac{P^2a^2}{6EI}(3L - 4a)$ ◀

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PROBLEM 11.27

Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam *AB* for the loading shown.

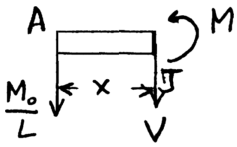
SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L - M_0 = 0 \qquad R_A = \frac{M_0}{L}\downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - M_0 = 0 \qquad R_B = \frac{M_0}{L}\uparrow$$

A to D:

$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{M_0 x}{L} + M = 0$$

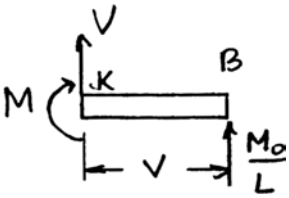


$$M = -\frac{M_0 x}{L}$$

$$U_{AD} = \int_0^a \frac{M^2 dx}{2EI} = \frac{M_0^2}{2EIL^2}\int_0^a x^2 dx = \frac{M_0^2 a^3}{6EIL^2}$$

D to B:

$$+\circlearrowleft \Sigma M_K = 0: \quad -M + \frac{M_0 v}{L}$$



$$M = \frac{M_0 v}{L}$$

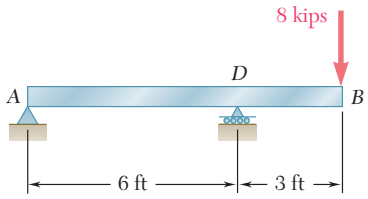
$$U_{DB} = \int_0^b \frac{M^2 dv}{2EI} = \frac{M_0^2}{2EIL^2}\int_0^b v^2 dv = \frac{M_0^2 b^3}{6EIL^2}$$

Total:

$$U = U_{AD} + U_{DB}$$

$$U = \frac{M_0^2 (a^3 + b^3)}{6EIL^2} \blacktriangleleft$$

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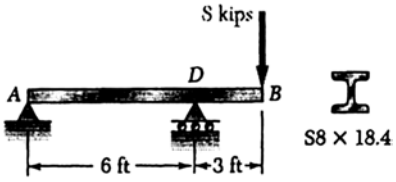




PROBLEM 11.28

Using $E = 29 \times 10^6$ psi, determine the strain energy due to bending for the steel beam and loading shown. (Neglect the effect of shearing stresses.)

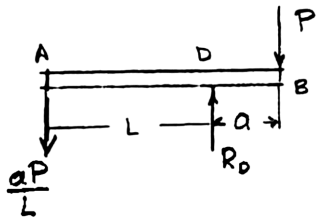
SOLUTION

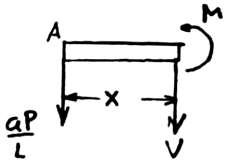


$$+\circlearrowleft \Sigma M_D = 0: \quad -R_A L - aP = 0$$

$$R_A = \frac{aP}{L} \downarrow$$

Over portion *AD*:





$$M = -\frac{aP}{L}x$$

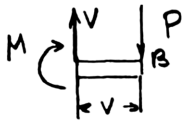
$$U_{AD} = \int_0^L \frac{M^2}{2EI}dx = \frac{1}{2EI}\int_0^L \left(\frac{aP}{L}x\right)^2 dx$$

$$= \frac{P^2a^2}{2EIL^2}\int_0^L x^2 dx$$

$$= \frac{P^2a^2L}{6EI}$$

Over portion *DB*:

$$M = -Pv$$



$$U_{BD} = \int_0^a \frac{M^2}{2EI}dv = \frac{1}{2EI}\int_0^a P^2x^2 dx = \frac{P^2a^3}{6EI}$$

Total:

$$U = U_{AD} + U_{DB} = \frac{P^2a^2}{6EI}(a + L)$$

Data:

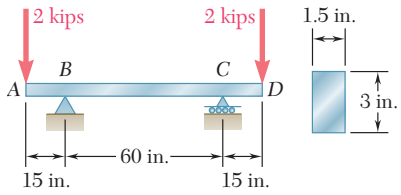
$$P = 8000 \text{ lb}, \quad L = 6 \text{ ft} = 72 \text{ in.}, \quad a = 3 \text{ ft} = 36 \text{ in.}, \quad E = 29 \times 10^6 \text{ psi}$$

$$I = 57.5 \text{ in}^4$$

$$U = \frac{(8000)^2(36)^2(72 + 36)}{(6)(29 \times 10^6)(57.5)}$$

$$U = 895 \text{ in.} \cdot \text{lb} \blacktriangleleft$$

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PROBLEM 11.29

Using $E = 29 \times 10^6$ psi, determine the strain energy due to bending for the steel beam and loading shown. (Neglect the effect of shearing stresses.)

SOLUTION

Over *A* to *B*:

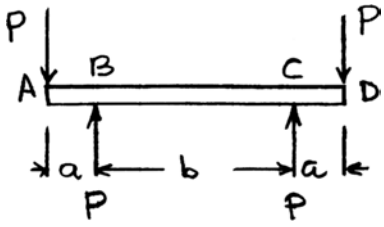
$$M = -Px$$

$$U_{AB} = \int_0^a \frac{M^2 dx}{2EI} = \frac{P^2}{2EI}\int_0^a x^2 dx = \frac{P^2 a^3}{6EI}$$

Over *B* to *C*:

$$-M = Pa = \text{constant}$$

$$U_{BC} = \frac{M^2 b}{2EI} = \frac{P^2 a^2 b}{2EI}$$



By symmetry,

$$U_{CD} = U_{AB} = \frac{P^2 a^3}{6EI}$$

Total:

$$U = U_{AB} + U_{BC} + U_{CD} = \frac{P^2 a^2 (2a + 3b)}{6EI}$$

Data:

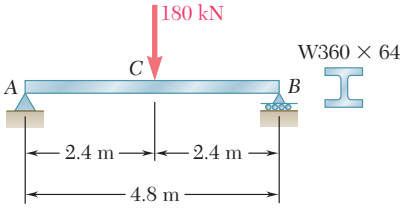
$$P = 2 \times 10^3 \text{ lb}, \quad a = 15 \text{ in.}, \quad b = 60 \text{ in.}$$

$$I = \frac{1}{12}(1.5)(3)^3 = 3.375 \text{ in}^4$$

$$U = \frac{(2 \times 10^3)^2 (15)^2 [(2)(15) + (3)(60)]}{(6)(29 \times 10^6)(3.375)}$$

$$U = 322 \text{ in.} \cdot \text{lb} \blacktriangleleft$$

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PROBLEM 11.30

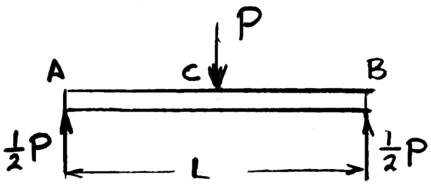
Using $E = 200$ GPa, determine the strain energy due to bending for the steel beam and loading shown. (Neglect the effect of shearing stresses.)

SOLUTION

Over portion *AC*, $M = \frac{1}{2}Px$

$$U_{AC} = \int_0^{L/2} \frac{M^2}{2EI}\,dx = \frac{P^2}{8EI}\int_0^{L/2} x^2\,dx$$

$$= \frac{P^2}{8EI}\frac{x^3}{3}\bigg|_0^{L/2} = \frac{P^2L^3}{192EI}$$



By symmetry, $U_{CB} = U_{AC} = \frac{P^2L^3}{192EI}$

Total: $U = U_{AC} + U_{CB} = \frac{P^2L^3}{96EI}$

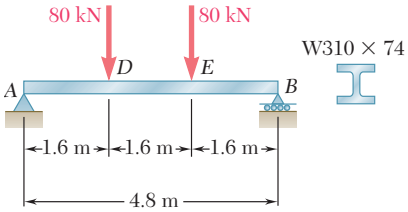
Data: $P = 180\times10^3$ N, $L = 4.8$ m, $E = 200\times10^9$ Pa

$I = 178\times10^6\,\text{mm}^4 = 178\times10^{-6}\,\text{m}^4$

$$U = \frac{(180\times10^3)^2(4.8)^3}{(96)(200\times10^9)(178\times10^{-6})} = 1048\ \text{N}\cdot\text{m}$$

$U = 1048$ J ◀

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PROBLEM 11.31

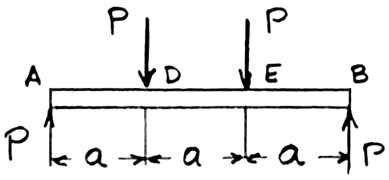
Using $E = 200$ GPa, determine the strain energy due to bending for the steel beam and loading shown. (Neglect the effect of shearing stresses.)

SOLUTION

Over portion *AD*, $M = Px$

$$U_{AD} = \int_0^a \frac{M^2}{2EI}\,dx = \frac{1}{2EI}\int_0^a (Px)^2\,dx$$

$$= \frac{P^2}{2EI}\frac{x^3}{3}\bigg|_0^a = \frac{P^2a^3}{6EI}$$



Over portion *DE*, $M = Pa$

$$U_{DE} = \frac{(Pa)^2 a}{2EI} = \frac{P^2a^3}{2EI}$$

By symmetry, $$U_{EB} = U_{AD} = \frac{P^2a^3}{6EI}$$

$$U = U_{AD} + U_{DE} + U_{EB} = \frac{5}{6}\frac{P^2a^3}{EI}$$

Data: $P = 80 \times 10^3$ N, $a = 1.6$ m, $E = 200 \times 10^9$ Pa

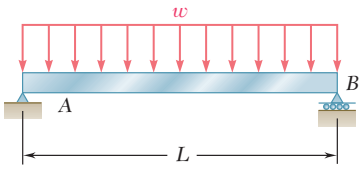
$I = 163 \times 10^6$ mm^4 $= 163 \times 10^{-6}$ m^4

$$U = \frac{5}{6}\frac{(80 \times 10^3)^2(1.6)^3}{(200 \times 10^9)(163 \times 10^{-6})} = 670 \text{ N} \cdot \text{m}$$

$U = 670$ J ◀

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PROBLEM 11.32



Assuming that the prismatic beam *AB* has a rectangular cross section, show that for the given loading the maximum value of the strain-energy density in the beam is

$$u_{\max} = \frac{45}{8}\frac{U}{V}$$

where U is the strain energy of the beam and V is its volume.

SOLUTION

$$+\circlearrowleft M_B = 0: \quad -R_A L + (wL)\frac{L}{2} = 0 \quad R_A = \frac{1}{2}wL$$

$$M = R_A x - \frac{1}{2}wL^2 = \frac{1}{2}w(Lx - x^2)$$

$$U = \int_0^L \frac{M}{2EI}dx = \frac{w^2}{8EI}\int_0^L (L^2x^2 - 2Lx^3 + x^4)dx$$

$$= \frac{w^2}{8EI}\left[\frac{L^2x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5}\right]_0^L$$

$$= \frac{w^2L^5}{8EI}\left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right]$$

$$= \frac{w^2L^5}{240EI}$$

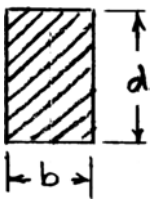
$$M_{\max} = \frac{1}{2}w\left[L\cdot\frac{L}{2} - \left(\frac{L}{2}\right)^2\right] = \frac{1}{8}wL^2$$

$$\sigma_{\max} = \frac{M_{\max}c}{I} = \frac{wL^2c}{8I}$$

$$u_{\max} = \frac{\sigma_{\max}^2}{2E} = \frac{w^2L^4c^2}{128EI^2}$$

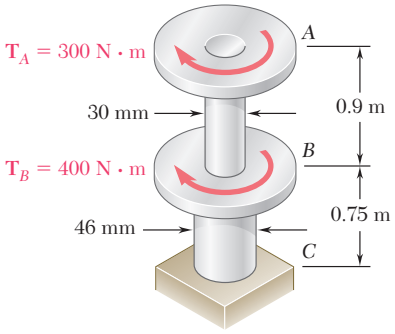
$$\frac{U}{u_{\max}} = \frac{8LI}{15c^2} = \frac{8L\left(\frac{1}{12}bd^3\right)}{15\left(\frac{d}{2}\right)^2}$$

$$= \frac{8}{45}Lbd = \frac{8}{45}V$$



$$u_{\max} = \frac{45}{8}\frac{U}{V} \blacktriangleleft$$

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PROBLEM 11.33

In the assembly shown, torques $\mathbf{T}_A$ and $\mathbf{T}_B$ are exerted on disks *A* and *B*, respectively. Knowing that both shafts are solid and made of aluminum (G = 73 GPa), determine the total strain energy acquired by the assembly.

SOLUTION

Over portion *AB*:

$$T_{AB} = T_A = 300 \text{ N} \cdot \text{m}$$

$$J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2}\left(\frac{30}{2}\right)^4 = 79.52 \times 10^3 \text{ mm}^4 = 79.52 \times 10^{-9} \text{ m}^4$$

$$L_{AB} = 0.9 \text{ m}$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} = \frac{(300)^2(0.9)}{(2)(73 \times 10^9)(79.52 \times 10^{-9})}$$

$$= 6.977 \text{ J}$$

Over portion *BC*:

$$T_{BC} = T_A + T_B = 300 + 400 = 700 \text{ N} \cdot \text{m},$$

$$L_{BC} = 0.75 \text{ m}$$

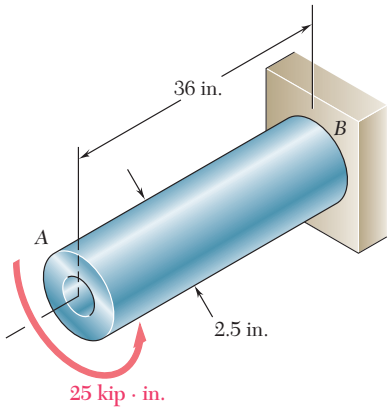
$$J_{BC} = \frac{\pi}{2}\left(\frac{46}{2}\right)^4 = 439.57 \times 10^3 \text{ mm}^4 = 439.57 \times 10^{-9} \text{ m}^4$$

$$U_{BC} = \frac{T_{BC}^2 L_{BC}}{2GJ_{BC}} = \frac{(700)^2(0.75)}{(2)(73 \times 10^9)(439.57 \times 10^{-9})} = 5.726 \text{ J}$$

Total:

$$U = U_{AB} + U_{BC} = 6.977 + 5.726 = 12.70 \text{ J}$$ ◀

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PROBLEM 11.34

The design specifications for the steel shaft *AB* require that the shaft acquire a strain energy of 400 in. · lb as the 25-kip · in. torque is applied. Using $G = 11.2 \times 10^6$ psi, determine (*a*) the largest inner diameter of the shaft that can be used, (*b*) the corresponding maximum shearing stress in the shaft.

SOLUTION

$$U = 400 \text{ in.} \cdot \text{lb}$$

$$T = 25 \text{ kip} \cdot \text{in.} = 25 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$L = 48 \text{ in.}$$

$$U = \frac{T^2 L}{2GJ}$$

$$J = \frac{T^2 L}{2GU} = \frac{(25 \times 10^3)^2 (48)}{(2)(11.2 \times 10^6)(400)} = 3.3482 \text{ in}^4$$

But

$$J = \frac{\pi}{2}\left[\left(\frac{d_0}{2}\right)^4 - \left(\frac{d_i}{2}\right)^4\right] = \frac{\pi}{32}\left(d_0^4 - d_i^4\right)$$

(*a*)
$$d_i^4 = d_0^4 - \frac{32}{\pi} J$$

$$= 2.5^4 - \frac{32}{\pi}(3.3482)$$

$$= 4.9580 \text{ in}^4$$

$$d_i = 1.492 \text{ in.} \blacktriangleleft$$

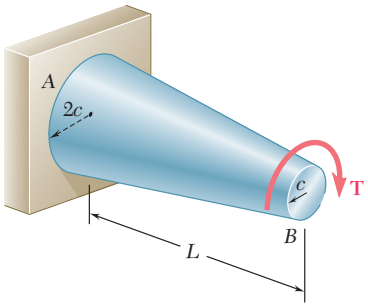
(*b*)
$$\tau = \frac{Tc_0}{J}$$

$$= \frac{(25 \times 10^3)(1.25)}{2.5112}$$

$$= 9.33 \times 10^3 \text{ psi}$$

$$\tau = 9.33 \text{ ksi} \blacktriangleleft$$

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PROBLEM 11.35

Show by integration that the strain energy in the tapered rod *AB* is

$$U = \frac{7}{48}\frac{T^2 L}{GJ_{\min}}$$

where $J_{\min}$ is the polar moment of inertia of the rod at end *B*.

SOLUTION

$$r = \frac{cx}{L}$$

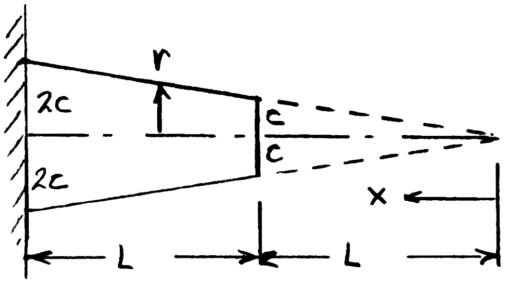
$$J = \frac{\pi}{2}r^4 = \frac{\pi}{2}\frac{c^4}{L^4}x^4, \quad J_{\min} = \frac{\pi}{2}c^4$$

$$U = \int_L^{2L}\frac{T^2 dx}{2GJ} = \int_L^{2L}\frac{T^2 dx}{2G\left(\frac{\pi}{2}\frac{c^4}{L^4}x^4\right)}$$

$$= \frac{T^2 L^4}{2GJ_{\min}}\int_L^{2L}\frac{dx}{x^4}$$

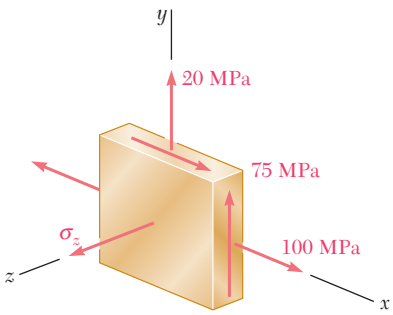
$$= \frac{T^2 L^4}{2GJ_{\min}}\left(-\frac{1}{3x^3}\right)\Bigg|_L^{2L}$$

$$U = \frac{T^2 L^4}{2GJ_{\min}}\left(-\frac{1}{3(2L)^3} + \frac{1}{3L^3}\right)$$



$$U = \frac{7}{48}\frac{T^2 L}{GJ_{\min}} \blacktriangleleft$$

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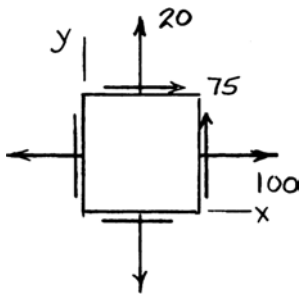


PROBLEM 11.36

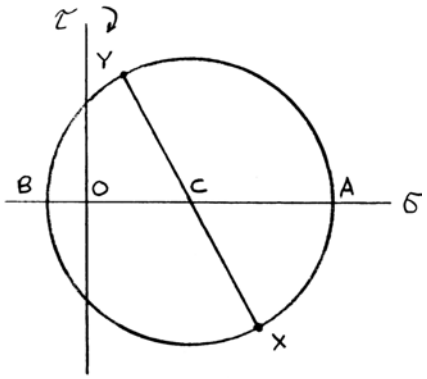
The state of stress shown occurs in a machine component made of a brass for which $\sigma_Y = 160$ MPa. Using the maximum-distortion-energy criterion, determine the range of values of σ_z for which yield does not occur.

SOLUTION

$$\sigma_{\text{ave}} = \frac{1}{2}(100 + 20)$$
$$= 60 \text{ MPa}$$
$$\frac{\sigma_x - \sigma_y}{2} = \frac{100 - 20}{2}$$
$$= 40 \text{ MPa}$$
$$\tau_{xy} = 75 \text{ MPa}$$



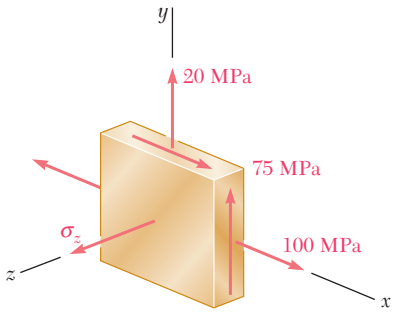
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \sqrt{40^2 + 75^2}$$
$$= 85 \text{ MPa}$$
$$\sigma_a = \sigma_{\text{ave}} + R$$
$$= 145 \text{ MPa}$$
$$\sigma_b = \sigma_{\text{ave}} - R$$
$$= -25 \text{ MPa}$$
$$\sigma_c = \sigma_z$$



$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\sigma_Y^2$$
$$(145 + 25)^2 + (-25 - \sigma_z)^2 + (\sigma_z - 145)^2 = (2)(160)^2$$
$$28{,}900 + (625 + 50\sigma_z + \sigma_z^2) + (\sigma_z^2 - 290\sigma_z + 21{,}025) = 51{,}200$$
$$2\sigma_z^2 - 240\sigma_z - 650 = 0$$
$$\sigma_z = \frac{240 \pm \sqrt{240^2 + (4)(2)(650)}}{(2)(2)} = 60 \pm 62.65$$
$$\sigma_z = 122.65 \text{ MPa}, \ -2.65 \text{ MPa}$$

$$-2.65 \text{ MPa} < \sigma_z < 122.65 \text{ MPa} \blacktriangleleft$$

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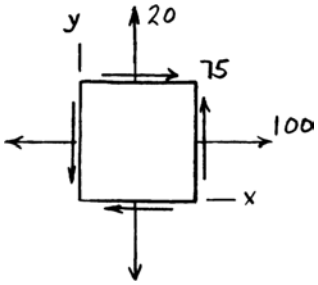


PROBLEM 11.37

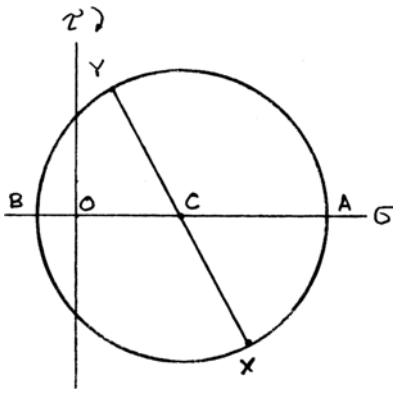
The state of stress shown occurs in a machine component made of a brass for which $\sigma_Y = 160$ MPa. Using the maximum-distortion-energy criterion, determine whether yield occurs when (*a*) $\sigma_z = +45$ MPa, (*b*) $\sigma_z = -45$ MPa.

SOLUTION

$$\sigma_{\text{ave}} = \frac{1}{2}(100 + 20)$$
$$= 60 \text{ MPa}$$
$$\frac{\sigma_x - \sigma_y}{2} = \frac{100 - 20}{2}$$
$$= 40 \text{ MPa}$$
$$\tau_{xy} = 75 \text{ MPa}$$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \sqrt{40^2 + 75^2}$$
$$= 85 \text{ MPa}$$
$$\sigma_a = \sigma_{\text{ave}} + R = 145 \text{ MPa}$$
$$\sigma_b = \sigma_{\text{ave}} - R = -25 \text{ MPa}$$
$$\sigma_c = \sigma_z$$



$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 \overset{?}{<} 2\sigma_Y^2$$

(*a*) $\sigma_c = \sigma_z = +45$ MPa

$$(145 + 25)^2 + (-25 - 45)^2 + (45 - 145)^2 \overset{?}{<} 2(160)^2 = 51{,}200$$
$$28{,}900 + 4900 + 10{,}000 = 43{,}800 < 51{,}200$$

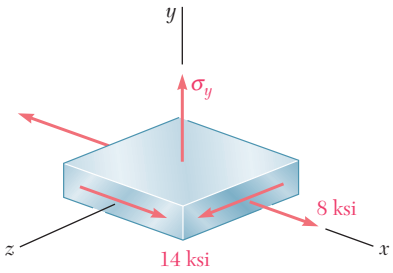
(No yield.) ◀

(*b*) $\sigma_c = \sigma_z = -45$ MPa

$$(145 + 25)^2 + (-25 + 45)^2 + (-45 - 145)^2 \overset{?}{<} 51{,}200$$
$$28{,}900 + 400 + 36{,}100 = 65{,}400 > 51{,}200$$

(Yield occurs.) ◀

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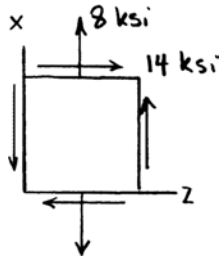


PROBLEM 11.38

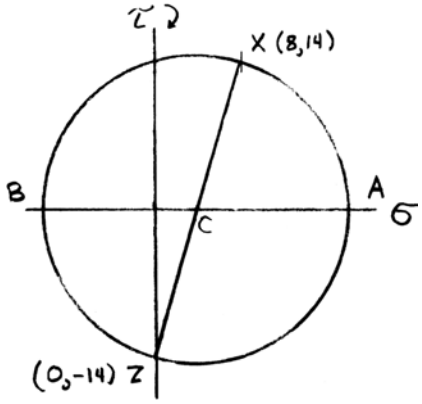
The state of stress shown occurs in a machine component made of a grade of steel for which $\sigma_Y = 65$ ksi. Using the maximum-distortion-energy criterion, determine the range of values of σ_y for which the factor of safety associated with the yield strength is equal to or larger than 2.2.

SOLUTION

$$\sigma_{\text{ave}} = \frac{1}{2}(0+8)$$
$$= 4 \text{ ksi}$$
$$\frac{\sigma_x - \sigma_z}{2} = \frac{8-0}{2}$$
$$= 4 \text{ ksi}$$
$$\tau_{xz} = 14 \text{ ksi}$$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$
$$= \sqrt{4^2 + 14^2}$$
$$= 14.56 \text{ ksi}$$
$$\sigma_a = \sigma_{\text{ave}} + R$$
$$= 18.56 \text{ ksi}$$
$$\sigma_b = \sigma_{\text{ave}} - R$$
$$= -10.56 \text{ ksi}$$
$$\sigma_c = \sigma_y$$



$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_Y}{F.S.}\right)^2$$

$$(18.56 + 10.56)^2 + (-10.56 - \sigma_y)^2 + (\sigma_y - 18.56)^2 = 2\left(\frac{65}{2.2}\right)^2$$

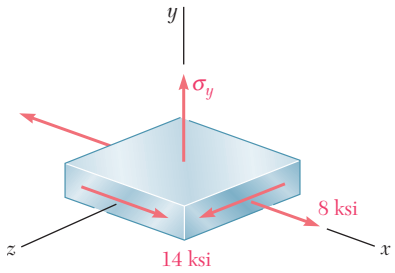
$$847.97 + (111.51 + 21.12\sigma_y + \sigma_y^2) + (\sigma_y^2 - 37.12\sigma_y + 344.47) = 1745.87$$

$$2\sigma_y^2 - 16\sigma_y - 441.92 = 0$$

$$\sigma_y = \frac{16 \pm \sqrt{16^2 + (4)(2)(441.92)}}{(2)(2)}$$
$$= 4 \pm 15.39$$
$$\sigma_y = 19.39 \text{ ksi}, \quad -11.39 \text{ ksi}$$

$$-11.39 \text{ ksi} \le \sigma_y \le 19.39 \text{ ksi} \blacktriangleleft$$

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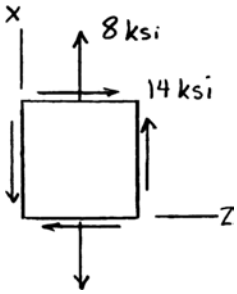


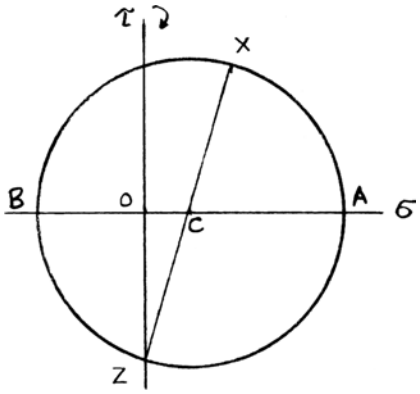
PROBLEM 11.39

The state of stress shown occurs in a machine component made of a grade of steel for which $\sigma_Y = 65$ ksi. Using the maximum-distortion-energy criterion, determine the factor of safety associated with the yield strength when (*a*) $\sigma_y = +16$ ksi, (*b*) $\sigma_y = -16$ ksi.

SOLUTION

$$\sigma_{\text{ave}} = \frac{1}{2}(0+8)$$
$$= 4 \text{ ksi}$$
$$\frac{\sigma_x - \sigma_z}{2} = \frac{8-0}{2}$$
$$= 4 \text{ ksi}$$
$$\tau_{xz} = 14 \text{ ksi}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$
$$= \sqrt{4^2 + 14^2}$$
$$= 14.56 \text{ ksi}$$
$$\sigma_a = \sigma_{\text{ave}} + R$$
$$= 18.56$$
$$\sigma_b = \sigma_{\text{ave}} - R$$
$$= -10.56$$
$$\sigma_c = \sigma_y$$





$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_Y}{F.S.}\right)^2$$

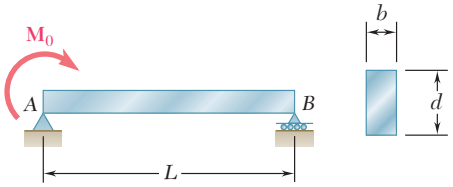
(*a*) $\sigma_c = \sigma_y = 16$ ksi

$$(18.56 + 10.56)^2 + (-10.56 - 16)^2 + (16 - 18.56)^2 = 2\left(\frac{65}{F.S.}\right)^2$$
$$847.97 + 705.43 + 6.55 = \frac{8450}{(F.S.)^2}$$
$$F.S. = 2.33 \blacktriangleleft$$

(*b*) $\sigma_c = \sigma_y = -16$ ksi

$$(18.56 + 10.56)^2 + (-10.56 + 16)^2 + (-16 - 18.56)^2 = 2\left(\frac{65}{F.S.}\right)^2$$
$$847.97 + 29.59 + 1194.39 = \frac{8450}{(F.S.)^2}$$
$$F.S. = 2.02 \blacktriangleleft$$

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PROBLEM 11.40

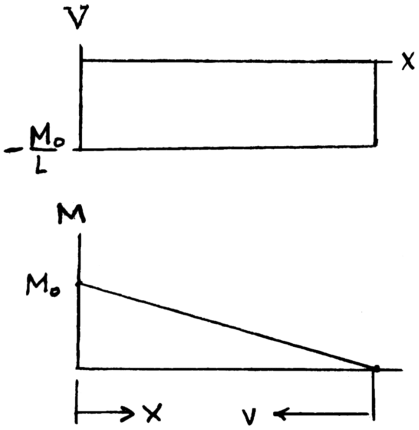
Determine the strain energy of the prismatic beam *AB*, taking into account the effect of both normal and shearing stresses.

SOLUTION

Reactions: $R_A = \frac{M_0}{L}\downarrow, \quad R_B = \frac{M_0}{L}\uparrow$

Shear: $V = -\frac{M_0}{L}$

Bending moment: $M = \frac{M_0}{L}v$



For bending,

$$U_1 = \int_0^L \frac{M^2}{2EI}dv = \frac{M_0^2}{2EIL^2}\int_0^L v^2 dv$$

$$= \frac{M_0 L^3}{6EIL^2} = \frac{M_0^2 L}{6EI}$$

For shear,

$$\tau_{xy} = \frac{3}{2}\frac{V}{A}\left(1-\frac{y^2}{c^2}\right) \qquad c = \frac{1}{2}d$$

$$u = \frac{\tau_{xy}^2}{2G} = \frac{9V^2}{8GA^2}\left(1-\frac{y^2}{c^2}\right)^2 = \frac{9M_0^2}{8G(bd)^2L^2}\left(1-2\frac{y^2}{c^2}+\frac{y^4}{c^4}\right)$$

$$U_2 = \int u\,dv = \int_0^L\int_{-c}^{c} ub\,dy\,dx = \frac{9M_0^2 b}{8Gb^2d^2L^2}\int_0^L\int_{-c}^{c}\left(1-2\frac{y^2}{c^2}+\frac{y^4}{c^4}\right)dy\,dx$$

$$= \frac{9M_0^2}{8Gbd^2L^2}\int_0^L\left(y-\frac{2}{3}\frac{y^3}{c^2}+\frac{1}{5}\frac{y^5}{c^4}\right)\Bigg|_{-c}^{c} dx = \frac{9M_0^2}{8Gbd^2L^2}\int_0^L\left(2c-\frac{4}{3}c+\frac{2}{5}c\right)dx$$

$$\frac{9M_0^2}{8Gbd^2L^2}\left(\frac{16}{15}c\right)L = \frac{6}{5}\frac{M_0^2 c}{Gbd^2L} = \frac{3}{5}\frac{M_0^2}{GbdL}$$

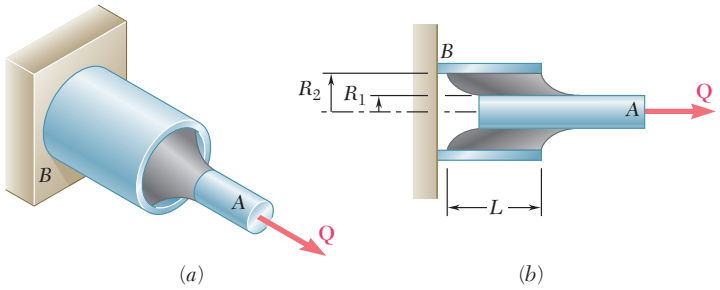
Total: $U = U_1 + U_2 = \frac{M_0^2 L}{6EI} + \frac{3}{5}\frac{M_0^2}{GbdL}$

with $I = \frac{1}{12}bd^3$

$$U = \frac{2M_0^2 L}{Ebd^3} + \frac{3}{5}\frac{M_0^2}{GbdL}$$

$$U = \frac{2M_0^2 L}{Ebd^3}\left\{1+\frac{3Ed^2}{10GL^2}\right\} \blacktriangleleft$$

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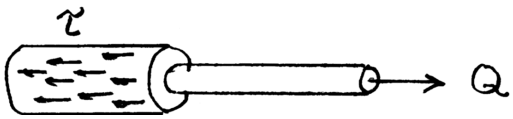
PROBLEM 11.41*

A vibration isolation support is made by bonding a rod *A*, of radius R_1, and a tube *B*, of inner radius R_2, to a hollow rubber cylinder. Denoting by *G* the modulus of rigidity of the rubber, determine the strain energy of the hollow rubber cylinder for the loading shown.

SOLUTION

$$+\longrightarrow \Sigma F_x = 0: \quad -\tau(2\pi rL) + Q = 0$$

$$\tau = \frac{Q}{2\pi rL}$$



$$u = \frac{\tau^2}{2G} = \frac{Q^2}{8\pi^2 r^2 L^2 G}$$

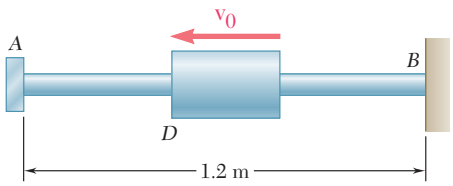
$$U = \int u\,dV = \frac{Q^2}{8\pi^2 GL^2}\int \frac{dV}{r^2} = \frac{Q^2}{8\pi^2 GL^2}\int_0^L \int_{R_1}^{R_2} \frac{2\pi r\,dr}{r^2}\,dx$$

$$= \frac{Q^2}{4\pi GL^2}\int_0^L \int_{R_1}^{R_2} \frac{dr}{r}\,dx = \frac{Q^2}{4\pi GL^2}\int_0^L \left(\ln r\Big|_{R_1}^{R_2}\right)dx$$

$$U = \frac{Q^2}{4\pi GL}\ln\frac{R_2}{R_1} \blacktriangleleft$$

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PROBLEM 11.42



A 5-kg collar D moves along the uniform rod AB and has a speed $v_0 = 6$ m/s when it strikes a small plate attached to end A of the rod. Using $E = 200$ GPa and knowing that the allowable stress in the rod is 250 MPa, determine the smallest diameter that can be used for the rod.

SOLUTION

$$U_m = \frac{1}{2}mv_0^2 = \frac{1}{2}(5)(6)^2 = 90 \text{ J}$$

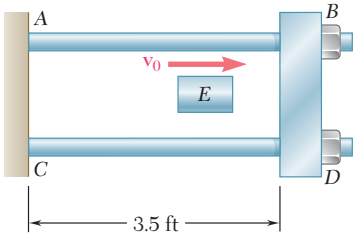
$$U_m = \frac{P_m^2 L}{2EA} = \frac{(A\sigma_{\max})^2 L}{2EA}$$

$$A = \frac{2EU_m}{\sigma_{\max}^2 L} = \frac{(2)(200\times10^9)(90)}{(250\times10^6)^2(1.2)} = 480\times10^{-6}\text{ m}^2$$

$$\frac{\pi}{4}d^2 = A \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(480\times10^{-6})}{\pi}} = 24.7\times10^{-3}\text{ m}$$

$$d = 24.7 \text{ mm} \blacktriangleleft$$

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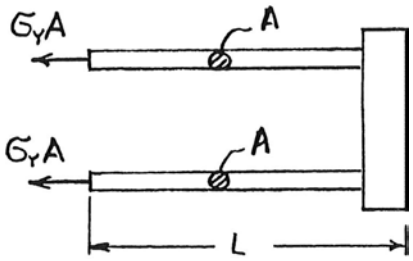
PROBLEM 11.43

The 18-lb cylindrical block *E* has a horizontal velocity $\mathbf{v_0}$ when it strikes squarely the yoke *BD* that is attached to the $\frac{7}{8}$-in.-diameter rods *AB* and *CD*. Knowing that the rods are made of a steel for which $\sigma_Y = 50$ ksi and $E = 29 \times 10^6$ psi, determine the maximum allowable speed v_0 if the rods are not to be permanently deformed.

SOLUTION

At the onset of yielding, the force in each rod is

$$F = \sigma_Y A.$$



Corresponding strain energy:

$$U_{AB} = \frac{F_{AB}^2 L_{AB}}{2EA_{AB}} = \frac{\sigma_Y^2 A^2 L}{2EA} = \frac{\sigma_Y^2 AL}{2E}$$

$$U_{CD} = \frac{F_{CD}^2 L_{CD}}{2EA_{CD}} = \frac{\sigma_Y^2 AL}{2E}$$

Total: $$U_m = U_{AB} + U_{CD} = \frac{\sigma_Y^2 AL}{E}$$

$$U_m = \frac{1}{2} m v_0^2 = \frac{1}{2} \frac{W}{g} v_0^2$$

Solving for v_0^2, $$v_0^2 = \frac{2gU_m}{W} = \frac{2g\sigma_Y^2 AL}{EW}$$

$$v_0 = \sqrt{\frac{2g\sigma_Y^2 AL}{EW}}$$

Data: $g = 32.17 \text{ ft/s}^2 = 386 \text{ in./s}^2$ $\sigma_Y = 50 \times 10^3$ psi

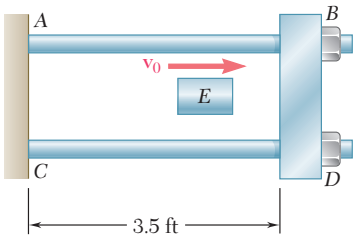
$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$, $E = 29 \times 10^6$ psi

$L = 3.5 \text{ ft} = 42 \text{ in.}$ $W = 18$ lb

$$v_0 = \sqrt{\frac{(2)(386)(50 \times 10^3)^2 (0.60132)(42)}{(29 \times 10^6)(18)}} = 305.6 \text{ in./sec}$$

$v_0 = 25.5$ ft/sec ◀

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PROBLEM 11.44

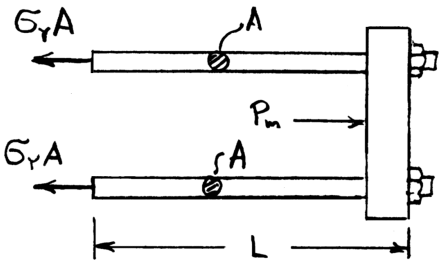
The cylindrical block *E* has a speed $v_0 = 16$ ft/s when it strikes squarely the yoke *BD* that is attached to the $\frac{7}{8}$-in.-diameter rods *AB* and *CD*. Knowing that the rods are made of a steel for which $\sigma_Y = 50$ ksi and $E = 29 \times 10^6$ psi, determine the weight of block *E* for which the factor of safety is five with respect to permanent deformation of the rods.

SOLUTION

At the onset of yielding, the force in each rod is

$$F = \sigma_Y A.$$

Corresponding strain energy:



$$U_{AB} = \frac{F_{AB}^2 L_{AB}}{2EA_{AB}} = \frac{\sigma_Y^2 A^2 L}{2EA} = \frac{\sigma_Y^2 AL}{2E}$$

$$U_{CD} = U_{AB} = \frac{\sigma_Y^2 AL}{2E}$$

$$U_m = U_{AB} + U_{CD} = \frac{\sigma_Y^2 AL}{E}$$

$$U_m = \left(\frac{1}{2} m v_0^2\right)(F.S.) = \left(\frac{1}{2}\frac{W}{g} v_0^2\right)(F.S.)$$

Solving for *W*,

$$W = \frac{2gU_m}{v_0^2 (F.S.)} = \frac{2g\sigma_Y^2 AL}{v_0^2 (F.S.) E}$$

Data:

$$g = 32.17 \text{ ft/sec}^2 = 386 \text{ in./sec}^2, \qquad \sigma_Y = 50 \times 10^3 \text{ psi},$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2 \qquad E = 29 \times 10^6 \text{ psi}$$

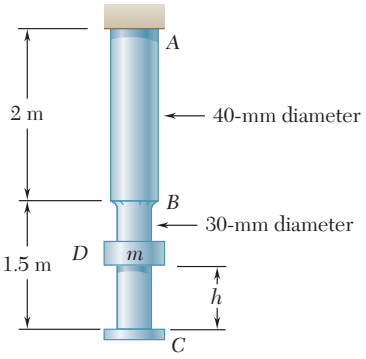
$$L = 3.5 \text{ ft} = 42 \text{ in.} \qquad F.S. = 5$$

$$v_0 = 16 \text{ ft/sec} = 192 \text{ in/sec}$$

$$W = \frac{(2)(386)(50 \times 10^3)^2 (0.60132)(42)}{(192)^2 (5)(29 \times 10^6)}$$

$$W = 9.12 \text{ lb} \blacktriangleleft$$

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PROBLEM 11.45

The 35-kg collar *D* is released from rest in the position shown and is stopped by a plate attached at end *C* of the vertical rod *ABC*. Knowing that $E = 200$ GPa for both portions of the rod, determine the distance h for which the maximum stress in the rod is 250 MPa.

SOLUTION

Portion *BC* has smaller cross section, thus

$$P_m = \sigma_{\text{all}} A_{BC} = (250\text{ MPa})\frac{\pi}{4}(0.030\text{ m})^2 = 176.7\text{ kN}$$

Maximum strain energy (for $P_m = 176.7$ kN)

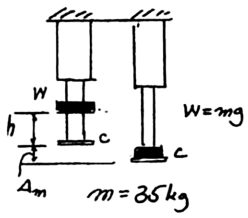
$$U_m = \sum \frac{P_m^2 L_i}{2A_i E} = \frac{P_m^2}{2E}\sum \frac{L_i}{A_i} = \frac{(176.7\text{ kN})^2}{2(200\text{ GPa})}\left[\frac{2\text{ m}}{\frac{\pi}{4}(0.040\text{ m})^2} + \frac{1.5\text{ m}}{\frac{\pi}{4}(0.030\text{ m})^2}\right]$$

$$U_m = 78.06 \times 10^{-3}[1591.6 + 2122.1] = 289.9\text{ J}$$

Max. deflection: $\frac{1}{2}P_m\Delta_m = U_m$

$$\frac{1}{2}(176.7\text{ kN})\Delta_m = 289.9\text{ J}$$

$$\Delta_m = 3.28 \times 10^{-3}\text{ m} = 3.28\text{ mm}$$



Work of weight = U_m

$$W(h + \Delta_m) = U_m$$

$$(mg)(h + \Delta_m) = U_m$$

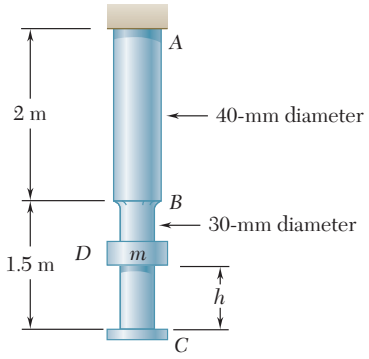
$$(35\text{ kg})(9.81\text{ m/s}^2)(h + \Delta_m) = 289.9\text{ J}$$

$$h + \Delta_m = 0.8443\text{ m} = 844.3\text{ mm}$$

$$h = 844.3 - \Delta_m = 844.3 - 3.28$$

$$h = 841\text{ mm} \blacktriangleleft$$

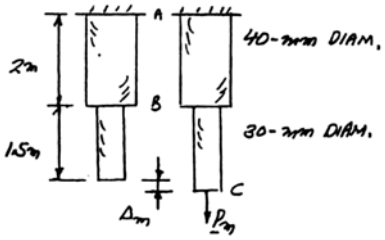
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PROBLEM 11.46

The 15-kg collar *D* is released from rest in the position shown and is stopped by a plate attached at end *C* of the vertical rod *ABC*. Knowing that $E = 200$ GPa for both portions of the rod, determine (*a*) the maximum deflection of end *C*, (*b*) the equivalent static load, (*c*) the maximum stress that occurs in the rod.

SOLUTION



$E = 200$ GPa

(*a*) $$\Delta_m = \sum \frac{P_m L_i}{A_i E} = \frac{P_m}{E} \sum \frac{L_i}{A_i} = \frac{P_m}{200\text{ GPa}} \left[\frac{2\text{ m}}{\frac{\pi}{4}(0.04\text{ m})^2} + \frac{1.5\text{ m}}{\frac{\pi}{4}(0.03\text{ m})^2} \right]$$

$$\Delta_m = 18.57 \times 10^{-9} P_m$$

$$P_m = 53.85 \times 10^6 \Delta_m$$

Strain energy: $$U_m = \frac{1}{2} P_m \Delta_m = \frac{1}{2}(53.85 \times 10^6)\Delta_m^2$$

Weight falls distance of $h + \Delta_m$

Work of weight = strain energy

$$W(h + \Delta_m) = \frac{1}{2}\left(53.85 \times 10^6 \Delta_m^2\right) \qquad (1)$$

$$W = mg = (35\text{ kg})(9.81\text{ m/s}) = 343.4\text{ N}$$

For $h = 0.5$ m

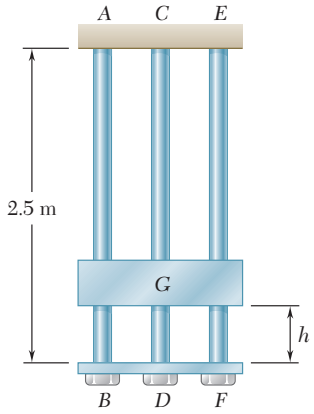
Eq. (1): $343.4(0.5 + \Delta_m) = 26.93 \times 10^6 \Delta_m^2$

Solve quadratic: $\Delta_m = 2.531 \times 10^{-3}\text{ m} = 2.53\text{ mm}$ ◀

(*b*) $P_m = 53.85 \times 10^6 \Delta_m = (53.85 \times 10^6)(2.531 \times 10^{-3}\text{ m})$ $P_m = 136.3\text{ kN}$ ◀

(*c*) $$\sigma_m = \frac{P_m}{A_{BC}} = \frac{136.3\text{ kN}}{\frac{\pi}{4}(0.030\text{ m})^2} = 192.82\text{ MPa}$$ $\sigma_m = 192.8\text{ MPa}$ ◀

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PROBLEM 11.47

The 48-kg collar G is released from rest in the position shown and is stopped by plate BDF that is attached to the 20-mm-diameter steel rod CD and to the 15-mm-diameter steel rods AB and EF. Knowing that for the grade of steel used $\sigma_{all} = 180$ MPa and $E = 200$ GPa, determine the largest allowable distance h.

SOLUTION

Let Δ_m be the maximum elongation. $\Delta_m = \dfrac{\sigma_{AB}L}{E} = \dfrac{\sigma_{CD}L}{E} = \dfrac{\sigma_{EF}L}{E}$

$$\sigma_{AB} = \sigma_{CD} = \sigma_{EF} = 180 \text{ MPa} = 180 \times 10^6 \text{ Pa}$$

$$L = 2.5 \text{ m} \qquad E = 200 \times 10^9 \text{ Pa}$$

$$\Delta_m = \frac{(180 \times 10^6)(2.5)}{200 \times 10^9} = 0.00225 \text{ m}$$

For each rod, $U = \dfrac{F_m^2 L}{2EA} = \dfrac{(EA\Delta_m/L)^2 L}{2EA} = \dfrac{EA\Delta_m^2}{2L}$

Rod CD: $A_{CD} = \dfrac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16 \times 10^{-6} \text{ m}^2$

$$U_{CD} = \frac{(200 \times 10^9)(314.16 \times 10^{-6})(0.00225)^2}{(2)(2.5)} = 63.617 \text{ J}$$

Rods AB and EF: $A_{AB} = A_{EF} = \dfrac{\pi}{4}(15)^2 = 176.71 \text{ mm}^2 = 176.71 \times 10^{-6} \text{ m}^2$

$$U_{AB} = U_{EF} = \frac{(200 \times 10^9)(176.71 \times 10^{-6})(0.00225)^2}{(2)(2.5)} = 35.674 \text{ J}$$

Total strain energy: $U_m = U_{AB} + U_{CD} + U_{EF} = 134.97 \text{ J}$

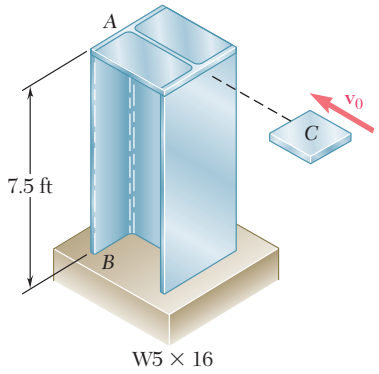
Work of falling collar:

$$U_m = mg(h + \Delta_m) = (48)(9.81)(h + \Delta_m)$$

Equating, $(48)(9.81)(h + \Delta_m) = 134.97 \quad h + \Delta_m = 0.28662 \text{ m}$

$$h = 0.28662 - 0.00225 = 0.285 \text{ m} \qquad h = 285 \text{ mm} \blacktriangleleft$$

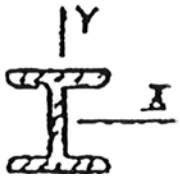
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PROBLEM 11.48

A 25-lb block C moving horizontally with a velocity $\mathbf{v_0}$ hits the post AB squarely as shown. Using $E = 29 \times 10^6$ psi, determine the largest speed v_0 for which the maximum normal stress in the pipe does not exceed 18 ksi.

SOLUTION

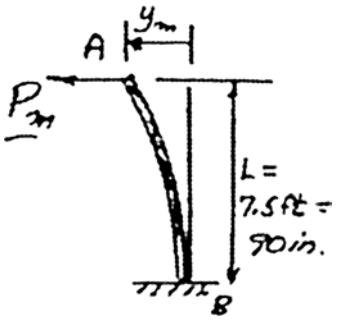


W5 × 16: $I_x = 21.4 \text{ in}^4, \quad S_x = 8.55 \text{ in}^3$

Maximum stress: $\sigma_m = 18 \text{ ksi}$

Maximum bending moment:

$$M_m = \sigma_m S_x = (18 \text{ ksi})(8.55 \text{ in}^3) = 153.9 \text{ kip} \cdot \text{in.}$$



Equivalent force: $P_m L = M_m$

$$P_m = \frac{M_m}{L} = \frac{153.9 \text{ kip} \cdot \text{in.}}{90 \text{ in.}} = 1.71 \text{ kips} = 1710 \text{ lb}$$

From Appendix D,

$$y_m = \frac{P_m L^3}{3EI} = \frac{(1710)(90)^3}{(3)(29 \times 10^6)(21.4)} = 0.66956 \text{ in.}$$

$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2}(1710)(0.66956) = 572.48 \text{ in.} \cdot \text{lb}$$

$$= 47.706 \text{ ft} \cdot \text{lb}$$

Kinetic energy: $T = \frac{1}{2}\frac{W}{g} v_0^2$

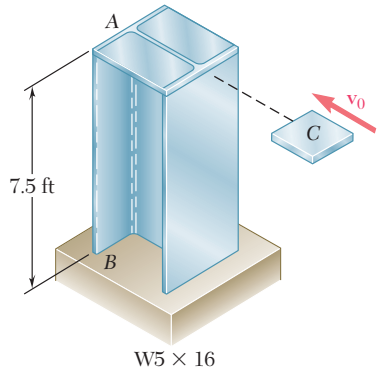
$$T = \frac{25}{(2)(32.2)} v_0^2 = 0.3882 v_0^2 \text{ ft} \cdot \text{lb}$$

Equating, $T = U_m$

$$0.3882 v_0^2 = 47.706$$

Maximum speed. $v_0 = 11.09 \text{ ft/s} \blacktriangleleft$

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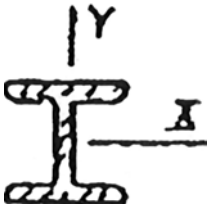


PROBLEM 11.49

Solve Prob. 11.48, assuming that the post *AB* has rotated 90° about its longitudinal axis.

PROBLEM 11.48 A 25-lb block *C* moving horizontally with a velocity $\mathbf{v_0}$ hits the post *AB* squarely as shown. Using $E = 29 \times 10^6$ psi, determine the largest speed v_0 for which the maximum normal stress in the pipe does not exceed 18 ksi.

SOLUTION

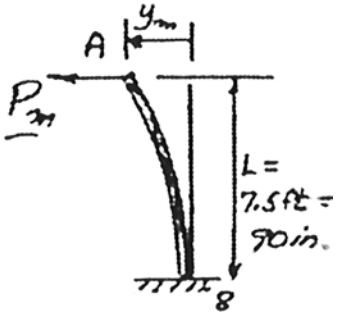


W5 × 16: $I_y = 7.51 \text{ in}^4, \quad S_y = 3.00 \text{ in}^3$

Maximum stress: $\sigma_m = 18 \text{ ksi}$

Maximum bending moment:

$$M_m = \sigma_m S_y = (18 \text{ ksi})(3.00 \text{ in}^3) = 54.0 \text{ kip} \cdot \text{in.}$$



Equivalent force: $P_m L = M_m$

$$P_m = \frac{M_m}{L} = \frac{54.0 \text{ kip} \cdot \text{in.}}{90 \text{ in.}} = 0.600 \text{ kips} = 600 \text{ lb}$$

From Appendix D,

$$y_m = \frac{P_m L^3}{3EI} = \frac{(600)(90)^3}{(3)(29 \times 10^6)(7.51)} = 0.66945 \text{ in.}$$

$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2}(600)(0.66945) = 200.83 \text{ in.} \cdot \text{lb}$$

$$= 16.736 \text{ ft} \cdot \text{lb}$$

Kinetic energy: $T = \frac{1}{2}\frac{W}{g} v_0^2$

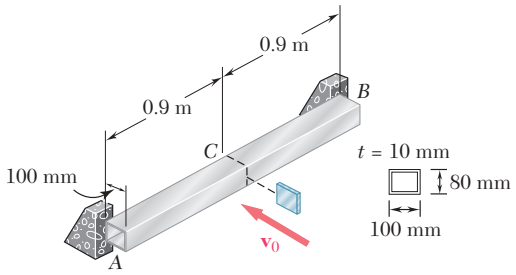
$$T = \frac{25}{(2)(32.2)} v_0^2 = 0.3882 v_0^2 \text{ ft} \cdot \text{lb}$$

Equating, $T = U_m$

$$0.3882 v_0^2 = 16.736$$

Maximum speed. $v_0 = 6.57 \text{ ft/s} \blacktriangleleft$

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PROBLEM 11.50

An aluminum tube having the cross section shown is struck squarely in its midsection by a 6-kg block moving horizontally with a speed of 2 m/s. Using $E = 70$ GPa, determine (*a*) the equivalent static load, (*b*) the maximum stress in the beam, (*c*) the maximum deflection at the midpoint *C* of the beam.

SOLUTION

Kinetic energy:

$$T = \frac{1}{2}mv_0^2 = \frac{1}{2}(6\text{ kg})(2\text{ m/s})^2 = 12\text{ J}$$

Moment of inertia.



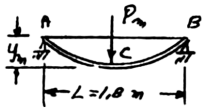
Aluminium

$E = 70$ GPa

$$I_{aa} = \frac{1}{12}[80 \times 100^3 - 60 \times 80^3]$$

$$= 4.1067 \times 10^6\text{ mm}^4$$

$$= 4.1067 \times 10^{-6}\text{ m}^4$$



From Appendix *D*: $y_m = \dfrac{P_m L^3}{48EI}$

$$U_m = \frac{1}{2}P_m y_m = \frac{P_m^2 L^3}{96EI}$$

(*a*) $U_m = T$: $\dfrac{P_m^2(1.8\text{ m})^3}{96(70\text{ GPa})(4.1067 \times 10^{-6}\text{ m}^4)} = 12\text{ J}$ $\qquad P_m = 7535.5\text{ N}$

$$P_m = 7.54\text{ kN} \blacktriangleleft$$

(*b*) $M_m = \dfrac{1}{4}P_m L = \dfrac{1}{4}(7535.5\text{ N})(1.8\text{ m}) = 3391\text{ N} \cdot \text{m}$

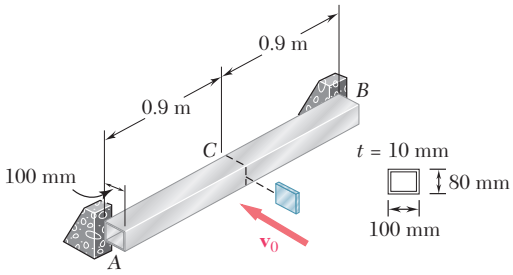
$$\sigma_m = \frac{M_m c}{I} = \frac{(3391\text{ N} \cdot \text{m})(0.050\text{ m})}{4.1067 \times 10^{-6}\text{ m}^4} = 41.28\text{ MPa}$$

$$\sigma_m = 41.3\text{ MPa} \blacktriangleleft$$

(*c*) $y_m = \dfrac{P_m L^3}{48EI} = \dfrac{(7535.5\text{ N})(1.8\text{ m})^3}{48(70\text{ GPa})(4.1067 \times 10^{-6}\text{ m}^4)} = 3.184 \times 10^{-3}\text{ m}$

$$y_m = 3.18\text{ mm} \blacktriangleleft$$

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PROBLEM 11.51

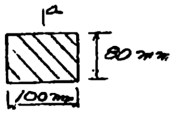
Solve Prob. 11.50, assuming that the tube has been replaced by a solid aluminum bar with the same outside dimensions as the tube.

PROBLEM 11.50 An aluminum tube having the cross section shown is struck squarely in its midsection by a 6-kg block moving horizontally with a speed of 2 m/s. Using $E = 70$ GPa, determine (*a*) the equivalent static load, (*b*) the maximum stress in the beam, (*c*) the maximum deflection at the midpoint *C* of the beam.

SOLUTION

For solid aluminum bar,

$$I_a = \frac{1}{12}80 \times 100^3 = 6.667 \times 10^{-6}\ \text{m}^4$$



Follow solution of Prob. 10.50: $T = 12$ J

$$y_m = \frac{P_m L^2}{48EI}; \quad U_m = \frac{1}{2}P_m y_m = \frac{P_m^2 L^3}{96EI}$$

(*a*) $$U_m = T: \frac{P_m^2(1.8\ \text{m})^3}{96(70\ \text{GPa})(6.667 \times 10^{-6}\ \text{m}^4)} = 12\ \text{J} \qquad P_m = 9601\ \text{N}$$

$$P_m = 9.60\ \text{kN} \blacktriangleleft$$

(*b*) $$M_m = \frac{1}{4}P_m L = \frac{1}{4}(9601\ \text{N})(1.8\ \text{m}) = 4320.5\ \text{N} \cdot \text{m}$$

$$\sigma_m = \frac{M_m c}{I} = \frac{(4320.5\ \text{N} \cdot \text{m})(0.05\ \text{m})}{6.667 \times 10^{-6}\ \text{m}^4} = 32.40\ \text{MPa}$$

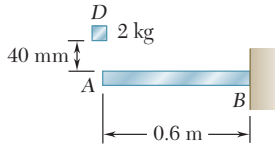
$$\sigma_m = 32.4\ \text{MPa} \blacktriangleleft$$

(*c*) $$y_m = \frac{P_m L^3}{48EI} = \frac{(9601\ \text{N})(1.8\ \text{m})^3}{48(70\ \text{GPa})(6.667 \times 10^{-6}\ \text{m}^4)} = 2.500 \times 10^{-3}\ \text{m}$$

$$y_m = 2.50\ \text{mm} \blacktriangleleft$$

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PROBLEM 11.52



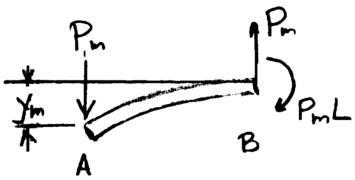
The 2-kg block *D* is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (*a*) the maximum deflection of end *A*, (*b*) the maximum bending moment in the rod, (*c*) the maximum normal stress in the rod.

SOLUTION

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{16}{2}\right)^4 = 3.2170\times10^{3}\ \text{mm}^4$$

$$= 3.2170\times10^{-9}\ \text{m}^4$$

$$c = \frac{d}{2} = 8\ \text{mm} = 8\times10^{-3}\ \text{m} \quad L_{AB} = 0.6\ \text{m}$$



Appendix *D*, Case 1:

$$y_m = \frac{P_m L_{AB}^3}{3EI} \quad M_m = P_m L_{AB}$$

$$P_m = \frac{3EI}{L_{AB}^3} y_m = \frac{(3)(200\times10^{9})(3.217\times10^{-9})}{(0.6)^3} = 8.9361\times10^{3} y_m$$

$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2}(8.9361\times10^{3}) y_m^2 = 4.4681\times10^{3} y_m^2$$

Work of dropped weight: $mg(h + y_m) = (2)(9.81)(0.040 + y_m)$

$$= 0.7848 + 19.62 y_m$$

Equating work and energy,

$$0.7848 + 19.62 y_m = 4.4681\times10^{3} y_m^2$$

$$y_m^2 - 4.3911\times10^{-3} y_m - 175.645\times10^{-6} = 0$$

(*a*) $$y_m = \frac{1}{2}\left\{4.3911\times10^{-3} + \sqrt{(4.3911\times10^{-3})^2 + (4)(175.645\times10^{-6})}\right\}$$

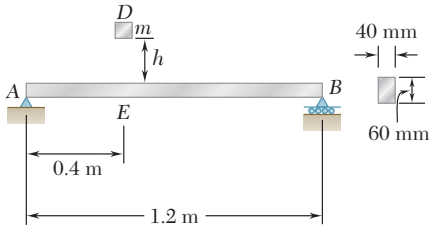
$$= 15.629\times10^{-3}\ \text{m} \qquad y_m = 15.63\ \text{mm} \blacktriangleleft$$

$$P_m = (8.9361\times10^{3})(15.629\times10^{-3}) = 139.66\ \text{N}$$

(*b*) $$M_m = -P_m L_{AB} = -(139.66)(0.6) \qquad |M_m| = 83.8\ \text{N}\cdot\text{m} \blacktriangleleft$$

(*c*) $$\sigma_m = \frac{|M_m|\, c}{I} = \frac{(83.8)(8\times10^{-3})}{3.2170\times10^{-9}} = 208\times10^{6}\ \text{Pa} \qquad \sigma_m = 208\ \text{MPa} \blacktriangleleft$$

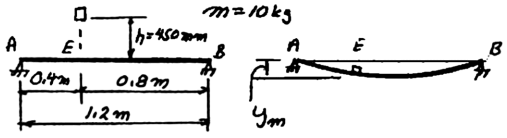
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PROBLEM 11.53

The 10-kg block D is dropped from a height $h = 450$ mm onto the aluminum beam AB. Knowing that $E = 70$ GPa, determine (*a*) the maximum deflection of point E, (*b*) the maximum stress in the beam.

SOLUTION

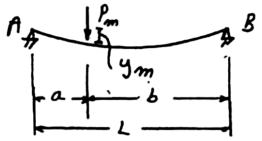




$$I = \frac{1}{12}(40)(60)^3 = 720 \times 10^3\ \text{mm}^4 = 720 \times 10^{-9}\ \text{m}^4$$

$$S = \frac{1}{6}(40)(60)^2 = 24 \times 10^3\ \text{mm}^3 = 24 \times 10^{-6}\ \text{m}^3$$

Appendix D:



$$y_m = \frac{P_m a^2 b^2}{3EIL}$$

$$y_m = \frac{P_m (0.4\ \text{m})^2 (0.8\ \text{m})^2}{3(70\ \text{GPa})(720 \times 10^{-9})(1.2\ \text{m})}$$

(*a*) $y_m = 0.5644 \times 10^{-6} P_m$ or $P_m = 1.772 \times 10^6 y_m$

$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2}(1.772 \times 10^6 y_m) y_m = 885.9 \times 10^3 y_m^2$$

Work of weight $= W(h + y_m) = mg\ (h + y_m)$

$$(10\ \text{kg})(9.81\ \text{m/s}^2)(0.45\ \text{m} + y_m) = 885.9 \times 10^3 y_m^2$$

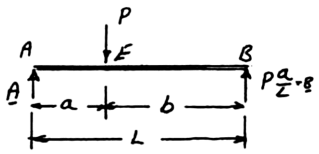
$$885.9 \times 10^3 y_m^2 - 98.1 y_m - 44.145 = 0$$

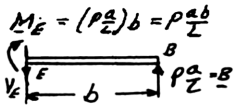
Solve quadratic: $y_m = 7.114 \times 10^{-3}\ \text{m}$ $\qquad y_m = 7.11\ \text{mm}$ ◀

(*b*) $P_m = 1.772 \times 10^6 y_m = 1.772 \times 10^6 (7.114 \times 10^{-3}\ \text{m})$

$$P_m = 12.61\ \text{kN}$$

$$M_m = M_E = P_m \frac{ab}{L} = (12.61\ \text{kN}) \frac{(0.4\ \text{m})(0.8\ \text{m})}{1.2\ \text{m}}$$





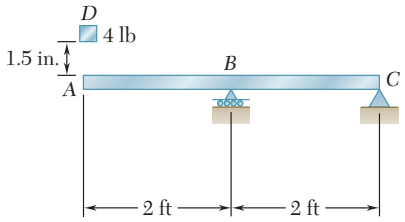
$$M_m = 3.363\ \text{kN} \cdot \text{m}$$

$$\sigma_m = M_m \frac{c}{I} = \frac{M_m}{S} = \frac{3.363\ \text{kN} \cdot \text{m}}{24 \times 10^{-6}\ \text{m}^3}$$

$$\sigma_m = 140.11\ \text{MPa}$$

$\sigma_m = 140.1\ \text{MPa}$ ◀

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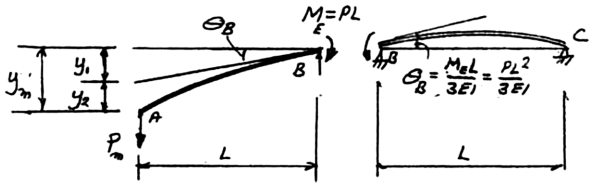


PROBLEM 11.54

The 4-lb block *D* is dropped from the position shown onto the end of a $\frac{5}{8}$-in.-diameter rod. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the maximum deflection at point *A*, (*b*) the maximum bending moment in the rod, (*c*) the maximum normal stress in the rod.

SOLUTION

Use Appendix D.



$$y_2 = \frac{P_m L^3}{3EI} \qquad y_1 = L\theta_B = \frac{P_m L^3}{3EI}$$

$$y_m = y_1 + y_2 = \frac{2P_m L^3}{3EI} \qquad y_m = \frac{2P_m L^3}{3EI}$$

$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2}\left(\frac{3EI}{2L^3}\right) y_m = \frac{3EI}{4L^3} y_m^2$$

$$W(h + y_m) = U_m$$

$$W(h + y_m) = \frac{3EI}{4L^3} y_m^2 \qquad (1)$$

Substitute given data:

$$(4\text{ lb})(1.5\text{ in.} + y_m) = \frac{3(29 \times 10^6\text{ psi})(7.49 \times 10^{-3}\text{ in}^4)}{4(24\text{ in.})^3} y_m^2$$

(*a*) $6 + 4y_m = 11.784 y_m^2$

Solve quadratic: $y_m = 0.9032$ in. $\qquad y_m = 0.903$ in. ◀

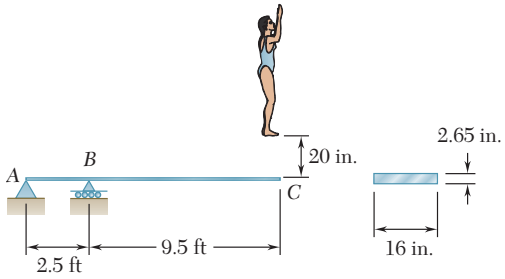
(*b*) $P_m = \frac{3EI}{2L^3} y_m = \frac{3(29 \times 10^6\text{ psi})(7.49 \times 10^{-3}\text{ m}^4)}{2(24\text{ in.})^3}(0.9032\text{ in.}) = 21.29\text{ lb}$

$M_m = M_E = P_m^L = (21.29\text{ lb})(24\text{ in.}) = 510.96\text{ lb} \cdot \text{in.}$

$M_m = 511\text{ lb} \cdot \text{in.}$ ◀

(*c*) $\sigma_m = \frac{M_m c}{I} = \frac{M_m}{S} = \frac{510.96\text{ lb} \cdot \text{in.}}{23.97 \times 10^{-3}\text{ in}^3}$ $\qquad \sigma_m = 21.3\text{ ksi}$ ◀

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PROBLEM 11.55

A 160-lb diver jumps from a height of 20 in. onto end *C* of a diving board having the uniform cross section shown. Assuming that the diver's legs remain rigid and using $E = 1.8\times10^6$ psi, determine (*a*) the maximum deflection at point *C*, (*b*) the maximum normal stress in the board, (*c*) the equivalent static load.

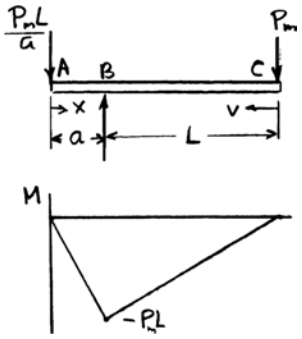
SOLUTION

$$I = \frac{1}{12}(16)(2.65)^3 = 24.813 \text{ in}^4$$

$$L = 9.5 \text{ ft} = 114 \text{ in.}$$

$$a = 2.5 \text{ ft} = 30 \text{ in.}$$

$$c = \frac{1}{2}(2.65) = 1.325 \text{ in.}$$



Over portion *AB*: $M = -\dfrac{P_m L}{a}x$

$$U_{AB} = \int_0^a \frac{M^2}{2EI}dx = \frac{P_m^2 L^2}{2EIa^2}\int_0^a x^2 dx = \frac{P_m^2 L^2 a}{6EI}$$

Over portion *BC*: $M = -P_m v$

$$U_{BC} = \int_0^L \frac{M^2}{2EI}dv = \frac{P_m^2}{2EI}\int_0^L v^2 dv = \frac{P_m^2 L^3}{6EI}$$

Total: $$U = U_{AB} + U_{BC} = \frac{P_m^2 L^2 (a+L)}{6EI}$$

$$\frac{1}{2}P_m y_m = U_m \qquad y_m = \frac{2U_m}{P_m} = \frac{P_m L^2 (a+L)}{3EI}$$

$$P_m = \frac{3EI}{L^2(a+L)}y_m = \frac{(3)(1.8\times10^6)(24.813)}{(114)^2(114+30)}y_m = 71.598 y_m$$

$$U_m = \frac{1}{2}P_m y_m = 35.799 y_m^2$$

Work of weight: $$= W(h + y_m) = (160)(20 + y_m) = 3200 + 160 y_m$$

Equating: $$3200 + 160 y_m = 35.799 y_m^2$$

$$y_m^2 - 4.4694 y_m - 89.388 = 0$$

(*a*) $$y_m = \frac{1}{2}\left\{4.4694 + \sqrt{4.4694^2 + (4)(89.388)}\right\}$$ $y_m = 11.95$ in. ◀

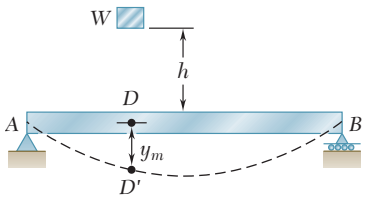
(*c*) $$P_m = (71.598)(11.95) = 856 \text{ lb}$$ $P_m = 856$ lb ◀

$$M_m = -(856)(114) = 97{,}535 \text{ lb}\cdot\text{in.}$$

(*b*) $$\sigma_m = \frac{|M_m|c}{I} = \frac{(97{,}535)(1.325)}{24.813} = 5210 \text{ psi}$$ $\sigma_m = 5.21$ ksi ◀

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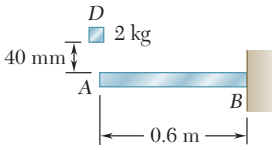
PROBLEM 11.56



A block of weight W is dropped from a height h onto the horizontal beam AB and hits it at point D. (*a*) Show that the maximum deflection y_m at point D can be expressed as

$$y_m = y_{st}\left(1+\sqrt{1+\frac{2h}{y_{st}}}\right)$$

where y_{st} represents the deflection at D caused by a static load W applied at that point and where the quantity in parenthesis is referred to as the *impact factor*. (*b*) Compute the impact factor for the beam and the impact of Prob. 11.52.

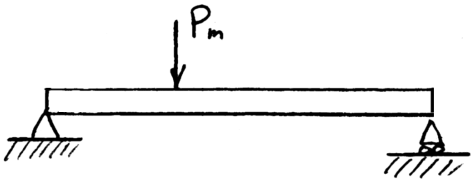


PROBLEM 11.52 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that E = 200 GPa, determine (*a*) the maximum deflection of end A, (*b*) the maximum bending moment in the rod, (*c*) the maximum normal stress in the rod.

SOLUTION

Work of falling weight: $\text{Work} = W(h+y_m)$

Strain energy: $U = \frac{1}{2}Py_m = \frac{1}{2}ky_m^2$



where k is the spring constant for a load applied at point D.

Equating work and energy,

$$W(h+y_m) = \frac{1}{2}ky_m^2$$

$$y_m^2 - \frac{2W}{k}y_m - \frac{2W}{k}h = 0$$

$$y_m^2 - 2y_{st}y_m - 2y_{st}h = 0 \quad \text{where} \quad y_{st} = \frac{W}{k}$$

(*a*) $$y_m = \frac{2y_{st}+\sqrt{4y_{st}^2+8y_{st}h}}{2}$$

$$y_m = y_{st}\left(1+\sqrt{1+\frac{2h}{y_{st}}}\right) \blacktriangleleft$$

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PROBLEM 11.56 *(Continued)*

For Problem 11.52, $W = \text{mg} = (2)(9.81) = 19.62 \text{ N}$

$$E = 200 \times 10^9 \text{ Pa} \quad I = \frac{\pi}{4}\left(\frac{16}{2}\right)^4 = 3.217 \times 10^3 \text{ mm}^4 = 3.217 \times 10^{-9} \text{ m}^4$$

$$L = 0.6 \text{ m} \quad h = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

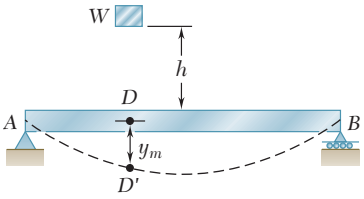
Using Appendix D, Case 1, $y_{st} = \dfrac{WL^3}{3EI}$

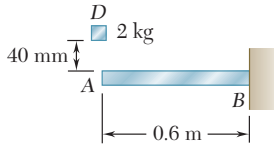
$$y_{st} = \frac{(19.62)(0.6)^3}{(3)(200 \times 10^9)(3.217 \times 10^{-9})} = 2.196 \times 10^{-3} \text{ m}$$

$$\frac{2h}{y_{st}} = \frac{(2)(40 \times 10^{-3})}{2.196 \times 10^{-3}} = 36.44$$

(*b*) Impact factor. $= 1 + \sqrt{1 + 36.44}$ Impact factor $= 7.12$ ◀

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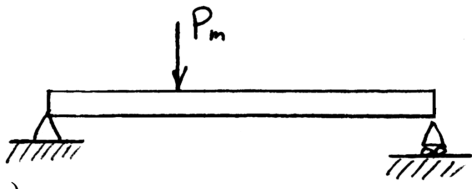
PROBLEM 11.57

A block of weight W is dropped from a height h onto the horizontal beam AB and hits point D. (*a*) Denoting by y_m the exact value of the maximum deflection at D and by y'_m the value obtained by neglecting the effect of this deflection on the change in potential energy of the block, show that the absolute value of the relative error is $(y'_m - y_m)/y_m$, never exceeding $y'_m/2h$. (*b*) Check the result obtained in part *a* by solving part *a* of Prob. 11.52 without taking y_m into account when determining the change in potential energy of the load, and comparing the answer obtained in this way with the exact answer to that problem.

PROBLEM 11.52 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (*a*) the maximum deflection of end A, (*b*) the maximum bending moment in the rod, (*c*) the maximum normal stress in the rod.

SOLUTION

$$U = \frac{1}{2}P_m y_m = \frac{1}{2}ky_m^2$$



where k is the spring constant for a load at point D.

Work of falling weight: exact: $\text{Work} = W(h + y_m)$

approximate: $\text{Work} \approx Wh$

Equating work and energy,

$$\frac{1}{2}ky_m^2 = W(h + y_m) \quad (1) \quad \text{exact}$$

$$\frac{1}{2}ky'^2_m = Wh \quad (2) \quad \text{approximate}$$

where y'_m is the approximate value for y_m.

Subtracting,

$$\frac{1}{2}k\left(y_m^2 - y'^2_m\right) = Wy_m$$

$$y_m^2 - y'^2_m = (y_m - y'_m)(y_m + y'_m) = \frac{2W}{k}y_m$$

Relative error:

$$\frac{y_m - y'_m}{y_m} = \frac{2W}{k(y_m + \tilde{y}_m)}$$

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PROBLEM 11.57 *(Continued)*

But $\dfrac{2W}{k} = \dfrac{y_m'^2}{h}$ from Eq. (2).

(*a*) Relative error $= \dfrac{y_m - y_m'}{y_m} = \dfrac{y_m'^2}{h(y_m + y_m')} < \dfrac{y_m'}{2h}$

(*b*) From the solution to Problem 11.52, $y_m = 15.63$ mm

Approximate solution:

$$W = \text{mg} = (2)(9.81) = 19.62 \text{ N}$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{10}{2}\right)^4 = 3.217 \times 10^3 \text{ mm}^4$$

$$= 3.217 \times 10^{-9} \text{ m}^4$$

$$L = 0.6 \text{ m}, \quad h = 40 \text{ mm}$$

$$= 40 \times 10^{-3} \text{m}$$

$$k = \frac{3EI}{L^3} = \frac{(3)(200 \times 10^9)(3.217 \times 10^{-9})}{(0.6)^3}$$

$$= 8.936 \times 10^3 \text{ N/m}$$

$$y_m'^2 = \frac{2Wh}{k} = \frac{(2)(19.62)(40 \times 10^{-3})}{8.936 \times 10^3}$$

$$= 175.65 \times 10^{-6} \text{ m}^2$$

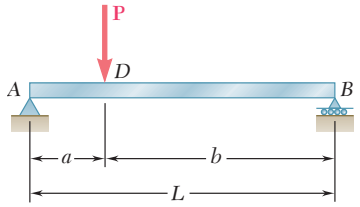
$$y_m' = 13.25 \times 10^{-3} \text{ m} = 13.25 \text{ mm}$$

Relative error: $= \dfrac{15.63 - 13.25}{15.63}$

relative error $= 0.152$ ◀

$\dfrac{y_m'}{2h} = 0.166$ ◀

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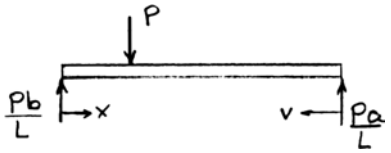
PROBLEM 11.58

Using the method of work and energy, determine the deflection at point *D* caused by the load **P.**

SOLUTION

Reactions: $R_A = \frac{Pb}{L}, \quad R_B = \frac{Pa}{L}$

Over *AD*: $M = R_A x = \frac{Pbx}{L}$



$$U_{AD} = \int_0^a \frac{M^2}{2EI}dx = \frac{P^2b^2}{2EIL^2}\int_0^a x^2dx$$

$$= \frac{P^2b^2a^3}{6EIL^2}$$

Over *DB*: $M = R_B v = \frac{Pav}{L}$

$$U_{DB} = \int_0^b \frac{M^2}{2EI}dv = \frac{P^2a^2}{2EIL^2}\int_0^b v^2dv$$

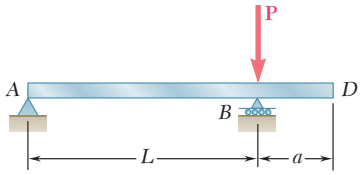
$$= \frac{P^2a^2b^3}{6EIL^2}$$

Total: $U = U_{AB} + U_{BC} = \frac{P^2a^2b^2(a+b)}{6EIL^2} = \frac{P^2a^2b^2}{6EIL}$

$$\frac{1}{2}P\delta_D = U \qquad \delta_D = \frac{2U}{P}$$

$$\delta_D = \frac{Pa^2b^2}{3EI}\downarrow \blacktriangleleft$$

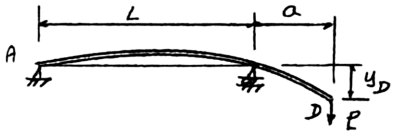
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PROBLEM 11.59

Using the method of work and energy, determine the deflection at point *D* caused by the load **P**.

SOLUTION

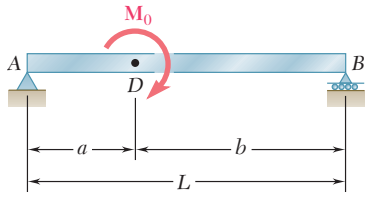


See solution of Prob. 11.28 for beam of length L with overhang of length a, load at end of overhang.

$$U = \frac{P^2a^2}{6EI}(a + L)$$

$$\frac{1}{2}Py_D = U: \quad \frac{1}{2}Py_D = \frac{P^2a^2}{6EI}(a + L) \qquad y_D = \frac{Pa^2}{3EI}(a + L) \downarrow \blacktriangleleft$$

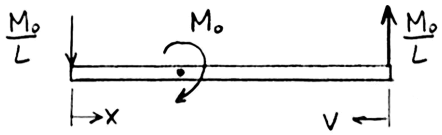
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PROBLEM 11.60

Using the method of work and energy, determine the slope at point *D* caused by the couple $\mathbf{M}_0$.

SOLUTION



Reactions: $R_A = \frac{M_0}{L}\downarrow \qquad R_B = \frac{M_0}{L}\uparrow$

Over portion *AD*: $M = -\frac{M_0 x}{L}$

$$U_{AD} = \int_0^a \frac{M^2}{2EI}dx = \frac{M_0^2}{2EIL^2}\int_0^a x^2 dx$$

$$= \frac{M_0 a^3}{6EIL^2}$$

Over portion *DB*: $M = \frac{M_0 v}{L}$

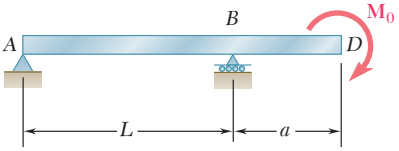
$$U_{DB} = \int_0^b \frac{M^2}{2EI}dv = \frac{M_0^2}{2EIL^2}\int_0^b v^2 dv$$

$$= \frac{M_0^2 b}{6EIL^2}$$

Total: $U = U_{AD} + U_{DB} = \frac{M_0^2(a^3+b^3)}{6EIL^2}$

$$\frac{1}{2}M_0\theta_D = U \qquad \theta_D = \frac{2U}{M_0} \qquad \theta_D = \frac{M_0(a^3+b^3)}{3EIL^2}$$ ◀

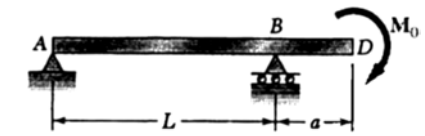
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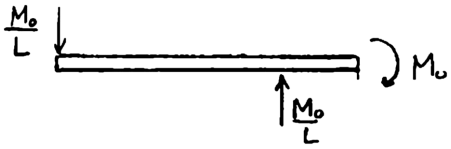


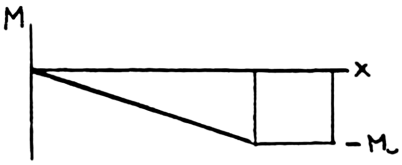
PROBLEM 11.61

Using the method of work and energy, determine the slope at point *D* caused by the couple $\mathbf{M}_0$.

SOLUTION







Reactions: $R_A = \frac{M_0}{L} \downarrow \qquad R_B = \frac{M_0}{L} \uparrow$

Over portion *AB*: $M = -\frac{M_0 x}{L}$

$$U_{AB} = \int_0^L \frac{M^2}{2EI}dx = \frac{M_0^2}{2EIL^2}\int_0^L x^2 dx$$

$$= \frac{M_0^2 L}{6EI}$$

Over portion *BD*: $M = -M_0$

$$U_{BD} = \frac{M_0^2 a}{2EI}$$

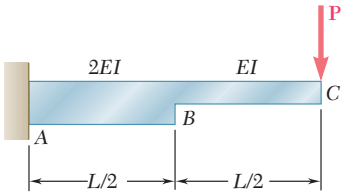
Total: $U = U_{AB} + U_{BD}$

$$= \frac{M_0^2(L+3a)}{6EI}$$

$$\frac{1}{2}M_0\theta_0 = U$$

$$\theta_D = \frac{2U}{M_0} \qquad \theta_D = \frac{M_0(L+3a)}{3EI} \measuredangle \blacktriangleleft$$

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PROBLEM 11.62

Using the method of work and energy, determine the deflection at point *C* caused by the load **P.**

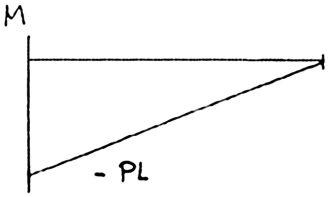
SOLUTION

Bending moment: $M = -Pv$

Over *AB*:

$$U_{AB} = \int_{L/2}^{L} \frac{M^2}{4EI}dv = \frac{P^2}{4EI}\int_{L/2}^{L} v^2 dv$$

$$= \frac{P^2}{12EI}\left[L^3 - \left(\frac{L}{2}\right)^3\right] = \frac{7}{96}\frac{P^2L^3}{EI}$$



Over *BC*:

$$U_{BC} = \int_{0}^{L/2} \frac{M^2}{2EI}dv = \frac{P^2}{2EI}\int_{0}^{L/2} v^2 dv$$

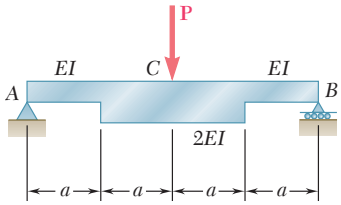
$$= \frac{1}{48}\frac{P^2L^3}{EI}$$

Total:

$$U = U_{AB} + U_{BC} = \frac{3}{32}\frac{P^2L^3}{EI}$$

$$\frac{1}{2}P\delta_C = 0 \qquad \delta_C = \frac{2U}{P} \qquad \delta_C = \frac{3PL^3}{16EI}\downarrow \blacktriangleleft$$

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PROBLEM 11.63

Using the method of work and energy, determine the deflection at point C caused by the load **P.**

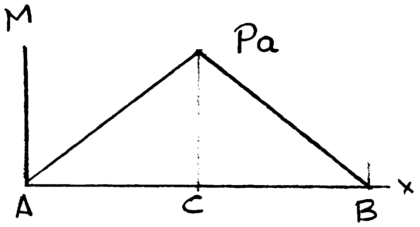
SOLUTION

Symmetric beam and loading:

$$R_A = R_B = \frac{1}{2}P$$

From A to C,

$$M = R_A x = \frac{1}{2}Px$$



$$U_{AC} = \int_0^a \frac{M}{2EI}dx + \int_a^{2a} \frac{M^2}{4EI}dx$$

$$= \frac{P^2}{8EI}\int_0^a x^2 dx + \frac{P^2}{16EI}\int_a^{2a} x^2 dx$$

$$\frac{P^2a^3}{24EI} + \frac{P^2}{48EI}\left[(2a)^3 - a^3\right] = \frac{3}{16}\frac{P^2a^3}{EI}$$

By symmetry,

$$U_{CB} = U_{AB} = \frac{3}{16}\frac{P^2a^3}{EI}$$

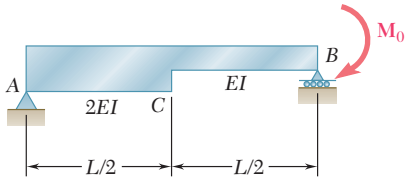
Total:

$$U = U_{AB} + U_{BC} = \frac{3}{8}\frac{P^2a^3}{EI}$$

$$\frac{1}{2}P\delta_C = U \qquad \delta_C = \frac{2U}{P}$$

$$\delta_C = \frac{3Pa^3}{4EI}\downarrow \blacktriangleleft$$

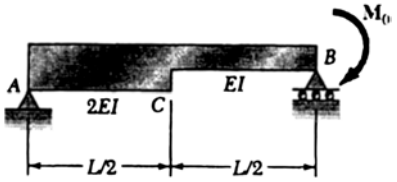
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PROBLEM 11.64

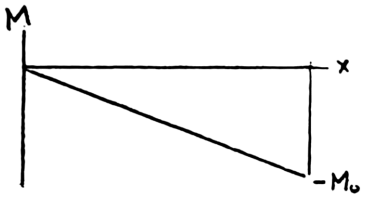
Using the method of work and energy, determine the slope at point B caused by the couple $\mathbf{M}_0$.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L - M_0 = 0 \qquad R_A = -\frac{M_0}{L}$$

$$M = R_A X = -\frac{M_0}{L} X$$



Over portion AC: $\quad U_{AC} = \int_0^{L/2} \frac{M^2}{2(2EI)}\,dx$

$$U_{AC} = \frac{M_0^2}{4EIL^2}\int_0^{L/2} x^2\,dx = \frac{1}{96}\frac{M_0^2}{EI}$$

Over portion CB: $\quad U_{CB} = \int_{L/2}^{L} \frac{M^2}{2EI}\,dx$

$$U_{CB} = \frac{M_0^2}{2EIL^2}\int_{L/2}^{L} x^2\,dx$$

$$= \frac{M_0^2}{6EIL^2}\left[L^3 - \left(\frac{L}{2}\right)^3\right]$$

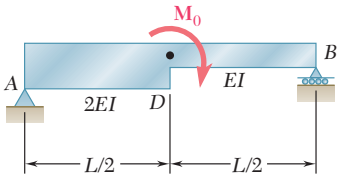
$$= \frac{7}{48}\frac{M_0^2 L}{EI}$$

Total: $\quad U = U_{AC} + U_{CB} = \frac{5}{32}\frac{M_0^2 L}{EI}$

$$\frac{1}{2}M_0\theta_B = U \qquad \theta_B = \frac{2U}{M_0}$$

$$\theta_B = \frac{5M_0 L}{16EI} \curvearrowright$$

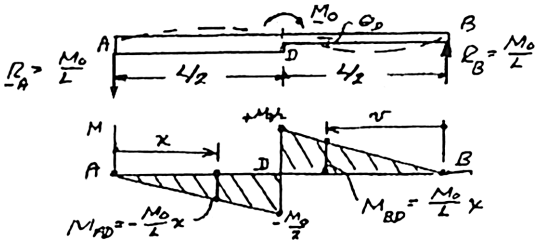
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PROBLEM 11.65

Using the method of work and energy, determine the slope at point *D* caused by the couple $\mathbf{M}_0$.

SOLUTION



Reactions: $R_A = \dfrac{M_0}{L} \downarrow, \qquad R_B = \dfrac{M_0}{L} \uparrow$

Bending moment diagram.

Over $0 < x < \dfrac{L}{2}$, $\qquad M = -\dfrac{M_0 x}{L}$

Over $0 < v < \dfrac{L}{2}$, $\qquad M = \dfrac{M_0 v}{L}$

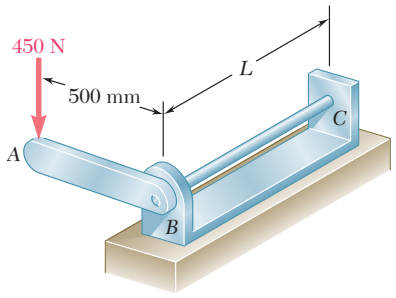
Strain energy: $\quad U = U_{AD} + U_{BD} = \displaystyle\int_0^{L/2} \frac{M_{AD}^2}{2(2EI)}\,dx + \int_0^{L/2} \frac{M_{BD}^2}{2EI}\,dv$

$$U = \int_0^{L/2} \frac{\left(-\frac{M_0}{L}x\right)^2}{2(2EI)}\,dx + \int_0^{L/2} \frac{\left(\frac{M_0}{L}v\right)^2}{2EI}\,dv = \frac{M_0}{EIL}\left(\frac{1}{4}\right)\frac{(L/2)^3}{3} + \frac{M_0}{EIL}\left(\frac{1}{2}\right)\frac{(L/2)^3}{3}$$

$$= \frac{M_0^2 L}{EI}\left(\frac{1}{96} + \frac{1}{48}\right) = \frac{3}{96}\frac{M_0^2 L}{EI} = \frac{1}{32}\frac{M_0^2 L}{EI}$$

$$\frac{1}{2}M_0\theta_D = U: \quad \frac{1}{2}M_0\theta_D = \frac{1}{32}\frac{M_0^2 L}{EI} \qquad \theta_D = \frac{1}{16}\frac{M_0 L}{EI} \curvearrowright$$

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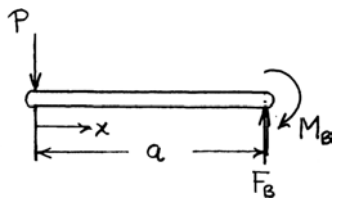


PROBLEM 11.66

The 20-mm-diameter steel rod *BC* is attached to the lever *AB* and to the fixed support *C*. The uniform steel lever is 10 mm thick and 30 mm deep. Using the method of work and energy, determine the deflection of point *A* when $L = 600$ mm. Use $E = 200$ GPa and $G = 77.2$ GPa.

SOLUTION

Member *AB*. (Bending)



$$I = \frac{1}{12}(10)(30)^3 = 22.5\times10^3\ \text{mm}^4$$
$$= 22.5\times10^{-9}\ \text{m}^4$$
$$a = 500\ \text{mm} = 0.500\ \text{m}$$
$$M_B = Pa = (450)(0.500) = 225\ \text{N}\cdot\text{m}$$
$$M = Px$$
$$U_{AB} = \int_0^a \frac{M}{2EI}dx = \int_0^a \frac{P^2x^2dx}{2EI} = \frac{P^2a^3}{6EI}$$
$$= \frac{(450)^2(0.500)^3}{(6)(200\times10^9)(22.5\times10^{-9})} = 0.9375\ \text{J}$$

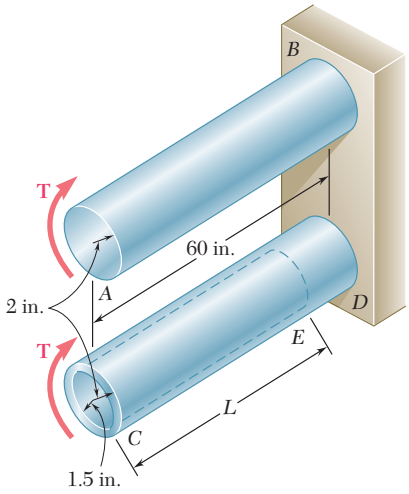
Member *BC*. (Torsion)

$$T = M_B = 225\ \text{N}\cdot\text{m} \qquad c = \frac{1}{2}d = 10\ \text{mm}$$
$$J = \frac{\pi}{2}c^4 = 15.708\times10^3\ \text{mm}^4 = 15.708\times10^{-9}\ \text{m}^4$$
$$L = 600\ \text{mm} = 0.600\ \text{m}$$
$$U_{BC} = \frac{T^2L}{2GJ} = \frac{(225)^2(0.600)}{(2)(77.2\times10^9)(15.708\times10^{-9})} = 12.5242\ \text{J}$$

Total:
$$U = U_{AB} + U_{BC} = 0.9375 + 12.5242 = 13.4617\ \text{J}$$
$$\frac{1}{2}P\delta_A = U \qquad \delta_A = \frac{2U}{P} = \frac{(2)(13.4617)}{450} = 59.8\times10^{-3}\ \text{m}$$

$$\delta_A = 59.8\ \text{mm}\downarrow \blacktriangleleft$$

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PROBLEM 11.67

Torques of the same magnitude T are applied to the steel shafts AB and CD. Using the method of work and energy, determine the length L of the hollow portion of shaft CD for which the angle of twist C is equal to 1.25 times the angle of twist at A.

SOLUTION

T is the same for each shaft.

$$\theta_C = 1.25\theta_A \qquad U_{AB} = \frac{1}{2}T\theta_A \qquad \text{and} \qquad U_{CD} = \frac{1}{2}T\theta_C$$

Then $U_{CD} = 1.25\,U_{AB}$ (1)

Shaft AB: $L_{AB} = 60$ in. $\quad c = 2$ in. $\quad J_{AB} = \dfrac{\pi}{2}c^4 = \dfrac{\pi}{2}(2)^4 = 25.133\ \text{in}^4$

$$U_{AB} = \frac{T^2L_{AB}}{2GJ_{AB}} = \frac{T^2(60)}{(2G)(25.133)} = 2.3873\frac{T^2}{2G}$$

Shaft portion CE: $L_{CE} = L, \quad c_o = 2$ in., $\quad c_i = 1.5$ in.

$$J_{CE} = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) = \frac{\pi}{2}(2^4 - 1.5^4) = 17.1806\ \text{in}^4$$

$$U_{CE} = \frac{T^2L_{CE}}{2GJ_{CE}} = \frac{T^2L}{(2G)(17.1806)} = 0.058205\frac{T^2L}{2G}$$

Shaft portion ED: $L_{ED} = 60 - L, \quad J_{DE} = J_{AB} = 25.133\ \text{in}^4$

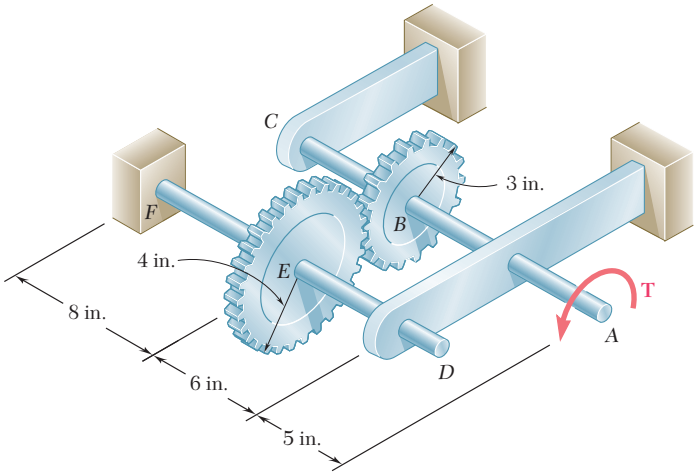
$$U_{ED} = \frac{T^2L_{ED}}{2GJ_{ED}} = \frac{T^2(60-L)}{2G(25.138)} = 2.3873\frac{T^2}{2G} - 0.039789\frac{T^2L}{2G}$$

Shaft CD: $U_{CE} + U_{ED} = 2.3873\dfrac{T^2}{2G} + 0.018416\dfrac{T^2L}{2GJ}$

Using Eq. (1), $2.3873\dfrac{T^2}{2G} + 0.018416\dfrac{T^2L}{2G} = (1.25)(2.3873)\dfrac{T^2}{2G}$

$0.018416L = (0.25)(2.3873)$ $\qquad L = 32.4$ in. ◀

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PROBLEM 11.68

Two steel shafts, each of 0.75-in. diameter, are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi and that shaft *DF* is fixed at *F*, determine the angle through which end *A* rotates when a 750-lb · in. torque is applied at *A*. (Neglect the strain energy due to the bending of the shafts.)

SOLUTION

Work-energy equation:

$$\frac{1}{2} T_A \varphi_A = U$$

$$\varphi_A = \frac{2U}{T_A}$$

Portion *AB* of shaft *ABC*:

$$T_{AB} = T_A = 750 \text{ lb} \cdot \text{in.}$$

$$L_{AB} = 5 + 6 = 11 \text{ in.}$$

$$J_{AB} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{2}\left(\frac{0.75}{2}\right)^4 = 31.063 \times 10^{-3} \text{in}^4$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} = \frac{(750)^2(11)}{(2)(11.2 \times 10^6)(31.063 \times 10^{-3})} = 8.892 \text{ in.} \cdot \text{lb}$$

Portion *BC* of shaft *ABC*: $U_{BC} = 0$

Gear *B*: $F_{BE} = \frac{T_B}{r_B} = \frac{T_{AB}}{r_B} = \frac{750}{3} = 250 \text{ lb}$

Gear *E*: $T_E = r_E F_{BE} = (4)(250) = 1000 \text{ lb} \cdot \text{in.}$

Portion *DE* of shaft *DEF*: $U_{DE} = 0$

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PROBLEM 11.68 *(Continued)*

Portion *EF* of shaft *DEF*:

$$T_{EF} = T_E = 1000 \text{ lb} \cdot \text{in.}$$

$$L_{EF} = 8 \text{ in.} \qquad J_{EF} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = 31.063 \times 10^{-3} \text{ in}^4$$

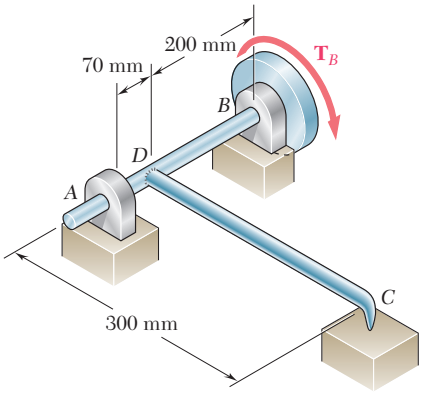
$$U_{EF} = \frac{T_{EF}^2 L_{EF}}{2GJ_{EF}} = \frac{(1000)^2(8)}{(2)(11.2 \times 10^6)(31.063 \times 10^{-3})} = 11.497 \text{ lb} \cdot \text{in.}$$

Total:

$$U = U_{AB} + U_{BC} + U_{DE} + U_{EF} = 20.389 \text{ in.} \cdot \text{lb}$$

$$\varphi_A = \frac{2U}{T_A} = \frac{(2)(20.389)}{750} = 54.4 \times 10^{-3} \text{ rad} \qquad \varphi_A = 3.12° \blacktriangleleft$$

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PROBLEM 11.69

The 20-mm-diameter steel rod *CD* is welded to the 20-mm-diameter steel shaft *AB* as shown. End *C* of rod *CD* is touching the rigid surface shown when a couple $\mathbf{T}_B$ is applied to a disk attached to shaft *AB*. Knowing that the bearings are self aligning and exert no couples on the shaft, determine the angle of rotation of the disk when $T_B = 400\ \text{N}\cdot\text{m}$. Use $E = 200$ GPa and $G = 77.2$ GPa. (Consider the strain energy due to both bending and twisting in shaft *AB* and to bending in arm *CD*.)

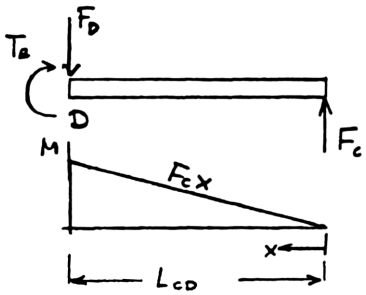
SOLUTION

$$+\circlearrowleft \Sigma M_{AB} = 0: \quad r_{CD}\,F_C = T_B \quad F_C = \frac{T_B}{r_{CD}}$$

$$F_C = \frac{400}{300\times10^{-3}} = 1333.3\ \text{N},$$

$$F_D = 1333.3\ \text{N}$$

Bending of rod *CD*:



$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4$$

$$= \frac{\pi}{4}\left(\frac{20}{2}\right)^4$$

$$= 7.854\times10^{3}\ \text{mm}^4$$

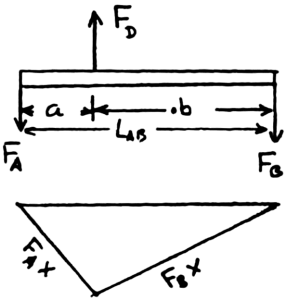
$$= 7.854\times10^{-9}\ \text{m}^4$$

$$M = F_C x$$

$$U = \int_0^{L_{CD}} \frac{(F_C x)^2}{2EI} = \frac{F_C^2 L_{CD}^3}{6EI}$$

$$= \frac{(1333.3)^2(300\times10^{-3})^3}{(6)(200\times10^{9})(7.854\times10^{-9})} = 5.093\ \text{J}$$

Bending of shaft *ADB*:



$$+\circlearrowleft \Sigma M_B = 0: \quad -F_A L_{AB} + F_D b = 0 \qquad F_A = \frac{F_D b}{L_{AB}}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad +F_A L_{AB} - F_D b = 0 \qquad F_A = \frac{F_D a}{L_{AB}}$$

$$U = \frac{1}{2EI}\left\{\int_0^a \left(\frac{F_D b}{L_{AB}}\right)^2 dx + \int_0^b \left(\frac{F_D a}{L_{AB}}\right)^2 dx\right\} = \frac{F_D^2 a^2 b^2}{6EIL_{AB}}$$

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PROBLEM 11.69 *(Continued)*

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = 7.854 \times 10^3 \text{ mm}^4 = 7.854 \times 10^{-9} \text{ m}^4$$

$$L_{AB} = (270 \times 10^{-3}) \text{ m}$$

$$U = \frac{(1333.3)^2 (70 \times 10^{-3})^2 (200 \times 10^{-3})^2}{(6)(200 \times 10^9)(7.854 \times 10^{-9})(270 \times 10^{-9})} = 0.137 \text{ J}$$

Torsion: Only portion *DB* carries torque. $J = 2J = 15.708 \times 10^{-9} \text{ m}^4$

$$U = \frac{T_B^2 L_{DB}}{2GJ} = \frac{(400)^2 (200 \times 10^{-3})}{(2)(77.2 \times 10^9)(15.708 \times 10^{-9})} = 13.194 \text{ J}$$

Total: $U = 5.093 + 0.137 + 13.194 = 18.424 \text{ J}$

$$\frac{1}{2} T_B \varphi_B = U$$

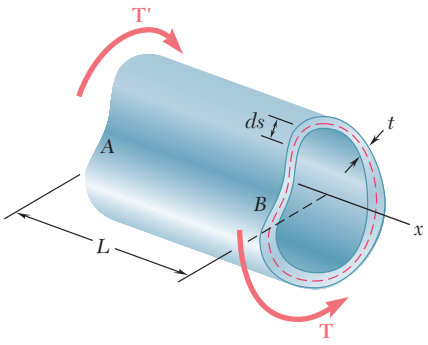
$$\varphi_B = \frac{2U}{T_B} = \frac{(2)(18.424)}{400}$$

$$= 92.1 \times 10^{-3} \text{ rad}$$

$$\varphi_B = 5.28° \blacktriangleleft$$

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PROBLEM 11.70



The thin-walled hollow cylindrical member *AB* has a noncircular cross section of nonuniform thickness. Using the expression given in Eq. (3.50) of Sec. 3.10 and the expression for the strain-energy density given in Eq. (11.17), show that the angle of twist of member *AB* is

$$\phi = \frac{TL}{4\mathcal{A}^2 G}\oint \frac{ds}{t}$$

where ds is the length of a small element of the wall cross section and $\mathcal{A}$ is the area enclosed by the center line of the wall cross section.

SOLUTION

From Eq. (3.53),

$$\tau = \frac{T}{2t\mathcal{A}}$$

Strain energy density:

$$u = \frac{\tau^2}{2G} = \frac{T^2}{8Gt^2\mathcal{A}^2}$$

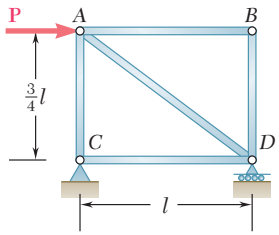
$$U = \int_0^L \oint ut\, ds\, dx$$

$$= \int_0^L \frac{T^2}{8G\mathcal{A}^2}\oint \frac{ds}{t}\, dx = \frac{T^2 L}{8G\mathcal{A}^2}\oint \frac{ds}{t}$$

Work of torque:

$$= \frac{1}{2}T\varphi = \frac{T^2 L}{8G\mathcal{A}^2}\oint \frac{ds}{t} \qquad \varphi = \frac{TL}{4G\mathcal{A}^2}\oint \frac{ds}{t}$$

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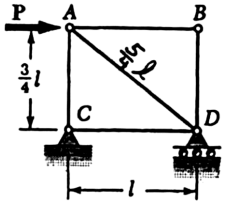


PROBLEM 11.71

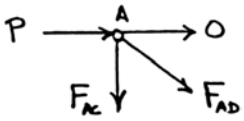
Each member of the truss shown has a uniform cross-sectional area *A*. Using the method of work and energy, determine the horizontal deflection of the point of application of the load **P**.

SOLUTION

Members *AB* and *BD* are Zero force members.



Joint *A*:

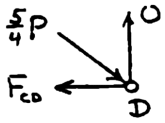


$$+\rightarrow \Sigma F_x = 0: \quad \frac{4}{5}F_{AD} + P = 0 \qquad F_{AD} = -\frac{5}{4}P$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{AC} - \frac{3}{5}F_{AD} = 0 \qquad F_{AC} = \frac{3}{4}P$$

Member	F	L	F^2L
AB	0	l	0
BD	0	$\frac{3}{4}l$	0
AD	$-\frac{5}{4}P$	$\frac{5}{4}l$	$\frac{125}{64}P^2l$
CD	P	l	P^2l
AC	$\frac{3}{4}P$	$\frac{3}{4}l$	$\frac{27}{64}P^2l$
Σ			$\frac{27}{8}P^2l$

Joint *D*:



$$+\rightarrow \Sigma F_x = 0: \quad \frac{4}{5}\cdot\frac{5}{4}P - F_{CD} = 0$$

$$F_{CD} = P$$

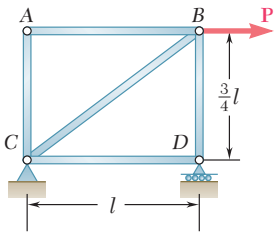
$$U = \Sigma\frac{F^2L}{2EA} = \frac{1}{2EA}\Sigma F^2L$$

$$= \frac{27}{16}\frac{P^2l}{EA} \quad \text{Work of } P = \frac{1}{2}P\Delta = U$$

$$\Delta = \frac{2U}{P} = \frac{27}{8}\frac{Pl}{EA}$$

$$\Delta = 3.38\frac{Pl}{EA} \rightarrow \blacktriangleleft$$

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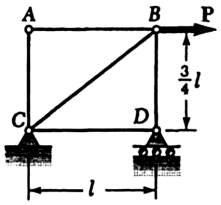


PROBLEM 11.72

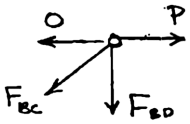
Each member of the truss shown has a uniform cross-sectional area A. Using the method of work and energy, determine the horizontal deflection of the point of application of the load **P**.

SOLUTION

Members AB, AC and CD are zero force members.



Joint B:



$$+\rightarrow \Sigma F_x = 0: \quad P - \frac{4}{5}F_{BC} = 0 \qquad F_{BC} = \frac{5}{4}P$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{BD} - \frac{3}{5}F_{BC} = 0 \qquad F_{BD} = -\frac{3}{4}P$$

$$U = \Sigma\frac{F^2L}{2EA} = \frac{1}{2EA}\Sigma F^2L$$

$$= \frac{19}{16}\frac{P^2l}{EA}$$

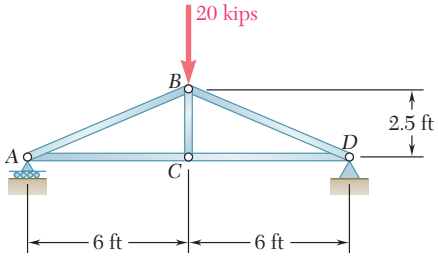
Work of $P = \frac{1}{2}P\Delta = U$

$$\Delta = \frac{2U}{P} = \frac{19}{8}\frac{Pl}{EA}$$

Member	F	L	F^2L
AB	0	l	0
AC	0	$\frac{3}{4}l$	0
CD	0	l	0
BC	$\frac{5}{4}P$	$\frac{5}{4}l$	$\frac{125}{64}P^2l$
BD	$-\frac{3}{4}P$	$\frac{3}{4}l$	$\frac{27}{64}P^2l$
Σ			$\frac{19}{8}P^2l$

$$\Delta = 2.38\frac{Pl}{EA} \rightarrow \blacktriangleleft$$

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PROBLEM 11.73

Each member of the truss shown is made of steel and has a uniform cross-sectional area of 5 in^2. Using $E = 29 \times 10^6$ psi, determine the vertical deflection of joint B caused by the application of the 20-kip load.

SOLUTION

$$R_A = R_B = 10 \text{ kips} \uparrow$$

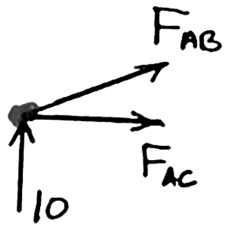
$$L_{AC} = L_{CD} = 6 \text{ ft} = 72 \text{ in.}$$

$$L_{BC} = 2.5 \text{ ft} = 30 \text{ in.}$$

$$L_{AB} = L_{BC} = \sqrt{6^2 + 2.5^2} = 6.5 \text{ ft} = 78 \text{ in.}$$

Equilibrium of joint A.

Joint A



$$+\uparrow \Sigma F_y = 0: \quad \frac{2.5}{6.5} F_{AB} + 10 = 0 \qquad F_{AB} = -26 \text{ kips}$$

$$\pm \rightarrow \Sigma F_x = 0: \quad \frac{6}{6.5} F_{AB} + F_{AB} = 0 \qquad F_{AC} = 24 \text{ kips}$$

Equilibrium of joint C. $F_{BC} = 0$, $F_{CD} = 24$ kips

By symmetry, $F_{BD} = F_{AB} = -26$ kips

Strain energy: $$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2EA} \Sigma F^2 L$$

Member	F (kips)	L (in.)	F^2L (kip^2 · in.)
AB	–26	78	52,728
AC	24	72	41,472
BC	0	30	0
CD	24	72	41,472
BD	–26	78	52,728
Σ			188,400

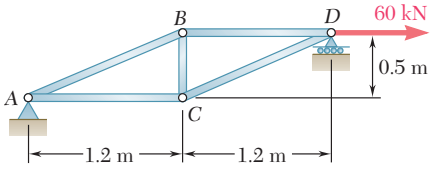
$$U = \frac{188{,}400}{(2)(29 \times 10^3)(5)} = 0.64966 \text{ kip} \cdot \text{in.}$$

Vertical deflection of point B.

$$\frac{1}{2} P \delta_B = U$$

$$\delta_B = \frac{2U}{P} = \frac{(2)(0.64966)}{20} \qquad \delta_B = 0.0650 \text{ in.} \downarrow \blacktriangleleft$$

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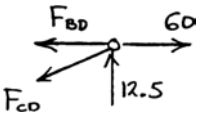


PROBLEM 11.74

Each member of the truss shown is made of steel. The cross-sectional area of member *BC* is 800 mm^2 and for all other members the cross-sectional area is 400 mm^2. Using $E = 200$ GPa, determine the deflection of point *D* caused by the 60-kN load.

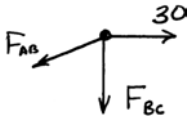
SOLUTION

Entire truss: $+\circlearrowleft \Sigma M_A = 0: \quad 2.4R_D - (0.5)(60) = 0 \qquad R_D = 12.5 \text{ kN}$



Joint *D*: $+\uparrow \Sigma F_y = 0: \quad 12.5 - \frac{0.5}{1.3}F_{CD} = 0 \qquad F_{CD} = 32.5 \text{ kN}$

$\rightarrow + \Sigma F_x = 0: \quad 60 - F_{BD} - \frac{1.2}{1.3}F_{CD} = 0 \qquad F_{BD} = 30 \text{ kN}$



Joint *B*: $+\rightarrow \Sigma F_x = 0: \quad 30 - \frac{1.2}{1.3}F_{AB} = 0 \qquad F_{AB} = 32.5 \text{ kN}$

$+\uparrow \Sigma F_y = 0: \quad -\frac{0.5}{1.3}F_{AB} + F_{BC} = 0 \qquad F_{BC} = 12.5 \text{ kN}$

Joint *C*: $+\rightarrow \Sigma F_x = 0: \quad -F_{AC} + \frac{1.2}{1.3}(32.5) = 0 \qquad F_{AC} = 30 \text{ kN}$



$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2E}\Sigma \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	$A(10^{-6}\text{m}^2)$	$F^2L/A(\text{N}^2/\text{m})$
CD	32.5	1.3	400	3.4328×10^{12}
BD	30	1.2	400	2.7×10^{12}
AB	32.5	1.3	400	3.4328×10^{12}
BC	12.5	0.5	800	0.0977×10^{12}
AC	30	1.2	400	2.7×10^{12}
Σ				12.3633×10^{12}

$$E = 200\times10^9 \text{ Pa}$$

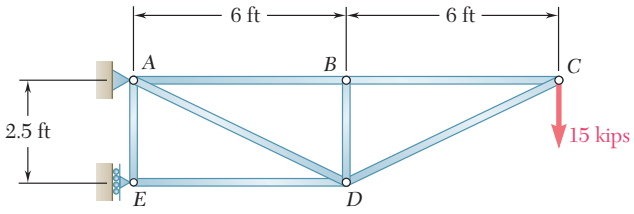
$$U = \frac{12.3633\times10^{12}}{(2)(200\times10^9)} = 30.908 \text{ J}$$

Work-energy:

$$\frac{1}{2}P\Delta = U \qquad \Delta = \frac{2U}{P} = \frac{(2)(30.908)}{60\times10^3} = 1.030\times10^{-3}\text{m}$$

$$\Delta = 1.030 \text{ mm} \rightarrow \blacktriangleleft$$

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PROBLEM 11.75

Each member of the truss shown is made of steel and has a cross-sectional area of 5 in^2. Using $E = 29\times10^6$ psi, determine the vertical deflection of point C caused by the 15-kip load.

SOLUTION

Members BD and AE are zero force members.

For entire truss, $+\circlearrowleft M_A = 0$: $2.5R_E - (12)(15) = 0$ $R_E = 72$ kips

For equilibrium of joint E, $F_{ED} = -R_E = -72$ kips

Joint C:



$+\uparrow \Sigma F_y = 0$: $-\frac{2.5}{6.5}F_{CD} - 15 = 0$ $F_{CD} = -39$ kips

$+\rightarrow \Sigma F_x = 0$: $-\frac{6}{6.5}F_{CD} - F_{BC} = 0$ $F_{BC} = 36$ kips

Joint D:



$+\rightarrow \Sigma F_x = 0$:

$72 - \frac{6}{6.5}(F_{AD} + 39) = 0$

$F_{AD} = 39$ kips

Joint B:

$\Sigma F_x = 0$:

$-F_{AB} + F_{BC} = 0$

$F_{AB} = 36$ kips

Strain energy: $U_m = \Sigma\frac{F^2L}{2EA} = \frac{1}{2EA}\Sigma F^2L$

Member	F (kips)	L (in.)	F^2L (kip$^2\cdot$in.)
AB	36	72	93,312
BC	36	72	93,312
CD	–39	78	118,638
DE	–72	72	373,248
BD	0	30	0
AE	0	30	0
AD	39	78	118,638
Σ			797,148

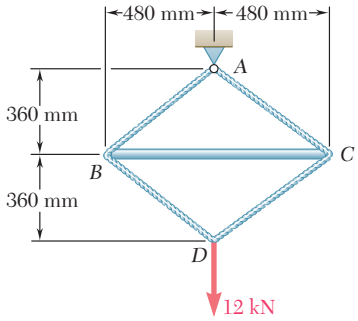
Data: $E = 29\times10^3$ ksi $A = 5$ in^2

$$U_m = \frac{797{,}148}{(2)(29\times10^3)(5)} = 2.7488 \text{ kip}\cdot\text{in.}$$

$$\frac{1}{2}P_m\Delta_m = U_m \qquad \Delta_m = \frac{2U_m}{P_m} = \frac{(2)(2.7488)}{15}$$

$\Delta_m = 0.366$ in.$\downarrow$ ◀

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PROBLEM 11.76

The steel rod *BC* has a 24-mm diameter and the steel cable *ABDCA* has a 12-mm diameter. Using $E = 200$ GPa, determine the deflection of joint *D* caused by the 12-kN load.

SOLUTION

Owing to symmetry,

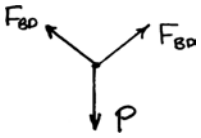
$$F_{AB} = F_{BD} = F_{DC} = F_{CA}$$
$$U_{AB} = U_{BD} = U_{DC} = U_{CA}$$

$$U = 4U_{BD} + U_{BC} = 4\frac{F_{BD}^2 L_{BD}}{2EA_{BD}} + \frac{F_{BC}^2 L_{BC}}{2EA_{BC}}$$

Let *P* be the load at *D*.

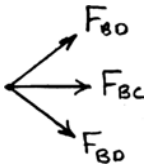
$$\frac{1}{2}P\delta_D = U \qquad \delta_D = \frac{2U}{P} = 4\frac{F_{BD}{}^2 L_{BD}}{EA_{BD}P} + \frac{F_{BC}{}^2 L_{BC}}{EA_{BC}P}$$

Joint *D*:



$$+\uparrow \Sigma F_y = 0:$$
$$2\frac{3}{5}F_{BD} - P = 0$$
$$F_{BD} = \frac{5}{6}\text{P}$$

Joint *B*:



$$\xrightarrow{+} \Sigma F_x = 0:$$
$$F_{BC} + (2)\frac{4}{5}F_{BD} = 0$$
$$F_{BC} = -\frac{8}{5}F_{BD} = -\frac{4}{3}\text{P}$$

$$\delta_D = 4\left(\frac{5}{6}\right)^2 \frac{PL_{BD}}{EA_{BD}} + \left(\frac{4}{3}\right)^2 \frac{PL_{BC}}{EA_{BC}} = \frac{P}{E}\left\{\frac{25}{9}\frac{L_{BD}}{A_{BD}} + \frac{16}{9}\frac{L_{BC}}{A_{BC}}\right\}$$

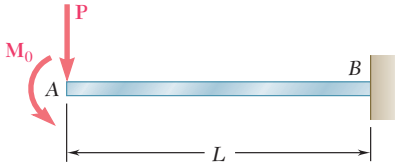
Data: $P = 12\times10^3\,\text{N}$ $\qquad E = 200\times10^9\,\text{Pa}$

$$L_{BD} = 600\times10^{-3}\,\text{m} \qquad A_{BD} = \frac{\pi}{4}(12)^2 = 113.097\ \text{mm}^2 = 113.097\times10^{-6}\,\text{m}^2$$

$$L_{BC} = 960\times10^{-3}\,\text{m} \qquad A_{BC} = \frac{\pi}{4}(24)^2 = 452.39\ \text{mm}^2 = 452.39\times10^{-6}\,\text{m}^2$$

$$\delta_D = \frac{12\times10^3}{200\times10^9}\left\{\frac{25}{9}\cdot\frac{600\times10^{-3}}{113.097\times10^{-6}} + \frac{16}{9}\cdot\frac{960\times10^{-3}}{452.39\times10^{-6}}\right\} = 1.111\times10^{-3}\,\text{m} \qquad \delta_D = 1.111\ \text{mm}\downarrow \blacktriangleleft$$

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PROBLEM 11.77

Using the information in Appendix D, compute the work of the loads as they are applied to the beam (*a*) if the load **P** is applied first, (*b*) if the couple **M** is applied first.

SOLUTION

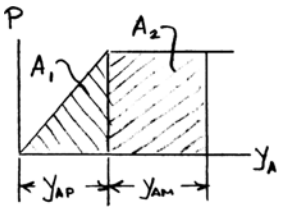
From Appendix D, Case 1,

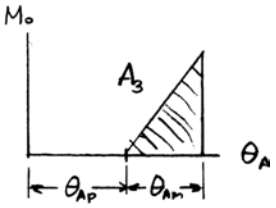
$$y_{AP} = \frac{PL^3}{3EI} \qquad \theta_{AP} = -\frac{PL^2}{2EI}$$

From Appendix D, Case 3,

$$y_{AM} = \frac{M_0 L^2}{2EI} \qquad \theta_{AM} = \frac{M_0 L}{EI}$$

(*a*) First P, then M_0.





$$U = A_1 + A_2 + A_3$$

$$= \frac{1}{2}Py_{AP} + Py_{AM} + \frac{1}{2}M_0\theta_{AM}$$

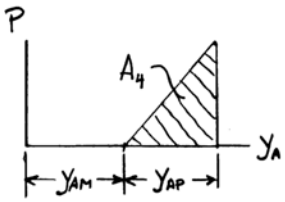
$$U = \frac{P^2L^3}{6EI} + \frac{PM_0L^2}{2EI} + \frac{M_0^2L}{2EI} \blacktriangleleft$$

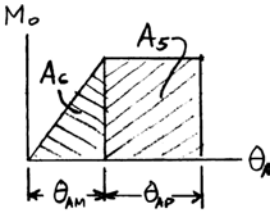
(*b*) First M_0, then P.

$$U = A_4 + A_5 + A_6$$

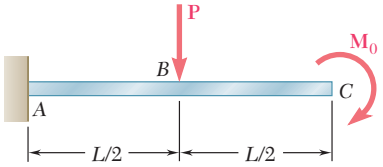
$$= \frac{1}{2}Py_{AP} + M_0\theta_{AP} + \frac{1}{2}M_0\theta_{AM}$$

$$U = \frac{P^2L^3}{6EI} + \frac{M_0PL^2}{2EI} + \frac{M_0^2L}{2EI} \blacktriangleleft$$





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PROBLEM 11.78

Using the information in Appendix D, compute the work of the loads as they are applied to the beam (*a*) if the load **P** is applied first, (*b*) if the couple **M** is applied first.

SOLUTION

Appendix D, Cases 1 and 3,

$$y_{BP} = \frac{P(L/2)^3}{3EI} = \frac{PL^3}{24EI} \qquad \theta_{CP} = \frac{P(L/2)^2}{2EI} = \frac{PL^2}{8EI}$$

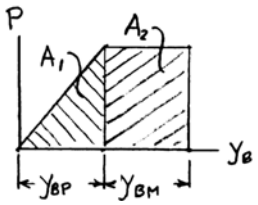
$$y_{BM} = \frac{M_0(L/2)^2}{2EI} = \frac{M_0L^2}{8EI} \qquad \theta_{BM} = \frac{M_0L}{EI}$$

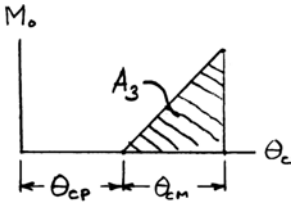
(*a*) First P, then M_0.

$$U = A_1 + A_2 + A_3$$

$$= \frac{1}{2}Py_{BP} + Py_{BM} + \frac{1}{2}M_0\theta_{CM}$$

$$U = \frac{P^2L^3}{48EI} + \frac{PM_0L^2}{8EI} + \frac{M_0^2L}{2EI} \quad \blacktriangleleft$$



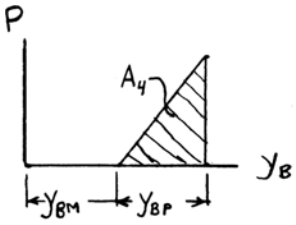


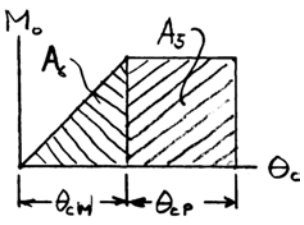
(*b*) First M_0, then P.

$$U = A_4 + A_5 + A_6$$

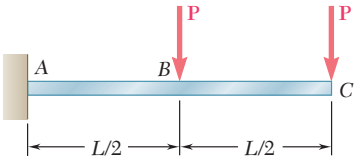
$$= \frac{1}{2}Py_{BP} + M_0\theta_{CP} + \frac{1}{2}M_0\theta_{CM}$$

$$U = \frac{P^2L^3}{48EI} + \frac{M_0PL^2}{8EI} + \frac{M_0^2L}{2EI} \quad \blacktriangleleft$$





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PROBLEM 11.79

For the beam and loading shown, (*a*) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (*b*) compute the strain energy of the beam by the method of Sec. 11.2A and show that it is equal to the work obtained in part *a*.

SOLUTION

(*a*) Label the forces P_B and P_C.

Using Appendix D, Case 1,

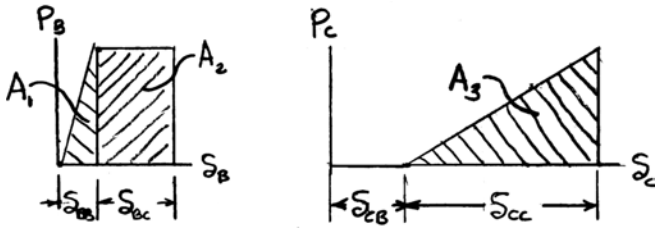
$$\delta_{BB} = \frac{P_B (L/2)^3}{3EI} = \frac{1}{24}\frac{P_B L^3}{EI}$$

$$\delta_{CB} = \delta_{BB} + \frac{L}{2}\theta_B = \frac{1}{24}\frac{P_B L^3}{3EI} + \frac{L}{2}\frac{P_B (L/2)^2}{2EI}$$

$$= \left(\frac{1}{24} + \frac{1}{16}\right)\frac{P_B L^3}{EI} = \frac{5}{48}\frac{P_B L^3}{EI}$$

$$\delta_{CC} = \frac{1}{3}\frac{P_C L^3}{EI}$$

$$\delta_{BC} = \frac{P_C}{6EI}(3Lx^2 - x^3) = \frac{P_C}{6EI}\left(3L\left(\frac{L}{2}\right)^2 - \left(\frac{L}{2}\right)^3\right) = \frac{5}{48}\frac{P_C L^3}{EI}$$



Apply P_B first, then P_C.

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2}P_B \delta_{BB} + P_B \delta_{BC} + \frac{1}{2}P_C \delta_{CC}$$

$$= \frac{1}{48}\frac{P_B L^3}{EI} + \frac{5}{48}\frac{P_B P_C L^3}{EI} + \frac{1}{6}\frac{P_C^2 L^3}{EI}$$

with $P_B = P_C = P$, $U = \left(\frac{1}{48} + \frac{5}{48} + \frac{1}{6}\right)\frac{P^2 L^3}{EI}$ $\qquad U = \frac{7P^2 L^3}{24EI}$ ◀

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PROBLEM 11.79 *(Continued)*

(*b*) Over *AB*: $M = Pv + P\left(v - \frac{L}{2}\right) = P\left(2v - \frac{L}{2}\right)$

$$U_{AB} = \int_{L/2}^{L} \frac{M^2}{2EI}\,dv = \frac{P^2}{2EI}\int_{L/2}^{2}\left(4v^2 - 2Lv + \frac{1}{4}L^2\right)dv$$

$$= \frac{P^2}{2EI}\left\{\frac{4}{3}\left[L^3 - \left(\frac{L}{2}\right)^3\right] - 2L\cdot\frac{1}{2}\left[L^2 - \left(\frac{L}{2}\right)^2\right] + \frac{1}{4}L^2\left[L - \frac{L}{2}\right]\right\}$$

$$= \frac{P^2}{2EI}\left\{\frac{7}{6}L^3 - \frac{3}{4}L^3 + \frac{1}{8}L^3\right\} = \frac{13}{48}\frac{P^2L^3}{EI}$$

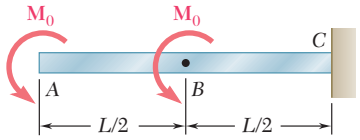
Over *BC*: $M = Pv$ $\quad U_{BC} = \int_0^{L/2}\frac{M^2}{2EI}\,dv = \frac{P^2}{2EI}\int_0^{L/2} v^2\,dv = \frac{P^2}{2EI}\cdot\frac{1}{3}\left(\frac{L}{2}\right)^3$

$$= \frac{P^2L^3}{48EI}$$

Total: $U = U_{AB} + U_{BC} = \left(\frac{13}{48} + \frac{1}{48}\right)\frac{P^2L^3}{EI}$ $\qquad U = \frac{7P^2L^3}{24EI}$ ◀

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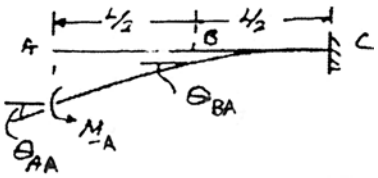
PROBLEM 11.80



For the beam and loading shown, (*a*) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (*b*) compute the strain energy of the beam by the method of Sec.11.2A and show that it is equal to the work obtained in part *a*.

SOLUTION

(*a*) Label the applied couples M_A and M_B. Apply M_A at point A first. Note that $M_B = 0$ during this phase.

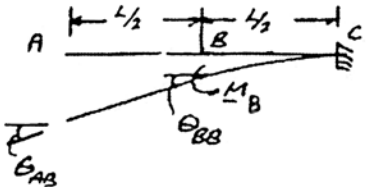


From Appendix D,

$$\theta_{AA} = \frac{M_A L}{EI}$$

$$U_1 = \frac{1}{2} M_B \theta_{AA} = \frac{M_A^2 L}{2EI}$$

Now apply M_B at point B. Note that M_A remains constant during this second phase.



From Appendix D,

$$\theta_{BB} = \frac{M_B (L/2)}{EI} = \frac{M_B L}{2EI}$$

Since the curvature of portion AB does not change as M_B is applied,

$$\theta_{AB} = \theta_{BB} = \frac{M_B L}{2EI}$$

$$U_2 = \frac{1}{2} M_B \theta_{BB} + M_A \theta_{AB}$$

$$= \frac{M_B^2 L}{4EI} + \frac{M_A M_B}{2EI}$$

Total strain energy:

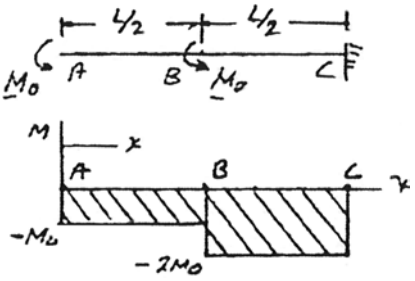
$$U = U_1 + U_2 = \frac{M_A^2 L}{2EI} + \frac{M_B^2 L}{4EI} + \frac{M_A M_B L}{2EI}$$

Set $M_A = M_B = M_0$.

$$U = \frac{5 M_0^2 L}{4EI} \blacktriangleleft$$

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PROBLEM 11.80 ***(Continued)***



(*b*) Bending moment diagram.

Over portion *AB*: $\left(0 < x < \frac{L}{2}\right)$

$$M = -M_0$$

$$U_{AB} = \int_0^{L/2} \frac{M_0^{\,2}}{2EI} dx = \frac{M_0^{\,2} L}{4EI}$$

Over portion *BC*: $\left(\frac{L}{2} < x < L\right)$

$$M = -2M_0$$

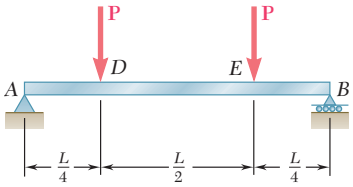
$$U_{BC} = \int_{L/2}^{L} \frac{(2M_0)^2}{2EI} dx = \frac{M_0^{\,2} L}{EI}$$

Total strain energy:

$$U = U_{AB} + U_{BC}$$

$$U = \frac{5M_0^{\,2} L}{4EI} \blacktriangleleft$$

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PROBLEM 11.81

For the beam and loading shown, (*a*) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (*b*) compute the strain energy of the beam by the method of Sec. 11.2A and show that it is equal to the work obtained in part *a*.

SOLUTION

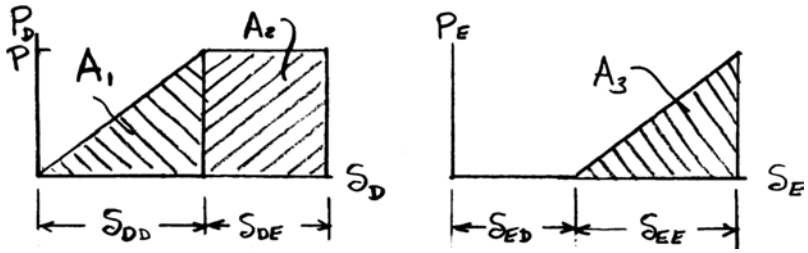
(*a*) Label the forces P_D and P_E.

Using Appendix D, Case 5,

$$\delta_{EE} = \frac{P_E a^2 b^2}{3EIL} = \frac{P_E \left(\frac{3L}{4}\right)^2 \left(\frac{L}{4}\right)^2}{3EIL} = \frac{3}{256}\frac{P_E L^3}{EI}$$

$$\delta_{DE} = \frac{P_E b}{6EIL}[(L^2 - b^2)x - x^3] = \frac{P_E \left(\frac{L}{4}\right)}{6EIL}\left[\left(L^2 - \left(\frac{L}{4}\right)^2\right)\left(\frac{L}{4}\right) - \left(\frac{L}{4}\right)^3\right] = \frac{7}{768}\frac{P_E L^3}{EI}$$

Likewise, $\delta_{DD} = \frac{3}{256}\frac{P_D L^3}{EI}$ and $\delta_{ED} = \frac{7}{768}\frac{P_D L^3}{EI}$



Let P_D be applied first.

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2}P_D\delta_{DD} + P_D\delta_{DE} + \frac{1}{2}P_E\delta_{EE}$$

$$= \frac{3}{512}\frac{P_D^2 L^3}{EI} + \frac{7}{768}\frac{P_D P_E L^3}{EI} + \frac{3}{512}\frac{P_E^2 L^3}{EI}$$

with $P_D = P_E = P$

$$U = \frac{1}{48}\frac{P^2 L^3}{EI}$$

$$U = \frac{P^2 L^3}{48EI} \blacktriangleleft$$

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PROBLEM 11.81 *(Continued)*

(*b*) Reactions: $R_A = R_B = P$

Over portion *AD*: $\left(0 < x < \frac{L}{4}\right)$: $M = Px$

$$U_{AD} = \int_0^{L/4} \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^{L/4} x^2 dx = \frac{P^2}{2EI} \cdot \frac{1}{3}\left(\frac{L}{4}\right)^3 = \frac{1}{384} \frac{P^2 L^3}{EI}$$

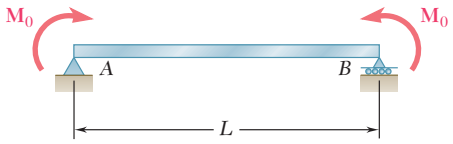
Over portion *DE*: $M = \frac{PL}{4}$ $\qquad U_{DE} = \frac{M^2 \left(\frac{L}{2}\right)^2}{2EI} + \frac{P^2 L^2}{2EI} \cdot \frac{1}{16} \cdot \frac{L}{2} = \frac{P^2 L^3}{64EI}$

Over portion *EB*: By symmetry, $U_{EB} = U_{AD} = \frac{1}{384} \frac{P^2 L^3}{EI}$

Total: $U = U_{AB} + U_{DE} + U_{EB} = \left(\frac{1}{384} + \frac{1}{64} + \frac{1}{384}\right) \frac{P^2 L^3}{EI}$ $\qquad U = \frac{1}{48} \frac{P^2 L^3}{EI}$ ◀

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PROBLEM 11.82



For the beam and loading shown, (*a*) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (*b*) compute the strain energy of the beam by the method of Sec. 11.2A and show that it is equal to the work obtained in part *a*.

SOLUTION

(*a*) Label the couples M_A and M_B.

Using Appendix D, Case 7.

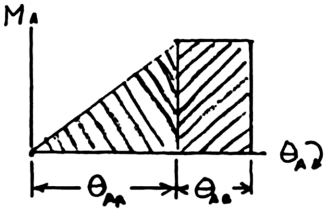
$$\theta_{AA} = \frac{M_A L}{3EI} \qquad \theta_{BA} = \frac{M_A L}{6EI} \qquad \theta_{BB} = \frac{M_B L}{3EI} \qquad \theta_{AB} = \frac{M_B L}{6EI}$$

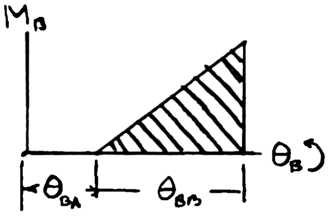
Apply M_A first, then M_B.

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2} M_A \theta_{AA} + M_A \theta_{AB} + \frac{1}{2} M_B \theta_{BB}$$

$$= \frac{1}{6} \frac{M_A^2 L}{EI} + \frac{1}{6} \frac{M_A M_B L}{EI} + \frac{1}{6} \frac{M_B^2 L}{EI}$$





With $M_A = M_B = M_0$

$$U = \frac{1}{2} \frac{M_0^2 L}{EI}$$

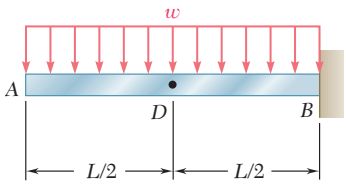
$$U = \frac{M_0^2 L}{2EI} \blacktriangleleft$$

(*b*) Bending moment: $M = M_0$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{M_0^2 L}{2EI}$$

$$U = \frac{M_0^2 L}{2EI} \blacktriangleleft$$

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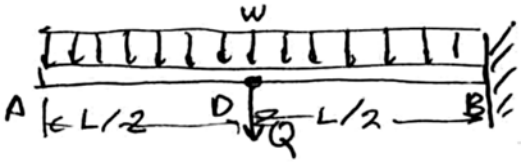


PROBLEM 11.83

For the prismatic beam shown, determine the deflection of point *D*.

SOLUTION

Add force Q at point D.



$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\delta_D = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int_0^L M\,\frac{\partial M}{\partial Q}\,dx$$

Over portion *AD*: $\left(0 < x < \frac{L}{2}\right)$ $\quad M = -\frac{1}{2}wx^2 \quad \frac{\partial M}{\partial Q} = 0$

Over portion *DB*: $\left(\frac{L}{2} < x < L\right)$ $\quad M = -\frac{1}{2}wx^2 - Q\left(x - \frac{L}{2}\right) \quad \frac{\partial M}{\partial Q} = -\left(x - \frac{L}{2}\right)$

Set $Q = 0$.

$$\delta_D = \frac{1}{EI}\int_0^{L/2}\left(-\frac{1}{2}wx^2\right)(0)\,dx + \frac{1}{EI}\int_{L/2}^{L}\left(-\frac{1}{2}wx^2\right)\left[-\left(x - \frac{L}{2}\right)\right]dx$$

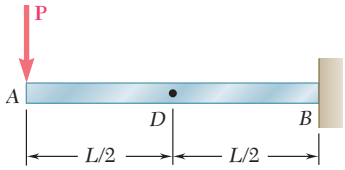
$$= \frac{w}{2EI}\int_{L/2}^{L}\left(x^3 - \frac{L}{2}x^2\right)dx$$

$$= \frac{w}{2EI}\left\{\frac{1}{4}L^4 - \frac{1}{4}\left(\frac{L}{2}\right)^4 - \left(\frac{L}{2}\right)\frac{1}{3}L^3 + \left(\frac{L}{2}\right)\frac{1}{3}\left(\frac{L}{2}\right)^3\right\}$$

$$= \frac{1}{2}\left(\frac{1}{4} - \frac{1}{64} - \frac{1}{6} + \frac{1}{48}\right)\frac{wL^4}{EI} = \frac{17}{384}\frac{wL^4}{EI}$$

$$\delta_D = 0.0443\frac{wL^4}{EI}\downarrow \blacktriangleleft$$

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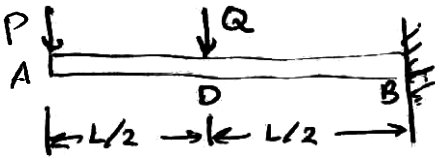


PROBLEM 11.84

For the prismatic beam shown, determine the deflection of point *D*.

SOLUTION

Add force *Q* at point *D*.



$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\delta_D = \frac{\partial U}{\partial Q} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q}\,dx = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial Q}\,dx$$

Over portion *AD*: $\left(0 < x < \frac{L}{2}\right)$ $\quad M = -Px, \quad \frac{\partial M}{\partial Q} = 0$

Over portion *DB*: $\left(\frac{L}{2} < x < L\right)$ $\quad M = -Px - Q\left(x - \frac{L}{2}\right), \quad \frac{\partial M}{\partial Q} = -\left(x - \frac{L}{2}\right)$

Set $Q = 0$.

$$\delta_D = \frac{1}{EI}\int_0^{L/2}(-Px)(0)\,dx + \frac{1}{EI}\int_{L/2}^{L}(-Px)\left[-\left(x - \frac{L}{2}\right)\right]dx$$

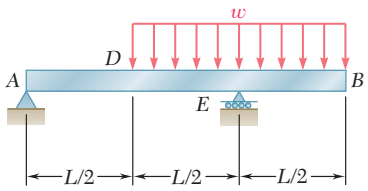
$$= \frac{P}{EI}\int_{L/2}^{L}\left(x^2 - \frac{L}{2}x\right)dx$$

$$= \frac{P}{EI}\left\{\frac{1}{3}L^3 - \frac{1}{3}\left(\frac{L}{2}\right)^3 - \left(\frac{L}{2}\right)\frac{1}{2}L^2 + \frac{L}{2}\frac{1}{2}\left(\frac{L}{2}\right)^2\right\}$$

$$= \left(\frac{1}{3} - \frac{1}{24} - \frac{1}{4} + \frac{1}{16}\right)\frac{PL^3}{EI}$$

$$\delta_D = \frac{5PL^3}{48EI}\downarrow \blacktriangleleft$$

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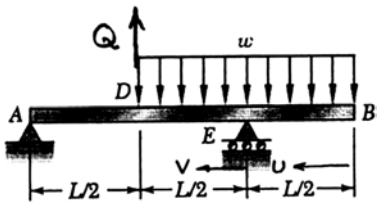


PROBLEM 11.85

For the prismatic beam shown, determine the deflection of point *D*.

SOLUTION

Add force Q at point D.



Reactions: $R_A = -\frac{1}{2}Q, \quad R_B = wL - \frac{1}{2}Q$

$$U = U_{AD} + U_{DE} + U_{EB} \qquad \delta_D = \frac{\partial U}{\partial Q}$$

Over portion AD, with $Q = 0$ $\quad M = 0 \quad \dfrac{\partial U_{AD}}{\partial Q} = 0$

Over portion DE,

$$M = R_B v - \frac{1}{2}w\left(v + \frac{L}{2}\right)^2$$

$$= wLv - \frac{1}{2}w\left(v + \frac{L}{2}\right)^2 - \frac{1}{2}Qv$$

$$\frac{\partial M}{\partial Q} = -\frac{1}{2}v \qquad U_{DE} = \frac{1}{2EI}\int_0^{\frac{L}{2}} M^2 dv \qquad \text{Set } Q = 0$$

$$\frac{\partial U_{DE}}{\partial Q} = \frac{1}{EI}\int_0^{\frac{L}{2}} M\frac{\partial M}{\partial Q}dv$$

$$= \frac{1}{EI}\int_0^{\frac{L}{2}}\left[wLv - \frac{1}{2}w\left(v + \frac{L}{2}\right)^2\right]\left(-\frac{1}{2}v\right)dv$$

$$= \frac{w}{2EI}\int_0^{\frac{L}{2}}\left[-Lv^2 + \frac{1}{2}\left(v^3 + Lv^2 + \frac{1}{4}L^2 v\right)\right]dv$$

$$= \frac{w}{2EI}\left[-L\cdot\frac{1}{3}\left(\frac{L}{2}\right)^3 + \frac{1}{2}\left(\frac{1}{4}\left(\frac{L}{2}\right)^4 + L\cdot\frac{1}{3}\left(\frac{L}{2}\right)^3 + \frac{1}{4}L^2\frac{1}{2}\left(\frac{L}{2}\right)^2\right)\right]$$

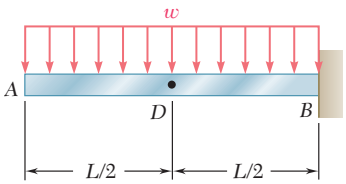
$$= \frac{1}{2}\left(-\frac{1}{24} + \frac{1}{128} + \frac{1}{48} + \frac{1}{64}\right)\frac{wL^4}{EI} = \frac{1}{768}\frac{wL^4}{EI}$$

Over portion EB, $\quad M = -\dfrac{1}{2}wu^2 \qquad \dfrac{\partial M}{\partial Q} = 0 \qquad \dfrac{\partial U_{EB}}{\partial Q} = 0$

$$\delta_D = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q} = 0 + \frac{1}{768}\frac{wL^4}{EI} + 0 = \frac{1}{768}\frac{wL^4}{EI} \uparrow$$

◀

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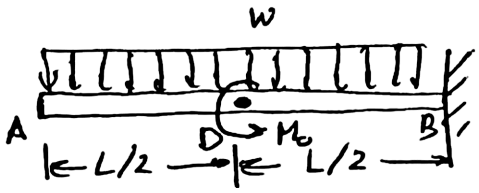


PROBLEM 11.86

For the prismatic beam shown, determine the slope at point *D*.

SOLUTION

Add couple M_0 at point *D*.



$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\theta_D = \frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_0}\,dx = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial M_0}\,dx$$

Over portion *AD*: $\left(0 < x < \frac{L}{2}\right)$ $\quad M = -\frac{1}{2}wx^2 \qquad \frac{\partial M}{\partial M_0} = 0$

Over portion *DB*: $\left(\frac{L}{2} < x < L\right)$ $\quad M = -\frac{1}{2}wx^2 - M_0 \qquad \frac{\partial M}{\partial M_0} = -1$

Set $M_0 = 0$.

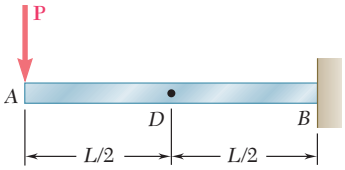
$$\theta_D = \frac{1}{EI}\int_{L/2}^{L}\left(-\frac{1}{2}wx^2\right)(0)\,dx + \frac{1}{EI}\int_{L/2}^{L}\left(-\frac{1}{2}wx^2\right)(-1)\,dx$$

$$= \frac{w}{2EI}\int_{L/2}^{L} x^2\,dx = \frac{w}{2EI}\left[\frac{1}{3}L^3 - \frac{1}{3}\left(\frac{L}{2}\right)^3\right]$$

$$= \frac{1}{6}\left(1 - \frac{1}{8}\right)\frac{wL^3}{EI}$$

$$\theta_D = \frac{7wL^3}{48EI} \curvearrowright \blacktriangleleft$$

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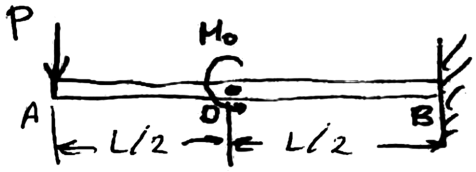


PROBLEM 11.87

For the prismatic beam shown, determine the slope at point *D*.

SOLUTION

Add couple M_0 at point *D*.



$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\theta_D = \frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_0}\,dx = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial M_0}\,dx$$

Over portion *AD*: $\left(0 < x < \frac{L}{2}\right)$ $\quad M = -Px \quad \frac{\partial M}{\partial M_0} = 0$

Over portion *DB*: $\left(\frac{L}{2} < x < L\right)$ $\quad M = -Px - M_0 \quad \frac{\partial M}{\partial M_0} = -1$

Set $M_0 = 0$.

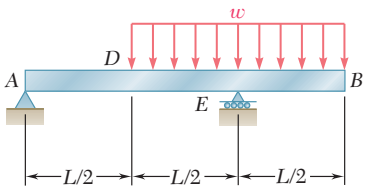
$$\theta_D = \frac{1}{EI}\int_{L/2}^L (-Px)(0)\,dx + \frac{1}{EI}\int_{L/2}^L (-Px)(-1)\,dx$$

$$= \frac{P}{EI}\int_{L/2}^L x\,dx = \frac{P}{EI}\left[\frac{1}{2}L^2 - \frac{1}{2}\left(\frac{L}{2}\right)^2\right]$$

$$= \left(\frac{1}{2} - \frac{1}{8}\right)\frac{PL^2}{EI}$$

$$\theta_D = \frac{3PL^2}{8EI} \blacktriangleleft$$

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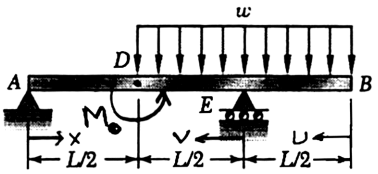


PROBLEM 11.88

For the prismatic beam shown, determine the slope at point D.

SOLUTION

Add couple M_0 at point D.



Reactions: $R_A = \frac{M_0}{L} \quad R_E = wL - \frac{M_0}{L}$

$$U = U_{AD} + U_{DE} + U_{EB}, \qquad \theta_D = \frac{\partial U}{\partial M_0}$$

Over portion AD, $M = \frac{M_0}{L}x = 0$ with $M_0 = 0 \quad \frac{\partial U_{AD}}{\partial M_0} = 0$

Over portion DE, $M = R_B v - \frac{1}{2}w\left(v + \frac{L}{2}\right)^2 = wLv - \frac{1}{2}w\left(v + \frac{L}{2}\right)^2 - \frac{M_0}{L}v$

$$\frac{\partial M}{\partial M_0} = -\frac{1}{L}v, \qquad U_{DE} = \frac{1}{2EI}\int_0^{\frac{L}{2}} M^2 dv \qquad \text{Set } M_0 = 0$$

$$\frac{\partial U_{DE}}{\partial M_0} = \frac{1}{EI}\int_0^{\frac{L}{2}} M\frac{\partial M}{\partial M_0}dv = \frac{1}{EI}\int_0^{\frac{L}{2}}\left[wLv - \frac{1}{2}w\left(v + \frac{L}{2}\right)^2\right]\left(-\frac{1}{L}v\right)dv$$

$$= \frac{w}{EIL}\int_0^{\frac{L}{2}}\left[-Lv^2 + \frac{1}{2}\left(v^3 + Lv^2 + \frac{1}{4}L^2 v\right)\right]dv$$

$$= \frac{w}{EIL}\left[-L\cdot\frac{1}{3}\left(\frac{L}{2}\right)^3 + \frac{1}{2}\left(\frac{1}{4}\left(\frac{L}{2}\right)^4 + L\cdot\frac{1}{3}\left(\frac{L}{2}\right)^3 + \frac{1}{4}L^2\frac{1}{2}\left(\frac{L}{2}\right)^2\right)\right]$$

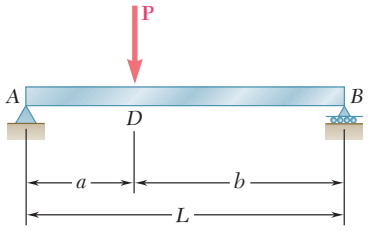
$$= \left(-\frac{1}{24} + \frac{1}{128} + \frac{1}{48} + \frac{1}{64}\right)\frac{wL^3}{EI} = \frac{1}{384}\frac{wL^3}{EI}$$

Over portion EB, $M = -\frac{1}{2}wu^2 \qquad \frac{\partial M}{\partial M_0} = 0 \qquad \frac{\partial U_{EB}}{\partial M_0} = 0$

$$\delta_D = \frac{\partial U_{AD}}{\partial M_0} + \frac{\partial U_{DE}}{\partial M_0} + \frac{\partial U_{EB}}{\partial M_0} = 0 + \frac{1}{384}\frac{wL^3}{EI} + 0 = \frac{1}{384}\frac{wL^3}{EI}$$

◀

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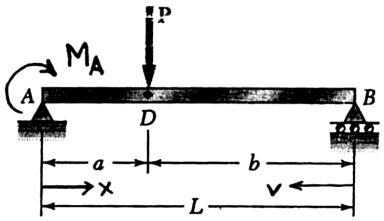


PROBLEM 11.89

For the prismatic beam shown, determine the slope at point *A*.

SOLUTION

Add couple M_A at point *A*.



Reactions: $R_A = \frac{Pb}{L} - \frac{M_A}{L}, \quad R_B = \frac{Pa}{L} + \frac{M_A}{L}$

$$U = \int_0^L \frac{M^2}{2EI}dx = \frac{1}{2EI}\int_0^a M^2 dx + \frac{1}{2EI}\int_0^b M^2 dv$$

$$\theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI}\int_0^a M\frac{\partial M}{\partial M_A}dx + \frac{1}{EI}\int_0^b M\frac{\partial M}{\partial M_A}dv$$

Over portion *AD* $(0 < x < a)$, $M = M_A + R_A x = M_A\left(1 - \frac{x}{L}\right) + \frac{Pbx}{L}, \quad \frac{\partial M}{\partial M_A} = 1 - \frac{x}{L}$

Over portion *DB* $(0 < v < b)$, $M = R_B v = \frac{Pav}{L} + \frac{M_A v}{L}, \quad \frac{\partial M}{\partial M_A} = \frac{v}{L}$

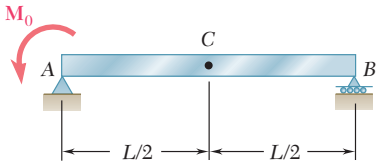
Set $M_A = 0$

$$\theta_A = \frac{1}{EI}\int_0^a \left(\frac{Pbx}{L}\right)\left(1 - \frac{x}{L}\right)dx + \frac{1}{EI}\int_0^b \frac{Pav}{L}\frac{v}{L}dv$$

$$= \frac{P}{EIL^2}\left(\frac{1}{2}bLa^2 - \frac{1}{3}ba^3 + \frac{1}{3}ab^3\right)$$

$$\theta_A = \frac{Pab}{6EIL^2}(3La - 2a^2 + 2b^2) \curvearrowright \blacktriangleleft$$

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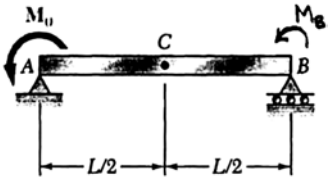


PROBLEM 11.90

For the prismatic beam shown, determine the slope at point *B*.

SOLUTION

Add couple M_B at point *B* as shown.



Reactions. $$R_A = \frac{1}{L}(M_0 + M_B) \uparrow$$

Strain energy. $$U = \int_0^L \frac{M^2}{2EI}\,dx$$

Slope at point *B*. $$\theta_B = \frac{\partial U}{\partial M_B}$$

$$M = R_A x - M_0 = (M_0 + M_B)\frac{x}{L} - M_0$$

$$\frac{\partial M}{\partial M_B} = \frac{X}{L} \qquad \text{With } M_B = 0 \quad M = M_0\left(\frac{x}{L} - 1\right)$$

$$\frac{\partial U}{\partial M_A} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_B}\,dx$$

$$= \frac{M_0}{EI}\int_0^L \left(\frac{x}{L} - 1\right)\frac{x}{L}\,dx$$

$$= \frac{M_0}{EIL^2}\int_0^L (x - L)x\,dx$$

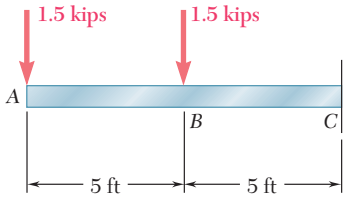
$$= \frac{M_0}{EIL^2}\int_0^L (x^2 - Lx)dx$$

$$= \frac{M_0}{EI}\left(\frac{x^3}{3} - \frac{Lx^2}{2}\right)\Bigg|_0^L = -\frac{M_0 L}{6EI}$$

$$\theta_B = -\frac{M_0 L}{6EI}$$

$$\theta_B = \frac{M_0 L}{6EI} \curvearrowright \blacktriangleleft$$

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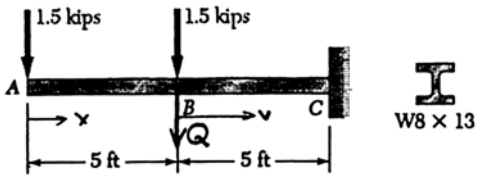


PROBLEM 11.91

For the beam and loading shown, determine the deflection of point *B*. Use $E = 29 \times 10^6$ psi.

SOLUTION

Add force Q at point B.



Units: forces in kips, lengths in ft

$$E = 29 \times 10^3 \text{ ksi}$$

$$I = 39.6 \text{ in}^4$$

$$EI = (29 \times 10^3)(39.6)$$

$$= 1.148 \times 10^6 \text{ kip} \cdot \text{in}^2$$

$$= 7975 \text{ kip} \cdot \text{ft}^2$$

$$U = \int_0^5 \frac{M^2}{2EI}\,dx + \int_0^5 \frac{M^2}{2EI}\,dv$$

$$\delta_B = \frac{\partial U}{\partial Q} = \frac{1}{EI}\left\{\int_0^5 M\frac{\partial M}{\partial Q}\,dx + \int_0^5 M\frac{\partial M}{\partial Q}\,dv\right\}$$

Over *AB*: $M = -1.5x, \quad \dfrac{\partial M}{\partial Q} = 0 \quad \int_0^5 M\dfrac{\partial M}{\partial Q}\,dx = 0$

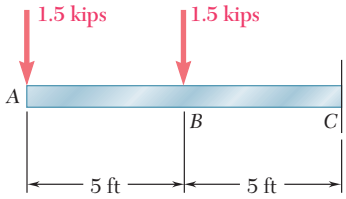
Over *BC*: $M = -1.5(v + 5) - 1.5v - Qv = -3v - 7.5 - Qv; \quad \dfrac{\partial M}{\partial Q} = -v$

$$\int_0^5 \frac{\partial M}{\partial Q}\,dv = \int_0^5 (3v^2 + 7.5v)\,dv = (3)\left(\frac{1}{3}\right)(5)^3 + (7.5)\left(\frac{1}{2}\right)(5)^2 = 218.75$$

$$\delta_B = \frac{1}{EI}\{0 + 143.75\} = \frac{218.75}{7975} = 27.43 \times 10^{-3} \text{ ft}$$

$$\delta_B = 0.329 \text{ in.} \downarrow \blacktriangleleft$$

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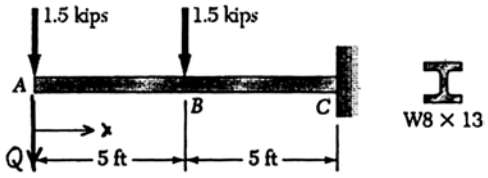


PROBLEM 11.92

For the beam and loading shown, determine the deflection of point A. Use $E = 29 \times 10^6$ psi.

SOLUTION

Add force Q at point A.



Units: forces in kips, length in ft

$$E = 29 \times 10^3 \text{ ksi}, \quad I = 39.6 \text{ in}^4$$

$$EI = (29 \times 10^3)(39.6)$$

$$= 1.148 \times 10^6 \text{ kip} \cdot \text{in}^2$$

$$= 7975 \text{ kip} \cdot \text{ft}^2$$

$$U = \int_0^{10} \frac{M^2}{2EI} dx$$

$$\delta_A = \frac{\partial U}{\partial Q} = \frac{1}{EI} \int_0^{10} M \frac{\partial M}{\partial Q} dx$$

Over portion AB: $0 < x < 5, \quad M = -1.5x - Qx \qquad \dfrac{\partial M}{\partial Q} = -x$

$$\int_0^5 M \frac{\partial M}{\partial Q} dx = \int_0^5 (1.5x)(x)\, dx = 1.5 \int_0^5 x^2 dx = (1.5)\left(\frac{1}{3}\right)(5)^3 = 62.5$$

Over portion BC: $5 < x < 10 \quad M = -1.5x - 1.5(x - 5) - Qx$

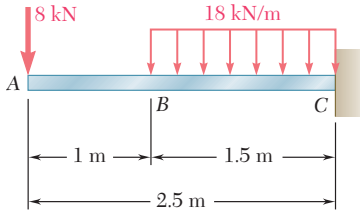
$$M = -3x + 7.5 - Qx \qquad \frac{\partial M}{\partial Q} = -x$$

$$\int_0^{10} M \frac{\partial M}{\partial Q} dx = \int_5^{10} (3x^2 - 7.5x)\, dx = (3)\left(\frac{1}{3}\right)(10^3 - 5^3) - (7.5)\left(\frac{1}{2}\right)(10^2 - 5^2)$$

$$= 593.75$$

$$\delta_A = \frac{1}{EI}\{62.5 + 593.75\} = \frac{656.29}{7975} = 82.29 \times 10^{-3} \text{ ft} \qquad \delta_A = 0.987 \text{ in.} \downarrow \blacktriangleleft$$

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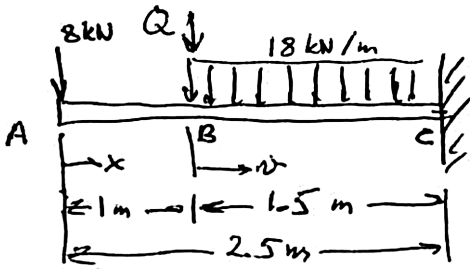


PROBLEM 11.93

For the beam and loading shown, determine the deflection at point *B*. Use $E = 200$ GPa.

SOLUTION

Add force Q at point B.



Units: Forces in kN; lengths in m.

Over *AB*: $M = -8x \qquad \dfrac{\partial M}{\partial Q} = 0$

Over *BC*: $M = -8(v+1) - \dfrac{1}{2}(18)v^2 - Qv \qquad \dfrac{\partial M}{\partial Q} = -v$

$$E = 200\times10^9\ \text{Pa}, \quad I = 28.7\times10^6\ \text{mm}^4 = 28.7\times10^{-6}\ \text{m}^4$$

$$EI = (200\times10^9)(28.7\times10^{-6}) = 5.74\times10^6\ \text{N}\cdot\text{m}^2 = 5740\ \text{kN}\cdot\text{m}^2$$

$$U = \int_0^1 \frac{M^2}{2EI}dx + \int_0^{1.5}\frac{M^2}{2EI}dv$$

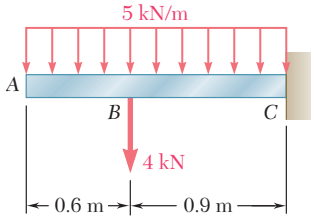
$$\delta_B = \frac{\partial U}{\partial Q} = \frac{1}{EI}\left\{\int_0^1 M\frac{\partial M}{\partial Q}dx + \int_0^{1.5} M\frac{\partial M}{\partial Q}dv\right\}$$

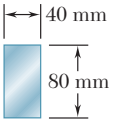
$$= \frac{1}{EI}\left\{0 + \int_0^{1.5}\left[-8(v+1) - \frac{1}{2}(18)v^2\right](-v)dv\right\} = \frac{1}{EI}\int_0^{1.5}(9v^3 + 8v^2 + 8v)dv$$

$$= \frac{1}{EI}\left\{\frac{9}{4}(1.5)^4 + \frac{8}{3}(1.5)^3 + \frac{8}{2}(1.5)^2\right\} = \frac{29.391}{EI} = \frac{29.391}{5740} = 5.12\times10^{-3}\ \text{m}$$

$$\delta_B = 5.12\ \text{mm}\downarrow \blacktriangleleft$$

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PROBLEM 11.94

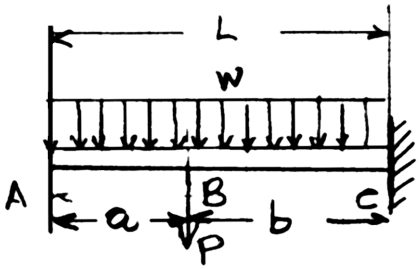
For the beam and loading shown, determine the deflection at point *B*. Use $E = 200$ GPa.

SOLUTION

$$U = \int_0^a \frac{M^2}{2EI}dx + \int_a^L \frac{M^2}{2EI}dx$$

$$\delta_B = \frac{\partial U}{\partial P}$$

$$= \int_0^a \frac{M}{EI}\frac{\partial M}{\partial P}dx + \int_a^L \frac{M}{EI}\frac{\partial M}{\partial P}dx$$



Portion *AB*: $(0 \le x \le a)$

$$M = -\frac{1}{2}wx^2 \qquad \frac{\partial M}{\partial P} = 0$$

$$\int_0^a \frac{M}{EI}\frac{\partial M}{\partial P}dx = 0$$

Portion *BC*: $(a < x \le L)$

$$M = -\frac{1}{2}wx^2 - P(x-a)$$

$$\frac{\partial M}{\partial P} = -(x-a)$$

$$\int_a^L \frac{M}{EI}\frac{\partial M}{\partial P}dx = \frac{w}{2EI}\int_a^L x^2(x-a)dx + \frac{P}{EI}\int_a^L (x-a)^2 dx$$

$$= \frac{w}{2EI}\int_a^L (x^3 - ax^2)dx + \frac{P}{EI}\int_0^b v^2 dv$$

$$= \frac{w}{2EI}\left(\frac{L^4}{4} - \frac{aL^3}{3} - \frac{a^4}{4} + \frac{a^4}{3}\right) + \frac{Pb^3}{3EI}$$

$$\delta_B = 0 + \frac{w}{2EI}\left(\frac{L^4}{4} - \frac{aL^3}{3} + \frac{a^4}{12}\right) + \frac{Pb^3}{3EI}$$

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PROBLEM 11.94 *(Continued)*

Data:

$$a = 0.6 \text{ m}, \quad b = 0.9 \text{ m}, \quad L = a + b = 1.5 \text{ m}$$

$$w = 5 \times 10^3 \text{ N/m}$$

$$P = 4 \times 10^3 \text{ N}$$

$$I = \frac{1}{12}(40)(80)^3 = 1.70667 \times 10^6 \text{ mm}^4$$

$$= 1.70667 \times 10^{-6} \text{ m}^4$$

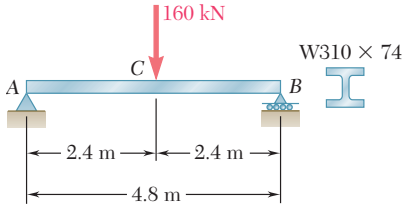
$$EI = (200 \times 10^9)(1.70667 \times 10^{-6}) = 341{,}333 \text{ N} \cdot \text{m}^2$$

$$\delta_B = 0 + \frac{5 \times 10^3}{(2)(341{,}333)}\left[\frac{(1.5)^4}{4} - \frac{(0.6)(1.5)^3}{3} + \frac{(0.6)^3}{12}\right] + \frac{(4 \times 10^3)(0.9)^3}{(3)(341{,}333)}$$

$$= 7.25 \times 10^{-3} \text{ m}$$

$$\delta_B = 7.25 \text{ mm} \downarrow \blacktriangleleft$$

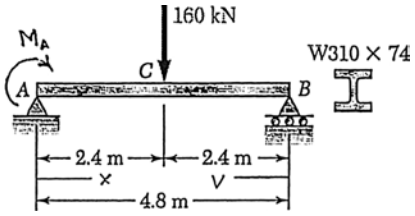
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PROBLEM 11.95

For the beam and loading shown, determine the slope at end *A*. Use $E = 200$ GPa.

SOLUTION



Add couple M_A at point *A*.

Units: Forces in kN; lengths in m.

$$E = 200 \times 10^9 \text{ Pa}, \quad I = 163 \times 10^6 \text{ mm}^4 = 163 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(163 \times 10^{-6}) = 32.6 \times 10^6 \text{ N} \cdot \text{m}^2 = 32{,}600 \text{ kN} \cdot \text{m}^2$$

Reactions: $R_A = 80 - \dfrac{M_A}{4.8}$ $\quad R_B = 80 + \dfrac{M_A}{4.8}$

$$U = U_{AB} + U_{BC} = \int_0^{2.4} \frac{M^2}{2EI}dx + \int_0^{2.4} \frac{M^2}{2EI}dv \qquad \theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AB}}{\partial M_A} + \frac{\partial U_{BC}}{\partial M_A}$$

Over *AB*: $M = M_A + R_A x = M_A + 80x - \dfrac{M_A}{4.8}x \quad \dfrac{\partial M}{\partial M_A} = \left(1 - \dfrac{x}{4.8}\right)$

Set $M_A = 0$. $$\frac{\partial U_{AB}}{\partial M_A} = \frac{1}{EI}\int_0^{2.4}(80x)\left(1-\frac{x}{4.8}\right)dx = \frac{1}{EI}\int_0^{2.4}(80x - 16.6667x^2)dx$$

$$= \frac{1}{EI}\left\{(80)\left(\frac{1}{2}\right)(2.4)^2 - (16.6667)\left(\frac{1}{3}\right)(2.4)^3\right\} = \frac{153.6}{EI}$$

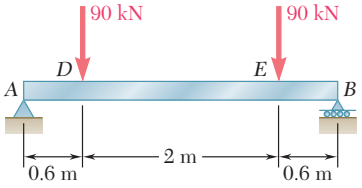
Over *BC*: $M = R_B v = 80v + \dfrac{M_A}{4.8}v, \quad \dfrac{\partial M}{\partial M_A} = \dfrac{1}{4.8}v$

Set $M_A = 0$. $$\frac{\partial U_{BC}}{\partial M_A} = \frac{1}{EI}\int_0^{2.4}(80v)\left(\frac{1}{4.8}v\right)dv = \frac{16.6667}{EI}\int_0^{2.4}v^2 dv$$

$$= \frac{(16.6667)(2.4)^3}{3EI} = \frac{76.8}{EI}$$

$$\theta_A = \frac{1}{EI}\{153.6 + 76.8\} = \frac{230.4}{32{,}600} \qquad \theta_A = 7.07 \times 10^{-3} \text{rad} \curvearrowright \blacktriangleleft$$

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PROBLEM 11.96

For the beam and loading shown, determine the deflection at point *D*. Use $E = 200$ GPa.

SOLUTION

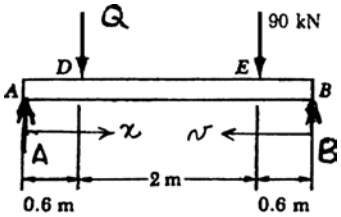
Units: Forces in kN, lengths in m.

$$E = 200\times10^{9}\,\text{Pa} = 200\times10^{6}\,\text{kN/m}^2$$

$$I = 51.2\times10^{6}\,\text{mm}^4 = 51.2\times10^{-6}\,\text{m}^4$$

$$EI = (200\times10^{6})(51.2\times10^{-6}) = 10{,}240\ \text{kN}\cdot\text{m}^2$$

Let Q be the force applied at D. It will be set equal to 90 kN later.



Reactions:

$$+\circlearrowleft \Sigma M_B = 0: \quad -3.2A + 2.6Q + (0.6)(90) = 0$$

$$A = 16.875 + 0.8125Q \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 3.2B - 0.6Q - (2.6)(90) = 0$$

$$B = 73.125 + 0.1875Q \uparrow$$

Strain energy:

$$U = \frac{1}{2EI}\int_0^{3.2} M^2\,dx$$

Deflection at point D: (formula)

$$\delta_D = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int_0^{3.2} M\frac{\partial M}{\partial Q}\,dx$$

Over portion AD: $(0 \le x \le 0.6\ \text{m})$

$$M = (16.875 + 0.8125Q)x$$

$$\frac{\partial M}{\partial Q} = 0.8125x; \qquad \text{Set } Q = 90\ \text{kN}.$$

$$M = 90\,x$$

$$\int_0^{0.6} M\frac{\partial M}{\partial Q} = \int_0^{0.6} (90x)(0.8125x) = 73.125\frac{x^3}{3}\bigg|_0^{0.6} = 5.265\ \text{kN}\cdot\text{m}^3$$

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PROBLEM 11.96 *(Continued)*

Over portion *DE*: $(0.6\text{ m} \le x \le 2.6\text{ m})$

$$M = (16.875 + 0.8125Q)x - Q(x - 0.6)$$
$$= 16.875x - 0.1875Qx + 0.6Q$$
$$\frac{\partial M}{\partial Q} = -0.1875x + 0.6; \qquad \text{Set } Q = 90\text{ kN}.$$
$$M = (0.6)(90) = 54\text{ kN}\cdot\text{m}$$
$$\int_{0.6}^{2.6} M\frac{\partial M}{\partial Q}dx = \int_{0.6}^{2.6}(54)(-0.1875x + 0.6)dx$$
$$= -10.125\frac{x^2}{2}\bigg|_{0.6}^{2.6} + 32.4x\big|_{0.6}^{2.6} = 32.4\text{ kN}\cdot\text{m}^3$$

Over portion *ED*: $(2.6\text{ m} \le x \le 3.2\text{ m};\quad 0 \le v \le 0.6\text{ m})$

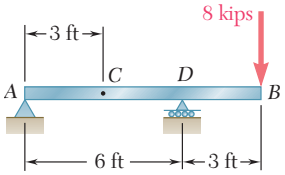
$$M = Bv = (73.125 + 0.1875Q)v$$
$$\frac{\partial M}{\partial Q} = 0.1875v \quad \text{Set } Q = 90\text{ kN}.$$
$$M = 90v$$
$$\int_{2.6}^{3.2} M\frac{\partial M}{\partial Q}dx = \int_{0}^{0.6} M\frac{\partial M}{\partial Q}dv = \int_{0}^{0.6}(90v)(0.1875v)dv$$
$$= 16.875\frac{v^3}{3}\bigg|_{0}^{0.6} = 1.215\text{ kN}\cdot\text{m}^3$$

Deflection at point *D*: (calculated)

$$\delta_D = \frac{5.265 + 32.4 + 1.215}{EI} = \frac{38.88}{10{,}240} = 3.797\times10^{-3}\text{ m}$$

$$\delta_D = 3.80\text{ mm}\downarrow \blacktriangleleft$$

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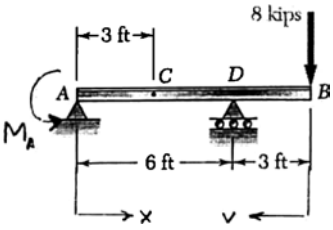




PROBLEM 11.97

For the beam and loading shown, determine the slope at end *A*. Use $E = 29\times10^6$ psi.

SOLUTION



Units: Forces in kips; lengths in ft.

$$E = 29\times10^3 \text{ ksi} \qquad I = 57.5 \text{ in}^4$$

$$EI = (29\times10^3)(57.5) = 1.6675\times10^6 \text{ kip}\cdot\text{in}^2 = 11{,}580 \text{ kip}\cdot\text{ft}^2$$

Add couple M_A at end *A*.

Reactions: $$R_A = -4 + \frac{M_A}{6}, \quad R_B = 12 - \frac{M_A}{6}$$

$$U = U_{AD} + U_{DB} \qquad \left(\theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AD}}{\partial M_A} + \frac{\partial U_{DB}}{\partial M_A}\right)$$

Over *AD*: $(0 < x < 6)$ $$M = -M_A + R_A x = -M_A - 4x + \frac{M_A}{6}x$$

$$\frac{\partial M}{\partial M_A} = -\left(1 - \frac{x}{6}\right) \qquad \text{Set } M_A = 0.$$

$$\frac{\partial U_{AD}}{\partial M_A} = \frac{1}{EI}\int_0^6 (4x)\left(1-\frac{x}{6}\right)dx = \frac{1}{EI}\int_0^6 \left(4x - \frac{2}{3}x^2\right)dx = \frac{1}{EI}\left\{(4)\frac{6^2}{2} - \frac{2}{3}\frac{6^3}{3}\right\}$$

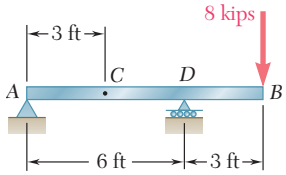
$$= \frac{24}{EI}$$

Over *DB*: $(0 < u < 3)$ $$M = -8u \qquad \frac{\partial M}{\partial M_A} = 0 \qquad \frac{\partial U_{DB}}{\partial M_A} = 0$$

$$\theta_A = \frac{24}{EI} + 0 = \frac{24}{11{,}580} = 0$$

$$\theta_A = 2.07\times10^{-3} \text{ rad} \blacktriangleleft$$

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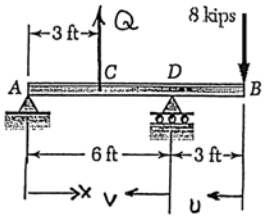


S8 × 18.4

PROBLEM 11.98

For the beam and loading shown, determine the deflection at point *C*. Use $E = 29\times10^6$ psi.

SOLUTION



S8 × 18.4

Units: Forces in kips; lengths in ft.

$E = 29\times10^3$ ksi $\quad I = 57.5$ in^4

$EI = (29\times10^3)(57.5) = 1.6675\times10^6$ kip·in^2 $= 11{,}580$ kip·ft^2

Add force *Q* at point *C*.

Reactions: $R_A = 4 + \frac{1}{2}Q\downarrow, \quad R_D = 12 - \frac{1}{2}Q\uparrow$

$$U = U_{AC} + U_{CD} + U_{DB} \qquad \delta_C = \frac{\partial U}{\partial Q} = \frac{\partial U_{AC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q} + \frac{\partial U_{DB}}{\partial Q}$$

Over *AC*: $(0 < x < 3)$ $\quad M = -\left(4 + \frac{1}{2}Q\right)x \qquad \frac{\partial M}{\partial Q} = -\frac{1}{2}x \qquad$ Set $Q = 0$.

$$\frac{\partial U_{AC}}{\partial Q} = \frac{1}{EI}\int_0^3 (4x)\left(\frac{1}{2}x\right)dx = \frac{2}{EI}\int_0^3 x^2 dx = \frac{(2)(3)^3}{3EI} = \frac{18}{EI}$$

Over *CD*: $(0 < v < 3)$ $\quad M = R_B v - 8(v+3) = 12v - \frac{1}{2}Q - 8v - 24 = 4v - 24 - \frac{1}{2}Qv$

$$\frac{\partial M}{\partial Q} = -\frac{1}{2}v \qquad \text{Set } Q = 0.$$

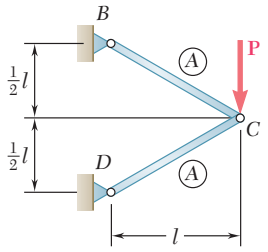
$$\frac{\partial U_{CD}}{\partial Q} = \frac{1}{EI}\int_0^3 (24 - 4v)\left(\frac{1}{2}v\right)dv = \frac{1}{EI}\int_0^3 (12v - 2v^2)dv = \frac{1}{EI}\left\{(12)\frac{(3)^2}{2} - (2)\frac{(3)^3}{3}\right\}$$

$$= \frac{36}{EI}$$

Over *DB*: $(0 < u < 3)$ $\quad M = -8u \qquad \frac{\partial M}{\partial Q} = 0 \qquad \frac{\partial U_{DB}}{\partial Q} = 0$

$$\delta_C = \frac{18}{EI} + \frac{36}{EI} + 0 = \frac{54}{11{,}580} = 4.663\times10^{-3}\text{ ft} \qquad \delta_C = 0.0560\text{ in.}\uparrow \blacktriangleleft$$

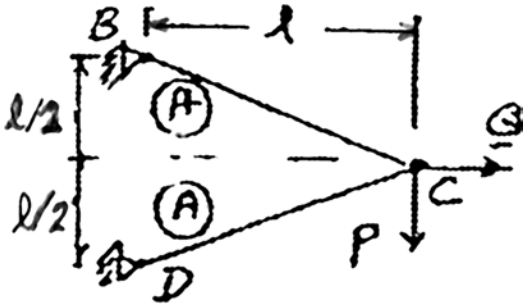
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PROBLEM 11.99

For the truss and loading shown, determine the horizontal and vertical deflection of joint *C*.

SOLUTION



Add horizontal force Q at point C.

From geometry, $L_{BC} = L_{CD} = \dfrac{\sqrt{5}}{2}\ell$

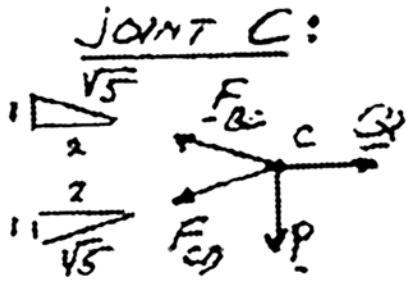
Equilibrium of joint C.

$$\pm \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}}(F_{BC} + F_{CD}) - Q = 0$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}(F_{BC} - F_{CD}) - P = 0$$

Solving simultaneously,

$$F_{BC} = \frac{\sqrt{5}}{2}P + \frac{\sqrt{5}}{4}Q \qquad F_{CD} = -\frac{\sqrt{5}}{2}P + \frac{\sqrt{5}}{4}Q$$



Strain energy: $U = \Sigma \dfrac{F_i^2 L_i}{2EA_i}$

Deflections. Horizontal: $x_C = \dfrac{\partial U}{\partial Q} = \dfrac{1}{E}\Sigma \dfrac{F_i L_i}{A_i}\dfrac{\partial F_i}{\partial Q}$

Vertical: $y_C = \dfrac{\partial U}{\partial P} = \dfrac{1}{E}\Sigma \dfrac{F_i L_i}{A_i}\dfrac{\partial F_i}{\partial P}$

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PROBLEM 11.99 *(Continued)*

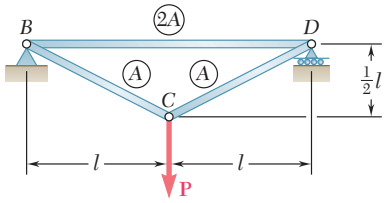
In the table, Q is set equal to zero in the last two columns.

	F_i	L_i	A_i	$\partial F_i / \partial P$	$\partial F_i / \partial Q$	$\frac{F_i L_i}{A}\frac{\partial F_i}{\partial P}$	$\frac{F_i L_i}{A}\frac{\partial F_i}{\partial Q}$
BC	$\frac{\sqrt{5}}{2}P + \frac{\sqrt{5}}{4}Q$	$\frac{\sqrt{5}}{2}\ell$	A	$\frac{\sqrt{5}}{2}$	$\frac{\sqrt{5}}{4}$	$\frac{5}{8}\sqrt{5}\frac{P\ell}{A}$	$\frac{5}{16}\sqrt{5}\frac{P\ell}{A}$
CD	$-\frac{\sqrt{5}}{2}P + \frac{\sqrt{5}}{4}Q$	$\frac{\sqrt{5}}{2}\ell$	A	$-\frac{\sqrt{5}}{2}$	$\frac{\sqrt{5}}{4}$	$\frac{5}{8}\sqrt{5}\frac{P\ell}{A}$	$-\frac{5}{16}\sqrt{5}\frac{P\ell}{A}$
					Σ	$\frac{5}{4}\sqrt{5}\frac{P\ell}{A}$	0

$$x_C = \frac{1}{E}\Sigma\frac{F_i L_i}{A}\frac{\partial F_i}{\partial Q} = 0 \qquad x_C = 0 \blacktriangleleft$$

$$y_C = \frac{1}{E}\Sigma\frac{F_i L_i}{A_i}\frac{\partial F_i}{\partial P} = \frac{5}{4}\sqrt{5}\frac{P\ell}{EA} \qquad y_C = 2.80\frac{P\ell}{EA}\downarrow \blacktriangleleft$$

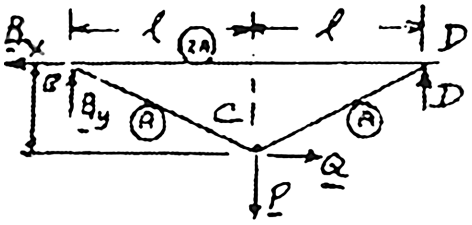
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PROBLEM 11.100

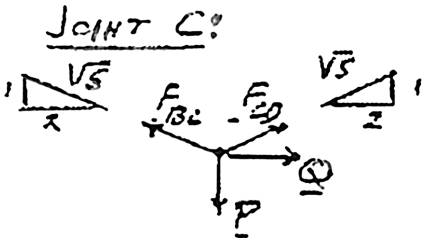
For the truss and loading shown, determine the horizontal and vertical deflection of joint *C*.

SOLUTION



Add horizontal force *Q* at point *C*.

From geometry, $L_{BC} = L_{CD} = \frac{\sqrt{5}}{2} l$



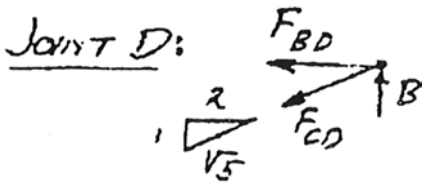
Equilibrium of joint *C*.

$$\xrightarrow{+} \sum F_x = 0: \quad -\frac{2}{\sqrt{5}} F_{BC} + \frac{2}{\sqrt{5}} F_{CD} + Q = 0$$

$$+\uparrow \sum F_y = 0: \quad \frac{1}{\sqrt{5}} F_{BC} + \frac{1}{\sqrt{5}} F_{CD} - P = 0$$

Solving simultaneously,

$$F_{BC} = \frac{\sqrt{5}}{2} P + \frac{\sqrt{5}}{4} Q \qquad F_{CD} = \frac{\sqrt{5}}{2} P - \frac{\sqrt{5}}{4} Q$$



Equilibrium of joint *D*.

$$\xleftarrow{+} \sum F_x = 0: \quad F_{BD} + \frac{2}{\sqrt{5}} + \left(\frac{\sqrt{5}}{2} P - \frac{\sqrt{5}}{4} Q \right) = 0$$

$$F_{BD} = -P + \frac{Q}{2}$$

Strain energy:

$$U = \sum \frac{F_i^2 L_i}{2EA_i}$$

Deflections. Horizontal:

$$x_C = \frac{\partial U}{\partial Q} = \frac{1}{E} \sum \frac{F_i L_i}{A_i} \frac{\partial F_i}{\partial Q}$$

Vertical:

$$y_C = \frac{\partial U}{\partial P} = \frac{1}{E} \sum \frac{F_i L_i}{A_i} \frac{\partial F_i}{\partial P}$$

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PROBLEM 11.100 *(Continued)*

In the table, Q is set equal to zero in the last two columns.

	F_i	L_i	A_i	$\partial F_i/\partial P$	$\partial F_i/\partial Q$	$\frac{F_iL_i}{A_i}\frac{\partial F_i}{\partial P}$	$\frac{F_iL_i}{A_i}\frac{\partial F_i}{\partial Q}$
BC	$\frac{\sqrt{5}}{2}P+\frac{\sqrt{5}}{4}Q$	$\frac{\sqrt{5}}{2}l$	A	$\frac{\sqrt{5}}{2}$	$\frac{\sqrt{5}}{4}$	$\frac{5}{8}\sqrt{5}\frac{Pl}{A}$	$\frac{5}{16}\sqrt{5}\frac{Pl}{A}$
CD	$\frac{\sqrt{5}}{2}P-\frac{\sqrt{5}}{4}Q$	$\frac{\sqrt{5}}{2}l$	A	$\frac{\sqrt{5}}{2}$	$-\frac{\sqrt{5}}{4}$	$\frac{5}{8}\sqrt{5}\frac{Pl}{A}$	$\frac{-5}{16}\sqrt{5}\frac{Pl}{A}$
BD	$-P+\frac{1}{2}Q$	$2l$	$2A$	-1	$+\frac{1}{2}$	$\frac{Pl}{A}$	$-\frac{1}{2}\frac{Pl}{A}$
					$\sum\left(\frac{5}{4}\sqrt{5}+1\right)\frac{Pl}{A}$		$-\frac{1}{2}\frac{Pl}{A}$

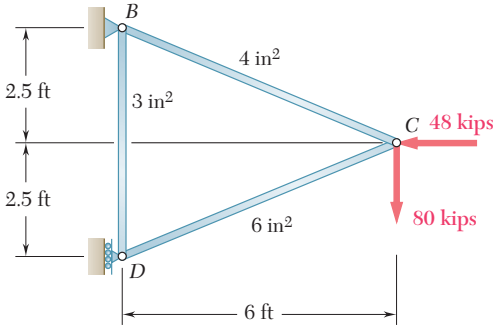
$$x_C=\frac{1}{E}\sum\frac{F_iL_i}{A_i}\frac{\partial F_i}{\partial Q}=-\frac{Pl}{2EA}$$

$$x_C=\frac{Pl}{2EA}\leftarrow\blacktriangleleft$$

$$y_C=\frac{1}{E}\sum\frac{F_iL_i}{A_i}\frac{\partial F_i}{\partial P}=\left(\frac{5}{4}\sqrt{5}+1\right)\frac{Pl}{EA}$$

$$y_C=3.80\frac{Pl}{EA}\downarrow\blacktriangleleft$$

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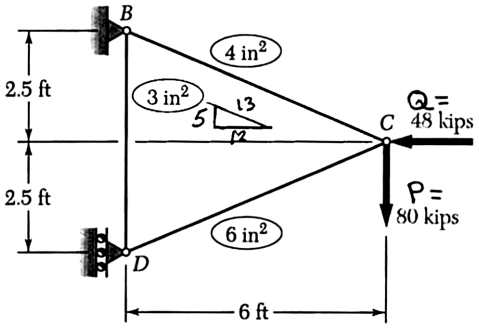


PROBLEM 11.101

Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 29 \times 10^6$ psi, determine the deflection indicated.

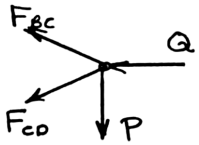
Vertical deflection of joint C.

SOLUTION



Joint C:

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0: \quad -\frac{12}{13}F_{BC} - \frac{12}{13} \qquad F_{CD} - Q = 0$$



$$F_{BC} + F_{CD} = -\frac{13}{12}Q \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{13}F_{BC} - \frac{5}{13}F_{CD} - P = 0$$

$$F_{BC} - F_{CD} = \frac{13}{5}\mathrm{P} \tag{2}$$

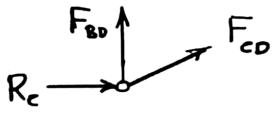
Solving (1) and (2) simultaneously,

$$F_{BC} = \frac{13}{10}P - \frac{13}{24}Q$$

$$F_{CD} = -\frac{13}{10}P - \frac{13}{24}Q$$

Joint D:

$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{13}F_{CD} + F_{BD} = 0$$



$$F_{BD} = -\frac{5}{13}F_{CD} = \frac{1}{2}P + \frac{5}{24}Q$$

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PROBLEM 11.101 *(Continued)*

Lengths of members: $L_{BC} = 78$ in.

$L_{CD} = 78$ in.

$L_{BC} = 60$ in.

$$U = \Sigma \frac{F^2 L}{2EA}$$

$$\delta_P = \frac{\partial U}{\partial P} = \Sigma \frac{FL}{EA}\frac{\partial F}{\partial P} = \frac{1}{E}\Sigma \frac{FL}{A}\frac{\partial F}{\partial P}$$

Member	F	L (in.)	$\frac{\partial F}{\partial P}$	A (in^2)	$\frac{FL}{A}\frac{\partial F}{\partial P}$
BC	$\frac{13}{10}P - \frac{13}{24}Q$	78	$\frac{13}{10}$	4	$32.955P - 13.73125Q$
CD	$-\frac{13}{10}P - \frac{13}{24}Q$	78	$-\frac{13}{10}$	6	$21.97P + 9.15417Q$
BD	$\frac{1}{2}P + \frac{5}{24}Q$	60	$\frac{1}{2}$	3	$5.00P + 2.08333Q$
Σ					$59.975P - 2.49375Q$

Further data: $E = 29 \times 10^6$ psi

$= 29{,}000$ ksi

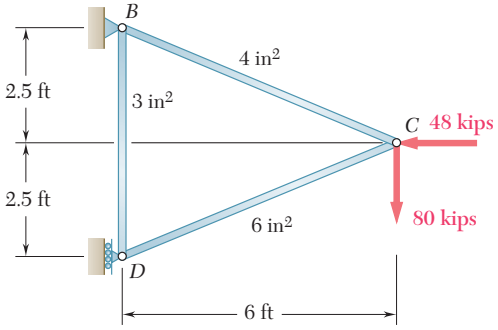
$P = 80$ kips

$Q = 48$ kips

$$\delta_P = \frac{(59.975)(80) + (-2.49375)(48)}{29{,}000} = 0.1613 \text{ in.} \downarrow$$

◀

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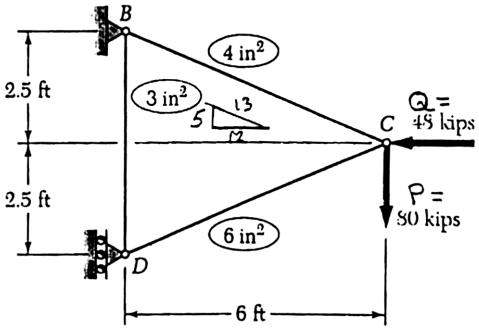


PROBLEM 11.102

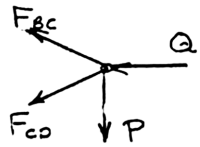
Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 29 \times 10^6$ psi, determine the deflection indicated.

Horizontal deflection of joint C.

SOLUTION



Joint C:



$$\pm\Sigma F_x = 0: \quad -\frac{12}{13}F_{BC} - \frac{12}{13} \qquad F_{CD} - Q = 0$$

$$F_{BC} + F_{CD} = -\frac{13}{12}Q \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{13}F_{BC} - \frac{5}{13}F_{CD} - P = 0$$

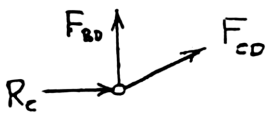
$$F_{BC} - F_{CD} = \frac{13}{5}P \tag{2}$$

Solving (1) and (2) simultaneously,

$$F_{BC} = \frac{13}{10}P - \frac{13}{24}Q$$

$$F_{CD} = -\frac{13}{10}P - \frac{13}{24}Q$$

Joint D:



$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{13}F_{CD} + F_B = 0$$

$$F_{BD} = -\frac{5}{13}F_{CD} = \frac{1}{2}P + \frac{5}{24}Q$$

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PROBLEM 11.102 *(Continued)*

Lengths of members: $L_{BC} = 78$ in.

$L_{CD} = 78$ in.

$L_{BC} = 60$ in.

$$U = \Sigma \frac{F^2 L}{2EA}$$

$$\delta_P = \frac{\partial U}{\partial Q} = \Sigma \frac{FL}{EA}\frac{\partial F}{\partial P} = \frac{1}{E}\Sigma \frac{LF}{A}\frac{\partial F}{\partial Q}$$

Member	F	L (in.)	$\frac{\partial F}{\partial Q}$	A (in^2)	$\frac{FL}{A}\frac{\partial F}{\partial Q}$
BC	$\frac{13}{10}P - \frac{13}{24}Q$	78	$-\frac{13}{24}$	4	$-13.73125P + 5.72135Q$
CD	$-\frac{13}{10}P - \frac{13}{24}Q$	78	$-\frac{13}{24}$	6	$9.15467P + 3.81424Q$
BD	$\frac{1}{2}P + \frac{5}{24}Q$	60	$\frac{5}{24}$	3	$2.08333P + 0.86806Q$
Σ					$-2.49325P + 10.40365Q$

Further data: $E = 29 \times 10^6$ psi

$= 29{,}000$ ksi

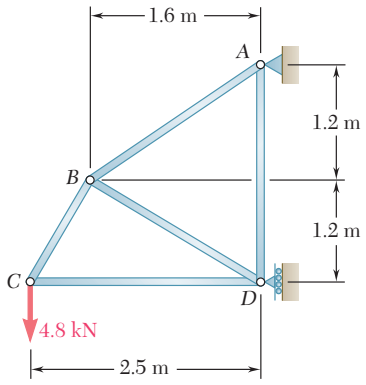
$P = 80$ kips

$Q = 48$ kips

$$\delta_Q = \frac{(-2.49325)(80) + (10.40365)(48)}{29{,}000} = 0.01034 \text{ in.} \leftarrow$$

◀

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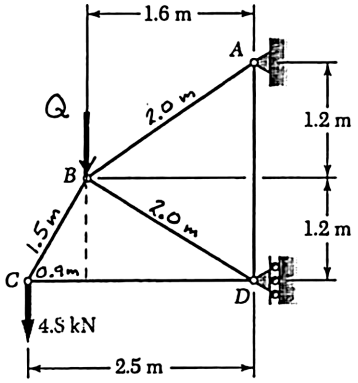
PROBLEM 11.103

Each member of the truss shown is made of steel and has a cross-sectional area of 500 mm^2. Using $E = 200$ GPa, determine the vertical deflection of joint B.

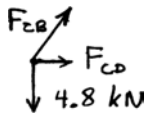
SOLUTION

Find the length of each member as shown.

Add vertical force Q at joint B.



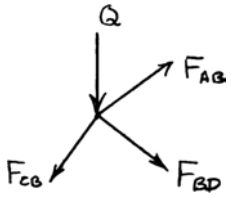
$$\delta_B = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q}\Sigma\frac{F^2L}{2EA} = \frac{1}{EA}\Sigma F\frac{\partial F}{\partial Q}L$$



Joint C:

$$+\uparrow \Sigma F_y = 0: \quad \frac{4}{5}F_{CB} - 4.8 = 0 \quad F_{CB} = 6.0 \text{ kN}$$

$$+\rightarrow \Sigma F_x = 0: \quad \frac{3}{5}F_{CB} + F_{CD} = 0 \quad F_{CD} = -3.6 \text{ kN}$$



Joint B:

$$+\rightarrow \Sigma F_x = 0: \quad \frac{4}{5}F_{AB} + \frac{4}{5}F_{BD} - 3.6 = 0$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{AB} - \frac{3}{5}F_{BD} - 4.8 - Q = 0$$

Solving simultaneously,

$$F_{AB} = 6.25 + 0.8333Q \quad \text{kN}$$

$$F_{BD} = -1.75 - 0.8333Q \quad \text{kN}$$



Joint D:

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} + F_{AD} = 0$$

$$F_{AD} = -\frac{3}{5}F_{BD} = 1.05 + 0.5Q$$

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PROBLEM 11.103 *(Continued)*

Member	$F(10^3\,\text{N})$	$\partial F/\partial Q$	$L\,(\text{m})$	with $Q=0$ $F(\partial F/\partial Q)L\ (10^3\,\text{N}\cdot\text{m})$
AB	$6.25+0.8333Q$	0.8333	2.0	10.4167
AD	$1.05+0.5Q$	0.5	2.4	1.26
BD	$-1.75-0.8333Q$	–0.8333	2.0	2.9167
BC	6.0	0	1.5	0
CD	–3.6	0	2.5	0
Σ				14.593

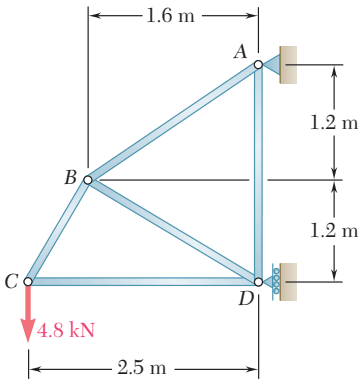
$$\delta_B = \frac{1}{EA}\Sigma F(\partial F/\partial Q)L$$

$$= \frac{14.593\times10^3}{(200\times10^9)(500\times10^{-6})}$$

$$= 145.9\times10^{-6}\,\text{m}$$

$$\delta_B = 0.1459\ \text{mm}\downarrow \blacktriangleleft$$

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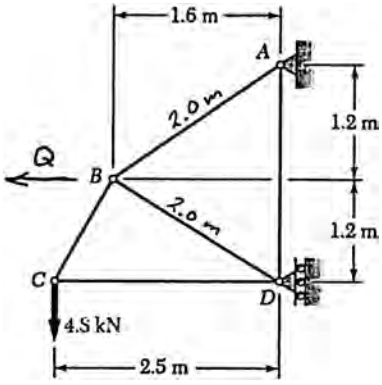
PROBLEM 11.104

Each member of the truss shown is made of steel and has a cross-sectional area of $500\ \text{mm}^2$. Using $E = 200$ GPa, determine the horizontal deflection of joint B.

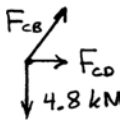
SOLUTION

Find the length of each member as shown.

Add horizontal force Q at joint B.

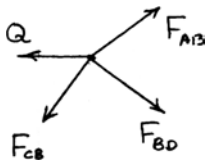


$$\delta_B = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q}\Sigma\frac{F^2L}{2EA} = \frac{1}{EA}\Sigma F\frac{\partial F}{\partial Q}L$$



Joint C: $+\uparrow \Sigma F_y = 0$: $\frac{4}{5}F_{CB} - 4.8 = 0$ $\quad F_{CB} = 6.0$ kN

Joint B: $+\rightarrow \Sigma F_x = 0$: $\frac{3}{5}F_{CB} + F_{CD} = 0$ $\quad F_{CD} = -3.6$ kN



$+\rightarrow \Sigma F_x = 0$: $\frac{4}{5}F_{AB} + \frac{4}{5}F_{BD} - 3.6 - Q = 0$

$+\uparrow \Sigma F_y = 0$: $\frac{3}{5}F_{AB} - \frac{3}{5}F_{BD} - 4.8 = 0$

Solving simultaneously, $F_{AB} = 6.25 + 0.625Q$ kN

$F_{BD} = -1.75 + 0.625Q$ kN



Joint D: $+\uparrow \Sigma F_y = 0$: $\frac{3}{5}F_{BD} + F_{AD} = 0$

$F_{AD} = -\frac{3}{5}F_{BD} = 1.05 - 0.375Q$

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PROBLEM 11.104 *(Continued)*

Member	$F(10^3\,\text{N})$	$\partial F/\partial Q$	$L\,(\text{m})$	$F(\partial F/\partial Q)L$ $(10^3\,\text{N}\cdot\text{m})$
AB	$6.25+0.625Q$	0.625	2.0	7.8125
AD	$1.05+0.375Q$	–0.375	2.4	–0.9450
BD	$-1.75+0.625Q$	0.625	2.0	–2.1875
BC	6.0	0	1.5	0
CD	–3.6	0	2.5	0
Σ				4.680

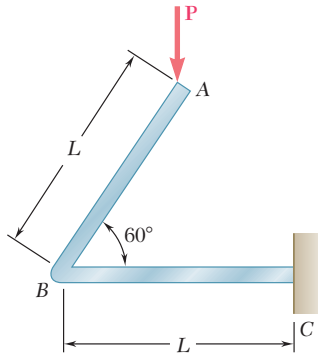
$$\delta_B = \frac{1}{EA}\Sigma F(\partial F/\partial Q)L$$

$$= \frac{4.680\times10^3}{(200\times10^9)(500\times10^{-6})}$$

$$= 46.8\times10^{-6}\,\text{m}$$

$$\delta_B = 0.0468\ \text{mm} \leftarrow \blacktriangleleft$$

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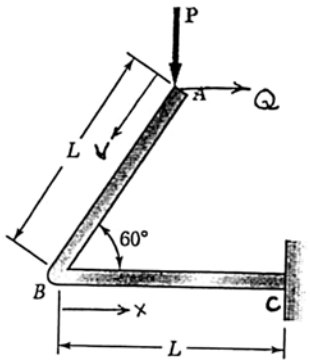


PROBLEM 11.105

A uniform rod of flexural rigidity *EI* is bent and loaded as shown. Determine (*a*) the vertical deflection of point *A*, (*b*) the horizontal deflection of point *A*.

SOLUTION

Add horizontal force Q at point A.



Over *AB*: $$M = \frac{1}{2}Pv + \frac{\sqrt{3}}{2}Qv$$

$$\frac{\partial M}{\partial P} = \frac{1}{2}v \qquad \frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2}v$$

$$U_{AB} = \int_0^L \frac{M^2}{2EI}dx$$

Set $Q = 0$.

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial P}dv = \frac{1}{EI}\int_0^L \left(\frac{1}{2}Pv\right)\left(\frac{1}{2}v\right)dv = \frac{1}{12}\frac{PL^3}{EI}$$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial Q}dv = \frac{1}{EI}\int_0^L \left(\frac{1}{2}Pv\right)\frac{\sqrt{3}}{2}dv = \frac{\sqrt{3}}{12}\frac{PL^3}{EI}$$

Over *BC*: $$M = -P\left(x - \frac{L}{2}\right) + \frac{\sqrt{3}}{2}QL, \qquad \frac{\partial M}{\partial P} = -\left(x - \frac{L}{2}\right), \qquad \frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2}L$$

$$U_{BC} = \int_0^L \frac{M^2}{2EI}dx$$

Set $Q = 0$.

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial P}dx = \frac{1}{EI}\int_0^L P\left(x - \frac{L}{2}\right)^2 dx = \frac{P}{3EI}\left(x - \frac{L}{2}\right)^3\Bigg|_0^L = \frac{1}{12}\frac{PL^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial Q}dx = \frac{-1}{EI}\int_0^L P\left(x - \frac{L}{2}\right)\left(\frac{\sqrt{3}}{2}\right)L\,dx = -\frac{\sqrt{3}P}{4EI}\left(x - \frac{L}{2}\right)^2\Bigg|_0^L = 0$$

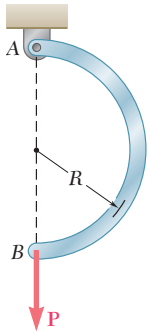
(*a*) Vertical deflection of point *A*.

$$\delta_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} \qquad \delta_P = \frac{PL^3}{6EI}\downarrow \blacktriangleleft$$

(*b*) Horizontal deflection of point *A*.

$$\delta_Q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{\sqrt{3}}{12}\frac{PL^3}{EI} \qquad \delta_Q = 0.1443\frac{PL^3}{EI}\rightarrow \blacktriangleleft$$

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PROBLEM 11.106

For the uniform rod and loading shown, and using Castigliano's theorem, determine the deflection of point *B*.

SOLUTION

Use polar coordinate φ.

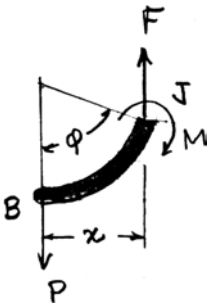
Calculate the bending moment $M(\varphi)$ using free body *BJ*.

$$+\circlearrowleft \Sigma M_J = 0: \quad Px - M = 0$$

$$M = Px = PR\sin\varphi$$

Strain energy:

$$U = \int_0^L \frac{M^2}{2EI}\,ds$$



$$U = \int_0^{\pi} \frac{(PR\sin\varphi)^2}{2EI}(Rd\varphi)$$

$$= \frac{P^2R^3}{2EI}\int_0^{\pi}\sin^2\varphi\,d\varphi$$

$$= \frac{P^2R^3}{2EI}\int_0^{\pi}\frac{1-\cos 2\varphi}{2}\,d\varphi$$

$$= \frac{P^2R^2}{2EI}\left(\frac{1}{2}\varphi\Big|_0^{\pi} - \frac{1}{4}\sin 2\varphi\Big|_0^{\pi}\right)$$

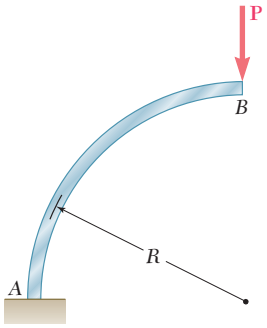
$$= \frac{\pi P^2 R}{4EI}$$

By Castigliano's theorem,

$$\delta = \frac{\partial U}{\partial P}$$

$$\delta = \frac{\pi PR^3}{2EI}\downarrow \blacktriangleleft$$

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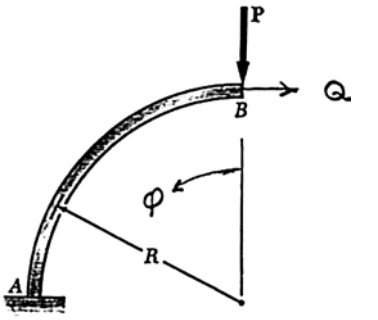
PROBLEM 11.107

For the beam and loading shown, and using Castigliano's theorem, determine (*a*) the horizontal deflection of point *B*, (*b*) the vertical deflection of point *B*.

SOLUTION

Add horizontal force Q at point B.

Use polar coordinate φ.



$$U = \int_0^{\pi/2} \frac{M^2}{2EI} R d\varphi$$

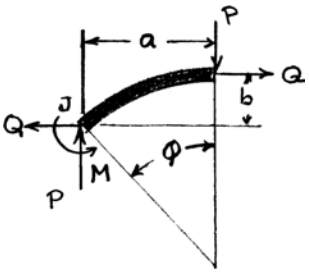
Bending moment.

$$+\circlearrowleft \Sigma M_J = 0: \quad M - Pa - Qb = 0$$

$$M = Pa + Qb$$

$$= PR\sin\varphi + QR(1-\cos\varphi)$$

$$\frac{\partial M}{\partial P} = R\sin\varphi \qquad \frac{\partial M}{\partial Q} = R(1-\cos\varphi)$$



Set $Q = 0$.

(*a*) $$\delta_Q = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int_0^{\pi/2} M\frac{\partial M}{\partial Q} R d\varphi = \frac{1}{EI}\int_0^{\pi/2} PR\sin\varphi\, R(1-\cos\varphi) R d\varphi$$

$$= \frac{PR^3}{EI}\int_0^{\pi/2}(\sin\varphi - \sin\varphi\cos\varphi)d\varphi = \frac{PR^3}{EI}\left(-\cos\varphi - \frac{1}{2}\sin^2\varphi\right)\Big|_0^{\pi/2}$$

$$= \frac{PR^3}{EI}\left(-\cos\frac{\pi}{2} + \cos 0 - \frac{1}{2}\sin^2\frac{\pi}{2} + \frac{1}{2}\sin^2 0\right)$$

$$= \frac{PR^3}{EI}\left(0 + 1 - \frac{1}{2} + 0\right)$$

$$\delta_Q = \frac{PR^3}{2EI} \rightarrow \blacktriangleleft$$

(*b*) $$\delta_P = \frac{\partial U}{\partial P} = \frac{1}{EI}\int_0^{\pi/2} M\frac{\partial M}{\partial P} R d\varphi = \frac{1}{EI}\int_0^{\pi/2} PR\sin\varphi\, R\sin\varphi\, R d\varphi$$

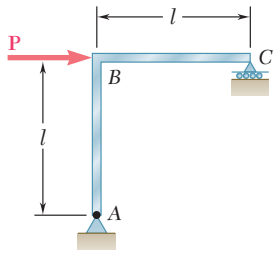
$$= \frac{PR^3}{EI}\int_0^{\pi/2}\sin^2\varphi\, d\varphi = \frac{PR^3}{EI}\int_0^{\pi/2}\frac{1}{2}(1-\cos 2\varphi)d\varphi$$

$$= \frac{PR^3}{EI}\left(\frac{1}{2}\varphi - \frac{1}{2}\sin 2\varphi\right)\Big|_0^{\pi/2} = \frac{PR^3}{EI}\left(\frac{1}{2}\cdot\frac{\pi}{2} - \frac{1}{2}\cdot 0 - \frac{1}{2}\sin\pi + \frac{1}{2}\cdot\sin 0\right)$$

$$= \frac{PR^3}{EI}\left(\frac{\pi}{4} - 0 - 0 + 0\right)$$

$$\delta_P = \frac{\pi PR^3}{4EI} \downarrow \blacktriangleleft$$

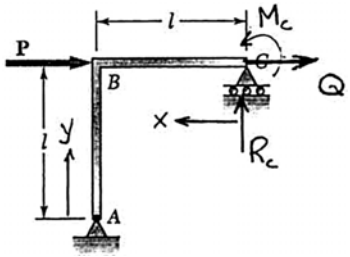
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PROBLEM 11.108

Two rods *AB* and *BC* of the same flexural rigidity *EI* are welded together at *B*. For the loading shown, determine (*a*) the deflection of point *C*, (*b*) the slope of member *BC* at point *C*.

SOLUTION



Add horizontal force Q and couple M_C at C.

$$+\circlearrowleft \Sigma M_A = 0: \quad R_C l + M_C - (P+Q)l = 0$$

$$R_C = P + Q + \frac{M_C}{l}$$

$$+\rightarrow \Sigma F_x = 0: \quad P + Q + R_{Ax} = 0 \qquad R_{Ax} = P + Q \leftarrow$$

Member *AB*:

$$M = R_{Ax} y = (P+Q)y, \qquad \frac{\partial M}{\partial Q} = y, \qquad \frac{\partial M}{\partial M_C} = 0$$

$$U_{AB} = \int_0^l \frac{M^2}{2EI}\,dy$$

Set $Q = 0$ and $M_C = 0$.

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial Q}\,dy = \frac{1}{EI}\int_0^l (Py)(y)dy = \frac{1}{3}\frac{Pl^3}{EI}$$

$$\frac{\partial U_{AB}}{\partial M_C} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial M_A}\,dx = 0$$

Member *BC*:

$$M = M_C + R_C x = M_C + \left(P + Q + \frac{M_C}{l}\right)x$$

$$\frac{\partial M}{\partial Q} = x, \qquad \frac{\partial M}{\partial M_C} = 1 - \frac{x}{l}$$

$$U_{BC} = \int_0^l \frac{M^2}{2EI}\,dx$$

Set $Q = 0$ and $M_C = 0$.

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial Q}\,dx = \frac{1}{EI}\int_0^l (Px)x\,dx = \frac{1}{3}\frac{Pl^3}{EI}$$

$$\frac{\partial U}{\partial M_A} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial M_A}\,dx = \frac{1}{EI}\int_0^l (Px)\left(1 - \frac{x}{l}\right)dx$$

$$= \frac{P}{EI}\int_0^l \left(x - \frac{x^2}{l}\right)dx$$

$$= \frac{P}{EI}\left(\frac{1}{2}l^2 - \frac{1}{3}l^2\right) = \frac{1}{6}\frac{Pl^2}{EI}$$

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PROBLEM 11.108 *(Continued)*

(*a*) Deflection at *C*.

$$\delta_C = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q}$$

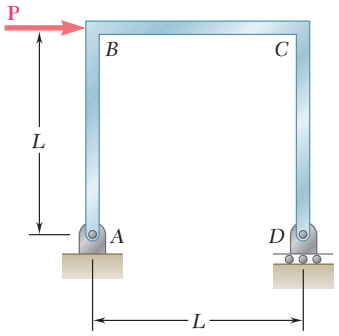
$$\delta_C = \frac{2Pl^3}{3EI} \rightarrow \blacktriangleleft$$

(*b*) Slope at *C*.

$$\theta_C = \frac{\partial U_{AB}}{\partial M_A} + \frac{\partial U_{BC}}{\partial M_C}$$

$$\theta_C = \frac{Pl^2}{6EI} \curvearrowright \blacktriangleleft$$

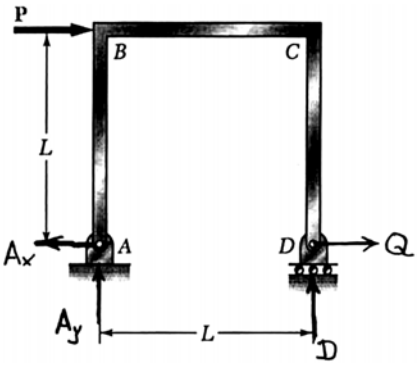
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PROBLEM 11.109

Three rods, each of the same flexural rigidity *EI*, are welded to form the frame *ABCD*. For the loading shown, determine the deflection of point *D*.

SOLUTION



Add dummy force Q at point D as shown.

Statics $+\circlearrowleft \Sigma M_A = 0: \quad DL - PL = 0 \qquad D = P \uparrow$

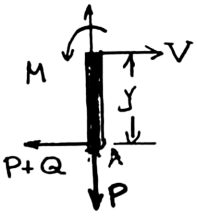
$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad -A_x + P + Q = 0 \qquad A_x = (P + Q) \leftarrow$

$+\uparrow \Sigma F_y = 0: \quad A_y + D = 0 \qquad A_y = P \downarrow$

$$U = U_{AB} + U_{BC} + U_{CD}$$

By Castigliano's theorem, $\delta_D = \dfrac{\partial U}{\partial Q}$

$$\delta_D = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q}$$

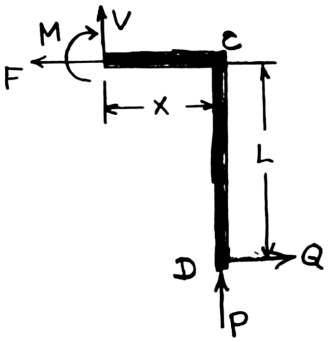


Member *AB*: $M = (P + Q)y \qquad \dfrac{\partial M}{\partial Q} = y$

Set $Q = 0 \quad M = P_y$

$$U_{AB} = \int_0^L \frac{M^2 dy}{2EI}$$

$$\frac{\partial U_{AB}}{\partial Q} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q}dy = \frac{p}{EI}\int_0^L y^2 dy = \frac{PL^3}{3EI}$$



Member *BC*:

$$M = Px + QL \qquad \frac{\partial M}{\partial Q} = L$$

Set $Q = 0 \quad M = Px$

$$U_{BC} = \int_0^L \frac{M^2 dx}{2EI}$$

$$\frac{\partial U_{BC}}{\partial Q} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q}dx = \frac{PL}{EI}\int_0^L x\,dx = \frac{PL^3}{2EI}$$

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PROBLEM 11.109 *(Continued)*

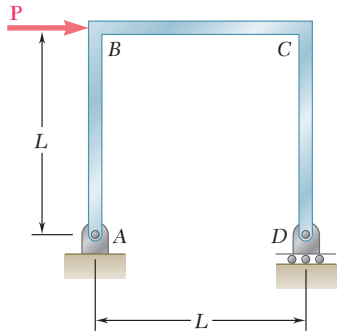
Member *CD*: $M = Qy \qquad \dfrac{\partial M}{\partial Q} = y$

Set $Q = 0 \quad M = 0$

$$U_{CD} = \int_0^L \frac{M^2 dy}{2EI} \qquad \frac{\partial U_{CD}}{\partial Q} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q} dy = 0$$

$$\delta_D = \frac{PL^3}{3EI} + \frac{PL^3}{2EI} + 0 = \frac{5PL^3}{6EI} \rightarrow \blacktriangleleft$$

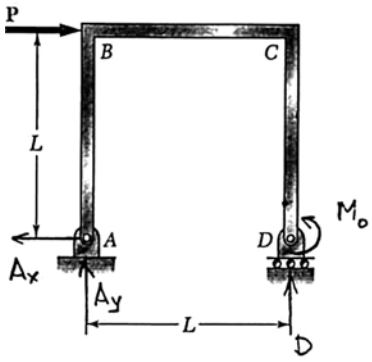
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PROBLEM 11.110

Three rods, each of the same flexural rigidity *EI*, are welded to form the frame *ABCD*. For the loading shown, determine the angle formed by the frame at point *D*.

SOLUTION



Add couple M_0 at point *D*.

Statics: $+\circlearrowleft \Sigma M_A = 0: \quad M_0 + DL - PL = 0$

$$D = P - \frac{M_0}{L} \uparrow$$

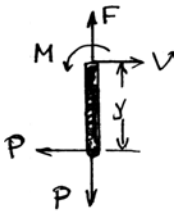
$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + P = 0 \quad A_x = P \leftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + D = 0 \quad A_y = P - \frac{M_0}{L} \downarrow$$

Strain energy: $U = U_{AB} + U_{BC} + U_{CD}$

By Castigliano's theorem, $\theta_D = \dfrac{\partial U}{\partial M_0}$

$$\theta_D = \frac{\partial U_{AB}}{\partial M_0} + \frac{\partial U_{BC}}{\partial M_0} + \frac{\partial U_{CD}}{\partial M_0}$$



Member *AB*: $M = Py \qquad \dfrac{\partial M}{\partial M_0} = 0 \qquad U_{AB} = \displaystyle\int_0^L \frac{M^2}{2EI} dy$

$$\frac{\partial U_{AB}}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dy = 0$$

Member *BC*: $M = M_0 + Dx = M_0 + Px - \dfrac{M_0 x}{L}$

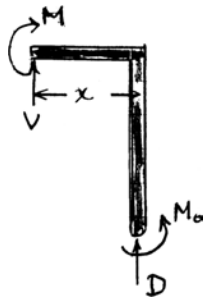
Set $M_0 = 0$ $\qquad M = Px \qquad \dfrac{\partial M}{\partial M_0} = 1 - \dfrac{x}{L}$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dx$$

$$\frac{\partial U_{BC}}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = \frac{P}{EI} \int_0^L x \left(1 - \frac{x}{L}\right) dy = \frac{PL^2}{6EI}$$

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PROBLEM 11.110 ***(Continued)***



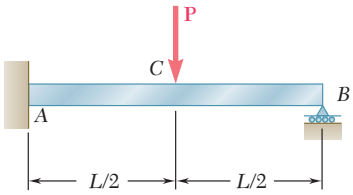
Member *CD*: $M = M_0 \qquad \dfrac{\partial M}{\partial M_0} = 1$

Set $M_0 = 0$ $\qquad M = 0$

$$U = \int_0^L \frac{M^2}{2EI}\,dy \qquad \frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_0}\,dx = 0$$

$$\theta_D = 0 + \frac{PL^2}{6EI} + 0 \qquad \theta_D = \frac{PL^2}{6EI} \circlearrowright \blacktriangleleft$$

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PROBLEM 11.111

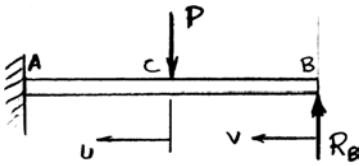
Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION

Remove support B and add reaction R_B as a load.

$$U = U_{AC} + U_{CB} = \int_0^{L/2} \frac{M^2}{2EI}du + \int_0^{L/2} \frac{M^2}{2EI}dv$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{\partial U_{AB}}{\partial R_B} + \frac{\partial U_{CB}}{\partial R_B} = 0$$



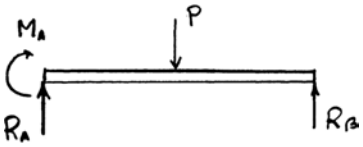
Over AC: $M = R_B\left(u + \frac{L}{2}\right) - Pu, \quad \frac{\partial M}{\partial R_B} = \left(u + \frac{L}{2}\right)$

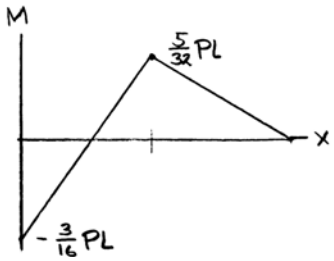
$$\frac{\partial U_{AB}}{\partial R_B} = \frac{1}{EI}\int_0^{L/2}\left[R_B\left(u + \frac{L}{2}\right) - Pu\right]\left(u + \frac{L}{2}\right)du$$

$$= \frac{R_B}{EI}\int_0^{L/2}\left(u + \frac{L}{2}\right)^2 dv - \frac{P}{EI}\int_0^{L/2} u\left(u + \frac{L}{2}\right)du$$

$$= \frac{R_B}{3EI}\left[L^3 - \left(\frac{L}{2}\right)^3\right] - \frac{P}{EI}\left[\frac{1}{3}\left(\frac{L}{2}\right)^3 + \frac{L}{2}\cdot\frac{1}{2}\left(\frac{L}{2}\right)^2\right]$$

$$= \frac{7}{24}\frac{R_B L^3}{EI} - \frac{5}{48}\frac{PL^3}{EI}$$





Over CB: $M = R_B v \qquad \frac{\partial M}{\partial R_B} = v$

$$\frac{\partial U_{CB}}{\partial R_B} = \frac{1}{EI}\int_0^{L/2}(R_B v)v\,dv = \frac{R_B}{3EI}\left(\frac{L}{2}\right)^3 = \frac{1}{24}\frac{R_B L^3}{EI}$$

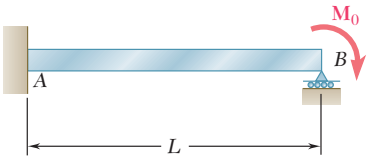
$$y_B = \left(\frac{7}{24} + \frac{1}{24}\right)\frac{R_B L^3}{EI} - \frac{5}{48}\frac{PL^3}{EI} = 0 \qquad R_B = \frac{5}{16}P \uparrow \blacktriangleleft$$

$$M_C = R_B\frac{L}{2} \qquad M_C = \frac{5}{32}PL \blacktriangleleft$$

$$M_A = R_B L - P\frac{L}{2} = \left(\frac{5}{16} - \frac{1}{2}\right)PL \qquad M_A = -\frac{3}{16}PL \blacktriangleleft$$

$$M_B = 0 \blacktriangleleft$$

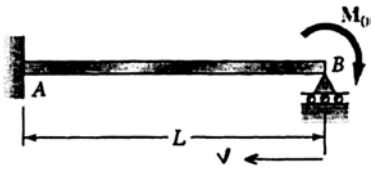
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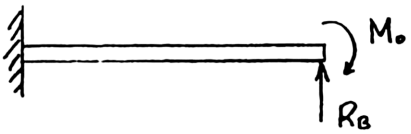
PROBLEM 11.112

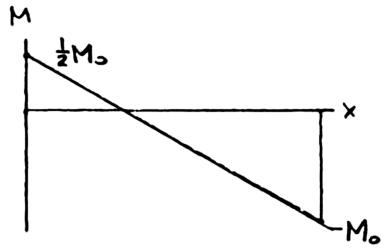
Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.

SOLUTION



Remove support B and add reaction R_B as a load.





$$U = \int_0^L \frac{M^2}{2EI}\,dv$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{1}{EI}\int_0^L M\,\frac{\partial M}{\partial R_B}\,dv = 0$$

$$M = R_B v - M_0 \qquad \frac{\partial M}{\partial R_B} = v$$

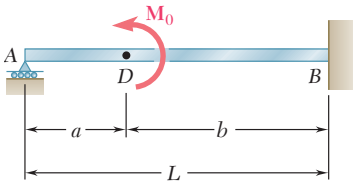
$$y_B = \frac{1}{EI}\int_0^L (R_B v M_0)\,v\,dv$$

$$= \frac{R_B}{EI}\int_0^L v^2 dv - \frac{M_0}{EI}\int_0^L v\,dv$$

$$= \frac{R_B L^3}{3EI} - \frac{M_0 L^2}{2EI} = 0 \qquad R_B = \frac{3}{2}\frac{M_0}{L} \uparrow \quad \blacktriangleleft$$

$$M_A = R_B - M_0 = \frac{3}{2}M_0 - M_0 = \frac{1}{2}M_0 \quad \blacktriangleleft$$

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PROBLEM 11.113

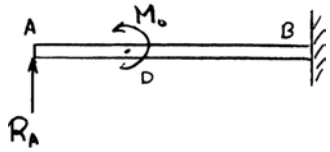
Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION

Remove support A and add reaction R_A as a load.

$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI}\int_0^L M\,\frac{\partial M}{\partial R_A}\,dx = 0$$



Portion *AD*: $(0 < x < a)$ $M = R_A x$ $\dfrac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AD}}{\partial R_A} = \frac{1}{EI}\int_0^a (R_A x)(x)\,dx = \frac{R_A a^3}{3EI}$$

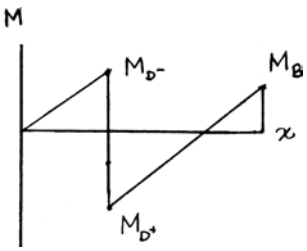
Portion *DB*: $(a < x < L)$ $M = R_A x - M_0$ $\dfrac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI}\int_a^L (R_A x - M_0)(x)\,dx = \frac{1}{EI}\left\{\frac{1}{3}R_A(L^3 - a^3) - \frac{1}{2}M_0(L^2 - a^2)\right\}$$

$$\delta_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI}\left\{R_A\left(\frac{1}{3}a^3 + \frac{1}{3}L^3 - \frac{1}{3}a^3\right) - \frac{1}{2}M_0(L^2 - a^2)\right\} = 0$$

$$R_A = \frac{3}{2}\frac{M_0(L^2 - a^2)}{L^3} \qquad R_A = \frac{3M_0 b(L+a)}{2L^3} \uparrow \blacktriangleleft$$

$$M_A = 0 \blacktriangleleft$$



$$M_{D-} = R_A a \qquad M_{D-} = \frac{3}{2}\frac{M_0 ab(L+a)}{L^3} \blacktriangleleft$$

$$M_{D+} = M_{D-} - M_0 \qquad M_{D+} = \frac{3}{2}\frac{M_0 ab(L+a)}{L^3} - M_0 \blacktriangleleft$$

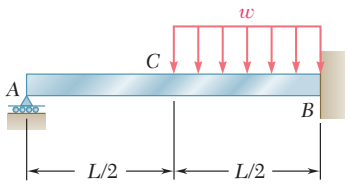
$$M_B = R_A L - M_0 \qquad M_B = \frac{3}{2}\frac{M_0 b(L+a)}{L^2} - M_0 \blacktriangleleft$$

Bending moment diagram drawn to scale for $a = \dfrac{1}{3}L$.

By singularity functions,

$$M = 3M_0 b(L+a)x/2L^3 - M_0\langle L - a\rangle^0 \blacktriangleleft$$

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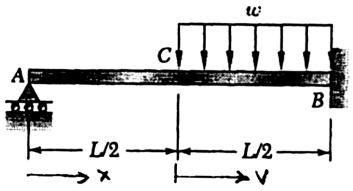


PROBLEM 11.114

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION

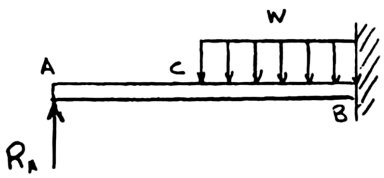
Remove support A and add reaction R_A as a load.



$$U = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}dx + \int_0^{\frac{L}{2}} \frac{M^2}{2EI}dv$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI}\int_0^{\frac{L}{2}} M\frac{\partial M}{\partial R_A}dx + \frac{1}{EI}\int_0^{\frac{L}{2}} M\frac{\partial M}{\partial R_A}dv = 0$$

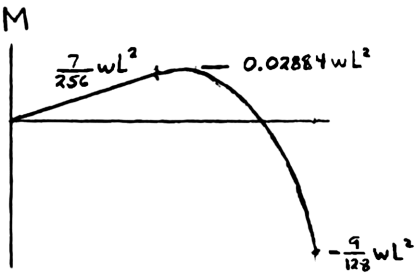
Portion AC: $\left(O < x < \frac{L}{2}\right)$ $\quad M = R_A x \quad \frac{\partial M}{\partial R_A} = x$



$$\frac{\partial U_{AC}}{\partial R_A} = \frac{1}{EI}\int_0^{\frac{L}{2}} (R_A x)(x)dx = \frac{R_A L^3}{24EI}$$

Portion CB: $\left(0 < v < \frac{L}{2}\right)$

$$M = R_A\left(v + \frac{L}{2}\right) - \frac{1}{2}wv^2 \qquad \frac{\partial M}{\partial R_A} = \left(v + \frac{L}{2}\right)$$



$$\frac{\partial U_{CB}}{\partial R_A} = \frac{1}{EI}\int_0^L \left[R_A\left(v + \frac{L}{2}\right) - \frac{1}{2}wv^2\right]\left(v + \frac{L}{2}\right)dv$$

$$= \frac{1}{EI}\left\{R_A\int_0^{\frac{L}{2}}\left(v + \frac{L}{2}\right)^2 dv - \frac{1}{2}w\int_0^{\frac{L}{2}}\left(v^3 + \frac{L}{2}v^2\right)dv\right\}$$

$$= \frac{R_A}{EI}\left[\frac{1}{3}L^3 - \frac{1}{3}\left(\frac{L}{2}\right)^3\right] - \frac{w}{2EI}\left[\frac{1}{4}\left(\frac{L}{2}\right)^4 + \frac{L}{2}\frac{1}{3}\left(\frac{L}{2}\right)^3\right]$$

$$= \left(\frac{1}{3} - \frac{1}{24}\right)\frac{R_A L^3}{EI} - \frac{7}{384}\frac{wL^4}{EI}$$

$$\delta_A = \frac{\partial U_{AC}}{\partial R_A} + \frac{\partial U_{CB}}{\partial R_A} = \frac{1}{3}\frac{R_A L^3}{EI} - \frac{7}{384}\frac{wL^4}{EI} = 0$$

$$R_A = \frac{7}{128}wL \uparrow \blacktriangleleft$$

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PROBLEM 11.114 *(Continued)*

Bending moments. Over *AC*:

$$M = \frac{7}{128} wLx$$

$$M_C = \frac{7}{256} wL^2 = 0.02734 wL^2 \quad \blacktriangleleft$$

Over *CB*:

$$M = \frac{7}{128} wL\left(v + \frac{L}{2}\right) - \frac{1}{2} wv^2$$

$$M_B = \frac{7}{128} wL^2 - \frac{1}{2} w\left(\frac{L}{2}\right)^2 = -\frac{9}{128} wL^2$$

$$M_B = -0.07031 wL^2 \quad \blacktriangleleft$$

$$\frac{dM}{dv} = \frac{7}{128} wL - wv_m = 0 \qquad v_m = \frac{7}{128} L$$

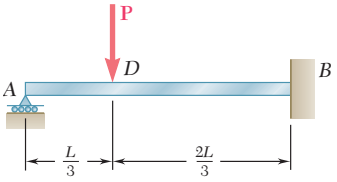
$$M_m = \frac{7}{128} wL\left(\frac{7}{128} L + \frac{L}{2}\right) - \frac{1}{2} w\left(\frac{7}{128} L\right)^2$$

$$= \frac{945}{32{,}768} wL^2 = 0.02884 wL^2 \quad \blacktriangleleft$$

or

$$M = \frac{7}{128} wLx - w\langle x - L/2\rangle^2/2 \quad \blacktriangleleft$$

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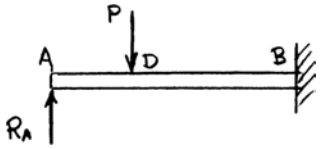


PROBLEM 11.115

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION

Remove support *A* and add reaction R_A as a load.



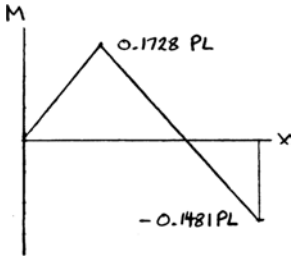
$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI}\int_0^L M\,\frac{\partial M}{\partial R_A}\,dx$$

Portion *AD*: $\left(0 < x < \frac{L}{3}\right)$ $M = R_A x$ $\frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AD}}{\partial R_A} = \frac{1}{EI}\int_0^{L/3} M\,\frac{\partial M}{\partial R_A}\,dx = \frac{1}{EI}\int_0^{L/3} (R_A x)(x)\,dx$$

$$= \frac{R_A}{3EI}\left(\frac{L}{3}\right)^3 = \frac{1}{81}\frac{R_A L^3}{EI}$$



Portion *DB*: $\left(\frac{L}{3} < x < L\right)$ $M = R_A x - P\left(x - \frac{L}{3}\right)$

$$\frac{\partial M}{\partial R_A} = x$$

$$\frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI}\int_{L/3}^{L} M\,\frac{\partial M}{\partial R_A}\,dx = \frac{1}{EI}\int_{L/3}^{L}\left[R_A - P\left(x - \frac{L}{3}\right)\right]x\,dx$$

$$= \frac{R_A}{EI}\int_{L/3}^{L} x^2\,dx - \frac{P}{EI}\int_{L/3}^{L}\left(x^2 - \frac{L}{3}x\right)dx$$

$$= \frac{R_A}{3EI}\left[L^3 - \left(\frac{L}{3}\right)^3\right] - \frac{P}{EI}\left[\frac{1}{3}\left(L^3 - \left(\frac{L}{3}\right)^3\right) - \frac{L}{6}\left(L^2 - \left(\frac{L}{3}\right)^2\right)\right]$$

$$= \left(\frac{1}{3} - \frac{1}{81}\right)\frac{R_A L^3}{EI} - \left(\frac{1}{3} - \frac{1}{81} - \frac{1}{6} + \frac{1}{54}\right)\frac{PL^3}{EI}$$

$$\delta_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \left(\frac{1}{81} + \frac{1}{3} - \frac{1}{81}\right)\frac{R_A L^3}{EI} - \frac{14}{81}\frac{PL^3}{EI}$$

$$= \frac{1}{3}\frac{R_A L^3}{EI} - \frac{14}{81}\frac{PL^3}{EI}$$

$$= 0$$

$$R_A = \frac{14}{27}P \uparrow \blacktriangleleft$$

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PROBLEM 11.115 ***(Continued)***

Bending moments:

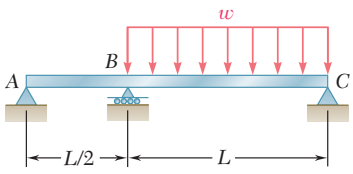
$$M_D = R_A\left(\frac{L}{3}\right) = \frac{14}{81}PL \qquad M_D = 0.1728PL \blacktriangleleft$$

$$M_B = R_A L - P\left(\frac{2L}{3}\right) = -\frac{4}{27}PL \qquad M_B = -0.1481PL \blacktriangleleft$$

By singularity functions,

$$M = \frac{14}{27}Px - P\langle x - L/3\rangle^1 \blacktriangleleft$$

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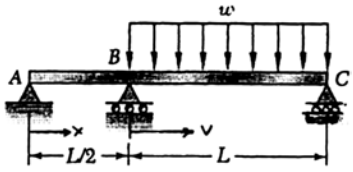


PROBLEM 11.116

For the uniform beam and loading shown, determine the reaction at each support.

SOLUTION

Remove support A and add reaction R_A as a load.

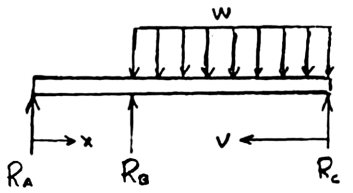


$$\Sigma M_B = 0 \quad -R_A\frac{L}{2} - \frac{1}{2}wL^2 + R_C L = 0$$

$$R_C = \frac{1}{2}R_A + \frac{1}{2}wL$$

$$U = U_{AB} + U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}dx + \int_0^{L} \frac{M^2}{2EI}dv$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = 0$$



Portion *AB*: $M = R_A x, \quad \dfrac{\partial M}{\partial Q_A} = x$

$$\frac{\partial U_{AB}}{\partial R_A} = \frac{1}{EI}\int_0^{\frac{L}{2}} M\frac{\partial M}{\partial R_A}dx = \frac{1}{EI}\int_0^{\frac{L}{2}} (R_A x)(x)dx = \frac{R_A}{3EI}\left(\frac{L}{2}\right)^3 = \frac{1}{24}\frac{R_A L^3}{EI}$$

Portion *BC*: $M = R_C v - \dfrac{1}{2}wv^2 = \dfrac{1}{2}R_A v + \dfrac{1}{2}wLv - \dfrac{1}{2}wLv^2$

$$\frac{\partial M}{\partial R_A} = \frac{1}{2}v$$

$$\frac{\partial U_{BC}}{\partial R_A} = \frac{1}{EI}\int_0^{L}\left[\frac{1}{2}R_A v + \frac{1}{2}w(Lv - v^2)\right]\left(\frac{1}{2}v\right)dv$$

$$= \frac{1}{4EI}\int_0^{L}\left[R_A v^2 + w(Lv^2 - v^3)\right]dv$$

$$= \frac{1}{4EI}\left[R_A\frac{L^3}{3} + w\left(\frac{L^4}{3} - \frac{L^4}{4}\right)\right] = \frac{R_A L^3}{12EI} + \frac{wL^4}{48EI}$$

$$\delta_A = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = \left(\frac{1}{24} + \frac{1}{12}\right)\frac{R_A L^3}{EI} + \frac{wL^4}{48EI} = 0$$

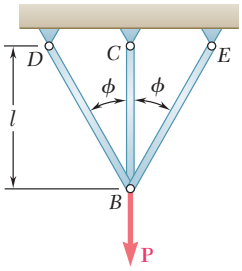
$$R_A = -\frac{1}{6}wL \qquad R_A = -\frac{1}{6}wL \downarrow \blacktriangleleft$$

$$R_C = \frac{1}{2}\left(-\frac{1}{6}wL\right) + \frac{1}{2}wL \qquad R_C = \frac{5}{12}wL \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B + R_C - wL = 0$$

$$-\frac{1}{6}wL + R_B + \frac{5}{12}wL - wL = 0 \qquad R_B = \frac{3}{4}wL \uparrow \blacktriangleleft$$

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PROBLEM 11.117

Three members of the same material and same cross-sectional area are used to support the load **P.** Determine the force in member *BC*.

SOLUTION

Detach member *BC* at support *C*.

Add reaction R_C as a load.

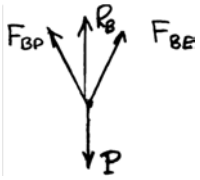
$$U = \Sigma \frac{F^2 L}{2EA} \qquad y_C = \frac{\partial U}{\partial R_C} = \Sigma \frac{FL}{EA}\frac{\partial F}{\partial R_C} = 0$$

Joint *C*: $F_{BC} = R_C$

Joint *B*: $+\rightarrow \Sigma F_x = 0: \quad F_{BE}\sin\varphi - F_{BD}\sin\varphi = 0 \qquad F_{BE} = F_{BD}$

$+\uparrow \Sigma F_y = 0: \quad F_{BD}\cos\varphi + F_{BE}\cos\varphi + R_B - P$

$$F_{BD} = F_{BE} = \frac{P - R_B}{2\cos\varphi}$$

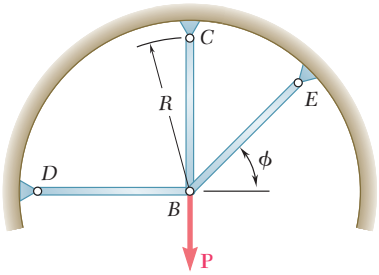


Member	F	$\partial F/\partial R_B$	L	$(FL/EA)\,(\partial F/\partial R_B)$
BD	$(P - R_B)/2\cos\varphi$	$-1/2\cos\varphi$	$l/\cos\varphi$	$(R_B - P)l/4EA\cos^3\varphi$
BE	$(P - R_B)/2\cos\varphi$	$-1/2\cos\varphi$	$l/\cos\varphi$	$(R_B - P)l/4EA\cos^3\varphi$
BC	R_B	1	l	$R_B l/EA$

$$y_B = -Pl/2EA\cos^3\varphi + R_B l/2EA\cos^3\varphi + R_B l/EA = 0$$

$$R_B = \frac{P}{1 + 2\cos^3\varphi} \qquad F_{BC} = R_B \qquad F_{BC} = \frac{P}{1 + 2\cos^3\varphi} \blacktriangleleft$$

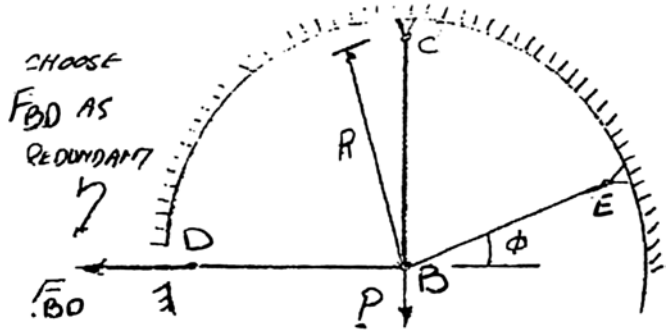
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PROBLEM 11.118

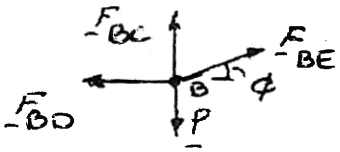
Three members of the same material and same cross-sectional area are used to support the load **P**. Determine the force in member *BC*.

SOLUTION



$$L_{BD} = L_{CB} = L_{BE} = R$$
$$A = \text{Constant}$$
$$E = \text{Constant}$$

Joint *E*:



$$\overset{+}{\rightarrow}\Sigma F_x = 0: \quad F_{BE}\cos\phi - F_{BD} = 0$$
$$F_{BE} = F_{BD}/\cos\phi$$
$$+\uparrow \Sigma F_y = 0: \quad F_{BC} + (F_{BD}/\cos\phi)\sin\phi - P = 0; \quad F_{BC} = P - F_{BD}\tan\phi$$
$$x_B = \Sigma \frac{F_i L_i}{EA_i}\frac{\partial F_i}{\partial F_{BD}} = \frac{L}{AE}\Sigma F_L \frac{\partial F_i}{\partial F_{BD}} = 0$$

We have $\Sigma F_i \dfrac{\partial F_i}{\partial F_{BD}} = 0$

	F_i	$\partial F_i/\partial F_{BD}$	$F_i\,\partial F_i/\partial F_{BD}$
BD	F_{BD}	1	F_{BD}
BC	$P - F_{BD}\tan\phi$	$-\tan\phi$	$-P\tan\phi + F_{BE}\tan^2\phi$
BE	$F_{BD}/\cos\phi$	$\dfrac{1}{\cos\phi}$	$F_{BD}/\cos^2\phi$

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PROBLEM 11.118 *(Continued)*

$$\Sigma F_i \frac{\partial F_i}{\partial F_{BD}} = F_{BD}\left(1 + \tan^2\phi + \frac{1}{\cos^2\phi}\right) - P\tan\phi = 0$$

$$F_{BD} = P\frac{\tan\phi}{1 + \tan^2\phi + \frac{1}{\cos^2\phi}} = \frac{\frac{\sin\phi}{\cos\phi}}{1 + \frac{\sin^2\phi}{\cos^2\phi} + \frac{1}{\cos^2\phi}} = \frac{\sin\phi\cos\phi}{\cos^2\phi + \sin^2\phi + 1}$$

$$F_{BD} = \frac{P}{2}\sin\phi\cos\phi = \frac{P}{4}\sin 2\phi$$

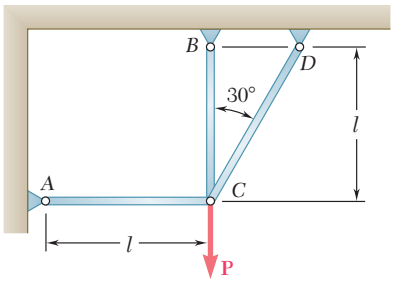
$$F_{BC} = P - F_{BD}\tan\phi$$

$$= P - \left(\frac{1}{2}\sin\phi\cos\phi\right)\frac{\sin\phi}{\cos\phi}$$

$$= P\left(1 - \frac{1}{2}\sin^2\phi\right)$$

$$F_{BC} = P\left(1 - \frac{1}{2}\left(\frac{1}{2}\right)^2\right) = \frac{7}{8}P \blacktriangleleft$$

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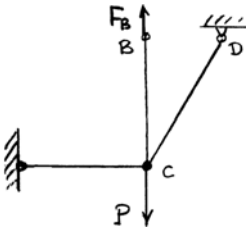


PROBLEM 11.119

Three members of the same material and same cross-sectional area are used to support the load **P**. Determine the force in member *BC*.

SOLUTION

Cut member *BC* at end *B* and replace member force F_{BC} by load F_B acting on member *BC* at *B*.

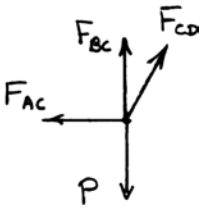


$$\delta_B = \frac{\partial U}{\partial F_B} = \frac{\partial}{\partial F_B}\Sigma\frac{F^2L}{EA} = \frac{1}{EA}\Sigma F\frac{\partial F}{\partial F_B}L = 0$$

Joint *C*:

$$+\uparrow \Sigma F_y = 0: \quad \frac{\sqrt{3}}{2}F_{CD} + F_{BC} - P = 0$$

$$F_{CD} = \frac{2}{\sqrt{3}}P - \frac{2}{\sqrt{3}}F_B$$



$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{AC} - \frac{1}{2}F_{CD} = 0$$

$$F_{AC} = \frac{1}{\sqrt{3}}P - \frac{1}{\sqrt{3}}F_B$$

Member	F	$\partial F/\partial F_B$	L	$F(\partial F/\partial F_B)L$
AC	F_B	1	l	$F_B l$
BC	$\frac{1}{\sqrt{3}}P - \frac{1}{\sqrt{3}}F_B$	$-\frac{1}{\sqrt{3}}$	l	$-\frac{1}{3}Pl + \frac{1}{3}F_B l$
CD	$\frac{2}{\sqrt{3}}P - \frac{2}{\sqrt{3}}F_B$	$-\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}l$	$-\frac{8}{\sqrt{3}}Pl + \frac{8}{\sqrt{3}}F_B l$
Σ				$-\left(\frac{1}{3}+\frac{8}{\sqrt{3}}\right)Pl + \left(\frac{4}{3}+\frac{8}{\sqrt{3}}\right)F_B l$

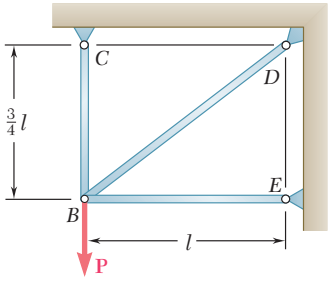
$$\delta_B = -\left(\frac{1}{3}+\frac{8}{\sqrt{3}}\right)\frac{Pl}{EA} + \left(\frac{4}{3}+\frac{8}{\sqrt{3}}\right)\frac{F_B l}{EA} = 0$$

$$F_B = \frac{\frac{1}{3}+\frac{8}{\sqrt{3}}}{\frac{4}{3}+\frac{8}{\sqrt{3}}}P = \frac{8+\sqrt{3}}{8+4\sqrt{3}}P = 0.652P$$

$$F_{BC} = F_B$$

$$F_{BC} = 0.652P \blacktriangleleft$$

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PROBLEM 11.120

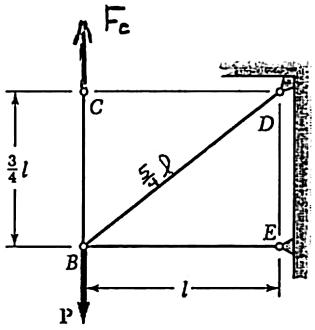
Three members of the same material and same cross-sectional area are used to support the load **P**. Determine the force in member *BC*.

SOLUTION

Detach member *BC* from support *C*. Add reaction F_C as a load.

$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2EA}\Sigma F^2 L$$

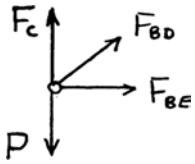
$$\delta_C = \frac{\partial U}{\partial F_C} = \frac{1}{EA}\Sigma F \frac{\partial F}{\partial F_C} L$$



Joint *B*:

$$+\uparrow \Sigma F_y = 0: \quad F_C - P + \frac{3}{5}F_{BD} = 0 \qquad F_{BD} = \frac{5}{3}P - \frac{5}{3}F_C$$

$$\pm\rightarrow \Sigma F_x = 0: \quad F_{BE} + \frac{4}{5}F_{BD} = 0 \qquad F_{BE} = -\frac{4}{3}P + \frac{4}{3}F_C$$

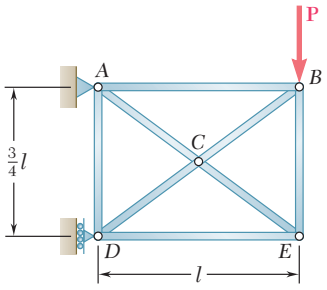


Member	F	$\partial F/\partial F_C$	L	$F(\partial F/\partial F_C)L$
BC	F_C	1	$\frac{3}{4}l$	$\frac{3}{4}F_C l$
BD	$\frac{5}{3}P - \frac{5}{3}F_C$	$-\frac{5}{3}$	$\frac{5}{4}l$	$-\frac{125}{36}Pl + \frac{125}{36}F_C l$
BE	$-\frac{4}{3}P + \frac{4}{3}F_C$	$\frac{4}{3}$	l	$-\frac{16}{9}Pl + \frac{16}{9}F_C l$
Σ				$-\frac{21}{4}Pl + 6F_C l$

$$\delta_C = \frac{1}{EA}\left(-\frac{21}{4}Pl + 6F_C l\right) = 0 \qquad F_C = \frac{7}{8}P \qquad F_{BC} = F_C$$

$$F_{BC} = \frac{7}{8}P \blacktriangleleft$$

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PROBLEM 11.121

Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member *AB*.

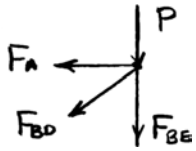
SOLUTION

Cut member *AB* at end *A* and replace member force F_{AB} by load $F_A \leftarrow$ acting on member *AB* at end *A*.

$$\delta_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A}\Sigma\frac{F^2L}{2EA} = \frac{1}{EA}\Sigma F\frac{\partial F}{\partial F_A}L = 0$$

Joint *B*:

$$+\rightarrow \Sigma F_x = 0: \quad -F_A - \frac{4}{5}F_{BD} = 0 \qquad F_{BD} = -\frac{5}{4}F_A$$



$$+\uparrow \Sigma F_y = 0: \quad -P - F_{BE} - \frac{3}{5}F_{BD} = 0 \qquad F_{BE} = -P + \frac{3}{4}F_A$$

Joint *E*:

$$+\uparrow \Sigma F_y = 0: \quad F_{BE} + \frac{3}{5}F_{AE} = 0 \qquad F_{AE} = \frac{5}{3}P - \frac{5}{4}F_A$$

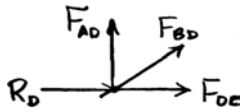


$$+\rightarrow \Sigma F_x = 0: \quad -\frac{4}{5}F_{AE} - F_{DE} = 0 \qquad F_{DE} = -\frac{4}{3}P + F_A$$

Joint *D*:

$$+\uparrow \Sigma F_y = 0: \quad F_{AD} + \frac{3}{5}F_{BD} = 0$$

$$F_{AD} = \frac{3}{4}F_A$$



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PROBLEM 11.121 *(Continued)*

Member	F	$\partial F/\partial F_A$	L	$F(\partial F/\partial F_A)L$
AB	F_A	1	l	$F_A l$
AD	$\frac{3}{4}F_A$	$\frac{3}{4}$	$\frac{3}{4}l$	$\frac{27}{64}F_A l$
AE	$\frac{5}{3}P-\frac{5}{4}F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$-\frac{125}{48}Pl+\frac{125}{64}F_A l$
BD	$-\frac{5}{4}F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$\frac{125}{64}F_A l$
BE	$-P+\frac{3}{4}F_A$	$\frac{3}{4}$	$\frac{3}{4}l$	$-\frac{9}{16}Pl+\frac{27}{64}F_A l$
DE	$-\frac{4}{3}P+F_A$	1	l	$-\frac{4}{3}Pl+F_A l$
Σ				$-\frac{9}{2}Pl+\frac{27}{4}F_A l$

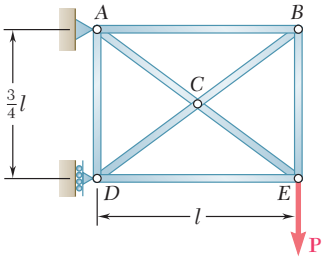
$$\delta_A=\frac{1}{EA}\left(-\frac{9}{2}Pl+\frac{27}{4}F_A l\right)=0$$

$$F_A=\frac{2}{3}P$$

$$F_{AB}=F_A$$

$$F_{AB}=\frac{2}{3}P \blacktriangleleft$$

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PROBLEM 11.122

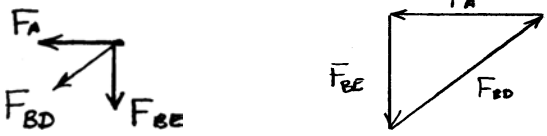
Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member *AB*.

SOLUTION

Cut member *AB* at end *A* and replace member force F_{AB} by load $F_A \leftarrow$ acting on member *AB* at end *A*.

$$\delta_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A}\Sigma\frac{F^2L}{2EA} = \frac{1}{EA}\Sigma F\frac{\partial F}{\partial F_A}L = 0$$

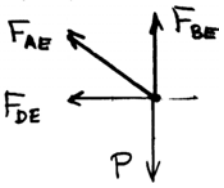
Joint *B*: $F_{BD} = -\frac{5}{4}F_A \qquad F_{BE} = \frac{3}{4}F_A$



Joint *E*: $+\uparrow \Sigma F_y = 0: \quad F_{BE} - P + \frac{3}{5}F_{AE}$

$$F_{AE} = \frac{5}{3}P - \frac{5}{3}F_{BE}$$

$$= \frac{5}{3}P - \frac{5}{4}F_A$$

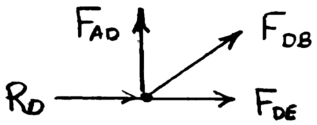


$$+\rightarrow \Sigma F_x = 0: \quad -\frac{4}{5}F_{AE} - F_{DE} = 0$$

$$F_{DE} = -\frac{4}{5}F_{AE} = -\frac{4}{3}P + F_A$$

Joint *D*: $+\uparrow \Sigma F_y = 0: \quad F_{AD} + \frac{3}{5}F_{DB} = 0$

$$F_{AD} = -\frac{3}{5}F_{DB} = -\frac{3}{4}F_A$$



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PROBLEM 11.122 *(Continued)*

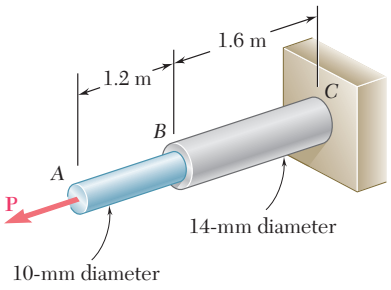
Member	F	$\partial F/\partial F_A$	L	$F(\partial F/\partial F_A)L$
AB	F_A	1	l	$F_A l$
AD	$-\frac{3}{4}F_A$	$-\frac{3}{4}$	$\frac{3}{4}l$	$\frac{27}{64}F_A l$
AE	$\frac{5}{3}P-\frac{5}{4}F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$-\frac{125}{48}Pl+\frac{125}{64}F_A l$
BD	$-\frac{5}{4}F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$\frac{125}{64}F_A l$
BE	$\frac{3}{4}F_A$	$\frac{3}{4}$	$\frac{3}{4}l$	$\frac{27}{64}F_A l$
DE	$-\frac{4}{3}P+F_A$	1	l	$-\frac{4}{3}Pl+F_A l$
Σ				$-\frac{63}{16}Pl+\frac{27}{4}F_A l$

$$\delta_A=\frac{1}{EA}\left(-\frac{63}{16}Pl+\frac{27}{4}F_A l\right)=0$$

$$F_A=\frac{7}{12}P$$

$$F_{AB}=F_A \qquad F_{AB}=\frac{7}{12}P \blacktriangleleft$$

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PROBLEM 11.123

Rod AB is made of a steel for which the yield strength is $\sigma_Y = 450$ MPa and $E = 200$ GPa; rod BC is made of an aluminum alloy for which $\sigma_Y = 280$ MPa and $E = 73$ GPa. Determine the maximum strain energy that can be acquired by the composite rod ABC without causing any permanent deformations.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(10)^2 = 78.54 \text{ mm}^2$$
$$= 78.54 \times 10^{-6} \text{ m}^2$$
$$A_{BC} = \frac{\pi}{4}(14)^2 = 153.94 \text{ mm}^2$$
$$= 153.94 \times 10^{-6} \text{ m}^2$$

$$P_{\text{all}} = \sigma_Y A \text{ for each portion.}$$

AB: $$P_{\text{all}} = (450 \times 10^6)(78.54 \times 10^{-6})$$
$$= 35.343 \times 10^3 \text{ N}$$

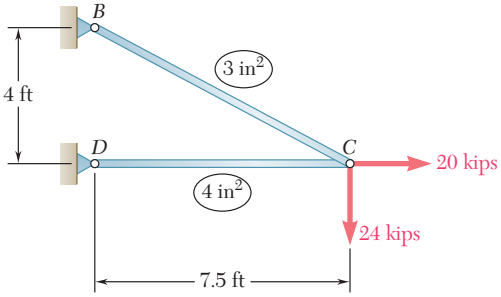
BC: $$P_{\text{all}} = (280 \times 10^6)(153.94 \times 10^{-6})$$
$$= 43.103 \times 10^3 \text{ N}$$

Use the smaller value. $$P = 35.343 \times 10^3 \text{ N}$$

$$U = \frac{P^2 L_{AB}}{2E_{AB}A_{AB}} + \frac{P^2 L_{BC}}{2E_{BC}A_{BC}} = \frac{(35.343 \times 10^3)^2 (1.2)}{(2)(200 \times 10^9)(78.54 \times 10^{-6})}$$
$$+ \frac{(35.343 \times 10^3)^2 (1.6)}{(2)(73 \times 10^9)(153.94 \times 10^{-6})}$$

$$U = 136.6 \text{ J} \blacktriangleleft$$

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PROBLEM 11.124

Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 29 \times 10^6$ psi, determine the strain energy of the truss for the loading shown.

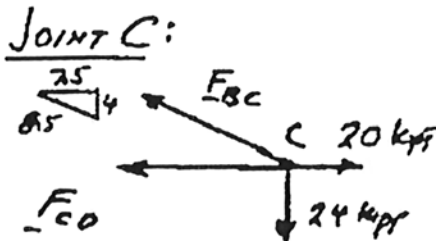
SOLUTION

$$L_{BC} = \sqrt{7.5^2 + 4^2} = 8.5 \text{ ft} = 102 \text{ in.}$$

$$L_{CD} = 7.5 \text{ ft} = 90 \text{ in.}$$

$$A_{BC} = 3 \text{ in}^2, \quad A_{CD} = 4 \text{ in}^2$$

$$E = 29{,}000 \text{ ksi}$$



Equilibrium at joint C.

$$+\uparrow \sum F_y = 0: \quad \frac{4}{8.5} F_{BC} - 24 = 0 \qquad F_{BC} = 51 \text{ kips}$$

$$+\rightarrow \sum F_x = 0: \quad -F_{CD} - \frac{7.5}{8.5}(51 \text{ kips}) + 20 \text{ kips} = 0$$

$$F_{CD} = -25 \text{ kips}$$

Strain energy.

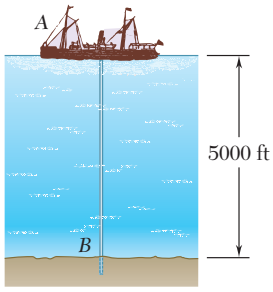
$$U = \sum \frac{F_i^2 L_i}{2EA_i} = \frac{F_{BC}^2 L_{BC}}{2EA_{BC}} + \frac{F_{CD}^2 L_{CD}}{2EA_{CD}}$$

$$= \frac{(51)^2(102)}{(2)(29{,}000)(3)} + \frac{(25)^2(90)}{(2)(29{,}000)(4)}$$

$$= 1.5247 + 0.2425$$

$$U = 1.767 \text{ in.} \cdot \text{kip} \blacktriangleleft$$

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PROBLEM 11.125

The ship at *A* has just started to drill for oil on the ocean floor at a depth of 5000 ft. The steel drill pipe has an outer diameter of 8 in. and a uniform wall thickness of 0.5 in. Knowing that the top of the drill pipe rotates through two complete revolutions before the drill bit at *B* starts to operate and using $G = 11.2 \times 10^6$ psi, determine the maximum strain energy acquired by the drill pipe.

SOLUTION

$$\varphi = (2)(2\pi) = 4\pi \text{ rad}$$

$$L = 5000 \text{ ft} = 60 \times 10^3 \text{ in.}$$

$$c_o = \frac{d_o}{2} = 4 \text{ in.} \qquad c_i = c_o - t = 3.5 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) = 166.406 \text{ in}^4$$

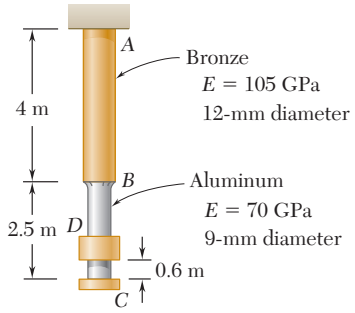
$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L}$$

$$U = \frac{T^2 L}{2GJ} = \left(\frac{GJ\varphi}{L}\right)^2 \frac{L}{2GJ} = \frac{GJ\varphi^2}{2L}$$

$$U = \frac{(11.2 \times 10^6)(166.406)(4\pi)^2}{(2)(60 \times 10^3)}$$

$$U = 2.45 \times 10^6 \text{ in.} \cdot \text{lb} \blacktriangleleft$$

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PROBLEM 11.126

Collar *D* is released from rest in the position shown and is stopped by a small plate attached at end *C* of the vertical rod *ABC*. Determine the mass of the collar for which the maximum normal stress in portion *BC* is 125 MPa.

SOLUTION

Portion *BC*:

$$\sigma_m = 125\times10^6\ \text{Pa}$$

$$A_{BC} = \frac{\pi}{4}(9)^2 = 63.617\ \text{mm}^2 = 63.617\times10^{-6}\ \text{m}^2$$

$$P_m = \sigma_m A_{BC} = 7952\ \text{N}$$

Corresponding strain energy:

$$U_{BC} = \frac{P_m^2 L_{BC}}{2E_{BC}A_{BC}} = \frac{(7952)^2(2.5)}{(2)(70\times10^9)(63.617\times10^{-6})} = 17.750\ \text{J}$$

$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.907\ \text{mm}^2 = 113.907\times10^{-6}\ \text{m}^2$$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2E_{AB}A_{AB}} = \frac{(7952)^2(4)}{(2)(105\times10^9)(113.907\times10^{-6})} = 10.574\ \text{J}$$

$$U_m = U_{BC} + U_{AB} = 28.324\ \text{J}$$

Corresponding elongation Δ_m:

$$\frac{1}{2}P_m\Delta_m = U_m$$

$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(28.324)}{7952} = 7.12\times10^{-3}\ \text{m}$$

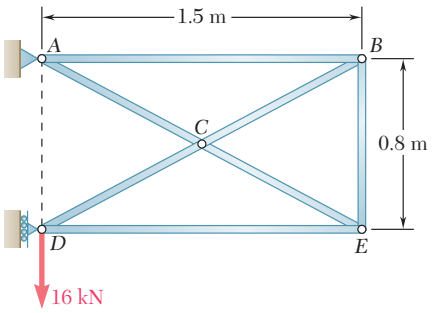
Falling distance: $h = 0.6 + 7.12\times10^{-3} = 0.60712\ \text{m}$

Work of weight $= U_m$ $\quad Wh = mgh = U_m$

$$m = \frac{U_m}{gh} = \frac{28.324}{(9.81)(0.60712)}$$

$m = 4.76\ \text{kg}$ ◀

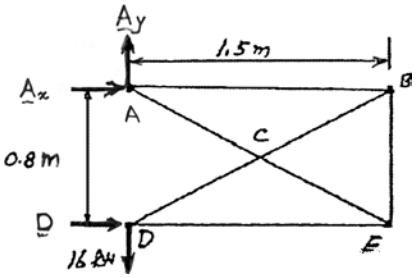
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PROBLEM 11.127

Each member of the truss shown is made of steel and has a cross-sectional area 400 mm^2. Using $E = 200$ GPa, determine the deflection of point D caused by the 16-kN load.

SOLUTION

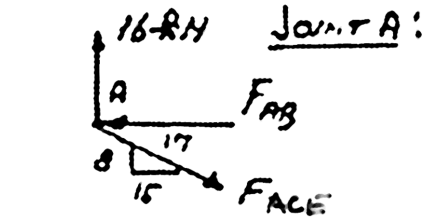


Equilibrium of entire truss.

$$\sum M_A = 0: \quad D = 0$$
$$\sum F_x = 0: \quad A_x = 0$$
$$\sum F_y = 0: \quad A_y = 16\,\text{kN} \uparrow$$

Equilibrium of joint A.

From the force triangle,

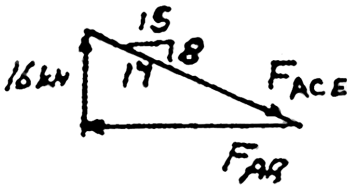


$$\frac{F_{ACE}}{17} = \frac{F_{AB}}{15} = \frac{16\ \text{kN}}{8}$$
$$F_{AB} = 30\ \text{kN} \quad \text{(compression)}$$
$$F_{ACE} = 34\ \text{kN} \quad \text{(tension)}$$

By symmetry,

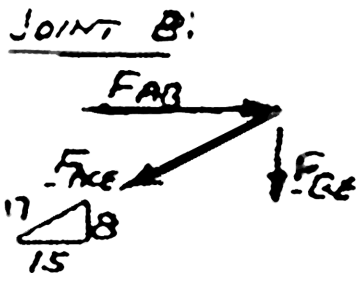
$$F_{DE} = 30\ \text{kN} \quad \text{(compression)}$$
$$F_{BCD} = 34\ \text{kN} \quad \text{(tension)}$$

Equilibrium of joint B.



$$+\uparrow \sum F_y = 0: \quad -\frac{8}{17}F_{ACE} - F_{BE} = 0$$

$$-\frac{8}{17}(34\ \text{kN}) - F_{BE} = 0 \quad F_{BE} = -16\ \text{kN}$$



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PROBLEM 11.127 *(Continued)*

Members: $A = 400\ \text{mm}^2 = 400 \times 10^{-6}\ \text{m}^2$

$E = 200\ \text{GPa} = 200 \times 10^9\ \text{Pa}$

$EA = (200 \times 10^9)(400 \times 10^{-6}) = 80 \times 10^6\ \text{N}$

Strain energy: $U = \Sigma \dfrac{F_i^2 L_i}{2EA} = \dfrac{1}{2EA} \Sigma F_i^2 L_i$

	F_i(kN)	L_i (m)	$F_i^2 L_i$
AB	–30	1.5	1350
DE	–30	1.5	1350
ACE	+34	1.7	1965.2
BCD	+34	1.7	1965.2
BE	–16	0.8	204.8

$$\Sigma = 6835.2\ (\text{kN})^2 \cdot \text{m}$$
$$= 6.8352 \times 10^9\ \text{N}^2 \cdot \text{m}$$

$$U = \frac{6.8352 \times 10^9}{(2)(80 \times 10^6)} = 42.72\ \text{N} \cdot \text{m}$$

Principle of work and energy:

$$\frac{1}{2} P\delta = U \qquad (P = 16\ \text{kN} = 16 \times 10^3\ \text{N})$$

$$\frac{1}{2}(16 \times 10^3)\delta = 42.72 \qquad \delta = 5.34 \times 10^{-3}\ \text{m}$$

Deflection of point *D*. $\delta = 5.34\ \text{mm} \downarrow$ ◀

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PROBLEM 11.128

A block of weight W is placed in contact with a beam at some given point D and released. Show that the resulting maximum deflection at point D is twice as large as the deflection due to a static load W applied at D.

SOLUTION

Consider dropping the weight from a height h above the beam. The work done by the weight is

$$\text{Work} = W(h + y_m)$$

Strain energy: $$U = \frac{1}{2} P_m y_m = \frac{1}{2} k y_m^2$$

where k is the spring constant of the beam for loading at point D.

Equating work and energy, $$W(h + y_m) = \frac{1}{2} k y_m^2.$$

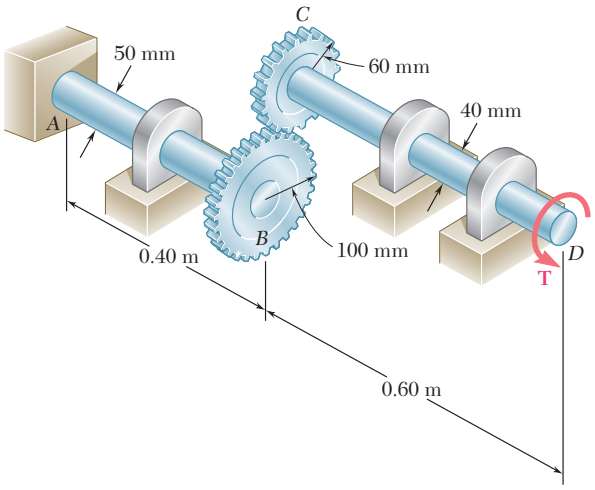
Setting $h = 0$, $$W y_m = \frac{1}{2} k y_m^2, \qquad y_m = \frac{2W}{k}.$$

The static deflection at point D due to weight applied at D is

$$\delta_{st} = \frac{W}{k}.$$

$$y_m = 2\delta_{st} \blacktriangleleft$$

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PROBLEM 11.129

Two solid steel shafts are connected by the gears shown. Using the method of work and energy, determine the angle through which end D rotates when $T = 820\ \text{N} \cdot \text{m}$. Use $G = 77.2\ \text{GPa}$.

SOLUTION

Shaft CD:

$$T = 820\ \text{N} \cdot \text{m} \qquad c = \frac{1}{2}d = 20\ \text{mm} = 0.020\ \text{m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33 \times 10^{-9}\ \text{m}^4$$

$$G = 77.2 \times 10^9\ \text{Pa}, \qquad L = 0.60\ \text{m}$$

$$U_{CD} = \frac{T^2 L}{2GJ} = \frac{(820)^2(0.60)}{(2)(77.2 \times 10^9)(251.33 \times 10^{-9})} = 10.397\ \text{J}$$

Equilibrium of shafts:

$$\frac{T_{CD}}{r_C} = \frac{T_{AB}}{r_B}$$

$$T_{AB} = \frac{r_B}{r_C}T_{CD} = \frac{100\ \text{mm}}{60\ \text{mm}}(820\ \text{N} \cdot \text{m}) = 1366.67\ \text{N} \cdot \text{m}$$

Shaft AB:

$$T = 1366.67\ \text{N} \cdot \text{m} \qquad c = \frac{1}{2}d = 25\ \text{mm} = 0.025\ \text{m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.025)^4 = 613.59 \times 10^{-6}\ \text{m}^4$$

$$G = 77.2 \times 10^9\ \text{Pa}, \qquad L = 0.40\ \text{m}$$

$$U_{AB} = \frac{T^2 L}{2GJ} = \frac{(1366.67)^2(0.40)}{(2)(77.2 \times 10^9)(613.59 \times 10^{-9})} = 7.886\ \text{J}$$

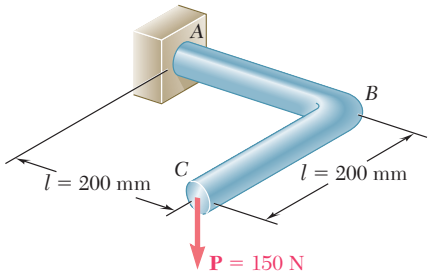
Total strain energy:

$$U = U_{CD} + U_{AB} = 18.283\ \text{J}$$

$$\frac{1}{2}T\theta = U \qquad \theta = \frac{2U}{T_A} = \frac{(2)(18.283)}{820} = 0.04459\ \text{rad}$$

$$\theta = 2.55^\circ \blacktriangleleft$$

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PROBLEM 11.130

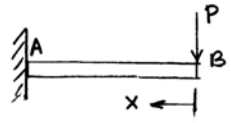
The 12-mm-diameter steel rod *ABC* has been bent into the shape shown. Knowing that $E = 200$ GPa and $G = 77.2$ GPa, determine the deflection of end *C* caused by the 150-N force.

SOLUTION

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{12}{2}\right)^4 = 2.0358\times10^{3}\,\text{mm}^4$$
$$= 2.0358\times10^{-9}\,\text{m}^4$$
$$I = \frac{1}{2}J = 1.0179\times10^{-9}\,\text{m}^4$$

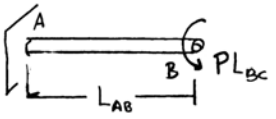
Portion *AB*: bending $M = -Px$

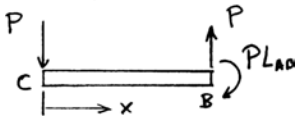
$$U_{AB,b} = \int_0^{L_{AB}} \frac{M^2}{2EI}dx = \frac{P^2}{2EI}\int_0^{L_{AB}} x^2dx$$
$$= \frac{P^2L_{AB}^3}{6EI} = \frac{(150)^2(200\times10^{-3})^3}{(6)(200\times10^{9})(1.0179\times10^{-9})}$$
$$= 0.14736\ \text{J}$$



torsion $T = PL_{BC}$

$$U_{AB,t} = \frac{T^2L_{AB}}{2GJ} = \frac{P^2L_{BC}^2L_{AB}}{2GJ}$$
$$= \frac{(150)^2(200\times10^{-3})^2(200\times10^{-3})}{(2)(77.2\times10^{9})(2.0358\times10^{-9})}$$
$$= 0.57265\ \text{J}$$





Portion *BC*: $M = -Px$

$$U_{BC} = \int_0^{L_{BC}} \frac{M^2}{2EI}dx = \frac{P^2}{3EI}\int_0^{L_{BC}} x^2dx = \frac{P^2L_{BC}^3}{6EI}$$
$$= \frac{(150)^2(200\times10^{-3})^3}{(6)(200\times10^{9})(1.0179\times10^{9})} = 0.14736\ \text{J}$$

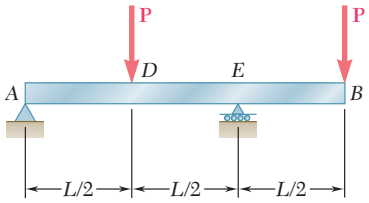
Total: $U = U_{AB,b} + U_{AB,t} + U_{BC} = 0.86737\ \text{J}$

Work-energy: $\frac{1}{2}P\delta = U \qquad \delta = \frac{2U}{P} = \frac{(2)(0.86737)}{150}$

$$= 11.57\times10^{-3}\,\text{m}$$

$$\delta = 11.57\ \text{mm}\downarrow \blacktriangleleft$$

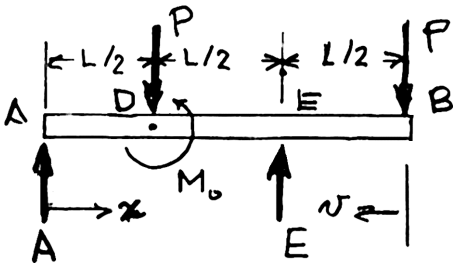
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PROBLEM 11.131

For the prismatic beam shown, determine the slope at point *D*.

SOLUTION



Add counterclockwise couple M_0 at point *D*.

Reactions:

$$\sum M_E = 0: \quad -AL + P\frac{L}{2} - P\frac{L}{2} + M_0 = 0$$

$$A = \frac{M_0}{L} \uparrow$$

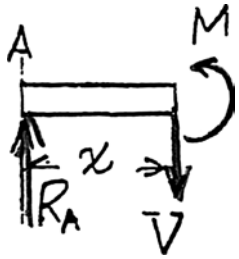
$$\sum M_A = 0: \quad EL - P\left(L + \frac{L}{2}\right) - P\frac{L}{2} + M_0 = 0$$

$$E = 2P - \frac{M_0}{L} \uparrow$$

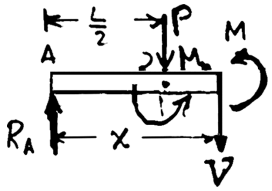
Strain energy: $U = U_{AD} + U_{DE} + U_{EB}$

Slope at point *D* (formula).

$$\theta_D = \frac{\partial U}{\partial M_0} = \frac{\partial U_{AD}}{\partial M_0} + \frac{\partial U_{DE}}{\partial M_0} + \frac{\partial U_{EB}}{\partial M_0}$$



Portion *AD*: $\left(0 \le x \le \frac{L}{2}\right)$

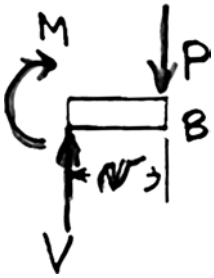


$$U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx$$

$$M = \frac{M_0}{L}x \qquad \frac{\partial M}{\partial M_0} = \frac{x}{L}$$

Set $M_0 = 0$ so that $M = 0$.

$$\frac{\partial U_{AD}}{\partial M_0} = \frac{1}{EI}\int_0^{L/2} M\frac{\partial M}{\partial M_0} dx = 0$$



Portion *DE*: $\left(\frac{L}{2} \le x \le L\right)$

$$U_{DE} = \int_{L/2}^{L} \frac{M^2}{2EI} dx$$

$$M = Ax - P\left(x - \frac{L}{2}\right) - M_0 = M_0\left(\frac{x}{L} - 1\right) - P\left(x - \frac{L}{2}\right)$$

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PROBLEM 11.131 *(Continued)*

$$\frac{\partial M}{\partial M_0} = \frac{x}{L} - 1; \qquad \text{Set } M_0 = 0 \text{ so that } M = -P\left(x - \frac{L}{2}\right).$$

$$\frac{\partial U_{DE}}{\partial M_0} = \frac{1}{EI}\int_{L/2}^{L} M\,\frac{\partial M}{\partial M_0}\,dx = \frac{P}{EI}\int_{L/2}^{L}\left(x - \frac{L}{2}\right)\left(1 - \frac{x}{L}\right)dx$$

$$= \frac{P}{EI}\int_{L/2}^{L}\left(\frac{3}{2}x - \frac{x^2}{L} - \frac{L}{2}\right)dx$$

$$= \frac{P}{EI}\left\{\frac{3}{2}\frac{x^2}{2}\bigg|_{L/2}^{L} - \frac{x^3}{3L}\bigg|_{L/2}^{L} - \frac{Lx}{2}\bigg|_{L/2}^{L}\right\}$$

$$= \frac{PL^2}{EI}\left\{\frac{3}{4} - \frac{3}{16} - \frac{1}{3} + \frac{1}{24} - \frac{1}{2} + \frac{1}{4}\right\} = \frac{PL^3}{48EI}$$

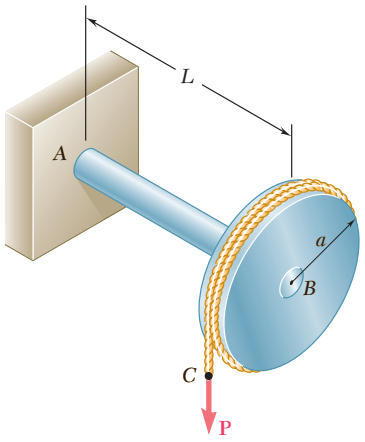
Portion *EB*: $\left(0 \le v \le \frac{L}{2}\right)$ $\qquad U_{EB} = \int_0^{L/2}\frac{M^2}{2EI}\,dx$

$$M = -Pv \qquad \frac{\partial M}{\partial M_0} = 0 \qquad \frac{\partial U_{EB}}{\partial M_0} = \frac{1}{EI}\int_0^{L/2} M\,\frac{\partial M}{\partial M_0}\,dv = 0$$

Slope at point *D*. $\qquad \theta_D = 0 + \frac{PL^2}{48EI} + 0$

$$\theta_D = \frac{PL^2}{48EI} \curvearrowright \blacktriangleleft$$

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PROBLEM 11.132

A disk of radius *a* has been welded to end *B* of the solid steel shaft *AB*. A cable is then wrapped around the disk and a vertical force **P** is applied to end *C* of the cable. Knowing that the radius of the shaft is *a* and neglecting the deformations of the disk and of the cable, show that the deflection of point *C* caused by the application of **P** is

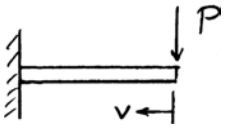
$$\delta_C = \frac{PL^3}{3EI}\left(1 + 1.5\frac{Ea^2}{GL^2}\right).$$

SOLUTION

Torsion:

$$T = Pa$$

$$U_t = \frac{T^2 L}{2GJ} = \frac{P^2 a^2 L}{2GJ}$$



Bending:

$$M = Pv$$

$$U_b = \int_0^L \frac{M^2 dv}{2EI} = \int_0^L \frac{P^2 v^2 dv}{2EI}$$

$$= \frac{P^2 L^3}{6EI}$$

Total:

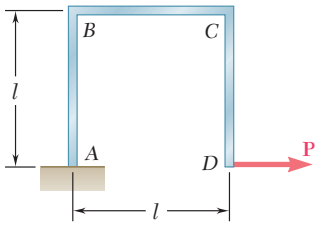
$$U = \frac{P^2 a^2 L}{2GJ} + \frac{P^2 L^3}{6EI} = \frac{1}{2} P\delta_C$$

$$\delta_C = \frac{Pa^2 L}{GJ} + \frac{PL^3}{3EI} = \frac{PL^3}{3EI}\left(1 + \frac{3EIa^2}{GJL^2}\right)$$

Since $J = 2I$,

$$\delta_C = \frac{PL^3}{3EI}\left(1 + 1.5\frac{Ea^2}{GL^2}\right) \blacktriangleleft$$

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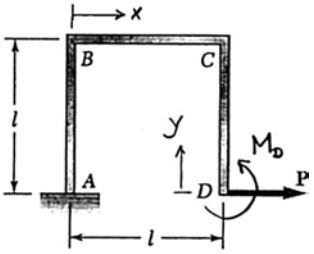


PROBLEM 11.133

A uniform rod of flexural rigidity *EI* is bent and loaded as shown. Determine (*a*) the horizontal deflection of point *D*, (*b*) the slope at point *D*.

SOLUTION

Add couple M_D at point *D*.



Reactions at *A*: $R_{Ay} = 0, \quad R_{Ax} = P \leftarrow, \quad M_A = M_0 \circlearrowleft$

Member *AB*: $M = M_A + R_A y = M_D + Py \qquad \dfrac{\partial M}{\partial P} = y, \qquad \dfrac{\partial M}{\partial M_D} = 1$

$$U_{AB} = \int_0^l \frac{M^2}{2EI}\,dy$$

Set $M_D = 0$.

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial P}\,dy = \frac{1}{EI}\int_0^l (Py)\,y\,dy = \frac{Pl^3}{3EI}$$

$$\frac{\partial U_{AB}}{\partial M_0} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial M_0}\,dy = \frac{1}{EI}\int_0^l (Py)(1)\,dy = \frac{Pl^2}{2EI}$$

Member *BC*: $M = M_A + R_A l = M_D + Pl \qquad \dfrac{\partial M}{\partial P} = l, \qquad \dfrac{\partial M}{\partial M_D} = 1$

$$U_{BC} = \int_0^l \frac{M^2}{2EI}\,dx$$

Set $M_D = 0$.

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial P}\,dx = \frac{1}{EI}\int_0^l (Pl)(l)\,dx = \frac{Pl^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial M_D} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial M_D}\,dx = \frac{1}{EI}\int_0^l (Pl)(1)\,dx = \frac{Pl^2}{EI}$$

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PROBLEM 11.133 *(Continued)*

Member *CD*: $M = M_D + Py \qquad \frac{\partial M}{\partial P} = y \qquad \frac{\partial M}{\partial M_D} = 1$

$$U_{CD} = \int_0^l \frac{M^2}{2EI}\,dy$$

Set $M_D = 0$.

$$\frac{\partial U_{CD}}{\partial P} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial P}\,dy = \frac{1}{EI}\int_0^l (Py)(y)\,dy = \frac{Pl^3}{3EI}$$

$$\frac{\partial U_{CD}}{\partial M_D} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial M_D}\,dy = \frac{1}{EI}\int_0^l (Py)(1)\,dy = \frac{Pl^2}{2EI}$$

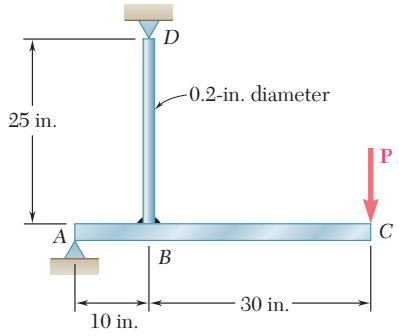
(*a*) Horizontal deflection of point *D*.

$$\delta_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} + \frac{\partial U_{CD}}{\partial P} = \left(\frac{1}{3} + 1 + \frac{1}{3}\right)\frac{Pl^3}{EI} \qquad \delta_P = \frac{5Pl^3}{3EI} \rightarrow \blacktriangleleft$$

(*b*) Slope at point *D*.

$$\theta_D = \frac{\partial U_{AB}}{\partial M_D} + \frac{\partial U_{BC}}{\partial M_D} + \frac{\partial U_{CD}}{\partial M_D} = \left(\frac{1}{2} + 1 + \frac{1}{2}\right)\frac{Pl^2}{EI} \qquad \theta_D = \frac{2Pl^2}{EI} \circlearrowright \blacktriangleleft$$

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PROBLEM 11.134

The steel bar *ABC* has a square cross section of side 0.75 in. and is subjected to a 50-lb load **P.** Using $E = 29\times10^6$ psi for the rod *BD* and the bar, determine the deflection of point *C*.

SOLUTION

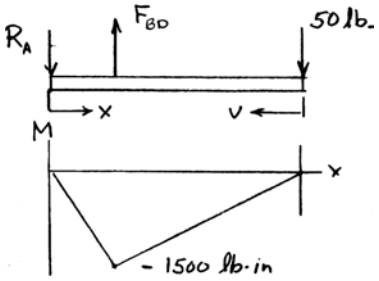
Assume member *BD* is a two-force member.

$$+\circlearrowleft \Sigma M_A = 0: \quad 10F_{BD} - (40)(50) = 0 \qquad F_{BD} = 200 \text{ lb}$$

$$A_{BD} = \frac{\pi}{4}(0.2)^2 = 31.416\times10^{-3}\text{in}^2$$

$$U_{BD} = \frac{F_{BD}^2 L_{BD}}{2EA} = \frac{(200)^2(25)}{(2)(29\times10^6)(31.416\times10^{-3})}$$

$$= 0.5488 \text{ in.}\cdot\text{lb}$$



Member *ABC*: $$I = \frac{1}{12}(0.75)(0.75)^3 = 26.367\times10^{-3}\text{ in}^4$$

Portion *AB*: $$M = -1500\frac{x}{10} = -150x$$

$$U_{AB} = \int_0^{10}\frac{M^2}{2EI}dx = \frac{150^2}{2EI}\int_0^{10}x^2dx$$

$$= \frac{(150)^2(10^3)}{(2)(29\times10^6)(26.367\times10^{-3})(3)}$$

$$= 4.904 \text{ in.}\cdot\text{lb}$$

Portion *BC*: $$M = -50v$$

$$U_{BC} = \int_0^{30}\frac{M^2}{2EI}dv = \frac{50^2}{2EI}\int_0^{30}v^2dv$$

$$= \frac{(50)^2(30)^3}{(2)(29\times10^6)(26.367\times10^{-3})(3)} = 14.713 \text{ in.}\cdot\text{lb}$$

Total: $$U = U_{BD} + U_{AB} + U_{BD} = 20.166 \text{ in.}\cdot\text{lb}$$

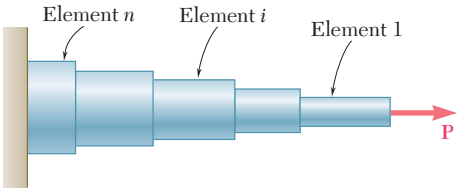
$$\frac{1}{2}P\delta_C = U$$

$$\delta_C = \frac{2U}{P} = \frac{(2)(20.166)}{50}$$

$$\delta_C = 0.807 \text{ in.}\downarrow \blacktriangleleft$$

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PROBLEM 11.C1



A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to a load **P** applied at its free end. The length of element i is denoted by L_i and its diameter by d_i. (*a*) Denoting by E the modulus of elasticity of the material used in the rod, write a computer program that can be used to determine the strain energy acquired by the rod and the deformation measured at its free end. (*b*) Use this program to determine the strain energy and deformation for the rods of Probs. 11.9 and 11.10.

SOLUTION

Enter: P and E

For each element

Enter A_i and D_i

Compute:

Normal stress: $$\sigma_i = \frac{P}{A_i}$$

Strain energy: $$U_i = \frac{P^2 L_i}{2A_i E}$$

Strain energy density: $$u = \frac{\sigma_i^2}{2E}$$

Total strain energy.

Update through n elements.

$$U = U + U_i$$

Total deformation.

$$\frac{1}{2}P\Delta = U: \quad \Delta = \frac{2U}{P}$$

Program Outputs

Problem 11.9

Axial load $= 8.000$ kips Modulus of elasticity $= 29 \times 10^6$ psi

Element	Length in.	ΔL in.	Stress ksi	Strain Energy in. · lb	Strain Energy Density lb · in./in^3
1	24.000	0.022	26.08	86.32	11.72
2	36.000	0.022	18.11	89.92	5.65

Total strain energy $= 176.24$ in. · lb

Total deformation $= 0.0441$ in.

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PROBLEM 11.C1 *(Continued)*

Program Outputs *(Continued)*

Problem 11.10

Axial load = 25.000 kN Modulus of elasticity = 200 GPa

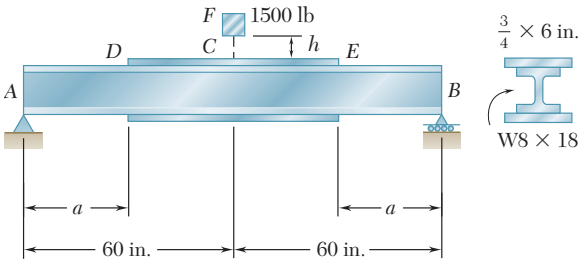
Element	Length m	ΔL mm	Stress MPa	Strain Energy J	Strain Energy Density kJ/m^3
1	0.80	0.497	124.34	6.22	38.65
2	1.20	0.477	79.58	5.97	15.83

Total strain energy = 12.1853 J

Total deformation = 0.9748 mm

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PROBLEM 11.C2



Two 0.75×6-in. cover plates are welded to a W8×18 rolled-steel beam as shown. The 1500-lb block is to be dropped from a height $h = 2$ in. onto the beam. (*a*) Write a computer program to calculate the maximum normal stress on transverse sections just to the left of *D* and at the center of the beam for values of *a* from 0 to 60 in. using 5-in. increments. (*b*) From the values considered in part *a*, select the distance *a* for which the maximum normal stress is as small as possible. Use $E = 29\times10^6$ psi.

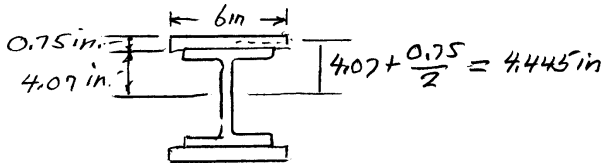
SOLUTION

Compute and enter moments of inertia and section moduli.

For *AD* and *EB*: W8×18:

$$I_1 = 61.9 \text{ in}^4 \quad S_1 = 15.2 \text{ in}^3$$ ◀

For *DCE*: W8×18 plus cover plates:



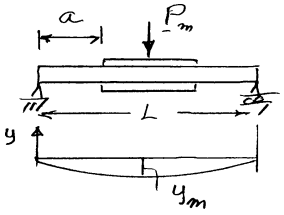
$$I_2 = 61.9 + 2(6\times0.75)(4.445)^2$$
$$= 239.72 \text{ in}^4$$ ◀

$$S_2 = \frac{I_2}{(4.07 + 0.75)}$$
$$= \frac{239.72}{4.82}$$
$$= 49.7 \text{ in}^3$$ ◀

$y_m = P_m\alpha$ where $\alpha =$ influence coefficient. See next page for determination of α.

$P_m =$ equivalent static load

$$U_2 = \frac{1}{2}P_m y_m = \frac{1}{2}\frac{y_m^2}{\alpha}$$



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PROBLEM 11.C2 *(Continued)*

Work done by *w* is $w(h + y_m)$

$$\frac{1}{2}\frac{y_m^2}{\alpha} = wh + wy_m$$

or $$y_m^2 - 2w\alpha y_m - 2wh\alpha \quad \text{Ⓐ}$$



Position 1 Position 2

Program solution of Ⓐ for y_m

Enter $L = 120$ in., $h = 2$ in., $W = 1500$ lb, $E = 29 \times 10^6$ psi

For $a = 0$ to 60 in., Step 5 in.:

Solve Ⓐ for y_m, $P_m = y_m/\alpha$, $y_{st} = w\alpha$

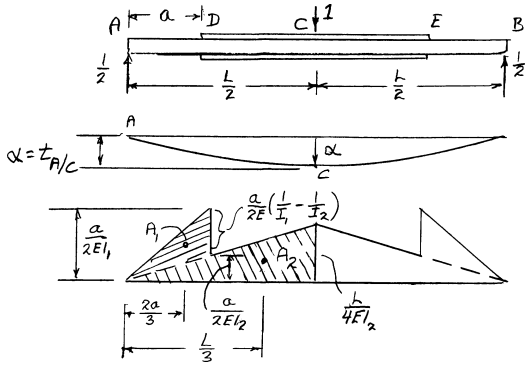
$$\sigma_D = \sigma_1 = \frac{1}{2}P_m a/S_1; \quad \sigma_c = \sigma_2 = \frac{1}{4}P_m L/S_2$$

Print: a, y_{st}, y_m, P_m, σ_1, σ_2, and $(\sigma_1 - \sigma_2)$

Repeat with smaller intervals to find a for $(\sigma_1 - \sigma_2) = 0$

This is the distance a for σ_{max} as small as possible.

Determination of α: α is deflection at c for a unit load at C.



$$\alpha = t_{A/C} = A_1\left(\frac{2a}{3}\right) + A_2\left(\frac{L}{3}\right)$$

$$= \left[\frac{1}{2}\frac{a}{EI_1}\left(\frac{1}{I_1} - \frac{1}{I_2}\right)\frac{a}{2}\right]\frac{2a}{3} + \left[\frac{1}{2}\cdot\frac{L}{4EI_2}\cdot\frac{L}{2}\right]\frac{L}{3}$$

$$\alpha = \left[\left(\frac{1}{I_1} - \frac{1}{I_2}\right)a^3 + \frac{1}{8I_2}L^3\right] \Big/ 6E \quad \blacktriangleleft$$

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PROBLEM 11.C2 *(Continued)*

Program Output

Beam = W8×18 with two 6 by 0.75-in. cover plates

$h = 2$ in. $W = 1500$ lb $L = 120$ in.

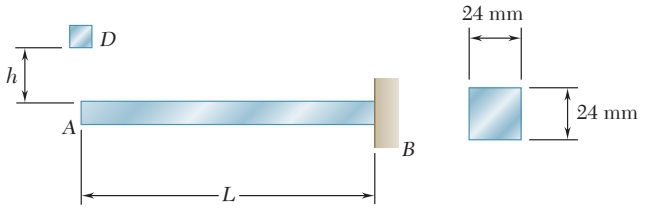
a in.	y_{stat} in.	y_{max} in.	P_{max} lb	σ_1 ksi	σ_2 ksi	$\sigma_1 - \sigma_2$ ksi	
0.00	0.00777	0.1842	35,572	0.00	21.46	–21.46	
5.00	0.00778	0.1844	35,544	5.85	21.44	–15.59	
10.00	0.00787	0.1855	35,348	11.63	21.32	–9.69	
15.00	0.00812	0.1885	34,834	17.19	21.01	–3.82	
20.00	0.00859	0.1942	33,896	22.30	20.45	1.85	←
25.00	0.00938	0.2033	32,509	26.73	19.61	7.13	
30.00	0.01056	0.2163	30,736	30.33	18.54	11.79	
35.00	0.01220	0.2334	28,706	33.05	17.32	15.73	
40.00	0.01438	0.2546	26,563	34.95	16.02	18.93	
45.00	0.01718	0.2799	24,436	36.17	14.74	21.43	
50.00	0.02068	0.3090	22,415	36.87	13.52	23.35	
55.00	0.02496	0.3419	20,550	37.18	12.40	24.78	
60.00	0.03008	0.3783	18,862	37.23	11.38	25.85	
Use smaller increments to seek the smallest maximum normal stress.							
18.33	0.00840	0.1919	34,259	20.657	20.665	–0.01	
18.34	0.00840	0.1920	34,257	20.667	20.664	0.00	
18.35	0.00841	0.1920	34,255	20.677	20.663	0.01	

Max. stress small as possible for $a = 18.34$ in.

Smallest max. stress $= 20.67$ ksi

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PROBLEM 11.C3



The 16-kg block *D* is dropped from a height *h* onto the free end of the steel bar *AB*. For the steel used $\sigma_{all} = 120$ MPa and $E = 200$ GPa. (*a*) Write a computer program to calculate the maximum allowable height *h* for values of the length *L* from 100 mm to 1.2 m, using 100-mm increments. (*b*) From the values considered in part *a*, select the length corresponding to the largest allowable height.

SOLUTION

Enter $\sigma_{all} = 120$ MPa, $E = 200$ GPa, $d = 0.024$ m

$m = 16$ kg, $g = 9.81$ m/s^2

$I = d^4/12 \quad S = \dfrac{I}{c} = \dfrac{I}{d/2} = d^3/b$

For $L = 100$ m to 1200 m, Step 100 mm

$L = L/1000$

$y_{st} = mgL^3/3EI$

$M_{max} = \sigma_{all} S$

$P_{max} = M_{max}/L$

$y_{max} = P_{max} L^3/3EI$

From Problem 11.69, Page 705,

$$y_m = y_{st}\left[1 + \sqrt{1 + \frac{2h}{y_{st}}}\right] \xrightarrow{\text{Solve for}} h = \left[\left(\frac{y_{max}}{y_{st}} - 1\right)^2 - 1\right]\frac{y_{st}}{2}$$

Print: L, y_{st}, y_{max}, P_{max}, M_{max}, h

Return

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PROBLEM 11.C3 *(Continued)*

Program Output

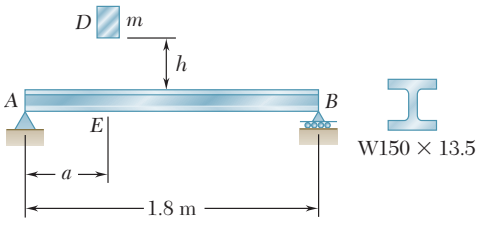
Problem 11.C3

$m = 16.0$ kg $d = 24$ mm $\sigma = 120$ MPa $G = 200$ GPa

L mm	y_{stat} mm	y_{max} mm	P_{max} N	M_{max} N · m	h mm
100	0.00946	0.167	2764.8	276.48	1.301
200	0.07569	0.667	1382.4	276.48	2.269
300	0.25547	1.500	921.6	276.48	2.904
400	0.60556	2.667	691.2	276.48	3.205
500	1.18273	4.167	553.0	276.48	3.173
600	2.04375	6.000	460.8	276.48	2.807
700	3.24540	8.167	395.0	276.48	2.109
800	4.84445	10.667	345.6	276.48	1.076
900	6.89766	13.500	307.2	276.48	–0.289
1000	9.46181	16.667	276.5	276.48	–1.988
1100	12.59367	20.167	251.3	276.48	–4.020
1200	16.35000	24.000	230.4	276.48	–6.385

Use smaller increments to seek the largest height h.					
435	0.77883	3.154	635.6	276.48	3.2316
440	0.80599	3.227	628.4	276.48	3.2320
445	0.83378	3.300	621.3	276.48	3.2317

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PROBLEM 11.C4

The block D of mass $m = 8$ kg is dropped from a height $h = 750$ mm onto the rolled-steel beam AB. Knowing that $E = 200$ GPa, write a computer program to calculate the maximum deflection of point E and the maximum normal stress in the beam for values of a from 100 to 900 mm, using 100-mm increments.

SOLUTION

Enter $L = 1.8$ m, $E = 200$ GPa, $h = 0.75$ m

$m = 8$ kg, $g = 9.81$ m/s^2

$I = 6.87 \times 10^{-6}$ m^4

$S = 91.6 \times 10^{-6}$ m^4

For $a = 100$ mm to 900 mm, Step 100 mm

$a = a/1000$

$b = L - a$

See Prob. 11.71, page 705 → $y_{st} = mga^2b^2/3EIL$

Influence coefficient for Δ_E for unit load at E → $\alpha = a^2b^2/3EIL$

See Prob. 11.69, page 705 → $y_m = y_{st}\left[1 + \sqrt{1 + \dfrac{2h}{y_{st}}}\right]$

$P_{\max} = y_m/\alpha$

$M_{\max} = P_{\max}ab/L$

$\sigma_{\max} = M_{\max}/S$

Print: a, y_{st}, y_m, $P_{\max}$, $\sigma_{\max}$

Return

Problem 11.C4

Beam: W150 × 13.5

$I = 6.87 \times 10^{-6}$ m^4

$S = 91.6 \times 10^{-6}$ m^3

$L = 1.8$ m

$h = 750$ mm

$m = 8$ kg

$g = 9.81$ m/s^2

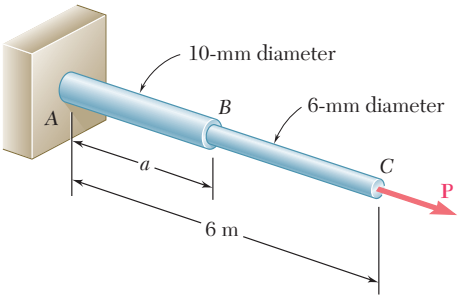
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PROBLEM 11.C4 *(Continued)*

a mm	y_{stat} mm	y_{max} mm	P_{max} N	σ_{max} MPa
100	0.0003	0.6775	173.93	179.33
200	0.0011	1.2757	92.43	179.40
300	0.0021	1.7946	65.75	179.46
400	0.0033	2.2339	52.85	179.51
500	0.0045	2.5936	45.55	179.55
600	0.0055	2.8734	41.13	179.59
700	0.0063	3.0734	38.46	179.61
800	0.0068	3.1934	37.02	179.63
900	0.0069	3.2334	36.56	179.63

Note the small variation in σ_{max}. This is due to the energy acquired by the mass as it falls through y_{max}. See Prob. 11.147, page 731, for a case where energy delivered is constant and σ_{max} is also constant.

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PROBLEM 11.C5

The steel rods *AB* and *BC* are made of a steel for which $\sigma_Y = 300$ MPa and $E = 200$ GPa. (*a*) Write a computer program to calculate for values of *a* from 0 to 6 m, using 1-m increments, the maximum strain energy that can be acquired by the assembly without causing any permanent deformation. (*b*) For each value of *a* considered, calculate the diameter of a uniform rod of length 6 m and of the same mass as the original assembly, and the maximum strain energy that could be acquired by this uniform rod without causing permanent deformation.

SOLUTION

Enter:

$$\sigma_Y = 300 \text{ MPa}, \quad E = 200 \text{ GPa}, \quad L = 6 \text{ m}$$

$$\text{Area}_{AB} = \frac{\pi}{4}(0.010 \text{ m})^2, \quad \text{Area}_{BC} = \frac{\pi}{4}(0.006 \text{ m})^2$$

$$P_m = \sigma_Y \text{Area}_{BC}$$

For $a = 0$ to 6 m, Step 1 m

$$U = \frac{P_m^2}{2E}\left(\frac{a}{\text{Area}_{AB}} + \frac{L-a}{\text{Area}_{BC}}\right)$$

For uniform rod of same volume,

$$\text{vol} = a(\text{Area}_{AB}) + (L-a)(\text{Area}_{BC})$$

$$d = \sqrt{\frac{4\text{vol}}{\pi L}}$$

$$\text{Area}_{\text{new}} = \frac{\pi}{4}d^2$$

$$P_{\text{new}} = \sigma_Y(\text{Area}_{\text{new}})$$

$$U_{\text{new}} = \frac{P_{\text{new}}^2 L}{2E(\text{Area}_{\text{new}})}$$

Print: a, U, vol, d, $P_{\text{new}}, U_{\text{new}}$

Return

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PROBLEM 11.C5 *(Continued)*

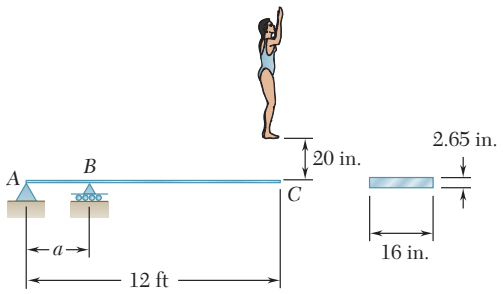
Program Output

Problem 11.C5

$\sigma_Y = 300$ MPa, $P_m = 8482$ N, $L = 6$ m, $E = 200$ GPa

a m	*U* J	Vol m^3	*d* mm	New *P* N	New *U* J
0.00	38.17	169.65	6.00	8482.30	38.17
1.00	34.10	219.91	6.83	10,995.58	49.48
2.00	30.03	270.18	7.57	13,508.85	60.79
3.00	25.96	320.44	8.25	16,022.12	72.10
4.00	21.88	370.71	8.87	18,535.40	83.41
5.00	17.81	420.97	9.45	21,048.67	94.72
6.00	13.74	471.24	10.00	23,561.95	106.03

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PROBLEM 11.C6

A 160-lb diver jumps from a height of 20 in. onto end *C* of a diving board having the uniform cross section shown. Write a computer program to calculate for values of *a* from 10 to 50 in., using 10-in. increments, (*a*) the maximum deflection of point *C*, (*b*) the maximum bending moment in the board, (*c*) the equivalent static load. Assume that the diver's legs remain rigid and use $E = 1.8 \times 10^6$ psi.

SOLUTION

Enter: $L = 12$ ft, $h = 20$ in., $W = 160$ lb

$E = 1.8 \times 10^6$ psi

$I = (16 \text{ in.})(2.65 \text{ in.})^3/12$

$S = (16 \text{ in.})(2.65 \text{ in.})^2/6$

$y_m = P_m \alpha$ where $\alpha =$ influence coefficient. See below for determination of α

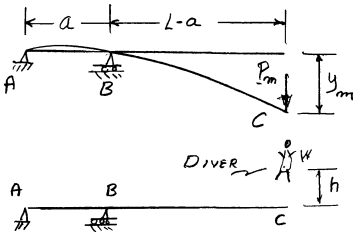
where $P_m =$ equivalent static load.

$$U_2 = \frac{1}{2} P_m y_m = \frac{1}{2} \frac{y_m^2}{\alpha}$$

$$\text{work} = w(h + y_m)$$

$$\text{work} = U_2$$

$$w(h + y_m) = \frac{1}{2} \frac{y_m^2}{\alpha} \text{A}$$



Position 1



Position 2

Program solution of *a* for y_m. Enter α.

For $a = 10$ in. to 50 in., Step 10 in.

Solve A for y_m, $\quad P_m = y_m/\alpha$

$$M_{\max} = M_B = P_m(L - a)$$

$$\sigma = M_{\max}/S$$

Print a, y_m, P_m, M_m, σ

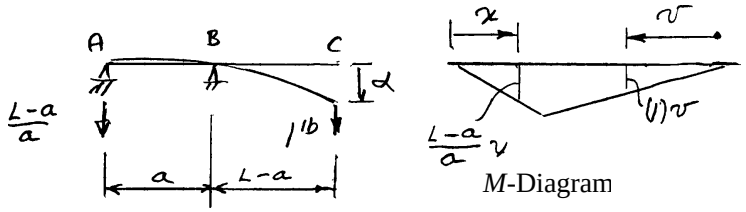
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PROBLEM 11.C6 *(Continued)*

Program Output

a in.	y_m in.	P_m lb	Max. M kip · in.	σ psi
10	14.622	757.7	101.532	5422
20	13.262	802.6	99.519	5314
30	11.950	855.6	97.536	5208
40	10.683	919.1	95.583	5104
50	9.462	996.4	93.661	5001

Determination of influence of coefficient α:



$$U = \frac{1}{2}(1\text{ lb})\alpha = \Sigma\int \frac{M^2}{2EI}dx$$

$$\frac{\alpha}{2} = \frac{1}{2EI}\left[\int_0^a \left(\frac{L-a}{a}\right)^2 x^2 dx + \int_0^{L-a} v^2 dv\right.$$

$$\alpha = \frac{1}{EI}\left[\frac{(L-a)^2}{a^2}\frac{a^3}{3} + \frac{(L-a)^3}{3}\right]$$

$$\alpha = \frac{1}{3EI}\left[(L-a)^2 a + (L-a)^3\right]$$ ◀

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